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**Does Competition Affect Evolutionary  
Dynamics? Evidence from a Collegiate  
University**

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# 1 Introduction

Evolutionary games were first developed by biologists (e.g., Maynard Smith and Price 1973) and later adapted and extended by economists (see, e.g., Friedman 1991). While classical game theory makes strong assumptions about the player's cognitive capacities, evolutionary game theory replaces rationality by natural selection. Specifically, rather than maximizing expected payoffs, the players in evolutionary games are replicators who reproduce their kind. The well-known replicator dynamic equation (Taylor and Jonker 1978) predicts that the rate of growth in the distribution of population playing each strategy would be roughly proportional to the deviation of that strategy's payoff from the population's average payoff.

There is, however, an important difference between evolutionary dynamics and standard repeated games in the following sense. In evolutionary games, players are essentially programmed to follow some rule with no appeal to rationality. On the other hand, the distribution of strategies across a population can be interpreted as a sequence of mixed-strategy Nash equilibrium. In the latter case, a necessary implication is that there should be no serial correlation or dependence in the distribution of strategies played in each period. In this note, I present evidence that the strength of evolutionary dynamics is inversely related to the degree of competition among players. This suggests that competition favors rational-choice game theory.

A relatively few papers tested mixed-strategy equilibrium play using nonexperimental data (Chiappori et al. 2002 and Palacios-Huerta 2003 using penalty kicks in soccer; Walker and Wooders 2001 and Hsu et al. 2007 using serve directions in tennis). The authors considered whether players switch from one direction to another too often or too infrequently to be random. One observation is that serial independence seems to hold more often for penalty kicks than for tennis matches. One possibility is that the kicker is under more pressure than a tennis player considering the nature of these two sports. Substantiating this conjecture would require a set of quasi-experiments across settings that only differ by the level of competition.

I argue that application statistics from a collegiate university (where governing authorities are divided between a number of constituent colleges) can provide a testing ground for the relation between competition and evolutionary dynamics, where payoffs in the previous year can potentially affect the current distribution of strategies. In the next Section, I explain why Cambridge University’s admissions system generates a unique data set of quasi-experiments from a large high-stake game with adequate information environment. Section 3 estimates the replicator dynamic equation for each undergraduate subject offered and show the main result that the coefficient of the dynamic is inversely related to the applications-to-offers ratio.

## 2 Data

In Cambridge, the Colleges are responsible for admitting students to the University’s degree programs. Unlike applying to (multiple) universities, a prospective student can only apply to a single College in a given year. All applications, where interviewing process is an important factor, are handled by the Colleges. It is fair to say that applicants are highly motivated and decide carefully which College to apply. It seems reasonable to assume that applicants are randomly drawn from a single population, and they do not attempt to influence the future applicant’s action.<sup>1</sup>

The information environment is that the University started publishing application statistics on their website from the academic year 2004-05. In particular, application statistics were broken down for each subject by College (and for each College by subject). This means that the prospective students could see the number of applicants and offers in the previous few years for all College and the subject that they were considering to study.<sup>2</sup> Similarly, one of the control variables, the College’s ranking, was also published by the press and

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<sup>1</sup>Cambridge Colleges are committed to non-discrimination and equal opportunity, so it is hard to think of any reason why observed applicant characteristics should matter.

<sup>2</sup>Oxford University operates a similar College-based admissions system; however, Oxford University has not published disaggregated application statistics for each subject and College combination, which renders the information environment unsuitable for the present study.

well-advertised nationally.

There are a couple of institutional details that need to be taken into account when defining the strategies and payoffs. First, applicants can make an open application without specifying a College.<sup>3</sup> However, this option is not included in the strategy set because each open application is allocated to a College using a computer algorithm, and Colleges treat them in exactly the same manner as direct applications to the College. Accordingly, the University has not published statistics for open applications category, and we do not observe the payoffs from this strategy.<sup>4</sup>

Second, the Colleges jointly operate a pool system, where Colleges can make an offer to those who applied to other Colleges and were subsequently placed in the pool. That is, the system provides a second chance for those who were marginally rejected by their College of choice. The application statistics that are available on the University's website contained such disaggregated numbers for all categories (i.e., offers, of which offers through pool, and offers through pool by another College), which allowed applicants to calculate the success rate from choosing any College.

There are 29 constituent Colleges that admit undergraduate students, which represent the strategy set of the players. Among these, three Colleges are only for women, and another four are exclusively for mature students (i.e., 21 years or older); however, this would not pose a problem unless these applicants behave very differently from the general pool of the applicants. The data runs from academic year 2004 to 2009, and there are 24 subjects in the sample, which represent 24 quasi-experimental rounds. That is, the strength of replicator dynamic is assumed to differ across subjects.<sup>5</sup>

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<sup>3</sup>Of the total applications received during the sample period, 12.9 percent were originally open applications.

<sup>4</sup>The information on the number of open applications and their assignments to Colleges were obtained from the University's central administration by request under the Freedom of Information Act 2000.

<sup>5</sup>I excluded 4-year Classics because the program is relatively small compared to the regular 3-year Classics, and also Linguistic because the program started only in 2009.

### 3 Estimation

The following exposition is standard in the literature. The set of strategies is  $S = \{1, \dots, n\}$ , where  $i \in S$  represents a College. A population state is  $x \in X = \{x \in R_+^n \mid \sum_{i \in S} x_i = 1\}$ , where  $x_i$  represents the proportion of applicants choosing College  $i$ . The observed state space includes only direct applications to each College. (One may be reminded that payoffs from open applications are not directly observed.) Thus,  $x$  represents the population state that can be modelled as an evolutionary dynamic. The expected payoff of strategy  $i$  is a function  $F_i(\hat{x}) : X \rightarrow R$ , where  $\hat{x}$  is the payoff-relevant population state, which will be explained below. The average payoff experienced by the population in each round is denoted as  $\bar{F}(\hat{x}) = \sum_{i \in S} \hat{x}_i F_i(\hat{x})$ .

I assume that the applicant's goal is to get an offer. The total number of offers from College  $i$  is the number of places offered directly by College  $i$ , excluding offers made to applicants through the pool, but including offers received by its applicants through the pool. That is, from the standpoint of a prospective applicant, an offer received through the pool has the same weight as an offer made by the College directly, when calculating the expected probability of an offer. Then, the expected payoff of choosing College  $i$  is the proportion of total offers to the number of applications (which includes open allocations). Since applicants do not separately observe direct and open applications, this is how the applicants perceive the relevant payoffs.

A well-known replicator dynamic equation (Maynard Smith 1982) can be written as follows:

$$\frac{\dot{x}_i}{x_i} = \frac{\tilde{F}_i(\hat{x})}{\bar{F}(\hat{x})},$$

where  $\dot{x}$  is the time derivative and  $\tilde{F}_i(\hat{x}) = F_i(\hat{x}) - \bar{F}(\hat{x})$  denotes the excess payoff of strategy  $i$  relative to the population average.

The empirical counterpart can be specified as

$$\ln x_{i,t+1} - \ln x_{i,t} = \alpha + \beta [\ln F_i(\hat{x}) - \ln \bar{F}(\hat{x})]_t + \gamma X_{i,t} + \tau_t + c_i + \varepsilon_{i,t},$$

where  $\tau_t$  is the year dummy,  $c_i$  is the College dummy, and  $\varepsilon_{i,t}$  is the error term.  $X_{i,t}$  is a set of time-varying covariates, which include the annual College ranking and the total number of offers made in year  $t$ , both of which are casually suspected to affect the number of applications.<sup>6</sup>

I estimate the above equation for each of 24 subjects using OLS and fixed-effects specifications. The key parameter is  $\beta$ , which measures the strength of evolutionary dynamics. Table 1 shows only the  $\beta$  coefficients from each estimation, where the four columns are self-explanatory. Not surprisingly, the results give some support for the central prediction of evolutionary dynamics in the sense that the  $\beta$  coefficients are generally positive and sometimes statistically significant. Fixed-effects and the covariates separately do not seem to change the estimates significantly; however, jointly they increase the estimates substantially. In column (4), in 20 out of 24 subjects the coefficient is positive and statistically significant.<sup>7</sup>

The question of interests is what explains this relatively large variation in the estimated coefficients across subjects. As previously mentioned, the level of competition in each subject may explain the differential strength of evolutionary dynamics. Therefore, I regress the estimated coefficients  $\beta$  on the (average) offers-to-applications ratio for each subject. The results are shown in Table 2 along with scatterplots in Figure 1. They show a clear, strong and robust relationship between the two variables with upward-sloping fitted lines. This means that the previous year's excess payoff is a weaker predictor for the applicants' choice of Colleges in the current year for subjects that have on average been more competitive than others.

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<sup>6</sup>The selection of covariates is based on the author's informal conversation with Admissions Tutors at Cambridge.

<sup>7</sup>The results are very similar to FE specifications where an AR(1) error correction model is estimated.

## 4 Conclusion

This note empirically measures the strength of evolutionary dynamics in college applications and relates it to the degree of competition in each subject discipline. The evidence suggests a negative relationship between the two, which is in line with the general notion that competitive pressures tend to favor players' rational choice behavior rather than naive adaptive dynamics. That is, in terms of conventional game theory, the previous round's payoffs tend to be a predictor for the choice of current strategy only when the ex ante payoffs are generally high. One potentially avenue for future research is to examine the relationship between the strength of evolutionary dynamics and the level of competition across markets or industries.

## References

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Subject	OLS	OLS	FE	FE
Anglo-Saxon, Norse and Celtic	0.451**	0.729***	0.458	1.465***
Archaeology and Anthropology	0.251	0.579***	0.257	0.776***
Architecture	-0.020	-0.031	0.081	0.633***
Asian and Middle Eastern Studies	0.562***	0.769***	0.718***	1.294***
Classics	0.372**	0.756***	0.361*	0.785***
Computer Science	0.087	0.159	0.173	0.602***
Economics	0.072	0.146*	0.305**	0.709***
Education	0.665**	0.826**	1.329***	1.328**
Engineering	0.051	0.085	0.030	0.445***
English	0.197**	0.268***	0.161	0.519***
Geography	0.092	0.146	0.101	0.458*
History	0.232***	0.335***	0.454***	0.805***
History of Art	0.486***	0.552***	0.760***	1.023***
Land Economy	0.273**	0.401***	0.473**	1.109***
Law	0.061	0.189**	-0.028	0.544***
Mathematics	0.208*	0.246**	0.123	0.541***
Medicine	-0.012	-0.043	-0.040	0.214
Modern and Medieval Languages	-0.008	0.120	-0.098	0.272*
Music	0.430***	0.489***	0.623***	1.118***
Natural Sciences	0.07	0.113	0.073	0.257*
Philosophy	0.347***	0.490***	0.393**	1.095***
Politics, Psychology and Sociology	0.195**	0.291***	0.263**	0.747***
Theology and Religious Studies	0.045	0.406**	0.061	0.809***
Veterinary Medicine	0.196*	0.351**	0.227	0.721**
Year dummies included?	yes	yes	yes	yes
Covariates included?	no	yes	no	yes

Table. 1: Estimation of Replicator Dynamic Equation for Each Subject. (\* significant at 10% level, \*\* significant at 5% level, \*\*\* significant at 1% level.)

Variable	Dependent variable			
	$\beta$ (OLS)	$\beta$ (OLS w/ X)	$\beta$ (FE)	$\beta$ (FE w/ X)
Offers-to-applications ratio	0.788 (0.011)	1.391 (0.000)	1.021 (0.055)	1.042 (0.069)
$R^2$	0.260	0.463	0.158	0.143
$N$	24	24	24	24

Table. 2: Regression of Coefficients. (p-values are in the parentheses.)

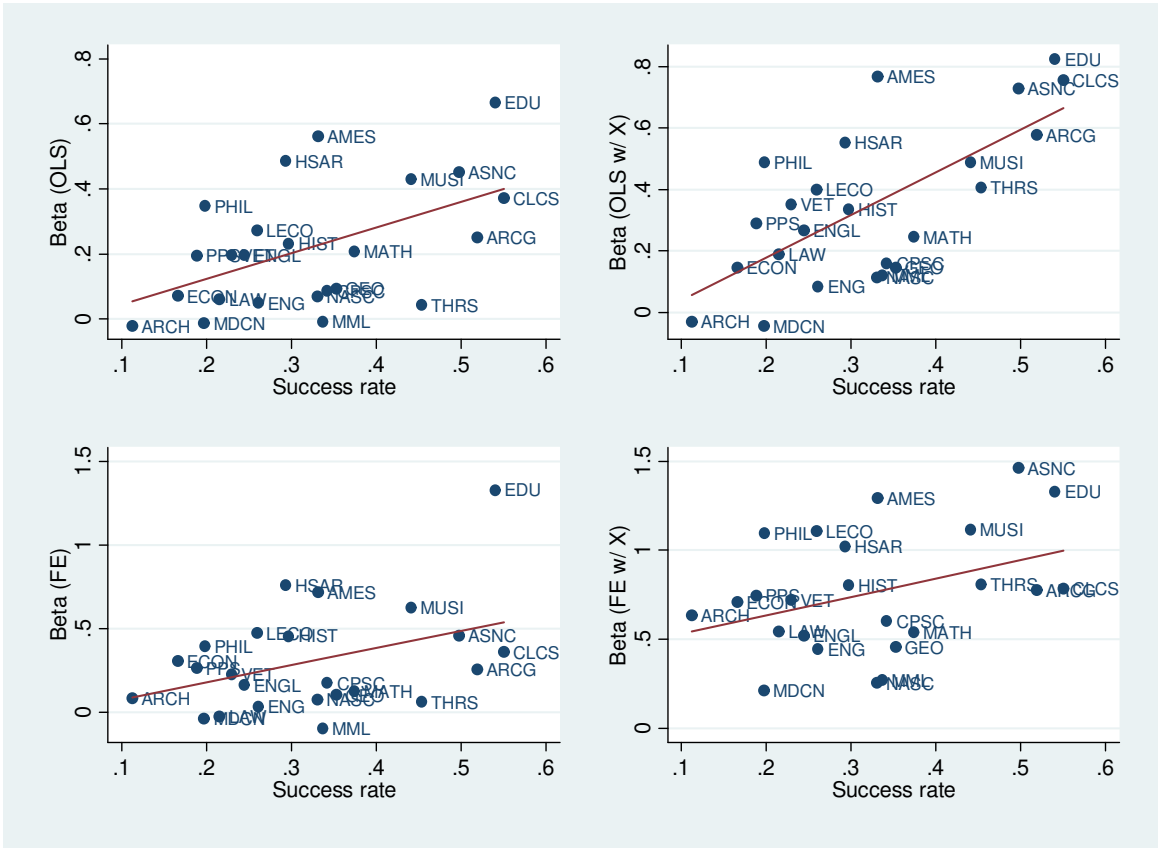


Figure 1: Relationship between Beta and Success Rate