Performance of Markov-Switching GARCH Model Forecasting Inflation Uncertainty

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Abstract

This paper seeks to uncover the non-linear characteristics of uncertainty underlying the US inflation rates over the period 1971-2015 within a regime-switching framework. Accordingly, we employ two variants of a Markov regime-switching GARCH model, one with normally distributed errors (MS-GARCH-N) and another with t-distributed errors (MS-GARCH-t), and compare their relative in-sample as well as out-of-sample performances with those of their standard single-regime counterparts. Consistent with the findings in existing studies, both of our regime-switching models are successful in identifying the year 1984 as the breakpoint in inflation volatility. Among other interesting results is a new finding that the process of switching to the low volatility regime started around April, 1979 and continued until mid 1983. This time frame is matched with the period of aggressive monetary policy changes implemented by the then Fed chairman Paul Volcker. As regards the performance in forecasting uncertainty, for shorter horizons spanning 1 to 5 months, MS-GARCH-N forecasts are found to outperform all other models whereas for 8 to 12-month ahead forecasts MS-GARCH-t appears superior.

Key words: Markov-switching GARCH, inflation uncertainty, forecasting.
JEL classification: E31, C01, C53.

1 Introduction

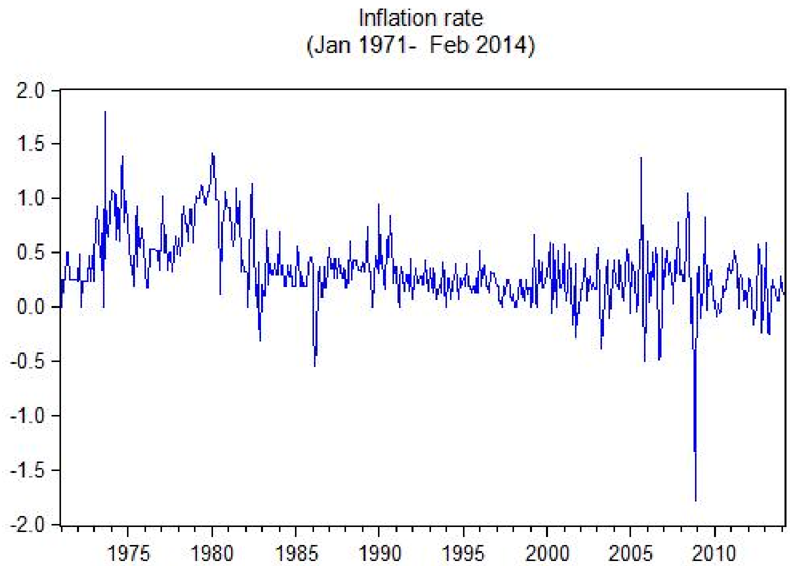
Uncertainties of various macroeconomic and financial variables have garnered special attention of both academic researchers and practitioners because of the nontrivial role they play in influencing policy making and financial market decisions. For example, during the period 1979-1982, the Federal Reserve switched from targeting interest rates to using nonborrowed reserves as a monetary policy tool which led to unprecedented interest rate volatility. This rise in volatility might have distorted the relationship between nominal interest rates and other explanatory variables which are important ingredients in the policy making process (Gray, 1996). Another example of a macroeconomic variable which is susceptible to uncertainty is exchange rate. Financial market exploits exchange rate's volatility to determine the price of currency options which in turn is used for risk management. It is not difficult to find other variables that the portfolio managers, option traders and market makers all are interested in forecasting to either increase profit or hedge against risk. Hence, the importance of an accurate estimation and forecast of volatility cannot be overstated.

Such forecasts typically hinge on the stylized facts that high frequency time series data exhibit time-varying volatility and volatility clustering. The latter means that volatility periods of similar magnitude tend to cluster together. To capture these features, the most commonly used model in the literature is GARCH (Generalized Autoregressive Conditional Heteroskedasticity) first introduced by Bollerslev (1986) who generalized the idea of ARCH (Autoregressive Conditional Heteroskedasticity) by Engle (1982). Although GARCH models produces a better fit than a constant variance model and also yields good volatility forecasts as maintained by Andersen and Bollerslev (1998), there is a caveat. As Gray (1996) has argued these models maybe misspecified due to the reason that the structural form of conditional means and variances is relatively inflexible. In other words, the models are held fixed throughout the entire sample period and thus ignore possible structural changes in mean and variances. The latter may lead to estimated high persistence of individual shocks resulting in high volatility persistence as shown by Lamoureux and Lastrapes (1990). This high volatility persistence may be the reason behind excessive GARCH forecasts in volatile periods. To solve this problem, researchers have recently generalized the GARCH model by allowing for multiple regimes with varying volatility levels. This is called the Markov-Switching GARCH (MS-GARCH) model.

The main objective of this paper is to examine the forecasting performance of a two-regime MS-GARCH model with respect to inflation uncertainty in U.S over the period January 1971- March 2015 using multiple statistical loss functions. Performances of two variants of an MS-GARCH model, one with normally distributed errors and another with t-distributed errors are juxtaposed with the performances of their standard non-regime switching counterparts. The existing literature so far has produced evidences on forecasting the volatility of exchange rates and stock returns using MS-GARCH. But surprisingly, the performance of MS-GARCH model forecasting inflation uncertainty has not been examined yet. It is important to put MS-GARCH to test to see how well it performs while forecasting inflation certainty for at least two reasons. First of all, it will shed light on the method's appropriate applicability in terms of forecasting. Second, inflation uncertainty is itself a very important macroeconomic variable which affects a society's welfare. It, in fact, was the first variable modeled using ARCH (Engle, 1982, 1983).

As far as the first reason is concerned, obviously the same model cannot be expected to be equally good in characterizing and forecasting different variables. Therefore, testing MS-GARCH's forecasting capability with respect to different variables will yield a better understanding of the method's usefulness. The appropriateness of the application of MS-GARCH to inflation uncertainty can be primarily ascertained by eyeballing the data on U.S. inflation rate from 1971 to early 2014 (Figure 1). It seems that inflation rate was very volatile from early 1970s to mid 1980s. After that it remained relatively stable until before 2006 which coincides with the onset of the recent financial crisis. Therefore, a casual observation of the data suggests that the U.S. inflation rate might be characterized by at two regimes: a high volatility regime and a low volatility regime. While a standard GARCH model is not capable of distinguishing between these two regimes an MS-GARCH model is better suited at this task.

Figure 1:



On the other hand, inflation uncertainty's being a variable of great interest to many parties is related to the general consensus that its future values are a major reason behind the welfare loss associated with inflation. Engle (1983) has argued that inflation uncertainty causes loss to risk averse economic agents even if the prices and quantities are perfectly flexible in all markets. It also distorts the efficiency of the current period's resource allocation decisions. In his Nobel lecture, Friedman (1977) has stressed that higher variability of inflation may even lead to decreased output, ceteris paribus. Inflation uncertainty's pervasive effect becomes specially more pronounced due to the use of nominal contracts. This is because future price level uncertainty induces risk premia for long-term contracts and increases costs for hedging against inflation. Hence, in order to minimize hedging cost and loss of wealth, it is important to be able to forecast inflation uncertainty as accurately as possible.

After modeling US inflation volatility using both the regime-switching and non-regime switching versions of the GARCH model, several key results emerge. The paper finds that US inflation volatility can be characterized by two regimes, high volatility and low volatility regimes. In the high volatility regime, shock persistence is lower compared to the low volatility regime. However, the immediate impact of an individual shock is higher in the high volatility regime. There is evidence that the main source of volatility clustering in the high volatility regime is the persistence of the regime itself, not the persistence of an individual inflationary shock. The paper also finds that the regime switch of inflation uncertainty took place in mid 1983. This result is consistent with the general agreement in the literature that there was a structural break around 1984. A related but a novel finding of this paper is that the process of the regime switch started much earlier around April, 1979. This date is very close to when Paul Volcker was nominated as the chairman of the Board of Governors of the Federal Reserve System on July, 1979. The regime switching process seemed to have coincided with the aggressive monetary policy changes implemented by the newly appointed Fed chairman.

As regards forecasting performances, this paper provides evidences that for a forecast horizon of 1 to 5 months, a Markov regime-switching GARCH model with normally distributed errors performs better than both standard GARCH models and a Markov regime-switching GARCH model with t distributed errors. However, for longer horizons such as 8 to 12 months, a Markov regime-switching GARCH model with t distributed errors outperforms all other models.

The contribution of this paper is mainly twofold. This is the first paper which models US inflation uncertainty within a Markov regime-switching GARCH framework and thus uncovers inflation uncertainty's underlying regime-dependent characteristics. It is also the first attempt in the literature at forecasting US inflation uncertainty. The organization of the paper is as follows. Section 2 discusses the existing relevant studies in the literature. Section 3 describes the data and the methodology used. Then section 4 discusses the results. Finally, section 5 concludes.

2 Literature review

This paper is concerned with two strands of the literature. The first is Markov-switching GARCH models and the second is inflation uncertainty. Cai (1994) and Hamilton and Susmel

(1994) are the first to extend the seminal idea of regime-switching parameters by (Hamilton, 1988, 1989) to an ARCH specification to control for possible structural breaks which may bias the estimates. However, the authors have argued that regime-switching GARCH models are intractable and impossible to estimate due to the dependence of the conditional variance on previous regime-dependent conditional variances. In other words, the conditional variance at time t depends on the entire sequence of regimes up to time $t-1$. Since the number of possible regime paths grows exponentially with t, an econometrician, who does not observe regimes, will have to deal with a large number of paths to t. This renders the estimation of the likelihood function constructed by integrating over all possible paths, intractable for large sample sizes.

To remove path-dependence, Gray (1996) first proposed the idea of aggregating conditional variances from the two regimes at each time step as he developed a generalized regime-switching model of the short-term interest rate. This single regime-aggregated conditional variance is then used as the input to compute the conditional variance at the next step. To be precise, Gray's specification involves formulating the conditional variance equation in the GARCH(1,1) model in a regime-switching framework in the following manner:

$$h_{it} = \alpha_{0i} + \alpha_{1i}\varepsilon_{t-1}^2 + \alpha_{2i}h_{t-1} \tag{1}$$

where h_{it} denotes conditional variance at period t in regime $i = (1, 2)$, and h_{t-1} is a state-independent average of past conditional variances. Gray (1996) makes use of the information observable at time $t-2$ to integrate out the unobserved regimes as follows:

$$\begin{aligned} h_{t-1} = E_{t-2}\{h_{it-1}\} = p_{1t-1}[\mu_{1t-1}^2 + h_{1t-1}] + (1-p_{1t-1})[\mu_{2t-1}^2 + h_{2t-1}] \\ - [p_{1t-1}\mu_{1t-1} + (1-p_{1t-1})\mu_{2t-1}]^2 \end{aligned} \tag{2}$$

where $p_{1t-1} = Pr(S_{t-1} = 1|I_{t-2})$ and I_{t-2} is the information available until time $t-1$. However, the main drawback of this model specification is that it is rather complicated to compute multi-period ahead volatility forecasts since this model does not make use of all the information. Dueker (1997) also estimated Markov-switching models to forecast stock market volatility by adopting Kim's (1994) collapsing procedure to avoid the path-dependence problem. The collapsing procedure involves treating the conditional variance as a function of at most the most recent M values of the state variable S. Similar to Gray's specification, this method essentially leads to not using all the information. To use more observable information when integrating out the previous regime, alternative to equation 2 Klaassen (2002) proposed the following specification for the conditional variance:

$$h_{t-1} = E_{t-1}\{h_{it-1}|s_t\} = \tilde{p}_{ii,t-1}[\mu_{it-1}^2 + h_{it-1}] + \tilde{p}_{ji,t-1}[\mu_{jt-1}^2[\mu_{t-1}^2 + h_{t-1}^j] - [\tilde{p}_{ii,t-1}\mu_{it-1} + \tilde{p}_{ji,t-1}\mu_{jt-1}]^2 \quad (3)$$

where

$$\tilde{p}_{ji,t-1} = Pr(s_{t-1} = j|s_t = i, I_{t-2}) = \frac{p_{ji}Pr(s_{t-1=j}|I_{t-2})}{Pr(s_t = i|I_{t-2})} = \frac{p_{ji}p_{jt-1}}{p_{it}} \quad (4)$$

with $i, j = 1, 2$ and p_{ji} is the transition probability of switching from state j in period $t-1$ to state i in period t i.e. $p_{ji} = Pr(s_t = i|s_{t-1} = j)$. Equation 3 makes the distinction between Gray's and Klaassen's specification clear. It shows that Klaassen (2002) takes the information from the current state, s_t into account while calculating the conditional probability of the previous state being in a particular regime whereas, Gray (1996) incorporates information observable only at period $t-2$. Klaassen (2002) has argued that if regimes are highly persistent, current regime provides useful information about the previous regime and this information should be incorporated in the probability calculation. Another advantage of Klaassen's method is it provides a straightforward expression for the multi-step ahead volatility forecasts that can be calculated recursively as in standard GARCH models (Marcucci, 2005).

The second strand of the literature that this paper contributes to, as mentioned above is concerned with the importance and measures of inflation uncertainty. A vast literature has extensively analyzed these specially in the context of the inflation uncertainty's possible dependence on inflation rate and its potential harmful effect on real economic activity. For example, with regards to the latter, on the theoretical side some authors have pointed out that inflation uncertainty reduces the rate of investment by hindering long-term contracts (see Fischer and Modigliani 1978), or by increasing the option value of delaying an irreversible investment (Pindyck, 1991). Contrasting results are reported by Dotsey and Sarte (2000) who using a cash-in-advance constraint in their model show that inflation uncertainty may increase investment through its impact on precautionary savings. Motivated by these theoretical suggestions, a number of studies have empirically examined the relationship between inflation and other macroeconomic variables. But a measure of uncertainty needs to be employed to carry out these investigations.

Early studies use unconditional volatility measures as a proxy for uncertainty; for example Fischer (1981) employs the moving standard deviation of inflation. However, such measures fail to capture inflation uncertainty which is actually the variance of the stochastic, or unpredictable component of inflation rate (Grier and Perry, 1998). To clarify this point, suppose that agents have very little information about inflation. In this case, they may deem the future as highly uncertain even though econometricians observe small ex post variability. If however, agents possess adequate information in advance, then there may be very little uncertainty associated with large change in actual inflation (Evans, 1991). Therefore, higher variability does not necessarily imply higher uncertainty. Rather, it will imply higher uncertainty only if agents do not possess the relevant information to predict part of the increased variability (Kontonikas, 2004).

The second type of measures of uncertainty that has been used in the literature is based on surveys for instance, Survey of Professional Forecasters (SPF). SPF is a quarterly survey of professional forecasters' views on key economic variables. Studies that have used survey data to construct inflation uncertainty include Barnea et al. (1979), Melvin (1982), Holland (1995), Lahiri and Sheng (2010) among others. Typically, survey based measures summarize the dispersion of forecasts of individual forecasters at a point in time (see Giordani and Söderlind 2003 for different types of uncertainty measures based on survey data). However, Grier and Perry (1998) has argued that these measures do not provide information about individual forecaster's uncertainty about their own forecasts. In a given time period, it is possible that each forecaster is extremely uncertain about inflation and yet submit very similar point estimates. This would lead to a significant underestimation of actual inflation uncertainty.

In contrast to these ad hoc measures of inflation uncertainty, GARCH provides a parametric technique to estimate a model of time-varying variance of stochastic innovations. This is a more sophisticated method than simply constructing a variability measure from past outcomes or from range of disagreement among individual forecasters at a point in time. With a view to examining the relationship between inflation and inflation uncertainty in the G7 countries., Grier and Perry (1998) employ an AR(12)-GARCH(1,1) model to estimate inflation uncertainty over the period 1948-1993. A similar study is conducted by Nas & Perry (2000) for Turkey which also measures inflation uncertainty using an ARMA-GARCH(1,1) model. In the context of the relationship between inflation uncertainty and real output, bi-variate GARCH models have been utilized to construct estimates of inflation uncertainty (see Grier et al., 2004; BREDIN and FOUNTAS, 2005; Fountas et al., 2006). However, none of these papers take into account structural shifts in their models which may ultimately lead to biased estimation of inflation uncertainty. This potential problem is partially addressed by Caporale et al. (2010) who employ an AR(k)-GARCH(1,1) model with time-varying parameters only in the mean equation to estimate inflation uncertainty. But they do not incorporate regime shifts in the conditional variance model, parameters of which too are susceptible to such shifts.

With a view to accounting for structural changes in both the conditional mean and variance equations, Chang and He (2010) have first applied a bi-variate Markov-switching ARCH model to analyze the relationship among inflation, inflation uncertainty and output growth using quarterly data from U.S. over the period 1960Q1-2003Q3. They have shown how allowing for possible regime switches culminates in uncovering effects or results that are either in contradiction with the conclusions from a single-regime GARCH model or are not captured by the latter at all. Nevertheless, to avoid the problem of path dependence this model omitted the potentially important GARCH term which could be used to parsimoniously

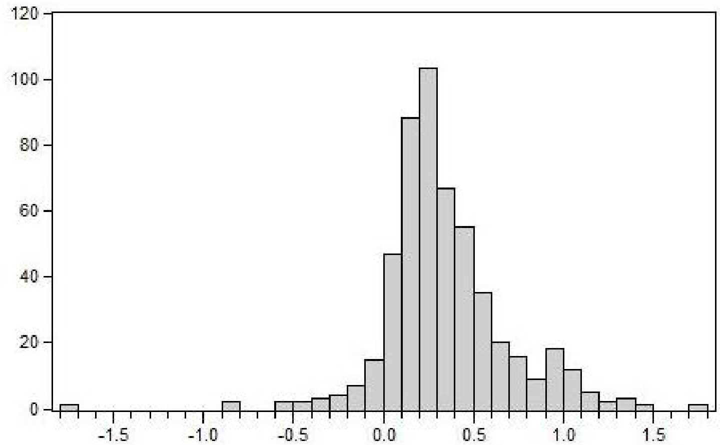


Figure 2: Histogram for monthly inflation rates from January 1, 1971 to February 1, 2014

represent a high-order ARCH process.

3 Data and methodology

This paper analyzes monthly U.S. inflation rates calculated as the differences in the log of monthly consumer price indices (CPI) collected from the Federal Reserve Economic Data (FRED). Monthly data has been chosen as opposed to quarterly ones since GARCH models are not well-suited for the latter ones. The sample period consists of two parts. The first part contains 518 observations from the period between January 1, 1971 and February 1, 2014. It is used for the purpose of in-sample estimation. The second part extends from March 1, 2014 to March 1, 2015 and is used for out-of-sample forecasting.

Figure 2 displays the histogram and Table 1 contains the descriptive statistics of the in-sample data. The mean inflation rate is small and around 0.34%. Both the histogram and the skewness coefficient suggest that U.S. monthly inflation rates are positively skewed. This implies that extreme positive inflation rates are more likely than extreme negative rates. However, the value of the skewness coefficient is not statistically significant at the 5% significance level.[1] On the other hand, positive excess kurtosis provides evidence of a fatter

[1]Skewness coefficient/Standard error of skewness $= 0.109/\sqrt{6/518} = 1.01$ which is between -2 and $+2$.

Table 1: Summary statistics of monthly inflation rates

Statistic	Estimate
Mean	0.34
Median	0.28
Maximum	1.79
Minimum	-1.78
Standard deviation	0.33
Skewness	0.109
Kurtosis	7.37
Jarque-Bera	414.83*

Note: Inflation rates are reported in percentage terms for the sample period January 1, 1971 to February 1, 2014. *P-value = 0.

right tail. This result is statistically significant at the 5% significance level.[2] Overall, there is a strong indication of a non-normal distribution of inflation rates which is confirmed by a statistically significant large value of Jarque-Bera statistic.

We estimate four different types of GARCH(1,1) models. The first two are standard GARCH models, one with normally distributed errors and another with t-distributed errors to capture the potential fat-tailed behavior of the empirical distribution of inflation rate. Since our main focus is on volatility forecasting, we make use of a simplified GARCH model consisting of a mean equation of the following AR(1) form:

$$\pi_t = \delta + \beta_1 \pi_{t-1} + \varepsilon_t \tag{5}$$

[2]Excess kurtosis/Standard error of kurtosis = $4.37/\sqrt{\frac{24}{518}} = 20.3 > 2$.

and a conditional volatility equation of the following form:

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 h_{t-1} \tag{6}$$

where $\alpha_0 > 0$, $\alpha_1 \geq 0$ and $\alpha_2 \geq 0$ to ensure a positive conditional variance. With a t-distribution, the probability density function of the innovations becomes:

$$f(\varepsilon_t) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi}\Gamma(\frac{\nu}{2})}(\nu-2)^{-\frac{1}{2}} h_t^{-\frac{1}{2}} \left[1 + \frac{\varepsilon_t^2}{h_t(\nu-2)}\right]^{-\frac{\nu+1}{2}} \tag{7}$$

The other two models are Markov-switching GARCH (MS-GARCH) models with two regimes, again one with normally distributed errors and another with t-distributed errors. We follow Klaassen's (2002) specification of MS-GARCH which consists of the following conditional mean equation along with equations 1, 3 and 4:

$$\pi_t = \delta_i + \beta_{1i}\pi_{t-1} + \eta_t \sqrt{h_{it}} \tag{8}$$

where $i = 1, 2$ and η_t is an i.i.d process with zero mean and unit variance. Because of the absence of serial correlation in the monthly inflation rates, the m-step ahead volatility forecast at time T-1 can be computed in the following manner:

$$\widehat{h}_{T,T+m} = \sum_{\tau}^{m} \widehat{h}_{T,T+\tau} = \sum_{\tau=1}^{m} \sum_{i=1}^{2} Pr(s_\tau = i | I_{\tau-1}) \widehat{h}_{iT,T+\tau} \tag{9}$$

where $\widehat{h}_{T,T+m}$ denotes the time aggregated volatility forecast for the next m steps calculated at time T, and $\widehat{h}_{iT,T+\tau}$ denotes the τ-step ahead volatility forecast in regime i made at time T that can be obtained recursively from the following:

$$\widehat{h}_{iT,T+\tau} = \alpha_{0i} + (\alpha_{1i} + \beta_{1i}) E_T\{h_{iT,T+\tau-1} | s_{T+\tau}\} \tag{10}$$

This formula is analogous to the one derived for the standard, single-regime GARCH model and the probability to be used here to calculate the expected value comes from equation 4. Equation 9 suggests that the multi-step ahead volatility forecasts are computed as a weighted-average of the multi-step-ahead volatility forecasts in each regime estimated , where the weights are the prediction probabilities. Using the theory of Markov processes, to compute the volatility forecasts the filter probability at τ periods ahead $Pr\left(s_{t+\tau}=i|I_t\right)=p_{it+\tau}=M^{\tau}p_{it}$ is required where

$$M=\begin{bmatrix} p_{11} & 1-p_{22} \\ 1-p_{11} & p_{22} \end{bmatrix} \tag{11}$$

The substantial simplification of the computation of the conditional variance due to the specification in equation 10 stands as one of the main advantages of Klaassen's MS-GARCH model over Gray's (1996) one. To estimate the Markov regime-switching model parameters, a quasi-maximum likelihood approach is undertaken with the aid of the ex-ante probability $p_{1t}=Pr\left(s_t=1|I_{t-1}\right)$ which can be calculated from:

$$p_{1t}=p_{11}\left[\frac{f(\pi_{t-1}|s_{t-1}=1)(1-p_{1t-1})}{f(\pi_{t-1}|s_{t-1}=1)p_{1t-1}+f(\pi_{t-1}|s_{t-1}=2)(1-p_{1t-1})}\right] + (1-p_{22})\left[\frac{f(\pi_{t-1}|s_{t-1}=2)(1-p_{1t-1)}}{f(\pi_{t-1}|s_{t-1}=1)p_{1t-1}+f(\pi_{t-1}|s_{t-1}=2)(1-p_{1t-1})}\right]. \tag{12}$$

Here $f\left(\cdot|s_t=i\right)$ denotes one of the possible conditional distributions from Normal and Student's t given that regime i occurs at time t. With the input in equation 11 the log-likelihood function can be written as:

$$l=\sum_{t=-R+w+1}^{T+w} log\left[p_{1t}f\left(\pi_t|s_t=1\right)+\left(1-p_{1t}\right)f\left(\pi_t|s_t=2\right)\right] \tag{13}$$

where $w=0,1,....,n$. The maximimum likelihood estimates are obtained by maximizing equation 13 using quasi-Newton algorithm in the Matlab numerical optimization routines. The estimation is carried out on a moving window of 492 monthly observations.

In this paper, following Marcucci (2005) we evaluate the forecasting performances of competing models with respect to seven statistical loss functions which are listed below:

$$MSE_1 = n^{-1}\sum_{t=1}^{n}(\widehat{\sigma}_{t+1} - \widehat{h}_{t+1|t}^{1/2})^2 \tag{14}$$

$$MSE_2 = n^{-1}\sum_{t=1}^{n}(\widehat{\sigma}_{t+1} - \widehat{h}_{t+1|t})^2 \tag{15}$$

$$QLike = n^{-1}\sum_{t=1}^{n}(log\widehat{h}_{t+1|t} + \widehat{\sigma}_{t+1}\widehat{h}_{t+1|t}^{-1}) \tag{16}$$

$$R2Log = n^{-1}\sum_{t=1}^{n}[log(\widehat{\sigma}_{t+1}^2\widehat{h}_{t+1|t}^{-1})]^2 \tag{17}$$

$$MAD_1 = n^{-1}\sum_{t=1}^{n}|\widehat{\sigma}_{t+1} - \widehat{h}_{t+1|t}^{1/2}| \tag{18}$$

$$MAD_2 = n^{-1}\sum_{t=1}^{n}|\widehat{\sigma}_{t+1}^2 - \widehat{h}_{t+1|t}| \tag{19}$$

$$HMSE = T^{-1}\sum_{t=1}^{T}(\widehat{\sigma}_{t+1}^2\widehat{h}_{t+1|t}^{-1} - 1)^2 \tag{20}$$

where $\widehat{\sigma}^2$ is an estimate of realized volatility and $\widehat{h}$ is volatility forecast from GARCH models. Equations 14 and 15 are loss functions based on typical mean squared error metrics. The loss function in equation 16 computes loss implied by a gaussian likelihood and is suggested by Bollerslev et al. (1994). Equation 17 which is called the Logarithmic Loss Function, penalizes volatility forecasts asymmetrically in low volatility and high volatility periods (Pagan and Schwert, 1990). Loss functions in 18 and 19 are particularly useful as they are more robust to outliers than MSEs. However, these functions do not differentiate between over and under-predictions while applying the penalty. They are also sensitive to scale transformations. Bollerslev and Ghysels (1996) have argued that MSE criterion might not be appropriate in heteroskedastic environment and therefore, suggested heteroskedasticity-adjusted MSE (HMSE) in equation 20.

In addition to the above statistical loss functions, two non-parametric measures of directional accuracy are also employed: (i) Success Ratio (SR) and (ii) Directional Accuracy (DA) test. These measures are generally aimed at computing the number of times a given model correctly predicts the directions of change of the actual volatility. As Marcucci (2005) has argued, directional accuracy of volatility forecasts bears special significance since they can be used as inputs to construct various trading strategies such as straddles. SR is defined as the fraction of the demeaned volatility forecasts that have the same direction of change as the corresponding demeaned actual volatility. Thus it measures the number of times the volatility forecast accurately captures the direction of the true volatility process. Formally, SR can be computed in the following manner:

$$SR = \frac{\sum_{j=1}^{m} I_{\{\bar{\sigma}_{t+j}\bar{h}_{t+j|t+j-1}\}>0}}{m} \tag{21}$$

where $I_{g>0}$ is an indicator function such that it takes the value of one when the function g is positive and zero otherwise.

The second test statistic, DA proposed by Pesaran and Timmermann (1992) is computed as follows:

$$DA = \frac{SR - SRI}{\sqrt{\mathrm{Var}(SR) - \mathrm{Var}(SRI)}} \tag{22}$$

where

$$\begin{aligned} SRI &= P\widehat{P} + (1-P)(1-\widehat{P}) \\ \mathrm{Var}(SR) &= m^{-1}SRI(1-SRI) \\ \mathrm{Var}(SRI) &= m^{-1}(2P-1)^2\widehat{P}(1-\widehat{P}) + m^{-1}(2\widehat{P}-1)^2P(1-P) \\ &\qquad + 4m^{-2}P\widehat{P}(1-P)(1-\widehat{P}) \end{aligned} \tag{23}$$

$$\begin{aligned} P &= m^{-1}\sum_{j=1}^{m} I(\bar{\sigma}_{t+j}) \\ \widehat{P} &= m^{-1}\sum_{j=1}^{m} I(\widehat{h}_{t+j|t+j-1}) \end{aligned} \tag{24}$$

$$I(g) = \begin{cases} 1 & \text{if } g > 0 \\ 0 & \text{otherwise} \end{cases} \tag{25}$$

In words, P represents the fraction of times that $\bar{\sigma}_{t+j} > 0$ and $\widehat{P}$ gives the proportion of demeaned volatility forecasts that are positive. The square of the DA statistic has a χ^2 distribution with one degree of freedom. To compute equations 14 - 22, an estimate of realized volatilities, $\widehat{\sigma}^2$ is required. We compute that as squared inflation rates. This classical approach is used to calculate various financial series' realized volatilities including stock market returns.

4 Results

4.1 Single-regime GARCH

Estimation results of standard single-regime GARCH models with both normal and t-distributions are presented in Table 2. The t-statistics are calculated using asymptotic standard errors. Across the two models, all of the coefficients in the conditional mean and variance equations appear to be very similar and are statistically significant. Since the summation of the estimated ARCH and GARCH parameters, $\alpha_1 + \alpha_2 < 1$ for both models, the assumption of stationarity is satisfied though this violation is common when applying GARCH models on financial variables for e.g. short-run interest rates. Given these facts, it can be argued that at least the in-sample performance of standard GARCH models is quite good. Furthermore, in terms of log-likelihood, GARCH-t performs better than GARCH-n. This is not entirely unexpected since the histogram and summary statistics provided above suggested non-normality of inflation rate.

Also, notice that the estimated sum of α_1 and α_2 is relatively large which is indicative of high volatility persistence of individual shocks, as argued in the introduction.[3] For example, a shock of 1% to the inflation rate increases the conditional variance at times t+1 to t+5 by respectively 0.203, 0.135, 0.089, 0.059 and 0.039. Whether this high volatility persistence is spurious can be confirmed by estimating the regime-switching GARCH model. Further, the excess kurtosis of a t-distribution is given by $6/(\nu - 4)$ which gives a value of 3.97. This again confirms that the U.S. inflation rate exhibits fat-tailed behavior.

[3] $\alpha_1 + \alpha_2 = 0.87$ for normally distributed errors and $\alpha_1 + \alpha_2 = 0.847$ for t-distributed errors.

Parameters	GARCH-N	GARCH-t
δ	0.1106*	.1084*
	(6.50)	(6.1425)
β_1	0.630*	0.629*
	(9.64)	(9.031)
α_0	0.008*	0.008*
	(2.86)	(2.855)
α_1	0.203*	0.232*
	(4.15)	(4.412)
α_2	0.663*	0.639*
	(10.34)	(9.261)
ν		5.51*
		(63.53)
Log-Likelihood	14.603	26.318

Table 2: Maximum Likelihood Estimates of Standard GARCH models with normal and t distributions

4.2 Markov-switching GARCH

Table 3 reports estimates of the Markov-switching GARCH models. The second and the third columns contain the results respectively for the models with normally distributed errors and t-distributed errors. As characterized by unconditional standard deviations σ_i, regime 1 has a slightly higher volatility than regime 2.[4] All of the coefficients in the conditional mean equation of both models appear statistically significant except the intercept term δ_2 in the second regime of MS-GARCH-N. But in the conditional variance equations, four of the total twelve parameters arise as statistically insignificant, three of which correspond to the MS-GARCH model with a t distribution. The t-statistics associated with α_{21} suggest that for

[4] Regime-specific unconditional standard deviations are calculated as $\sigma_i = \sqrt{\alpha_{0i}/(1-\alpha_{1i}-\alpha_{2i})}$ where $i = 1, 2$.

both models in regime 1, the GARCH terms are probably not necessary, but in regime 2 they are useful. In fact, MS-GARCH with a t distribution suggests that unlike regime 2, regime 1 is characterized by a constant variance since both the ARCH coefficient α_{11} and the GARCH coefficient α_{21} are statistically insignificant. With respect to persistence, both MS-GARCH-N and MS-GARCH-t indicate lower value for regime 1 (higher volatility regime) than regime 2 (lower volatility regime).

The above results highlight the superior capability of Markov-switching GARCH models in identifying and distinguishing between different sources of volatility clustering. As Gray (1996) has argued, volatility clustering has two main sources. The first one is within-regime persistence and the second one is the persistence of regimes. The implication of regime persistence is that if the unconditional variance is higher in one regime than the other, then periods of high volatility tend to cluster together during episodes of high volatility-regime given that the regimes are persistent. This implies that for US inflation rates, volatility clustering in regime 1 is caused by the persistence of the high volatility regime and in regime 2 it is caused by both regime persistence and within-regime persistence. After all, the estimates of regime persistence as given by the transition probabilities p and q in Table 3 are both quite high and statistically significant.

The log-likelihood gives an initial idea of whether regime persistence is an important source of volatility persistence. For each error distribution, the log-likelihoods corresponding to the regime-switching models are higher than their single-regime counterpart. Hence, incorporating regimes can be an important mechanism to capture volatility clustering. Also as expected, estimates of persistence from standard GARCH models fall between the estimates from the high and low volatility regimes produced by the Markov-switching models. Another interesting result is that the immediate impact of an individual shock seems to be greater during the higher volatility regime (regime 1) as captured by higher values for the ARCH term in regime 1, α_{11} in comparison with the values for the ARCH term in regime 2, α_{12}. This means that for both Markov-switching models in the high volatility regime, inflationary shocks have a large immediate impact that dies out quickly. But the second regime's sensitivities to an individual shock are comparatively low and similar to the ones obtained under standard GARCH models.

The top panel in Figure 3 displays the time series plots of the smoothed, filter and ex ante probabilities that the inflation rate is in regime 1 at time t as estimated by the MS-GARCH-N model. MS-GARCH-t model also produces similar plots and therefore, they are not presented here. According to smoothed probabilities (blue dotted line), there was a 100% probability of the inflation rate being in the high volatility regime until April, 1979. Eventually, there was a switch to a low volatility regime around mid 1983. These results are consistent with the finding in the literature that inflation volatility was high in the 1970s but declined around 1984 during the period of Great Moderation (see Gordon (2007); Blanchard and Simon (2001); Stock and Watson (2002); Sensier and van Dijk (2004)). This consistency of result indicates the reliability of our choice of a simple AR(1) conditional mean equation in the Markov-switching GARCH model.

While the existing studies in the literature only report the break date of 1984, this paper is the first to present evidence on exactly when the process of structural break in inflation uncertainty started. According to the smoothed probability plot, the process started around April 1979 which marginally precedes the nomination of Paul Volcker to serve as the chairman of the Board of Governors of the Federal Reserve System on July, 1979. Upon the confirmation of the Senate, Paul Volcker took office on August 6, 1979 and started a series of contractionary monetary policies including shifting the Fed's focus to managing the volume of bank reserves from trying to manage the day-to-day level of the federal funds rate (Lindsey et al., 2013). Therefore, it can be argued that the process of volatility moderation closely followed the time frame of the drastic monetary policy changes implemented by the Fed under Paul Volcker. Nevertheless, to what extent Volcker's policy changes impacted inflation volatility or if they affected inflation volatility at all is a separate debate which we do not seek to settle here.

As a final point before moving on to discuss in-sample goodness-of-fit statistics, both ex-ante and filter probabilities suggest occurrences of high volatility regimes between (i) late 1987 and late 1990 and (ii) around the onset of the 2007 recession. However, once information

Table 3: Maximum Likelihood Estimates of Markov-switching GARCH models with normal and t distributions

Parameters	MS-GARCH-N	MS-GARCH-t
δ_1	0.162* (9.41)	0.157* (9.31)
δ_2	0.638 (0.44)	0.471* (6.80)
β_{11}	0.739* (19.64)	0.7706* (20.74)
β_{12}	0.330* (5.23)	0.3458* (6.36)
α_{01}	0.039* (3.52)	0.050* (3.72)
α_{02}	0.004* (1.98)	0.005 (1.66)
α_{11}	0.442* (2.95)	0.488 (1.12)
α_{12}	0.230* (2.75)	0.196* (2.16)
α_{21}	0.025 (0.82)	0.005 (0.23)
α_{22}	0.69* (7.45)	0.708* (6.22)
p	0.997* (226.61)	0.996* (215.79)
q	0.998* (495.93)	0.998* (690.17)
ν		4.216* (9.84)
σ_1	0.27	0.31
σ_2	0.22	0.23
Log Likelihood	52.08	70.69

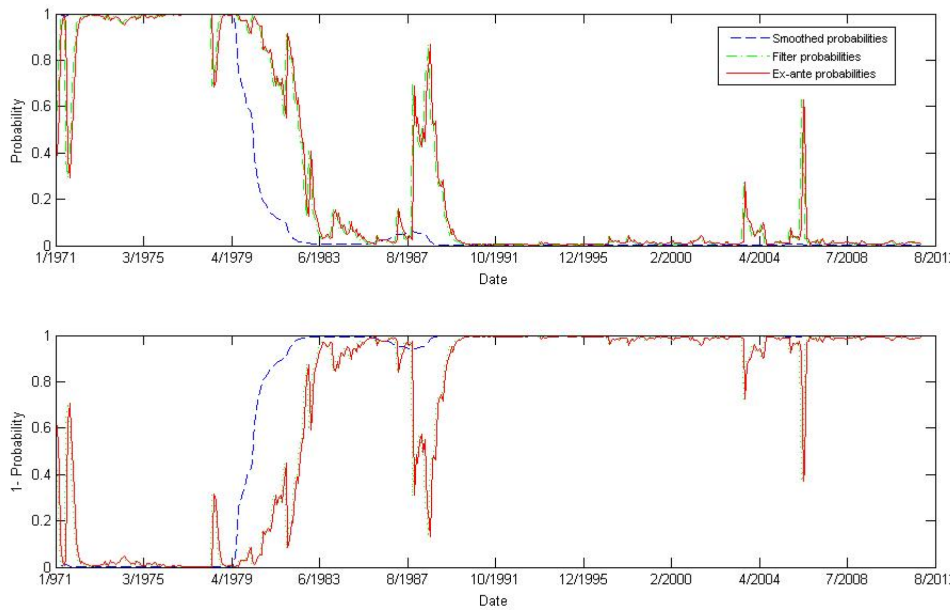


Figure 3: The top panel contains a time series plot of the smoothed, filter and ex ante probabilities that the inflation rate is in regime 1 at time t according to the MS-GARCH-N model. The bottom panel displays the same probabilities for regime 2.

from the whole sample is taken into account by smoothed probabilities, it becomes clear that neither of these periods actually correspond to high volatility regimes. Another alternative explanation based on ex-ante probabilities with respect to the period around the onset of the 2007 recession is possible. (Klaassen, 2002) has argued that some large shocks are are not persistent at all and have a rather "pressure relieving" effect. Since the within-regime persistence estimated in this paper for the high volatility regime is low, the effect of the shock to inflation volatility dies out quickly before switching to the low volatility regime. In that sense, the shock to the inflation volatility before the recession of 2007 imparted a "pressure relieving" effect. This is depicted in Figure 4 as a spike in the conditional volatility around the time of the recession in 2007.

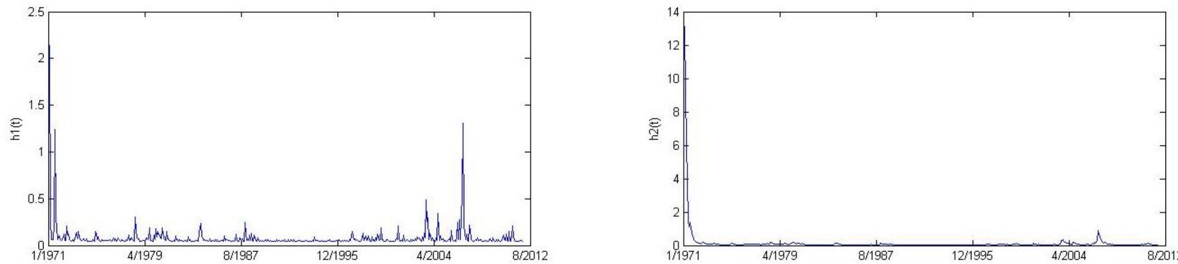


Figure 4: Conditional volatilities of US inflation rates over the period 1971-2012

4.3 In-Sample Goodness-of-Fit

First of all, it has to be clarified that testing the null hypothesis of a linear model or single-regime model against a regime-switching model is a non-trivial task. The difficulty mainly arises because conventional likelihood-based inference is invalid since the regime-staying probabilities remain as unidentified parameters under the null. This results in a likelihood ratio whose asymptotic distribution is not the usual χ^2 anymore and therefore, may lead to misleading conclusions (Klaassen, 2002). Although there are some papers which have sought to circumvent this problem (see for example Hansen (1992); Dufour and Luger (2017)), we do not seek to formally test for the significance of the second regime here. Rather we only report some in-sample goodness-of-fit statistics in Table 4 as our main focus is on the forecasting performance.

It is evident from Table 4 that GARCH-N has the poorest performance of all. On the other hand, MS-GARCH-N that is, the Markov-switching model with normally distributed errors outperforms all other models by ranking first according to 7 out of 10 statistical loss functions. Based on the rest of the statistical loss functions, the MS-GARCH-t model ranks first which means that together the two Markov-switching models share between them 100% of the top places in the ranks. The superiority of the MS-GARCH-N model is consistent with the finding of Marcucci (2005) who examined the performance with respect to stock market volatility.

Table 4: In-sample goodness-of-fit statistics

Model	NumPar	AIC	Rank	BIC	Rank	LogL	Rank	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank
GARCH-N	5	-0.03	4.00	0.02	4.00	11.23	4.00	0.06	3.00	0.12	3.00	-1.34	3.00	5.79	3.00	0.12	3.00	0.16	3.00	6.81	3.00
GARCH-t	6	-0.09	3.00	-0.04	3.00	28.71	3.00	0.06	2.00	0.12	2.00	-1.37	2.00	5.81	4.00	0.12	2.00	0.16	2.00	6.38	2.00
MS-GARCH-N	12	-0.16	2.00	-0.06	2.00	52.04	2.00	0.05	1.00	0.08	1.00	-1.43	1.00	5.37	1.00	0.11	1.00	0.15	1.00	6.12	1.00
MS-GARCH-t	13	-0.23	1.00	-0.12	1.00	70.69	1.00	0.07	4.00	0.23	4.00	-1.23	4.00	5.70	2.00	0.13	4.00	0.17	4.00	9.20	4.00

Note: NumPar is the number of parameters estimated in each model, AIC is Akaike Information Criterion calculated as $-2log(L)/T + 2k/T$ where k is the number of parameters and T is the total number of observations. BIC is the Bayesian Information Criterion or Schwarz Criterion calculated as $-2log(L)/T + (k/T)$. The rest of the statistical loss functions MSE_1, MSE_2, QLike, R2Log, MAD_1, MAD_2, and HMSE are defined in Section 3.

4.4 Out-of-Sample Forecasting Performance

One particular caveat about the previous section's results is that highly parameterized models tend to produce good in-sample fits. Therefore, one needs to be careful about the apparent superiority of Markov-switching models in terms of their in-sample performance since they are inherently highly parameterized. In contrast, out-of-sample tests are capable of controlling either possible over-fitting or over-parameterization problems (Marcucci, 2005). Therefore, in this section we examine and compare with each other the out-of-sample performances of the previous four variants of GARCH models in forecasting inflation volatility. Out-of-sample volatility forecasting performance is important also because of its relevance to researchers and practitioners.

Tables 5 to 10 report 1 to 12-month ahead inflation uncertainty forecasting performances in terms of the seven statistical loss functions defined in Section 3. They also report estimates for Success Ratio (SR) and Directional Accuracy (DA) test statistic. It is clear that MS-GARCH-N clearly outperforms all other models in forecasting inflation uncertainty 1 to 5-month ahead. For the same forecasting horizon, MS-GARCH-t ranks second best while GARCH-N fares worst. These rankings are consistent with in-sample performances found in Section 4.2. However, note that unlike for other models the DA test statistic for MS-GARCH-N is not statistically significant.[5] Nevertheless, MS-GARCH-N has the highest SR value for each forecast horizon from 1 to 5 months.

For forecast horizons of 6 and 7 months, both Markov-switching GARCH models have comparable performances. Standard GARCH models still perform worse than their regime-switching counterparts. From 8-month ahead horizon onward, MS-GARCH-t starts exceeding all other models in forecasting performance. In fact, for the 12-month ahead volatility forecasts, MS-GARCH-t ranks 1 in 6 out of 7 statistical loss functions. Also notice that beyond 5-month forecast horizon, MS-GARCH-N has a statistically significant DA test statistic. However, its performance clearly declines from 10-month forecast horizon onward when

[5] The square of DA test statistics for MS-GARCH-N are less than the 5% significance level χ^2 critical value 3.84. Therefore, we fail to reject the null hypothesis that forecasted conditional volatility cannot predict realized volatility.

even standard GARCH-t performs better than MS-GARCH-N. In a nutshell, for short-term forecast horizon spanning 1 to 5-months, Markov-switching GARCH model with normally distributed errors (MS-GARCH-N) performs better than the other three GARCH models. But for longer horizons, MS-GARCH-t performs better in terms of out-of-sample forecasting evaluation.

5 Conclusion

Volatility of inflation rate or inflation uncertainty is as important a variable as the level of inflation rate. It has serious welfare loss implications for risk averse economic agents even if all the prices in the economy are fully flexible. Therefore, being able to forecast inflation uncertainty as accurately as possible is of paramount importance. Coupled with that is the fact that a casual “eyeballing” of the data on US inflation rates from 1971 to present suggests that its volatility might have undergone regime changes multiple times. Existing studies in the literature also confirm at least one structural break in 1984. Therefore, it might be appropriate to forecast inflation uncertainty using Markov-switching GARCH models which are capable of handling regime changes unlike standard GARCH models.

Modeling inflation uncertainty using regime-switching GARCH models also provides the opportunity to evaluate the forecasting performance of these models relative to standard ones. In this paper, we seize that opportunity to augment the existing evidences which already support regime-switching GARCH models' superior shorter horizon forecasting performance. However, those evidences are based on only stock market and exchange rate data. Following Marcucci (2005), this paper employs a broad set of statistical loss functions to evaluate the relative performances of Markov-switching GARCH models in forecasting US inflation uncertainty.

One of the first major findings of this paper is that a Markov-regime switching GARCH model consisting of a simple AR(1) conditional mean equation does remarkably well in identifying US inflation uncertainty's structural shift in the year 1984. This result is consistent with the general agreement in the literature on the break date. In addition, this paper has identified April, 1979 as the time when the regime switching process might have started before culminating in a complete switch in 1984. The whole switching process mirrors the time line which follows a specific period that starts from the nomination of Paul Volcker as the new chairman of the Federal Reserve System to his implementation of various drastic monetary policy initiatives until 1984.

Another important result of this paper is that in the high volatility regime, shock persistence is lower compared to the low volatility regime. But the immediate impact of an individual inflationary shock is higher in the high volatility regime. New evidences are presented which show that the main source of volatility clustering in the high volatility regime is caused by the persistence of the regime itself. Finally, a comparison of the forecasting performances of the four different GARCH models indicates that for a forecasting horizon of 1 to 5 months, a Markov regime-switching GARCH model with normally distributed errors (MS-GARCH-N) outperforms all other three models. However, for longer forecasting horizon such as 8 to 12 months, a Markov regime-switching GARCH model with t distributed errors (MS-GARCH-t) performs the best. For the same longer horizon, MS-GARCH-N performs poorly even compared to a standard GARCH model with t distributed errors.

The results and analyses of this paper can be extended in the future to explore the relationship between inflation and inflation uncertainty within a regime-switching framework. Also, forecasting exercises similar to the ones in this paper can also be carried out for other countries' inflation rates. It will be interesting to further evaluate the relative performances of Markov regime-switching GARCH models in the contexts of different economic settings.

Table 5: Out-of-sample evaluation of one and two-month ahead volatility forecasts

1-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.0048	4	0.0088	4	-2.7139	3	1.5208	2	0.1453	4	0.0594	4	1.2004	2	0.36	-3.9767
GARCH-t	0.003	3	0.0088	3	-2.9989	4	0.0026	1	0.1372	3	0.057	3	1.4516	3	0.38	-3.8858
MS-GARCH-N	0.0189	1	0.0042	1	-2.1548	2	5.2561	3	0.1116	1	0.0467	1	0.7784	1	0.51	-1.9727
MS-GARCH-t	0.0212	2	0.0047	2	-1.9906	1	5.6603	4	0.1208	2	0.0495	2	1.7038	4	0.44	-3.7334

2-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.0048	4	0.0088	4	-2.7139	3	1.5208	2	0.1453	4	0.0594	4	1.2004	2	0.36	-3.9767
GARCH-t	0.003	3	0.0088	3	-2.9989	4	0.0026	1	0.1372	3	0.057	3	1.4516	3	0.38	-3.8858
MS-GARCH-N	0.0189	1	0.0042	1	-2.1548	2	5.2561	3	0.1116	1	0.0467	1	0.7784	1	0.51	-1.9727
MS-GARCH-t	0.0212	2	0.0047	2	-1.9906	1	5.6603	4	0.1208	2	0.0495	2	1.7038	4	0.44	-3.7334

Table 6: Out-of-sample evaluation of three and four-month ahead volatility forecasts

3-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.0198	4	0.0409	4	-4.3265	4	0.5748	2	0.2155	4	0.1399	4	344.0068	4	0.41	-2.5931
GARCH-t	0.0157	3	0.0399	3	0.4847	1	0.4252	1	0.2064	3	0.1358	3	75.8866	3	0.44	-2.4503
MS-GARCH-N	0.0334	1	0.021	1	-0.9934	3	1.5323	3	0.1456	1	0.1107	1	0.4615	1	0.51	-1.4759
MS-GARCH-t	0.0366	2	0.0222	2	-0.9135	2	1.675	4	0.153	2	0.113	2	0.7898	2	0.49	-2.172

4-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.0401	4	0.0706	4	-0.9952	3	-0.0826	1	0.2382	4	0.1849	4	2.1482	4	0.38	-2.7801
GARCH-t	0.0378	3	0.0692	3	-1.0669	4	-0.2367	2	0.2309	3	0.1808	3	1.8226	3	0.41	-2.627
MS-GARCH-N	0.0442	1	0.0344	1	-0.6642	2	1.3966	3	0.1661	1	0.1453	1	0.4674	1	0.49	-1.6718
MS-GARCH-t	0.0468	2	0.0355	2	-0.5998	1	1.4809	4	0.1806	2	0.1537	2	0.7232	2	0.41	-3.1542

Table 7: Out-of-sample evaluation of five and six-month ahead volatility forecasts

5-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.062	4	0.0991	4	-1.1493	3	0.0616	1	0.2661	4	0.232	4	6.9225	3	0.33	-3.1563
GARCH-t	0.0602	3	0.0971	3	-1.3805	4	0.0097	2	0.2593	3	0.228	3	10.4287	4	0.36	-2.9824
MS-GARCH-N	0.055	1	0.0509	2	-0.3972	2	1.3432	3	0.1818	1	0.1738	1	0.5602	1	0.49	-1.2888
MS-GARCH-t	0.0559	2	0.05	1	-0.36	1	1.3832	4	0.1988	2	0.1867	2	0.6854	2	0.41	-2.6436

6-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.0756	4	0.1227	4	-0.5359	3	0.2616	2	0.2823	4	0.2706	4	3.3074	4	0.38	-2.4236
GARCH-t	0.0745	3	0.12	3	-0.6228	4	0.1868	1	0.2744	3	0.2644	3	3.0855	3	0.41	-2.2368
MS-GARCH-N	0.0657	2	0.0702	2	-0.1885	2	1.3152	3	0.199	1	0.2043	1	0.5717	1	0.44	-2.0503
MS-GARCH-t	0.0647	1	0.0667	1	-0.1724	1	1.321	4	0.2165	2	0.2209	2	0.6123	2	0.36	-3.4247

Table 8: Out-of-sample evaluation of seven and eight-month ahead volatility forecasts

7-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.0858	4	0.1345	4	-0.6945	3	0.2278	1	0.2851	4	0.2874	4	5.0744	3	0.36	-2.6304
GARCH-t	0.0859	3	0.1313	3	-0.8356	4	0.2232	2	0.278	3	0.2813	3	7.102	4	0.38	-2.4333
MS-GARCH-N	0.0781	2	0.0974	2	-0.0201	2	1.3184	4	0.221	1	0.2453	1	0.5687	1	0.41	-2.2368
MS-GARCH-t	0.0742	1	0.089	1	-0.0175	1	1.2886	3	0.2302	2	0.2532	2	0.5944	2	0.33	-3.5687

8-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.0936	4	0.1446	4	-1.5086	3	0.3767	1	0.2862	4	0.3043	4	41.4512	3	0.38	-2.133
GARCH-t	0.0934	3	0.1412	3	36.3074	4	1.9969	4	0.2802	3	0.2997	3	37865.0154	4	0.41	-1.913
MS-GARCH-N	0.0898	2	0.1295	2	0.1192	2	1.2825	3	0.246	2	0.2952	2	0.5445	1	0.38	-2.4236
MS-GARCH-t	0.0822	1	0.1131	1	0.1105	1	1.2208	2	0.2416	1	0.2842	1	0.5613	2	0.31	-3.719

Table 9: Out-of-sample evaluation of nine and ten-month ahead volatility forecasts

9-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.0942	4	0.1558	3	0.438	4	0.6243	2	0.2902	4	0.3268	3	13.7268	4	0.36	-2.3527
GARCH-t	0.0932	3	0.1528	2	0.1126	3	0.5168	1	0.2854	3	0.3243	2	5.0627	3	0.38	-2.122
MS-GARCH-N	0.1019	2	0.1648	4	0.239	2	1.2903	4	0.2716	2	0.3487	4	0.5054	1	0.36	-2.612
MS-GARCH-t	0.0906	1	0.1386	1	0.2227	1	1.2054	3	0.2533	1	0.3154	1	0.5239	2	0.28	-3.8759

10-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.1064	3	0.1843	3	0.115	2	0.5126	1	0.2971	4	0.3571	3	4.9909	4	0.36	-2.3527
GARCH-t	0.1077	4	0.1828	2	-0.0347	1	0.5078	2	0.2943	3	0.3561	2	4.3118	3	0.41	-1.6576
MS-GARCH-N	0.1127	2	0.2041	4	0.3437	4	1.2571	4	0.2886	2	0.3941	4	0.4921	1	0.33	-2.8294
MS-GARCH-t	0.0975	1	0.1654	1	0.3198	3	1.1527	3	0.2617	1	0.3445	1	0.5012	2	0.28	-3.8759

Table 10: Out-of-sample evaluation of eleven and twelve-month ahead volatility forecasts

11-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.1136	2	0.2021	2	-0.1639	1	0.5188	1	0.2969	4	0.3738	2	7.9816	3	0.41	-1.8895
GARCH-t	0.1171	3	0.2033	3	-0.6314	4	0.6516	2	0.2955	2	0.3744	3	22.4232	4	0.41	-1.6576
MS-GARCH-N	0.1214	4	0.2446	4	0.4396	3	1.1945	4	0.296	3	0.4278	4	0.4719	2	0.33	-2.8294
MS-GARCH-t	0.1019	1	0.1909	1	0.4087	2	1.0713	3	0.2607	1	0.3615	1	0.4696	1	0.28	-3.8759

12-month ahead volatility forecasts

Model	MSE_1	Rank	MSE_2	Rank	QLike	Rank	R2Log	Rank	MAD2	Rank	MAD1	Rank	HMSE	Rank	SR	DA
GARCH-N	0.1192	3	0.2337	2	2.8951	4	1.2391	4	0.3038	3	0.4095	2	185.8821	4	0.33	-3.1633
GARCH-t	0.1184	2	0.2372	3	0.9242	3	0.8923	1	0.3012	2	0.4132	3	10.5512	3	0.31	-3.3477
MS-GARCH-N	0.133	4	0.2972	4	0.569	2	1.1636	3	0.3126	4	0.4771	4	0.5014	2	0.23	-4.6233
MS-GARCH-t	0.1086	1	0.225	1	0.5292	1	1.0204	2	0.2709	1	0.3968	1	0.4775	1	0.18	-5.8629

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