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The Effects of Secondary Markets for Government Bonds on Inflation Dynamics*

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Abstract

We analyze how trading in secondary markets for public debt change the inherent links between monetary and fiscal policy, by studying both inflation and debt dynamics. When agents do not trade in these markets, there exists a unique steady state and traditional passive/active policy prescriptions are useful in delivering determinate equilibria. In contrast, when agents trade in secondary markets and bonds are scarce, there exist a liquidity premium on public debt and bonds affect inflation dynamics. There are different combinations of inflation and debt that deliver the same tax revenues. Thus, self-fulfilling beliefs that deliver multiple steady states are possible. We also find that, with a low inflation target, active monetary policies are more likely to deliver real and nominal determinacy and further amplify the effectiveness of these policies in reducing steady state inflation. Finally, we show that a spread-adjusted Taylor rule delivers a unique steady state where active monetary policies yield locally determinate equilibria.

JEL Codes: E40, E61, E62, H21.

Keywords: taxes; inflation; secondary markets, liquidity premium.

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1 Introduction

Traditional dynamic general equilibrium macroeconomic models based on representative agent frameworks imply various neutrality results.[1] In the context of policy, Ricardian equivalence is the most prominent one. Barro (1974) shows how a government finances its expenditures does not alter real allocations whenever the economy has rational homogenous agents, lump sum taxes, no liquidity constraints and complete markets. Environments where a central bank is also included, Ricardian equivalence typically holds, greatly restricting how monetary and fiscal policies interact in equilibrium. As a result, the channels through which fiscal policy might influence inflation dynamics is rather limited.[2]

Our objective in this paper is to provide new insights of how monetary and fiscal policy interactions change once households do not always have access complete asset markets. We do so in the context of the Great Moderation. Over this period we have witnessed several financial innovations, suggesting the existence of some market incompleteness. Among the various financial innovations, here we focus the increasing importance of on secondary markets for public debt.[3] These markets other than providing an additional opportunity for household's to re-adjust their portfolios, they also change the inherent links between monetary and fiscal policies. This is the case as prices of the primary issuance of public debt also incorporates the value associated of trading these assets in the future in secondary markets.[4] This additional feature greatly alters relative prices.

To study monetary and fiscal policy interactions, we consider simple policy rules. We do so in the context of a frictional, stochastic and incomplete market framework, where agents can trade in secondary markets for public debt. As a result, the liquidity services for public debt are an equilibrium outcome that not only depends on the primitives of the environment but also on the policy rules.[5] We find that inflation and bond dynamics crucially depend on whether agents participate in secondary markets or not. When there is no trade in these markets, we show that there exists a unique monetary steady state, where public debt does not affect inflation dynamics. However, when there is trade and bonds are scarce, public debt exhibits a liquidity premium. Agents are willing to buy additional bonds to increase their consumption possibilities in frictional goods markets. As result, Ricardian equivalence breaks down. By issuing less bonds, the government can affect the premium and reduce the inflation rate. Thus, the resulting equilibrium

[1]We refer to Akerlof (2007) for a detailed discussion of five neutrality results in macroeconomics.

[2]We refer to Sargent and Wallace (1981) and Leeper (1991) among others for more on such interactions.

[3]From 1986 to 1993, the volume of secondary market sovereign debt sales in the U.S. increased from \$7 to \$273 Billion. We refer to Power (1996) for more on the evolution of secondary markets.

[4]Wallace (1981) shows that when markets are complete, open market operations do not have real effects.

[5]The way monetary and fiscal policies interact critically depends on the beliefs about future inflation. These beliefs are not only influenced by fiscal and monetary policies, as noted by Sargent and Wallace (1981) and Leeper (1991), but also by financial frictions, as highlighted by Fernández-Villaverde (2010), Leeper and Nason (2015), and Gomes and Seoane (2015), among others.

open market operations in this economy are quite different compared to environments with no premiums. Thus the traditional prescriptions of active/passive monetary and fiscal policies based in complete and frictionless financial markets do not always deliver locally determinate equilibria in our environment.

When bonds are scarce and agents trade in secondary markets, the government is able to affect the real return on public debt through changes in the inflation rate as well as the issuance of public debt. As a result, there are different combinations of inflation and real public debt that deliver the same tax revenues. Thus, self-fulfilling beliefs that are consistent with existence of multiple steady states are possible. Changes in policies can then imply very different equilibrium allocations. Finally, regardless of how many steady states exist, we show that traditional active monetary policies decrease the steady state inflation, while passive monetary policies increase it.

In our numerical exercise, calibrated to the Pre Great Moderation period, we find that regardless of the fiscal policy stance, active monetary policies are more likely to deliver a unique monetary steady state. Whenever the steady state is unique, we find that a passive monetary policy delivers locally indeterminate equilibria regardless of the fiscal stance. At the same time, active monetary policies deliver determinacy independent of fiscal policy being active or passive. In contrast, passive monetary policy can lead to multiple steady states, one is stable while the other is unstable. These findings critically depend on the long run inflation target.[6] When the inflation target is high, two steady states may exist even under active monetary policies, one of them being locally indeterminate. However, when the central bank follows an active policy and has a low inflation target, then these policies are likely to deliver a unique and stable monetary equilibrium, regardless of the fiscal stance. Lastly, we find that secondary markets tend to reduce the stabilizing effect of monetary policy and depending on the stance of monetary policy, they strengthen or weaken the stabilizing effect of fiscal policy.

Finally, in the spirit of Canzoneri and Diba (2005) and Cúrdia and Woodford (2010), we analyze an interest spread-adjusted Taylor rule. Once the monetary authority explicitly takes into account the additional value that public debt gives to buyers, it can then internalize the additional value that public debt has. As a result, the central bank is able to rule out self-fulling beliefs regarding interest rates and public debt that deliver the same revenue, effectively eliminating real indeterminacies. Typically, when the central bank follows an active policy we find a stable monetary equilibrium.

The paper is organized as follows. Section 2 offers a literature review. Section 3 illustrates the mechanism by presenting a simple model with an adhoc bond premium. Section 4 describes the environment with an endogenous liquidity premium and characterizes the monetary equilibria. In

[6]In environments with sticky prices and complete financial markets Ascari and Ropele (2009), among others, also show that the long run inflation target affects the usefulness of the Taylor principle. We obtain similar insights in a flexible price environment with incomplete markets.

Section 5 we perform a numerical analysis. A conclusion then follows.

2 Literature Review

(Beg no se si es necesario incur el primer parrafo, en mi opinion lo podriamos quitar).

Conventional stabilization policy suggests that monetary policy controls inflation, while fiscal policy stabilizes debt through an appropriate adjustment in current or future taxation.[7] In contrast, proponents of the fiscal theory of the price level emphasize that fiscal policy can also determine the path of the price level.[8] These different views (the ones proposed by Friedman and proponents of the fiscal theory of the price level) critically depend on having rational expectations, lump sum taxation, government bonds not providing liquidity services and having frictionless financial markets. Once agents are boundedly rational, as in Evans and Honkapohja (2007) or Eusepi and Preston (2011, 2017), taxes are distortionary as in Canzoneri et al. (2016), government bonds provide liquidity services, as in Canzoneri et al. (2005, 2016) and Andolfatto and Williamson (2015), when there is uncertainty regarding the underlying policy regime, as in Davig and Leeper (2011), financial markets are not complete, as in Gomis-Porqueras (2016), or when and how central bank revenues are transferred to the fiscal authority as in Bassetto and Cui (2017), traditional stabilization policies fail to be useful.[9] Here we add to the literature by considering an endogenous liquidity premium in an economy with incomplete markets and several financial markets where agents can rebalance their portfolio.

Our paper relates to a growing literature that consider environments with a bond premium and studies monetary and fiscal policy interactions. One of the earlier works is that of Canzoneri and Diba (2005) who consider an endowment economy with a modified cash in advance constraint framework, where bonds can be used to pay for goods, They do so by specifying an exogenous bond liquidity service function. Once bonds provide liquidity, fiscal policy becomes a key determinant for inflation dynamics. As a result a peg interest rate and a passive fiscal rule can yield locally determinate equilibria. Using a similar environment, Andolfatto and Williamson (2015) allow government debt to be used as payment in some states of the world. The authors show that under an indefinite zero interest rate policy non-deflationary periods are possible when bonds have a liquidity premium. As a result, traditional monetary and fiscal policy interactions at the zero

[7] We refer to Friedman (1968) for more discussion on this point.

[8] The Fiscal Theory of the Price Level was developed primarily by Leeper (1991), Sims (1994), Woodford (1994) and Cochrane (2001). This literature highlights that bonds are denominated in nominal terms so that they may be fully backed by real resources or backed only by nominal cash flows. We refer to Canzoneri et al. (2011) and Leeper and Leith (2016) for excellent surveys of the Fiscal Theory of the Price Level. When real resources fully back debt, Sargent and Wallace (1981) obtain equilibria where fiscal policy is inflationary only if the central bank monetizes deficits. But when nominal debt is not backed by real resources, the government can trade current for future inflation through debt operations and then fiscal policies can help stabilize the price level.

[9] In particular, in all these previous cases, public debt matters for inflation dynamics.

lower bound are quite different from traditional frameworks. Within the same spirit, Bassetto and Cui (2017) show that when there is a liquidity premium on government debt, additional Taylor rule perils emerge when the economy faces persistently low real interest rate. When agents face frictional and stochastic trading opportunities and nominal government bonds as collateral in secured lending arrangements, Berensten and Waller (21016) show that if the collateral constraint binds, agents price in a liquidity premium on bonds that lowers the real rate on bonds. As a result, the market value of the government debt can fluctuate even though there are no changes to current or future taxes or spending. The price dynamics can be driven solely by the liquidity premium on the debt.

In contrast to Canzoneri and Diba (2005) and Andolfatto and Williamson (2015), our framework considers trading in a decentralized financial market for government debt, which can deliver an endogenous liquidity premium. The features of this over the counter market (search and bargaining frictions) directly impact the resulting equilibrium liquidity premia. These are considerations that have not being explored in previous work, when examining monetary and fiscal policy interactions. Here we show that these details are not as innocuous as it may seem a priori. Finally, in our environment agents can adjust their consumption through changes in their labor income, thus we do not impose a negative relationship between fiat money and bonds, which is what is implied by the *augmented* cash constraint in an endowment economy as in Canzoneri and Diba (2005). Such restriction is important as it ensures a unique monetary equilibrium as well as directly affects the potential open market operations that are consistent with implementing a Taylor rule. As these operations change the relative prices, the class of monetary and fiscal policies consistent with determinate equilibria are generally going to be different.

3 A Motivating Example

Let us consider an endowment economy where an infinitely lived household that discounts the future at a rate β derives utility from the consumption of a perishable good. Other than the representative household, there is a government that needs to finance a constant stream of expenditures, G, by issuing nominal debt, B_t, and collecting lump sum taxes, τ_t. To implement policies, the government follows simple rules. In particular, the central bank implements a Taylor rule so that nominal interest rates, R_t, are linked to inflation, Π_t. On the other hand, the fiscal authority considers a rule that links taxes to real public debt. In particular, we have that

$$R_t = \alpha_0 + \alpha \Pi_t, \tag{1}$$

$$\tau_t = \gamma_0 + \gamma \frac{B_{t-1}}{P_{t-1}}, \tag{2}$$

where P_{t-1} is the price level at time $t-1$, α_0 and α (γ_0 and γ) are the monetary (fiscal) policy parameters.

To smooth consumption, the representative household has access to nominal government bonds, while facing some market incompleteness. Other than storing value, public debt also provides liquidity services.[10] In what follows we take a reduced form approach and are not explicit about the underlying frictions of the environment that generate the bond premium and how it depends on the economic environment. In particular, we allow the premia to depend on inflation and real public debt, which we denote by $\tilde{s}_{t+1}(\Pi_{t+1}, b_t) > 0$; where b_t denotes real bonds.

In this simple and stylized environment, it is easy to show that the resulting equilibrium is characterized by the evolution of inflation and real debt, which are given by

$$\Pi_{t+1} = \beta \left[\alpha_0 + \alpha \Pi_t + \tilde{s}_{t+1}(\Pi_{t+1}, b_t)\right], \tag{3}$$

$$b_t = G - \gamma_0 + \left(\frac{1}{\beta} - \gamma - \frac{\tilde{s}_t(\Pi_t, b_{t-1})}{\Pi_t}\right) b_{t-1}; \tag{4}$$

where equation (3) corresponds to the household's first order condition for public debt, and equation (4) describes the evolution of real bonds implied by the government budget constraint.

Within this environment, we analyze how monetary and fiscal policy interactions change once different types of debt premiums are considered. We first analyze an economy where the representative agent faces a constant premium.[11] Then we analyze an economy where the premium depends on inflation and real bonds. To establish comparisons with the previous literature, we define traditionally active/passive policies termed by Leeper (1991), as follows.

Definition 1 *Monetary policy is defined as traditionally active (passive) when $\beta\alpha > 1$ ($\beta\alpha < 1$) and $\alpha_0 < 0$ ($\alpha_0 > 0$).*

Definition 2 *Fiscal policy is defined as traditionally active (passive) when $\frac{1}{\beta} - \gamma > 1$ ($\frac{1}{\beta} - \gamma < 1$) and $\gamma_0 > G$ ($\gamma_0 < G$).*

Constant Premium

When bonds provide some liquidity services that are constant overtime, we have that $\tilde{s}_{t+1}(\Pi_{t+1}, b_t) = \tilde{s}\ \forall t$. In this environment we can establish the following results.

Lemma 1 *The stationary monetary equilibria has the following properties:*

[10] These services can be viewed as stemming from two sources. Bonds can be used as a medium of exchange or they can be used as collateral for secured loans.

[11] Note that when there is no premium (the constant is zero), we recover the complete market and frictionless environment.

(i) The steady state is unique where the long run inflation and real debt are given by $\Pi_0 = \frac{\beta(\alpha_0+\tilde{s})}{(1-\alpha\beta)}$ *and* $b_0 = \frac{G-\gamma_0}{1-\left(\frac{1}{\beta}-\gamma\right)+\frac{\tilde{s}}{\Pi}}$, *respectively.*

(ii) Traditional active/passive monetary policies deliver locally determinate equilibria.

As we can see, when the premium is constant, the economy does not have real indeterminacies and we recover the same active/passive policy prescriptions that deliver local determinate equilibria as in Leeper (1991) and Woodford (1998). We can then conclude that the type of market incompleteness that delivers a constant premium does not alter the properties of the equilibrium nor the traditional policy prescriptions that deliver locally determinate equilibria.

Time Varying Premium

We now explore the equilibrium properties once agents face a premium on public debt that evolves over time. In particular, the premium on public debt depends on both real bonds, b_t, and gross inflation, Π_{t+1}, so that $\tilde{s}_{t+1}(b_t, \Pi_{t+1})$ is generally a non-linear function in both arguments.

As we can see from equation (3), once bonds provide liquidity services that are not constant over time, the evolution of inflation depends on the additional value that bonds provide. As a result, inflation dynamics are affected by real debt. Now, the fiscal authority has a direct impact on the evolution of inflation through the amount of bonds that it issues via the liquidity premium. In such environment *fiscal policy matters*, breaking the traditional dichotomy of monetary and fiscal policies observed in frictionless and complete market environments. Thus we expect to have drastically different equilibrium properties, which are summarized in the following Proposition.

Proposition 1 *With a time varying premium* $\tilde{s}_{t+1}(b_t, \Pi_{t+1})$, *the stationary monetary equilibria has the following properties:*

(i) The steady state is generically is not unique.

(ii) Traditional active/passive monetary policies are not useful in delivering locally determinate equilibria.

All proofs can be found in the Appendix.

Bego, los resultado de active/pasive no estoy 100% seguro que sean correctos, pues cuando el premium depende de (b_t, Π_{t+1}), el valor en el estado estacionario de Π y b puede ser mas grande o mas pequenyo dependiendo de la forma funcional de $\tilde{s}(b, \Pi)$.

Once public debt provides liquidity services, government liabilities exhibit a premium. This is the case as agents are willing to buy government bonds above their fundamental value. Moreover, when the bond premium depends on inflation and real debt, Ricardian equivalence does not hold anymore. As a result, the underlying wealth and substitution effects when revaluing public debt, through changes in price levels, are drastically different to environments without a liquidity premium. Thus it is not surprising that the resulting equilibrium properties and traditional active/passive monetary and fiscal policy prescriptions that deliver locally determinate equilibria are different from environments without a premium.

In this environment where *fiscal policy matters*, observing real indeterminacies is a direct consequence of agents trading in an economy that is not Ricardian. In this setting the government is able to affect the real return on public debt through changes in the inflation rate as well as the issuance of public debt. As a result, there are different combinations of inflation and real public debt that deliver the same tax revenues.[12] Thus, self-fulfilling beliefs that generate multiple steady states are possible. Moreover, dynamic indeterminacies can be observed when traditional active/passive monetary and fiscal policies are followed. It is important to highlight that the underlying mechanism delivering the multiplicity of steady states is in sharp contrast to economies with no public debt premiums. In this latter environments departures from Ricardian equivalence are a result of distortionary taxation. In contrast, here we deliver such departure by considering the liquidity services that bonds can provide.

This motivating example illustrates the importance of providing explicit frictions in the economic environment that yield bond premia when studying how monetary and fiscal policy interact. This is the case as the details of the premium can deliver quite different equilibrium properties. In the next sections we present a frictional framework that delivers a bond premia as an equilibrium outcome. In particular, we consider a frictional, stochastic and incomplete market environment based on Berentsen and Waller (2011). Such framework allow us to nest various economies that differ in terms of the severity of the market incompleteness and the development of secondary markets, which can give rise to the premium on government debt. Within this environment we study the properties of the resulting monetary equilibria and analyze the underlying monetary and fiscal policy interactions.

4 The environment

The basic structure builds on the frictional and incomplete market framework of Berentsen and Waller (2011). Time is discrete and there is a continuum of infinitively-lived agents of measure one that discount the future at a rate $\beta \in (0, 1)$. These agents have access to fiat money and nominal government bonds. These are the only durable assets in the economy. As in Lagos and Wright

[12] It is easy to rearrange equation (4) in steady state so that the tax revenue is equal to the bond seignorage

$$\tau = \gamma_0 + \gamma b = G - \left(1 - \left(\frac{1}{\beta} - \frac{\tilde{s}(\Pi, b)}{\Pi}\right)\right) b.$$

It is then possible that various combinations of Π and b yield the same tax revenue.

(2005), agents face preference shocks, have stochastic trading opportunities and sequentially trade in various markets that are characterized by different frictions. In particular, each period has three sub-periods. In the first one, after the preference shocks are realized, agents have access to a decentralized secondary market for government debt (SM).[13] In this market, government debt is traded for money in an over the counter (OTC) market, which is characterized by search frictions and bargaining. Due to search frictions, a buyer (seller) is matched with a seller (buyer) with probability $\kappa \in [0, 1]$.[14] With complementary probability, a buyer and a seller are not matched, so they cannot trade. In the second sub-period, agents can trade goods for fiat money in a decentralized frictional goods market (DM). In this market, anonymous buyers and sellers are also randomly and bilaterally matched. In particular, matches in DM are such that with probability $\sigma \in (0, 1)$, a buyer (seller) is matched with a seller (buyer).[15] Finally, in the last sub-period, agents trade in a frictionless centralized market (CM), where they can produce and consume a general good, re-adjust their portfolio as well as pay their taxes.[16]

4.1 Preferences and Technologies

Agents have preferences over consumption of the general CM perishable good (x_t), effort to produce the CM good (h_t), consumption of the specialized DM perishable good (q_t) and effort to produce the DM good (e_t). Their expected utility is then given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(x_t) - h_t + \chi \frac{q_t^{1-\xi}}{1-\xi} - e_t \right], \tag{5}$$

where $\chi>0$ captures the relative weight on DM consumption and $\xi \in (0, 1)$ is the inverse of the inter-temporal elasticity of substitution of DM consumption. Finally, E_0 denotes the linear expectation operator with respect to an equilibrium distribution of idiosyncratic agent types.

All perishable goods in the economy are produced according to a linear technology where labor is the only input. The production function is such that one unit of labor yields one unit of output.

[13] In contrast to Berentsen and Waller (2011), we consider an over the counter market rather than a competitive market. This is the case as secondary markets for public debt around the world are mostly structured as OTC markets with bargaining and search/informational frictions, rather than competitive financial markets.

[14] The value of κ captures the accessibility of these secondary markets faced by agents.

[15] The magnitude of σ give us the degree of market incompleteness faced by agents.

[16] An alternative specification to the DM/CM structure to generate a demand for money would be a cash/credit framework, as in Lucas and Stokey (1983). However, such environments imply a constant velocity of money and no variability in the demand for liquidity, features that we do not want to impose in our environment. Such properties restrict how the government implements open market operations (exchanging bonds for money) that are going to be consistent with a central bank following a Taylor rule and having a fiscal rule that links taxes to government debt. Given that the underlying economy is frictional and incomplete, this allows the possibility for open market operation to have real effects. Thus imposing the underlying restrictions of the cash/credit framework are not innocuous when thinking about monetary and fiscal policies that deliver determinate equilibria.

4.2 Government

The government must finance a constant stream of exogenous expenditures, $G>0$, through lump sum CM taxes and by issuing nominal bonds as well as fiat money. The corresponding per period government budget constraint is given by

$$\tau_t^{CM} + \phi_t M_t + \phi_t B_t = G + \phi_t M_{t-1} + \phi_t R_{t-1} B_{t-1}; \tag{6}$$

where M_t denotes money supply at time t, B_t represents nominal bonds, R_{t-1} is the gross nominal interest rate on bonds issued at $t-1$, τ_t^{CM} represents lump sum taxes in CM and ϕ_t is the real price of money in terms of the CM good. The real value of all bond issues at every period is assumed to be bounded above by a sufficiently large constant as to avoid Ponzi schemes.

To implement monetary policy, the central bank follows a Taylor rule so that nominal interest rates are linked to inflation. This can be achieved through appropriate open market operations in CM. The fiscal authority considers a rule, whereby taxes are related to the previous level of real debt. These rules are given by

$$R_t = \alpha_0 + \alpha \, \Pi_t, \tag{7}$$

$$\tau_t^{CM} = \gamma_0 + \gamma \, \phi_{t-1} B_{t-1}, \tag{8}$$

where $\Pi_t = \frac{\phi_{t-1}}{\phi_t}$ denotes the gross inflation rate at time t and $\alpha_0, \alpha, \gamma_0$ and γ are constants that determine the responsiveness of monetary and fiscal rules to inflation and real debt, respectively. While typically these policy rules may not be optimal, these rules have been extensively analyzed in the macroeconomic literature as stabilization tools. More precisely, particular combinations of monetary (α) and fiscal (γ) policies are known to deliver locally determinate equilibria under various environments with frictionless financial markets.

It is important to highlight that the underlying open market operations consistent with the implementation of monetary and fiscal policy can have real effects when agents trade in frictional and incomplete markets. Thus not having an equilibrium money to bond ratio is not without loss of generality.

4.3 Agent's Problem

Given the sequential nature of the problem, we solve the representative agent's problem backwards. Thus we first solve the CM problem, then the DM and finally solve the SM problem, respectively.

4.3.1 CM Problem

In this market, all agents can produce and consume the general consumption good, x_t and trade in a frictionless competitive market. Thus, a medium of exchange is not essential in CM. Agents can settle their CM trades with any assets, CM goods or CM labor.

An agent in period t enters CM with a portfolio of fiat money ($\tilde{M}_{t-1}$) and nominal government bonds ($\tilde{B}_{t-1}$). This portfolio is different across agents, depending on the type of preference shock they have previously received. In particular, the portfolio when entering CM reflects whether they were able to trade in SM or not and if they had the opportunity to trade in the previous DM. We refer the reader to the Appendix for the various initial CM portfolios before trade occurs that agents can have.

Given the portfolio ($\tilde{M}_{t-1}$, $\tilde{B}_{t-1}$), the problem of the representative agent in CM can be written as follows

$$\begin{aligned} W(\tilde{M}_{t-1}, \tilde{B}_{t-1}) &= \max_{x_t, h_t, M_t, B_t} \left\{ \ln(x_t) - h_t + \ \beta \, V^{SM}(M_t, B_t) \right\} \\ \text{s.t. } \ x_t + \phi_t M_t + \phi_t B_t &= h_t - \tau_t^{CM} + \phi_t \tilde{M}_{t-1} + \phi_t R_{t-1} \tilde{B}_{t-1}, \end{aligned} \tag{9}$$

where V^{SM} is the expected value function of an agent for the next period SM. After the preference shock has been realized, agents may have the possibility to trade in SM and adjust their liquidity, by trading fiat money for nominal bonds.

The corresponding first order conditions are given by

$$\frac{1}{x_t} - 1 = 0, \tag{10}$$

$$-\phi_t + \beta \, \frac{\partial V^{SM}(M_t, B_t)}{\partial M_t} = 0, \tag{11}$$

$$-\phi_t + \beta \, \frac{\partial V^{SM}(M_t, B_t)}{\partial B_t} = 0, \tag{12}$$

and the associated envelope conditions are $\frac{\partial W_t}{\partial M_{t-1}} = \phi_t$ and $\frac{\partial W_t}{\partial B_{t-1}} = \phi_t R_{t-1}$.

4.3.2 DM Problem

Before CM and right after SM, buyers/sellers enter DM. This market is characterized by random and bilateral trading opportunities as well as a lack of record-keeping services. Matches in DM are such that with probability $\sigma \in (0, 1)$, a buyer (seller) is matched with a seller (buyer). As in Aruoba and Chugh (2010), Berentsen and Waller (2011) and Martín (2011), among others, government bonds are viewed as book-entries in the government's record.[17] Since sellers do not have access to record-keeping services in this market, nominal bonds will not be accepted as a means of payment in DM. Moreover, since agents are anonymous, sellers are not going to extend unsecured credit to buyers when purchasing DM goods. Thus, the only feasible trade is the exchange of DM goods for fiat money.

An agent in period t enters DM with a portfolio of fiat money ($\hat{M}_{t-1}$) and nominal government bonds ($\hat{B}_{t-1}$). These will differ across agents depending on the preference shock they have received at the beginning of the period as well as their trading opportunities in SM. We refer the reader to the Appendix for these various portfolios.

The expected utility of a buyer that has traded in the previous SM and enters DM with a portfolio ($\hat{M}_{t-1}, \hat{B}_{t-1}$) is then given by

$$V_{b,\kappa}^{DM}(\hat{M}_{t-1}, \hat{B}_{t-1}) = \sigma \left[\chi \frac{{q_t^S}^{1-\xi}}{1-\xi} + W(\hat{M}_{t-1} - D_t^{M_s}, \hat{B}_{t-1}) \right] + (1-\sigma) W(\hat{M}_{t-1}, \hat{B}_{t-1}),$$

where q_t^S denotes the DM quantity of goods purchased in DM when the buyer has traded in SM and $D_t^{M_s}$ represents the corresponding cash payment. By feasibility, buyers cannot pay more than the fiat money they brought into the match so that $D_t^{M_s} \leq \hat{M}_{t-1}$.

When the buyer has not been able to trade in SM, his expected utility entering DM with a portfolio ($\hat{M}_{t-1}, \hat{B}_{t-1}$) is then given by

$$V_{b,1-\kappa}^{DM}(\hat{M}_{t-1}, \hat{B}_{t-1}) = \sigma \left[\chi \frac{{q_t}^{1-\xi}}{1-\xi} + W(\hat{M}_{t-1} - D_t^{M}, \hat{B}_{t-1}) \right] + (1-\sigma) W(\hat{M}_{t-1}, \hat{B}_{t-1}),$$

where q_t denotes the DM quantity of goods consumed in DM when the buyer has not traded in SM and D_t^M is the corresponding cash payment. As in the previous state of the world, buyers cannot pay more than the fiat money they brought into the match, thus we have that $D_t^M \leq \hat{M}_{t-1}$. Note that these buyers will have fewer cash balances to buy in DM, as they did not have an opportunity to rebalance their portfolio in the secondary market.

Similarly, the expected utility of a seller that has traded in the previous SM and enters DM with a portfolio ($\hat{M}_{t-1}, \hat{B}_{t-1}$) is given by

$$V_{s,\kappa}^{DM}(\hat{M}_{t-1}, \hat{B}_{t-1}) = \sigma \left[-q_t^S + W(\hat{M}_{t-1} + D_t^{M_s}, \hat{B}_{t-1}) \right] + (1-\sigma) W(\hat{M}_{t-1}, \hat{B}_{t-1}),$$

while the expected utility of a seller that has not traded in the previous SM and enters DM with

[17] Alternatively, this could be interpreted as a fraction of sellers where government bonds are not recognized as in Shi (2014) or Rocheteau, Wright and Xiao (2016). This could be endogenized as in Lester et al. (2012) or as Li et al. (2012). This treatment is beyond the scope of this paper.

a portfolio $(\hat{M}_{t-1}, \hat{B}_{t-1})$ is given by

$$V^{DM}_{s,1-\kappa}(\hat{M}_{t-1}, \hat{B}_{t-1}) = \sigma\left[-q_t + W(\hat{M}_{t-1} + D^M_t, \hat{B}_{t-1})\right] + (1-\sigma)W(\hat{M}_{t-1}, \hat{B}_{t-1}).$$

The terms of trade in DM are determined ex-post by a buyer take it or leave it offer. In order to induce trade in DM, buyers need to offer terms of trade that satisfy the seller's participation constraint and their cash feasibility constraint. For buyers that have not been able to trade in the previous SM, the terms of trade solve the following problem

$$\max_{q_t, D^M_t}\left\{\chi\frac{{q_t}^{1-\xi}}{1-\xi} + W(M_{b,t-1} - D^M_t, B_{b,t-1})\right\} \text{ s.t.}$$

$$M_{b,t-1} - D^M_t \geq 0,$$

$$-q_t + W(M_{s,t-1} + D^M_t, B_{s,t-1}) \geq W(M_{s,t-1}, B_{s,t-1}),$$

where $M_{b,t-1}(M_{s,t-1})$ and $B_{b,t-1}(B_{s,t-1})$ represent the buyer's (seller's) fiat money and nominal bond holdings, respectively, when trading in DM. The previous problem yields the following first order conditions

$$\frac{\chi}{{q_t}^{\xi}} = 1 + \lambda_t,$$

$$\lambda_t(M_{b,t-1} - D^M_t) = 0,$$

$$q_t = \phi_t D^M_t,$$

where λ_t denotes the Lagrange multiplier associated with the payment feasibility constraint. It is important to note that the optimal terms of trade do not depend on whether the seller has previously traded in SM or not. This is the case as the CM value function is linear.

Similarly, for buyers that have been able to trade in the previous SM, the terms of trade in DM are given by

$$\frac{\chi}{q_t^{S\xi}} = 1 + \lambda^s_t,$$

$$\lambda^s_t(M_{b,t-1} + D^{M^o}_t - D^{M_s}_t) = 0,$$

$$q^S_t = \phi_t D^{M_s}_t;$$

where λ^s_t represents the Lagrange multiplier associated with the payment feasibility constraint when the agent has previously traded in SM. Relative to the previous case, here buyers have access to more fiat money as they have been able to trade some bonds for fiat money in the previous SM.

These various terms of trade imply the following envelope conditions for fiat money

$$\frac{\partial V^{DM}_{b,\kappa}}{\partial M_{b,t-1}} = \sigma\left[\frac{\chi}{q^{S\xi}_t}\frac{\partial q^S_t}{\partial M_{b,t-1}} - \phi_t\frac{\partial D^{M_s}_t}{\partial M_{b,t-1}} + \phi_t\right] + (1-\sigma)\phi_t,$$

$$\frac{\partial V^{DM}_{b,1-\kappa}}{\partial M_{b,t-1}} = \sigma\left[\frac{\chi}{q_t{}^{\xi}}\frac{\partial q_t}{\partial M_{b,t-1}} - \phi_t\frac{\partial D^{M}_t}{\partial M_{b,t-1}} + \phi_t\right] + (1-\sigma)\phi_t,$$

while for bonds we have that

$$\frac{\partial V^{DM}_{b,\kappa}}{\partial B_{b,t-1}} = \frac{\partial V^{DM}_{b,1-\kappa}}{\partial B_{b,t-1}} = \phi_t R_{t-1}.$$

For the seller, we obtain similar envelope expressions, which are given by

$$\frac{\partial V^{DM}_{s,\kappa}}{\partial M_{s,t-1}} = \phi_t,\ \ \frac{\partial V^{DM}_{s,1-\kappa}}{\partial M_{s,t-1}} = \phi_t,\ \ \frac{\partial V^{DM}_{s,\kappa}}{\partial B_{s,t-1}} = \frac{\partial V^{DM}_{s,1-\kappa}}{\partial B_{s,t-1}} = \phi_t R_{t-1}.$$

Throughout the rest of the paper we focus on monetary equilibria with positive nominal interest rates so that $R_t > 1$. This type of equilibria then implies that $\lambda_t > 0$, so that buyers that have not been able to trade in the previous SM spend all their money when purchasing DM goods. Thus we have that $\frac{\partial D^M_t}{\partial M_{t-1}} = 1$. For buyers that were able to trade in the SM, their cash constraint may or not bind.

4.3.3 SM Problem

At the beginning of each period, agents experience a preference shock that determines whether they are a buyer or a seller in the ensuing DM. After this preference shock is realized, agents enter a secondary market for government debt where they can re-adjust their portfolio according to their new liquidity needs. The SM is an OTC financial market that is characterized by random trading opportunities and bargaining.[18] Matches in this market are such that with probability $\kappa \in [0,1]$, a buyer (seller) is matched with a seller (buyer). With complementary probability, a buyer (seller) is not matched, thus cannot trade in SM.

The expected utility of an agent entering SM with a portfolio (M_{t-1}, B_{t-1}) is then given by

$$\begin{aligned} V^{SM}(M_{t-1},B_{t-1}) = \frac{1}{2}\left[\kappa\, V^{DM}_{b,\kappa}(M_{t-1}{+}a_tD^{B^o}_t, B_{t-1}{-}D^{B^o}_t) + (1-\kappa)\, V^{DM}_{b,1-\kappa}(M_{t-1},B_{t-1})\right] + \\ +\frac{1}{2}\left[\kappa\, V^{DM}_{s,\kappa}(M_{t-1}{-}a_tD^{B^o}_t, B_{t-1}{+}D^{B^o}_t) + (1-\kappa)\, V^{DM}_{s,1-\kappa}(M_{t-1},B_{t-1})\right], \end{aligned}$$

where $\frac{1}{2}$ reflects that an agent has equal probability to be either a buyer or a seller in the ensuing DM and $V^{DM}_{j,n}$ represents the value function of trading in DM where $j = \{b, s\}$ and $n = \{\kappa, 1-\kappa\}$.

[18] Berentsen et al. (2014) consider a similar environment where agents face an exogenous probability that dictates whether they can participate or not in a competitive and Walrasian secondary market for government debt.

The terms of trade in the OTC market are $(a_t, D_t^{B^o})$, where a_t denotes the price per unit of bonds and $a_t D_t^{B^o}$ represents the total units of money received by the buyer. These terms of trade are determined ex-post by a buyer take it or leave it offer. It is important to note that when determining the terms of trade, agents do not know if they will have an opportunity to trade in the ensuing DM. Moreover, the threat point of both the buyer and seller is to not trade in the OTC. This is equivalent to the value of not having had the opportunity to trade in the OTC. Thus, the terms of trade in the OTC solves the following problem

$$\begin{aligned} \max_{a_t, D_t^{B^o}} \left\{ V_{b,\kappa}^{DM} - V_{b,1-\kappa}^{DM} \right\} \text{ s.t.} \\ V_{s,\kappa}^{DM} - V_{s,1-\kappa}^{DM} \geq 0, \\ a_t D_t^{B^o} \leq M_{s,t-1}, \\ D_t^{B^o} \leq B_{b,t-1}. \end{aligned}$$

Using the previous expressions for the DM value functions, the OTC terms of trade can then be written as follows

$$\begin{aligned} \max_{a_t, D_t^{B^o}} \left\{ \sigma \left[\chi \frac{{q_t^S}^{1-\xi} - {q_t}^{1-\xi}}{1-\xi} + \phi_t \left(D_t^M - D_t^{M_s} \right) \right] + \phi_t \left(a_t D_t^{B^o} - D_t^{B^o} \right) \right\} \text{ s.t.} \\ \sigma \left[-q_t^S + q_t - \phi_t \left(D_t^M - D_t^{M_s} \right) \right] - \phi_t \left(a_t D_t^{B^o} - D_t^{B^o} \right) \geq 0, \\ a_t D_t^{B^o} \leq M_{s,t-1}, \\ D_t^{B^o} \leq B_{b,t-1}. \end{aligned}$$

Using the fact that the differential payment in DM for the two different states of the world in SM is $D_t^M - D_t^{M_s} = -a_t D_t^{B^o}$, and that the amount produced in DM for buyers that have traded in SM is $q_t^S = \phi_t \left(M_{t-1} + a_t D_t^{B^o} \right)$, the corresponding first-order conditions for a_t and $D_t^{B^o}$ are given by

$$a_t : \quad \sigma \left\{ \chi \left(q_t^S \right)^{-\xi} - 1 \right\} + 1 - \epsilon_t^o - \mu_t^s = 0,$$

$$D_t^{B^o} : \; a_t \sigma \left\{ \chi \left(q_t^S \right)^{-\xi} - 1 \right\} + (a_t - 1) - \epsilon_t^o \left(a_t - 1 \right) - \mu_t^s a_t - \mu_t^b = 0,$$

where μ_t^s (μ_t^b) corresponds to the Lagrange multiplier of the seller (buyer) when trading in SM.

From the first order condition, we have that $\epsilon_t^o = \sigma \left\{ \chi \left(q_t^S \right)^{-\xi} - 1 \right\} + 1 - \mu_t^s$. From the second optimality condition, we can then establish the following

$$\sigma \left\{ \chi \left(q_t^S \right)^{-\xi} - 1 \right\} - \mu_t^s - \mu_t^b = 0.$$

Since μ_t^s and μ_t^b are non-negative, we have that $\sigma(\chi \left(q_t^S \right)^{-\xi} - 1) - \mu_t^s \geq 0$, which in turn implies that $\epsilon_t^o \geq 1 > 0$. Thus, the terms of trade in the OTC market are such that the seller just gets the outside option. Using the fact that $q_t^S = \phi_t \left(M_{t-1} + a_t D_t^{B^o}\right)$, $q_t = \phi_t M_{t-1}$ and that $D_t^M - D_t^{M_s} = -a_t D_t^{B^o}$, it is easy to show that in equilibrium the price is such that $a_t = 1$. In addition, in equilibrium, $(D_t^{B^o} \mu_t^s, \mu_t^b)$ must satisfy the following conditions

$$\begin{aligned}\left(M_{s,t-1} - D_t^{B^o}\right) \mu_t^s = 0, \text{ and } D_t^{B^o} \leq M_{s,t-1},\\ \left(B_{b,t-1} - D_t^{B^o}\right) \mu_t^b = 0, \text{ and } D_t^{B^o} \leq B_{b,t-1},\\ \sigma \left\{\chi \left(q_t^S\right)^{-\xi} - 1\right\} - \mu_t^s - \mu_t^b = 0.\end{aligned}$$

There are three possible terms of trade that can be observed in equilibrium.

Case 1. The bond and fiat money payments bind in SM, which implies

$$\begin{aligned}\mu_t^s > 0, \text{ and } D_t^{B^o} = M_{s,t-1},\\ \mu_t^b > 0, \text{ and } D_t^{B^o} = B_{b,t-1},\\ \sigma \left\{\chi \left(q_t^S\right)^{-\xi} - 1\right\} = \mu_t^s + \mu_t^b.\end{aligned}$$

Case 2. Only the fiat money payment in SM is binding, which implies

$$\begin{aligned}\mu_t^s = \sigma \left\{\chi \left(q_t^S\right)^{-\xi} - 1\right\} > 0, \text{ and } D_t^{B^o} = M_{s,t-1},\\ \mu_t^b = 0, \text{ and } D_t^{B^o} < B_{b,t-1}.\end{aligned}$$

Case 3. Finally, when only the bond payment in SM is binding, the terms of trade are such that

$$\begin{aligned}\mu_t^s = 0, \text{ and } D_t^{B^o} < M_{s,t-1},\\ \mu_t^b = \sigma \left\{\chi \left(q_t^S\right)^{-\xi} - 1\right\} > 0, \text{ and } D_t^{B^o} = B_{b,t-1}.\end{aligned}$$

Having characterized all possible terms of trade, we can determine the properties of the SM value function. An agent at the beginning of the period, before the preference shocks and trading opportunities have been realized, has an expected SM value function that is given by

$$\begin{aligned}V^{SM}(M_{t-1}, B_{t-1}) = \frac{1}{2} &\left[\kappa \, V_{b,\kappa}^{DM}(M_{t-1}{+}a_t D_t^{B^o}, B_{t-1}{-}D_t^{B^o}) + (1-\kappa) \, V_{b,1-\kappa}^{DM}(M_{t-1}, B_{t-1})\right]\\ \frac{1}{2} &\left[\kappa \, V_{s,\kappa}^{DM}(M_{t-1}{-}a_t D_t^{B^o}, B_{t-1}{+}D_t^{B^o}) + (1-\kappa) \, V_{s,1-\kappa}^{DM}(M_{t-1}, B_{t-1})\right]\\ +\frac{1}{2}\epsilon_t^o &\left[V_{s,\kappa}^{DM} - V_{s,1-\kappa}^{DM}\right] + \frac{1}{2}\mu_t^s \phi_t \left(M_{s,t-1} - a_t D_t^{B^o}\right) + \frac{1}{2}\mu_t^b \phi_t \left(B_{b,t-1} - D_t^{B^o}\right).\end{aligned}$$

To be able to determine the optimal portfolio allocation, given by equations (11) and (12), we need to calculate the marginal effect of bringing an additional unit of money and nominal bonds in SM. Using previous results, we have that

$$\frac{\partial V^{SM}(M_{t-1},B_{t-1})}{\partial M_{t-1}} = \phi_t + \phi_t\frac{1}{2}\sigma\left[\kappa\left(\frac{\chi}{{q_t^S}^{\xi}}-1\right)+(1-\kappa)\left(\frac{\chi}{{q_t}^{\xi}}-1\right)\right]+\frac{1}{2}\phi_t\mu_t^s,$$

$$\frac{\partial V^{SM}(M_{t-1},B_{t-1})}{\partial B_{t-1}} = \phi_t R_{t-1} + \frac{1}{2}\phi_t\mu_t^b,$$

which imply the following CM inter-temporal Euler equations

$$\phi_t = \beta\phi_{t+1}\left\{1+\frac{1}{2}\sigma\left[\kappa\left(\frac{\chi}{{q_{t+1}^S}^{\xi}}-1\right)+(1-\kappa)\left(\frac{\chi}{{q_{t+1}}^{\xi}}-1\right)\right]+\frac{1}{2}\mu_{t+1}^s\right\}, \tag{13}$$

$$\phi_t = \beta\phi_{t+1}\left(R_t+\frac{1}{2}\mu_{t+1}^b\right). \tag{14}$$

5 Monetary Equilibrium

Given the policy rules ($R_t = \alpha_0 + \alpha\Pi_t$ & $\tau_t^{CM} = \gamma_0 + \gamma\phi_{t-1}B_{t-1}$) public spending $\{G\}_{t=0}^{\infty}$ and initial conditions (M_{-1}, B_{-1}), a dynamic monetary equilibrium is a sequence of consumptions $\{x_t, q_t, q_t^S\}_{t=0}^{\infty}$, assets and prices $\{M_t, B_t, D_t^{B^o}, \phi_{t+1}, a_t, \mu_{b,t}, \mu_{s,t}\}_{t=0}^{\infty}$ satisfying market clearing and agents' problem, which imply the following conditions

$$x_t = 1, \tag{15}$$

$$q_t = \phi_t M_{t-1}, \tag{16}$$

$$q_t^S = \phi_t(M_{t-1} + a_t D_t^{B^o}), \tag{17}$$

$$a_t = 1, \tag{18}$$

$$\left(M_{s,t-1} - D_t^{B^o}\right)\mu_t^s = 0, \text{ and } D_t^{B^o} \leq M_{s,t-1}, \tag{19}$$

$$\left(B_{b,t-1} - D_t^{B^o}\right)\mu_t^b = 0, \text{ and } D_t^{B^o} \leq B_{b,t-1}, \tag{20}$$

$$\sigma\left\{\chi\left(q_t^S\right)^{-\xi}-1\right\} - \mu_t^s - \mu_t^b = 0, \tag{21}$$

$$\phi_t = \beta\phi_{t+1}\left(R_t+\frac{1}{2}\mu_{t+1}^b\right), \tag{22}$$

$$\phi_t = \beta\phi_{t+1}\left\{1+\frac{1}{2}\sigma\left[\kappa\left(\frac{\chi}{{q_{t+1}^S}^{\xi}}-1\right)+(1-\kappa)\left(\frac{\chi}{{q_{t+1}}^{\xi}}-1\right)\right]+\frac{1}{2}\mu_{t+1}^s\right\}, \tag{23}$$

$$\tau_t^{CM} + \phi_t M_t + \phi_t B_t = G + \phi_t M_{t-1} + \phi_t R_{t-1} B_{t-1}. \tag{24}$$

Depending whether agents, face market incompleteness, have the possibility to trade in SM, and, if they do, whether the various multipliers are strictly positive or not, we are going to observe different prices and interest rates. These various scenarios will result in vastly different inflation and bond dynamics.

5.1 No Trading in Secondary Markets

Here we analyze two extreme situations that are consistent with agents not trading in secondary markets for public debt.

Incomplete and Less Developed Financial Markets

Here we characterize a monetary equilibrium for an economy where agents are not able to trade in SM; i.e, $\kappa = 0$.[19] The resulting monetary equilibrium is described by the evolution of inflation, Π_{t+1}, and real bond holdings $b_t = \phi_t B_t$. These are given by

$$\Pi_{t+1} = \beta(\alpha_0 + \alpha \Pi_t),$$

$$b_t = G - \gamma_0 + \left(\frac{1}{\beta} - \gamma\right) b_{t-1} + \frac{m_{t-1}}{\Pi_t} - m_t,$$

where $m_t = \phi_t M_t$ denotes real money balances that satisfy the following condition

$$\frac{1}{2}\sigma \left(\chi \frac{\Pi_{t+1}^{\xi}}{m_t{}^{\xi}} - 1\right) = \alpha_0 + \alpha \Pi_t - 1.$$

As we can see, the evolution of future inflation is independent of real government bonds, as in Leeper (1991), among others. For our environment, we find the following results. From now on, we refer the reader to the Appendix for all the proofs.

Proposition 2 *The stationary monetary equilibrium of an economy where agents cannot trade in SM is unique. Traditional active/passive monetary and fiscal policy prescriptions deliver locally determinate equilibria.*

In an incomplete market economy where agents cannot trade in secondary markets, bonds are priced fundamentally and Ricardian equivalence holds. As a result, the steady state inflation is unique and equal to $\Pi = \frac{\beta \alpha_0}{(1-\alpha\beta)}$. For comparison purposes, from now we denote such long run inflation as Π_0. For this equilibrium, we obtain the same stabilization policy prescription as in

[19] This environment would correspond to an economy with fewer financial innovations.

Leeper (1991) or Woodford (1994, 1998); where traditionally active (passive) monetary policy $\beta\alpha > 1$ ($\beta\alpha < 1$) together with passive (active) fiscal policy $\frac{1}{\beta} - \gamma < 1$ ($\frac{1}{\beta} - \gamma > 1$) yield locally determinate equilibria.

Complete Markets

Here we characterize an equilibrium for an economy where agents do not face market incompleteness; i.e, $\sigma = 0$. The only goods market is CM and in such market, any medium of exchange is available to agents. Note that in such environment, agents will decide not to carry real balances across periods, as it is costly. As a result, agents will choose not to trade in secondary markets for public debt. The resulting monetary equilibrium is given by

$$\Pi_{t+1} = \beta(\alpha_0 + \alpha\Pi_t),$$

$$b_t = G - \gamma_0 + \left(\frac{1}{\beta} - \gamma\right) b_{t-1}.$$

As we can see, we recover the same decoupled dynamic monetary equilibrium as in the frictionless and cashless environments of Woodford (1998). Moreover, the evolution of future inflation is independent of real government bonds, as in Leeper (1991) and Woodford (1998), among others.

Proposition 3 *The stationary monetary equilibrium of a complete market economy where agents do not trade in SM is unique. Traditional active/passive monetary and fiscal policy prescriptions deliver locally determinate equilibria.*

In this economy bonds are also priced fundamentally and Ricardian equivalence holds. Thus inflation expectations generated in this monetary equilibrium are the same as those observed when agents face incomplete markets but do not have access to secondary markets. The resulting properties are also consistent with models with frictionless and perfect financial markets of Leeper (1991) and others. We can conclude that not having a premium in bonds is key in delivering traditional results, not the severity of the market incompleteness.

5.2 Trading in Secondary Markets

We now explore the implications for the resulting monetary equilibrium for economies with $\kappa > 0$ and $\sigma > 0$, so that agents can trade in SM. When characterizing the monetary equilibria, we established that depending on the fundamentals of the economy, we can observe three different types of monetary equilibria. Case 1 is consistent with a wide range of interest rates. In contrast, Cases 2 and 3 occur only for a small measure of nominal interest rates. In what follows, we analyze the dynamic monetary equilibrium for Case 1, where agents trade SM and their corresponding cash and bond constraints bind.[20]

This dynamic monetary equilibrium is given by the following evolution of inflation and bonds

$$\Pi_{t+1} = \beta\alpha_0 + \beta\alpha\Pi_t + \beta\tilde{s}_{t+1}, \tag{25}$$

$$b_t = \frac{1}{2}(G - \gamma_0) + \frac{1}{2}\left(\frac{1}{\beta} - \gamma + \frac{(1 - \tilde{s}_t)}{\Pi_t}\right) b_{t-1}, \tag{26}$$

where the liquidity premium $\tilde{s}_{t+1} \equiv \frac{1}{2}\mu^b_{t+1}$ equals $\tilde{s}_{t+1} = \frac{1}{2}\left(\hat{\theta}_{t+1} + (1-\sigma) - (\alpha_0 + \alpha\Pi_t)\right)$, with $\hat{\theta}_{t+1} = \frac{1}{2}\sigma\chi\left(\frac{\Pi_{t+1}}{2b_t}\right)^{\xi}\left[1 + \kappa + (1-\kappa)2^{\xi}\right]$. After repeated substitution, the dynamic monetary equilibrium can be written as

$$\Pi_{t+1} = \frac{\beta}{2}\left(\alpha_0 + \alpha\Pi_t + \hat{\theta}_{t+1} + 1 - \sigma\right), \tag{27}$$

$$b_t = \frac{1}{2}(G - \gamma_0) + \left[\frac{1}{\beta} - \frac{1}{2}\gamma + \frac{1}{2}\left(1 - \hat{\theta}_t - (1-\sigma)\right)\frac{1}{\Pi_t}\right] b_{t-1}. \tag{28}$$

Note that $\hat{\theta}_{t+1}$ depends negatively on the ratio $\left(\frac{b_t}{\Pi_{t+1}}\right)$. We can then conclude that an economy with market incompleteness and trading in secondary markets for debt can generate a premia on bonds that depends on both bonds and inflation. We have thus provided some structure that generate the properties we consider in the motivating example.[21]

As we can see, when buyers and sellers trade in SM and both of their payment constraints bind, the evolution of inflation depends on the bond liquidity premium. As a result, inflation dynamics are affected by real government bonds. Now, the fiscal authority has a direct impact on the evolution of inflation through the amount of bonds that it issues.[22] This is a direct consequence of having an incomplete frictional goods market, where the only feasible trade is one where goods are exchanged with fiat money. As a result, by previously trading government bonds, buyers can expand their consumption possibilities in the incomplete and frictional goods market. Then, the price that agents are willing to pay is above its fundamental value, thus delivering a liquidity premium and breaking the Ricardian equivalence. Note that the fiscal authority, by changing the amount of bonds that are issued, can directly affect the liquidity premium and therefore affect the inflation dynamics. This in sharp contrast to cashless environments with frictionless financial markets (Woodford (1998)) or even monetary economies with frictionless financial markets (Leeper (1991), Woodford (1994) or Sims (1994), among others). In these different environments, the

[20]For this equilibrium, the cash constraint in DM binds even for those who traded in SM, i.e. $\lambda^s_t > 0$. This implies that bonds are scarce.

[21]We refer the reader to the Appendix for the details of the derivation found in the text.

[22]In environments with frictionless and complete financial markets, as in Leeper (1991), Woodford (1994), Sims (1994) among others, bonds do not affect for inflation dynamics.

implied open market operations required to implemented the Taylor rule are quite different.

Finally, as in Berensten and Waller (2016), when public bonds exhibit a liquidity premium, the market value of government liabilities can fluctuate even though there are no changes to current or future taxes or spending. Price dynamics can be driven solely by the liquidity premium on public debt.

Given these equilibrium properties, it is not too surprising that in this non-Ricardian environment, the underlying wealth and substitution effects when revaluing public debt, through changes in price levels, are drastically different to environments without a liquidity premium. Thus we expect that traditional active/passive monetary and fiscal policy prescriptions are unlikely to deliver locally determinate equilibria.

Next we examine some of the properties of these monetary equilibria. We first consider the implications for stationary monetary equilibria.

Lemma 2 *Consider the monetary equilibrium where buyers and sellers in SM trade and are constrained. When monetary policy is traditionally passive (active), the steady state inflation,* Π*, is higher (lower) than the one with no SM,* Π_0*.*

This result is independent of the underlying mechanism that leads to a positive government debt premium. All is needed is that there exists such premium and that monetary policy follows the Taylor rule. This finding then suggests that trading in secondary markets, consistent with the Great Moderation, can further amplify the effectiveness of active monetary policies in reducing steady state inflation. This has been an aspect that has not been highlighted by the literature and is partly driven by the liquidity premium on public debt.

Proposition 4 *The stationary monetary equilibrium of an economy where buyers and sellers in SM trade and are constrained is generically not unique.*

It is important to note that this multiplicity result is similar to the real indeterminacy found in Benhabib et al. (2001) and Bassetto and Cui (2017). As in Benhabib et al. (2001), the non-linearities in the inflation dynamics are key in delivering real indeterminacies. However, the mechanism that generates the multiplicity of steady states in this paper is different. In our environment, it is a direct consequence of the liquidity properties of government bonds. Given that bonds are scarce, the liquidity premium is affected by bonds outstanding. This implies that the nominal interest rate depends on the level of real bonds. As a result, the total interest payment on bonds is non-linear, generating a *relative* bond seigniorage that is entirely driven by the liquidity needs of DM buyers. In this equilibrium, buyers are willing to pay prices for government bonds that are above their fundamental value.[23] Now the government cannot only affect the *relative*

[23] This bond liquidity Laffer curve effect is also found in Gomis-Porqueras (2016).

bond seigniorage through inflation, now the actual size of the public debt also affects it. This new fiscal environment critically alters the expectations about future inflation, as the fiscal backing of bonds is different to an economy without a liquidity premium on government bonds. These liquidity features have important implications for the evolution of inflation and public debt. In this section we examine these consequences.

When multiple steady states are possible, we are faced with real indeterminacies. Moreover, increased volatility can be observed as one can always construct sunspot equilibria between those steady states.[24] Are there any policies that can help rule-out real indeterminacies and reduce the scope for additional volatility?

Proposition 5 *When buyers and sellers in SM trade and are constrained, there exist adequate monetary and fiscal policies* ($\frac{2}{\beta} - \alpha = 0$, *and* $2 - \alpha + \gamma = 0$) *that deliver a unique monetary steady state.*

As we can see, a traditional aggressive monetary policy ($\alpha > \frac{2}{\beta}$) alone or an aggressive monetary coupled with an adequate fiscal policies ($\alpha = 2+\gamma$) are able to rule-out real indeterminacies. These results also imply, cetirus paribus, that aggressive monetary policies are more likely to generate a unique monetary steady state. This finding then suggests that having an aggressive monetary policy is even more important during the Great Moderation, which was characterized by increased trading in secondary markets.

Lemma 3 *Traditional monetary and fiscal active/passive policy prescriptions are not useful in delivering locally determinate equilibria.*

It is easy to check that the values outside of the main diagonal in the Jacobian may not be zero. In addition, the values in the main diagonal are very different to the ones found when there was no trade in SM. Both results are a direct consequence of the liquidity premium on public debt, as it affects both inflation dynamics and the tax burden. This feature creates a link between the path of government debt, taxes and inflation. As a result, the effectiveness of government policies cannot be independent of each other as both fiscal and monetary policies simultaneously affect the monetary and fiscal eigenvalue. This is in sharp contrast to environments where financial markets are complete and frictionless.

Lemma 4 *The specifics of the monetary and fiscal rules* ($\alpha_0, \alpha, \gamma_0, \gamma$) *critically affect the steady state values for inflation and real debt, which ultimately affect the effectiveness of traditional active and passive policies* (α, γ) *in delivering locally determinate equilibria.*

[24] We refer the reader to Azariadis (1981) and Cass and Shell (1983), among others, for more detailed discussion on sunspot equilibria.

In our environment, the steady state levels of inflation and real debt affect the nature of the stabilization policies that rule out indeterminate equilibria. This is the case as the values of the specific parameters of the monetary and fiscal rules $(\alpha_0, \alpha, \gamma_0, \gamma)$ affect the position of the economy in the *bond liquidity Laffer* curve. This in turn changes the potential for self-fulfilling values of real bonds that are consistent with a balanced government budget constraint. More precisely, when a liquidity premium exists, both inflation and the level of real debt affect the real rate of return on public debt. As a result, there are different combinations of steady state inflation rates and real debt that are consistent with the same total seignorage. This is sharp contrast with environments with complete financial markets and flexible prices, where the steady state levels of inflation and real debt do not affect the local stability properties of the monetary equilibrium.[25] Thus it is not surprising that the traditional prescriptions of active/passive monetary and fiscal policies that deliver locally determinate equilibria are not going to be operative in economies where agents trade in secondary markets.

Spread-Adjusted Taylor Rules

In this section we explore the usefulness of alternative Taylor rules in eliminating real indeterminacies. The monetary equilibrium is such that buyers are willing to buy public debt above their fundamental value. This is the case as trading them for fiat money in secondary markets can help expand their consumption possibilities when trading in DM. This additional value is captured by the interest spread between the natural rate in the economy and the total return (takes into account the store of value and liquidity services) on government debt.

Within the spirit of Cúrdia and Woodford (2010) and Canzoneri and Diba (2005), here we consider a spread-adjusted Taylor Rule. However, in contrast to Cúrdia and Woodford (2010), the equilibrium spread in this environment does not reflect any differential risk properties, as in Canzoneri and Diba (2005). In our setting, we consider the following spread-adjusted Taylor rule

$$R_t = \alpha_0 + \alpha \Pi_t - \tilde{s}_{t+1}. \tag{29}$$

Under this new monetary rule, the dynamic monetary equilibrium is given by

$$\Pi_{t+1} = \beta \left(\alpha_0 + \alpha \Pi_t \right), \tag{30}$$

$$2b_t = G - \gamma_0 + \left(\frac{1}{\beta} - \gamma + \frac{1 - \tilde{s}_t}{\Pi_t} \right) b_{t-1}. \tag{31}$$

As we can see, with this spread-adjusted Taylor rule, public debt does not affect inflation. We

[25] Ascari and Ropele (2009) show that in the standard New Keynesian model the Taylor principle remains valid in its more general formulation; however, its implications are radically different as the level of inflation affects the local stability properties.

can now establish the following results.

Proposition 6 *Under the spread-adjusted Taylor rule (29), the monetary steady state is unique and the steady state inflation is identical to the one where there is no SM. However, traditional active/passive policy prescriptions may not yield locally determinate equilibria.*

Proposition 6 highlights that once the monetary authority explicitly takes into account the additional value that public debt gives to buyers, it can then internalize the *bond liquidity Laffer curve*. In other words, the central bank takes into account that agents are willing to purchase bonds above their fundamental value. As a result, the central bank is able to rule out self-fulling beliefs regarding interest rates and public debt that deliver the same *revenue*, effectively eliminating real indeterminacies.

Even though public debt does not affect inflation, traditional policies that deliver local determinacy based on frictionless financial markets are not operative. While the monetary eigenvalue is the standard one, the fiscal eigenvalue depends on the spread-adjusted Taylor rule (α_0 and α), on the over the counter market conditions (κ and σ) as well as on the fiscal stance (γ_0 and γ). This implies that in order to rule out dynamic sunspot equilibria, the level of inflation and real debt need to be taken into account. This is the case as they critically depend on the specifics of the monetary and fiscal rules (α_0 , α, γ_0 and γ). However, in general it is unclear how this augmented Taylor rule can be used to rule out locally indeterminate equilibria.

6 A Numerical Exploration

In this Section, we resort to numerical analysis to determine when the monetary equilibria is locally determinate and unique. To do so, we need to parametrize the model. As a benchmark, we consider an economy with incomplete markets ($\sigma \neq 0$) and no trade in secondary markets ($\kappa = 0$). This scenario roughly captures the era before the Great Moderation, which we take to be from 1960 to 1984. Then we explore what are the consequences for monetary equilibria if agents in the economy are able to trade in the secondary market for government debt. We do so for a variety of monetary and fiscal stances.

To provide some discipline when deciding the parameter values, we proceed as follows. To determine the underlying discount factor, we compute the average annual real interest rate from 1960 to 1984, which is 2.5%. This results in $\beta = 0.9758$. To pin down preferences parameters for the DM utility, we calibrate ξ and χ to yield the ratio of M1 to GDP at two different interest rates. Specifically, we consider the ratios equal to 22% and 40%, which correspond to interest rates equal to 5% and 2.5%, respectively.[26] To determine G, γ_0 and α_0, we match the long-run average from

[26]In terms of CM output, these ratios are equivalent to 23% and 42% respectively. The money demand data is taken from Berentsen et al. (2014) for the period 1950-1989.

1960-1984 of government spending to GDP, government debt to GDP and the annual CPI inflation rate to be 20%, 34% and 5.27%, respectively.[27] Finally, to be closer to the previous literature, we consider an environment without stochastic trading opportunities, which imply $\sigma = \kappa = 1$. We also analyze alternative parametrizations that allow the possibility of not always finding a counter-party in DM and SM so that $\sigma \neq 1$ and $\kappa \neq 1$.

To analyze the consequences for inflation dynamics when changing the aggressiveness of monetary and fiscal rules, we consider a range of values for α and γ. To further discipline the model and to provide a meaningful comparison, for each pair of α and γ, the policy parameters α_0 and γ_0 are re-calibrated so that, without secondary markets, they deliver the same steady state values for real bonds and inflation. We refer to this inflation Π_0 as the inflation target. Table 1 summarizes our calibration and targets.

Table 1: Calibration Targets

Parameter	Target
$\beta = 0.9758$	Annual real interest rate of 2.5 %
χ and ξ	Real money holdings of 23 (41.8) % of CM GDP when $R - 1$ is 5 (2.5) %
$G = 0.21$	Government spending of 21 % of CM GDP
γ_0	Government debt of 35.7 % of CM GDP
α_0	Inflation rate of 5.27 %

With this benchmark calibration, we first explore the effects of active and passive monetary policies on the long-run characteristics of the monetary equilibrium in economies with and without secondary markets for public debt. We then study the robustness of active monetary policies in delivering a unique steady state and locally stable equilibria for a wide range of fiscal policies and changes in the economic environment. Finally, we analyze the equilibrium properties for economies that have a spread-adjusted Taylor rule.

6.1 Active and Passive Policies

In this section, for our benchmark calibration, we analyze the resulting monetary equilibria for a combination of active and passive monetary policies (MP) and fiscal policies (FP).

Table 2 reports real money balances, real bond holdings, the interest spread ($\tilde{s}$), and the monetary and fiscal eigenvalues, which are denoted by λ_M and λ_F, respectively.[28] We show the corresponding values for an economy with no SM and for Case 1. In particular, we consider an active monetary policy, $\alpha = 1.50$, and a passive one, $\alpha = 0.90$. For fiscal policy, we consider an active one policy, $\gamma = 0.024$, and a passive one, $\gamma = 0.030$.

[27] In terms of CM output, the first two correspond to 21% and 36%.

[28] We name the monetary eigenvalue as the one that would be commonly the monetary one. Similarly, we denote the other eigenvalue as the fiscal one.

Table 2: Active/Passive MP and FP

	No SM				SM			
	Active MP		Passive MP		Active MP		Passive MP	
	Active FP	Passive FP	Active FP	Passive FP	Active FP	Passive FP	Active FP	Passive FP
Π	1.0527	1.0527	1.0527	1.0527	1.0090	1.0298	1.1172	1.1644
b	0.3569	0.3569	0.3569	0.3569	0.1986	0.1755	0.0669	0.0373
$\tilde{s}$	0	0	0	0	0.0208	0.0109	0.0080	0.0141
λ_M	1.4638	1.4638	0.8782	0.8782	0.7596	0.7599	0.4567	0.4567
λ_F	1.0008	0.9948	0.9948	1.0008	1.0265	1.0174	0.9791	0.9615

Benchmark parameters: $\sigma = 1.00$, and $\kappa = 1.00$. MP active: $\alpha = 1.50$, passive: $\alpha = 0.90$. FP active: $\gamma = 0.24$, passive: $\gamma = 0.30$.

As we can see from Table 2, there exists a unique steady state regardless whether agents trade in SM or not. This is the case across all policies considered. When there is no trade in SM, and consistent with Lemma 2, an active (passive) monetary policy induces a lower (higher) steady state inflation relative to an economy with SM trade.

For an active MP and passive FP, our benchmark delivers a steady state inflation equal to 2.98%, which is close to the annual average inflation observed between 1985 and 2006 (3.06%), when there was trade in secondary markets for public debt. The resulting equilibrium interest rate spread is equal to 1.09%, which is less than half of the one experienced during the Great Moderation (2.48%).[29] This difference is not surprising as our spread is solely driven by the buyer's liquidity needs in DM and does not consider any potential differential risk among assets. In contrast, when the fiscal policy is active, $\gamma = 0.024$, we find that inflation is further reduced and spreads are higher.

In the literature, passive monetary policy has been widely suggested as a culprit for the inflation episodes and great volatility before the Great Moderation.[30] Consistent with this conventional wisdom, Table 2 shows that with a passive monetary policy and regardless of the fiscal stance, steady state inflation is higher when there is trade in SM relative to an economy when there is no trade. When this passive monetary policy is paired with active fiscal policy, steady state inflation and spreads are lower than when paired with a passive fiscal policy.

In terms of local stability, Table 2 highlights that an active monetary policy induces stability in all steady states regardless of the stance of fiscal policy. Against conventional wisdom, an active monetary policy paired with an active fiscal policy does not necessarily lead to locally indeterminate equilibria. When agents trade in SM, the liquidity premium reduces the monetary eigenvalue, λ_M, while strengthens the fiscal one, λ_F, enough to deliver determinacy. This is not the case when agents do not trade in SM, as such policies always deliver indeterminate equilibria.

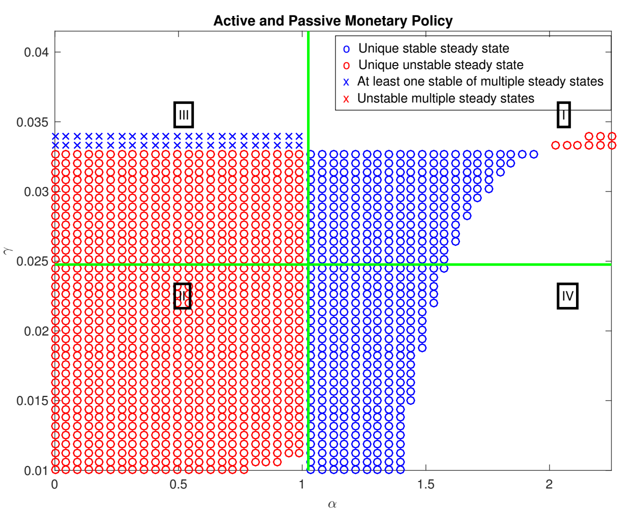
[29] The interest rate spread data has been calculated as the difference between the AAA corporate bond yield and the 1-year treasury constant maturity rate.

[30] We refer to by Clarida et al. (1999) and Lubik and Schorfheide (2004), among others, for more on this issue. Eusepi and Preston (2017), on the other hand, emphasize the role of learning and the maturity of structure in delivering the inflation experiences during the Great Moderation. More in line with this paper is De Blas (2009) who emphasizes the role of financial frictions.

Table 2 also points out that regardless of the fiscal stance, a passive monetary policy dampens both the fiscal and the monetary eigenvalues. With passive monetary and fiscal policies, the equilibrium is always indeterminate regardless whether agents trade in SM or not. However, even when the fiscal policy is active, a passive monetary policy leads to indeterminate equilibria.[31] These results highlight the importance of explicitly modeling the liquidity services that bonds provide. When the economy has a premium on public debt, the Ricardian equivalence breaks down. This ultimately alters the fiscal backing of bonds drastically changing inflation expectations relative to a model without a premium. In the Appendix, we illustrate that our qualitative results are robust to changes in search frictions in DM (σ) and SM (κ).

Are steady states unique regardless of the particulars of the Taylor rule? Does an active monetary policy always lead to a unique locally determinate equilibria? Does a passive monetary policy always deliver local instability? Figure 1 illustrates the existence, stability and uniqueness of monetary equilibria for a range of fiscal and monetary policies when agents trade in SM. These include both active and passive policies. Following the traditional policy prescriptions and the nomenclature used in Leeper (1991), Area I in Figure 1 represents the parameter space consistent with traditionally active monetary and passive fiscal policies. Area II corresponds to a traditionally passive monetary policy paired with a traditionally active fiscal policy. Area III captures traditionally passive fiscal and monetary policies. Finally, Area IV represents traditionally active monetary policy paired with traditionally active fiscal policy.

Figure 1: Uniqueness and Stability of Steady States



Benchmark parameters: $\sigma = 1.00$, and $\kappa = 1.00$.

As we can see from Figure 1, the possibility of real indeterminacy is observed when monetary and fiscal polices are passive. When there are multiple steady states, one of them is locally

[31] This result is in sharp contrast to Canzoneri and Diba (2005), who find that their exogenous liquidity premium makes the equilibrium determinate when monetary policy is passive. This is the case even when monetary policy follows an interest rate peg.

determinate and the other one is unstable. Moreover, under a passive monetary policy, if there exist a unique steady state, it is indeterminate. This is the case irrespective of the fiscal stance. On the other hand, real determinacy is observed when the central bank follows an active monetary policy. The unique steady state can be stable or unstable, depending on the fiscal stance. There seems to be a threshold level of fiscal policy, γ, above which passive monetary policy leads to multiple steady states and active monetary policy leads to either non-existence of equilibrium or uniqueness of equilibrium where agents trade in SM.

These real indeterminacy findings are robust to alternative parameterizations, as shown in Figure 9, which can be found in the Appendix. Different structural parameters modify the degree of passiveness of fiscal policy for which multiple steady states may exist. In a later part of the paper, we also explore the sensitivity of these results to alternative inflation targets. Finally, Table 3 illustrates the results obtained in Proposition 1 under passive monetary policy.

Table 3: Changes in MP Stance and Multiplicity of Equilibria

	No SM			Case 1				
	$\alpha = 0.00$	$\alpha = 0.95$	$\alpha = \frac{2}{\beta}$	$\alpha = 0.00$		$\alpha = 0.95$		$\alpha = \frac{2}{\beta}$
	One SS	One SS	One SS	SS1	SS2	SS1	SS2	One SS
Π	1.0527	1.0527	1.0527	1.0534	1.0733	1.0540	1.0900	1.0346
b	0.3569	0.3569	0.3569	0.1507	0.0936	0.1508	0.0964	0.1507
$\tilde{R} - R$	0	0	0	0.0007	0.0211	0.0001	0.0028	0.0186
λ_M	0	0.9270	2.0000	0	0	0.4815	0.4819	1.0408
λ_F	0.9914	0.9914	0.9914	1.0084	0.9904	1.0084	0.9908	1.0079

Benchmark parameters: $\gamma = 0.0333$, $\sigma = 1.00$, and $\kappa = 1.00$.

As we can see from Table 3, both a moderately passive, $\alpha = 0.95$, or an interest rate peg, $\alpha = 0$, induces multiplicity of steady states. However, an adequate active monetary policy, as those suggested in Proposition 1; i.e, $\alpha = (2/\beta)$, delivers a unique steady state when agents trade in SM. Even though, the real indeterminacy has been ruled out, the corresponding unique monetary equilibrium is locally undetermined.

Unless specific coordinated monetary and fiscal policies are considered, real indeterminacy under passive monetary policies is a robust phenomena. This can generate another source of volatility, as sunspot equilibria can be easily constructed. These results are in sharp contrast to Canzeroni and Diba (2005), who find that passive monetary policy paired with passive fiscal policy can lead to locally stable monetary steady states. This is the case even when the monetary policy follows an interest peg, $\alpha = 0$. Figure 1 shows that this only happens in combination with the existence of multiple steady states, and therefore, the potential for real indeterminacy. When unique, our numerical results show that a passive monetary policy leads to an unstable equilibrium. These findings are consistent with the multiplicity of steady states generated by a bond liquidity Laffer curve. One steady state is stable and the other one is unstable. In the stable steady state, the fiscal eigenvalue is above unity and provides local determinacy.

6.2 Exploring the Mechanism

Given the benchmark calibration, we now explore the mechanism driving the previous real and local indeterminacy results.

Real Indeterminacies

In this section we examine how the liquidity premium on bond interacts with monetary and fiscal policies in generating multiple stationary equilibria. Recall that a stationary equilibria when buyers and sellers are constrained when trading in SM is given by

$$\Pi = \frac{1}{(\frac{2}{\beta} - \alpha)} \left(\alpha_0 + 1 - \sigma + \hat{\theta} \right), \tag{32}$$

$$b = \frac{(G - \gamma_0)\,\Pi}{(2 - \alpha + \gamma)\,\Pi - (1 + \alpha_0)}. \tag{33}$$

We now explore whether there is a *bond Laffer curve.*[32] Using (33), one can solve for $\frac{\Pi}{b}$ as a function of only the inflation rate, Π. Plugging that ratio into $\hat{\theta}$ in (32), we get the following expression in terms of the inflation rate

$$\alpha_0 + (1 - \sigma) + \hat{\theta} - \left(\frac{2}{\beta} - \alpha \right) \Pi = 0. \tag{34}$$

Whenever this expression is equal to zero, we have a steady state solution for Π. Then through (33), we can then determine the corresponding level for real bonds b. For $G - \gamma_0 > 0$, the bond premium, $\hat{\theta}$, increases in Π. Then the slope of (34) may be positive or negative. Differentiating (34) with respect to inflation, we find that it is given by

$$(2 + \gamma - \alpha) \left(\frac{\xi \hat{\theta} \frac{b}{\Pi}}{(G - \gamma_0)} \right) - \left(\frac{2}{\beta} - \alpha \right). \tag{35}$$

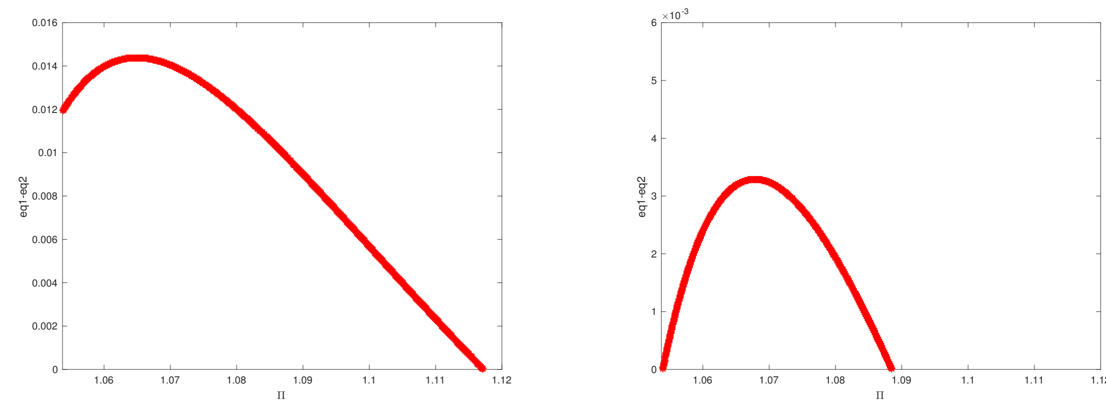
It is easy to show that the second derivative is negative. Moreover, from the previous expression, it is easy to see that the adequate monetary and fiscal policies of Proposition 3 eliminate one of the possible stationary equilibria.

[32]Note that the real bonds implied by the government budget constraint, equation (33), directly depend on the fiscal stance (γ_0 or γ). This is the case as taxes are linked to bonds and the fact that the fiscal backing affects bond issuance. Moreover, bonds are also affected by the inflation rate as they impact their real return.

Figure 2 draws the fix point equation (34) for a passive monetary policy paired with two different passive fiscal policies.

Figure 2: Bond Laffer Curve for Passive MP

$\gamma = 0.030$ $\gamma = 0.033$

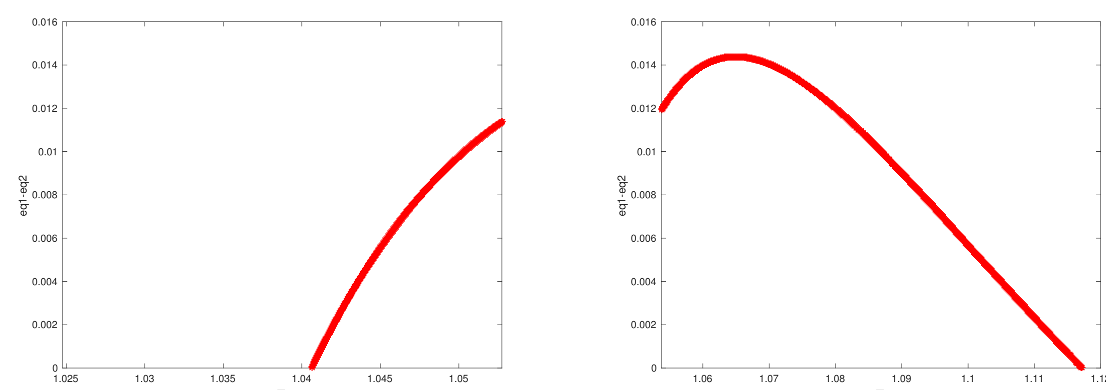


Benchmark parameters: $\sigma = 1.00$, and $\kappa = 1.00$. MP passive: $\alpha = 0.90$.

As we can see from Figure 2, as fiscal policy becomes more passive, γ increases, we find two inflation rates that are consistent with stationary equilibria. Under the benchmark calibration, we can then conclude that a *bond liquidity Laffer curve* exists when traditional passive monetary policies are implemented. Given the same passive fiscal policy, Figure 3 plots the *bond Laffer curve* for both passive and active monetary policies.

Figure 3: Bond Laffer Curve for Active/Passive MP

$\gamma = 0.030$ - Active MP $\gamma = 0.030$ - Passive MP



Benchmark parameters: $\sigma = 1.00$, and $\kappa = 1.00$. MP passive: $\alpha = 0.90$. MP active: $\alpha = 1.50$.

As we can see, with active and passive monetary policy the corresponding stationary equilibria are located in different sides of the *bond Laffer curve*. In particular, with an active monetary policy, the steady state is unique, locally stable and to the left of the Laffer curve. In contrast, with passive monetary policies, the unique steady state is locally unstable and to the right of the Laffer curve.

Local Stability Properties

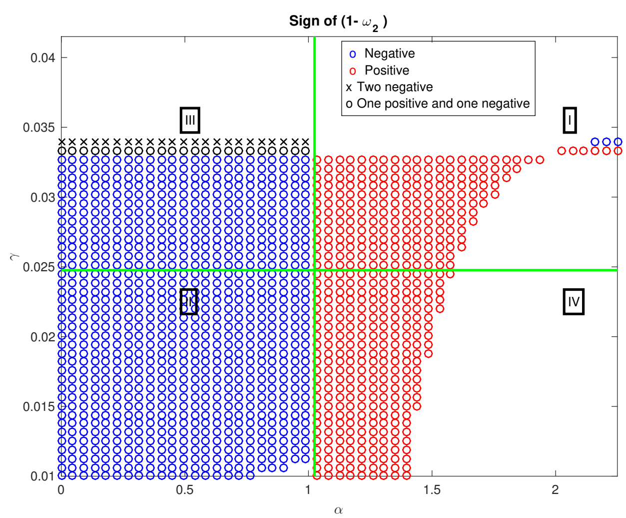
In our benchmark calibration, the value of the lower off diagonal term in the Jacobian tends to be quite close to zero.[33] This corresponds to a situation where changes in inflation do not affect much seignorage, and therefore, government bonds. Under these circumstances, we can approximate the eigenvalues by the diagonal elements of the Jacobian, which are given by

$$\lambda_M \approx \omega_1 \beta \alpha + \omega_3 (1 - \omega_2) \frac{1}{\Pi} \frac{b}{\Pi}, \quad \& \quad \lambda_F \approx \frac{1}{\beta} - \frac{1}{2}\gamma + \frac{1}{2}\frac{1}{\Pi} (1 - \omega_2),$$

where $\omega_1 = \frac{1}{2 - \sigma\left(\frac{\beta}{\Pi}\right)\left(\frac{\xi}{1-\xi}\right)}$ and $\omega_2 = 1 - \sigma + (1 - \xi)\hat{\theta}$. Since we are considering equilibria with positive nominal interest rates, if $\xi \leq \frac{1}{1+\sigma}$, then ω_1 dampens the monetary eigenvalue for both passive and active monetary policies.[34] This situation reflects the fact that when trading in secondary markets, money and bonds co-move one to one. This co-movement in the nominal government liabilities reduces the stabilizing effect of monetary policy.

Both fiscal and monetary eigenvalues are affected by the sign of $(1 - \omega_2)$, shown in Figure 4. Note that $-\frac{1}{2}(1 - \omega_2)\frac{b}{\Pi^2}$ indicates how inflation affects bond issuance.

Figure 4: Sign of $(1 - \omega_2)$.



Benchmark parameters: $\sigma = 1.00$, and $\kappa = 1.00$.

For an active monetary policy, we have that $(1 - \omega_2) > 0$, while for passive monetary policy we have that $(1 - \omega_2) < 0$. This change of sign does not drastically affect the monetary eigenvalue, as the effect of ω_1 dominates. However, when the fiscal stance is such that γ is small, it does affect

[33] In our numerical exercises, C is smaller than 0.002 in absolute value. See Figure 8 in the Appendix.

[34] In our numerical simulations, there are parameter values for which $\omega_2 \approx 1$ and where $\xi \leq \frac{1}{1+\sigma}$.

the fiscal eigenvalue. In such circumstances we have that this eigenvalue can be written as follows

$$\lambda_F \approx \frac{1}{\beta} - \gamma + \frac{1}{2}\left[\gamma + \frac{1}{\Pi}(1-\omega_2)\right].$$

For active monetary policies, we find that in general $(1-\omega_2) > 0$, and secondary markets amplify the fiscal eigenvalue. While for passive monetary policies, $\gamma + \frac{1}{\Pi}(1-\omega_2)$ is generically negative and secondary markets dampen the fiscal eigenvalue. Thus according to our numerical exercises, we can conclude that secondary markets tend to reduce the stabilizing effect of monetary policy and depending on the stance of monetary policy, they strengthen or weaken the stabilizing effect of fiscal policy.[35]

Inspecting the slope of the fix point equation, given by (35), we see that whether the equilibrium is at the left or at the right of the peak of the *Laffer* curve depends on the stances of both monetary and fiscal policies. Consider a passive monetary policy and/or moderately active one so that $(2+\gamma-\alpha)$ and $\left(\frac{2}{\beta}-\alpha\right)$ are positive and approximately equal. For these policies, the economy is at the left (right) of the peak of the Laffer curve whenever $\left(\frac{\xi\hat{\theta}\frac{b}{\Pi}}{(G-\gamma_0)}\right) > (<) 1$. By re-arranging (33), one can easily show that $\lambda_F > (<) 1$, which yields determinacy (indeterminacy), implies $\left(\frac{\xi\hat{\theta}\frac{b}{\Pi}}{(G-\gamma_0)}\right) > (<) 1$.

How does fiscal policy stabilize a sudden increase in inflation? By looking at the inflation dynamics equation (27), one can see that decreases in the ratio $\left(\frac{b_t}{\Pi_{t+1}}\right)$ increase the term $\hat{\theta}_{t+1}$ of the liquidity premium, which helps lower inflation. Differentiating the right hand side of the bond dynamics equation (28) with respect to Π_t we get $-\frac{1}{2}(1-\omega_2)\frac{b}{\Pi^2}$, which is our term C in the Jacobian. If future inflation is fixed, increases in inflation Π_t lower (raise) the issuance of bonds b_t when $(1-\omega_2) > (<) 0$. In our numerical exercise, we find this at the left (right) of the peak of the liquidity *Laffer* curve and when monetary policy is active (passive). Then active monetary policy is stabilizing as it decreases bond issuance, which decreases the ratio $\frac{b_t}{\Pi_{t+1}}$, which increases the term $\hat{\theta}_{t+1}$ in the liquidity premium, which in turns anchors inflation.[36] When passive monetary policy delivers two steady states, the stable one shares the same pattern of determinacy. The only difference between a stable active monetary policy and a stable passive monetary policy is in the premium dynamics. For both the term $\hat{\theta}_{t+1}$ increases, but for active monetary policy the premium decreases while for passive monetary policy it increases.

Summarizing, the endogenous liquidity premium can generate a liquidity *Laffer* curve, which is

[35] There are exceptions. When passive monetary policy induces multiplicity, one of the steady states displays $(1-\omega_2) > 0$ so that $\gamma + \frac{1}{\Pi}(1-\omega_2)$ is positive in one of the associated eigenvalues and negative in the other. When monetary policy is active but induces nominal indeterminacy, then the steady state is such that $(1-\omega_2) < 0$.

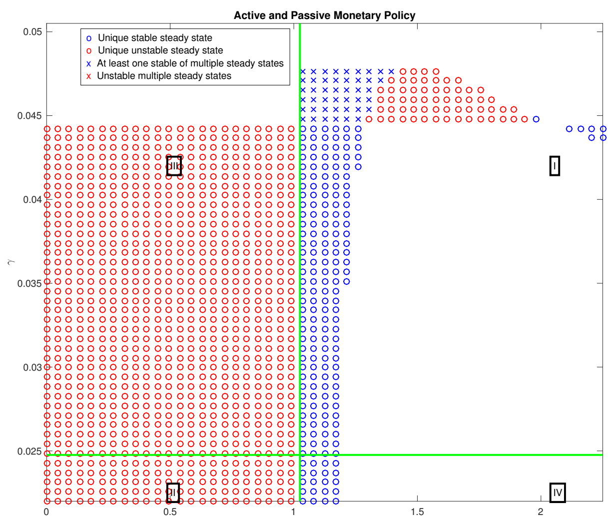
[36] This mechanism is similar to the one shown in Yun (2011). In our paper, nominal bonds make the premium depend also on inflation. This generates the liquidity *Laffer* curve and make inflation have non-monotonic effects on bonds. In our case and in contrast to Yun (2011), a stable equilibrium can also be found when monetary policy is passive.

critical for stability and is intimately connected with the source of multiplicity. Fiscal policy can stabilize inflation through the effects of bond issuance on the liquidity premium. This stabilization occurs only when the equilibrium is at the left of the peak of the liquidity curve. The stance of monetary policy can help select that equilibrium

Importance of the Steady State Inflation Target

Here, we explore how sensitive our results are to different inflation targets. Thus, rather than considering a long run inflation rate of 5.2%, as in the pre Great Moderation era, let us consider a situation where the inflation target is 10% instead. Our findings are summarized in Figure 5.

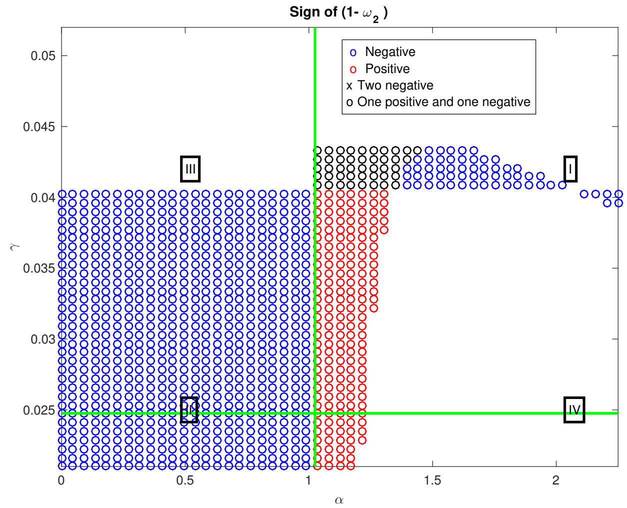
Figure 5: Active and Passive FP/MP when $\Pi_0 = 1.10$



Benchmark parameters: $\sigma = 1.00$, and $\kappa = 1.00$.

As we can see, once the inflation target is higher, multiple steady states are possible even with an active monetary policy, not just with passive ones. These real indeterminacy results are consistent with our liquidity Laffer curve explanation. More precisely, with a higher inflation target, the area for possible equilibria with an active monetary policy is larger, allowing for two steady states. As we saw, the off diagonal term in the Jacobian is very close to zero and the change in stability and multiplicity is connected to the sign of $1 - \omega_2$ as shown in Figure 6.

Figure 6: Sign of $(1 - \omega_2)$ when $\Pi_0 = 1.10$



Benchmark parameters: $\sigma = 1.00$, and $\kappa = 1.00$.

These results are in sharp contrast with those of Leeper (1991) and others, that consider a flexible price environment where the steady state level of inflation does not affect the multiplicity nor determinacy of monetary equilibria. As before, and consistent with Proposition 1, we find that a very aggressive monetary policy can eliminate one of the steady states. Even though the underlying frictions are different, our results are in line with Ascari and Ropele (2009), among others. These authors find that in basic New Keynesian neither the Taylor principle nor the generalized Taylor principle is a sufficient condition for local determinacy of equilibrium when the long run inflation is positive.

6.3 Spread-Adjusted Taylor Rules

Here we analyze the monetary equilibria that one obtains when the monetary authority follows a spread-adjusted Taylor rule. Table 4 reports our numerical findings for both passive and active fiscal policies. Recall that in this environment the central bank internalizes the liquidity premium on bonds when implementing their Taylor rule.

Table 4: A Spread-Adjusted Taylor Rule

	No SM				Case 1			
	$\alpha = 0.90$		$\alpha = 1.50$		$\alpha = 0.90$		$\alpha = 1.50$	
	$\gamma = 0.024$	$\gamma = 0.030$	$\gamma = 0.024$	$\gamma = 0.030$	$\gamma = 0.024$	$\gamma = 0.030$	$\gamma = 0.024$	$\gamma = 0.030$
Π	1.0527	1.0527	1.0527	1.0527	1.0527	1.0527	1.0527	1.0527
b	0.3569	0.3569	0.3569	0.3569	0.0248	0.0410	0.0248	0.0410
λ_M	0.8783	0.8783	1.4638	1.4638	0.8783	0.8783	1.4638	1.4638
λ_F	1.0008	0.9948	1.0008	0.9948	0.9301	0.9475	0.9301	0.9475

Benchmark parameters: $\sigma = 1.00$, and $\kappa = 1.00$.

As Table 4 shows, with a spread-adjusted Taylor rule, there is a unique steady state inflation that is equal to the one when there no SM trade. In terms of stability, the monetary eigenvalue is identical to the one corresponding to an economy when there is no SM trade. The fiscal eigenvalue however, is much lower, and below one. This is independent of whether fiscal policy is traditionally active or not. Therefore, active monetary policies with a spread-adjusted Taylor rule deliver the expected target inflation as well as real and nominal determinacy. These results are robust to different parameterizations.[37]

These findings then suggest the potential benefits for real and dynamic determinacy of considering spread-adjusted Taylor rule when thinking about stabilization policies. Once the monetary authority explicitly incorporates interest rate spreads in their decision making, it can help internalize the additional value that agents obtain when purchasing bonds, helping anchor inflation expectations.

7 Conclusions

Using an environment where public debt is used not only as a store of value but also as an asset that can help enlarge consumption possibilities in frictional markets, this paper provides new insights on how active monetary policies can be useful in ruling out real and dynamic indeterminacies.

When agents trade in secondary markets, we can observe a liquidity premium on public debt. When it exists, government bonds matter for inflation dynamics and the Ricardian equivalence does not hold anymore. After a sudden increase in inflation, the fiscal authority can change their bond issuance to affect the liquidity premium and, in turn, stabilize inflation. This liquidity premium makes the total interest payment on governments bonds non-linear, generating a *bond liquidity Laffer* curve. This allows for the possibility of real indeterminacies and drastically changes inflation expectations and the appropriate monetary and fiscal policies that deliver locally determinate equilibria.

To rule out real indeterminacies, we show that active monetary policy is more likely to deliver a unique monetary steady states regardless of the fiscal stance. Moreover, a spread-adjusted Taylor rule ensures a unique steady state. Our analytical and numerical results also show that trading in secondary markets amplify steady state inflation when monetary policy is passive. In contrast, it dampens steady state inflation when monetary policy is active. Moreover, trading in secondary markets change the stability properties of the economy. Traditional policy prescriptions that rule out local indeterminacy are no longer useful. Finally, we show that a spread-adjusted Taylor rule can rule out real indeterminacies and with active monetary policies one can generate a locally determinate equilibria.

Improved monetary policy or declining volatility of economic disturbances are unlikely to be the sole contributors of delivering the inflation experiences of the Great Moderation in the US. This paper shows the role of trading in secondary markets for public debt when bonds exhibit

[37] Changes in σ and κ have a small quantitative impact relative to the benchmark calibration.

a premia in amplifying the effects of active monetary policy and the usefulness of having a low inflation target. Our findings suggest that, with a more developed secondary market for public debt, ceteris paribus, monetary policy does not need to be as aggressive to achieve lower inflation. To anchor inflation expectations monetary policy must respond less aggressively to changes in inflation, over and above adjustments prescribed by the Taylor principle relative to economies without a liquidity premium for government debt.

Finally, in this paper we have considered government policies that are dictated by pre-determined rules and have found that these rules have very different implications once individuals trade in these markets. In future work, we plan to take a normative approach and study how the properties of optimal fiscal and monetary policies change once secondary markets provide a liquidity premium.

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Appendix

Beginning of sub-period portfolios

An agent in period t enters CM with a portfolio of fiat money ($\tilde{M}_{t-1}$) and nominal government bonds ($\tilde{B}_{t-1}$). In particular, we have that

$$\phi_t \tilde{M}_{t-1} = \begin{cases} \phi_t \left(M_{t-1} + a_t D_t^{B^o} \right), & \text{if DM buyer has traded in SM but not in DM in } t, \\ \phi_t \left(M_{t-1} + a_t D_t^{B^o} - D_t^{M_s} \right), & \text{if DM buyer has traded in SM and in DM in } t, \\ \phi_t \; M_{t-1}, & \text{if DM buyer has not traded in SM nor in DM in } t, \\ \phi_t \left(M_{t-1} - D_t^{M} \right), & \text{if DM buyer has not traded in SM but has in DM in } t, \\ \phi_t \left(M_{t-1} - a_t D_t^{B^o} \right), & \text{if DM seller has traded in SM but not in DM in } t, \\ \phi_t \left(M_{t-1} - a_t D_t^{B^o} + D_t^{M_s} \right), & \text{if DM seller has traded in SM but not in DM in } t, \\ \phi_t \; M_{t-1}, & \text{if DM seller has not traded in SM nor in DM in } t, \\ \phi_t \left(M_{t-1} + D_t^{M} \right), & \text{if DM seller has not traded in SM but has traded in DM in } t, \end{cases}$$

$$\phi_t \tilde{B}_{t-1} = \begin{cases} \phi_t \left(B_{t-1} - D_t^{B^o} \right), & \text{if DM buyer has traded in SM in } t, \\ \phi_t \; B_{t-1}, & \text{if DM buyer has not traded in SM in } t, \\ \phi_t \left(B_{t-1} + D_t^{B^o} \right), & \text{if DM seller has traded in SM in } t, \\ \phi_t \; B_{t-1}, & \text{if DM seller has not traded in SM in } t. \end{cases}$$

An agent in period t enters DM with a portfolio of fiat money ($\hat{M}_{t-1}$) and nominal government bonds ($\hat{B}_{t-1}$). In particular, we have that

$$\phi_t \hat{M}_{t-1} = \begin{cases} \phi_t \left(M_{t-1} + a_t D_t^{B^o} \right), & \text{if DM buyer has traded in SM in } t, \\ \phi_t \; M_{t-1}, & \text{if DM buyer has not traded in SM in in } t, \\ \phi_t \left(M_{t-1} - a_t D_t^{B^o} \right), & \text{if DM seller has traded in SM in } t, \\ \phi_t \; M_{t-1}, & \text{if DM seller has not traded in SM in } t, \end{cases}$$

$$\phi_t \hat{B}_{t-1} = \begin{cases} \phi_t \left(B_{t-1} - D_t^{B^o} \right), & \text{if DM buyer has traded in SM in } t, \\ \phi_t \; B_{t-1}, & \text{if DM buyer has not traded in SM in } t, \\ \phi_t \left(B_{t-1} + D_t^{B^o} \right), & \text{if DM seller has traded in SM in } t, \\ \phi_t \; B_{t-1}, & \text{if DM seller has not traded in SM in } t. \end{cases}$$

Note that $D_t^{B^o}$ denotes the units of government bonds that sellers transfer to buyers in SM, a_t the price per unit of bonds in SM, $D_t^{M_s}$ the cash payment in DM for goods when the buyer traded in the previous SM and D_t^{M} the DM payment when the buyer did not trade in SM.

Proof of Proposition 1

Imposing steady state conditions on equations (3) and (4), the equilibrium inflation rate and real debt solves the following fixed point

$$\Pi = \beta\left[\alpha_0 + \alpha\Pi + \tilde{s}(\Pi, b)\right],$$

$$b = G - \gamma_0 + \left(\frac{1}{\beta} - \gamma - \frac{\tilde{s}(\Pi, b)}{\Pi}\right) b.$$

where $\tilde{s}(\Pi, b)$ is a nonlinear function in both arguments. Note that generically a system of non-linear equations will typically have more than one solution.

In terms of policies that deliver local determinacy, it is important to highlight that the bond premia drastically changes the properties of the associated Jacobian. In particular, we have that

$$J = \begin{bmatrix} \frac{1}{\left(1-\beta\frac{\partial \tilde{s}_{t+1}}{\partial \Pi_{t+1}}\right)}\left(\beta\alpha + \beta\frac{\partial \tilde{s}_{t+1}}{\partial b_t}\left(\tilde{s}_t - \Pi\frac{\partial \tilde{s}_t}{\partial \Pi_t}\right)\frac{b}{\Pi^2}\right) & \frac{\beta\frac{\partial \tilde{s}_{t+1}}{\partial b_t}}{\left(1-\beta\frac{\partial \tilde{s}_{t+1}}{\partial \Pi_{t+1}}\right)}\left(\frac{1}{\beta} - \gamma - \frac{\tilde{s}}{\Pi} - \frac{b}{\Pi}\frac{\partial \tilde{s}_t}{\partial b_{t-1}}\right) \\ \left(\tilde{s}_t - \Pi\frac{\partial \tilde{s}_t}{\partial \Pi_t}\right)\left(\frac{b}{\Pi^2}\right) & \frac{1}{\beta} - \gamma - \frac{\tilde{s}}{\Pi} - \frac{b}{\Pi}\frac{\partial \tilde{s}_t}{\partial b_{t-1}} \end{bmatrix}.$$

Thus the resulting policy prescriptions for stability are likely to be different from those obtained with a constant premium as the economy is not dichotomous anymore. As a result, the underlying eigenvalues are going to be very different from the ones implied by an economy with a constant premium.

Proof of Proposition 2 and 3

After imposing stationarity, we have that the unique monetary steady state is given by

$$\Pi = \frac{\beta\alpha_0}{(1-\beta\alpha)}, \quad b = \frac{1}{\left(1 - \frac{1}{\beta} + \gamma\right)}\left\{(G - \gamma_0) + \left(\frac{1}{\Pi} - 1\right) m\right\},$$

where steady state real balances are given by $m = \Pi\left[\frac{\chi\sigma}{2(\alpha_0+\alpha\Pi-1)+\sigma}\right]^{\frac{1}{\xi}}$. The corresponding Jacobian is given by

$$\mathcal{J} = \begin{bmatrix} \beta\alpha & 0 \\ \omega_0 & \frac{1}{\beta} - \gamma \end{bmatrix},$$

where $\omega_0 = \frac{\partial b_t}{\partial \Pi_t} \neq 0$. Given that the dynamic system is decoupled, the corresponding eigenvalues are $\lambda_M = \beta\alpha$ & $\lambda_F = \frac{1}{\beta} - \gamma$. For the results in Proposition 2 we just need to set $\sigma = 0$, which implies that m_t=0 $\forall t$.

Derivation of Dynamic Monetary Equilibrium Case 1

When both constraints bind while trading in SM, the dynamic monetary equilibrium can be characterized by the sequence $\left\{\Pi_{t+1}, b_t, m_t, \mu_t^s, \mu_t^b\right\}_{t=0}^{\infty}$ satisfying the following equations

$$\Pi_{t+1} = \beta\left(\alpha_0 + \alpha\Pi_t + \frac{1}{2}\mu_{t+1}^b\right),$$

$$b_t = G - \gamma_0 + \left(\frac{1}{\beta} - \gamma - \frac{1}{2}\frac{\mu_t^b}{\Pi_t}\right)b_{t-1} + \frac{m_{t-1}}{\Pi_t} - m_t,$$

$$\sigma\left[\kappa\left(\chi\frac{\Pi_{t+1}^{\xi}}{(2m_t)^{\xi}} - 1\right) + (1-\kappa)\left(\chi\frac{\Pi_{t+1}^{\xi}}{m_t{}^{\xi}} - 1\right)\right] + \mu_{t+1}^s = 2\left(\alpha_0 + \alpha\Pi_t - 1\right) + \mu_{t+1}^b,$$

$$\sigma\left(\chi\frac{\Pi_t^{\xi}}{(2m_{t-1})^{\xi}} - 1\right) - \mu_t^s - \mu_t^b = 0,$$

and $b_t = m_t$, since $D_{t+1}^{B^o} = m_t$ and $D_{t+1}^{B^o} = b_t$. Solving for the Kuhn-Tucker multipliers, we get

$$\mu_{t+1}^b = \sigma\left(\chi\frac{\Pi_{t+1}^{\xi}}{(2b_t)^{\xi}} - 1\right) + (1-\kappa)\frac{1}{2}\sigma\chi\frac{\Pi_{t+1}^{\xi}}{b_t{}^{\xi}}\left(1 - \frac{1}{(2)^{\xi}}\right) - \left(\alpha_0 + \alpha\Pi_t - 1\right),$$

$$\mu_{t+1}^s = -(1-\kappa)\frac{1}{2}\sigma\chi\frac{\Pi_{t+1}^{\xi}}{b_t{}^{\xi}}\left(1 - \frac{1}{(2)^{\xi}}\right) + \left(\alpha_0 + \alpha\Pi_t - 1\right).$$

Substituting back these expressions we arrive to the dynamic equations found in the text.

Proof of Lemma 2

The Kuhn-Tucker multipliers in steady state are given by

$$\mu^b = 2\left[\left(\frac{1}{\beta} - \alpha\right)\Pi - \alpha_0\right],$$

$$\mu^s = -\frac{1}{2}\sigma\chi\frac{\Pi^{\xi}}{b^{\xi}}(1-\kappa)\left[1 - \frac{1}{(2)^{\xi}}\right] + \left(\alpha_0 + \alpha\Pi - 1\right).$$

Recall that when there is no trade in secondary markets, the steady state inflation equals $\frac{\alpha_0\beta}{1-\beta\alpha}$, which is the bound in conditions *(i)* and *(ii)*. For μ^b to be positive it requires that (i) if $\alpha_0 \geq 0$ and $\alpha\beta < 1$, then $\Pi \geq \frac{\alpha_0\beta}{1-\beta\alpha}$, or (ii) if $\alpha_0 \leq 0$ and $\alpha\beta > 1$, then $\Pi \leq \frac{\alpha_0\beta}{1-\beta\alpha}$. These two inequalities are consistent with the statement in the Lemma.

Proof of Proposition 4

After imposing steady state conditions, we have that the monetary steady state $\{\Pi, b, m\}$ satisfy $b = m$ and the following non-linear equations

$$\Pi = \frac{1}{(\frac{2}{\beta} - \alpha)} \left(\alpha_0 + 1 - \sigma + \sigma\chi \frac{1}{2} \frac{\Pi^{\xi}}{b^{\xi}} \left[\frac{1+\kappa}{(2)^{\xi}} + (1-\kappa) \right] \right),$$

$$b = \frac{(G - \gamma_0)\,\Pi}{(2 - \alpha + \gamma)\,\Pi - (1 + \alpha_0)}.$$

Once we substitute the steady state bond equation into the equation that defines the steady state inflation rate, we obtain the following

$$\left(\frac{2}{\beta} - \alpha \right) \Pi - (\alpha_0 + 1 - \sigma) = \sigma\chi \frac{1}{2} \left(\frac{\Pi}{\frac{2(G-\gamma_0)\Pi}{(2-\alpha+\gamma)\Pi-(1+\alpha_0)}} \right)^{\xi} \left[\frac{1+\kappa}{(2)^{\xi}} + (1-\kappa) \right],$$

As can be seen, the resulting equation characterizing the steady state inflation is highly non-linear. Thus multiple steady states can not be ruled out.

Proof of Proposition 5

Let us consider an economy where $G > \gamma_0$, $\alpha_0 < 0$ and $2 + \gamma > \alpha$. When $\alpha = \frac{2}{\beta}$, it is easy to show that the steady state inflation is unique and given by

$$\Pi = \frac{2\,(G - \gamma_0)}{\left(2 - \frac{2}{\beta} + \gamma\right)} \left(\frac{2\,(\sigma - \alpha_0 - 1)}{\sigma\chi \left[\frac{1+\kappa}{(2)^{\xi}} + (1-\kappa) \right]} \right)^{\frac{1}{\xi}} + \frac{(1 + \alpha_0)}{\left(2 - \frac{2}{\beta} + \gamma\right)}.$$

Let us consider an economy where $G > \gamma_0$, $\alpha_0 < 0$ and $\alpha\beta > 1$. When $\alpha = 2 + \gamma$, it is easy to show that the steady state inflation is unique and given by

$$\Pi = \frac{1}{\frac{2}{\beta} - \alpha} \left[(\alpha_0 + 1 - \sigma) + \sigma\chi \frac{1}{2} \left(\frac{-\alpha_0 - 1}{2\,(G - \gamma_0)} \right)^{\xi} \left(\frac{1+\kappa}{(2)^{\xi}} + (1-\kappa) \right) \right].$$

Proof of Lemma 3

Given our derivation of the dynamic monetary equilibrium in Case 1, the Jacobian is given by:

$$\mathcal{J} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \omega_1\beta\alpha + \omega_3\,(1 - \omega_2)\,\frac{1}{\Pi}\frac{b}{\Pi} & -\omega_3 \left[\frac{1}{\beta} - \frac{1}{2}\gamma + \frac{1}{2}\frac{1}{\Pi}\,(1 - \omega_2) \right] \\ -\frac{1}{2}\,(1 - \omega_2)\,\frac{1}{\Pi}\frac{b}{\Pi} & \frac{1}{\beta} - \frac{1}{2}\gamma + \frac{1}{2}\frac{1}{\Pi}\,(1 - \omega_2) \end{bmatrix},$$

with $\omega_1 = \frac{1}{2\left(1-\frac{\beta\frac{1}{2}\xi\hat{\theta}}{\Pi}\right)}$, $\omega_2 = 1-\sigma+(1-\xi)\,\hat{\theta}$, and $\omega_3 = \omega_1 \frac{\beta\xi\hat{\theta}}{b}$, where $\hat{\theta} = \frac{1}{2}\sigma\chi\frac{\Pi^{\xi}}{(2b)^{\xi}}\left(1+\kappa+(1-\kappa)(2)^{\xi}\right)$, which can be written as $\hat{\theta} = \left(\frac{2}{\beta}-\alpha\right)\Pi-\alpha_0-1+\sigma$.

In contrast to the traditional case, the values outside of the main diagonal in the Jacobian are in general not zero and, even if they were, the elements in the diagonal are different to the traditional ones. Therefore, traditional active/passive policies are not going to be useful in delivering locally determinate equilibria.

Proof of Proposition 6

Imposing stationarity on equation (24) yields a unique inflation rate. Given that steady state real debt and the buyer's Lagrange multiplier is linear in the inflation rate, we can then easily establish uniqueness. From the dynamic monetary equilibrium with the spread adjusted Taylor rule, the Jacobian is given by

$$\begin{bmatrix} \beta\alpha & 0 \\ \omega_1^s & \frac{1}{\beta}-\gamma+\omega_2^s \end{bmatrix},$$

where $\omega_1^s = \frac{1}{2}(\theta^s-1)\,\sigma\frac{b}{\Pi}\frac{1}{\Pi}$, $\omega_2^s = \frac{1}{2}\left[\gamma+\frac{\sigma}{\Pi}(1-\theta^s)\right]$, and $\theta^s = (1-\xi)\,\frac{1}{2}\chi\left(\frac{\Pi}{2b}\right)^{\xi}\left[(2-\kappa)+\kappa 2^{\xi}\right]$. While the monetary eigenvalue is the standard one, the fiscal eigenvalue is not. Therefore, traditional policy prescriptions may not be useful to induce determinacy.

Additional Figures and Robustness Analysis

Table 5: Active/Passive MP and FP: $\sigma = 0.75$

	No SM				SM			
	Active MP		Passive MP		Active MP		Passive MP	
	Active FP	Passive FP	Active FP	Passive FP	Active FP	Passive FP	Active FP	Passive FP
Π	1.0527	1.0527	1.0527	1.0527	1.0100	1.0317	1.1175	1.1679
b	0.3569	0.3569	0.3569	0.3569	0.1973	0.1734	0.0673	0.0372
$\tilde{s}$	0	0	0	0	0.0203	0.0100	0.0081	0.0144
λ_M	1.4638	1.4638	0.8782	0.8782	0.7594	0.7598	0.4571	0.4574
λ_F	1.0008	0.9948	0.9948	1.0008	1.0260	1.0167	0.9798	0.9617

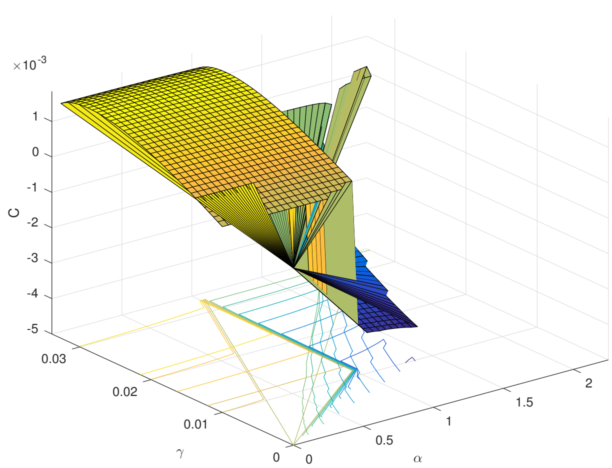
Benchmark parameters: $\sigma = 0.75$, and $\kappa = 1.00$. MP active: $\alpha = 1.50$, passive: $\alpha = 0.90$. FP active: $\gamma = 0.24$, passive: $\gamma = 0.30$.

Table 6: Active/Passive MP and FP: $\kappa = 0.75$

	No SM				SM			
	Active MP		Passive MP		Active MP		Passive MP	
	Active FP	Passive FP	Active FP	Passive FP	Active FP	Passive FP	Active FP	Passive FP
Π	1.0527	1.0527	1.0527	1.0527	1.0019	1.0143	1.1343	1.1788
b	0.3569	0.3569	0.3569	0.3569	0.2265	0.2102	0.0592	0.0348
$\tilde{s}$	0	0	0	0	0.00242	0.0182	0.0102	0.0157
λ_M	1.4638	1.4638	0.8782	0.8782	0.7594	0.7596	0.4567	0.4567
λ_F	1.0008	0.9948	0.9948	1.0008	1.0287	1.0219	0.9716	0.9560

Benchmark parameters: $\sigma = 1.00$, and $\kappa = 0.75$. MP active: $\alpha = 1.50$, passive: $\alpha = 0.90$. FP active: $\gamma = 0.24$, passive: $\gamma = 0.30$.

Figure 8: Values of C.



Benchmark parameters: $\sigma = 1.00$, and $\kappa = 1.00$.

Figure 9: Steady State Uniqueness and Stability in Region 1: Robustness Analysis

Figure 9a: Benchmark 1

Figure 9b: Benchmark 1 with $\kappa = 0.75$

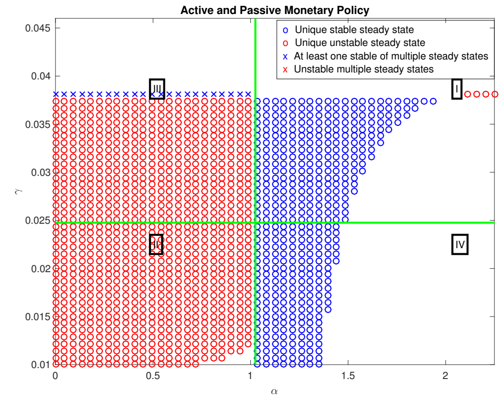


Figure 9c: Benchmark 1 with $\kappa = 0.50$

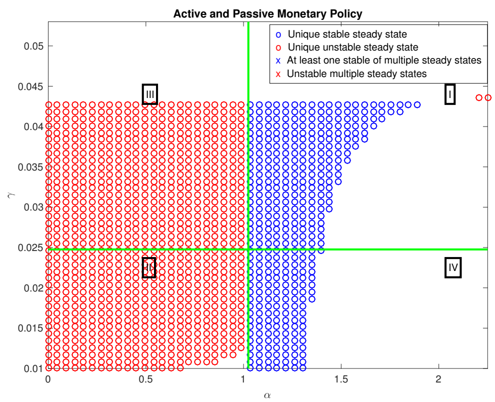


Figure 9d: Benchmark 1 with $\kappa = 0.50$ and $\sigma = 0.2$

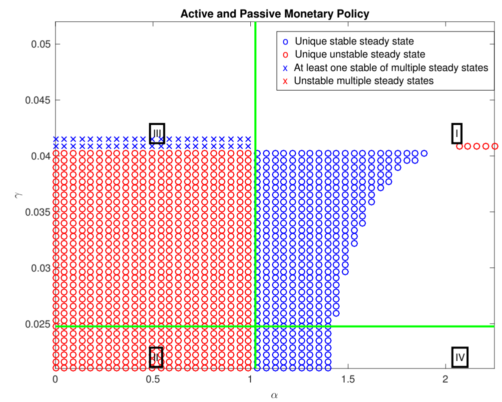


Figure 9e: Benchmark 1 with $M = 0.35$ at $R = 1.025$

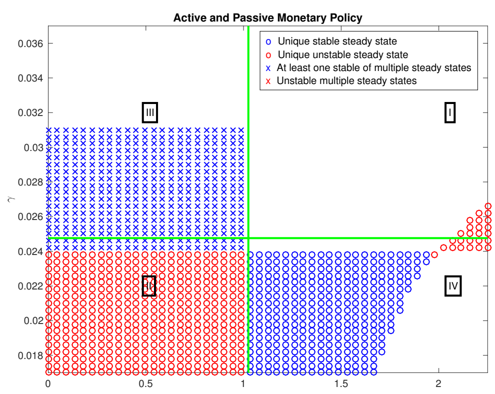
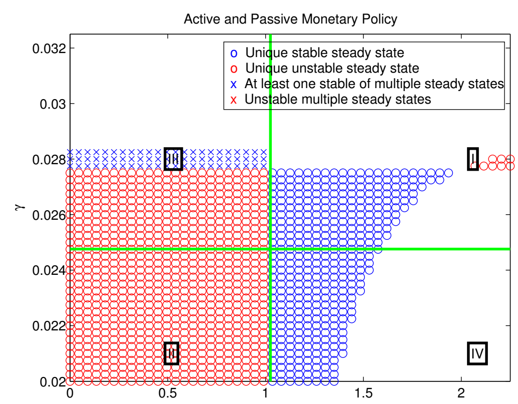


Figure 9f: Benchmark 1 with $b_0 = 0.75$



Benchmark 1: $\sigma = 1.0$, and $\kappa = 1.00$.