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Abstract

We investigate the effect of multimarket contacts on the privatization policy in mixed duopoly with price competition. There are two markets, one of which is served solely by the state-owned public firm, and the other is served by both public and private firms. Two markets are linked by the production technology of the public firm. In the general model, we first show that privatization is never optimal in the absence of multimarket contacts, i.e., if there is only one monopoly market or one duopoly market. Then, using a linear-quadratic specification, we show that a positive degree of privatization can be optimal in the presence of multimarket contacts. This result has an implication for the privatization policy in universal service sectors.

JEL classification H42, L33

Keywords Multimarket contacts, partial privatization, state-owned public enterprise

1 Introduction

Since the early 1980s, we have observed a worldwide wave of the privatization of state-owned public enterprises. Nevertheless, many public and semi-public enterprises (i.e., firms owned by both public and private sectors) are still active in planned and market economies in developed, developing, and transitional countries. While some public enterprises

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are traditional monopolists in natural monopoly markets, a considerable number of public (including semi-public) enterprises compete with private enterprises in a wide range of industries.[1] Optimal privatization policies in such mixed oligopolies have attracted extensive attention from economics researchers in such fields as industrial organization, public economics, financial economics, international economics, development economics, and political economy.[2]

Specifically, drawing on the result of Matsumura (1998) that full nationalization is never optimal in Cournot mixed duopoly, many studies on mixed oligopolies investigate how economic environments affect the optimal degree of privatization.[3] In this way, most studies of privatization policies in mixed oligopoly use quantity competition model to characterize the optimal privatization policies. However, there are many applications where it is more plausible to assume that firms compete in prices.[4] In addition, as shown by Matsumura and Ogawa (2012), when public and private enterprises can choose whether to compete in price or quantity, they choose to compete in price in the equilibrium. Therefore, discussing the optimal privatization policies under price competition is also important in both practical and theoretical perspectives. That said, there is a conventional wisdom in the literature of mixed oligopoly that the privatization policy, as an device to change the public firm's objective toward profit maximization, does not improve the welfare. The reason is that the privatization increases the public firm's price, and it also increases the price of private firms through the strategic interaction, both of which harm welfare.

We argue that this reasoning holds only if firms compete in a single market. If the public firm provides in multiple markets, the result changes. To show this, we consider a variation of model of Kawasaki and Naito (2017). There are two markets, one of which is served solely by the state-owned public firm, and the other is served by both public and private firms. Two markets are linked by the production cost of public firm. In this environment, we show that a positive degree of privatization can be optimal in the presence of multimarket contacts. As we explain in Section 3, this comes from the intra-

[1]Examples include United States Postal Service, Deutsche Post AG, Areva, Nippon Telecom and Telecommunication, Japan Tobacco, Volkswagen, Renault, Électricité de France, Japan Postal Bank, Kampo, Korea Development Bank, and Korea Investment Corporation.

[2]The idea of mixed oligopoly dates at least to Merrill and Schneider (1966). Recently, the literature on mixed oligopoly has become richer and more diverse. For examples of mixed oligopolies and recent developments in this field, see Ishibashi and Matsumura (2006), Ishida and Matsushima (2009), Colombo (2016), Chen (2017), Matsumura and Sunada (2013), and the papers cited therein.

[3]For example, see Lin and Matsumura, (2012) for the share of foreign investors who purchases the stock of public firm, Matsumura and Kanda (2005) for free entry, and Sato and Matsumura (2017) shadow cost of public funds

[4]For the anslyses of price competition in mixed oligopolies, see Bárcena-Ruiz, J. C. (2007), Matsumura. T. (2012), Cremer et al.(1991), and Andersion et al. (1997) for examples.

firm production substitution of public firm. An increase in the degree of privatization decreases the production of public firm in monopoly market. This decreases the marginal cost of production for the duopoly market, which raises the incentive to increase the production. When the degree of product differentiation between public and private firms are small, the latter effect tends to dominate the unilateral effect of privatization to decrease the production in duopoly market. Under the price competition, this decreases the equilibrium price of private firm through the strategic interaction. This improves the welfare. This is the reason why partial privatization can be optimal in the presence of multimarket contacts.

This result sheds light on an important aspect of privatization policy in mixed oligopoly. For example, in transportation industries, there are several situations where the public firm provides its services in both rural and urban areas probably due to universal service reasons, while private firms only provide services in urban areas. In such a situation, privatization of the public firm can stimulate the competition in urban area through the improved production efficiency of the public firm.

2 Model

Consider a model of multimarket mixed price competition. There are a state-owned public firm, firm 0, and a private firm, firm 1. There are two markets A and B. Market A is solely provided by firm 0, while market B is provided by both firm 0 and firm 1. This means that the public firm serves two markets, in one of which it competes with the private firm.

The representative consumer in market A is characterized by its relative size $\phi \in [0, 1]$ and the utility function $U^A(x_0^A) + y^A$, where x_0^A is the amount of the consumption of the products provided by firm 0 and y^A is the consumption of the composite goods. The representative consumer in market B is characterized by its relative size $(1-\phi)$ and the utility function $U^B(x_0^B, x_1^B) + y^B$, where x_0^B and x_1^B are the amount of the consumption of the products provided by firm 0 and firm 1, and y^B is the consumption of the composite goods. Assuming that the representative consumer in each market has enough income and U^A and U^B are concave, its consumption is derived from the first-order conditions

$$\frac{\partial U^A}{\partial x_0^A} = p_0^A, \ \frac{\partial U^B}{\partial x_0^B} = p_0^B, \ \text{and} \ \frac{\partial U^B}{\partial x_1^B} = p_1^B, \tag{1}$$

where p_0^A, p_0^B, and p_1^B are prices of products. We denote $D^A(p_0^A)$, $D_0^B(p_0^B, p_1^B)$, and $D_0^B(p_0^B, p_1^B)$ as the demand functions and $CS^A(p_0^A)$ and $CS^B(p_0^B, p_1^B)$ as the consumer surpluses. Note that, by the envelope theorem, $\partial CS^A/\partial p_0^A = -D_0^A$ and $\partial CS^B/\partial p_i^B = -D_i^B$, $i = 0, 1$ hold.

We assume that the products in market B are substitutes, i.e., $\partial D_i^B/\partial p_j^B < 0$ for $i \neq j$. We also assume that the demands are symmetric, that is, $D_0^B(x, x) = D_1^B(x, x)$. Further, we assume that the demand functions satisfy the following regularity condition

$$\frac{\partial D_i^B}{\partial p_i^B} + \frac{\partial D_i^B}{\partial p_j^B} < 0 \text{ for } i = 0, 1, j \neq i. \tag{2}$$

This condition means that if the prices of both firms simultaneously increase, the demands for both products decrease, which is natural to assume in many applications.

The production technologies of firms are given by cost functions $C_0(q_0^A, q_0^B)$ and $C_1(q_0^B)$. Then, the profit of each firm is given by

$$\Pi_0(p_0^A, p_0^B, p_1^B) = \phi D^A(p_0^A)p_0^A + (1-\phi)D_0^B(p_0^B, p_1^B)p_0^B - C_0(\phi D^A(p_0^A), (1-\phi)D_0^B(p_0^B, p_1^B)), \tag{3}$$

and

$$\Pi_1(p_0^B, p_1^B) = (1-\phi)D_0^B(p_0^B, p_1^B) - C_1((1-\phi)D_0^B(p_0^B, p_1^B)). \tag{4}$$

Social welfare SW is given by

$$SW = \phi CS^A(p_0^A) + (1-\phi)CS^B(p_0^B, p_1^B) + \Pi_0(p_0^A, p_0^B, p_1^B) + \Pi_1(p_0^B, p_1^B). \tag{5}$$

Firm 0 maximizes the weighted average of its own profit and social welfare

$$\Omega = \alpha \Pi_0 + (1-\alpha)SW, \tag{6}$$

where $\alpha \in [0, 1]$ is the degree of privatization.

3 Equilibrium

We adopt subgame-perfect equilibrium as the solution and solve the model by backward induction. In the market stage, the first-order conditions for firm 0 are given by[5]

$$\begin{aligned}
\frac{\partial \Omega}{\partial p_0^A} &= \alpha\left(\frac{\partial D_0^A}{\partial p_0^A}\left(p_0^A - \frac{\partial C_0}{\partial q_0^A}\right) + D_0^A\right) + (1-\alpha)\frac{\partial D_0^A}{\partial p_0^A}\left(p_0^A - \frac{\partial C_0}{\partial q_0^A}\right) = 0,\\
\frac{\partial \Omega}{\partial p_0^B} &= \alpha\left(\frac{\partial D_0^B}{\partial p_0^B}\left(p_0^B - \frac{\partial C_0}{\partial q_0^B}\right) + D_0^B\right) + (1-\alpha)\left(\frac{\partial D_0^B}{\partial p_0^B}\left(p_0^B - \frac{\partial C_0}{\partial q_0^B}\right) + \frac{\partial D_1^B}{\partial p_0^B}\left(p_1^B - \frac{\partial C_1}{\partial q_1^B}\right)\right) = 0,
\end{aligned} \tag{7}$$

[5]In the model of price competition with quadratic costs, firms may have incentives to serve the all the amount demanded. We ignore such possibilities in this model since as shown by Matsumura (2012), if the public firm faces the universal service obligation, there are no such incentives.

and the first-order condition for firm 1 is given by

$$\frac{\partial D_1^B}{\partial p_1^B}\left(p_1^B-\frac{\partial C_1}{\partial q_1^B}\right)+D_1^B=0. \tag{8}$$

We assume that the second-order conditions are satisfied, i.e., the Hessian matrix of Ω is negative definite, and $\partial^2\Pi_1/\partial p_1^B<0$. We also assume that the strategy of firm 1 exhibits strategic complementarity, that is,

$$\frac{\partial D_1^B}{\partial p_0^B}\left(1-\frac{\partial D_1^B}{\partial p_1^B}\frac{\partial^2 C_1}{\partial {q_1^B}^2}\right)+\frac{\partial^2 D_1^B}{\partial p_1^B\partial p_0^B}\left(p_1^B-\frac{\partial C_1}{\partial q_1^B}\right)>0. \tag{9}$$

A sufficient condition for the strategic complementarity is that $\partial^2 D_1^B/(\partial p_1^B\partial p_0^B)\geq 0$ and C_1 being weakly convex.

Further, to guarantee the uniqueness and the stability of the equilibrium, we put the following restriction. Let $R_0^A(p_1^B)$ and $R_0^B(p_1^B)$ be the best-response functions of firm 0 and $R_1^B(p_0^B)$ be the best-response function of firm 1. We assume that $|\partial R_0^A/\partial p_1^B|<1$, $|\partial R_0^B/\partial p_1^B|<1$, and $|\partial R_1^B/\partial p_0^B|<1$.

Let $p_0^A(\alpha)$, $p_0^B(\alpha)$, and $p_1^B(\alpha)$ be the equilibrium prices given α.

Next, in the privatization stage the government chooses $\alpha\in[0,1]$ to maximize SW. Let α^* be the welfare-maximizing value of α. In the case of interior solution, the first-order condition is given by

$$\begin{aligned}\left.\frac{dSW}{d\alpha}\right|_{\alpha=\alpha^*}=&\frac{dp_0^A}{d\alpha}\phi\frac{\partial D_0^A}{\partial p_0^A}\left(p_0^A-\frac{\partial C_0}{\partial q_0^A}\right)+(1-\phi)\frac{dp_0^B}{d\alpha}\left(\frac{\partial D_0^B}{\partial p_0^B}\left(p_0^B-\frac{\partial C_0}{\partial q_0^B}\right)+\frac{\partial D_1^B}{\partial p_0^B}\left(p_1^B-\frac{\partial C_1}{\partial q_1^B}\right)\right)\\&+\frac{dp_1^B}{d\alpha}(1-\phi)\left(\frac{\partial D_0^B}{\partial p_1^B}\left(p_0^B-\frac{\partial C_0}{\partial q_0^B}\right)+\frac{\partial D_1^B}{\partial p_1^B}\left(p_1^B-\frac{\partial C_1}{\partial q_1^B}\right)\right)=0.\end{aligned} \tag{10}$$

We assume that the second-order condition is satisfied. In the case of corner solution, we have either $(dSW/d\alpha)|_{\alpha=0}\leq 0$ or $(dSW/d\alpha)|_{\alpha=1}\geq 0$.

As a conventional wisdom, in the public monopoly or mixed oligopoly with price competition, positive degree of privatization would never be optimal. The following lemma and proposition formalize this conventional wisdom.

Lemma 1 *If $\phi=0$, $dp_0^B(0)/d\alpha>0$ and $dp_1^B(0)/d\alpha>0$.*

Proof: See Appendix.

Proposition 1 *If $\phi=0$ or $\phi=1$, full nationalization is optimal.*

Proof: See Appendix.

The reason for the above result is that an increase in the degree of privatization from full nationalization increases the public firm's price since it leans to its own profit, which also increases the price of private firms through the strategic interaction. The former change has negligible effect on the welfare since the public firm is welfare maximizer (envelope theorem), but the latter harms the welfare.

4 Partial Privatization with Multimarket Contact

In this section, we show that the result drastically changes when we take multimarket contacts into account. In order to maintain tractability in analyzing multimarket situation, we restrict our attention to quadratic utility and cost functions. Specifically, we assume that $U^A(x_0^A) = x_0^A - (x_0^A)^2/2$, $U^B(x_0^B, x_1^B) = x_0^B + x_1^B - ((x_0^B)^2 + 2\gamma x_0^B x_1^B + (x_1^B)^2)/2$ for $\gamma \in (0,1)$, $C_0(q_0^A, q_0^B) = (q_0^A + q_0^B)^2/2$, and $C_1(q_1^B) = (q_1^B)^2/2$.[6] Then we yield

$$\Pi_1 = p_1^B(1-\phi)\left(\frac{1-\gamma-p_1^B+\gamma * p_0^B}{1-\gamma^2}\right) - \frac{(1-\phi)^2}{2}\left(\frac{1-\gamma-p_1^B+\gamma p_0^B}{1-\gamma^2}\right)^2, \tag{11}$$

$$\begin{aligned}\Pi_o =& p_0^A\phi(1-p_0^A) + p_0^B(1-\phi)\left(\frac{1-\gamma-p_0^B+\gamma p_1^B}{1-\gamma^2}\right)\\ &-\frac{1}{2}\left(\frac{(1-\phi)(1-\gamma-p_0^B+\gamma p_1^B)}{1-\gamma^2}+\phi(1-p)\right)^2,\end{aligned}$$

$$CS^A = \frac{(1-p_0^A)^2}{2}, \tag{12}$$

and

$$CS^B = \frac{p_0^B 2 + p_1^B 2 + 2(1-p_0^B-p_1^B) - 2\gamma(1-p_1^B)(1-p_0^B)}{2(1-\gamma^2)}. \tag{13}$$

In this specification, we obtain the following lemma.

Lemma 2 *If* $\phi \in (0,1)$ *then,*

$$\exists\gamma^* \quad s.t. \quad \forall\gamma > \gamma^* \quad \left.\frac{\partial p_1^B}{\partial \alpha}\right|_{\alpha=0} < 0 \tag{14}$$

Proof: See Appendix.

The mechanism behind Lemma 2 is following. Departing from full nationalization to partial privatization makes a public enterprise lean to own profit, and that basically pulls

[6] In this specification, all the assumptions put in the general model hold.

up its prices in both markets. In a market solely supplied by the public firm especially, it leads to less production. Because of the less production in the one market, the public firm can have a room in its cost function to cut down the price in the other market. This sequence ends up slicing off the competitor's price through strategic complement relationship. The last effect gets stronger as their products being similar, and beyond some threshold it dominates the first pulling up effect.

Lemma 2 immediately yields our main proposition stating an optimality of the partial privatization in price competition situation, which never be optimal without multimarket contacts.

Proposition 2 *If $\phi \in (0,1)$ then for γ^* defined in Lemma 2,*

$$\forall \gamma > \gamma^* \quad \left.\frac{\partial SW}{\partial \alpha}\right|_{\alpha=0} > 0 \tag{15}$$

Proof: Suppose that $\gamma > \gamma^*$.

$$\frac{\partial SW}{\partial \alpha} = \frac{dp_0^A}{d\alpha}\frac{dSW}{dp_0^A} + \frac{dp_0^B}{d\alpha}\frac{dSW}{dp_0^B} + \frac{dp_1^B}{d\alpha}\frac{dSW}{dp_1^B} \tag{16}$$

At $\alpha = 0$, the first and second term of the right hand side are zero from the envelope theorem. The first factor of the third term is negative from Lemma 2, and the second factor is negative as shown in the proof of Proposition 1 in the Appendix. Thus the sign of the whole derivative is positive. Q.E.D.

5 Conclusion

In this paper, we have formally shown the conventional wisdom that under the price competition, privatization of public enterprises never improves welfare if they serve to a single market. Then we have shown that the partial privatization can be optimal if the public firm faces multimarket contacts. These results shed lights on the importance of taking multimarket interactions into account for the analysis of optimal privatization policies. In addition, these results have policy implications for the privatization policy in sectors such as transportation, in which public enterprises often solely serve to rural areas and compete with private enterprises in urban areas.

Appendix

Equilibrium Prices for Section 4

The equilibrium prices given α under the specification in Section 4 are as follows:

$$p_0^A(\alpha) = \frac{1}{\delta}[\alpha^2\left(\gamma^2-1\right)(\phi-3)+\alpha\left(\gamma\left(\gamma\left(\gamma^2+\gamma(\phi-1)+(\phi-2)\phi-4\right)-\phi+1\right)-2\phi+6\right)$$
$$+(\gamma-1)\gamma\left(\left(\gamma^2-2\right)\phi+\gamma\right)-2\gamma-\phi+3] \quad (17)$$

$$p_0^B(\alpha) = \frac{1}{\delta}[\alpha^2(\gamma^2-1)(2\gamma+\phi-3)+\alpha(\gamma(\gamma(\gamma^2+2\gamma\phi+\gamma+(\phi-1)\phi-5)-\phi-2)-2\phi+6)$$
$$+\gamma((\gamma-2)\gamma(\gamma+1)(\phi+1)+2\phi)-\phi+3] \quad (18)$$

$$p_1^B(\alpha) = \frac{\left(\gamma^2+\phi-2\right)\left(\alpha^2\left(\gamma^2-1\right)+\alpha\left(\gamma^2(\phi+1)+\gamma-3\right)+\gamma-2\right)}{\delta}, \quad (19)$$

where

$$\delta \equiv \alpha^2\left(\gamma^2-1\right)(\phi-3)+\alpha\left(\gamma^4+\gamma^2\left(\phi^2-\phi-7\right)-3\phi+9\right)+\gamma^4(\phi+1)-2\gamma^2(\phi+2)-2\phi+6. \quad (20)$$

Proof of Lemma 1

In the case where $\phi = 0$, the equilibrium prices given α is characterized by $\partial\Omega/\partial p_0^B = 0$ and $\partial\Pi_1/\partial p_1^B = 0$. Using the implicit function theorem, we have

$$H\begin{pmatrix} \frac{dp_0^B}{d\alpha} \\ \frac{dp_1^B}{d\alpha} \end{pmatrix} = -\begin{pmatrix} D_0^B - \frac{\partial D_1^B}{\partial p_0^B}\left(p_1^B - \frac{\partial C_1}{\partial q_1^B}\right) \\ 0 \end{pmatrix} \quad (21)$$

where

$$H = \begin{pmatrix} \frac{\partial^2\Omega}{\partial {p_0^B}^2} & \frac{\partial^2\Omega}{\partial p_0^B \partial p_1^B} \\ \frac{\partial^2\Pi_1}{\partial p_1^B \partial p_0^B} & \frac{\partial^2\Pi_1}{\partial {p_1^B}^2} \end{pmatrix} \quad (22)$$

At $\alpha = 0$, we have

$$\begin{aligned} & D_0^B - \frac{\partial D_1^B}{\partial p_0^B}\left(p_1^B - \frac{\partial C_1}{\partial q_1^B}\right) \\ =& D_0^B + \frac{\partial D_1^B/\partial p_0^B}{\partial D_1^B/\partial p_1^B} D_1^B > 0, \end{aligned} \quad (23)$$

which follows from the regularity condition, $p_0^B < p_1^B$, and the symmetry of demand function.

Then, using Cramer's rule, we have

$$\left.\frac{dp_0^B}{d\alpha}\right|_{\alpha=0} = \frac{-\left(D_0^B - \frac{\partial D_1^B/\partial p_0^B}{\partial D_0^B/\partial p_0^B}\frac{\partial D_1^B/\partial p_0^B}{\partial D_1^B/\partial p_1^B} D_1^B\right)\frac{\partial^2 \Pi_1}{\partial {p_1^B}^2}}{\det H} > 0$$

since

$$\begin{aligned}\det H &= \frac{\partial^2 \Omega}{\partial {p_0^B}^2}\frac{\partial^2 \Pi_1}{\partial {p_1^B}^2} - \frac{\partial^2 \Omega}{\partial p_0^B \partial p_1^B}\frac{\partial^2 \Pi_1}{\partial p_1^B \partial p_0^B} \\ &= \frac{\partial^2 \Omega}{\partial {p_0^B}^2}\frac{\partial^2 \Pi_1}{\partial {p_1^B}^2}\left(1 - \frac{\partial R_0^B}{\partial p_1^B}\frac{\partial R_1^B}{\partial p_0^B}\right) > 0\end{aligned} \tag{24}$$

from the stability condition.

Finally, the equation

$$\frac{\partial^2 \Pi_1}{\partial p_1^B \partial p_0^B}\frac{dp_0^B}{d\alpha} + \frac{\partial^2 \Pi_1}{\partial {p_1^B}^2}\frac{dp_1^B}{d\alpha} = 0$$

implies that $dp_1^B/d\alpha > 0$. This completes the proof. Q.E.D.

Proof of Proposition 1

For any $\phi \in [0,1]$, we have

$$\left.\frac{dSW}{d\alpha}\right|_{\alpha=0} = (1-\phi)\frac{dp_1^B}{d\alpha}\frac{1}{\partial D_0^B/\partial p_0^B}\left(p_1^B - \frac{\partial C_1}{\partial q_1^B}\right)\left(\frac{\partial D_1^B}{\partial p_1^B}\frac{\partial D_0^B}{\partial p_0^B} - \frac{\partial D_1^B}{\partial p_0^B}\frac{\partial D_0^B}{\partial p_1^B}\right). \tag{25}$$

When $\phi = 1$, this equals zero, which implies $\alpha^* = 0$. When $\phi = 0$, Lemma 1 implies that $dp_1^B/d\alpha|_{\alpha=0} > 0$. Since the term other than $dp_1^B/d\alpha|_{\alpha=0} > 0$, say dSW/dp_1^B, is negative from the stability condition and the first-order condition of firm 1, we have $(dp_1^B/d\alpha|_{\alpha=0})(dSW/dp_1^B) < 0$. Thus, we have $\alpha^* = 0$ in both cases. Q.E.D.

Proof of Lemma 2

$$\left.\frac{\partial p_1^B}{\partial \alpha}\right|_{\alpha=0} = \frac{\gamma(\gamma+1)\left(\gamma^2+\phi-2\right) f(\gamma,\phi)}{\left(\gamma^4(\phi+1) - 2\gamma^2(\phi+2) - 2\phi + 6\right)^2} \tag{26}$$

where

$$f(\gamma,\phi) \equiv \gamma^4(\phi+1)^2 - \gamma^3\left(\phi^2+\phi+1\right) - \gamma^2\left(\phi^2+8\phi+4\right) + 7\gamma(\phi+1) + \phi - 3. \tag{27}$$

Since $\gamma,\ \phi \in (0,1)$, the sign of the derivative is the same as of f. We have $f(0,\phi) < 0$, $f(1,\phi) > 0$ and $f(\gamma,\phi)$ belongs to C^∞ class. Then showing $f(\cdot,\phi)$ has at most one extremum in $\gamma \in (0,1)$ for any $\phi \in (0,1)$ proves Lemma 2.[7]

[7] Suppose that $f(\cdot,\phi)$ has at most one minimum or maximum. If the extremum is minimum at $\underline{\gamma}$, then $f_\gamma(\gamma,\phi) < 0$ for all $\gamma < \underline{\gamma}$, since otherwise there is some point $\gamma' \in (0,\underline{\gamma})$ such that $f_\gamma(\gamma',\phi) = 0$, contradicting the assumption that $f(\cdot,\phi)$ has at most one extremum. Similarly, $f_\gamma(\gamma,\phi) > 0$ for all $\gamma \in (\underline{\gamma},1)$. These imply that there exists γ^* such that $f(\gamma,\phi) < 0$ for any $\gamma \in [0,\gamma^*)$, $f(\gamma^*,\phi) = 0$, and $f(\gamma,\phi) > 0$ for any $\gamma \in (\gamma^*,1]$. The case where the extremum is maximum is analogous.

Let $f_\gamma(\gamma,\phi)$ and $f_{\gamma\gamma}(\gamma,\phi)$ be the first and second partial derivatives with respect to γ. Then,

$$f_\gamma(\gamma,\phi) = 4\gamma^3(1+\phi)^2 - 3\gamma^2(1+\phi+\phi^2) - 2\gamma(4+8\phi+\phi^2) + 7(1+\phi) \qquad (28)$$

$$f_{\gamma\gamma}(\gamma,\phi) = 12\gamma^2(1+\phi)^2 - \gamma(1+\phi+\phi^2) - 2(4+8\phi+\phi^2)\ . \qquad (29)$$

Since $f_{\gamma\gamma}(0,\phi) < 0$ and $f_{\gamma\gamma}$ is convex, f_γ has at most one extremum. In addition to it, $f_\gamma(0,\phi) > 0$ and $f_\gamma(1,\phi) < 0$ together show the solution of $f_\gamma(\cdot,\phi) = 0$ with respect to γ is unique, which implies so is the extremum of $f(\cdot)$. Q.E.D.

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