Elderly Labor and Unemployment

Watanabe, Minoru and Yasuoka, Masaya

Hokusei Gakuen University, Kwansei Gakuin University

13 November 2017

Elderly Labor and Unemployment†

Minoru WATANABE‡ Masaya YASUOKA§

Abstract

Economically developed countries are confronting the aging of society. Especially, their respective shares of elderly people among the total population in Japan are highest in the world. Moreover, the labor participation rates of older people in Japan are higher than in other OECD countries. Several reasons underlie the increased labor participation of older people in Japan. One reason is the subsidy for the labor supply of elderly people. This paper presents an examination of how this subsidization of the labor supply of elderly people affects the labor participation of young and elderly people and unemployment. As a result, in a temporary equilibrium, an increase in the subsidy raises the employment rate of elderly people. However, in a steady state in a closed economy, this subsidy has little effect on the employment rate of elderly people.

Key words: Elderly labor, Subsidy, Unemployment

JEL Classification Code: J21

† We thank Kazumitsu Sako and seminar participants for helpful comments. Research for this paper was supported financially by JSPS KAKENHI Grant Numbers 17K03746 and 17K03791. Nevertheless, any remaining errors are the authors' responsibility.

‡ Lecturer, Hokusei Gakuen University.

§ Corresponding to: Professor, Kwansei Gakuin University, 1-155 Uegahara, Ichiban-cho, Nishinomiya, Hyogo 662-8501 Japan, Tel.:+81-798-54-6993
E-mail: yasuoka@kwansei.ac.jp

1. Introduction

In an aging society, the labor participation rate of elderly people increases. Comparison with other OECD countries shows that the labor participation rate of elderly people is high in Japan, as shown in Fig. 1, but why is the labor participation rate of elderly people so high in Japan? Several reasons can be inferred. In Japan, the age at which pension benefits start is 65 years old. Therefore, older people must work until that age to obtain a labor income for subsistence. As another reason, government policy can be considered. In a society with fewer children, the working population size continues to decrease. The shortage of working age population represents a severe problem because the social security system cannot be maintained. The social security system is financed mainly as a pay-as-you-go system. Therefore, if the working population decreases, then sufficient revenues cannot be collected to provide for social security benefits.

[Insert Fig. 1 around here.]

Our paper sets a model with labor of two types: young labor and elderly labor. Using it, we examine how an aging society and a subsidy for elderly labor affect the employment rate and the labor participation of elderly people. Consequently, in a monopolistic labor union model, an aging society brought about by an increase in the survival rate and the subsidy for elderly labor raise the employment rate and labor participation rate of elderly people in a temporary equilibrium model. This result is consistent with the real world.

However, considering the steady state in a closed economy under which capital accumulation is considered, the effect of this subsidy on the employment rate and labor participation of elderly people is slight because of a decrease in the wage rate. This result demonstrates that the subsidy for elderly people has a positive short-run effect on the labor supply of elderly people.

In addition to the results described above, our manuscript presents consideration of the labor participation of young and elderly people: specifically, whether they are substitutive or complementary. If young labor and elderly labor are substitutive, then an increase in the demand for elderly labor reduces the demand for young labor, representing a crowding effect. Our paper presents derivation of the complementary relation between young and elderly labor in the temporary equilibrium: both the young labor supply and labor supply of elderly people rises. However, at the steady state in closed economy, although the young labor supply does not change, the elderly labor rises because of the subsidy. This result, which exhibits neither a substitutive nor a complementary relation, is partially consistent with the real world.

Some reports of the related literature describe examination of the labor participation of elderly people. Hu (1979) reports that the retirement age is earlier because of pension benefits. Christian, Keuschnigg, and Keuschnigg (2010) also examine how the pension system affects the retirement age. As demonstrated by Butrica, Smith, and Steuerle (2006), the extension of the retirement age has a positive effect on the income and pension benefit because the household can obtain more labor income and can pay a higher pension premium. Moreover, this extension contributes to pension system sustainability. However, Lachance (2007) shows the negative effects of the extension of retirement age on the welfare. Cocco and Gomes (2012) and Aísa, Pueyo, and Sanso (2012) examine how an increase in longevity affects individual saving and retirement decisions. Aísa, Pueyo, and Sanso (2012) derive that an increase in longevity raises the labor participation rate of elderly people. An increase in household savings accumulates capital stock. Then the wage rate of marginal labor productivity increases. Because of this effect, retirement occurs earlier, as demonstrated by Matsuyama (2008).

Some empirical studies examine the effects of the subsidy policy for elderly labor. Kangasharju (2007) shows a positive effect of the subsidy on the labor supply of elderly people. Acs and Toder (2007) derive that a subsidy policy for the labor supply of elderly people is more efficient than an income transfer policy. Ohta (2007) examines whether the relation between young and elderly labor is substitutive or complementary, and derives that the relation between the two groups of laborers is not always substitutive. The relation is determined based on the form of employment or the type of industry. The theoretical studies of our manuscript are consistent with the results of empirical studies.

The remainder of this paper includes the following. Section 2 describes setting of the model, derivation of the temporary equilibrium, and examination of the effect of an aging society. The subsidy of elderly labor is explained in section 3. Section 4 derives the steady state equilibrium in a closed economy with numerical examples. The final section presents conclusions of the studies described in this paper.

2. The Model

This model has agents of three types: households, firms, and the government. Moreover, the labor union exists. The population size is unity in each generation. No population growth exists.

2.1 Household

In this model, individuals in households exist in two periods: young and old periods. During the young period, the younger people work inelastically to obtain wage income w_t.

However, this model includes involuntary unemployment. If people cannot obtain a job, then they can receive unemployment benefit u_t. Individuals must provide for their own consumption during the old period c_{t+1}, in addition to leisure. Utility function U_t is assumed as

$$U_t = lnc_{t+1} + V, \tag{1}$$

where V denotes the utility from the leisure during the old period. If the individuals enjoy the leisure in old period, then they can obtain utility V. Actually, V is assumed to be distributed uniformly in $[0, \bar{V}]$.[1] There exists heterogeneity of the utility from the leisure in individuals. Our paper presents consideration of survival rate p. Presumably, some younger people cannot live in the older period.

Defining s_{tw} as saving, the budget constraint of working people in the young period is

$$s_{tw} = (1 - \tau - \theta)w_t, \tag{2}$$

[1] This setting is roughly equivalent to that reported by Maeda and Momota (2002). Maeda and Momota (2002) assume that the health condition is distributed uniformly. The utility from retirement differs among elderly people.

where τ and θ respectively denote the tax rate for the unemployment benefit and the policy for the subsidy for the labor supply of elderly people or pension benefit. The budget constraint of unemployed people during the young period, that is, the saving of unemployment people, s_{tu} is shown as

$$s_{tu} = u_t. \tag{3}$$

In the older period, older people choose whether to work or enjoy leisure. The budget constraint of the older people working both in young and old periods c_{t+1w} is

$$c_{t+1w} = R_{t+1}(1 - \tau - \theta)w_t + (1 + \sigma)w_{t+1} + p_{t+1}, \tag{4}$$

where R_t, σ and p_{t+1} respectively denote the interest rate for saving, the subsidy rate of elderly labor, and the pension benefit. Consumption in older people who work in the young period and unemployed people in the old period c_{t+1wu} are shown as

$$c_{t+1wu} = R_{t+1}(1 - \tau - \theta)w_t + p_{t+1}. \tag{5}$$

Consumption by older people who are unemployed status during the young period and who work in the old period c_{t+1uw} is

$$c_{t+1uw} = R_{t+1}u_t + (1 + \sigma)w_{t+1} + p_{t+1}. \tag{6}$$

Consumption by older people who are unemployed in both the young and old period c_{t+1u} is shown as

$$c_{t+1u} = R_{t+1}u_t + p_{t+1}. \tag{7}$$

Considering the distribution of V, the share of older people who are unemployed in the young period and who hope to work in the old period is given as $\frac{V_t^*}{\bar{V}}$. V_t^* denotes older people V who are indifferent between working and enjoying leisure during the older period. V_t^* is given by the following condition:

$$V_t^* = ln\frac{R_t u_{t-1} + (1 + \sigma)w_t + p_t}{R_t u_{t-1} + p_t}. \tag{8}$$

Therein, V_t^{**} denotes older people V who are indifferent between working and enjoying leisure in the older period in the case of older people who work in the young period and who hope to work in the older period. Then, the share of older people who work when young and who want to work in the old period is given as $\frac{V_t^{**}}{\bar{V}}$. Also, V_t^{**} is given as the following condition.

$$V_t^{**} = ln\frac{R_t(1-\tau-\theta)w_{t-1} + (1+\sigma)w_t + p_t}{R_t(1-\tau-\theta)w_{t-1} + p_t}. \tag{9}$$

With $u_t < w_t$, $V_t^* < V_t^{**}$ is obtainable.[2] Many older people who are in employment as young people hope to work because consumption is at a low level without working in the older period. However, people who work in the young period obtain some consumption and want to enjoy leisure.

2.2 Firms

Firms produce the final goods to input capital stock K_t and labor L_t. The product function is assumed as

$$Y_t = AK_t^{\varepsilon}L_t^{1-\varepsilon}, \; 0 < \varepsilon < 1, 0 < A. \tag{10}$$

Therein, Y_t denotes the output. The demand for labor and capital stock are given as

$$w_t = A(1-\varepsilon)K_t^{\varepsilon}L_t^{-\varepsilon}, \tag{11}$$

$$r_t = A\varepsilon K_t^{\varepsilon-1}L_t^{1-\varepsilon}, \tag{12}$$

where w_t and r_t respectively denote the wage rate and rental rate for capital stock. Household saving is regarded as an annuity. Then, $R_t = \frac{1+r_t}{p}$ is obtainable: the capital income and the savings that deceased people have are equally distributed for living older people. For simplicity, these analyses assume no depreciation of capital stock.

2.3 Government

The government provides benefits for unemployed people. Then, the budget constraint of unemployment benefit is shown as

$$u_t(1-l_t) = \tau l_t w_t. \tag{13}$$

Therein, l_t denotes the employment rate. The population size of younger people who want to work in the young period is unity. The population size of older people who want to work in older people is given as $l_{t-1}\frac{V_t^{**}}{\bar{V}} + (1-l_{t-1})\frac{V_t^*}{\bar{V}}$. However, only l_t of younger people and l_t of older people who want to work can work in t period. Then, the employment rate is given as shown below.

2 We assume that $\bar{V}$ is sufficiently large so as not to be $\frac{V_t^{**}}{\bar{V}} > 1$ or $\frac{V_t^*}{\bar{V}} > 1$.

$$l_t = \frac{L_t}{1 + p\left(l_{t-1}\frac{V_t^{**}}{\bar{V}} + (1 - l_{t-1})\frac{V_t^{*}}{\bar{V}}\right)}. \tag{14}$$

Defining $k_t = \frac{K_t}{1+p\left(l_{t-1}\frac{V_t^{**}}{\bar{V}}+(1-l_{t-1})\frac{V_t^{*}}{\bar{V}}\right)}$, the wage rate (11) and interest rate (12) are

$$w_t = A(1-\varepsilon)k_t^{\varepsilon} l_t^{-\varepsilon}, \tag{15}$$

$$r_t = A\varepsilon k_t^{\varepsilon-1} l_t^{1-\varepsilon}. \tag{16}$$

The budget constraint of the policy of pension benefit and the subsidy for elderly labor is given by the following:

$$pp_t + p\sigma l_t\left(l_{t-1}\frac{V_t^{**}}{\bar{V}} + (1 - l_{t-1})\frac{V_t^{*}}{\bar{V}}\right) = \theta l_t w_t. \tag{17}$$

2.4 Labor Union

In this model, there exists a labor union, which decides the wage rate and employment rate to maximize the following objective function:[3]

$$v_t = \big((1-\tau-\theta)w_t + p(1+\sigma)w_t\big)l_t + u_t(1-l_t) + p_t. \tag{18}$$

This equation shows the household average income in t period. Substituting (15) into (18), the employment rate to maximize v_t is derived as follows for given u_t and p_t.[4]

$$l_t = \frac{A(1-\varepsilon)\left(1-\tau-\theta+p(1-l_{t-1})\frac{V_t^{*}}{\bar{V}} + pl_{t-1}\frac{V_t^{**}}{\bar{V}}\right)(1+\sigma)}{\tau + A(1-\varepsilon)\left(1-\tau-\theta+p(1-l_{t-1})\frac{V_t^{*}}{\bar{V}} + pl_{t-1}\frac{V_t^{**}}{\bar{V}}\right)(1+\sigma)} \tag{19}$$

3. Temporary Equilibrium

We consider the temporary equilibrium by which k_t, k_{t-1}, l_{t-1} are given as constant. Then, the employment rate in t period is given as (19).

First, we examine how an increase in survival rate p, that is, an increase in the aging process, affects the employment rate in t period. For simplicity, without pension benefit p_{t+1} and with the subsidy for elderly labor σ, an increase in p raises the employment

[3] This setting is the same as that with Daveri and Tabellini (2000) and Ikeda (2016). Ono (2010) considers the labor union function including the household lifetime income.

[4] In this maximization, the labor union selects the employment rate for given V^{*}, V^{**}, u_t.

rate l_t as shown by (19).

Second, we examine the effects of a subsidy for elderly labor on the employment rate. In this case, we assume no pension benefit: $p_t = 0$. Considering (8), (9), (17), and (19) and with total differentiation with respect to $l_t, \sigma, \theta, V_t^*, V_t^{**}$ at the approximation of $\sigma = 0$ and $\theta = 0$, $\frac{dl_t}{d\sigma}$ is given as

$$\frac{dl_t}{d\sigma} = \frac{\frac{p(1-\varepsilon)(1-l_t)}{\bar{V}}\left((1-l_{t-1})\frac{\partial V_t^*}{\partial \sigma} + l_{t-1}\frac{\partial V_t^{**}}{\partial \sigma}\right)}{B - \frac{p(1-\varepsilon)(1-l_t)}{\bar{V}}\left((1-l_{t-1})\frac{\partial V_t^*}{\partial l_t} + l_{t-1}\frac{\partial V_t^{**}}{\partial l_t}\right)}, \tag{20}$$

where

$$B = \tau + \frac{1-\varepsilon}{\bar{V}}\left(1 - \tau + p(1-l_{t-1})\frac{V_t^*}{\bar{V}} + pl_{t-1}\frac{V_t^{**}}{\bar{V}}\right). \tag{21}$$

The denominator of (20) is always a positive sign. Signs of $\frac{\partial V_t^*}{\partial \sigma}$ and $\frac{\partial V_t^{**}}{\partial \sigma}$ are positive because an increase in the subsidy for elderly labor increases the opportunity cost of leisure. Considering (9) and (16), an increase in l_t reduces wage rate w_t. For that reason, both V_t^* and V_t^{**} decrease: we obtain $\frac{\partial V_t^*}{\partial l_t} < 0$ and $\frac{\partial V_t^{**}}{\partial l_t} < 0$. Therefore, $\frac{dl_t}{d\sigma} > 0$. An increase in subsidies for elderly labor raises the employment rate.

Third, we examine the effect of the pension benefit on the employment rate. In this case, we assume no subsidy for elderly labor $\sigma = 0$ and $p_t > 0$. Considering (8), (9), (17), and (19) and with total differentiation with respect to $l_t, \theta, V_t^*, V_t^{**}$ at the approximation of $\theta = 0$, $\frac{dl_t}{d\theta}$ is given as shown below.

$$\frac{dl_t}{d\theta} = \frac{-(1-\varepsilon)(1-l_t) + \frac{(1-\varepsilon)(1-l_t)l_t w_t}{\bar{v}}\left((1-l_{t-1})\frac{\partial V_t^*}{\partial p_t} + l_{t-1}\frac{\partial V_t^{**}}{\partial p_t}\right)}{B - \frac{p(1-\varepsilon)(1-l_t)}{\bar{V}}\left((1-l_{t-1})\frac{\partial V_t^*}{\partial l_t} + l_{t-1}\frac{\partial V_t^{**}}{\partial l_t}\right)} \tag{22}$$

An increase in p_t increases consumption. Older people want to enjoy leisure because the utility from the leisure is greater than that from consumption. Then, the signs of both $\frac{\partial V_t^*}{\partial p_t}$ and $\frac{\partial V_t^{**}}{\partial p_t}$ are negative. We can obtain $\frac{dl_t}{d\theta} < 0$. Therefore, we can establish the following proposition.

Proposition 1

An increase in the share of older people to total population size and the subsidy for elderly labor raises the employment rate of both young and elderly people. However, an increase in the pension benefit reduces the labor participation rate of both young and elderly people.

We would like to explain why this proposition is obtainable. An increase in p raises the first term of the labor union objective function (18). Then, an increase in l_t raises the labor union objective function v_t greatly before increasing p. Therefore, the labor union increases the employment rate l_t. Similarly, an increase in the subsidy for elderly labor raises the first term of the labor union objective function (18). Then, an increase in l_t raises the labor union objective function v_t to a greater level than before the increase in the subsidy rate σ, even if an increase in σ brings about a negative effect on l_t because of the tax burden.

An increase in θ to raise the pension benefit reduces the employment rate. As shown by the first term of the labor union objective function (18), an increase in θ decreases v_t for given l_t. This result shows that the positive effect of an increase in θ on v_t is weakened. Therefore, the labor union decreases l_t.

Moreover, we can infer the change of the share of older people who hope to work in the old period. Increases in p and σ raise both V_t^* and V_t^{**}, as shown by (8) and (9). Therefore, the number of older people hoping to work rises. Inversely, an increase in pension benefit increases both V_t^* and V_t^{**}, as shown by (8) and (9). The number of older people hoping to work decreases.

We notify the definition of employment rate l_t, which is given as (19). The employment rate of both young and elderly people is given as l_t. However, the labor supply size differs. Although the young labor supply is given as l_t, the labor supply of elderly people is given as

$$l_t\big((1-l_{t-1})V_t^* + l_{t-1}V_t^{**}\big). \qquad (23)$$

Proposition 1 shows that both young labor supply and labor supply of elderly people increases because of the subsidy for the labor supply of elderly people and decreases because of the pension benefit. Although the labor participation of younger people is given by l_t, the labor participation of elderly people is given by (23). This definition includes elderly people who want to work during the old period. An increase in p and

σ increases V_t^* and V_t^{**}. Then the labor participation of elderly people rises.

4. Steady State Equilibrium

This section presents examination of the steady state equilibrium. The aggregate saving is given as $l_t s_{tw} + (1 - l_t)u_t$. Considering the capital market equilibrium $K_{t+1} = l_t s_{tw} + (1 - l_t)u_t$, the dynamics of K_t are derived as

$$K_{t+1} = (1-\theta)A(1-\varepsilon)K_t^{\varepsilon}\left(1 + \frac{p}{\bar{V}}((1 - l_{t-1})V_t^* + l_{t-1}V_t^{**})\right)^{-\varepsilon} l_t^{1-\varepsilon}. \quad (24)$$

The dynamics of employment rate l_t is given by equation (19). For given K_t, l_t and considering (8), (9), (11), (12), (14), and (17) without pension benefit p_t, we obtain K_{t+1}, l_{t+1}.

Defining K, l as the capital stock and employment rate in the steady state, K, l are given by the following equations based on (8), (9), (11), (12), (14), (17), (19), and (24).

$$K = (1-\theta)A(1-\varepsilon)K^{\varepsilon}\left(1 + \frac{p}{\bar{V}}\big((1-l)V^* + lV^{**}\big)\right)^{-\varepsilon} l^{1-\varepsilon}. \quad (25)$$

$$l = \frac{A(1-\varepsilon)\left(1 - \tau - \theta + p(1-l)\frac{V^*}{\bar{V}} + pl\frac{V^{**}}{\bar{V}}\right)(1+\sigma)}{\tau + A(1-\varepsilon)\left(1 - \tau - \theta + p(1-l)\frac{V^*}{\bar{V}} + pl\frac{V^{**}}{\bar{V}}\right)(1+\sigma)}. \quad (26)$$

$$V^* = ln\frac{R\frac{\tau l}{1-l} + (1+\sigma)}{R}. \quad (27)$$

$$V^{**} = ln\frac{R + (1+\sigma)}{R}. \quad (28)$$

$$\mathrm{w} = A(1-\varepsilon)K^{\varepsilon}\left(1 + \frac{p}{\bar{V}}\big((1-l)V^* + lV^{**}\big)\right)^{-\varepsilon} l^{-\varepsilon}. \quad (29)$$

$$1 + r = A\varepsilon K^{\varepsilon-1}\left(1 + \frac{p}{\bar{V}}\big((1-l)V^* + lV^{**}\big)\right)^{1-\varepsilon} l^{1-\varepsilon}. \quad (30)$$

$$\frac{p\sigma}{\bar{V}}((1-l)V^* + lV^{**}) = \theta l w. \quad (31)$$

The linear approximation of (19) and (23) at the approximation of K, l is reduced to the following equations:

$$\begin{pmatrix} K_{t+1} - K \\ l_{t+1} - l \end{pmatrix} = \begin{pmatrix} \frac{\partial K_{t+1}}{\partial K_t} & \frac{\partial K_{t+1}}{\partial l_t} \\ \frac{\partial l_{t+1}}{\partial K_t} & \frac{\partial l_{t+1}}{\partial l_t} \end{pmatrix} \begin{pmatrix} K_t - K \\ l_t - l \end{pmatrix}. \tag{32}$$

Depending on the trace $\frac{\partial K_{t+1}}{\partial K_t} + \frac{\partial l_{t+1}}{\partial K_t}$ and the determinant $\frac{\partial K_{t+1}}{\partial K_t}\frac{\partial l_{t+1}}{\partial K_t} - \frac{\partial K_{t+1}}{\partial l_t}\frac{\partial l_{t+1}}{\partial K_t}$, the stability of the steady state equilibrium is determined. This section presents an examination of the effect of an aging society and the subsidy for elderly labor on the employment in the steady state as a long-term effect of policy. However, we cannot derive these effects analytically. Therefore, using numerical examples, we derive the subsidy policy effects.

In numerical examples, we set the parameter as the following table.

[Insert Table 1 around here.]

Then, we obtain $1 + r = 1.595$. If the interest rate per year is 1.5%, the interest rate of the overlapping generations model by which a period is regarded as 30 years is about 56%. This rate is nearly the same as that found from recent data of OECD countries. In this parameter setting, we obtain $\frac{u}{w} = 0.6$. The benefit rate of unemployment in Japan is set as 50–80%. Then, 0.6 is an inappropriate value for consideration in the case of Japan. p denotes the survival rate. We set $p = 0.9$. As reported from data of Ministry of Health, Labour and Welfare, Japan "Simple Life Table," the survival rate by which zero-year-old people can survive until sixty years old is about 0.9.

The following table presents the effects of the subsidy for elderly labor σ.

[Insert Table 2 around here.]

As shown by the table, the employment rate is almost unchanged by the subsidy for elderly labor. Labor opportunities of younger people are not prevented by the subsidy policy. The wage rate decreases. The subsidy policy is not valid to raise the employment rate of older people. The labor supply of elderly people given by (23) rises because of an increase in V^{**}. However, an increase in σ does not change V^{*}. Therefore, the subsidy policy is not valid for older people who were unemployed during the young period. Generally, unemployed people face a poor situation. The results of our paper demonstrate that a subsidy policy to facilitate labor supply is not effective.

[Insert Table 3 around here.]

We derive the degree to which an increase in p affects the employment rate and other parameters. The results are presented in Table 3. The employment rate is not changed, but the labor participation of elderly people given by (23) rises.

5. Conclusions

Our paper presents an examination of how an aging society and the subsidy for elderly labor affects the employment rate and labor supply of both young and elderly people. As a consequence of temporary effects described in our paper, an aging society increases the labor participation rate of both younger people and older people in a model in which the labor union decides the employment rate and wage rate. In addition, the subsidy for elderly labor raises the employment rate and labor supply of both younger people and older people. These results demonstrate that the factor or policy to increase the labor supply of older people does not crowd out the labor supply of younger people. By virtue of an increase in the labor participation rate of older people, the employment rate of younger people rises. Therefore, crowding out effect does not occur for the labor supply of younger people.

Considering the steady state equilibrium in the closed economy, even if the subsidy is provided, the employment rate does not change for either young or elderly people. However, the labor supply of elderly people rises. This result demonstrates that the effects of elderly labor differ between a temporal and steady state equilibrium. In addition, the relations between young and elderly labor change.

References

Acs G, Toder E (2007) Should we subsidize work? Welfare reform, the earned income tax credit and optimal transfers. International Tax and Public Finance 14:327–343

Aísa R, Pueyo F, Sanso M (2012) Life Expectancy and Labor Supply of the Elderly. Journal of Population Economics 25:545–568

Butrica BA, Smith KE, Steuerle CE (2006) Working for a Good Retirement. Economics Working Paper Archive wp_463, Levy Economics Institute.

Christian J, Keuschnigg C, Keuschnigg M (2010) Pension reform, retirement, and life-cycle unemployment. International Tax and Public Finance 17(5):556–585

Cocco JF, Gomes FJ (2012) Longevity Risk, Retirement Savings, and Financial Innovation. Journal of Financial Economics 103:507–529

Daveri F, Tabellini G (2000) Unemployment, Growth and Taxation in Industrial Countries. Economic Policy 15(30):48–104

Hu SC (1979) Social Security, the Supply of Labor, and Capital Accumulation. American Economic Review 69(3):274–283

Ikeda R (2016) Unemployment Benefit and Fertility. Journal of Social Security 1(3):642–655 (*in Japanese*)

Kangasharju A (2007) Do Wage Subsidies Increase Employment in Subsidized Firms? Economica 74:51–67

Lachance M-E (2008) Pension Reductions: Can Welfare be Preserved by Delaying Retirement? Journal of Pension Economics and Finance 7(2):157–177

Maeda Y, Momota A (2002) Pension, Early Retirement and Saving. Quarterly Journal of Social Security 37(4):360–370 (*in Japanese*).

Matsuyama K (2008) A One-Sector Neoclassical Growth Model With Endogenous Retirement. Japanese Economic Review 59(2):139–155

Ohta S (2012) Young and Old in the Labor Market: Are They Competing for Jobs? Japanese Journal of Labour Studies 54(9):60–74

Ono T (2010) Growth and Unemployment in an OLG Economy with Public Pensions. Journal of Population Economics 23(2):737–767

Appendix

A. Derivation of (20)

Total differentiation of (19) with respect to l_t, σ, θ at the approximation of $\sigma = 0$ and $\theta = 0$, we obtain the following equation of

$$\begin{aligned} &Bdl_t - (1-\varepsilon)l_t d\theta + X_1 l_t d\sigma + X_2 l_t d\sigma + X_3 l_t dl_t \\ &\quad = -(1-\varepsilon)d\theta + X_1 d\sigma + X_2 d\sigma + X_3 dl_t, \end{aligned} \tag{A.1}$$

where

$$X_1 = \frac{(1-\varepsilon)p}{\bar{v}}\left((1-l_{t-1})V_t^* + l_{t-1}V_t^{**}\right), \tag{A.2}$$

$$X_2 = \frac{(1-\varepsilon)p}{\bar{v}}\left((1-l_{t-1})\frac{\partial V_t^*}{\partial \sigma} + l_{t-1}\frac{\partial V_t^{**}}{\partial \sigma}\right), \tag{A.3}$$

$$X_3 = \frac{(1-\varepsilon)p}{\bar{v}}\left((1-l_{t-1})\frac{\partial V_t^*}{\partial l_t} + l_{t-1}\frac{\partial V_t^{**}}{\partial l_t}\right). \tag{A.4}$$

Total differentiation of (17) with respect to l_t, σ, θ at the approximation of $\sigma = 0$ and $\theta = 0$, we obtain the following equation as

$$d\theta = X_1 d\sigma. \tag{A.5}$$

We note that this section presents examination of the temporary effect for given the policy parameters in $t-1$ period: (9) is shown by the following equation as

$$V_t^{**} = ln\frac{R_t(1-\tau_{t-1}-\theta_{t-1})w_{t-1} + (1+\sigma)w_t}{R_t(1-\tau_{t-1}-\theta_{t-1})w_{t-1}}. \tag{A.6}$$

Then, we obtain $\frac{\partial V_t^*}{\partial \theta} = 0$ because θ_{t-1} shows the policy parameter in the prior period. Considering (A.1)–(A.6), we obtain (20).

B. Derivation of (22)

Total differentiation of (19) with respect to l_t, θ, p_t at the approximation of $p_t = 0$ and $\theta = 0$, we obtain the following equation of

$$\begin{aligned} &Bdl_t - (1-\varepsilon)l_t d\theta + X_3 l_t dl_t + X_4 l_t dp_t \\ &\quad = -(1-\varepsilon)d\theta + X_1 d\sigma + X_2 d\sigma + X_3 dl_t + X_4 dp_t, \end{aligned} \tag{B.1}$$

where

$$X_4 = \frac{(1-\varepsilon)p}{\bar{v}}\left((1-l_{t-1})\frac{\partial V_t^*}{\partial p_t} + l_{t-1}\frac{\partial V_t^{**}}{\partial p_t}\right), \tag{B.2}$$

Total differentiation of (17) with respect to l_t, p_t, θ at the approximation of $\sigma = 0$ and $\theta = 0$, we obtain

$$dp_t = \frac{l_t w_t}{p} d\theta. \tag{B.3}$$

Considering (A.2)–(A.6), (B.1)–(B.3), we obtain (22).

C. Dynamics of K_t and l_t

The dynamics of K_t and l_t with no pension and no subsidy for elderly labor are specified by the following equations for given K_{t-1} and l_{t-1}.

[Insert Fig.2 around here.]

Then, as shown in Fig. 2, the dynamics is locally stable for given parameters shown by Table 1.

$$K_{t+1} = (1-\theta)A(1-\varepsilon)K_t^{\varepsilon}\left(1+\frac{p}{\bar{V}}((1-l_t)V_t^* + l_{t-1}V_t^{**})\right)^{-\varepsilon} l_t^{1-\varepsilon}. \tag{C.1}$$

$$l_t = \frac{A(1-\varepsilon)\left(1-\tau-\theta+p(1-l_{t-1})\frac{V_t^*}{\bar{V}} + pl_{t-1}\frac{V_t^{**}}{\bar{V}}\right)(1+\sigma)}{\tau + A(1-\varepsilon)\left(1-\tau-\theta+p(1-l_{t-1})\frac{V_t^*}{\bar{V}} + pl_{t-1}\frac{V_t^{**}}{\bar{V}}\right)(1+\sigma)}. \tag{C.2}$$

$$V_t^* = ln\frac{R_t u_{t-1} + w_t}{R_t u_{t-1}}. \tag{C.3}$$

$$V_t^{**} = ln\frac{R_t(1-\tau)w_{t-1} + w_t}{R_t(1-\tau)w_{t-1}}. \tag{C.4}$$

$$w_t = A(1-\varepsilon)K_t^{\varepsilon}\left(1+\frac{p}{\bar{V}}((1-l_t)V_t^* + l_{t-1}V_t^{**})\right)^{-\varepsilon} l_t^{-\varepsilon}. \tag{C.5}$$

$$r_t = A\varepsilon K_t^{\varepsilon-1}\left(1+\frac{p}{\bar{V}}((1-l_t)V_t^* + l_{t-1}V_t^{**})\right)^{1-\varepsilon} l_t^{1-\varepsilon}. \tag{C.6}$$

$$R_t = \frac{1 + r_t}{p}. \tag{C.7}$$

$$u_t(1 - l_t) = \tau l_t w_t. \tag{C.8}$$

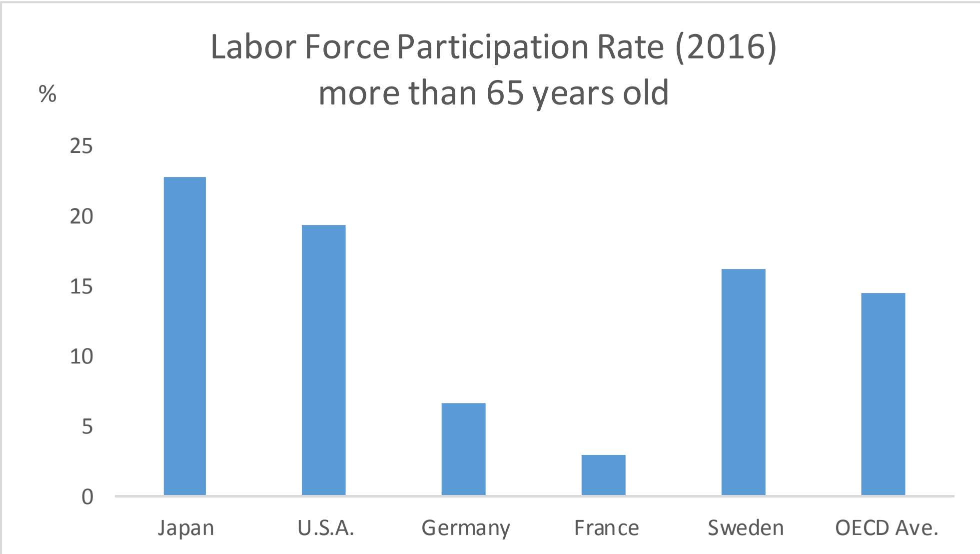

Fig. 1: Elderly labor force participation rate (2016; more than 65 years old) (Data: OECD data "Employment, labor force participation rate").

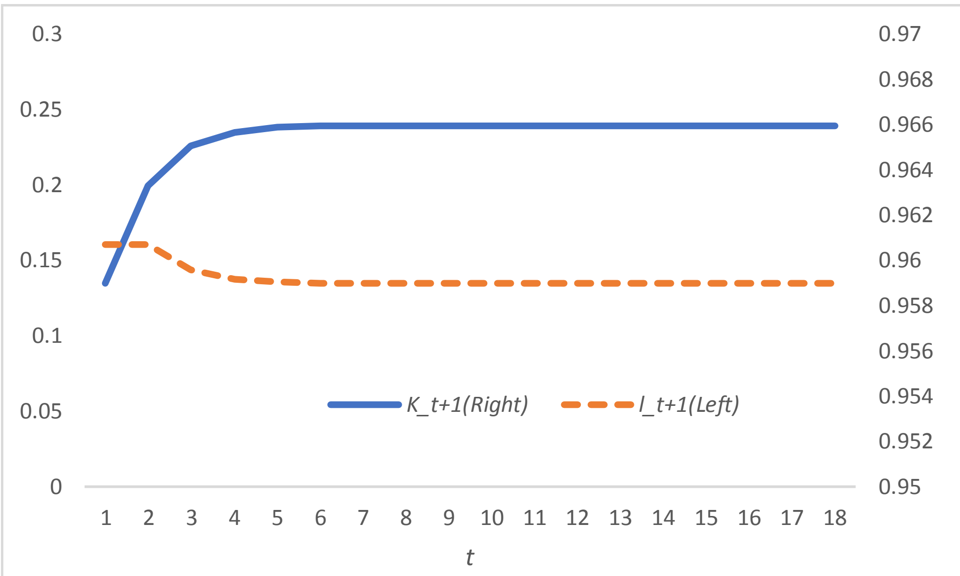

Fig. 2: Transition path of K_{t+1} and l_{t+1}.

σ	0
ε	0.3
τ	0.025
A	0.6
p	0.9
$\bar{V}$	1

Table 1: Parameter setting

σ	l	w	$V*$	$V**$	u/w	$1+r$	θ
0	0.9593	0.2486	0.6854	0.4670	0.5894	1.5509	0
0.025	0.9594	0.2421	0.6854	0.4871	0.5909	1.5857	0.0480
0.05	0.9593	0.2346	0.6847	0.5098	0.5896	1.6303	0.1033

Table 2: Effects of subsidies on employment, wage rate, and others

p	l	w	$V*$	$V**$	u/w	$1+r$
0.9	0.9593	0.2486	0.6854	0.4670	0.5894	1.5509
0.95	0.9602	0.2462	0.6870	0.4768	0.6031	1.5948

Table 3: Effects of an aging society on employment, wage rate, and others