Liquidity, Risk Taking, and the Lender of Last Resort

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28 February 2005

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This paper studies the strategic interaction between a bank whose deposits are randomly withdrawn and a lender of last resort (LLR) that bases its decision on supervisory information on the quality of the bank's assets. The bank is subject to a capital requirement and chooses the liquidity buffer that it wants to hold and the risk of its loan portfolio. The equilibrium choice of risk is shown to be decreasing in the capital requirement and increasing in the interest rate charged by the LLR. Moreover, when the LLR does not charge penalty rates, the bank chooses the same level of risk and a smaller liquidity buffer than in the absence of an LLR. Thus, in contrast with the general view, the existence of an LLR does not increase the incentives to take risk, while penalty rates do.

JEL Codes: E58, G21, G28.

From their inception, central banks have assumed as one of their key responsibilities the provision of liquidity to banks unable to find it elsewhere. The classical doctrine on the lender of last resort (LLR) was put forward by Bagehot (1873, 96–7): "Nothing, therefore, can be more certain than that the Bank of England . . . must in time of panic do what all other similar banks must do. . . . And for this purpose there are two rules: First. That these loans should only be made at a very high rate of interest. . . . Secondly. That at this rate these advances should be made on all good banking securities, and as largely as the public ask for them." The contemporary literature on the LLR has disagreed on whether the aim of "staying

*I would like to thank Andrés Almazán, Giovanni Dell'Ariccia, Arturo Estrella, Douglas Gale, Javier Suarez, and an anonymous referee for their comments, and Dante Amengual and Abel Elizalde for their excellent research assistance. Address for correspondence: CEMFI, Casado del Alisal 5, 28014 Madrid, Spain. Tel: (34) 914290551. Fax: (34) 914291056. E-mail: repullo@cemfi.es.

the panic" may be achieved by open market operations (see, for example, Goodfriend and King [1988] or Kaufman [1991]) or whether it requires lending to individual banks (see, for example, Flannery [1996] or Goodhart [1999]).[1] However, both sides seem to agree on the proposition that such lending creates a moral hazard problem. As argued by Solow (1982, 242): "The existence of a credible LLR must reduce the private cost of risk taking. It can hardly be doubted that, in consequence, more risk will be taken."

The purpose of this paper is to show that this proposition is not generally true. Specifically, we model the strategic interaction between a bank and an LLR. The bank is funded with insured deposits and equity capital, is subject to a minimum capital requirement, and can invest in two assets: a safe and perfectly liquid asset, and a risky and illiquid asset, whose risk is privately chosen by the bank. Deposits are randomly withdrawn. If the withdrawal is larger than the funds invested in the safe asset (the liquidity buffer), the bank will be forced into liquidation unless it can secure emergency lending from the LLR. In this setting, we show that when the LLR does not charge penalty rates, the bank chooses the same level of risk and a smaller liquidity buffer than in the absence of an LLR. Moreover, the equilibrium choice of risk is increasing in the penalty rate.

To explain the basic intuition for these results, consider a setup in which a risk-neutral bank raises a unit of insured deposits at an interest rate that is normalized to zero, and invests all these funds in an illiquid asset that yields a gross return $R_1 = R(p)$ with probability p, and $R_0 = 0$ otherwise. Moreover, suppose that p is chosen by the bank at the time of investment, and that the success return $R(p)$ is decreasing in p, so safer investments yield a lower success return.

Without deposit withdrawals, and hence without the need for an LLR, with probability p the bank gets the return $R(p)$ of its investment in the risky asset minus the amount due to the depositors, that is, $R(p) - 1$, and with probability $1 - p$ the bank fails. Under limited liability, the bank then maximizes $p[R(p) - 1]$, which gives $p^* = \arg\max p[R(p) - 1]$.

Suppose now that a certain fraction of the deposits are withdrawn, and that there is an LLR that only provides the required

[1] All these references (and more) are usefully collected in Goodhart and Illing (2002).

funding if its supervisory information on the quality of the bank's asset is good. Specifically, let s_1 denote the good supervisory signal, and let $q = \Pr(s_1 \mid R_1)$ denote the quality of the supervisory information. If the LLR only charges the zero deposit rate, the bank will get $R(p) - 1$ with probability $\Pr(s_1, R_1) = \Pr(s_1 \mid R_1)\Pr(R_1) = qp$. Since the constant q factors out of the maximization problem, we get the same $p^* = \arg\max p[R(p) - 1]$. Hence we conclude that the introduction of deposit withdrawals and an LLR does not affect the bank's incentives to take risk.

As for the result on penalty rates, the intuition is that they increase the expected interest payments in the high-return state and, consequently, push the bank toward choosing higher risk and higher return strategies (i.e., a lower p). This positive relationship between the bank's (expected) funding costs and its portfolio risk is not new, since it is a simple implication of the analysis in the classical paper on credit rationing of Stiglitz and Weiss (1981). In particular, they show (p. 393) how "higher interest rates induce firms to undertake projects with lower probabilities of success but higher payoffs when successful." Applying the same argument to banks instead of firms gives the key result.

To endogenize the decision of the LLR, we adopt a political economy perspective according to which government agencies have objectives that need not correspond with the maximization of social welfare. In particular, following Repullo (2000) and Kahn and Santos (2001), we assume that the LLR cares about (1) the revenues and costs associated with its lending activity and (2) whether the bank fails. This may be justified by relating the payoff of the officials in charge of LLR decisions with the surpluses or deficits of the agency, as well as with the possible reputation costs associated with a bank failure.

Specifying an objective function for the LLR would not be needed if the supervisory information were verifiable, because then the intervention rule could be specified ex-ante, possibly in order to implement a socially optimal decision. However, the information coming from bank examinations is likely to contain many subjective elements that are difficult to describe ex-ante, so it seems reasonable to assume that it is nonverifiable. In this case, the decision will have to be delegated to the LLR, which will simply compare its conditional expected payoff of supporting and not supporting the bank.

To facilitate the presentation, the analysis starts with a basic model in which the bank is fully funded with deposits and can only invest in the risky asset. Then the model is extended to the case where the bank can invest in a safe and perfectly liquid asset and can raise equity capital. In the general model, we also assume that the bank is subject to a minimum capital requirement, and that (in line with Basel bank capital regulation) investment in the safe asset does not carry a capital charge.

We characterize the Nash equilibrium of the game between the bank and the LLR, where the former chooses the level of risk (and, in the general model, its capital and liquidity buffer) and the latter its contingent lending policy. The LLR's equilibrium strategy is straightforward: it will support the bank if and only if the liquidity shortfall is smaller than or equal to a critical value that is decreasing in the ex-post (i.e., conditional on the supervisory signal) probability of bank failure. The bank's equilibrium strategy is, however, more difficult to characterize. The reason is that its objective function is likely to be convex in the capital decision, which leads to a corner solution where the bank's capital is equal to the minimum required by regulation. In this case, the equilibrium level of risk only depends on the capital requirement, with higher capital increasing the bank shareholders' losses in case of default and reducing their incentives for risk taking. We complete the analysis by deriving numerically, for a simple parameterization of the model, the bank's equilibrium liquidity. We show that in equilibrium, the bank chooses the same level of risk of its illiquid portfolio and a lower liquidity buffer than in the absence of an LLR.

Four extensions are then discussed. First, we derive the result that penalty rates increase the equilibrium choice of risk. Next, we examine the second of Bagehot's rules—namely, that last-resort lending be collateralized—and show that this protection translates into a lower liquidity buffer and therefore a higher probability that the bank will require emergency liquidity assistance, but without any effect on risk taking. Third, we consider the effects of introducing a higher discount rate for the LLR, which yields a forbearance result: in equilibrium, the bank is more likely to receive support from the LLR, and hence it will hold a lower liquidity buffer. Finally, we look at the case where the LLR shares a fraction of the deposit insurance payouts (which includes, in the limit, the case where the LLR is the deposit insurer) and show that in this case the LLR's decision becomes more sensitive to the supervisory information.

It is important to stress that the key result on the zero effect on risk taking of having an LLR crucially depends on the specification of the order of moves, in particular the fact that the bank cannot modify the level of risk after receiving the support of the LLR (or cannot borrow from the LLR to undertake new investments). But in such a situation, the LLR is likely to carefully monitor the bank, preventing it from engaging in any significant risk shifting, so this seems a reasonable assumption.

Although the literature on the LLR is extensive (see Freixas et al. [2000] for a recent survey), somewhat surprisingly there has been little formal modeling of the issues discussed in this paper. Most of the relevant papers invoke general results on the link between any form of insurance and moral hazard. Moreover, liquidity support is not always distinguished from capital support, which clearly has bad incentive effects whenever it translates into rescuing the shareholders of a distressed bank. This paper restricts attention to liquidity support based on supervisory information on the quality of the bank's assets, and shows that under fairly general conditions this support does not encourage risk taking.

The paper is organized as follows. Section 1 presents the basic model of the game between the bank and the LLR. Section 2 introduces equity capital and a minimum capital requirement, and allows the bank to invest in a safe asset, characterizing the equilibrium with and without an LLR and discussing its comparative statics properties. Section 3 analyzes the effects of Bagehot's rules of charging penalty rates and requiring collateral, as well as changing the objective function of the LLR to allow for higher discounting of future payouts and sharing deposit insurance payouts. Section 4 offers some concluding remarks.

1. The Basic Model

Consider an economy with three dates ($t = 0, 1, 2$) and two risk-neutral agents: a *bank* and a *lender of last resort* (LLR). At date 0 the bank raises one unit of deposits at an interest rate that is normalized to zero, and invests these funds in an asset that yields a random *return* R at date 2. The probability distribution of R is described by

$$R = \begin{cases} R_0, & \text{with probability } 1-p, \\ \\ R_1, & \text{with probability } p, \end{cases} \tag{1}$$

where $p \in [0,1]$ is a parameter chosen by the bank at date 0. We assume that $R_0 < 1 < R_1$, so $1-p$ measures the riskiness of the bank's portfolio. The risky asset is illiquid in that there is no secondary market where it can be traded at date 1. However, the asset can be fully liquidated at this date, which yields a *liquidation value* $L \in (0,1)$.[2] Deposits are fully insured and can be withdrawn at either date 1 or date 2. To simplify the presentation, deposit insurance premia are set equal to zero.

At date 1 a fraction $v \in [0,1]$ of the deposits are withdrawn. Since the bank's asset is illiquid, if $v > 0$ the bank faces a liquidity problem that can only be solved by borrowing from the LLR. If such funding is not provided, the bank is liquidated at date 1. Otherwise, the bank stays open until the final date 2. The liquidity shock v is observable, and we initially suppose that the LLR only charges the deposit rate, which has been normalized to zero.

In order to decide whether to provide this emergency funding, the LLR supervises the bank, which yields a *signal* $s \in \{s_0, s_1\}$ on the return of the bank's risky asset. Signal s is assumed to be nonverifiable, so the LLR's decision rule cannot be designed ex-ante, but will be chosen ex-post by the LLR in order to maximize an objective function that will be specified below.

We introduce the following assumptions.

Assumption 1. $R_0 = 0$ and $R_1 = R(p)$, where $R(p)$ is decreasing and concave, with $R(1) \geq 1$ and $R(1) + R'(1) < 0$.

Assumption 2. $\Pr(s_0 \mid R_0) = \Pr(s_1 \mid R_1) = q \in [\frac{1}{2}, 1]$.

Assumption 1, together with (1), implies that the expected final return of the risky asset, $E(R) = pR(p)$, reaches a maximum at $\widehat{p} \in (0,1)$, which is characterized by the first-order condition

$$R(\widehat{p}) + \widehat{p}R'(\widehat{p}) = 0. \tag{2}$$

[2]The liquidation value L could be correlated with the final return R, but this would not change the results.

To see this, notice that the first derivative of $pR(p)$ with respect to p equals $R(0) > 0$ for $p = 0$ and $R(1) + R'(1) < 0$ for $p = 1$, and the second derivative satisfies $2R'(p) + pR''(p) < 0$. Thus, increases in p below (above) $\widehat{p}$ increase (decrease) the expected final return of the risky asset. Moreover, we have $\widehat{p}R(\widehat{p}) > R(1) \geq 1$. Assumption 1 is borrowed from Allen and Gale (2000, chap. 8) and allows us to analyze in a continuous manner the risk-shifting effects of different institutional settings.[3]

Assumption 2 introduces a parameter q that describes the quality of the supervisory information.[4] This information is only about whether the final return of the risky asset will be low (R_0) or high (R_1), and not about the particular value $R(p)$ taken by the high return. By Bayes' law, it is immediate to show that

$$\Pr(R_1 \mid s_0) = \frac{p(1-q)}{p(1-q)+(1-p)q}, \tag{3}$$

and

$$\Pr(R_1 \mid s_1) = \frac{pq}{pq+(1-p)(1-q)}. \tag{4}$$

Hence when $q = \frac{1}{2}$ we have $\Pr(R_1 \mid s_0) = \Pr(R_1 \mid s_1) = p$, so the supervisory signal is uninformative, while when $q = 1$ we have $\Pr(R_1 \mid s_0) = 0$ and $\Pr(R_1 \mid s_1) = 1$, so the signal completely reveals whether the final return will be low or high. Since $\Pr(R_1 \mid s_0) < p < \Pr(R_1 \mid s_1)$ for $p < 1$ and $q > \frac{1}{2}$, s_0 and s_1 will be called the bad and the good signal, respectively.

From the point of view of the initial date 0, the deposit withdrawal v is a continuous random variable with support $[0, 1]$ and cumulative distribution function $F(v)$.[5] Since deposits are fully insured, it is natural to assume that the withdrawal v is independent of the final return R. Also, v is assumed to be independent of the supervisory signal s.

The bank and the LLR play a sequential game in which the bank chooses at date 0 the riskiness of its portfolio p, and if $v > 0$, the LLR

[3]This assumption has also been used by Cordella and Levy-Yeyati (2003) and Repullo (2005).

[4]More generally, we could have $\Pr(s_0 \mid R_0) \neq \Pr(s_1 \mid R_1)$, but this would not change the results.

[5]The distribution function $F(v)$ could have a mass point at $v = 0$, in which case $F(0) > 0$ would be the probability that the bank does not suffer a liquidity shock at date 1.

decides at date 1 whether to support the bank based on two pieces of information: the size of the liquidity shock v, and the supervisory signal s. Importantly, the LLR does not observe the bank's choice of p, so we have a game of complete but imperfect information.

In this game, the LLR is assumed to care about the expected value of its final wealth net of a share α of the *social cost* c incurred in the event of a bank failure. Such cost comprises the administrative costs of closing the bank and paying back depositors and the negative externalities associated with the failure (contagion to other banks, breakup of lending relationships, distortions in the monetary transmission mechanism, etc.). As noted above, the LLR's objective function may be justified by relating the payoff of the officials in charge of its decisions with the income generated or lost through its lending activity and the social cost associated with a bank failure. To simplify the presentation, we assume that $\alpha = 1$, so the LLR fully internalizes the social cost of bank failure.[6]

Consider a situation in which $v > 0$, and let s be the signal observed by the LLR. The payoff of the LLR if it supports the bank is computed as follows. With probability $\Pr(R_1 \mid s)$ the bank will be solvent at date 2 and the LLR will recover its loan v, while with probability $\Pr(R_0 \mid s)$ the bank will fail and the LLR will lose v and incur the cost c, so the LLR's expected payoff is $-(v+c)\Pr(R_0 \mid s)$. On the other hand, if the LLR does not provide the liquidity support, the bank will be liquidated at date 1, and the LLR's payoff will be $-c$. Hence the LLR will support the bank if

$$-(v + c)\Pr(R_0 \mid s) \geq -c.$$

Using the fact that $\Pr(R_1 \mid s) = 1 - \Pr(R_0 \mid s)$, this condition simplifies to

$$v \leq \frac{c\Pr(R_1 \mid s)}{\Pr(R_0 \mid s)}.$$

Substituting (3) and (4) into this expression, it follows that when the LLR observes the bad signal s_0, it will support the bank if

$$v \leq v_0 \equiv \frac{cp(1-q)}{(1-p)q}, \tag{5}$$

[6] Clearly, this assumption does not affect the characterization of the equilibrium of the game, since it is equivalent to a change in the cost c. Interestingly, Repullo (2000) assumes $\alpha < 1$, while Kahn and Santos (2001) assume $\alpha > 1$.

and when the LLR observes the good signal s_1, it will support the bank if

$$v \leq v_1 \equiv \frac{cpq}{(1-p)(1-q)}. \tag{6}$$

The critical values v_0 and v_1 defined in (5) and (6) satisfy

$$v_1 = \left(\frac{q}{1-q}\right)^2 v_0,$$

which implies $v_1 > v_0$ whenever $q > \frac{1}{2}$. Hence if the signal is informative, the LLR is more likely to provide support to the bank when it observes the good signal s_1 than when it observes the bad signal s_0. Moreover, the critical value v_0 is decreasing in the quality q of the supervisory information, with $\lim_{q\to 1} v_0 = 0$, and the critical value v_1 is increasing in q, with $\lim_{q\to 1} v_1 = \infty$.[7] Thus, when the signal is perfectly informative, the bank will never be supported if the signal is bad, and will always be supported if it is good.

The critical values v_0 and v_1 are increasing in the social cost of bank failure c, because when this cost is high, the LLR has a stronger incentive to lend to the bank in order to save c when the high return R_1 obtains. They are also increasing in $p = \Pr(R_1)$, because when this probability is high, the LLR is more likely to recover its loan v and save the cost c.

By limited liability, the bank gets a zero payoff if it is liquidated at date 1 or fails at date 2, and gets $R(p) - 1$ if it succeeds at date 2. This event happens when the high return R_1 obtains and either the LLR observes the bad signal s_0 and the liquidity shock satisfies $v \leq v_0$, or it observes the good signal s_1 and the liquidity shock satisfies $v \leq v_1$. By assumption 2 and the independence of v we have

$$\begin{aligned}\Pr(R_1, s_0, \text{ and } v \leq v_0) &= \Pr(R_1)\Pr(s_0 \mid R_1)\Pr(v \leq v_0)\\ &= p(1-q)F(v_0),\\ \Pr(R_1, s_1, \text{ and } v \leq v_1) &= \Pr(R_1)\Pr(s_1 \mid R_1)\Pr(v \leq v_1)\\ &= pqF(v_1).\end{aligned}$$

Hence, the bank's objective function is

$$U_B = p[(1-q)F(v_0) + qF(v_1)][R(p) - 1]. \tag{7}$$

[7] The fact that v_1 may be greater than one is not a problem, because since the support of v is $[0, 1]$, we have $\Pr(v \leq v_1) = F(v_1) = 1$, so in this case the bank would be supported with probability one.

A Nash equilibrium of the game between the bank and the LLR is a choice of risk p^* by the bank, and a choice of maximum liquidity support by the LLR contingent on the bad and the good signal, v_0^* and v_1^*, such that p^* maximizes

$$p[(1-q)F(v_0^*) + qF(v_1^*)][R(p) - 1],$$

and

$$v_0^* = \frac{cp^*(1-q)}{(1-p^*)q} \quad \text{and} \quad v_1^* = \frac{cp^*q}{(1-p^*)(1-q)}.$$

In this definition, it is important to realize that since the LLR does not observe the bank's choice of risk, the critical values v_0^* and v_1^* only depend on the equilibrium p^*. This in turn implies that the term $[(1-q)F(v_0^*) + qF(v_1^*)]$ factors out in the bank's objective function, so its problem reduces to maximize $p[R(p) - 1]$.[8]

The first-order condition that characterizes the equilibrium choice of risk p^* is

$$R(p^*) + p^*R'(p^*) = 1. \tag{8}$$

Since, by assumption 1, $R(p) + pR'(p)$ is decreasing in p, conditions (2) and (8) imply that p^* is strictly below the first-best $\widehat{p}$, so the bank will be choosing too much risk. This is just the standard risk-shifting effect that follows from debt financing under limited liability.

It should be noted that the bank's choice of p changes the probability distribution of the signals, increasing $\Pr(s_1) = pq+(1-p)(1-q)$ and decreasing $\Pr(s_0) = 1 - \Pr(s_1)$ (as long as $q > \frac{1}{2}$). However, by assumption 2, p does not affect the distribution of the signals conditional on the realization of the high return R_1, which implies that the bank's probability of getting $R(p) - 1$ is linear in $p = \Pr(R_1)$.

To sum up, we have set up a model of a bank and an LLR in which the former chooses the riskiness of its portfolio and the latter chooses whether to lend to the bank to cover random deposit withdrawals, a decision that depends on a signal on the ex-post quality of the portfolio. We have shown that the bank's equilibrium choice of risk is independent of the distribution of the liquidity shocks and the other parameters that determine the LLR's decision, such as the quality of the supervisory information or the social cost of bank failure.

[8]In a sequential game of complete information, the characterization of equilibrium would be more complicated, since the critical values v_0^* and v_1^* would depend on the bank's choice of p.

2. The General Model

We now introduce in our basic model two features of banking in the real world that are relevant to the problem under discussion. First, on the asset side of the bank's balance sheet, we suppose that, apart from the risky asset, the bank can invest in a safe and perfectly liquid asset that can be used as a buffer against liquidity shocks. Second, on the liability side, we suppose that the bank can be funded with both deposits and equity capital, and that the bank is subject to a minimum capital requirement. The LLR observes both the bank's equity capital and its investment in the liquid asset. We characterize the equilibrium of the new game between the bank and the LLR, compare it with that of a model without an LLR, and examine its comparative statics properties.

Specifically, suppose that at date 0 the bank raises k equity capital and $1-k$ deposits, and invests λ in the safe asset and $1-\lambda$ in the risky asset, so the size of its balance sheet is normalized to one.[9] Bank capital has to satisfy the constraint $k \geq \kappa(1-\lambda)$, where $\kappa \in (0,1)$. Thus, the capital requirement depends on the (observable) bank's investment in the risky asset, but not on the (unobservable) bank's choice of risk.

We assume that the return of the safe asset is equal to the deposit rate, which has been normalized to zero, and that bank capital is provided by a special class of agents, called bankers, who require an expected rate of return $\delta \geq 0$ on their investment. A strictly positive value of δ captures either the scarcity of bankers' wealth or, perhaps more realistically, the existence of a premium for the agency and asymmetric information problems faced by the bank shareholders.[10]

2.1 Characterization of Equilibrium

At date 1 a fraction $v \in [0,1]$ of the deposits are withdrawn. Since the bank has $1-k$ deposits, then $v(1-k)$ deposits are withdrawn

[9]This assumption is made without loss of generality. The same results would obtain if, for example, the bank raised one unit of deposits and k units of capital, and invested λ in the safe asset and $1+k-\lambda$ in the risky asset.

[10]See Holmström and Tirole (1997) and Diamond and Rajan (2000) for explicit models of why δ might be positive. The same assumption is made by Bolton and Freixas (2000), Hellmann, Murdock, and Stiglitz (2000), and Repullo and Suarez (2004), among others.

at this date. There are two cases to consider. First, if $v(1-k) \leq \lambda$, the bank can repay the depositors by selling the required amount of the safe asset, so it keeps $\lambda - v(1-k)$ invested in the safe asset. In this case the bank's payoff in the high-return state equals the return of its investment in the safe asset, $\lambda - v(1-k)$, plus the return of its investment in the risky asset, $(1-\lambda)R(p)$, minus the amount paid to the remaining depositors, $(1-v)(1-k)$, that is,

$$\lambda - v(1-k) + (1-\lambda)R(p) - (1-v)(1-k) = (1-\lambda)[R(p)-1] + k.$$

Second, if $v(1-k) > \lambda$, the bank needs to borrow $v(1-k) - \lambda$ from the LLR in order to avoid liquidation. If such funding is obtained, the bank's payoff in the high-return state equals the return of its investment in the risky asset, $(1-\lambda)R(p)$, minus the amount paid to the remaining depositors, $(1-v)(1-k)$, minus the amount paid to the LLR, $v(1-k) - \lambda$, that is,

$$(1-\lambda)R(p) - (1-v)(1-k) - [v(1-k) - \lambda] = (1-\lambda)[R(p)-1] + k.$$

In both cases, if the low-return state obtains, the bank's net worth is $\lambda - (1-k)$, which will be negative as long as the bank's investment in the liquid asset, λ, does not exceed its deposits, $1-k$, which will generally obtain in equilibrium.[11] Hence, by limited liability, the bank's payoff in the low-return state will be zero. Obviously, its payoff will also be zero when $v(1-k) > \lambda$ and the LLR does not support the bank.

The decision of the LLR in the case in which the bank requires emergency lending, $v(1-k) > \lambda$, is characterized as follows. If the LLR observes signal s and decides to support the bank, with probability $\Pr(R_1 \mid s)$ the bank will be solvent at date 2 and the LLR will recover its loan $v(1-k) - \lambda$, while with probability $\Pr(R_0 \mid s)$ the bank will fail and the LLR will lose $v(1-k) - \lambda$ and incur the cost c. If, on the other hand, the LLR does not provide the liquidity support, the bank will be liquidated at date 1, and the LLR's payoff will be $-c$. Hence the LLR will support the bank if

$$-[v(1-k) - \lambda + c] \Pr(R_0 \mid s) \geq -c.$$

[11] In particular, we show below that under plausible conditions the capital requirement will be binding, so $k = \kappa(1-\lambda)$, which implies $\lambda - (1-k) = -(1-\kappa)(1-\lambda) < 0$.

As before, substituting (3) and (4) into this expression, it follows that when the LLR observes the bad signal s_0, it will support the bank if the liquidity shortfall, $v(1-k)-\lambda$, is smaller than or equal to the critical value v_0 given by (5), that is, if

$$v \leq \frac{v_0+\lambda}{1-k}, \tag{9}$$

and when it observes the good signal s_1, it will support the bank if the liquidity shortfall, $v(1-k)-\lambda$, is smaller than or equal to the critical value v_1 given by (6), that is, if

$$v \leq \frac{v_1+\lambda}{1-k}. \tag{10}$$

Thus, the probability that the bank will reach the final date 2 is increasing in its investment in the safe asset λ and its equity capital k. This is explained by the role of the safe asset as a buffer against liquidity shocks, and by the fact that the higher the bank capital, the lower its deposits and hence the size of the liquidity shocks.

The bank's objective function is to maximize the expected value of the shareholders' payoff net of the opportunity cost of their initial infusion of capital. The latter is simply $(1+\delta)k$. To compute the former, notice that bank shareholders get a zero payoff if the bank is liquidated at date 1 or fails at date 2, and they get $(1-\lambda)(R(p)-1)+k$ if it succeeds at date 2. This event happens when the high return R_1 obtains and either the LLR observes the bad signal s_0 and the liquidity shock v satisfies (9), or it observes the good signal s_1 and the liquidity shock v satisfies (10). As before, we have

$$\begin{aligned}
\Pr\left(R_1, s_0, \text{ and } v \leq \frac{v_0+\lambda}{1-k}\right) &= p(1-q)F\left(\frac{v_0+\lambda}{1-k}\right), \\
\Pr\left(R_1, s_1, \text{ and } v \leq \frac{v_1+\lambda}{1-k}\right) &= pqF\left(\frac{v_1+\lambda}{1-k}\right).
\end{aligned}$$

Hence, the bank's objective function in the general model is

$$\begin{aligned}
U_B = p&\left[(1-q)F\left(\frac{v_0+\lambda}{1-k}\right) + qF\left(\frac{v_1+\lambda}{1-k}\right)\right] \\
&\times [(1-\lambda)(R(p)-1)+k] - (1+\delta)k.
\end{aligned} \tag{11}$$

Obviously, U_B coincides with the objective function (7) in the previous section when $\lambda = 0$ and $k = 0$.

A Nash equilibrium of the game between the bank and the LLR is a choice of liquidity λ^*, capital k^*, and risk p^* by the bank, and a choice of maximum liquidity support by the LLR contingent on the bad and the good signal, v_0^* and v_1^*, such that (λ^*, k^*, p^*) maximizes

$$p\left[(1-q)F\left(\frac{v_0^*+\lambda}{1-k}\right)+qF\left(\frac{v_1^*+\lambda}{1-k}\right)\right] \times [(1-\lambda)(R(p)-1)+k]-(1+\delta)k, \tag{12}$$

subject to the capital requirement $k \geq \kappa(1-\lambda)$, and

$$v_0^* = \frac{cp^*(1-q)}{(1-p^*)q} \quad \text{and} \quad v_1^* = \frac{cp^*q}{(1-p^*)(1-q)}. \tag{13}$$

As in the basic model, it is important to note that since the LLR does not observe the bank's choice of risk, the critical values v_0^* and v_1^* only depend on the equilibrium p^*. This in turn implies that the bank's problem reduces to maximize $p\left[(1-\lambda)(R(p)-1)+k\right]$. Thus, the bank's choice of risk is characterized by the first-order condition

$$(1-\lambda^*)[R(p^*)-1]+k^*+p^*(1-\lambda^*)R'(p^*)=0,$$

which simplifies to

$$R(p^*)+p^*R'(p^*)=1-\frac{k^*}{1-\lambda^*}. \tag{14}$$

Comparing this expression with (2) and (8), and taking into account that, by assumption 1, $R(p)+pR'(p)$ is decreasing in p, it follows that the bank's equilibrium choice of risk p^* will be closer to the first-best $\widehat{p}$ than in the model without the capital requirement. This is just the standard *capital-at-risk effect*: higher capital implies higher losses for the bank's shareholders in case of default and hence lower incentives for risk taking.[12]

If the bank's equilibrium choice of capital k^* is at the corner $\kappa(1-\lambda^*)$, then the first-order condition (14) further simplifies to

$$R(p^*)+p^*R'(p^*)=1-\kappa. \tag{15}$$

[12] See Hellmann, Murdock, and Stiglitz (2000) and Repullo (2004) for a recent discussion of this effect.

In this case, the equilibrium p^* only depends on the capital requirement κ. Moreover, assumption 1 implies that $R(p) + pR'(p)$ is decreasing in p, which gives $dp^*/d\kappa > 0$. Hence, the higher the capital requirement, the lower the risk chosen by the bank.[13]

In general, it is difficult to prove that this corner solution will obtain, since the properties of the bank's objective function (12) depend on the shape of the distribution function of the liquidity shock $F(v)$.[14] For this reason, in what follows we work with a specific parameterization of $F(v)$, namely $F(v) = v^\eta$, where $\eta \in (0,1)$.[15] In this case it can be checked that the bank's objective function (12) is convex in k, so we can only have either $k^* = \kappa(1 - \lambda^*)$ or $k^* = 1$. But for large k we have

$$F\left(\frac{v_0^* + \lambda}{1-k}\right) = F\left(\frac{v_1^* + \lambda}{1-k}\right) = 1,$$

so the derivative of (12) with respect to k is $p - (1 + \delta) < 0$. Hence $k = 1$ cannot be a solution, which gives $k^* = \kappa(1 - \lambda^*)$.

This result implies that the equilibrium of the game between the bank and the LLR is easy to characterize. The risk p^* chosen by the bank is the unique solution of the first-order condition (15). This in turn determines the critical values v_0^* and v_1^* that characterize the behavior of the LLR. Substituting $p = p^*$ and $F(v) = v^\eta$ into the bank's objective function (12), we then find the value of λ^* by maximizing

$$\begin{aligned} p^* \left[(1-q)\left(\frac{v_0^* + \lambda}{1-k}\right)^\eta + q\left(\frac{v_1^* + \lambda}{1-k}\right)^\eta\right] \\ \times [(1-\lambda)(R(p^*) - 1) + k] - (1+\delta)k, \end{aligned} \tag{16}$$

subject to $k = \kappa(1 - \lambda)$. Finally, we get $k^* = \kappa(1 - \lambda^*)$.

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[13]Notice that for $\kappa = 1$, that is, a 100% capital requirement, (2) and (15) imply $p^* = \widehat{p}$.

[14]However, finding that k is at the minimum required by regulation is standard in both static and dynamic models of banking; see, for example, Repullo and Suarez (2004) and Repullo (2004).

[15]Notice that this is a simple special case of a beta distribution for which the density function $F'(v) = \eta v^{\eta-1}$ is decreasing in v, so small liquidity shocks are more likely than large shocks.

Going analytically beyond this point is, however, complicated, because although the bank's objective function (16) is concave in λ, this is in general no longer the case once we substitute the constraint $k = \kappa(1 - \lambda)$ into (16), since this function is convex in k. For this reason, our results on equilibrium liquidity and capital will be derived from numerical solutions.

2.2 Equilibrium without an LLR

We now compare the equilibrium behavior of the bank when there is an LLR with its behavior when there is no LLR. The objective function of the bank in such a model is a special case of (11) when we set $v_0 = v_1 = 0$ (i.e., no last-resort lending), which gives

$$U_B = pF\left(\frac{\lambda}{1-k}\right)[(1-\lambda)(R(p)-1)+k] - (1+\delta)k.$$

Thus, the bank gets $(1 - \lambda)(R(p) - 1) + k$ in the high-return state, which obtains with probability p, but only if it has sufficient liquidity to cover the deposit withdrawals at date 1—that is, if $v(1 - k) \leq \lambda$, an event that happens with probability $F(\lambda/(1 - k))$.

From here we can follow our previous steps to conclude that when the bank's capital k is at the corner $\kappa(1 - \lambda^*)$, its choice of risk is characterized by the first-order condition (15), so we get exactly the same p^* as in the model with the LLR. In other words, contrary to what has been taken for granted in the banking literature, our model predicts that *the existence of an LLR does not have any effect on the bank's incentives to take risk.*

Computation of the effects of having an LLR on the liquidity decision of the bank requires us to specify the functional forms of the high return of the risky asset, $R(p)$, and the cumulative distribution function of the liquidity shock, $F(v)$, as well as the parameter values of the capital requirement κ, the cost of capital δ, the informativeness of the supervisory signal q, and the social cost of bank failure c. Since our focus is on qualitative results, we will not calibrate the model to obtain plausible numerical results, but instead choose simple functional forms and parameter values. Specifically, the functional forms $R(p) = 3 - 2p^2$ and $F(v) = v^\eta$, with $\eta = 0.25$,

Table 1. Equilibria with and without an LLR

	With LLR	Without LLR
λ^*	0.11	0.23
k^*	0.09	0.08
p^*	0.59	0.59
v_0^*	0.10	–
v_1^*	0.22	–

will be maintained in all our simulations,[16] and our baseline parameter values are $L = 0.50$, $\kappa = \delta = c = 0.10$, and $q = 0.60$.

The corresponding equilibria with and without an LLR are shown in table 1. As noted above, the level of risk p^* chosen by the bank is the same in both models, and may be obtained by substituting $R(p) = 3 - 2p^2$ into (15), which gives $p^* = \sqrt{(2+\kappa)/6} = 0.59$. Not surprisingly, the results in table 1 show that the liquidity buffer λ^* is much larger in the absence of an LLR.[17] Given that $k^* = \kappa(1 - \lambda^*)$, this in turn implies a lower level of capital.

When the deposit withdrawal $v(1-k^*)$ is below the bank's liquidity λ^* (an event that happens with probability 0.58 in the model with the LLR, and with probability 0.71 in the model without it), the bank will be able to repay the depositors by selling the required amount of the safe asset. Moreover, in the first model, when $v(1 - k^*) > \lambda^*$, the LLR will provide liquidity up to $v_0^* = 0.10$ when it observes the bad signal s_0, and up to $v_1^* = 0.22$ when it observes the good signal s_1.

The probability that the bank gets a positive payoff in the model with an LLR is

$$p^* \left[(1-q)F\left(\frac{v_0^* + \lambda^*}{1 - k^*}\right) + qF\left(\frac{v_1^* + \lambda^*}{1 - k^*}\right) \right] = 0.44,$$

[16]Observe that $R(p) = 3 - 2p^2$ is decreasing and concave, with $R(1) = 1$ and $R(1) + R'(1) = -3 < 0$, so assumption 1 is satisfied. Also, the median liquidity shock is $F^{-1}(0.5) = 0.0625$.

[17]Gonzalez-Eiras (2003) provides some interesting empirical evidence on this result. He shows that the contingent credit line agreement signed by the Central Bank of Argentina with a group of international banks in December 1996 enhanced that central bank's ability to act as an LLR and led to a significant decrease in the liquidity holdings of domestic Argentinian banks.

while the corresponding probability in the model without an LLR is

$$p^* F\left(\frac{\lambda^*}{1-k^*}\right) = 0.42.$$

Since in the first model the bank is investing a higher proportion of its portfolio in the risky asset, its equilibrium expected payoff is significantly higher with an LLR (0.45 against 0.37).

2.3 Comparative Statics

We next analyze the effect on the equilibrium of the game between the bank and the LLR of changes in the capital requirement κ, the cost of capital δ, the informativeness of the supervisory signal q, and the social cost of bank failure c. The results summarized in table 2 are derived by computing the equilibrium corresponding to deviations in κ, δ, q, and c from the baseline case.

As noted above, an increase in the capital requirement κ leads to an increase in p^*, which by (13) increases the maximum support provided by the LLR contingent on the bad and the good signal, v_0^* and v_1^*. The effect on λ^* is also positive. Two reasons explain this result. First, the higher capital requirement makes investment in the risky asset, which does not carry a capital charge, relatively less attractive for the bank than investing in the safe asset. Second, the higher p^* reduces the success payoff of the risky asset, $R(p^*)$, and also makes it relatively less attractive for the bank than the safe

Table 2. Equilibrium Effects of Changes in the Capital Requirement κ, the Cost of Capital δ, the Informativeness of the Supervisory Signal q, and the Social Cost of Bank Failure c

x	$\frac{dp^*}{dx}$	$\frac{d\lambda^*}{dx}$	$\frac{dk^*}{dx}$	$\frac{dv_0^*}{dx}$	$\frac{dv_1^*}{dx}$
κ	+	+	+	+	+
δ	0	+	−	0	0
q	0	−	+	−	+
c	0	−	+	+	+

asset. On the other hand, the higher liquidity support offered by the LLR reduces the bank's incentives to invest in the safe asset, but the numerical results show that this effect is dominated by the other two.

With regard to the other comparative statics results, note first that, as shown analytically, the value of p^* chosen by the bank only depends on the capital requirement κ, so the effect of the other three parameters is zero.

Since the cost of capital δ does not affect p^*, it does not have any effect either on the maximum liquidity support provided by the LLR contingent on the bad and the good signal, v_0^* and v_1^*. The cost of capital δ has a positive effect on equilibrium liquidity λ^*, because when capital is more expensive, investing in the safe asset (which does not carry a capital charge) is relatively more attractive than investing in the risky asset. Since $k^* = \kappa(1 - \lambda^*)$, this also explains why an increase in δ has a negative effect on k^*.

As noted in section 1, the critical value v_0^* is decreasing in the quality q of the supervisory information, while the critical value v_1^* is increasing in q, so with better information the bank is less (more) likely to be supported by the LLR when the signal is bad (good). The sign of the derivative of λ^* with respect to q is negative, which means that the positive effect of having less support when the signal is bad is dominated by the negative effect of having more support when the signal is good. Since $k^* = \kappa(1 - \lambda^*)$, this in turn explains why an increase in q has a positive effect on k^*. However, when q is sufficiently large, we may get to the corner $\lambda^* = 0$ and $k^* = \kappa$, where these derivatives become zero.

As also noted in section 1, the critical values v_0^* and v_1^* are increasing in the social cost of bank failure c, because when this cost is high, the LLR has a stronger incentive to lend to the bank in order to save c when the higher return state obtains. This explains why the bank wants to hold a lower liquidity buffer λ^*, so $k^* = \kappa(1 - \lambda^*)$ will be higher. However, as in the case of parameter q, when c is sufficiently large, we may get to the corner $\lambda^* = 0$ and $k^* = \kappa$, where these derivatives become zero.

If we consider that the social cost of failure increases more than proportionately with the size of the bank's balance sheet (which we have normalized to one), c will be higher for large banks, which implies a "too big to fail" result: large banks are more likely to be supported by the LLR, and consequently they will hold smaller liquidity buffers.

3. Extensions

3.1 Penalty Rates

The classical doctrine on the LLR put forward by Bagehot (1873) required "that these loans should only be made at a very high rate of interest." We now examine how the results in section 2 are modified when the LLR charges a penalty rate $r > 0$. Importantly, we assume that r is exogenously given ex-ante, and not chosen by the LLR ex-post.

To characterize the equilibrium of the new game between the bank and the LLR, suppose that $v(1-k)$ deposits are withdrawn at date 1. If $v(1-k) \leq \lambda$, the bank can repay the depositors by selling the required amount of the safe asset, so there is no change with respect to our previous analysis. If, on the other hand, $v(1-k) > \lambda$, the bank needs to borrow $v(1-k) - \lambda$ from the LLR. If such funding is obtained, the bank's payoff in the high-return state equals the return of its investment in the risky asset, $(1-\lambda)R(p)$, minus the amount paid to the remaining depositors, $(1-v)(1-k)$, minus the amount paid to the LLR, $(1+r)[v(1-k)-\lambda]$, that is,

$$\begin{aligned}&(1-\lambda)R(p) - (1-v)(1-k) - (1+r)[v(1-k)-\lambda] \\ &\quad = (1-\lambda)[R(p)-1] + k - r[v(1-k)-\lambda].\end{aligned}$$

The last term in this expression accounts for the interest payments to the LLR.

The decision of the LLR in the case when $v(1-k) > \lambda$ is now characterized as follows. If the LLR observes signal s and decides to support the bank, with probability $\Pr(R_1 \mid s)$ the bank will be solvent at date 2 and the LLR will recover its loan $v(1-k) - \lambda$ and net $r[v(1-k)-\lambda]$ in interest payments, while with probability $\Pr(R_0 \mid s)$ the bank will fail and the LLR will lose $v(1-k) - \lambda$ and incur the cost c, so the LLR's expected payoff is

$$r[v(1-k)-\lambda]\Pr(R_1 \mid s) - [v(1-k)-\lambda+c]\Pr(R_0 \mid s).$$

On the other hand, if the LLR does not provide the liquidity support, the bank will be liquidated at date 1, and the LLR's payoff will be $-c$. Hence the LLR will support the bank if

$$r[v(1-k)-\lambda]\Pr(R_1 \mid s) - [v(1-k)-\lambda+c]\Pr(R_0 \mid s) \geq -c.$$

Substituting (3) and (4) into this expression, it follows that when the LLR observes the bad signal s_0, it will support the bank if the liquidity shortfall, $v(1-k)-\lambda$, is smaller than or equal to the critical value

$$v_0 \equiv \frac{c\Pr(R_1 \mid s_0)}{\Pr(R_0 \mid s_0) - r\Pr(R_1 \mid s_0)} = \frac{cp(1-q)}{(1-p)q - rp(1-q)},$$

and when the LLR observes the good signal s_1, it will support the bank if the liquidity shortfall, $v(1-k)-\lambda$, is smaller than or equal to the critical value

$$v_1 \equiv \frac{c\Pr(R_1 \mid s_1)}{\Pr(R_0 \mid s_1) - r\Pr(R_1 \mid s_1)} = \frac{cpq}{(1-p)(1-q) - rpq}.$$

As before, it is easy to check that $v_1 > v_0$ whenever $q > \frac{1}{2}$. Also, notice that both v_0 and v_1 are increasing in r, so with penalty rates the LLR will be softer with the bank, providing emergency funding for a larger range of liquidity shocks.

To compute the bank's new objective function, we have to subtract from U_B in (11) the expected interest payments to the LLR. If the LLR observes the bad signal s_0, the bank borrows from the LLR when $0 < v(1-k)-\lambda \leq v_0$, that is, when

$$\frac{\lambda}{1-k} < v \leq \frac{v_0+\lambda}{1-k},$$

so the conditional expected cost of this borrowing is

$$r\left[\int_{\frac{\lambda}{1-k}}^{\frac{v_0+\lambda}{1-k}} [v(1-k)-\lambda]\; dF(v)\right]\Pr(s_0 \mid R_1).$$

Similarly, if the LLR observes the good signal s_1, the conditional expected cost of the bank's borrowing is

$$r\left[\int_{\frac{\lambda}{1-k}}^{\frac{v_1+\lambda}{1-k}} [v(1-k)-\lambda]\; dF(v)\right]\Pr(s_1 \mid R_1).$$

Hence the bank's new objective function is

$$\begin{aligned} U_B = {} & p\left[(1-q)F\left(\frac{v_0+\lambda}{1-k}\right) + qF\left(\frac{v_1+\lambda}{1-k}\right)\right] \\ & \times [(1-\lambda)(R(p)-1)+k] - (1+\delta)k \\ & - rp\left[(1-q)\int_{\frac{\lambda}{1-k}}^{\frac{v_0+\lambda}{1-k}} [v(1-k)-\lambda]\, dF(v)\right. \\ & \left. + q\int_{\frac{\lambda}{1-k}}^{\frac{v_1+\lambda}{1-k}} [v(1-k)-\lambda]\, dF(v)\right]. \end{aligned}$$

Assuming that $F(v) = v^{\eta}$, the integrals in this expression can be easily solved, and we can compute for the baseline parameters the equilibrium effects of charging a penalty rate r.[18] The results are presented in table 3.

Thus, an increase in the penalty rate r leads to a reduction in p^*, so the bank's portfolio becomes riskier. The reason for this result is that penalty rates increase the expected interest payments in the high-return state and, consequently, the bank tries to compensate this effect by choosing a higher risk and higher return portfolio (recall that by assumption 1, a decrease in p increases $R(p)$). The reduction in p^* would ceteris paribus lead to a decrease in both v_0^* and v_1^*, but this is more than compensated by the positive effect of the interest payments on the LLR's willingness to lend. The increase in v_0^* and v_1^* in turn explains why the bank chooses a lower liquidity buffer λ^*, so $k^* = \kappa(1-\lambda^*)$ will be higher.

Table 3. Equilibrium Effects of Changes in the Penalty Rate r

x	$\frac{dp^*}{dx}$	$\frac{d\lambda^*}{dx}$	$\frac{dk^*}{dx}$	$\frac{dv_0^*}{dx}$	$\frac{dv_1^*}{dx}$
r	$-$	$-$	$+$	$+$	$+$

[18] It should be noted that this computation is complicated because now p^* depends on r, and cannot be directly solved from the first-order condition (15). The equilibrium is obtained by numerical iteration of the best response functions of the two players.

3.2 Collateralized Lending

The classical doctrine on the LLR put forward by Bagehot (1873) not only required charging "a very high rate of interest," but also "that at this rate these advances should be made on all good banking securities." We now examine how the results in section 2 are modified when last-resort lending is collateralized, so the LLR becomes a senior claimant when it provides the liquidity support and the bank subsequently fails.[19] Obviously, for this to make any difference the failure return should be positive, so in this subsection we assume that $R_0 = l \in (0, L)$.[20]

To analyze the effect of this change, consider a situation in which $v(1-k) > \lambda$, and let s be the signal observed by the LLR. There are two cases to consider. First, if the liquidity shortfall, $v(1-k) - \lambda$, is smaller than or equal to the collateral l, the LLR is fully covered, so its expected payoff if it supports the bank, $-c \Pr(R_0 \mid s)$, is greater than the payoff if it does not, $-c$, so the bank will always be supported. Second, if the liquidity shortfall, $v(1-k) - \lambda$, is greater than the collateral l, the expected payoff of the LLR if it supports the bank is

$$-\left[v(1-k) - \lambda - l + c\right] \Pr(R_0 \mid s),$$

since with probability $\Pr(R_0 \mid s)$ the bank will fail at date 2 and the LLR will lose $v(1-k) - \lambda - l$ and incur the cost c. On the other hand, if the LLR does not provide the liquidity support, the bank will be liquidated at date 1, and the LLR's payoff will be $-c$. Hence the LLR will support the bank if

$$-\left[v(1-k) - \lambda - l + c\right] \Pr(R_0 \mid s) \geq -c.$$

[19] The referee criticized this interpretation, noting that "if we follow Bagehot's second rule and only provide liquidity backed by 'good banking securities,' it is not clear why the central bank will end up underwriting risky investments." In the referee's view, in the present model, "a fairer interpretation of Bagehot might imply no lending by the central bank at all." However, as noted by Goodhart (1999, 343), "Bagehot's proposal related simply to the collateral that the applicant could offer, and the effect of this rule in practice was to distinguish, in part, between those loans on which the central bank might expect with some considerable probability to make a loss and those on which little, or no, loss should eventuate."

[20] The assumption that the failure return l at date 2 is smaller than the liquidation value L at date 1 is not required for our analysis, but makes a lot of sense in the context of the model.

Table 4. Equilibrium Effects of Collateralization

x	$\frac{dp^*}{dx}$	$\frac{d\lambda^*}{dx}$	$\frac{dk^*}{dx}$	$\frac{d(v_0^*+l)}{dx}$	$\frac{d(v_1^*+l)}{dx}$
l	0	$-$	$+$	1	1

Substituting (3) and (4) into this expression, it follows that when the LLR observes the bad signal s_0, it will support the bank if the liquidity shortfall, $v(1-k)-\lambda$, is smaller than or equal to the critical value v_0+l, where v_0 is given by (5); when the LLR observes the good signal s_1, it will support the bank if the liquidity shortfall, $v(1-k)-\lambda$, is smaller than or equal to the critical value v_1+l, where v_1 is given by (6).

Hence the bank's objective function becomes

$$\begin{aligned} U_B = p &\left[(1-q)F\left(\frac{v_0+l+\lambda}{1-k}\right)+qF\left(\frac{v_1+l+\lambda}{1-k}\right)\right] \\ &\times[(1-\lambda)(R(p)-1)+k]-(1+\delta)k. \end{aligned}$$

Using the same arguments as in section 2, it follows that the bank's choice of risk p^* will also be characterized by the first-order condition (15), so it only depends on the capital requirement κ.

As for the effect of collateralization on the bank's liquidity and capital decisions, note that the case in which the LLR's loan is not collateralized (and it is junior to the claim of the deposit insurer) is equivalent to the case $l=0$ analyzed in section 2, so the signs of derivatives with respect to l in table 4 indicate the effect of collateralization on λ^* and k^*.

Thus, collateralization of last-resort lending does not have any effect on the bank's incentives to take risk, but increases the maximum support that the LLR is willing to provide contingent on the bad and the good signal, v_0^*+l and v_1^*+l. This explains why the bank wants to hold a lower liquidity buffer λ^*, so $k^*=\kappa(1-\lambda^*)$ will be higher. In other words, the protection for the LLR advocated by Bagehot translates into a lower liquidity buffer and hence a higher probability that the bank will require emergency liquidity assistance, but without any effect on risk taking.

3.3 Discounting of Future Payoffs

The LLR is a public institution that is run by officials that may have fixed terms of office. If these terms are short or the officials are close to finishing their terms, the officials may have an incentive to avoid current costs possibly at the expense of some larger future costs that would be assumed by their successors. Formally, we can incorporate this possibility into our model by introducing a discount factor $\beta < 1$ for the LLR.[21]

To analyze the effect of such discounting, consider a situation in which $v(1-k) > \lambda$, and let s be the signal observed by the LLR. The expected discounted payoff of the LLR if it supports the bank is now

$$-\beta\,[v(1-k) - \lambda + c]\,\Pr(R_0 \mid s),$$

since with probability $\Pr(R_0 \mid s)$ the bank will fail at date 2 and the LLR will lose $v(1-k) - \lambda$ and incur the cost c. On the other hand, if the LLR does not provide the liquidity support, the bank will be liquidated at date 1, and the LLR's payoff will be $-c$. Hence the LLR will support the bank if

$$-\beta\,[v(1-k) - \lambda + c]\,\Pr(R_0 \mid s) \geq -c.$$

Substituting (3) and (4) into this expression, it follows that when the LLR observes the bad signal s_0, it will support the bank if the liquidity shortfall, $v(1-k)-\lambda$, is smaller than or equal to the critical value

$$v_0 \equiv \frac{c[1-\beta\,\Pr(R_0 \mid s_0)]}{\beta\,\Pr(R_0 \mid s_0)} = \frac{c[p(1-q) + (1-\beta)(1-p)q]}{\beta(1-p)q},$$

and when the LLR observes the good signal s_1, it will support the bank if the liquidity shortfall, $v(1-k) - \lambda$, is smaller than or equal to the critical value

$$v_1 \equiv \frac{c[1-\beta\,\Pr(R_0 \mid s_1)]}{\beta\,\Pr(R_0 \mid s_1)} = \frac{c[pq + (1-\beta)(1-p)(1-q)]}{\beta(1-p)(1-q)}.$$

[21]This assumption is justified by Kaufman (1991) in the following terms: "The discount rate used by policy makers, who are under considerable political pressure to optimize economic performance in the short-term and whose terms of office are relatively short and not guaranteed to last until the next crisis, is likely to be overestimated."

Table 5. Equilibrium Effects of Changes in the LLR's Discount Factor β

x	$\frac{dp^*}{dx}$	$\frac{d\lambda^*}{dx}$	$\frac{dk^*}{dx}$	$\frac{dv_0^*}{dx}$	$\frac{dv_1^*}{dx}$
β	0	+	−	−	−

As before, it is easy to check that $v_1 > v_0$ whenever $q > \frac{1}{2}$. Also, notice that both v_0 and v_1 are decreasing in the discount factor β. This means that an LLR with $\beta < 1$ will be softer with the bank, providing funding for a larger range of liquidity shocks.

We can now compute for the baseline parameters the equilibrium effects of introducing a discount factor $\beta < 1$ for the LLR. As in the model in section 2, the bank's choice of risk p^* is again characterized by the first-order condition (15), so the discount factor β does not have any effect on the bank's incentives to take risk. The full comparative statics results are presented in table 5.

As expected, an increase in the discount factor β (that is, a decrease in the corresponding discount rate) makes the LLR more willing to incur the current costs of not supporting the bank in order to save some larger future costs, so the derivative of v_0^* and v_1^* with respect to β is negative. The reduction in v_0^* and v_1^* in turn explains why the bank chooses a higher liquidity buffer λ^*, so $k^* = \kappa(1 - \lambda^*)$ will be lower.

Thus we conclude that a high LLR discount rate leads to forbearance, but in line with our previous results, this only translates into a lower liquidity buffer, without any effect on risk taking.

3.4 Internalizing Deposit Insurance Payouts

We have assumed so far that the LLR is institutionally separated from the deposit insurer, so the former does not take into account deposit insurance payouts in deciding whether to support the bank. We now consider a situation in which the LLR either internalizes or assumes a fraction $\gamma \in [0, 1]$ of these payouts. When $\gamma = 0$, the LLR is completely independent from the deposit insurer (e.g., a central bank with no deposit insurance role), whereas when $\gamma = 1$, the LLR also acts as deposit insurer.[22]

To analyze the effect of such possible connection between the LLR and the deposit insurer, consider a situation in which $v(1-k) > \lambda$, and let s be the signal observed by the LLR. The expected payoff of the LLR if it supports the bank is now

$$-[v(1-k) - \lambda + c + \gamma(1-v)(1-k)] \Pr(R_0 \mid s),$$

since with probability $\Pr(R_0 \mid s)$ the bank will fail at date 2 and the LLR will lose $v(1-k) - \lambda$, incur the cost c, and assume a fraction γ of the deposit insurance payouts that are given by $(1-v)(1-k)$. On the other hand, if the LLR does not provide the liquidity support, the bank will be liquidated at date 1, and the LLR will incur the cost c and assume a fraction γ of the deposit insurance payouts that are given by $(1-k) - \lambda - (1-\lambda)L$, where $(1-\lambda)L$ is the liquidation value of the bank's risky asset.[23] Hence the LLR will support the bank if

$$\begin{aligned} &-[v(1-k) - \lambda + c + \gamma(1-v)(1-k)] \Pr(R_0 \mid s) \\ &\quad \geq -[c + \gamma[(1-k) - \lambda - (1-\lambda)L]]. \end{aligned}$$

Substituting (3) and (4) into this expression, it follows that when the LLR observes the bad signal s_0, it will support the bank if the liquidity shortfall, $v(1-k)-\lambda$, is smaller than or equal to the critical value

$$\begin{aligned} v_0 &\equiv \frac{[c + \gamma(1-k-\lambda)] \Pr(R_1 \mid s_0) - \gamma(1-\lambda)L}{(1-\gamma)\Pr(R_0 \mid s_0)} \\ &= \frac{[c + \gamma(1-k-\lambda)]\, p(1-q) - \gamma(1-\lambda)L[p(1-q) + (1-p)q]}{(1-\gamma)(1-p)q}, \end{aligned}$$

[22] Intermediate cases are also relevant. For example, until 1998 the Bank of Spain matched the contribution of the Spanish banks to the deposit insurance fund, so γ was $\frac{1}{2}$.

[23] We are implicitly assuming that the amount of deposits is greater than or equal to the liquidation value of the bank at date 1, that is, $1-k \geq \lambda + (1-\lambda)L$. Notice that if k is at the corner $\kappa(1-\lambda)$, this condition reduces to $(1-\lambda)(1-\kappa-L) \geq 0$. In our numerical analysis we take $\kappa = 0.10$ and $L = 0.50$, so it holds.

and when the LLR observes the good signal s_1, it will support the bank if the liquidity shortfall, $v(1-k)-\lambda$, is smaller than or equal to the critical value

$$v_1 \equiv \frac{[c+\gamma(1-k-\lambda)]\Pr(R_1 \mid s_1)-\gamma(1-\lambda)L}{(1-\gamma)\Pr(R_0 \mid s_1)}$$
$$= \frac{[c+\gamma(1-k-\lambda)]\,pq-\gamma(1-\lambda)L[pq+(1-p)(1-q)]}{(1-\gamma)(1-p)(1-q)}.$$

As before, one can check that $v_1 > v_0$ whenever $q > \frac{1}{2}$.

We can now compute for the baseline parameters the equilibrium effects of internalizing a fraction γ of the deposit insurance payouts. As before, the bank's choice of risk p^* is again characterized by the first-order condition (15), so the share γ does not have any effect on the bank's incentives to take risk. The full comparative statics results are presented in table 6.

An increase in the share γ makes the LLR tougher when it observes the bad signal s_0 (since the critical value v_0^* is decreasing in γ), and makes it softer when it observes the good signal s_1 (since the critical value v_1^* is increasing in γ). The sign of the derivative of λ^* with respect to γ is negative for small values of γ, for which the positive effect of having less support when the signal is bad is dominated by the negative effect of having more support when the signal is good. However, for sufficiently high values of γ, the critical value v_1^* reaches the value of 1, which means that the bank will always be supported when the signal is good, and so the only remaining effect will be the positive one associated with further reductions in v_0^*. Since $k^* = \kappa(1-\lambda^*)$, this explains the two possible signs of the effect of γ on k^*.

Table 6. Equilibrium Effects of Changes in the LLR's Share of Deposit Insurance Payouts γ

x	$\frac{dp^*}{dx}$	$\frac{d\lambda^*}{dx}$	$\frac{dk^*}{dx}$	$\frac{dv_0^*}{dx}$	$\frac{dv_1^*}{dx}$
γ	0	$-/+$	$+/-$	$-$	$+$

4. Concluding Remarks

Goodhart (1999, 339–40) has argued that "there are few issues so subject to myth, sometimes unhelpful myths that tend to obscure rather than illuminate real issues, as is the subject of whether a central bank . . . should act as a lender of last resort." The third myth in his list is that "moral hazard is everywhere and at all times a major consideration."[24] This paper provides a rationale for the claim that this is indeed a myth. Specifically, it shows that the existence of a lender of last resort does not have any effect on the risk of the banks' illiquid portfolios, but simply reduces their incentives to hold liquid assets.

Although our model is special in a number of respects, we believe that the results are fairly robust. In particular, neither full deposit insurance nor the assumption that deposit withdrawals are purely random is essential. To see this, suppose that in the context of our basic model (without liquidity λ and capital k) there is an exogenous fraction $u \in (0, L]$ of junior uninsured deposits that require an expected return equal to zero,[25] and let d denote the corresponding interest rate. We assume that uninsured depositors observe the same signal $s \in \{s_0, s_1\}$ as the LLR,[26] and run on the bank at date 1 if and only if they observe the bad signal s_0.

In this situation, the LLR will support the bank if the withdrawal u is smaller than the critical value v_0 given by (5), so the bank's objective function (7) becomes

$$U_B = p[(1-q)1(u \leq v_0) + q][R(p) - 1 - ud],$$

where $1(u \leq v_0)$ is an indicator function that takes the value 1 if $u \leq v_0$, and 0 otherwise. From here it follows that the first-order condition that characterizes the equilibrium choice of risk p^* is

$$R(p^*) + p^* R'(p^*) = 1 + ud^*, \tag{17}$$

[24] The other myths are that it is generally possible to distinguish between illiquidity and insolvency, that national central bank LLR capabilities are unrestricted whereas international bodies cannot function as LLRs, and that it is possible to dispense with an LLR altogether.

[25] The assumption that $u \leq L$ is made for simplicity, in order to ensure that the junior uninsured depositors get zero when the bank is liquidated at date 1.

[26] This assumption is also made for simplicity. See Repullo (2005) for a model where the depositors' signal is different (but coarser) than that of the LLR.

where d^* is the equilibrium deposit rate. Assuming that uninsured depositors can only claim at date 1 the principal (and not the interest), they receive $1(u \leq v_0^*)u$ at date 1 with probability $\Pr(s_0) = q + (1 - 2q)p^*$, and $u(1 + d^*)$ at date 2 with probability $\Pr(R_1, s_1) = qp^*$. Hence their participation constraint is

$$[q + (1 - 2q)p^*]1(u \leq v_0^*) + qp^*(1 + d^*) = 1. \tag{18}$$

Solving equations (17) and (18) gives the equilibrium values of p^* and d^*.[27]

On the other hand, in the absence of an LLR, the bank's objective function becomes $U_B = pq[R(p) - 1 - ud]$, so the first-order condition (17) does not change, while the participation constraint (18) simplifies to $qp^*(1 + d^*) = 1$. Hence we conclude that when $u > v_0^*$ the existence of an LLR does not have any effect on the bank's incentives to take risk. Moreover, when $u \leq v_0^*$, the existence of an LLR reduces the deposit rate that satisfies the participation constraint (18), which in turn, by the Stiglitz and Weiss (1981) argument noted in the introductory paragraphs of this paper, increases the equilibrium value of p^*. Hence having an LLR may actually reduce the bank's incentives to take risk.

The stark contrast between our results and the extant literature deserves further discussion. It is true that in general any form of insurance (e.g., against liquidity shocks) has the potential to create a moral hazard problem. In the context of our model, this clearly shows in the effect on the holding of liquid assets. But to have an effect on risk taking, something else is needed. One such case would be the following. Suppose that instead of observing a signal s on the return of the bank's risky asset, the uninsured depositors observe the bank's choice of p. Furthermore, suppose that, in the absence of an LLR, they can make the deposit rate d contingent on the choice of p (for example, by threatening to withdraw their funds). In this case, the bank would maximize $U_B = p[R(p) - 1 - ud(p)]$ subject to the uninsured depositors' participation constraint $p[1 + d(p)] = 1$. Substituting the constraint into the bank's objective function and maximizing the resulting expression with respect to p gives the first-order condition

[27] Since the relationship between p^* and d^* in both the first-order condition (17) and the participation constraint (18) is decreasing, we may have multiple equilibria, in which case we focus on the one that is closest to the first-best $\widehat{p}$—that is, the one with the highest p^*.

$$R(\widetilde{p}) + \widetilde{p}R'(\widetilde{p}) = 1 - u. \tag{19}$$

Since $R(p) + pR'(p)$ is decreasing, the bank's choice of risk $\widetilde{p}$ is increasing in the proportion u of uninsured deposits, and converges to the first-best $\widehat{p}$ when u tends to 1. So we conclude that the existence of uninsured depositors that observe the bank's choice of risk and use this information to renegotiate the terms of their contract ameliorates the bank's risk-shifting incentives. Moreover, the introduction of an LLR that facilitates the withdrawal of the funds at date 1 may upset this disciplining mechanism, bringing us back to the $p^* < \widetilde{p}$ characterized above.

Two objections can be made to this argument. The standard one is that small depositors do not have the ability or the incentives to monitor banks.[28] The nonstandard one that we are putting forward here is that one should distinguish between the *monitoring of actions* and the *monitoring of the consequences of those actions*.[29] In the absence of an LLR, the former ameliorates the moral hazard problem, but the latter does not, because it simply changes the bank's objective function from $p[R(p) - 1 - ud]$ to $pq[R(p) - 1 - ud]$. Clearly, multiplying the function by a constant does not have any effect on the first-order condition that characterizes the bank's choice of risk. And the same result obtains when there is an LLR. Since arguably the second is the most plausible type of monitoring,[30] we conclude that there should be no presumption that the existence of an LLR worsens the bank's risk-shifting incentives—except, as shown in section 3.1, when it charges penalty rates.

Finally, it is worth noting that our model also provides a rationale for a standard feature of LLR policy, namely the principle of "constructive ambiguity." This is taken to mean that LLRs do not typically spell out beforehand the procedural and practical details of their policy. One possible rationalization of this principle is based on the idea of the LLR committing to a mixed strategy; see

[28]As forcefully argued by Corrigan (1991, 49–50), "I think it is sheer fantasy to assume that individual investors and depositors—and perhaps even large and relatively sophisticated investors and depositors—can make truly informed credit judgements about highly complex financial instruments and institutions."

[29]See Prat (2003) for a detailed discussion of the related distinction between signals on actions and signals on the consequences of actions.

[30]This is, for example, the assumption made in the recent work of Rochet and Vives (2004) on the LLR.

Freixas (1999). Our model supports a different story, suggested by Goodfriend and Lacker (1999), according to which the policy is not random from the perspective of the LLR, but it is perceived as such by outsiders that cannot observe the supervisory information on the basis of which decisions are made. Thus, the randomness lies in the supervisory information, not in the policy rule.

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