Environmental policy in dynamic models with pollution by consumers: The impact of exogenous shocks and dozy politicians

Barthel, Jens

17 July 2007

Environmental Policy in Dynamic Models with Pollution by Consumers: The Impact of Exogenous Shocks and Dozy Politicians

Jens Barthel
Humboldt-Universität zu Berlin
School of Business and Economics
Discussion Paper - Economic Series
200

September 11, 2007

Abstract

The paper discusses questions resulting from a study of the interaction of exogenous shocks and environmental policy. In a model with pollution as a side effect of consumption environmental policy is introduced in the form of a consumption tax with or without a subsidy on eco-friendly investments. In simulations we observe the dynamic behavior of models before and after sudden changes of exogenous variables. These shocks are jumps in productivity or a sudden depreciation of capital. Additionally we examine the effect of a simultaneous appearance of both types of shocks. Furthermore we investigate the consequences of a lagged reaction of the policy agents.

1 Introduction

Undeniably, the preservation of global and local ecosystems is not only of essential importance for present but also for future generations. Nevertheless, in economics environmental policy is usually evaluated in static models. Exceptions can be found in the literature on the interaction between environmental policy and economic growth. Seminal studies were published by Bovenberg and de Mooij (1997), Bovenberg and Smulders (1995, 1996), Forster (1973), Gradus and Smulders (1993), Huang and Cai (1994), Lighthart and van der Ploeg (1994) and Smulders and Gradus (1996). Conrad (1999) summarizes the literature on computable general equilibrium models.

Bohm and Russell (1985) discuss among other criteria the flexibility and dynamic incentives of policy instruments. Flexibility is considered as the facility to adjust the chosen environmental policy instrument to changes of exogenous variables if a certain environmental target level is to be reached. Dynamic incentives of policy instruments are effects on the development of new technologies, on the impact on relative factor prices and their consequences on locational decisions.

In this paper, we focus on the aspect of flexibility. Using the model developed in Barthel (2005) we explore the consequences of exogenous shocks on the economy. In steady state equilibrium models, variables remain constant or change with a (common) constant rate over time (see Chiang (1984), p. 499). Here we investigate the consequences of a jump in productivity. Typically, innovations in an economy do not cause a jump in productivity since the diffusion of innovations usually takes time. The reason is that fundamental innovations created by basic research have to be "translated" into secondary innovations that bring about realizations of possibilities rather than new opportunities. Therefore, changes in productivity in reality are sequences of innovations on several levels.[1] Even fundamental innovations such as the steam engine, the transistor or integrated circuits took years - sometimes even decades - to become part of everyday life. What we have in mind are events that change the business environment "overnight". Examples are the German Unification or the enlargement of the European Union. Here, suddenly new technologies became available for all enterprises of the respective country. The resulting jump in productivity is often accompanied by an abrupt depreciation of the country's capital stock. This can be a sectoral problem, if only a small number of industries face an intensified competition, or a general problem, if out-dated capital vintages become inefficient in the whole economy. Of course, a sudden depreciation of a country's capital can also occur without productivity shocks. It can be the consequence of a change in regulation that simply makes the use of certain technologies - and therefore certain types of capital - illegal or inefficient.[2] A thinkable reason

[1] Innovations in real life always have fundamental as well as secondary attributes. For a discussion see Aghion and Howitt (1998).

[2] Although the acceptance of instruments that complete the set of markets is increasing, in past and present direct regulation seems the most popular instrument among politicians (see Bohm and Russell (1985), p. 436).

for the ban of technologies is the appearance of evidence about risks or adverse effects of these technologies.[3] On the other hand free trade agreements may also lead to an abrupt disappearance of those industries that were formerly protected by trade barriers.

In such tumultuous times, environmental concerns are often brushed aside. For this reason we analyze the consequences of a lag in the adjustment of environmental policy instruments to the new exogenous conditions.

The paper is organized as follows: In the next section we introduce the basic model. Section (3) discusses the impact of shocks in models with and without environmental policy. Section (4) addresses the effect of an abrupt capital depreciation. In Section (5), productivity shocks are accompanied by a sudden depreciation of capital. Section (6) investigates the consequences of a delayed adjustment of the environmental policy after a shock. Section (7) summarizes the results and gives a brief outlook on possible extensions and variations of the model.

2 The Basic Model

2.1 Environment

The environmental quality $N(t)$ depends only on the flow of pollution. There is no accumulation of pollutants. It is assumed that all pollutants that are not eliminated due to environmental protection vanish in the next moment. This is equal to a situation with infinite but somewhat lagged self-regenerating capacity of the environment. Examples of pollutants of this type are traffic noise, malodor from thinners or other chemical substances and - sometimes - food, and last but not least cigarettes and cigars, especially those produced in Cuba, notably Havana.[4] The burden on the environment depends on the share of income devoted to cleaning the environment $E(S)$, in the following refered to as "environmental expenditures". Pollution is a damaging side effect of consumption $P(C)$. Without economic activity the environmental quality is $\overline{N}$. It follows:

$$N = N\left(E(S), P(C), \overline{N}\right)$$

with:

$$N_E > 0 \qquad N_P < 0$$

2.2 Households and Preferences

The representative household exhibits preferences over consumption goods and environmental amenities. Population growth is zero. The rate of time preference

[3] Blatant examples are the widely use of heroine as pain killer and antitussive as well as in the treatment of abstinence phenomenon after opium and morphine abuse or the use of thalidomide in the treatment of morning sickness. The American prohibition (1919-1932) made a whole industry illegal.

[4] We ignore that especially cigars cause stench for days if you cannot open the window. Permanent smoking can make a room unusable for years.

is ρ. The elasticity of substitution, σ, and the relative weight of environmental amenities in utility, $\phi > 0$, are constant. The utility function of the individual household can be written as:

$$W_i = \int_0^\infty U(c_i, N, \phi) \cdot e^{-\rho \cdot t} dt \qquad (1)$$

with the household's consumption being c_i and the environmental quality N.

It is assumed that all n households are identical, especially of equal size, and small. For the average consumption and investment into the regenerative capacity of the environment follows:

$$C = \sum_{i=1}^{n} c_i \qquad c = \frac{C}{n}$$

$$S_{(N)} = \sum_{i=1}^{n} s_{(N)_i} \qquad s_{(N)} = \frac{S_{(N)}}{n}$$

Households supply one unit of labor and receive a wage w. Each household holds assets a with a rate of return r. Part of the household's income can be invested "into the nature" to improve the regenerative capacity of the environment. This investment could be thought of as something like trash collection for which one has to pay or as engagement in environmental activities. The endogenous rate of these investments is $s_{(N)}$. The remaining income can be used for consumption c and saving $\dot{a}$. The flow budget constraint for the household is:

$$w + r \cdot a = \dot{a} + c + s_{(N)} \qquad (2)$$

The household's optimization problem is to maximize (1), subject to the budget constraint (2). As derived in Appendix 8.1, the control variables change according to:

$$g_{(c)} \equiv \frac{\dot{c}}{c} = \frac{\xi_4 - \xi_2}{\xi_1 \cdot \xi_4 - \xi_2 \cdot \xi_3} \cdot \frac{\rho - r}{c} \qquad (3)$$

$$g_{(s)} \equiv \frac{\dot{s}}{s} = \frac{\xi_1 - \xi_3}{\xi_1 \cdot \xi_4 - \xi_2 \cdot \xi_3} \cdot \frac{\rho - r}{s} \qquad (4)$$

with

$$\xi_1 \equiv \frac{U_{cc} + U_N \cdot \left(N_P \cdot P_{CC} + P_C^2 \cdot N_{PP}\right) \cdot n + U_{cN} \cdot P_C \cdot N_P \cdot (n+1) + U_{NN} \cdot P_C^2 \cdot N_P^2 \cdot n}{U_N \cdot N_E \cdot E_S}$$

$$\xi_2 \equiv \frac{((U_{NN} \cdot N_E \cdot N_P + U_N \cdot N_{EP}) \cdot P_C + U_{cN} \cdot N_E) \cdot n}{U_N \cdot N_E}$$

$$\xi_3 \equiv \frac{(U_{NN} \cdot N_E \cdot N_P + U_N \cdot N_{EP}) \cdot P_C \cdot n + U_{cN} \cdot N_E}{U_N \cdot N_E}$$

$$\xi_4 \equiv \frac{\left(U_N \cdot E_{SS} \cdot N_E + U_{NN} \cdot E_S^2 \cdot N_E^2 + U_N \cdot E_S^2 \cdot N_{EE}\right) \cdot n}{U_N \cdot N_E \cdot E_S}$$

For the change in environmental quality we can write:

$$\begin{aligned} \dot{N} &= N_E \cdot E_S \cdot \dot{S} + N_P \cdot P_C \cdot \dot{C} \\ &= n \cdot \left(N_E \cdot E_S \cdot \dot{s} + N_P \cdot P_C \cdot \dot{c}\right) \end{aligned} \tag{5}$$

2.3 Production

The production technology in this economy can be described by a linear-homogenous production function with labor L and capital K in efficiency units.

$$Y = F(K, L) \tag{6}$$

Since each of the n households supplies one unit of labor and owns the same share of the total capital stock, K, it follows:

$$\begin{aligned} Y &= F(K, n) = n \cdot F\left(\frac{K}{n}, 1\right) \\ k &\equiv \frac{K}{L} \\ f(k) &\equiv F(k, 1) \end{aligned}$$

Output per capita can be expressed by:

$$y \equiv \frac{Y}{n} = f(k)$$

The marginal productivities are then given by:

$$\begin{aligned} \frac{\partial Y}{\partial K} &= n \cdot \frac{\partial f(k)}{\partial k} \cdot \frac{1}{n} \\ &= \frac{\partial f(k)}{\partial k} \\ \frac{\partial Y}{\partial L} &= f(k) + n \cdot \frac{\partial f(k)}{\partial k} \cdot \frac{\partial k}{\partial n} \\ &= f(k) - \frac{K}{n} \cdot \frac{\partial f(k)}{\partial k} \end{aligned}$$

Output is equal to the sum of the marginal factor productivities multiplied by the quantities:

$$\begin{aligned} Y &= \frac{\partial Y}{\partial K} \cdot K + \frac{\partial Y}{\partial L} \cdot L \\ &= \frac{\partial f(k)}{\partial \frac{K}{n}} \cdot K + \left[f(k) - \frac{K}{n} \cdot \frac{\partial f(k)}{\partial k}\right] \cdot n \\ &= f(k) \cdot n \end{aligned}$$

In equilibrium, supply and demand on the capital and on the labor market are equal. This results in factor payments equal to marginal productivities:

$$\begin{aligned} r &= \frac{\partial Y}{\partial K} = \frac{\partial f(k)}{\partial k} \\ w &= \frac{\partial Y}{\partial L} = f(k) - k \cdot \frac{\partial f(k)}{\partial k} \end{aligned}$$

Equilibrium on the capital market ensures that savings are equal to investments. The total capital stock equals the total amount of assets:

$$a \cdot n = K$$

Consequently,the interest rate is equal to the marginal return on investment; the wage rate is equal to the output per capita net of capital costs:

$$\begin{aligned} r &= \frac{\partial f(a)}{\partial a} \\ w &= f(a) - a \cdot \frac{\partial f(a)}{\partial a} \end{aligned}$$

Therefore, the wage and the interest rate in equilibrium only depend on the size of the capital stock. The household's budget constraint can be written as:

$$\dot{a} + c + s_{(N)} = f(a)$$

2.4 Steady State

In this model - with no other engine of growth than capital accumulation - a steady state is characterized by constant variables. It follows:

$$\begin{aligned} \frac{\dot{\theta}_{(a)}}{\theta_{(a)}} &= \rho - r = 0 \\ \rho &= r \qquad (7) \\ U_c + U_N \cdot N_P \cdot P_C &= U_N \cdot N_E \cdot E_S = \theta_{(a)} \\ \frac{U_c}{U_N} &= N_E \cdot E_S - N_P \cdot P_C \qquad (8) \\ \dot{a} &= 0 \\ c + s_{(N)} &= w + r \cdot a = f(a) \qquad (9) \end{aligned}$$

For given parameter values, these equations allow to compute solutions for the steady state values c^*, s^* and a^*.

2.5 The Optimal Solution and a First-Best Policy

As a benchmark we derive the optimal solution of the model.[5] The benevolent dictator considers the trade-offs between higher consumption and consequential increased pollution and between higher expenditures for environmental quality resulting in lower consumption but higher environmental quality. The behavior

[5] See Appendix 8.2.

of the system can be described by:

$$g_{(a)} \equiv \frac{\dot{a}}{a} = \frac{f(a) - c - s_{(N)}}{a} \tag{10}$$

$$g_{(c)} \equiv \frac{\dot{c}}{c} = \frac{\xi_2 - \xi_3}{\xi_2^2 - \xi_1 \cdot \xi_3} \cdot \frac{\rho - f_a}{c} \tag{11}$$

$$g_{(s)} \equiv \frac{\dot{s}}{s} = \frac{\xi_2 - \xi_1}{\xi_2^2 - \xi_1 \cdot \xi_3} \cdot \frac{\rho - f_a}{s} \tag{12}$$

with:

$$\xi_1 \equiv \frac{U_{cc} + \left[\begin{array}{c} U_N \cdot \left(N_P \cdot P_{CC} + P_C^2 \cdot N_{PP} \right) \\ +U_{NN} \cdot N_P^2 \cdot P_C^2 \end{array} \right] \cdot n^2 + 2 \cdot U_{cN} \cdot P_C \cdot N_P \cdot n}{U_N \cdot N_E \cdot E_S \cdot n}$$

$$\xi_2 \equiv \frac{U_{cN} + U_{NN} \cdot N_P \cdot P_C \cdot n}{U_N} + \frac{N_{EP} \cdot P_C \cdot n}{N_E}$$

$$\xi_3 \equiv \left(\frac{U_{NN} \cdot N_E \cdot E_S}{U_N} + \frac{N_{EE} \cdot E_S}{N_E} + \frac{E_{SS}}{E_S} \right) \cdot n$$

The steady state of the optimal solution is characterized by:

$$\frac{\dot{\theta}_{(a)}}{\theta_{(a)}} = \rho - f_a = 0$$

$$\rho = f_a \tag{13}$$

$$U_c + U_N \cdot N_P \cdot P_C \cdot n = U_N \cdot N_E \cdot E_S \cdot n = \theta_{(a)}$$

$$\frac{U_c}{U_N} = (N_E \cdot E_S - N_P \cdot P_C) \cdot n \tag{14}$$

$$\dot{a} = 0$$

$$c + s_{(N)} = w + r \cdot a = f(a) \tag{15}$$

As a first-best policy we introduce a combination of consumption taxes d and subsidies on environmental expenditures p that ensure the optimal level of consumption and environmental expenditures in the steady state.[6] As a consequence, the optimal environmental quality and the maximal utility level are reached. The budget constraint of household i is now given by:

$$w_i + r \cdot a_i = (1 + d) \cdot c_i + (1 - p) \cdot s_{(N)i}$$

The control variables change according to:

$$g_{(c)} \equiv \frac{\dot{c}}{c} = \frac{(\xi_5 - \xi_2) \cdot (\rho - r) - \xi_3 \cdot \xi_5 \cdot \dot{d} + \xi_2 \cdot \xi_6 \cdot \dot{p}}{\xi_1 \cdot \xi_5 - \xi_4 \cdot \xi_2} \cdot \frac{1}{c}$$

$$g_{(s)} \equiv \frac{\dot{s}}{s} = \frac{(\xi_1 - \xi_4) \cdot (\rho - r) + \xi_3 \cdot \xi_4 \cdot \dot{d} - \xi_1 \cdot \xi_6 \cdot \dot{p}}{\xi_1 \cdot \xi_5 - \xi_4 \cdot \xi_2} \cdot \frac{1}{s}$$

[6] See Appendix 8.3.

with:

$$\begin{aligned}
\xi_1 &\equiv \frac{\begin{array}{c} U_{cc} + U_N \cdot \left(N_P \cdot P_{CC} + N_{PP} \cdot P_C^2\right) \cdot n \\ + U_{NN} \cdot P_C^2 \cdot N_P^2 \cdot n + U_{cN} \cdot N_P \cdot P_C \cdot (n+1) \end{array}}{\frac{1+d}{1-p} \cdot U_N \cdot N_E \cdot E_S} \\
\xi_2 &\equiv \frac{\left(\left(U_N \cdot N_{EP} + U_{NN} \cdot N_E \cdot N_P\right) \cdot P_C + U_{cN} \cdot N_E\right) \cdot n}{\frac{1+d}{1-p} \cdot U_N \cdot N_E} \\
\xi_3 &\equiv -\frac{1}{1+d} \\
\xi_4 &\equiv \frac{\left(U_N \cdot N_{EP} + U_{NN} \cdot N_E \cdot N_P\right) \cdot P_C \cdot n + U_{cN} \cdot N_E}{U_N \cdot N_E} \\
\xi_5 &\equiv \frac{\left(U_N \cdot N_E \cdot E_{SS} + U_{NN} \cdot N_E^2 \cdot E_S^2 + U_N \cdot N_{EE} \cdot E_S^2\right) \cdot n}{U_N \cdot N_E \cdot E_S} \\
\xi_6 &\equiv \frac{1}{1-p}
\end{aligned}$$

To run numerical simulations, we have to specify the general equations used so far.

2.6 Specific Functions for Numerical Simulations

2.6.1 Utility Function

In the following we will concentrate on the case of an elasticity of substitution equal to one ($\sigma = 1$) - the Cobb-Douglas utility function:[7]

$$U = c^{\alpha} \cdot (\phi \cdot N)^{1-\alpha} \tag{16}$$

2.6.2 Environmental Quality

For simplicity, the following function is chosen to describe the environmental quality :

$$N = \overline{N} + E(S) - P(C) \tag{17}$$

We assume for the impact of economic activities on environmental quality:

$$\begin{aligned}
E(S) &= \tau_{(S)} \cdot S^{\gamma} && (18) \\
P(C) &= \tau_{(C)} \cdot C^{\beta} && (19) \\
0 &< \gamma < 1 < \beta \\
0 &\leq \tau_{(S)}, \tau_{(C)}
\end{aligned}$$

[7] The impact of the elasticity of substitution is discussed in Barthel (2005).

The relevant derivatives are:

$$E_S = \tau_{(S)} \cdot \gamma \cdot S^{\gamma-1} > 0 \tag{20}$$
$$E_{SS} = \tau_{(S)} \cdot \gamma \cdot (\gamma - 1) \cdot S^{\gamma-2} < 0 \tag{21}$$
$$P_C = \tau_{(C)} \cdot \beta \cdot C^{\beta-1} > 0 \tag{22}$$
$$P_{CC} = \tau_{(C)} \cdot \beta \cdot (\beta - 1) \cdot C^{\beta-2} > 0 \tag{23}$$

This implies decreasing marginal effects of investments into environmental quality and increasing marginal damages due to consumption. Decreasing marginal effects of environmentally friendly expenditures are analogous to decreasing marginal productivities in production. Increasing marginal damages result from the assumption of unfeasibility of life on earth if environmental quality is to low.

2.6.3 Production Function

We use a Cobb-Douglas production function:

$$Y = F(K, L) = A \cdot K^{\delta} \cdot L^{1-\delta} \tag{24}$$

In the Cobb-Douglas case, the output per capita and the interest rate are given by:

$$y = A \cdot k^{\delta}$$
$$r = \delta \cdot A \cdot k^{\delta-1}$$

It follows that the unique equilibrium is determined by the parameters. The equilibrium capital stock is given by:

$$k^* = \left(\frac{\delta \cdot A}{\rho}\right)^{\frac{1}{1-\delta}}$$

The labor supply is one unit per head. In the economy there are n households. This results in:

$$k = a$$

3 The Impact of Productivity Shocks

In the present chapter, we consider first the case of an unregulated market economy. A second benchmark model deals with the solution of a benevolent planner. Following that we compare the results with a model of a market economy regulated with an environmental policy consisting of a combination of consumption taxes and subsidies on environmentally friendly expenditures that ensures an optimal expenditure structure. The following parameter values are used: $A = 5$, $n = 1000$, $\alpha = 0.75$, $\beta = 1.1$, $\gamma = 0.9$, $\delta = 0.5$, $\rho = 0.05$, $\phi = 0.5$,

$\overline{N} = 1000$, $\tau_{(S)} = 5$ and $\tau_{(C)} = 0.05$.[8] Various methods can be applied to find correct initial values of the control variables.[9] However, we use the method of backward integration as described by Brunner and Strulik (2002). The trajectories correspond to a time path that approaches 99.5% of the equilibrium capital stock in $t = 0$. Note that the dashed trajectories in the figures correspond to the change of variables without shocks.

3.1 Numerical Results for an Unregulated Market Economy

In a first set of models we analyze the impact of a pure productivity shock. In $t = 0$, the productivity level A jumps to 5.1, 5.25 or 5.5, which corresponds to a change of 2%, 5% or 10%, respectively. The shocks are unanticipated, but once productivity is on the new level the change and all of its consequences are common knowledge. There is no stochastic element in the model. Consequently, sensible formation of expectations is impossible.

Using the specific functions we can rewrite Condition (8) for the unregulated economy in the following way:

$$U_c = U_N \cdot (E_S + P_C)$$

It follows:

$$\begin{aligned}
\xi_1 &\equiv \frac{U_{cc} - U_N \cdot P_{CC} \cdot n - U_{cN} \cdot P_C \cdot (n+1) + U_{NN} \cdot P_C^2 \cdot n}{U_N \cdot E_S} \\
\xi_2 &\equiv \frac{(U_{cN} - U_{NN} \cdot P_C) \cdot n}{U_N} \\
\xi_3 &\equiv \frac{U_{cN} - U_{NN} \cdot P_C \cdot n}{U_N} \\
\xi_4 &\equiv \frac{\left(U_N \cdot E_{SS} + U_{NN} \cdot E_S^2\right) \cdot n}{U_N \cdot E_S}
\end{aligned}$$

The path of the system is now determined by the equations:

$$g_{(c)} \equiv \frac{\dot{c}}{c} = \frac{\xi_4 - \xi_2}{\xi_1 \cdot \xi_4 - \xi_3 \cdot \xi_2} \cdot \frac{\rho - r}{c} \tag{25}$$

$$g_{(s)} \equiv \frac{\dot{s}}{s} = \frac{\xi_1 - \xi_3}{\xi_1 \cdot \xi_4 - \xi_2 \cdot \xi_3} \cdot \frac{\rho - r}{s} \tag{26}$$

$$g_{(a)} \equiv \frac{\dot{a}}{a} = \frac{f(a) - c - s_{(N)}}{a} \tag{27}$$

The following figures illustrate the behavior of the system in case of a productivity shock of 2% in $t = 0$.

[8] A rate of time preference of $\rho = 0.05$ results in equilibrium in an interest rate of $r = 0.05$. This corresponds to period length of one year. The other parameter values are more or less arbitrarily chosen provided that they fulfill the conditions mentioned above and result in a model that can be solved numerically in reasonable time.

[9] For an overview, see Barro and Sala-i-Martin (1995), pp. 471-491.

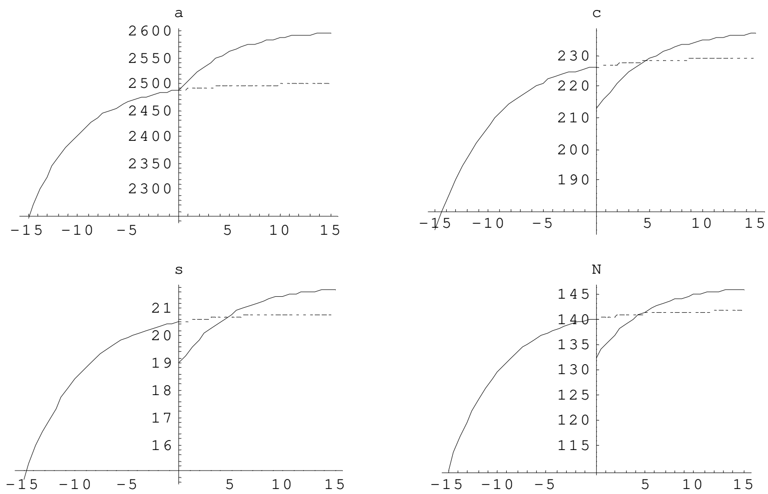

Figure 3.1: Productivity shock of 2% in $t = 0$: assets, consumption, environmental expenditures and environmental quality

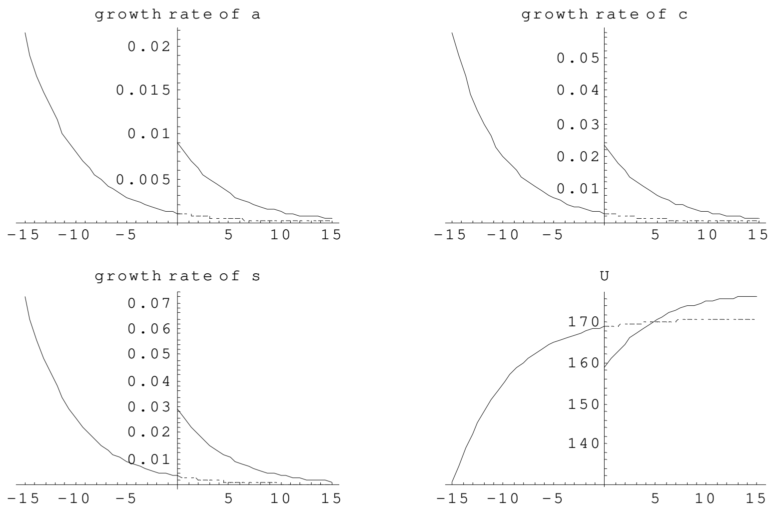

Figure 3.2: Productivity shock of 2% in $t = 0$: growth rates of assets, consumption and environmental expenditures, and utility level

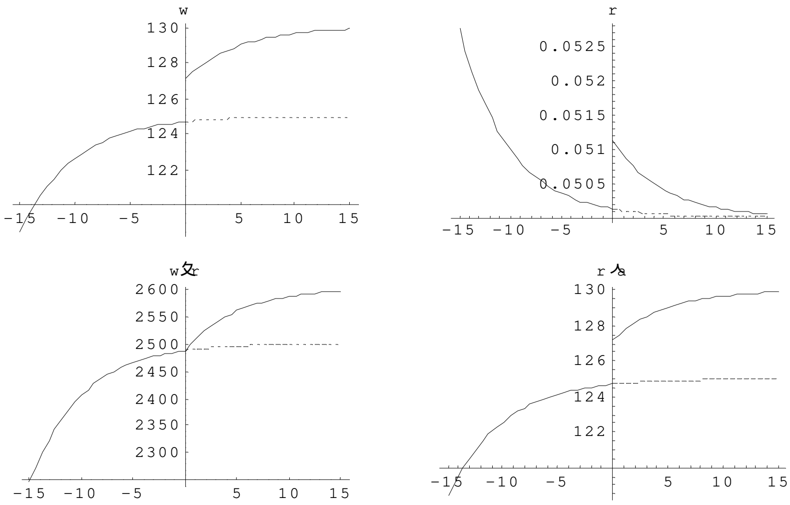

Figure 3.3: Productivity shock of 2% in $t = 0$: wage rate, interest rate, relation between wage and interest rate, and capital income

Due to the shock, the equilibrium asset level increases. This increases the opportunity costs of consumption and environmental expenditures. Therefore, growth rates of assets, consumption and environmental expenditures jump upward, whereas the level of consumption, of environmental expenditures, and consequently the utility level, drop down. Wage rate and capital income increase due to the productivity shock at 2%, 5% and 10%, respectively. The following table shows the size of these immediate effects for a number of other relevant variables in relation to the productivity shock.

Table 3.1: Percentage change of variables in t = 0 following a productivity shock

ΔA	Δc	Δs	ΔN	ΔU
+2	−5.9801	−7.4453	−5.3828	−5.8311
+5	−14.4212	−17.7732	−13.0364	−14.0771
+10	−27.1454	−32.9057	−24.7117	−26.5445

The consequence of a productivity shock is a shift in all equilibrium variable values except for the growth rates.[10] The percentage change of environmental expenditures is higher than that of all other variables (see Table 3.2). This general result is observed in all models of this study and is reflected in the highest jump of the associated growth rate. It results from the decreasing marginal effectivity of these expenditures (see Equation 21). The households try to compensate this effect. Furthermore, more consumption implies more pollution. It

[10] In equilibrium, all growth rates are equal to zero.

is therefore not surprising that the increase in environmental quality is smaller than the relative change of all other variables.

The households lower expenditures to attain the new equilibrium capital stock. This implies increased growth rates. As a consequence of a productivity shock of 2%, 5% and 10%, the growth rate of the capital stock jumps in $t = 0$ from 0.103% to 0.908%, 2.063% and 3.846%, respectively. After 15 periods, the capital stock approaches 99.794%, 99.519% and 99.062% of its new equilibrium value, respectively.

Table 3.2: Percentage change of equilibrium values of variables following a productivity shock in t = 0 in an unregulated economy

ΔA	Δa^*	Δc^*	Δs^*	ΔN^*	ΔU^*
+2	+4.040	+3.955	+4.978	+3.544	+3.852
+5	+10.250	+10.027	+12.703	+8.960	+9.759
+10	+21.000	+20.520	+26.278	+18.256	+19.950

3.2 Numerical Results for a Planned Economy

Now we look at the effects of the same type of shocks but in a planned economy.[11] Note that the benevolent dictator does not anticipate the productivity shocks. But his immediate reaction puts the economy on the new long-run optimal path. The following figures illustrate the consequences of a productivity shock of 2% for the relevant variables.

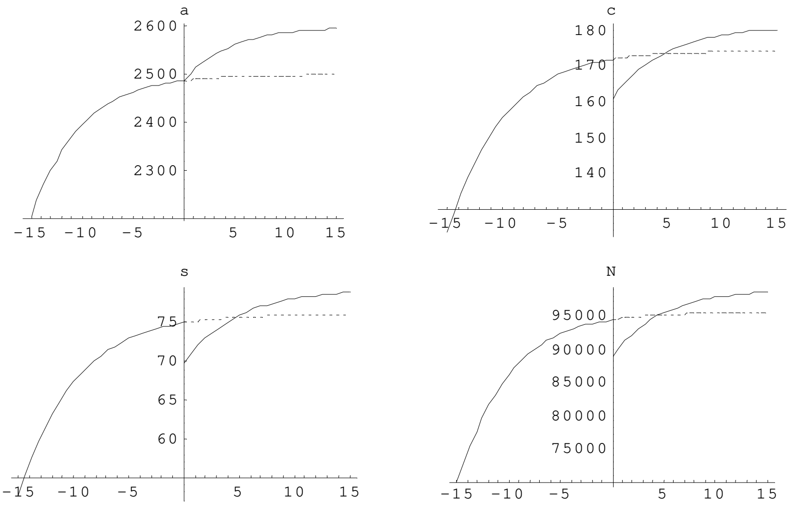

Figure 3.4: Productivity shock of 2% in $t = 0$: assets, consumption, environmental expenditures and environmental quality

[11] See Appendix 8.2 for the specification of the equations of motion for the state and control variables.

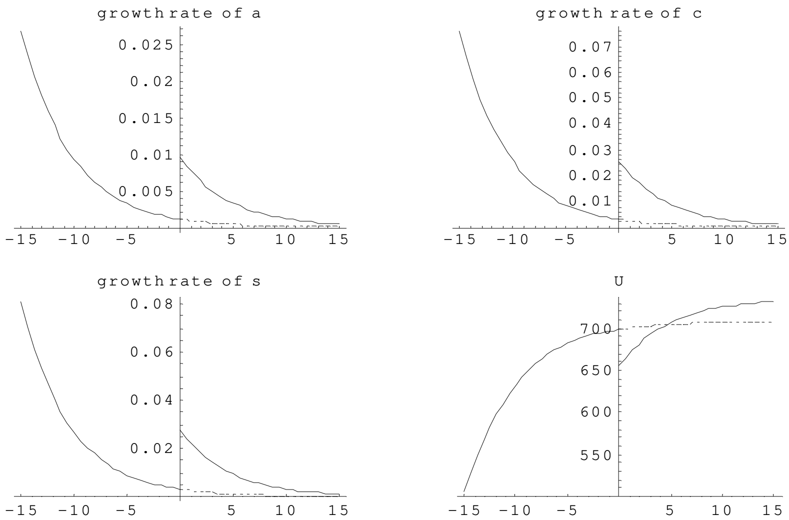

Figure 3.5: Productivity shock of 2% in $t = 0$: growth rates of assets, consumption and environmental expenditures, and utility level

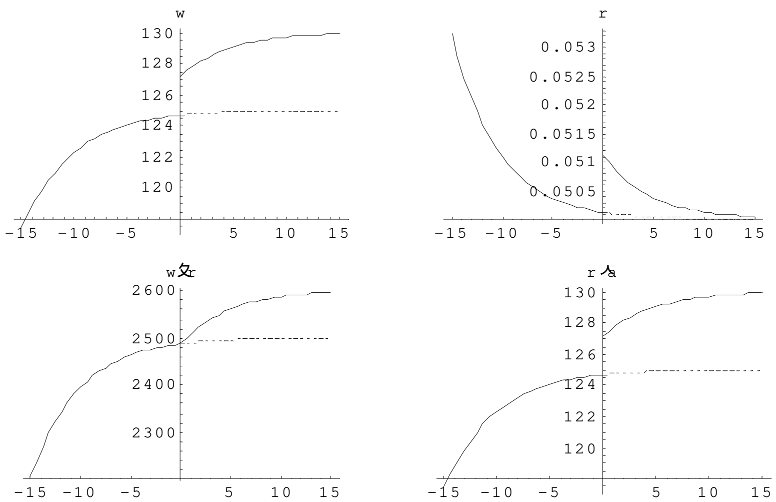

Figure 3.6: Productivity shock of 2% in $t = 0$: wage rate, interest rate, relation between wage and interest rate, and capital income

As one would expect, the planned economy is characterized by a different structure of the household expenditures. Compared to the unregulated economy, consumption is lower. Due to the higher environmental expenditures and lower pollution as a consequence of lower consumption environmental quality is much higher. This over-compensates the effect of lower consumption on utility so that utility in the planned economy is higher.

Following a productivity shock, the growth rates of assets and consumption are higher in a planned economy. This results in a higher speed of convergence; all variables including the wage and the interest rate approach to their equilibrium values faster than an unregulated economy.

Table 3.3: Percentage change of variables in t = 0 following a productivity shock in a planned economy

ΔA	Δc	Δs	ΔN	ΔU
+2	−6.396	−6.851	−5.872	−6.265
+5	−15.406	−16.444	−14.200	−15.106
+10	−28.941	−30.709	−26.848	−28.423

In $t = 0$, the relative change of consumption is slightly higher and the relative change of environmental expenditures slightly lower than in an unregulated economy. In absolute terms the initial loss in consumption is higher in the unregulated economy. Without regulation, consumption decreases at about 13.534, 32.637 and 61.435 units due to a productivity shock of 2%, 5% and 10%, respectively. The initial reductions in consumption prescribed by a planner are 10.982, 26.453 and 49.695 units, respectively. The environmental expenditures decrease in an unregulated economy at about 1.526, 3.642 and 6.744 units; in a planned economy at 5.136, 12.327 and 23.021 units, respectively.

Table 3.4: Percentage change of equilibrium values of variables following a productivity shock in t = 0 in a planned economy

ΔA	Δa^*	Δc^*	Δs^*	ΔN^*	ΔU^*
+2	+4.040	+3.949	+4.248	+3.611	+3.865
+5	+10.250	+10.013	+10.793	+9.134	+9.792
+10	+21.000	+20.490	+22.167	+18.621	+20.020

3.3 Numerical Results for an Optimal Environmental Policy

In the present model environmental policy has to internalize two external effects: the negative effect of pollution due to consumption and the positive effect of environmental expenditures. This can easily be done by a combination of instruments. Furthermore, it is possible to calculate consumption tax rates and subsidies on environmental expenditures that guarantee a balanced governmental budget in equilibrium by varying the size of the budget. To keep things simple we assume constant tax and subsidy rates.[12] That implies the possibil-

[12] The ideal first-best policy in this dynamic model is simply unrealistic. It would imply a permanent adjustment of tax and subsidy rates whenever the economy is off the steady state equilibrium. An alternative to the combination of instruments that we look at here is a constant tax rate with subsidies depending on the momentary tax revenue. Theoretically,

ity of a budget surplus or deficit in the short run. In the long run, the budget is balanced.

In this section we assume that tax rates and subsidies change immediately after the productivity shock. After the shock a larger capital stock is optimal. It follows that the external effects become more important: higher consumption leads to more pollution.

Compared with the solution in an unregulated economy, here in the optimal solution the equilibrium value of consumption is lower; environmental expenditures, environmental quality and the utility level are higher. In the following table, equilibrium levels of variables in the unregulated economy and an economy with optimal environmental policy are compared.

Table 3.5: Equilibrium values of variables of the reference model (no shock) and following a productivity shock in t = 0

	ΔA	a	c	s	N	U
unregulated economy	0%	2500.000	229.181	20.819	141.617	170.866
	2%	2601.000	238.244	21.856	146.635	177.447
	5%	2756.250	252.161	23.464	154.306	187.541
	10%	3025.000	276.210	26.290	167.470	204.954
optimal policy	0%	2500.000	173.973	76.027	95476.0	708.072
	2%	2601.000	180.844	79.256	98923.7	735.436
	5%	2756.250	191.392	84.232	104197	777.409
	10%	3025.000	209.620	92.880	113255	849.829

It is interesting that the necessary change of tax and subsidy rates after the productivity shock is comparatively small. A productivity shock of 2%, 5% and 10% should be accompanied by an increase of the tax rate of 0.287%, 0.709% and 1.392% as well as by an increase of the subsidy rate of 0.0000035%, 0.0000097% and 0.0000215%, respectively. Consequently, the utility gain due to a *change* in the regulation is also unspectacular; it amounts to 0.0000071%, 0.0000438%, 0.000170%, respectively. This is completely different from the huge gains resulting from the *introduction* of regulation as we can see in Table 3.5.

The initial reaction after the productivity shock is illustrated in Table 3.6. As in the unregulated economy, the values of the control variables consumption and environmental expenditures fall considerably. Compared with the unregulated economy, this decrease is more substantial (see Table 3.1). Consequently, environmental quality and the utility level decrease whereas all growth rates and savings increase.

the budget of the government would be balanced both in the long and in the short run. But given that policy there exists a set of initial points consistent with the first order conditions without trajectories to the equilibrium. As a consequence, corner solutions of the optimization problem have to be considered. For the sake of simplicity we abandon a deeper discussion of this problem.

Table 3.6: Percentage change of variables following a productivity shock in t = 0 in the case of an optimal environmental policy

ΔA	Δc	Δs	ΔN	ΔU
+2	−9.144	−9.562	−8.143	−8.895
+5	−21.625	−22.543	−19.419	−21.078
+10	−39.390	−40.854	−35.842	−38.522

The increase in savings results in increased growth rates. Following the productivity shock of 2%, 5% and 10%, the growth rate of the capital stock is 1.238%, 2.764% and 5.012%, respectively. After 15 periods, the capital stock approaches 99.935%, 99.847% and 99.694% of its new equilibrium value, respectively.

In a comparison of table 3.6. with variable values for the unregulated (Table 3.1) and planned (Table 3.3) economy it is noticeable that the initial decrease of total expenditures - and following that the decrease in environmental quality and utility - is higher in a planned economy than in an unregulated market. Furthermore, it is higher with an optimal policy than in a planned economy. Simultaneously, the speed of convergence is low in an unregulated economy and higher with an optimal policy than in a planned economy.[13] The planned economy converges faster than the unregulated economy since external effects are internalized - the positive externality of less consumption today on the utility level of other persons tomorrow are taken into account. The optimal policy leads to a faster reaction of the economy than the direct regulation by a planner since off equilibrium the tax rates are not optimal - they are too high if consumption is lower than its equilibrium value. That implies that consumption will be lower than in a planned economy, savings are higher and consequently so is the speed of convergence. But - as mentioned above - a permanent adjustment of tax and subsidy rates would cause prohibitively high transaction costs. Therefore, the scenario considered here seems to be more realistic.[14]

Table 3.7 shows the impact of a productivity shock on equilibrium levels of various variables. Since the equilibrium level of capital is determined by exogenous parameters, its change is exactly the same as in an unregulated economy (see Table 3.2). The changes of the other variables are similar to the changes calculated for the unregulated economy. The change of the level of consumption and the environmental expenditures are slightly smaller than in the models of the unregulated market, reflecting most notably different base values. Never-

[13] Here we understand speed of convergence as the possibility to close a gap between an initial value and a target level of a certain variable, i.e., in our case capital stock. The easiest way to evaluate the speed of convergence is by a comparison of growth rates: relatively high growth rates in the beginning and low growth rates at the end of the considered period indicate a high speed of convergence, provided that the variable converges at all.

[14] In the real world taxes are raised at some predetermined dates (see e.g. the German energy tax). This is something completely different, as it is no adjustment to real world data. In fact it is an attempt to give the individuals and firms the possibility to adjust their behavior in response to the expected change of the tax. The stepwise introduction of a tax is primarily a way to lower transaction costs.

theless, in the optimal policy case the increase of environmental quality and utility level is in spite of an elevated base value higher than in the unregulated market.

Table 3.7: Percentage change of equilibrium values of variables following a productivity shock in t = 0 in the case of an optimal environmental policy

ΔA	Δa^*	Δc^*	Δs^*	ΔN^*	ΔU^*
+2	+4.040	+3.949	+4.248	+3.611	+3.865
+5	+10.250	+10.013	+10.793	+9.134	+9.792
+10	+21.000	+20.490	+22.167	+18.621	+20.020

The impact of the internalization of the two external effects is shown in Table 3.8. Despite the decrease of consumption the utility level increases dramatically. This is mainly a consequence of the increase of environmental expenditures which leads to an enormous change in the environmental quality. There is no significant difference contingent on the size of the productivity shock.

Table 3.8: Equilibrium values of variables with optimal policy in percentage of values in an unregulated economy

ΔA	c	s	N	U
+2%	75.907	362.628	67462.5	414.454
+5%	75.901	358.984	67526.2	414.527
+10%	75.892	353.290	67627.0	414.644

The following figures show the graphs for various variables before and after a productivity shock. The dashed line indicates the behavior of the system without a shock.

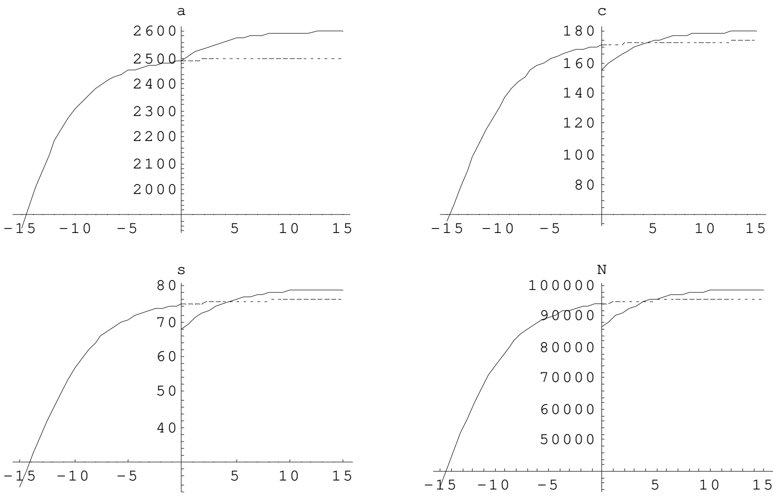

Figure 3.7: Productivity shock of 2% in $t = 0$: assets, consumption, environmental expenditures and environmental quality with an optimal policy

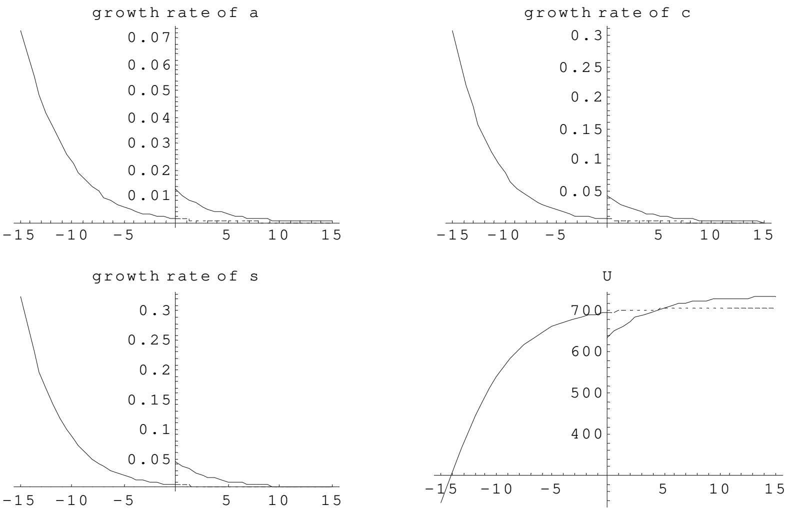

Figure 3.8: Productivity shock of 2% in $t = 0$: growth rates of assets, consumption and environmental expenditures, and utility level with an optimal policy

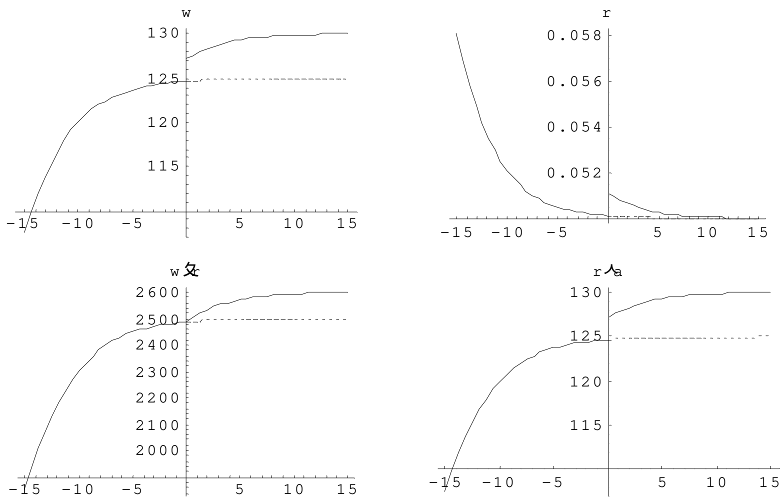

Figure 3.9: Productivity shock of 2% in $t = 0$: wage rate, interest rate, relation between wage and interest rate, and capital income with an optimal policy

As mentioned above, tax and subsidy rates are calculated so as to keep the government's budget balanced in equilibrium. Although in the case of $\Delta A = 2\%$ the budget surplus jumps from 0.070 to 0.570 in $t = 0$, this surplus is negligible compared to the overall tax revenue falling from 74.668 to 68.036 at the same time. The jump results from different magnitudes of change of consumption and environmental expenditures in $t = 0$. Since environmental expenditures decrease relatively more, tax revenues fall less than subsidies. The following figure displays the government's budget cash flow over time.

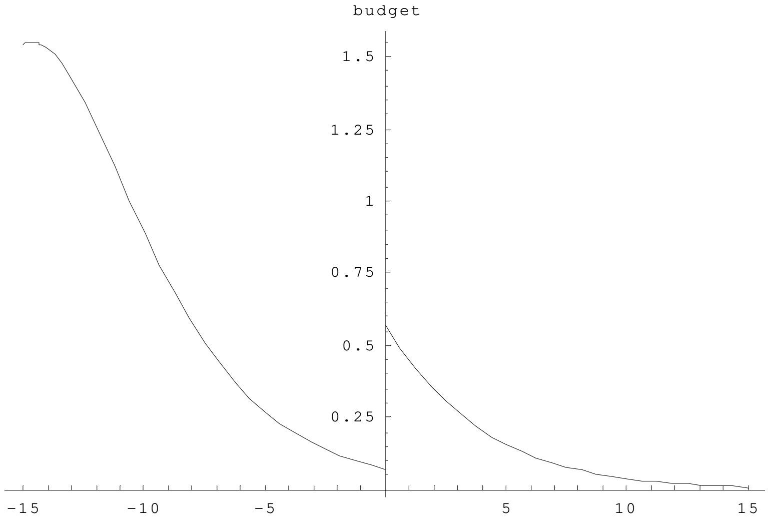

Figure 3.10: Productivity shock of 2% in $t = 0$: budget cash flow

In Table 3.9 we compare tax revenues (T), subsidies (S) and the budget cash flow (B) over 30 periods as well as the consequences of the sudden change in $t = 0$ in absolute terms.

Table 3.9: Tax revenues, subsidies and budget cash flow over time and change after shock in absolute values

$\Delta A\|_{t=0}$	$\int_{-15}^{15} T$	$\int_{-15}^{15} S$	$\int_{-15}^{15} B$	$\Delta T\|_{t=0}$	$\Delta S\|_{t=0}$	$\Delta B\|_{t=0}$
2%	2050.91	2038.80	13.108	−6.633	−7.133	0.500
5%	2069.73	2054.92	14.818	−15.733	−16.817	1.084
10%	2100.91	2081.93	18.981	−28.783	−30.477	1.694

Since the decrease in tax revenues in $t = 0$ due to lower consumption is accompanied by an even bigger decrease in subsidies due to lower environmental expenditures the budget is always nearly balanced. Over time, the budget surplus is less than 1% of the tax revenue.

4 The Sudden Impact of Capital Depreciation

A second type of shock is a sudden capital depreciation. Again we assume that this shock is unanticipated and that there is no sensible way to form expectations. One example for such a shock is the German reunification. Although it took roughly a year from the opening of the Berlin Wall to the official unification, the economic effects came faster. With a look at the average life cycle of physical capital it was virtually an overnight loss of capital.[15] Other examples

[15] In the former German Democratic Republic the lifespan of physical capital was a bit longer than in western countries. The author had vocational training with technology developed right

are bans of certain technolgies due to a change of attitudes toward their use or the consumption of their products. Catchwords in this context are: mad cow disease, nuclear power, bird flu, thalidomide, fur, stem-cell treatment, sizeable angled windows in airplanes etc. Most of these examples do not have measurable consequences for a whole country's economy but can influence the welfare of regions if they are specialized in certain industries. A last set of examples are catastrophes and wars.

In the following we analyze the consequences of a loss of capital of about 10%, 20%, 30%, 40% and 50% at $t = 0$ in an economy with and without an optimal environmental policy. As in the previous section, at this point of time the pre-shock capital stock attains 99.5% of its equilibrium level. The loss does not influence the equilibrium values of the economy. Therefore a change in environmental policy is not necessary. The following table illustrates the percentage change of relevant economic variables in $t = 0$. Note that as a consequence of the Cobb-Douglas production function and other specifications in our model the change in production per capita equals the change of the wage rate ($\Delta y = \Delta w$).

Table 4.1: Percentage change of variables in t = 0 after capital depreciation

	Δa	Δc	Δs	ΔN	ΔU	Δw	Δr
unregulated economy	−10	−23.852	−29.043	−21.673	−23.314	−5.1317	+5.409
	−20	−44.951	−53.072	−41.389	−44.081	−10.557	+11.803
	−30	−62.889	−71.914	−58.731	−61.890	−16.334	+19.523
	−40	−77.223	−85.531	−73.178	−76.273	−22.540	+29.099
	−50	−87.542	−94.151	−84.107	−86.760	−29.289	+41.421
optimal policy	−10	−30.779	−32.185	−28.038	−30.104	−5.141	+5.420
	−20	−55.745	−57.740	−51.765	−54.782	−10.566	+11.815
	−30	−74.719	−76.686	−70.757	−73.782	−16.342	+19.535
	−40	−87.711	−89.282	−84.637	−87.006	−22.548	+29.112
	−50	−95.226	−96.304	−93.361	−94.816	−29.296	+41.435
planned economy	−10	−24.830	−26.396	−22.987	−24.373	−5.131	+5.409
	−20	−46.714	−49.138	−43.766	−45.992	−10.557	+11.803
	−30	−65.222	−67.866	−61.894	−64.418	−16.334	+19.523
	−40	−79.895	−82.249	−76.855	−79.052	−22.540	+29.099
	−50	−90.345	−92.098	−88.091	−89.825	−29.289	+41.421

after the war - World War I. And it was not training for a job in a museum, but instead in telecommunication!

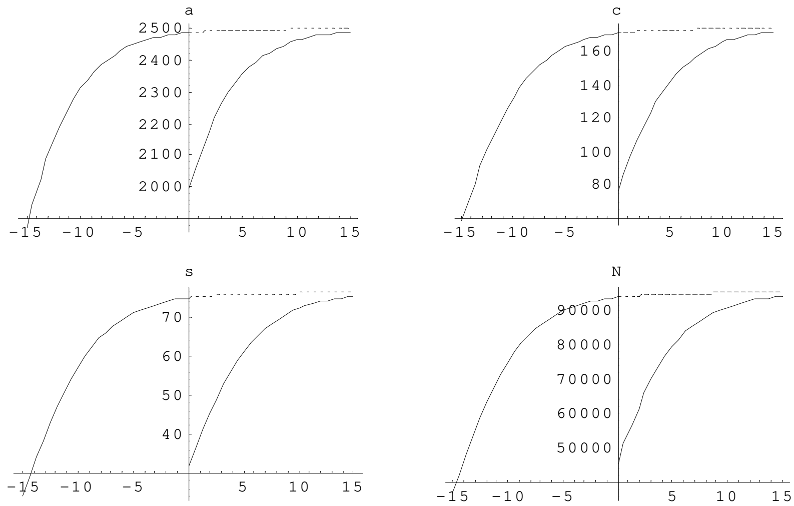

Figure 4.1: Capital depreciation of 20% in $t = 0$: assets, consumption, environmental expenditures and environmental quality with an optimal policy

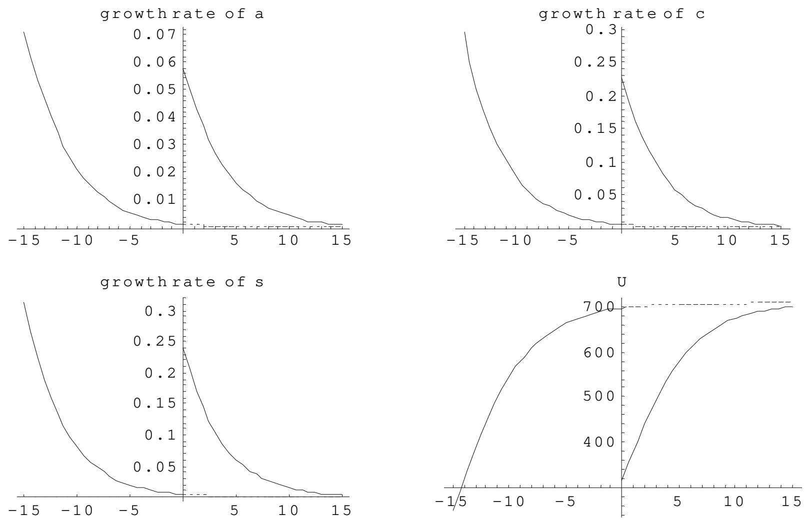

Figure 4.2: Capital depreciation of 20% in $t = 0$: growth rates of assets, consumption and environmental expenditures, and utility level with an optimal policy

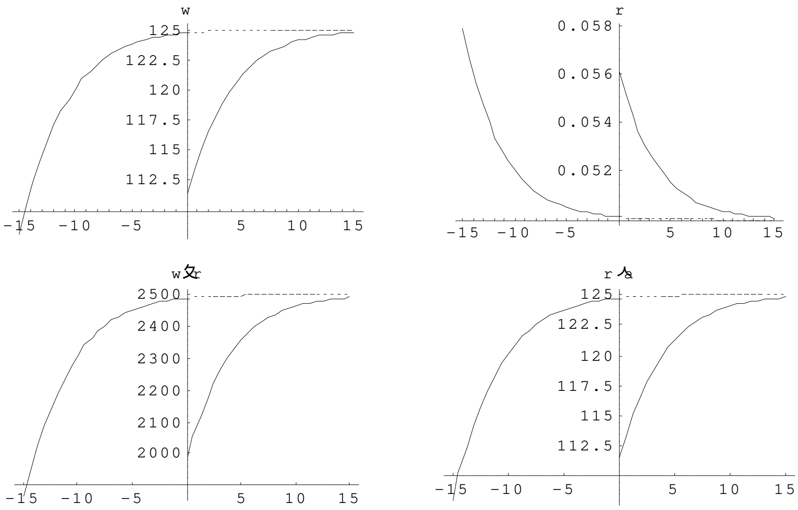

Figure 4.3: Capital depreciation of 20% in $t = 0$: wage rate, interest rate, relation between wage and interest rate and capital income with an optimal policy

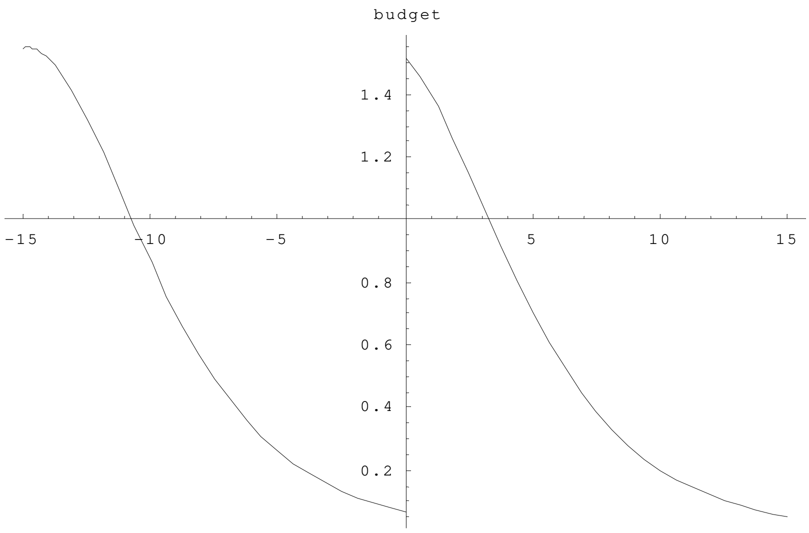

Figure 4.4: Capital depreciation of 20% in $t = 0$: budget cash flow with an optimal environmental policy

With an optimal policy we can observe the most prominent decrease of consumption and environmental expenditures. Consequently, the relative fall of the utility level is dramatic: a depreciation of 50% of the capital stock reduces utility to 5.184% of the original level. On the other hand, in this model the convergence rates are higher than in the other models. After a 20% depreciation, the capital stock reaches in $t = 15$ 99.637% with an optimal policy, 99.078% in a planned economy and 98.915% in an unregulated market of the equilibrium value.

Figure 4.5 illustrates the budget cash flow if the capital depreciation is 50%. After $t = 0$ the budget cash flow increases. The reason is - compared with consumption - a more heavy decrease of environmental expenditures, reflected also in the higher growth rate $g_{(s)}$ in $t = 0$. Therefore, in the beginning tax revenues increase faster than expenditures for subsidies. After some time, the higher growth rate $g_{(s)}$ results in a catch up of the environmental expenditures. Again, in the long run the budget cash flow approaches zero.

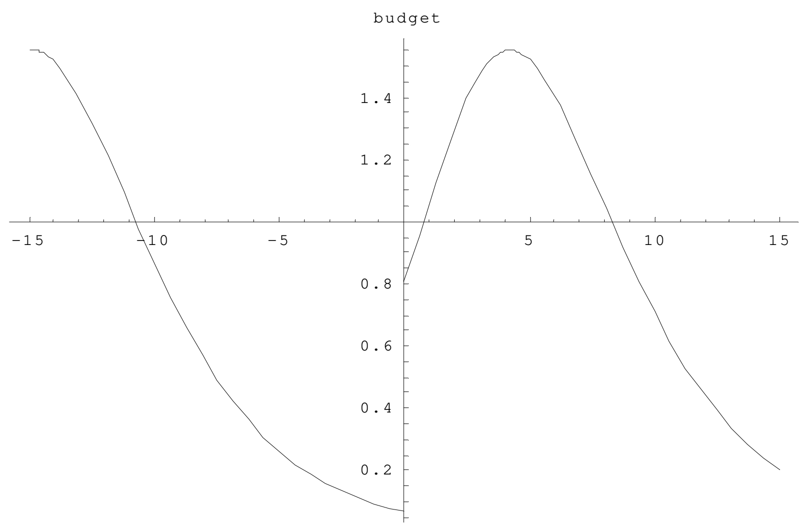

Figure 4.5: Capital depreciation of 50% in $t = 0$: budget cash flow

Although tax revenues and subsidy expenditures decrease with a higher capital depreciation the aggregated budget cash flow increases. The reason is again the faster and more prominent decrease of the environmental expenditures. Contrary to the results with productivity shocks in these models the change of the budget cash flow in $t = 0$ decreases if the degree of the distortion - here depreciation - increases (except for the case of a 10% depreciation). This is also a consequence of the relatively heavy decline of environmental expenditures (see Table 4.2).

Table 4.2: Tax revenues, subsidies and budget cash flow over time and change after a capital depreciation in absolute values

$\Delta a\|_{t=0}$	$\int_{-15}^{15} T$	$\int_{-15}^{15} S$	$\int_{-15}^{15} B$	$\Delta T\|_{t=0}$	$\Delta S\|_{t=0}$	$\Delta B\|_{t=0}$
−10%	1957.02	1942.60	14.424	−22.998	−24.026	1.029
−20%	1866.74	1848.71	18.036	−41.652	−43.104	1.452
−30%	1774.01	1753.19	20.821	−55.829	−57.247	1.418
−40%	1677.93	1655.12	22.802	−65.536	−66.650	1.114
−50%	1577.13	1553.08	24.049	−71.151	−71.892	0.741

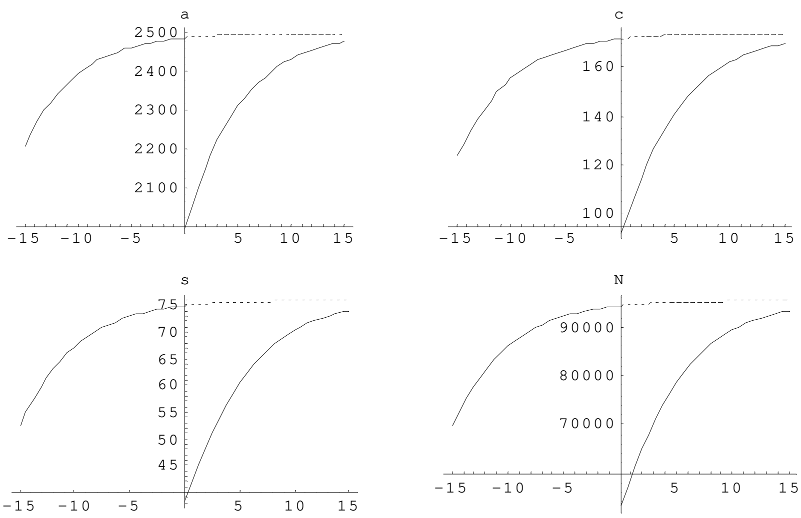

Figure 4.6: Capital depreciation of 20% in $t = 0$: assets, consumption, environmental expenditures and environmental quality in a planned economy

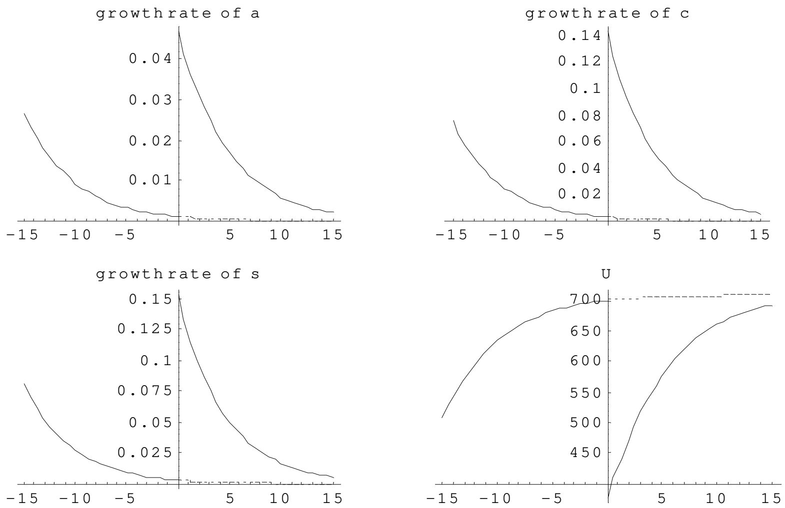

Figure 4.7: Capital depreciation of 20% in $t = 0$: growth rates of assets, consumption and environmental expenditures, and utility level in a planned economy

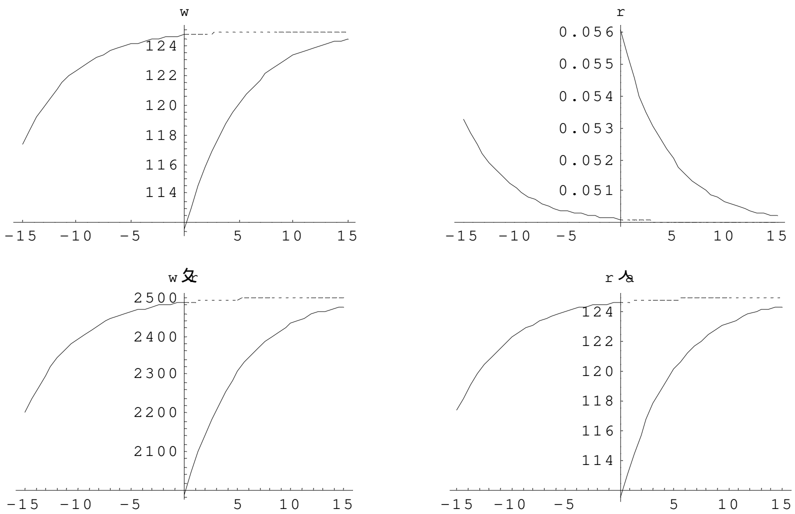

Figure 4.8: Capital depreciation of 20% in $t = 0$: wage rate, interest rate, relation between wage and interest rate and capital income in a planned economy

In comparison with the unregulated market the planned economy approaches its equilibrium values faster. Consequently, the growth rates of assets and related variables (consumption, production etc.) are higher in $t = 0$. The solution of the planner differs from the market solution reached in an unregulated economy especially in that the decrease of consumption and environmental expenditures are more alike.

5 Double Impact: Productivity Shock and Sudden Capital Depreciation

In this section we analyze the combined effect of productivity shocks of different sizes and a simultaneous depreciation of capital. Overnight, 20% of the existing capital stock - which has reached 99.5% of its equilibrium value in $t = 0$ - disappear.[16] At the same time, productivity jumps to 102%, while an immediate adaptation of environmental policy accounts for that. Although the capital stock drops only from 2487.5 to 1990.0 (-20%), the effects on consumption (-60.777%) and environmental expenditures (-62.785%) are dramatic. Environmental quality falls to 43.336% of its pre-shock value. The utility level decreases to 40.213%. Table 5.1 illustrates the percentage change of various variables in dependence of the size of the productivity shock in $t = 0$.

Table 5.1: Percentage change of variables following a productivity shock and capital depreciation in t = 0 in the case of an optimal environmental policy

ΔA	Δc	Δs	ΔN	ΔU	Δw	Δr
+2	−60.777	−62.785	−56.664	−59.787	−8.768	+14.039
+5	−67.471	−69.452	−63.329	−66.473	−6.085	+17.394
+10	−76.497	−78.351	−72.491	−75.554	−1.613	+22.984

Although production and thus the household's income do not change very much, consumption and environmental expenditures are slashed.[17] The reason is a jump in the savings rate from 0.014% to 77.02% in the model with optimal constant tax and subsidy rates. Hence, the growth rate of the capital stock jumps from 0.142% to 6.583%, 7.742% and 9.414% for an increase of productivity of 2%, 5% and 10%, respectively. The consequence of such a rapid growth is that after 15 periods 99.564% (2%-shock), 99.452% (5%-shock) and 99.256% (10%-shock) of the new equilibrium capital stock are reached.

In contrast to the expenditures, the environmental quality as well as utility level, the wage and the interest rate increase with an increasing productivity. The wage rate depends on the productivity and the capital endowment. The productivity shock nearly compensates the whole impact of capital depreciation if it is sufficiently large. Nevertheless, consumption is low since a high interest rate provides incentives to save a large fraction of the income in order to build up the new equilibrium capital stock.

[16] Note that the equilibrium values do not change due to capital depreciation. They are given in Table 3.3, the percentage changes due to the shock in Table 3.5.

[17] Note that a distribution parameter $\delta = 0.5$ implies in our case $w = r \cdot a$; and consequently $\Delta w = \Delta (r \cdot a) = \Delta y$.

The following figures display the trajectories of various variables for a productivity shock of +2%.

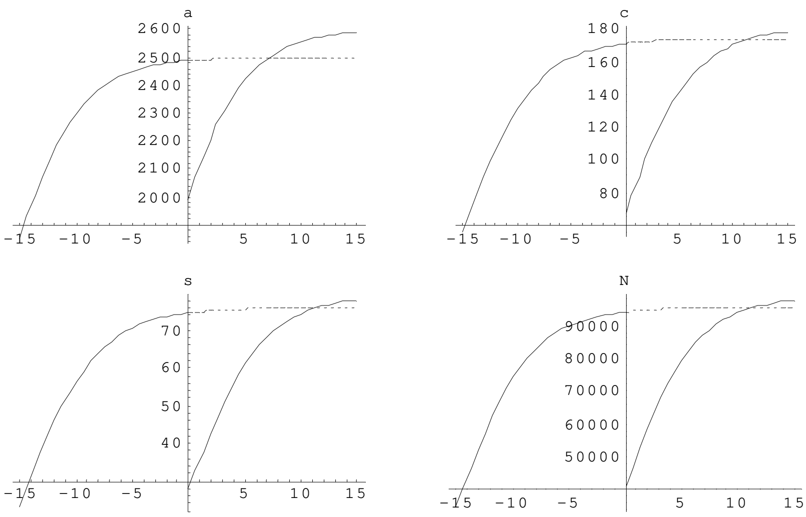

Figure 5.1: Productivity shock of 2% and capital depreciation of 20% in $t = 0$: assets, consumption, environmental expenditures and environmental quality with an optimal policy

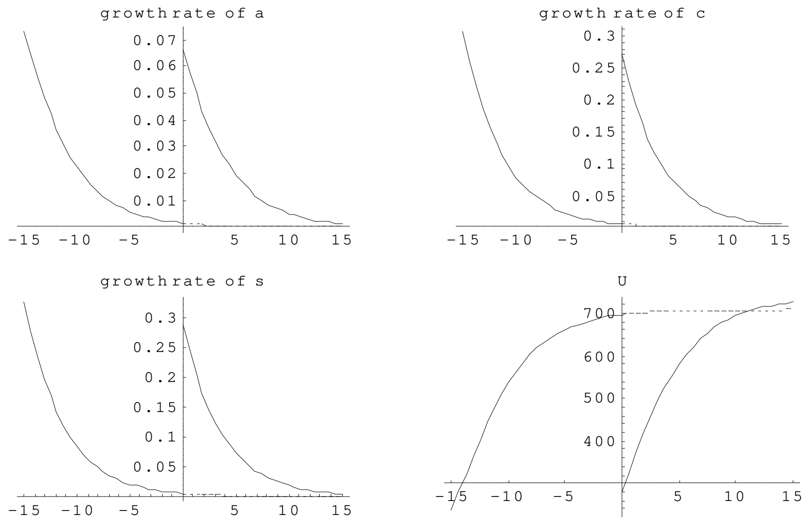

Figure 5.2: Productivity shock of 2% and capital depreciation of 20% in $t = 0$: growth rates of assets, consumption and environmental expenditures, and utility level with anoptimal policy

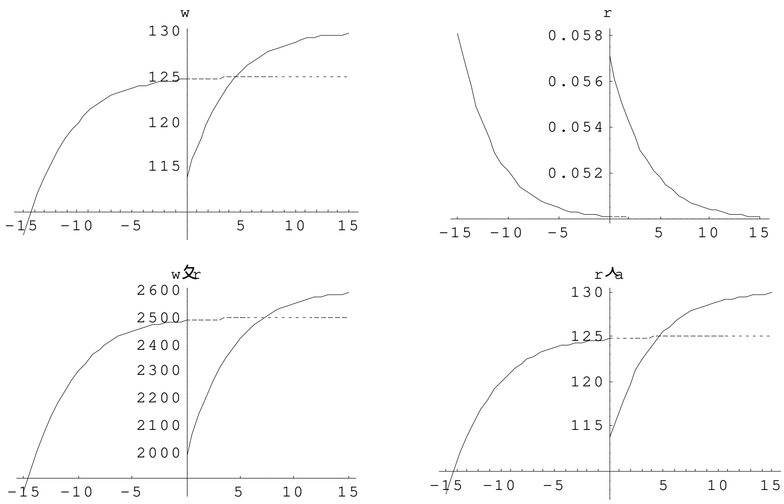

Figure 5.3: Productivity shock of 2% and capital depreciation of 20% in $t = 0$: wage rate, interest rate, relation between wage and interest rate and capital income with an optimal policy

As can be seen in Figure 5.4, the government's budget is again nearly balanced. Over 30 periods, the budget surplus is 19.578, 21.473 and 24.399 for shocks of 2%, 5% and 10%, respectively. This is roughly above 1% of the tax revenue.

Table 5.2: Tax revenues, subsidies and budget cash flow over time and change after a productivity shock and capital depreciation in absolute values

$\Delta A\|_{t=0}$	$\int_{-15}^{15} T$	$\int_{-15}^{15} S$	$\int_{-15}^{15} B$	$\Delta T\|_{t=0}$	$\Delta S\|_{t=0}$	$\Delta B\|_{t=0}$
2%	1870.48	1850.90	19.578	−45.298	−46.837	1.539
5%	1886.66	1865.19	21.473	−50.207	−51.811	1.604
10%	1913.18	1888.78	24.399	−56.875	−58.449	1.574

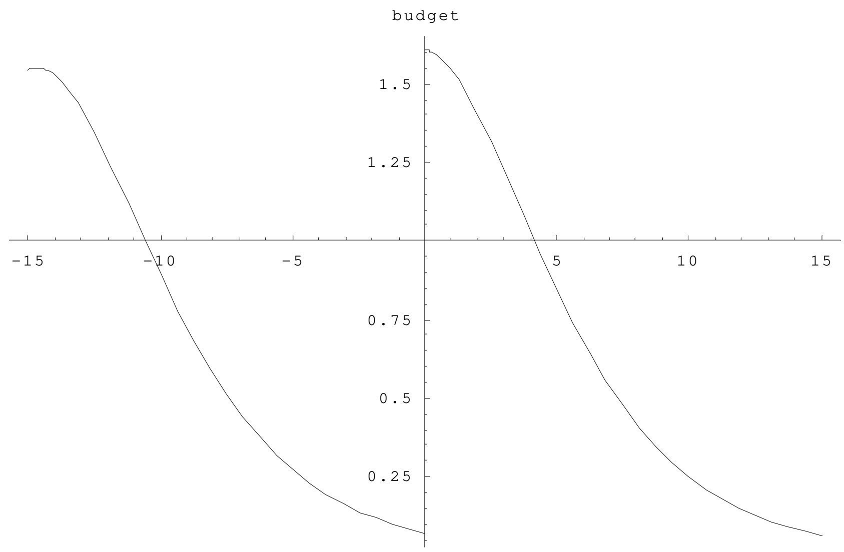

Figure 5.4: Productivity shock of 2% and capital depreciation of 20% in $t = 0$: budget cash flow

Figure 5.5 illustrates the change of the budget cash flow after a 10% productivity shock. Although the relative increase to the new steady state level of environmental expenditures (+22.167%) is higher than the increase in consumption (+20.490%) , the budget surplus increases even more in the first periods after the shock. But in $t = 0$ the change of environmental expenditures exceeds the change of consumption (see Table 5.1). As we could expect this is accompanied by a larger jump in the growth rate of environmental expenditures. The non-monotonic change of the budget cash flow over time reflects these effects and a contrary one: Initially, the budget surplus increases due to the fact that the tax base is reduced less than the subsidized expenditures. Later the higher growth rates of environmental expenditures result in a decrease of the budget surplus. Eventually, the budget is balanced in the long run.

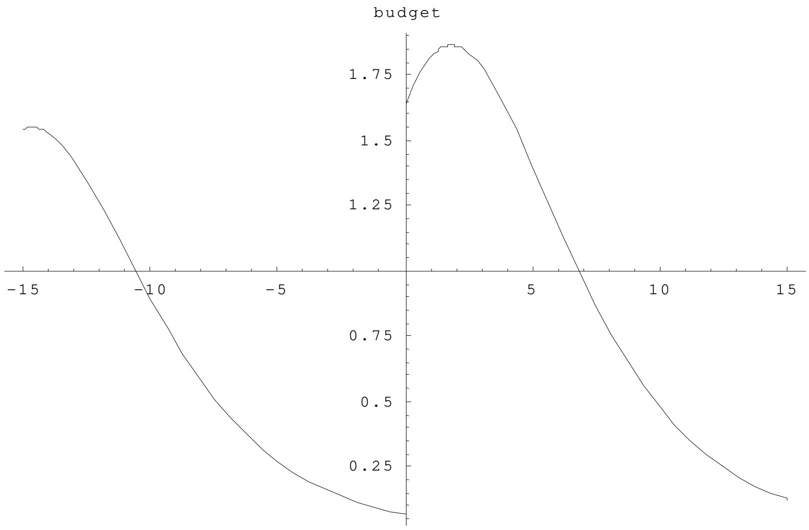

Figure 5.5: Productivity shock of 10% and capital depreciation of 20% in $t = 0$: budget cash flow

Again we can compare the solution with the results derived for a planned economy. Table 5.3 gives the change of variables in $t = 0$.

Table 5.3: Percentage change of variables following a productivity shock and capital depreciation in t = 0 in a planned economy

ΔA	Δc	Δs	ΔN	ΔU	Δw	Δr
+2	−50.934	−53.454	−47.844	−50.179	−8.768	+14.039
+5	−56.767	−59.378	−53.533	−55.980	−6.085	+17.394
+10	−65.255	−67.899	−61.928	−64.452	−1.613	+22.984

As in previous models, the drop of expenditures is slightly less dramatic than in an economy with constant optimal tax and subsidy rates (see Table 5.1). The explanation is again the too high tax rate given the reduced capital stock and thus reduced production. Wage and interest rates change in the same way as in the model with an optimal environmental policy. The reason is that both variables depend only on the capital endowment, productivity and exogenous parameters. These variables are constant or change initially irrespective of the type of regulation.

6 As Time Goes By: Delayed Adjustment of Environmental Policy

In the previously discussed models the government was able to adjust immediately tax and subsidy rates to changes of exogenous variables. This is only plausible if the shock is anticipated by the government - but not by the other economic agents[18] - or if the governmental power is exerted by a benevolent dictator who can act autonomously and therefore without any delay. In real life, however, adjustment is a bit more complicated and takes time.

Here we analyze the consequences of a delay of three periods in models with productivity shocks and with joint productivity shocks and capital depreciation. For illustrative purposes we also test in selected models the consequences of a delay of ten periods - which seems rather unrealistic - and compare the results with those of models with the faster reaction of the government.[19] As in previous sections, in $t = 0$ a productivity shock changed the overall productivity to 102%, 105% or 110% of the base level. In a second set this productivity shock is accompanied by a sudden depreciation of the capital stock.[20] The capital stock that reached 99.5% of its equilibrium level of $a^* = 2500$ dropped to 80% ($a_{t-0} = 2487.5$, $a_{t+0} = 1990$). In both sets of models households and firms adjust their behavior to changed exogenous variables immediately. However, due to the time consuming legislative or administrative processes the government reacts with a delay of three periods. Subsequently households and firms adjust their behavior again - now to the new tax and subsidy rates. For a productivity shock of +10% with or without capital depreciation we also calculate the consequences of a delay of ten periods as an "extreme" scenario.

The delay has no influence on the long run equilibrium values of variables.[21] It affects the speed of convergence - here simply defined as the time necessary to approach a certain percentage of the equilibrium capital stock or the percentage of the equilibrium capital stock reached in a specified term. Additionally, the welfare of households is influenced. Here, the long-run optimal path is given by the first-order conditions with a combination of taxes and subsidies that guarantees a socially optimal expenditure decision of households. But in the short run deviations from this path can result in higher welfare.[22] Therefore, the comparison of welfare in defined terms can shed light on the incentives to adjust policy variables quickly or to react "dozily". It should be pointed out here that the necessary change in tax and subsidy rates is rather small (see Section 3.3).

[18] Anticipated shocks would not cause jumps in consumption given standard utility functions; individuals would try to smooth the time path of utility and consequently of consumption.

[19] A change of tax rates is - as compared with a change of the tax system - not so difficult, therefore three periods is an ample term.

[20] It does not make sense to look at a delay in the case of a capital depreciation without productivity shock because in our model capital depreciation alone does not require a change of the tax or subsidy rates. The impact of capital depreciation is discussed in Section 4.

[21] See tables 3.3 and 3.5.

[22] This is not a big surprise. It is simply a fact of every-day-politics that necessary adjustments of policy instruments are postponed because they cause harms today, even if they bring benefits tomorrow.

6.1 The Case of a Productivity Shock

The following figures illustrate the time path of various variables after a productivity shock of 10% when the government reacts with a delay of three periods. The short dashes indicate the hypothetical path without a shock, the very short dashes the path without a government reaction, i.e., without adjustment of the tax and subsidy rates.

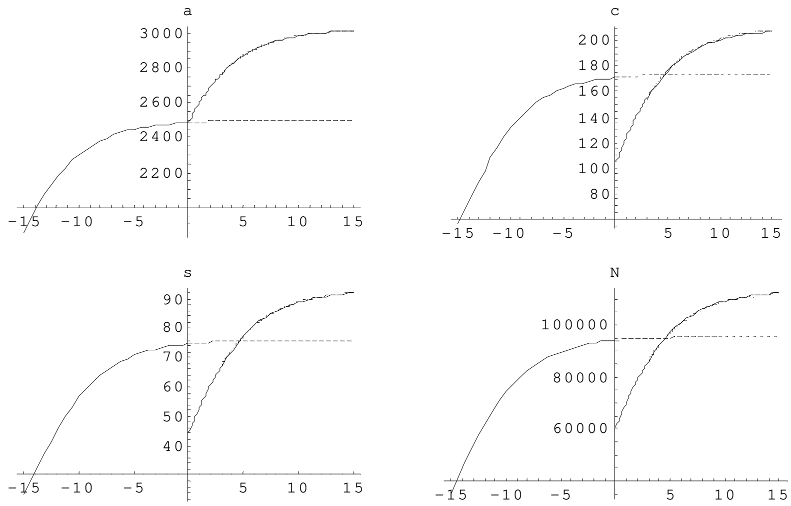

Figure 6.1: Productivity shock of 10% in $t = 0$ and 3-period delay in policy response: assets, consumption, environmental expenditures and environmental quality with an optimal policy

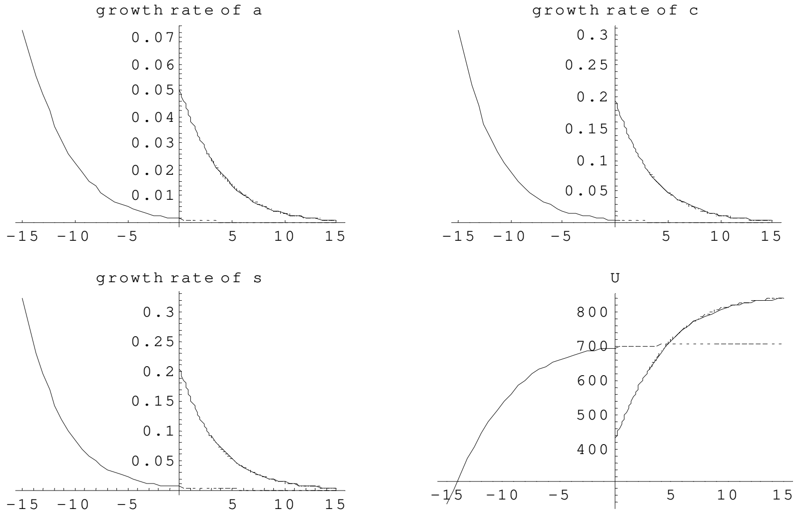

Figure 6.2: Productivity shock of 10% in $t = 0$ and 3-period delay in policy response: growth rate of assets, consumption and environmental expenditures, and utility level with an optimal policy

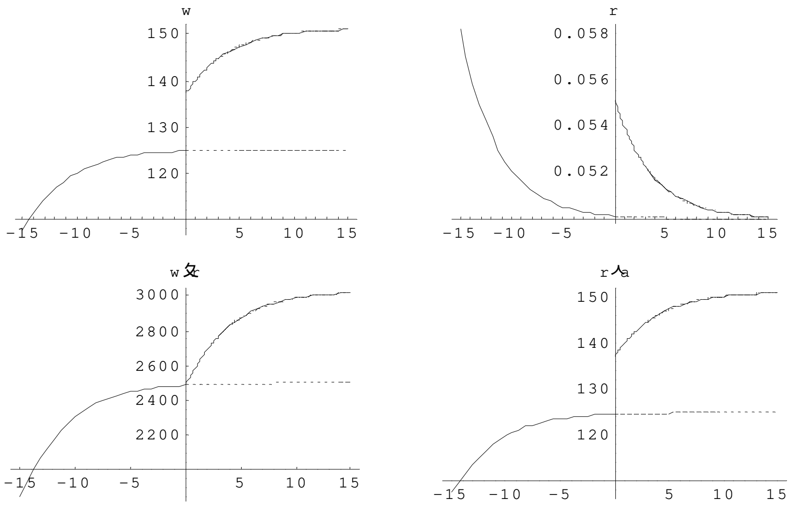

Figure 6.3: Productivity shock of 10% in $t = 0$ and 3-period delay in policy response: wage rate, interest rate, relation between wage and interest rate, and capital income with an optimal policy

At first glance, the trajectories after a policy adjustment do not differ very much from the hypothetical trajectories without adjusted policy instruments. The values of control and stock variables for points in time $t \geq 5$ of the models with lagged adjustment do not differ substantially from the values with immediate adjustment. The equilibrium utility level after a shock with unadjusted and with adjusted tax and subsidy rates is nearly the same; adjustment increases the equilibrium utility level by 0.00000714%, 0.0000438% and 0.0001701% for productivity shocks of 2%, 5% and 10%, respectively. But there is a more measurable difference in the expenditure structure: with adjusted policy instruments, consumption decreases and environmental quality increases slightly. Table 6.1 illustrates the differences.

Table 6.1: Change of equilibrium variables due to adjustment as percentage of values without adjustment

ΔA	c	s	N
+2%	−0.0213	+0.049	+0.064
+5%	−0.0526	+0.120	+0.158
+10%	−0.1036	+0.235	+0.312

Although the equilibrium is nearly unaffected, there is a qualitative difference in the path. The productivity shock increases the optimal capital stock and therefore the equilibrium consumption. The increased consumption is accompanied by increased pollution. Therefore, the tax and subsidy rate should increase, too. In $t = 0$, consumption is in the adjusted model lower than in the model with a delay. Table 6.2 delivers a comparison of the initial values of both types of models after the shock.

Table 6.2: Variables with adjustment as percentage of values without adjustment after a productivity shock in $\mathbf{t = 0}$

ΔA	c	s	N	U
+2%	99.912	99.979	100.004	99.936
+5%	99.784	99.950	100.011	99.841
+10%	99.574	99.904	100.021	99.685

Abstaining from an immediate adjustment of the tax and subsidy rates slightly increases the initial values of consumption and the utility level after the shock. Integrated over 15 periods after the shock, the aggregated utility reaches with immediate adjustment 99.988%, 99.973% and 99.954% of the aggregated utility with lagged adjustment for a productivity shock of 2%, 5% and 10%, respectively. That implies gains from postponing adjustment of 0.012%, 0.027% and 0.046% of the total utility over 15 periods. The values seem to be very small. But the aggregated total utility gain over that period due to the productivity shock is - with a lagged policy response - only 0.828%, 2.059% and 4.073% (for a productivity shock of 2%, 5% and 10%, respectively). Compared with the total utility gain over 15 periods, the difference between initial values after the shock is not completely negligible. At least there is a small incentive to postpone necessary changes in the policy variables. Nevertheless, compared with the absolute utility loss right after the shock, the actions of the government have hardly any impact.[23] The following table illustrates the relative size of variable changes due to the change of the tax rate. Note that the income effect of the tax results in decreasing expenditures for consumption and the environment. However, the substitution effect gives rise to an increasing environmental quality since the cutback of consumption expenditures is relatively larger. In addition we have the subsidy on environmental expenditures.[24] Above all the immediate utility loss due to a change of the tax rate is relatively large compared to the gains in the long run.[25] An only slightly myopic government will therefore do nothing.

Table 6.3: Percentage change of variables due to a change of tax and subsidy rates in t = 3 in the case of a productivity shock in t = 0

$\Delta A\|_{t=3}$	$\Delta c\|_{t=3}$	$\Delta s\|_{t=3}$	$\Delta N\|_{t=3}$	$\Delta U\|_{t=3}$
+2%	−0.0866	−0.0205	+0.0050	−0.0639
+5%	−0.2151	−0.0506	+0.0124	−0.1583
+10%	−0.4228	−0.0980	+0.0249	−0.3110

A look at the government's budget verifies this overall impression. Tax revenue and subsidy expenditures are similar to the values in models discussed in Section 3.[26] The main difference is the second point of discontinuity at $t = 3$ (see Figure 6.4). At this point the tax revenue increases and subsidy expenditures decrease. The tax revenue changes due to the increase of the tax rate. This compensates the decrease of the tax base (since consumption decreases at about 0.423%). Although the subsidy rate increases subsidy expenditures decrease simply because private environmental expenditures are reduced. Therefore, the budget cash flow increases a second time.

Table 6.4: Change of tax revenues, subsidies and budget cash flow after a productivity shock in absolute values in t = 0 and after adjustment of the tax and subsidy rates

$\Delta A\|_{t=0}$	$\Delta T\|_{t=0}$	$\Delta T\|_{t=3}$	$\Delta S\|_{t=0}$	$\Delta S\|_{t=3}$	$\Delta B\|_{t=0}$	$\Delta B\|_{t=3}$
2%	−6.768	+0.148	−7.120	−0.015	+0.351	+0.163
5%	−16.021	+0.384	−16.788	−0.036	+0.768	+0.390
10%	−29.218	+0.652	−30.435	−0.066	+1.216	+0.718

Again the budget cash flow is small and approaches zero in the long run. In $t = 15$ it is 0.0094, 0.0238 and 0.0523 for a productivity shock of 2%, 5% and 10%, respectively.

[23] Compare the values in Table 3.4.

[24] Note that the decrease in expenditures corresponds to the positive budget cash flow in Figure 6.4.

[25] See Section 3.3.

[26] See Table 3.7.

Table 6.5: Tax revenues, subsidies and budget cash flow over time after a productivity shock in t = 0 in absolute values

$\Delta A\|_{t=0}$	$\int_{-15}^{15} T$	$\int_{-15}^{15} S$	$\int_{-15}^{15} B$
2%	2050.48	2038.84	11.637
5%	2068.76	2055.01	13.746
10%	2099.24	2082.09	17.162

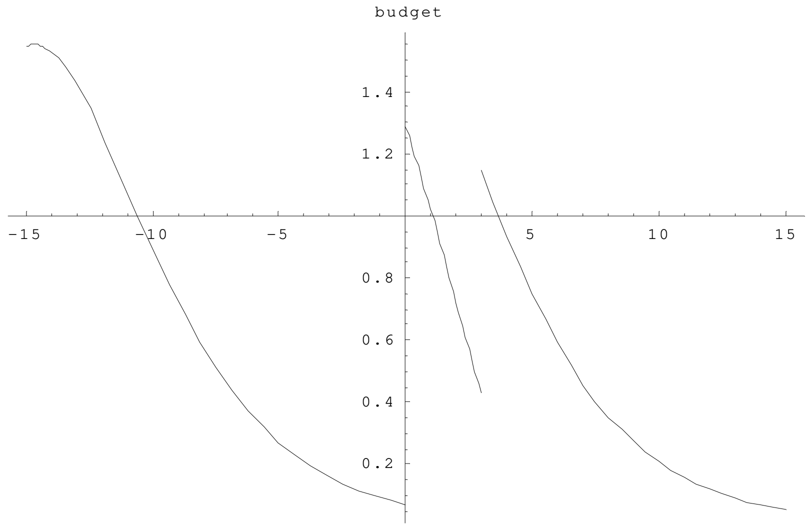

Figure 6.4: Productivity shock of 10% in $t = 0$ and 3-period delay in policy response: budget cash flow

To verify the impression we calculate an "extreme" scenario with a shock of 10% and a policy response delay of ten periods. Again, the aggregated utility levels are lower if the optimal tax and subsidy rates are introduced immediately after the shock. A simple explanation is that the strong and fast action of the government creates an overshooting effect in $t = 0$. Compared with the *actual* external effect, the tax rate at this point is just too high. An optimal path is characterized by a smooth adjustment of tax and subsidy rates. But this is a very unlikely scenario since its transaction costs are prohibitively high. Overall, the impact of the way and speed of adjustment on welfare is - compared with the effect of the introduction of environmental policy - in the present model astonishingly small.

6.2 The Case of a Productivity Shock and Capital Depreciation

As in the previous set of models, the changes of equilibrium variables due to the adjustment of the policy instruments are very small. The percentage changes of equilibrium variables due to changes of tax and subsidy rates are equal to the values given in Table 6.1 since equilibrium values are unaffected by the depreciation of capital.

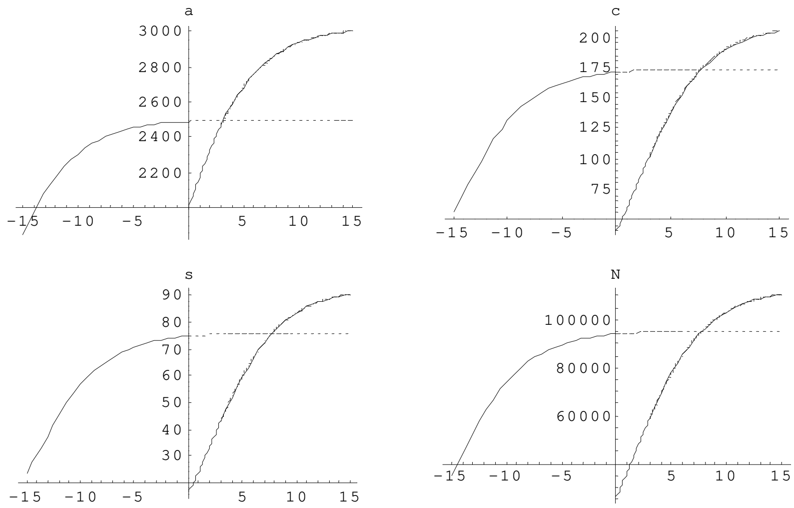

Figure 6.5: Productivity shock of 10% and capital depreciation of 20% in $t = 0$ and 3-period delay in policy response: assets, consumption, environmental expenditures and environmental quality with an optimal policy

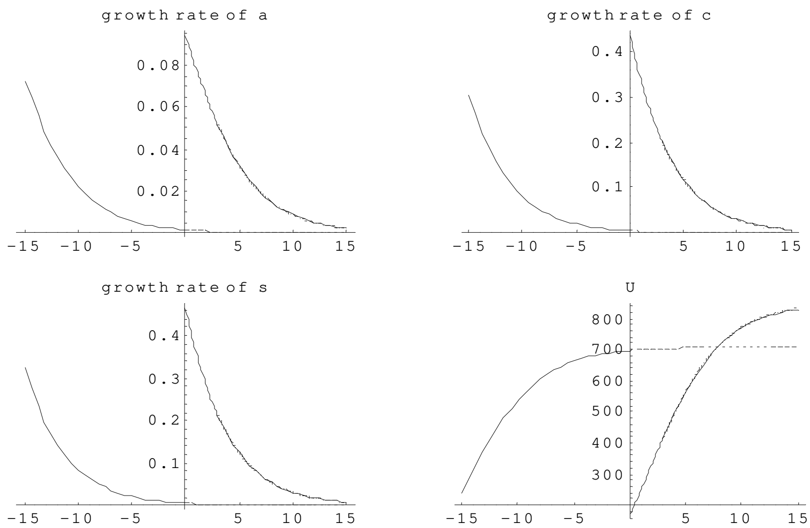

Figure 6.6: Productivity shock of 10% and capital depreciation of 20% in $t = 0$ and 3-period delay in policy response: growth rate of assets, consumption and environmental expenditures, and utility level with an optimal policy

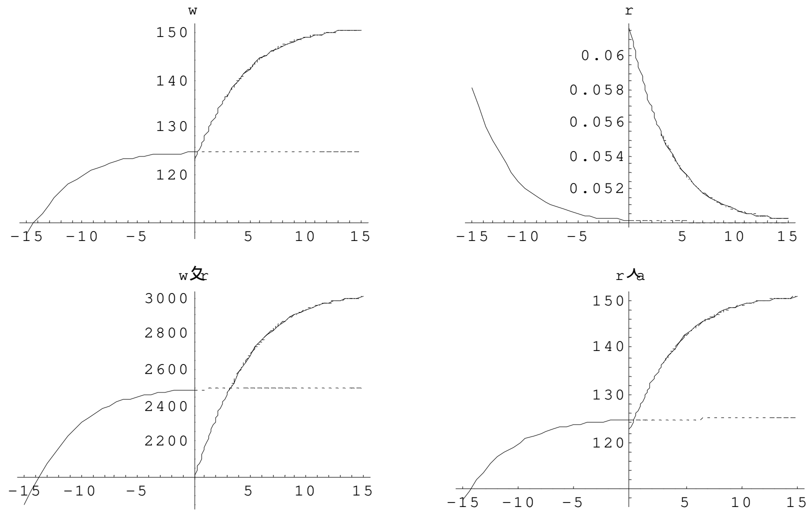

Figure 6.7: Productivity shock of 10% and capital depreciation of 20% in $t = 0$ and 3-period delay in policy response: wage rate, interest rate, relation between wage and interest rate, and capital income with an optimal policy

Again, we can compare the variables in the case of an immediate adjustment with the case of a delay in the political response. As can be seen in Table 6.6 the values are very similar to the case of the mere productivity shock.

Table 6.6: Variables with adjustment as percentage of values without adjustment after productivity shock in t = 0

ΔA	c	s	N	U
+2%	99.911	99.884	100.003	99.934
+5%	99.780	99.953	100.007	99.837
+10%	99.565	99.910	100.016	99.677

Integrated over 15 periods the utility gains from postponing changes of tax and subsidy rates are here 0.0082%, 0.0180% and 0.0289% for productivity shocks of 2%, 5% and 10%, respectively. Hence, a sudden depreciation of the capital stock slightly reduces the incentives to abstain from necessary changes of the tax and subsidy rates.

Table 6.7: Percentage change of variables due to a change of tax and subsidy rates in t = 3 in the case of a productivity shock and capital depreciation in t = 0

$\Delta A\|_{t=3}$	$\Delta c\|_{t=3}$	$\Delta s\|_{t=3}$	$\Delta N\|_{t=3}$	$\Delta U\|_{t=3}$
+2%	−0.0877	−0.0201	+0.0040	−0.0648
+5%	−0.2170	−0.0494	+0.0104	−0.1603
+10%	−0.4269	−0.0951	+0.0204	−0.3152

Once again, the income effect of the tax results in decreasing expenditures for all purposes, but the substitution effect and the subsidy imply an increasing environmental quality due to a change in the expenditure structure. The pattern of the changes of tax revenues, subsidies and budget cash flow is similar to the pattern in the model without capital depreciation (see Table 6.4) with the exception of an increased magnitude of all changes in $t = 0$ due to the more severe cuts in the individuals' expenditures.

Table 6.8: Change of tax revenues, subsidies and budget cash flow after a productivity shock and capital depreciation in t = 0 in absolute values and after adjustment of the tax and subsidy rates

$\Delta A\|_{t=0}$	$\Delta T\|_{t=0}$	$\Delta T\|_{t=3}$	$\Delta S\|_{t=0}$	$\Delta S\|_{t=3}$	$\Delta B\|_{t=0}$	$\Delta B\|_{t=3}$
2%	−45.356	+0.102	−46.831	−0.010	+1.476	+0.112
5%	−50.326	+0.239	−51.800	−0.023	+1.474	+0.262
10%	−57.043	+0.424	−58.434	−0.041	+1.392	+0.464

Compared with the model without capital depreciation, in $t = 0$ the tax revenues and expenditures for subsidies over time are lower. This is caused by the cutback in individual expenditures following a capital depreciation in $t = 0$. Again, the budget cash flow approaches zero in the long run; in $t = 15$ it is 0.0633, 0.0843 and 0.1256 for a productivity shock of 2%, 5% and 10%, respectively.

Table 6.9: Tax revenues, subsidies and budget cash flow over time after a productivity shock and capital depreciation in t = 0 in absolute values

$\Delta A\|_{t=0}$	$\int_{-15}^{15} T$	$\int_{-15}^{15} S$	$\int_{-15}^{15} B$
2%	1870.24	1850.92	19.312
5%	1886.12	1865.23	20.883
10%	1912.29	1888.85	23.441

Figure 6.8 illustrates the budget cash flow in the case of a productivity shock of 10% and a capital depreciation of 20% in $t = 0$. Similar to Figure 5.5, the budget surplus increases in the first periods after $t = 0$. In $t = 3$ the second discontinuity caused by the adjustment of the tax and subsidy rates is visible.

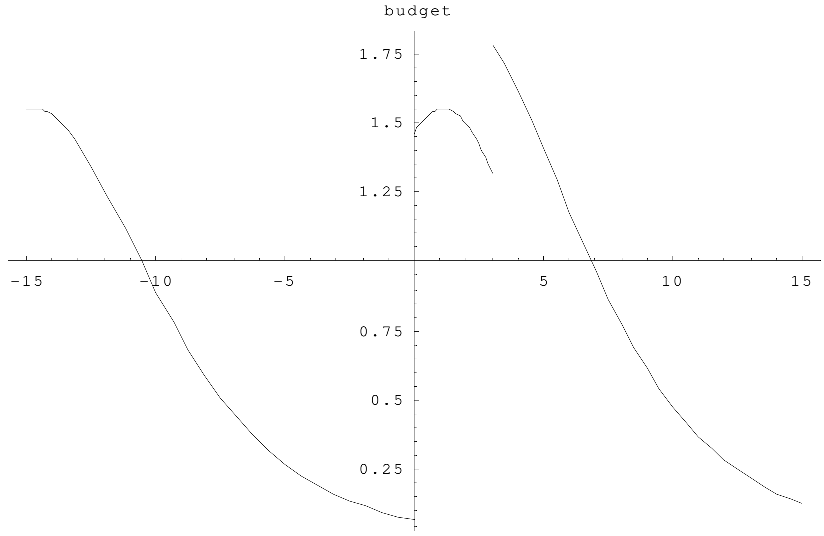

Figure 6.8: Productivity shock of 10% and capital depreciation in $t = 0$ and 3-period delay in the policy response: budget cash flow

7 Summary

We have examined the impact of productivity shocks and a sudden capital depreciation in a dynamic model in which pollution is modeled as a side effect of consumption. As reference points we have calculated the planner's solution and the outcome of an unregulated economy. The government takes care of the externalities by using two instruments. The negative externality caused by pollution is internalized by a tax on consumption, the positive externality generated by environmental expenditures is internalized via a subsidy on these expenditures. To keep things simple - and transaction costs low - we have assumed that tax and subsidy rates are fixed at the optimal level: the level that guarantees an optimal expenditure structure and a balanced government budget in the long-run equilibrium.[27] Additionally we have analyzed the consequences of delays in the response of the government after exogenous shocks - jumps in the productivity and sudden capital depreciation.

A productivity shock results in an increased equilibrium asset level. Consequently, consumption and environmental expenditures - and as a result environmental quality itself - decrease initially to provide resources to elevate the actual capital stock. The planned economy differs from the unregulated market and from an economy regulated by an optimal policy as described above. With an optimal policy, the decline of consumption and as a consequence the fall of the utility level is most drastic. Simultaneously the speed of convergence to the

[27] Deviations from optimal tax and subsidy rates are discussed in Barthel (2005).

new equilibrium is the highest. An unregulated economy reacts with the lowest decline of consumption and hence needs the longest term to converge to the new steady state. The initial change of variables in a planned economy is more prominent than in the market solution since the planner takes positive externalities of a reduced consumption on other households into account. The regime of constant tax and subsidy rates causes an even stronger shift of the variable values due to the fact that with a reduced output - and therefore consumption - the tax is too high. But a permanent adjustment of tax and subsidy rates would create prohibitively high transaction costs. Consequently we stick to this part of the model.

Sudden capital depreciation of a certain dimension results in a much more severe fall of consumption and environmental expenditures, and as a consequence of the utility level. Again, the reduction of consumption and, as a result, the speed of convergence is higher in the model with an optimal environmental policy. In an unregulated economy, the drop of the expenditures for environmental quality is unequal to that of consumption, whereas in a planned economy or with an optimal policy the reductions of both types of expenditures are very much alike.

In combination, a productivity shock and sudden capital depreciation have dramatic effects. In the case of an optimal environmental policy, a capital depreciation of 20% and a simultaneous productivity shock of 10% lead - although the wage rate decreases only by about 1.6% - to a drop of the utility level to 24.4%. At the same time, savings and interest rates as well as all growth rates jump up. Once again we can observe a higher speed of convergence in models with optimal policy than in a planned economy, caused by a more severe initial fall of expenditures due to a too high tax rate given the reduced consumption and therefore a higher savings rate.

An immediate adjustment of the tax and subsidy rates is not possible in the real world. Therefore, we have analyzed the consequences of a delay in the reaction of the government caused by the time necessary to change the tax and subsidy rates in a democratic system. For simplicity, we have assumed a delay of three periods (with a few exceptions). The delay does not influence the equilibrium values of variables but the trajectories towards these equilibria.

The first proposition is that in the models presented the necessary adjustments are very small, thus their impact will be rather small, too. Adjusting the regulative system has a very small influence on the utility level. In the long run, small gains due to a change in the tax and subsidy rates can be expected. But in the short run, there are incentives to abstain from adjustment. These incentives are small compared to the initial change in the utility level but not completely negligible if we take the size of the aggregated utility gain due to the productivity shock over 15 periods into account. Therefore an only slightly myopic government will not adjust the tax and subsidy rates, especially if this causes additional transaction costs.

If the productivity shock is accompanied by a sudden capital depreciation, the equilibrium values - and therefore the gains from adjustment of the regulation measure - do not change but so do the trajectories to the equilibrium. The utility gains from postponing the adjustment, aggregated over the 15 periods, are slightly reduced. Nevertheless, they remain positive.

Last but not least we have to emphasize that all models converge to a budget cash flow of zero in the long run. Productivity shocks, capital depreciation and also the adjustment of the tax system lead to discontinuities with a budget surplus.

The present model deals with - from the household's point of view - exogenous changes in the economic system. Since the household is the polluter in this setting[28] it would be interesting to look at changes in the household's preference structure. But this will be left open for future research.

[28] Here: in a physical sense. And, yes, I know the Coase theorem.

8 Appendix

8.1 Solution of the Household's Optimization Problem in the Basic Model

The Hamiltonian for the household i is:

$$J_H = U\left(c_i, N\left(E(S), P(C), \overline{N}\right), \phi\right) + \theta_{(a)} \cdot \left(r \cdot a + w - c_i - s_{(N)_i}\right) \tag{28}$$

The first-order conditions are:

1. $\frac{\partial J}{\partial c_i} = 0$
$$U_c + U_N \cdot N_P \cdot P_C = \theta_{(a)} \tag{29}$$

2. $\frac{\partial J}{\partial s_{(N)_i}} = 0$
$$U_N \cdot N_E \cdot E_S = \theta_{(a)} \tag{30}$$

3. $\frac{\partial J}{\partial a} = \rho \cdot \theta_{(a)} - \dot{\theta}_{(a)}$
$$\rho \cdot \theta_{(a)} - \dot{\theta}_{(a)} = \theta_{(a)} \cdot r \tag{31}$$

The transversality condition[29] is given by:

$$\lim_{t\to\infty}\left[\theta_{(a)} \cdot a\right] = 0$$

which is equivalent to:

$$\lim_{t\to\infty}\left[e^{-\rho \cdot t} \cdot a\right] = 0$$

From the Conditions (29) and (30) we can derive:

$$U_c = U_N \cdot (N_E \cdot E_S - N_P \cdot P_C)$$

After derivation of the Conditions (29) and (30) with respect to time follows:

$$\begin{aligned} \frac{\dot{\theta}_{(a)}}{\theta_{(a)}} &= \xi_1 \cdot \dot{c} + \xi_2 \cdot \dot{s} \\ &= \xi_3 \cdot \dot{c} + \xi_4 \cdot \dot{s} \end{aligned}$$

with

$$\begin{aligned} \xi_1 &\equiv \frac{\begin{array}{c} U_{cc} + U_N \cdot n \cdot \left(N_P \cdot P_{CC} + P_C^2 \cdot N_{PP}\right) \\ +U_{cN} \cdot P_C \cdot N_P \cdot (n+1) + U_{NN} \cdot P_C^2 \cdot N_P^2 \cdot n \end{array}}{U_N \cdot N_E \cdot E_S} \\ \xi_2 &\equiv \frac{((U_{NN} \cdot N_E \cdot N_P + U_N \cdot N_{EP}) \cdot P_C + U_{cN} \cdot N_E) \cdot n}{U_N \cdot N_E} \\ \xi_3 &\equiv \frac{(U_{NN} \cdot N_E \cdot N_P + U_N \cdot N_{EP}) \cdot P_C \cdot n + U_{cN} \cdot N_E}{U_N \cdot N_E} \\ \xi_4 &\equiv \frac{\left(U_N \cdot E_{SS} \cdot N_E + U_{NN} \cdot E_S^2 \cdot N_E^2 + U_N \cdot E_S^2 \cdot N_{EE}\right) \cdot n}{U_N \cdot N_E \cdot E_S} \end{aligned}$$

[29] See Barro and Sala-i-Martin (1995), p. 503-508.

Therefore, the control variables change according to:

$$\dot{c} = \frac{\xi_4 - \xi_2}{\xi_1 \cdot \xi_4 - \xi_3 \cdot \xi_2} \cdot (\rho - r) \tag{32}$$

$$\dot{s} = \frac{\xi_1 - \xi_3}{\xi_1 \cdot \xi_4 - \xi_2 \cdot \xi_3} \cdot (\rho - r) \tag{33}$$

8.2 Solution of the Planner's Optimization Problem

The Hamiltonian can now be written as:

$$J_P = U\left(c, N\left(E(S), P(C), \overline{N}\right), \phi\right) + \theta_{(a)} \cdot \left(f(a) - c - s_{(N)}\right) \tag{34}$$

The first-order conditions are:

1. $\frac{\partial J}{\partial c} = 0$

$$U_c + U_N \cdot N_P \cdot P_C \cdot n = \theta_{(a)} \tag{35}$$

2. $\frac{\partial J}{\partial s_{(N)}} = 0$

$$U_N \cdot N_E \cdot E_S \cdot n = \theta_{(a)} \tag{36}$$

3. $\frac{\partial J}{\partial a} = \rho \cdot \theta_{(a)} - \dot{\theta}_{(a)}$

$$\rho \cdot \theta_{(a)} - \dot{\theta}_{(a)} = \theta_{(a)} \cdot f_a \tag{37}$$

From Equations (35) and (36) follows:

$$U_c = U_N \cdot n \cdot (N_E \cdot E_S - N_P \cdot P_C)$$

After derivation of Conditions (35) and (36) with respect to time we arrive at:

$$\begin{aligned} \frac{\dot{\theta}_{(a)}}{\theta_{(a)}} &= \xi_1 \cdot \dot{c} + \xi_2 \cdot \dot{s} \\ &= \xi_2 \cdot \dot{c} + \xi_3 \cdot \dot{s} \end{aligned}$$

with:

$$\begin{aligned} \xi_1 &\equiv \frac{U_{cc} + \left[\begin{array}{c} U_N \cdot \left(N_P \cdot P_{CC} + P_C^2 \cdot N_{PP}\right) \\ +U_{NN} \cdot N_P^2 \cdot P_C^2 \end{array} \right] \cdot n^2 + 2 \cdot U_{cN} \cdot P_C \cdot N_P \cdot n}{U_N \cdot N_E \cdot E_S \cdot n} \\ \xi_2 &\equiv \frac{U_{cN} + U_{NN} \cdot N_P \cdot P_C \cdot n}{U_N} + \frac{N_{EP} \cdot P_C \cdot n}{N_E} \\ \xi_3 &\equiv \frac{U_{NN} \cdot N_E \cdot E_S \cdot n}{U_N} + \frac{N_{EE} \cdot E_S \cdot n}{N_E} + \frac{E_{SS} \cdot n}{E_S} \end{aligned}$$

It follows

$$\begin{aligned} \rho - f_a &= \xi_1 \cdot \dot{c} + \xi_2 \cdot \dot{s} \\ &= \xi_2 \cdot \dot{c} + \xi_3 \cdot \dot{s} \end{aligned}$$

Here, the growth rates are given by:

$$\dot{a} = f(a) - c - s_{(N)} \tag{38}$$

$$\dot{c} = \frac{\xi_2 - \xi_3}{\xi_2^2 - \xi_1 \cdot \xi_3} \cdot (\rho - f_a) \tag{39}$$

$$\dot{s} = \frac{\xi_2 - \xi_1}{\xi_2^2 - \xi_1 \cdot \xi_3} \cdot (\rho - f_a) \tag{40}$$

Using the assumed specifications the short-hand equations simplify to:

$$\xi_1 \equiv \frac{U_{cc} + \left(U_{NN} \cdot P_C^2 - U_N \cdot P_{CC}\right) \cdot n^2 - 2 \cdot U_{cN} \cdot P_C \cdot n}{U_N \cdot E_S \cdot n}$$

$$\xi_2 \equiv \frac{U_{cN} - U_{NN} \cdot P_C \cdot n}{U_N}$$

$$\xi_3 \equiv \frac{U_{NN} \cdot E_S^2 + U_N \cdot E_{SS}}{U_N \cdot E_S} \cdot n$$

8.3 Solution of the Household's Optimization Problem with Repayment of Tax Revenues as a Subsidy

The Hamiltonian for the household i is:

$$\begin{aligned} J_H = {} & U\left(c_i, N\left(E(S), P(C), \overline{N}\right), \phi\right) \\ & +\theta_{(a)} \cdot \left(r \cdot a_i + w_i - (1+d) \cdot c_i - (1-p) \cdot s_{(N)_i}\right) \end{aligned} \tag{41}$$

The first-order conditions are:

1. $\frac{\partial J}{\partial c_i} = 0$

$$U_c + U_N \cdot N_P \cdot P_C = \theta_{(a)} \cdot (1+d) \tag{42}$$

2. $\frac{\partial J}{\partial s_{(N)_i}} = 0$

$$U_N \cdot N_E \cdot E_S = \theta_{(a)} \cdot (1-p) \tag{43}$$

3. $\frac{\partial J}{\partial a} = \rho \cdot \theta_{(a)} - \dot{\theta}_{(a)}$

$$\rho \cdot \theta_{(a)} - \dot{\theta}_{(a)} = \theta_{(a)} \cdot r \tag{44}$$

Again, we can derive:

$$U_c = U_N \cdot \left(\frac{1+d}{1-p} \cdot N_E \cdot E_S - N_P \cdot P_C\right)$$

From the derivation of Conditions (42) and (43) with respect to time follows:

$$\begin{aligned} \frac{\dot{\theta}_{(a)}}{\theta_{(a)}} &= \xi_1 \cdot \dot{c} + \xi_2 \cdot \dot{s} + \xi_3 \cdot \dot{d} \\ &= \xi_4 \cdot \dot{c} + \xi_5 \cdot \dot{s} + \xi_6 \cdot \dot{p} \end{aligned}$$

with:

$$
\begin{aligned}
\xi_1 &\equiv \frac{\begin{array}{c} U_{cc} + U_N \cdot n \cdot \left(N_P \cdot P_{CC} + N_{PP} \cdot P_C^2\right) \\ + U_{NN} \cdot P_C^2 \cdot N_P^2 \cdot n + U_{cN} \cdot N_P \cdot P_C \cdot (n+1) \end{array}}{\frac{1+d}{1-p} \cdot U_N \cdot N_E \cdot E_S} \\
\xi_2 &\equiv \frac{((U_N \cdot N_{EP} + U_{NN} \cdot N_E \cdot N_P) \cdot P_C + U_{cN} \cdot N_E) \cdot n}{\frac{1+d}{1-p} \cdot U_N \cdot N_E} \\
\xi_3 &\equiv -\frac{1}{1+d} \\
\xi_4 &\equiv \frac{(U_N \cdot N_{EP} + U_{NN} \cdot N_E \cdot N_P) \cdot P_C \cdot n + U_{cN} \cdot N_E}{U_N \cdot N_E} \\
\xi_5 &\equiv \frac{\left(U_N \cdot N_E \cdot E_{SS} + U_{NN} \cdot N_E^2 \cdot E_S^2 + U_N \cdot N_{EE} \cdot E_S^2\right) \cdot n}{U_N \cdot N_E \cdot E_S} \\
\xi_6 &\equiv \frac{1}{1-p}
\end{aligned}
$$

This can be rewritten to:

$$
\begin{aligned}
\dot{c} &= \frac{(\xi_5 - \xi_2) \cdot (\rho - r) - \xi_3 \cdot \xi_5 \cdot \dot{d} + \xi_2 \cdot \xi_6 \cdot \dot{p}}{\xi_1 \cdot \xi_5 - \xi_4 \cdot \xi_2} \\
\dot{s} &= \frac{(\xi_1 - \xi_4) \cdot (\rho - r) + \xi_3 \cdot \xi_4 \cdot \dot{d} - \xi_1 \cdot \xi_6 \cdot \dot{p}}{\xi_1 \cdot \xi_5 - \xi_4 \cdot \xi_2}
\end{aligned}
$$

9 References

1. Aghion, Philippe, and Peter Howitt (1998): *Endogenous Growth Theory*, MIT Press, Cambridge (MA, USA) and London (UK).

2. Barro, Robert, and Xavier Sala-i-Martin (1995): *Economic Growth*, McGraw Hill, New York et al.

3. Barthel, Jens (2005): Environmental Policy in Dynamic Models: The Impact of the Elasticity of Substitution if Consumers Pollute, Humboldt-Universität zu Berlin, Discussion Paper - Economics Series 198.

4. Bohm, Peter, and Clifford S. Russell (1985): Comparative Analysis of Alternative Policy Instruments, In: Allan Kneese and James L. Sweeney (eds.), *Handbook of Natural Resource and Energy Economics*, North-Holland, 395-460.

5. Bovenberg, A. Lans, and Ruud A. de Mooij (1997): Environmental Tax Reform and Endogenous Growth, *Journal of Public Economics*, **63**, 207-237.

6. Bovenberg, A. Lans, and Sjak A. Smulders (1995): Environmental Quality and Pollution-augmenting Technical Change in a Two-sector Endogenous Growth Model, *Journal of Public Economics*, **57**, 369-391.

7. Bovenberg, A. Lans, and Sjak A. Smulders (1996): Transitional Impacts of Environmental Policy in an Endogenous Growth Model, *International Economic Review*, **37**, 861-891.

8. Brunner, Martin, and Holger Strulik (2002): Solution of Perfect Foresight Saddlepoint Problems: A Simple Method and Applications, *Journal of Economic Dynamics and Control*, **26**, 737-753.

9. Chiang, Alpha C. (1984): *Fundamental Methods of Mathematical Economics*, Third Edition, McGraw-Hill, New York et al.

10. Conrad, Klaus (1999): Computable General Equilibrium Models for Environmental Economics and Policy Analysis, in: Jeroen C. J. M. van den Bergh (ed.), *The Handbook of Environmental and Resource Economics*, Edward Elgar Publishing Comp., Cheltenham (UK).

11. Ferguson, Brian, and G. C. Lim (1998): *Introduction to Dynamic Economic Models*, Manchester University Press, Manchester and New York.

12. Forster, Bruce A. (1973): Optimal Capital Accumulation in a Polluted Environment, *Southern Economic Journal*, **39**, 544-547.

13. Gradus, Raymond, and Sjak A. Smulders (1993): The Trade-off Between Environmental Care and Long-term Growth - Pollution in Three Prototype Growth Models, *Journal of Economics / Zeitschrift für Nationalökonomie*, **58**, 25-51.

14. Huang, Chung-Huang, and Deqin Cai (1994): Constant Returns Endogenous Growth with Pollution Control, *Environmental and Resource Economics*, **4**, 383-400.

15. Lighthart, Jenny E., and Frederick van der Ploeg (1994): Pollution, the Cost of Public Funds and Endogenous Growth, *Economics Letters*, **46**, 351-361.

16. Smulders, Sjak A., and Raymond Gradus (1996): Pollution Abatement and Long-term Growth, *European Journal of Political Economy*, **12**, 505-532.