Patent Protection, Optimal Licensing, and Innovation with Endogenous Entry

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Abstract

How does patent policy affect innovation when patent licensing is crucial for firms? To address this question, the present paper incorporates voluntary patent licensing between an innovator and followers, which has been discussed in the literature of industrial organization, into a dynamic general equilibrium model. Unlike in existing studies, both the licensing fee and the number of licensees are endogenously determined by the innovator's maximization and the free-entry condition. Using this model, we show that strong patent protection does not enhance innovation, economic growth, and welfare. Furthermore, the extended analysis provides a policy implication that the effect of patent policy depends on how difficult further innovation is without patent licensing of the current leading technology.

Keywords: innovation, patent protection, optimal patent licensing, endogenous market structure.

JEL-Classification: L24, O31, O34.

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1 Introduction

Patent licensing between firms is a crucial process for technological diffusion and innovation in modern knowledge-intensive industries. If a firm in such markets is not licensed the leading technology, she will be hard-pressed to not only gain from the sale of products but also develop a new technology. In fact, Wang et al. (2013) find a "learning-by-licensing" process among licensees by showing that licensed firms perform better in a subsequent innovation than non-licensed firms do. From the perspective of patented firms, licensing has a growing importance as a way to earn revenue from intellectual properties. Robbins (2009) estimates that the U.S. firms' licensing revenue has rapidly been rising year after year. She also reports that the knowledge-intensive industries, such as electronics, chemical, and pharmaceuticals, where rapid technological advances have occurred, are the largest recipients of licensing revenue. This fact naturally requires us to explore the technological advance with patent licensing between firms.

How does patent policy affect innovation when patent licensing is crucial for firms? Licensing has been subjected to separate theoretical analyses in endogenous growth theory and industrial organization (IO). In endogenous growth theory, some existing studies have investigated this question in R&D-based growth models with patent licensing. In particular, O'donoghue and Zweimüller, (2004), Chu (2009), Chu et al. (2012), and Chu and Pan (2013) analyze the effect of "patent blocking" on innovation by considering a case of compulsory patent licensing involving a new innovator and the previous innovator(s) in which the new technology infringes the patent of previous holder.[1] For example, in Chu et al. (2012), a new innovator in a vertical innovation must transfer a fraction ($0 < s < 1$) of the profit to the previous innovator as a license fee. They assume that s is exogenously determined by an implicit bargaining between the firms and regard s as the strength of patent protection. Then, a stronger patent protection, a larger s, works to block the subsequent innovation. Using the setup, they show that the pro-patent policy stimulates horizontal innovation but hinders vertical innovation. As with their study, most of the literature on patent blocking simply assumes exogenous profit division between firms.[2]

On the other hand, numerous studies in IO have analyzed the optimal licensing strat-

[1]Instead, Iwaisako and Futagami (2003) analyze compulsory patent licensing between the innovator and non-innovative firms. They also interpret that the royalty rate indicates the level of patent protection.

[2]Chu and Pan (2013) is an exception. Although they consider a similar profit-division rule as in Chu et al. (2012), the fraction of profit transferred to the previous innovator, denoted by $0 < s < 1$, is endogenously determined. They assume that $s = \beta/z$, where $\beta \in [0, z]$ is the strength of patent protection and z is the size of quality improvement, determined by the R&D firm. This specification implies that, if an R&D firm succeeds in improving the previous technology dramatically (a larger z), the fraction of profit division, s, becomes smaller. In this setup, they find that there is a non-monotonic relationship between patent protection and growth.

egy for a patented firm. For example, as an application of non-cooperative game theory, Kamien and Tauman (1986), Muto (1993), Sen and Tauman (2007), and Chen (2017) investigate how the patented firm should charge the license fee for profit maximization in several situations.[3]

The present study provides new insights into the effect of patent policy on the optimal licensing fee, the innovation rate, and welfare by developing a unified framework that incorporates the feature of endogenous licensing fee in IO models into an R&D-based growth model. Unlike Kamien and Tauman (1986), which supposes a fixed number of firms in an industry, this study considers an endogenous market structure (EMS) where the number of firms is determined by a free-entry condition. To do so, this paper develops a three-stage game with an innovator and entrants. In the first stage, the innovator charges a licensing fee to maximize her profit. In the second stage, potential entrants decide on acceptance of license, entry into the goods market, and R&D investment to develop a superior good in the industry. In the third stage, they engage in Cournot competition. After one of the firms succeeds in R&D, she becomes a new innovator and charges the license fee of her patent in the first stage of a new game. Thus, the present study explicitly considers subsequent R&D undertaken by licensees under the *ex-post* licensing contract. This setup is clearly different from the existing studies in both IO and growth literature. In their models, the non-patented firms enter into the licensing agreement only to produce the patented goods legally, not to conduct R&D for quality improvement.

Using the unified framework, this study demonstrates that strengthening patent protection does not necessarily enhance innovation as empirically shown in some studies (e.g., Sakakibara and Branstetter 2001; Qian 2007). This conclusion is drawn from two opposite effects of patent licensing on innovation. In the model, stronger patent protection makes it difficult for non-licensed firms to imitate an innovative good without infringement of the patent, and then the innovator can charge a higher licensing fee. It increases the innovator's total profit which is the sum of profit in Cournot competition and license revenue, and this stimulates the incentive to innovate. However, under a certain condition, the policy simultaneously decreases the number of licensees who undertake R&D activities since they are charged a large fee, which negatively affects innovation. Therefore, innovation may show an inverted-U relationship with the strength of patent protection.

This study makes several contributions to the existing literature. First, this study can complement the results obtained in the existing studies in growth theory by shedding light on *voluntary* patent licensing. Although many studies have considered compulsory patent

[3] Kamien and Tauman (1986), a seminal paper, develop a three stage model that an innovator determines the licensing fixed fee or per-unit output royalty in the first stage, other firms individually decide whether contract the license or not in the second stage, and all of them engage in Cournot competition in the third stage. They show that the optimal mode of licensing contract for patented firm is a fixed fee, independent from the amount of production of licensee, rather than a royalty.

licensing between innovators, it is just one among numerous forms of patent licensing in practice. Instead, this study focuses on voluntary patent licensing between the innovator and other firms who are going to produce or improve the technology. Thus, this study can highlight another aspect of the effect of patent policy on innovation. Second, the present study provides a *micro-foundation* to the common assumption in existing studies that stronger patent protection increases the patent holder's share of profit. While previous studies implicitly assume that strong patent protection increases the patent-holder's power of negotiation, the present study demonstrates an equivalent effect by constructing the aforementioned three-stage game.

In addition, the present paper is also related to other studies that analyze the link between patent protection and innovation-driven growth in EMS models.[4] For example, Chu et al. (2016) develop a hybrid model of variety expansion and quality improvement where the number of firms is endogenously determined. Their model shows that strict patent protection enhances economic growth in the short run but hinders it in the long run because the larger the number of entrants, the lower the value of quality improvement innovation. By contrast, the present paper finds that strong patent protection has a non-monotonic effect on growth even in the long run. Although similar results are obtained in many theoretical studies, to our knowledge, no other study shows this effect in a model with EMS and endogenous licensing fee.[5]

The rest of the paper is organized as follows. After constructing the basic model in Section 2, we solve the equilibrium in Section 3. Then, we investigate how patent protection affects the optimal licensing fee, the innovation rate, and welfare in Section 4. An extended analysis is provided in Section 5. Mathematical proofs and derivations are given in the appendices.

2 The basic model

The model is based on the quality-ladder model of Grossman and Helpman (1991). Although the problem for households is unchanged, the setup of industries is substantially redesigned.

[4] Suzuki (2017) develops a growth model with EMS and demonstrates that strict patent protection can affect innovation both positively and negatively, depending on the level of market competition. In that model, all followers must incur an exogenous operating cost in each period, which works as an effective entry cost under the free-entry condition. Although that paper does not explicitly specify what the fixed operating cost is, the present paper considers it as the licensing fee of the leader's patent. See also Etro (2007) and Etro (2009) for variants of the EMS model and its applications.

[5] See Cysne and Turchick (2012), Furukawa (2007), Futagami and Iwaisako (2007), Horii and Iwaisako (2007), and Suzuki (2015).

2.1 Households

We consider an economy consisting of L identical and infinitely living households. Each household supplies a unit of labor inelastically and earns wage w in every period. Their intertemporal utility function is as follows:

$$U_t = \int_0^\infty \exp(-\rho t) \ln C_t dt, \tag{1}$$

where ρ is the subjective discount rate and C_t is an index of consumption at time t. The economy has a continuum of industries indexed by $i \in [0, 1]$. The households consume final goods across all industries. The period utility is

$$\ln C_t = \int_0^1 \ln \left(\sum_{k=0}^{\tilde{k}(i)} \lambda^k X_{kt}(i) \right) di, \tag{2}$$

where $X_{kt}(i)$ is the consumption of a good whose quality is k in industry i at time t. The quality of each good is represented as an integer k power of $\lambda > 1$, which means that the quality of the new good is λ times higher than that of the previous one. Industry i produces $\tilde{k}(i)$ types of goods that are perfect substitutes. We will show that, in equilibrium, households buy only the highest quality good in each industry. Therefore, we focus on the latest good, whose quality is $\lambda^{\tilde{k}(i)}$.

Under the logarithmic utility function, households spend their budget equally across the industries. Therefore, the demand of a good in industry i is $X_{\tilde{k}}(i) = E/p_{\tilde{k}}(i)$, where E is expenditure and $p_{\tilde{k}}$ is the price of the good whose quality is $\tilde{k}(i)$.

In this setting, the ideal price index associated with the consumption index C is

$$P = \exp \left[\int_0^1 \ln \left(\frac{p_{\tilde{k}}(i)}{\lambda^{\tilde{k}(i)}} \right) \right] di. \tag{3}$$

Given the aggregate price index, households spend to maximize their intertemporal utility. From the maximization result, the household's optimal time path for spending is represented by $\dot{E}/E = r - \rho$. Using aggregate expenditure as the *numéraire*, we get $E = 1$ and $r = \rho$. Hereinafter, we omit i from the notations in cases where no risk of misunderstanding is present.

2.2 Industries: A three-stage game

An industry includes a successful innovator of the highest quality good and many potential entrants. We consider a third-stage game that is a variant of Mankiw and Whinston (1986).

First, the innovator decides the licensing fee for her patent and informs it to the potential entrants. We assume that all licensees must pay a fixed licensing fee $F > 0$ in each period.[6] [7] Second, potential entrants individually decide (i) whether to be a licensee; (ii) whether to enter into the industry; and (iii) whether to conduct R&D investment for further innovation. Third, all firms engage in Cournot competition. The following parts solve the three-stage game with backward induction.

2.2.1 The Cournot equilibrium

Before we derive the Cournot equilibrium in an industry in the third stage of game, let us explain the difference in productivity between licensing and non-licensing firms. Non-licensing firms can partially imitate the innovator's good without infringing the patent because patent protection in the economy is imperfect. We call such firms "imitators." The productivity of imitators is lower than that of the innovator and the licensees because the imitators cannot perfectly copy the technology. While the innovator and the licensees can produce one state-of-the-art good by devoting one unit of labor, imitators must employ $\lambda\chi > 1$ units of labor to produce one unit of the same quality good. Parameter $\chi \in (1/\lambda, 1)$ indicates the degree of patent breadth, the extent to which patent holders can legally prevent imitators from copying their patented technologies. Patent breadth is the broadest when $\chi = 1$ and narrowest when $\chi = 1/\lambda$.

Consider an industry where an innovator, ℓ licensees, and m imitators produce the same good. The inverse demand function for goods in the industry is $p = 1/X$ as already derived. In the equilibrium, X equals the aggregate output in the industry. Given the inverse demand function and wage rate of one unit of labor, denoted by w, producer j maximizes her own profit, $\pi(j)$. The profit maximization problem is

$$\max_{x(j)} \pi(j) = \frac{1}{X} \cdot x(j) - c(j) \cdot w \cdot x(j), \tag{4}$$

where $x(j)$ is the output and $c(j)$ is the production cost. We have $c(j) = 1$ when producer j is the innovator or a licensee and $c(j) = \lambda\chi$ when she is an imitator. By solving this

[6]Such a mode of payment is called "fixed sum royalty." In practice, there are various forms of payment in patent licensing, such as percentage royalty, per-output royalty, minimum royalty, maximum royalty, fixed-sum royalty, paid-up royalty, and milestone royalty. Furthermore, a new form of licensing contract is often made by mixing two types. Of course, we cannot consider all styles, so this study assumes that firms pay the licensing fee only in the form of a fixed royalty.

[7]This setup contrasts with an exogenous profit-division rule in existing studies. In practice, a licensing contract with profit-division between firms is rare because the concept of "profit" is not clear. From the perspective of the licensor, it is hard to determine whether an expenditure incurred by the licensee, including the cost of the licensed technology, is necessary to produce the goods. A licensor who is concerned about the moral hazard of licensees would not like to sign such a contract.

problem, we obtain the output of producer j as follows:

$$\begin{aligned}\frac{\partial \pi(j)}{\partial x(j)} = 0 \quad &\Leftrightarrow \quad \frac{1}{X} - \frac{x(j)}{X^2} - c(j) \cdot w = 0 \\ &\Leftrightarrow \quad x(j) = X - c(j) \cdot w \cdot X^2. \end{aligned} \tag{5}$$

Let x_i, x_ℓ, and x_m denote the output of the innovator, a licensee, and an imitator, respectively. Assume that all licensees are symmetric, and so are all imitators. Then, the aggregate output in the industry is written as

$$X = x_i + \ell \cdot x_\ell + m \cdot x_m. \tag{6}$$

By using (5) and (6), we can derive the industry's aggregate output in the Cournot equilibrium as follows:

$$\begin{aligned} X = X - wX^2 + \ell \cdot \left(X - wX^2\right) + m \cdot \left(X - \lambda\chi w X^2\right) \\ \Leftrightarrow X = \left(\frac{\ell + m}{1 + \ell + m\lambda\chi}\right) \frac{1}{w}. \end{aligned} \tag{7}$$

Then, the price in the Cournot equilibrium is

$$p = \left(\frac{1 + \ell + m\lambda\chi}{\ell + m}\right) w. \tag{8}$$

The ratio of output between the two firms is equal to that of their markup $p - c(j) \cdot w$ from (5) and $X = 1/p$. Then, we have

$$x_i = x_\ell \text{ and} \tag{9}$$

$$x_m = \left[\frac{1 - \ell(\lambda\chi - 1)}{1 + m\lambda\chi}\right] x_i. \tag{10}$$

Using (6), (9), and (10), we obtain the equilibrium output of each producer as follows:

$$x_i = x_\ell = \frac{(\ell + m)[1 + m(\lambda\chi - 1)]}{(1 + \ell + m\lambda\chi)^2} \left(\frac{1}{w}\right) \text{ and} \tag{11}$$

$$x_m = \max\left[0, \frac{(\ell + m)[1 - \ell(\lambda\chi - 1)]}{(1 + \ell + m\lambda\chi)^2} \left(\frac{1}{w}\right)\right]. \tag{12}$$

In the Cournot equilibrium, the imitators do not produce the good when the number of licensees is sufficiently high because the equilibrium price becomes smaller than the unit cost of imitators by the entry of licensees. The equilibrium profit of the innovator and the licensee are

$$\pi_i = \pi_\ell = \left(1 - \frac{\ell + m}{1 + \ell + m\lambda\chi}\right)^2. \tag{13}$$

The equilibrium profit of the imitator is

$$\pi_m = \left[\frac{1 - \ell(\lambda\chi - 1)}{1 + \ell + m\lambda\chi}\right]^2 \tag{14}$$

when $\ell < 1/(\lambda\chi - 1)$ and is zero when $\ell \geq 1/(\lambda\chi - 1)$.

2.2.2 The entry and R&D processes

Before we consider the entry decision of the licensees, let us describe the R&D process in the model. We simply regard R&D investment as a binary choice, and its success follows a Poisson process.[8] A firm can draw a lottery that may succeed in creating a high-quality good with a small probability by employing a worker. We assume that the licensees' probability of R&D success is given by $a > 0$ while the non-licensees' probability is $b > 0$. Following Wang et al. (2013), we assume $a > b$ because patent licensing improves R&D productivity. The success or failure of R&D comes out after the game.

In the second stage, the potential entrants individually decide (i) whether to be a licensee; (ii) whether to enter into the industry; and (iii) whether to conduct R&D investment for further innovation. The decision process and the entrants' payoff are depicted in Fig.1. Although the decisions lead to $2^3 = 8$ results, some of them are not chosen in the equilibrium. For example, all licensees choose to enter into the goods market because $\pi_\ell > 0$ holds in the equilibrium as we will discuss later. Furthermore, in the basic model, we focus on a situation in which all non-licensees do not conduct R&D in the equilibrium by assuming b is sufficiently close to zero. This assumption will be relaxed in an extended model in Section 4. Note that the successful innovator does not conduct additional R&D because of Arrow's replacement effect. The innovator cannot increase her profit even if she succeeds in further R&D because potential entrants imitate the new good again. Therefore, all R&D are undertaken by licensees in the basic model.

Let us consider the licensees' decision on whether to conduct R&D investment or only produce the good. The licensees who undertake R&D are referred to as "innovative licensees" in this paper, and we call the licensees who only produce the goods "non-innovative licensees." Let $R \in [0, 1]$ denote the fraction of innovative licensees. Then, the number of innovative licenses is $R\ell$. The free-entry condition for non-innovative licensee is

$$\pi_\ell \leq F \quad \text{equality holds when} \quad (1 - R)\ell > 0. \tag{15}$$

[8] Marsiglio and Tolotti (2015) and Horii and Iwaisako (2007) also consider a binary choice of research.

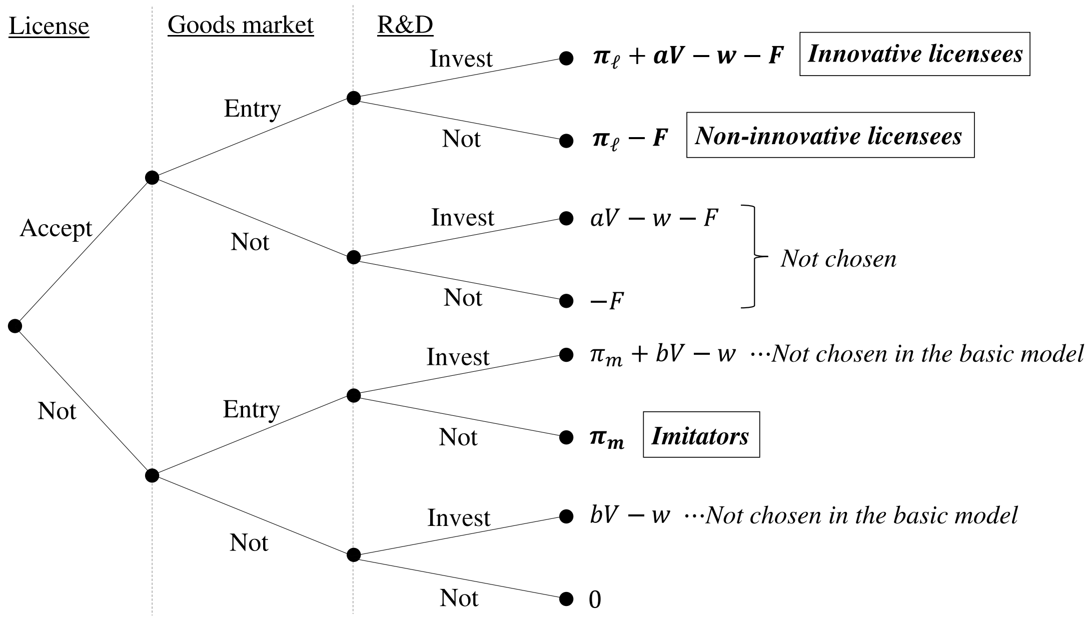

Figure 1: The licensing, entry, and R&D decisions in the second stage of the game. A potential entrant decides about acceptance of the license, entry into the goods market, and R&D investment. The decisions lead to eight results, and the payoff for each is written at the end of the tree.

Similarly, the free-entry condition for innovative licensee is,

$$\pi_\ell + a\bar{V} - w \leq F \;\text{ equality holds when }\; R\ell > 0, \tag{16}$$

where $\bar{V}$ is the *future* firm value that an innovative licensee can gain if she succeeds in R&D. These conditions are equivalent when $a\bar{V} = w$ holds.

The entry decision of imitators is quite simple. Since imitators do not pay license fee F, they enter the market infinitely as long as $\pi_m > 0$. Therefore, from (14), the number of imitators goes to infinity ($m \to \infty$) when the number of licensees is strictly smaller than $\bar{\ell} \equiv 1/(\lambda\chi - 1)$. On the other hand, when $\ell \geq \bar{\ell}$, no imitator enters ($m = 0$) since they cannot produce a positive amount of goods by (12). The entry decision of imitators is summarized as follows:

$$m = \begin{cases} \infty & \ell < 1/(\lambda\chi - 1) \\ 0 & \ell \geq 1/(\lambda\chi - 1) \end{cases} \tag{17}$$

Let us consider how many licensees are innovative. The licensees' decision obviously depends on $\bar{V}, w$, m, and F. However, we will see that the innovator decides F so as to invite some licensees in the market ($\ell > 0$) and exclude all imitators from the market

($m = 0$) in the first stage of the game. Hence, we focus on this case here. Then, we have the following lemma, which shows how many licensees are innovative.

Lemma 1. *Assume that $m = 0$ and $\ell > 0$ hold. Then, all licensees are innovative $(R = 1)$ if and only if $a\bar{V} > w$ holds. When $a\bar{V} = w$ holds, the number of innovative licensees R is indeterminate in $[0, 1)$.*

Proof. See Appendix A. □

Hereafter, the situation in which all licensees are innovative $(R = 1)$ is called case 1, and that in which non-innovative licensees and innovative licensees coexist $(0 < R < 1)$ is called case 2.

2.2.3 The optimal licensing fee

The innovator earns profit by production and from licensing revenue in each period. For simplicity, we assume that licensing contracts are terminated when one of the licensees succeeds in R&D because it makes the licensed technology obsolete.[9] Since all firms engage in Cournot competition, the previous innovator can remain in the industry after a further innovation occurs.[10] Appendix B shows that, after the innovation occurs, the previous innovator becomes a licensee of the successful firm or an imitator and produces the new good, instead of continuing to produce the old good. Since the firm value of licensees and that of imitators are zero under free entry, the innovator loses V when further innovation occurs.

The innovator's expected present value of profit V is derived by the following Bellman equation:

$$V = \max_F \left[(\pi_i(\ell, m) + \ell F)dt + \exp(-rdt)(1 - aR\ell dt)V\right]. \tag{18}$$

By approximating $(\exp(-rdt) \simeq 1 - rdt$ and $(dt)^2 \simeq 0)$, this equation can be rewritten as

$$rV = \max_F \left[\pi_i(\ell, m) + \ell F - aR\ell V\right]. \tag{19}$$

In the first stage of the game, the innovator chooses F so as to maximize the right-hand side (RHS) of the above equation, taking R, $\bar{V}$, and w as given. We assume that the innovator cannot distinguish whether a licensee is going to undertake R&D under the contract

[9] Similarly, Chu et al. (2012) also assume that only the most advanced technology is patented. Furthermore, we assume no "grant-back" clauses, which require the licensees to assign any improvements that originate from the licensed technology to the licensor. For the analyses of innovation and licensing with grant-back clauses, see Choi (2002).

[10] This is in contrast to the model of Grossman and Helpman (1991), where all firms engage in Bertrand competition. In their model, the old innovator must exit from the industry after further innovation since the new innovator practices a limit-pricing strategy.

because of information asymmetry. Therefore, she cannot practice price discrimination between innovative and non-innovative licensees and regards R as uncontrollable. However, she can decrease the number of innovative licensees ($R\ell$) by choosing F because ℓ depends on the licensing fee.

In the licensing fee decision, the innovator faces a trade-off between earning licensing revenue and the risk of further innovation by licensees. If she charges a sufficiently low licensing fee, she can earn licensing revenue but will eventually take the place of some of the innovative licensees. On the other hand, if she charges a sufficiently high licensing fee, she cannot earn licensing revenue but can earn a profit from production forever. Appendix C demonstrates the following result:

Proposition 1. *Assume that $a < \rho$ holds. Then, the optimal licensing fee for the innovator is $F^* \equiv [1 - 1/(\lambda\chi)]^2 + a\bar{V} - w$ in case 1 and $F^* \equiv [1 - 1/(\lambda\chi)]^2$ in case 2. In both cases, the number of licensees is $\ell = 1/(\lambda\chi - 1)$, with no imitator $(m = 0)$.*

The assumption of $a < \rho$ implies that $a > 0$ is small or ρ is large. Intuitively, these mean that the risk of further innovation is small and the innovator is sufficiently myopic, and then she prefers earning a high profit in the short run. In this paper, we adopt the parameter assumption.

In both cases, $m = 0$ holds as a result of the game. The imitators are harmful for the innovator since they do not pay the license fee and, furthermore, they push down the market price. Therefore, the innovator try to exclude the imitators from the market by setting F at a sufficiently small level since then some licensees enter into the market. This *exclusion of imitators* is a reason the innovator has an incentive to license her own patent.

3 The Equilibrium

We now solve the equilibrium in the economy in this section. In the labor market, aggregate labor demand must be equal to labor supply:

$$x_i + \ell x_\ell + R\ell = L. \tag{20}$$

Furthermore, $r = \rho$ and $V = \bar{V}$ hold in the equilibrium.

Case 1: All licensees are innovative ($R = 1$)

In case 1, we have $R = 1$ and $\ell^* = 1/(\lambda\chi - 1)$. Given this, V and w are determined in the equilibrium. From (11) and (20), we can derive

$$w^* = \left(1 - \frac{1}{\lambda\chi}\right)\left(\frac{1}{L(\lambda\chi - 1) - 1}\right). \tag{21}$$

We assume that the labor supply is sufficiently large (i.e., $L > 1/(\lambda - 1)$) so as to be $w^* > 0$. By using (19), (21), and F^*, we can also derive the equilibrium value of V as follows:

$$V^* = \frac{1}{\rho}\left[\left(1 - \frac{1}{\lambda\chi}\right) - \frac{1}{\lambda\chi}\left(\frac{1}{L(\lambda\chi - 1) - 1}\right)\right]. \tag{22}$$

Case 2: Coexistence ($0 < R < 1$)

This case happens when $aV = w$ from Lemma 1. Given this, V and R are determined in the equilibrium. From (11), (20), and $aV = w$, we can derive

$$V = \frac{1}{a\lambda\chi(L - R/(\lambda\chi - 1))}. \tag{23}$$

From (19), we also have

$$V = \frac{1 - 1/(\lambda\chi)}{\rho + aR/(\lambda\chi - 1)}. \tag{24}$$

Solving (23) and (24), the equilibrium value of R is

$$R^* = \left(1 - \frac{1}{\lambda\chi}\right)\left[L\left(\lambda\chi - 1\right) - \frac{\rho}{a}\right]. \tag{25}$$

This value is positive when $L(\lambda\chi - 1) > \rho/a$ holds. If $L(\lambda\chi - 1) > \rho/a$ does not hold, all licensees are non-innovative ($R = 0$). The critical value of χ at which $R = 0$ holds is defined as follows:

$$\chi_0 \equiv \frac{1}{\lambda}\left(\frac{\rho}{aL} + 1\right). \tag{26}$$

In other words, to let potential firms have an incentive to obtain the license, patent protection must be stronger than χ_0. To make the analysis interesting, we assume that $\chi_0 < 1$ holds. Then, case 2 ($0 < R < 1$) happens in the region of $\chi \in (\chi_0, \chi_1)$. Similarly, from (25), another critical value of χ at which $R = 1$ holds can be calculated as

$$\chi_1 \equiv \frac{1}{\lambda}\left[\frac{(\rho/a + 1) + \left((\rho/a + 1)^2 + 4L\right)^{1/2}}{2L} + 1\right]. \tag{27}$$

If $\chi_1 \leq 1$ holds, case 1 arises in the region of $\chi \in [\chi_1, 1]$.

3.1 Strengthening Patent Policy

This section analyzes the effect of strengthening patent protection ($\chi \uparrow$) on the innovation rate in the equilibrium ($aR^*\ell^*$). Note that the effect on economic growth rate has the same implication since the growth rate $g^* = \dot{C}/C$ is calculated as $g^* = aR^*\ell^* \ln \lambda$. Therefore, we also can interpret the results in the section as growth effects.

3.1.1 The effect on innovation

For convenience, we first consider the policy effect in case 2 ($0 < R < 1$). We can solve $R\ell$ in the equilibrium as follows:

$$R\ell^* = \left(1 - \frac{1}{\lambda\chi}\right) L - \frac{\rho}{a\lambda\chi}. \tag{28}$$

This is an increasing function of χ, which means that strengthening patent protection stimulates innovation in case 2. The intuition is as follows. With stronger patent protection, the innovator charges a higher license fee F^*, thereby excluding non-innovative licensees from the market.[11] This reduction in ℓ increases the Cournot equilibrium profit of the surviving licensees and the innovator, promoting the entry of innovative licensees ($R \uparrow$). Thus, strengthening patent protection encourages the entry of innovative licensees by discouraging the entry of non-innovative licensees. If $\chi_1 > 1$ holds, case 1 does not happen, and therefore the relationship between patent protection and innovation is always positive as shown in Fig.2.

Second, we consider the policy effect in case 1 ($R = 1$). In this case, since the innovation rate is $a\ell^* = a/(\lambda\chi - 1)$, strengthening patent protection negatively affects innovation. As in case 2, with stronger patent protection, the innovator charges a higher license fee, decreasing the number of licensees. However, unlike in case 1, R does not increase any more since all licensees are already innovative. Therefore, strengthening patent protection is harmful for innovation in case 1. Consequently, patent protection and innovation have an inverted-U relationship when χ_1 is smaller than 1, as shown in Fig.3.

These discussions are summarized as follows:

[11] Note that the total number of licensees, $\ell^* = 1/(\lambda\chi - 1)$, is decreased in χ. This implies that strengthening patent protection reduces the number of licensees. This result seems to be consistent with Arora and Ceccagnoli (2006), who find that patent protection decreases the licensing propensity when firms are able to commercialize technology themselves. In contrast, when firms lack specialized complementary assets required to commercialize new technologies, patent protection increases the licensing propensity. Our model corresponds to the former case since the innovator can commercialize her technology by herself.

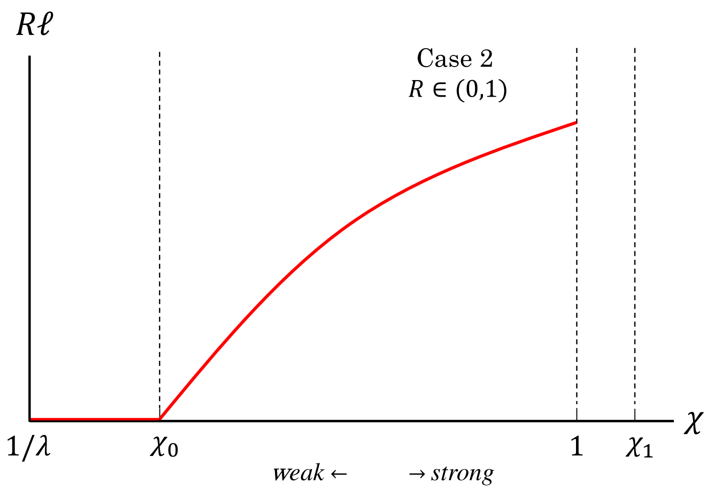

Figure 2: Effect of strengthening patent protection on innovation when χ_1 is larger than 1.

Proposition 2. *The relationship between the strength of patent protection and innovation assumes an inverted U-shape when χ_1 is smaller than 1. On the other hand, strengthening patent protection inevitably enhances innovation when χ_1 is larger than 1.*

Thus, how strengthening patent protection affects innovation depends on whether χ_1 is less than 1 or not. To understand this implication, let us go back to equations (24), (25), (27), and $aV = w$. They show that a large λ or a large L decreases both w and χ_1, which implies that an economy with a small wage rate is likely to be a case of $\chi_1 < 1$.[12] Recall that wage is the cost of R&D. Intuitively, when it is small, the licensees are likely to have the incentive to innovate even if χ is small. Therefore, the threshold value of χ at which $R = 1$ holds decreases.

3.1.2 Effect on welfare

We start to investigate the welfare effect of strengthening patent protection. To do so, we evaluate welfare in a case where the economy starts at the steady state. From equation (2) and the labor market equilibrium condition, we have $\ln C_t = g^* \cdot t + \ln(L - aR\ell)$. By using this and integrating the lifetime-utility function (1) with respect to time, we obtain welfare:

$$W = \frac{1}{\rho}\left[\frac{aR^*\ell^*\ln\lambda}{\rho} + \ln(L - R^*\ell^*)\right]. \tag{29}$$

[12] Some empirical studies find a similar result. For example, Thompson and Rushing (1996) find that patent protection is likely to be positively related to economic growth in high-income countries, but they see no similar relationship in low-income countries.

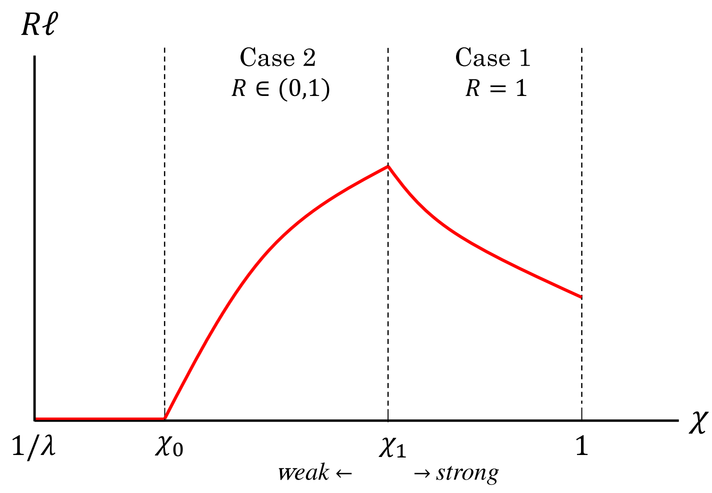

Figure 3: Effect of strengthening patent protection on innovation when χ_1 is smaller than 1.

By differentiating this with respect to $R^*\ell^*$, we obtain

$$\frac{\partial W}{\partial R\ell^*} > 0 \Leftrightarrow R^*\ell^* < L - \frac{\rho}{a \ln \lambda}. \tag{30}$$

When this inequality holds, the welfare effect of strengthening patent protection has the same sign as the effect on innovation. In this case, patent protection and welfare too have either a positive or inverted-U relationship, and the innovation-maximizing patent protection also maximizes welfare. However, when this inequality does not hold, they have either a negative or U-shape relationship. In this case, the innovation-maximizing patent protection minimizes welfare.

Parameters L, ρ, and a determine whether the condition (30) holds or not. Intuitively, the welfare effect is determined by (i) static distortion by higher price and (ii) dynamic gain from innovation. A large L decreases the wage rate in the labor market equilibrium, mitigating the static distortion by reducing the market price. A small ρ allows the households to be more patient, and then they regard the dynamic gain as more important. A large a and λ naturally magnify the dynamic gain of innovation.

Let us derive a sufficient condition that the welfare effect of strengthening patent protection has the same sign as the effect on innovation. The discussion in the previous section implicitly shows that the upper bound of $R^*\ell^*$ is $(1 - 1/\lambda)L - \rho/(a\lambda) \equiv \overline{R\ell}$, which happens in the special case of $\chi_1 = 1$. Therefore, $\overline{R\ell} < L - \rho/(a \ln \lambda)$ is the sufficient condition. By rewriting this, we can summarize the discussion in the following statement:

Proposition 3. *The welfare effect of strengthening patent protection is qualitatively the same as the effect on innovation if*

$$L > \frac{\rho}{a}\left(\frac{\lambda}{\ln \lambda} - 1\right).$$

4 Extension: Innovation by non-licensees

The basic model has focused on the case in which only licensees are innovative in the equilibrium. However, in this section, we consider a different situation in which non-licensed firms can also be innovative in the equilibrium by assuming a sufficiently large b. We here call such firms innovative non-licensees. Let us consider the entry of innovative non-licensees in the second stage of the game.[13] The free-entry condition for innovative non-licensee is

$$bV \leq w \ \text{ equality holds when } \ n > 0, \tag{31}$$

where n is the number of innovative non-licensees. Note that it does not matter whether an innovative non-licensee is an imitator in the goods market or a potential firm since $\pi_m = 0$ holds in the equilibrium of the entry game as already discussed.

The innovator's Bellman equation can be rewritten as

$$rV = \max_{F} \left[\pi_i(\ell, m) + \ell F - (aR\ell + bn)V\right]. \tag{32}$$

The innovator chooses F so as to maximize the RHS of the above equation, taking R, n, $\bar{V}$, and w as given.

When χ is sufficiently small, this extension does not change any results in the basic model. Suppose that the economy is in case 2 ($\chi < \chi_1$). Then, $bV < w$ always holds because we have $aV = w$ and $a > b$. Hence, there is no innovative non-licensee $(n = 0)$ in the equilibrium, and the above Bellman equation returns to (19). This implies that the optimal licensing fee is also F^*, obtained in the basic model, so the results are the same as in the basic model. Even in case 1 ($\chi_1 \leq \chi$ and $aV > w$), there is no innovative non-licensee ($bV < w$) when χ is sufficiently small because V is increasing in χ by (22) but w is decreasing in χ by (21). This case also yields the same results in the basic model.

In contrast, when χ is sufficiently large in case 1 ($aV > w$), some non-licensees may be innovative ($bV = w$) in the equilibrium. We call the situation case 1(b), and let χ_b denote the critical value at which $bV = w$ holds under $\chi \geq \chi_b$ (see Appendix D for the derivation of χ_b). In the following analysis, we assume $\chi_b < 1$. In addition, the optimal

[13] Note that this extension does not affect the Cournot equilibrium in the third stage of the game.

fee in case 1(b) is also F^*, obtained in the basic model.[14] Then, the number of licensees becomes $\ell^* = 1/(\lambda\chi - 1)$ as in the basic model.

Let us solve the equilibrium in the extended model. In the equilibrium, n and V are determined given $R = 1$ and $bV = w$. From the labor market equilibrium condition, we obtain

$$x_i + \ell x_\ell + R\ell + n = L$$
$$\Leftrightarrow V = \frac{1}{b}\left(1 - \frac{1}{\lambda\chi}\right)\left[\frac{1}{(L-n)(\lambda\chi - 1) - 1}\right]. \tag{33}$$

Furthermore, from the Bellman equation, we obtain

$$\begin{aligned} V &= \frac{(1 - 1/(\lambda\chi))^2 + \left[(1 - 1/(\lambda\chi))^2 + aV - w\right]/(\lambda\chi - 1)}{\rho + a/(\lambda\chi - 1) + bn} \\ &= \left(1 - \frac{1}{\lambda\chi}\right)\frac{1}{\rho + b[n + 1/(\lambda\chi - 1)]}. \end{aligned} \tag{34}$$

Solving these two equations, the number of innovative non-licensees in the equilibrium is

$$n^* = L\left(1 - \frac{1}{\lambda\chi}\right) - \left(\frac{\rho}{b\lambda\chi} + \frac{1}{\lambda\chi - 1}\right), \tag{35}$$

which is an increasing function of χ. Therefore, strengthening patent protection increases the number of innovative non-licensees. This is caused by two reasons. First, strengthening patent protection increases the gross profit of innovator ($\pi_\ell + \ell F$). This also increases the firm value of the innovator, which naturally stimulates the incentive to innovate. Second, strengthening patent protection decreases the R&D cost. As shown in the basic model, the number of licensees goes down since the innovator charges a higher license fee. Then, the labor demand of the licensees falls, reducing the wage rate in the labor market equilibrium. As a result, strengthening patent protection increases innovation by non-licensees.

However, the effect on total innovation rate is ambiguous since the strengthening patent protection simultaneously excludes the innovative licensees ($R\ell^* \downarrow$). The total innovation rate in case 1(b) can be calculated as follows:

$$aR\ell^* + bn^* = \left(\frac{a-b}{\lambda\chi - 1}\right) + \left[Lb\left(1 - \frac{1}{\lambda\chi}\right) - \frac{\rho}{\lambda\chi}\right]. \tag{36}$$

[14] Note that the RHS of (32) is a form which is only subtracted bnV, independent from F in the maximization, from the RHS of (19). This implies that the RHS of (32) also exhibits the same shape as Fig.5 and attains the maximum at $F = F^*$.

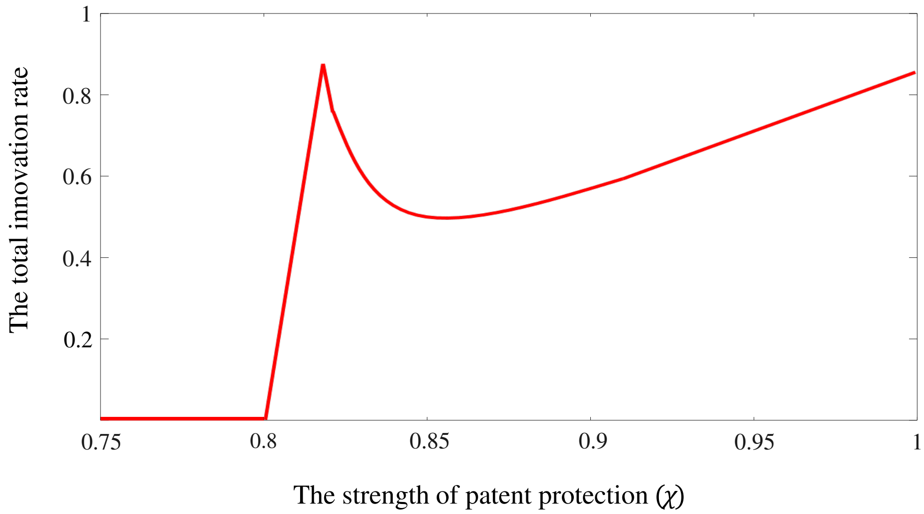

Figure 4: A numerical example of the relationship between patent protection and innovation. The parameters are $\lambda = 1.25$, $a = 0.02$, $b = 0.002$, $\rho = 0.03$, and $L = 2000$. The threshold values are $\chi_0 = 0.8006$, $\chi_1 = 0.8184$, and $\chi_b = 0.8214$.

Strengthening patent protection decreases the first term in the RHS and increases the second term, and the total effect is determined by the sum of these opposite effects. The key parameter here is b. In the extreme case of $b \to a$, the first term vanishes, and therefore the total effect is positive. However, equation (36) also shows that a smaller b amplifies the negative effect and weakens the positive effect. The total effect can then be negative when b is sufficiently small. Using a moderate value of b, Fig.4 shows a numerical example that exhibits a very complex relationship between the total innovation rate and patent protection. These results imply that, even if strengthening patent protection decreases the number of innovative licensees, the policy may enhance total innovation when b is not too small compared to a. In other words, the analysis in the present paper provides a policy implication that the effect of patent policy depends on how difficult further innovation would be without patent licensing of the current leading technology.

5 Conclusion

We analyzed the effect of patent policy on the licensing fee, innovation, and welfare in an endogenous growth model with a strategic environment involving the innovator and the entrants. In this model, the innovator can charge a patent licensing fee so as to maximize her profit. After that, the entrants individually decide whether to become a licensee. In other words, this paper endogenizes both (i) the licensing fee and (ii) the number of firms in the goods market. These features are not shown in previous studies. Using this setup, we demonstrate that strengthening patent protection does not always stimulate innovation. Although the policy makes the licensees more innovative, since the reward for innovation increases, it also discourages the entrant's incentive to become a licensee, by raising the license fee. The mechanism and intuition of the latter effect are as follows. Strengthening patent protection decreases the profit of firms that are not licensed. This implies that the value of the outside option decreases from the perspective of the entrant who considers whether or not to become a licensee. Then, since the entrant would like to be a licensee even if the license fee is high, the innovator charges a large license fee to maximize her profit, decreasing the number of licensees. As a result, the model shows an inverted-U relationship between patent protection and innovation under a certain condition.

Appendix A. The proof of Lemma 1

When $m = 0$ holds, the equilibrium profit of the licensee becomes $\pi_\ell = [1/(1+\ell)]^2$. Then, from (16), innovative licensees can enter the market as long as the number of licensees is less than $1/(F-a\bar{V}+w)^{1/2}-1$. On the other hand, from (15), a non-innovative licensee can enter only if the number of licensees is less than $1/F^{1/2}-1$.

First, suppose that $a\bar{V} > w$ holds. Then, we have $1/(F-a\bar{V}+w)^{1/2}-1 > 1/F^{1/2}-1$. This implies that the incentive to enter the market is stronger for innovative licensees than non-innovative licensees. As a result, in the equilibrium, the number of innovative licensees is $\ell = 1/(F-a\bar{V}+w)^{1/2}-1$, and the number of non-innovative licensees is zero ($R = 1$). Thus, when firms face the case of $a\bar{V} > w$, all licensees are innovative. Intuitively, $a\bar{V} > w$ means that R&D is a profitable investment for all licensees. Conversely, $a\bar{V} > w$ holds when $R = 1$ because the free-entry condition for innovative licensee (16) is binding while (15) is not.

Second, suppose that $a\bar{V} = w$. Then, the entry conditions for all licensees (15) and (16) are equivalent. Since $a\bar{V} = w$ implies that the expected benefit from innovation equals the R&D cost, licensees are indifferent to the amount of R&D activity. Therefore, non-innovative licensees and innovative licensees can coexist, and R is indeterminate in $[0, 1)$. In this case, R is determined in the general equilibrium. $\square$

Appendix B. The behavior of previous innovator

The previous innovator becomes a licensee of the successful firm or an imitator after the innovation occurs, instead of producing the old good, because she can reduce the qualityadjusted unit cost in production by doing so. To understand this, recall that the quality of the previous innovator's good is 1 and the quality of the new innovator's (as well as the new followers') good is $\lambda > 1$. The old innovator can produce her good (quality 1) by paying unit cost w. This can be interpreted that she now can produce the new innovator's good (quality λ) by paying unit cost λw. However, if she imitates the new innovator's good, she can produce the good by paying unit cost $\lambda\chi w$, which is lower than λw. Consequently, at least she never keeps producing the old good. This result also implies that all firms in the industry produce the latest good.

Appendix C. The proof of Proposition 1

To derive the optimal licensing fee, we investigate how the RHS of Bellman equation (19) changes in F in two cases: $R = 1$ and $0 < R < 1$. At first, we start to investigate the case of $R = 1$. In this proof, for simplicity, we assume that the economy is already in equilibrium.

Case 1: All licensees are innovative ($R = 1$)

Suppose that $a\bar{V} > w$ holds. Then, from Lemma 1, all licensees are innovative ($R = 1$).

First, suppose that the innovator chooses a licensing fee in $0 < F \leq a\bar{V} - w$. From (16), the licensees continue to enter infinitely even if $\pi_\ell = 0$, and then, from (15), no imitator enters into the market. Consequently, $m = 0$ and $\ell \to \infty$ hold as a result of the entry game. However, $R\ell \to \infty$ never happens in the labor market equilibrium because labor supply is infinite. Therefore, in equilibrium, the innovator never chooses F below $a\bar{V} - w$. So we do not need to analyze the range.

Second, suppose that the innovator chooses a licensing fee in $a\bar{V} - w < F \leq [1 - 1/(\lambda\chi)]^2 + a\bar{V} - w$. Then, from (13), (16), and the proof of Lemma 1, the entry decision of licensees given m and F is summarized as follows:

$$\ell = \begin{cases} 1/(F - a\bar{V} + w)^{1/2} - 1 & m = 0 \\ \infty & m \to \infty \text{ and } F < (1 - 1/(\lambda\chi))^2 + a\bar{V} - w \\ [0, \infty) & m \to \infty \text{ and } F = (1 - 1/(\lambda\chi))^2 + a\bar{V} - w. \end{cases} \tag{37}$$

The equilibrium of the entry stage is derived from this and (17) in the manner of solving two reaction functions. When $F < (1 - 1/(\lambda\chi))^2 + a\bar{V} - w$, the number of licensees is necessarily higher than $1/(\lambda\chi - 1)$. Therefore, the equilibrium is $m = 0$ and $\ell =$

$1/(F - a\bar{V} + w)^{1/2} - 1$. Then, the innovator's gross profit is

$$\pi_i + \ell \cdot F = \left(\frac{1}{1+\ell}\right)^2 + F\left(\frac{1}{(F - a\bar{V} + w)^{1/2}} - 1\right), \tag{38}$$

which is increasing in F since the number of licensees ℓ is decreasing in F. Consequently, the RHS of equation (19) is increasing in F. This means that the innovator raises F as long as $a\bar{V} - w < F < [1 - 1/(\lambda\chi)]^2 + a\bar{V} - w$. When the innovator chooses $F^* \equiv [1 - 1/(\lambda\chi)]^2 + a\bar{V} - w$, there are two equilibria: (i) $m = 0$ and $\ell = 1/(\lambda\chi - 1)$ and (ii) $m \to \infty$ and $\ell = 0$. To avoid the complexity of multiple equilibria, we assume that (i) is selected as the equilibrium in the second stage of the game. By using $\bar{V} = V$ and w^* in (21) in advance,[15] the RHS of equation (19) in equilibrium can be derived as follows:

$$\left(1 - \frac{1}{\lambda\chi}\right)^2 + \left(\frac{1}{\lambda\chi}\right)\left[\left(1 - \frac{1}{\lambda\chi}\right) - \frac{1}{L(\lambda\chi - 1) - 1}\right]. \tag{39}$$

Third, suppose that the innovator chooses a licensing fee strictly higher than F^*. Then, the entry decision of licensees is as follows:

$$\ell = \begin{cases} 1/(F - a\bar{V} + w)^{1/2} - 1 & m = 0 \\ 0 & m \to \infty. \end{cases} \tag{40}$$

Then, the number of licensees is necessarily strictly smaller than $1/(\lambda\chi - 1)$ by $F = F^*$. After that, imitators infinitely enter to the market by (17), and all licensees finally exit the market by (40). As a result, $m \to \infty$ and $\ell = 0$ hold in equilibrium. In this case, the RHS of equation (19) in equilibrium becomes $(1 - 1/(\lambda\chi))^2$. This implies that the innovator chooses F^* in the equilibrium if the second term in (39) is strictly positive. The condition is

$$\left(1 - \frac{1}{\lambda\chi}\right) - \frac{1}{L(\lambda\chi - 1) - 1} > 0, \tag{41}$$

which is increasing in χ. Therefore, if this condition is satisfied when $\chi = \chi_1$, then it is always satisfied in case 1 ($\forall\chi \in [\chi_1, 1]$). From (25), we obtain

$$\left(1 - \frac{1}{\lambda\chi_1}\right) = \frac{1}{L(\lambda\chi_1 - 1) - \rho/a}. \tag{42}$$

Therefore, if $a < \rho$ holds, condition (41) is necessarily satisfied in case 1, and the relationship between RHS of (19) and F can be drawn as Fig.5. As a result, when $a\bar{V} > w$, the optimal fee in equilibrium is F^*.

[15] Note that the equilibrium wage (21) is derived under $F^* \equiv [1 - 1/(\lambda\chi)]^2 + a\bar{V} - w$.

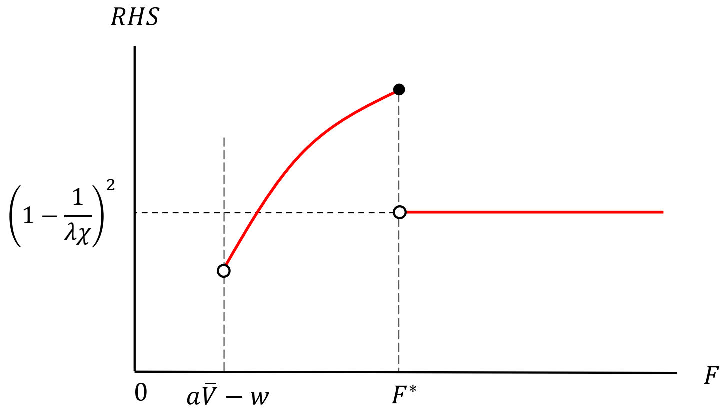

Figure 5: The RHS of (19) and the license fee in case 1.

Case 2: Coexistence ($0 < R < 1$)

Next, we consider the case of $a\bar{V} = w$. Then, from Lemma 1, the ratio of innovative licensees is $R < 1$. Following the same procedure as in case 1, we solve the optimal licensing fee in case 2.

First, suppose that the innovator chooses a licensing fee in $0 < F \leq [1 - 1/(\lambda\chi)]^2$. The entry decision of the licensees is the same as (37) in case 1:

$$\ell = \begin{cases} 1/F^{1/2} - 1 & m = 0 \\ \infty & m \to \infty \text{ and } F < (1 - 1/(\lambda\chi))^2 \\ [0, \infty) & m \to \infty \text{ and } F = (1 - 1/(\lambda\chi))^2 . \end{cases} \tag{43}$$

Using the above and (17), we can solve the equilibrium of the entry stage. When $F < (1 - 1/(\lambda\chi))^2$, as in case 1, the number of licensees is larger than $1/(\lambda\chi - 1)$ regardless of the number of imitators. Then, from (17), there is no imitator in the market ($m = 0$). As a result, $m = 0$ and $\ell = 1/F^{1/2} - 1$ hold in equilibrium. The RHS of equation (19) becomes

$$F^{1/2} - aR\left(\frac{1}{F^{1/2} - 1}\right)V. \tag{44}$$

Since this is an increasing function of F, the innovator will raise F as long as $0 < F < [1 - 1/(\lambda\chi)]^2$. When the innovator chooses $F^* = [1 - 1/(\lambda\chi)]^2$, there are two equilibria: (i) $m = 0$ and $\ell = 1/(\lambda\chi - 1)$ and (ii) $m \to \infty$ and $\ell = 0$. As in case 1, we assume that the first one is chosen as the equilibrium in this case. Then, the RHS of equation (19) becomes

$$\left(1 - \frac{1}{\lambda\chi}\right) + aR\lambda\chi V. \tag{45}$$

Second, suppose that the licensing fee is $F > [1 - 1/(\lambda\chi)]^2$. Then, by (16) and (17), $m \to \infty$ and $\ell = 0$ hold in the equilibrium of the game. In this case, the RHS of equation (19) becomes $(1 - 1/(\lambda\chi))^2$, and is necessarily lower than (45). Therefore, when $a\bar{V} = w$, the optimal fee is $F^* = [1 - 1/(\lambda\chi)]^2$. $\square$

Appendix D. The derivation of χ_b

The threshold value χ_b is given by substituting $n = 0$ into (35) and solving the equation. Then, we obtain

$$\chi_b = \frac{1}{\lambda}\left[\frac{(\rho/b + 1) + \left((\rho/b + 1)^2 + 4L\right)^{1/2}}{2L} + 1\right]. \tag{46}$$

This is strictly higher than χ_1 in (27) since $a > b$ holds. We assume $\chi_b < 1$ holds in the extended model. In other words, we assume this condition is violated in the basic model by considering a sufficiently low b.

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