Preventing Self-fulfilling debt crises

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Abstract

This paper asks whether a government can implement policies that help to avert a crisis driven by self-fulfilling expectations. I consider two policies that are often at the center of political discussions, namely austerity and fiscal stimulus. I find that under plausible conditions austerity tends to decrease the probability of a debt crisis, while stimulus tends to increase it. I also show that endogenous expectations amplify the effects of government policies so that even a small policy adjustment can have significant effects. Finally, I find that policy uncertainty further increases the attractiveness of austerity versus stimulus, but tends to decrease the overall impact of both policies.

Key words: sovereign debt crises, expectations, policy uncertainty, taxes, fiscal stimulus

JEL codes: D82, D84, F34

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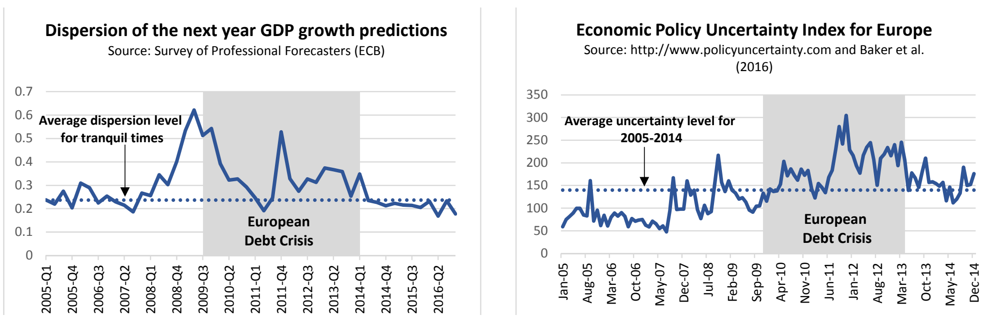
"[...] the assessment of the Governing Council is that we are in a situation now where you have large parts of the Euro Area in what we call a bad equilibrium, namely an equilibrium where you have self-fulfilling expectations. [...] So, there is a case for intervening, in a sense, to "break" these expectations."

Mario Draghi, Press Conference, Frankfurt am Main, September 6, 2012

Sovereign debt crises are a recurrent phenomenon. After the turbulent 1980s and a series of defaults in the late 1990s and early 2000s, sovereign defaults once again became a hotly debated topic. One of the leading views on the sovereign defaults, as exemplified by the above quote, is that they are the result of an interplay between poor economic fundamentals and self-fulfilling expectations.[1]



(a) Economic uncertainty

(b) Politicy uncertainty

Figure 1: Economic and policy uncertainty in Europe 2005-2014.

It is important to note that confidence crises do not appear out of nowhere, but rather are preceded by a deterioration of a debtor country economic situation and an increase in economic and political uncertainty. Since investors often have access to different sources of private information (or vary in their interpretation of common information), this increase in uncertainty translates into an increased dispersion of beliefs among investors. As the consequence, individual investors afraid that other investors hold more pessimistic beliefs about the debtor country's economic situation may choose not extend new loans, even if they believe that debtor country is

[1] See also Bocola and Dovis (2016), Conesa and Kehoe (2015), or De Grauwe and Ji (2013).

solvent, triggering a default. Indeed, as shown in Figure 1, the recent European debt crisis was accompanied by both an increase in dispersion of beliefs about the future economic prospects of EU countries (Panel A) and an increase in economic policy uncertainty (Panel B).

Motivated by these observations, in this paper I ask (1) whether a government can implement policies that help to avert a crisis driven by self-fulfilling expectations and (2) how the desirability of such policies depends on market participants' expectations and on the presence of economic policy uncertainty. I focus on two policies that have been at the center of political discussion in Europe during the recent debt crisis, namely austerity and fiscal stimulus (see Brunnermeier et al. (2016), Corsetti (2012) and Reinhart and Rogoff (2010)). My findings suggests that under plausible conditions austerity tends to decrease the probability of an imminent crisis, while stimulus tends to increase it.[2] I also show that endogenous expectations amplify the effects of government policies so that even a small policy adjustment can have significant effects. Finally, I find that presence policy uncertainty further increases the attractiveness of austerity versus stimulus, but tends to decreases the overall impact of government policies.

The paper consists of two parts. In the first part, I develop a model of self-fulfilling debt crises where crises arise as a result of an interplay between poor fundamentals, foreign lenders' expectations, and domestic households' expectations. To model dispersed beliefs and to endogenize expectations about sovereign default I assume that lenders and households do not observe the relevant fundamentals of the economy but instead only receive noisy private signals. This realistic assumption not only captures the uncertainty surrounding the state of the economy during crises episodes, but also transforms lenders' and households' expectations into endogenous equilibrium objects and restores the uniqueness of equilibrium within the class of monotone equilibria.[3] The resulting environment is rich enough to

[2] To be precise, I provide conditions under which austerity and fiscal stimulus decrease probability of default and conditions under which they increase it. However, I argue that the conditions under which stimulus work are unlikely to hold in practice, while those for austerity to work are likely to be satisfied.

[3] Even though the model has a unique equilibrium outcome, a debt crisis is still driven by expectations in the following sense: There is a region of the fundamentals

capture main trade-offs faced by governments during debt crises, but, in contrast to standard models of self-fulfilling sovereign debt crises, it also links beliefs and expectations to economic fundamentals.

In the second part of the paper, I use the model to analyze which policies available to the government can decrease the ex-ante likelihood of a debt crisis (i.e., prevent a debt crisis). I show first that a change in the probability of default implied by *any* policy adjustment can be decomposed into the product of the "direct effect" (the initial effect of the policy change on the government's incentive to default holding households' and lenders' beliefs constant) and the "multiplier effect" (the change in the government's default decision implied by the adjustment in households' and lenders' expectations). I show that the direct effect determines whether a given policy decreases or increases the likelihood of a crisis, while the multiplier effect, which captures the role played by expectations, acts like an amplification mechanism that always magnifies the initial response of the economy. These novel results indicate that if the government wants to avoid default, it can use expectations to its own advantage as even a small policy change, when amplified by adjustments in expectations can significantly decrease the likelihood of default.

I use the above observations to analyze the impact of an adjustment in a tax rate and the impact of a fiscal stimulus on the probability of default. In the model, increasing taxes decrease the government's incentives to default by filling the financing gap faced by the government when lenders are unwilling to provide the funding. On the other hand, higher taxes distort investment and decrease future output making it more difficult for the government to repay the debt later on. I find that an increase in a tax rate tends to decrease the probability of default as long as the initial level of taxes is not "very high" and argue that this condition is typically satisfied in practice. I model a fiscal stimulus as an increase in government investment financed with debt. A fiscal stimulus, by increasing the output of the economy, and hence government tax revenues, tends to decrease

where both crisis and no crisis outcomes are consistent with fundamentals and whether a crisis occurs depends only on agents' expectations. If agents expect default, then a crisis occurs, while if they expect repayment, then the government will indeed repay the debt; in that sense, a crisis is self-fulfilling (see Morris and Shin, 1998).

the government's incentives to default. On the other hand, the associated increase in the government debt makes defaulting more attractive. I show that the positive effect dominates if the ratio of the government debt to the initial stock of capital in the economy is sufficiently high. However, I argue that the conditions under which stimulus works are unlikely to hold in practice. It follows that austerity is typically a preferred option.

The above analysis was conducted under the assumption that the government always implements its announced policies. However, often debt crises are accompanied by a substantial uncertainty as to whether the government will go through with its plans (e.g., see Panizza et al. (2009)). Indeed, according to the recent index of economic political uncertainty constructed by Baker et al. (2016) this uncertainty reached historical heights in Europe during the recent debt crises (Panel B of Figure 1). Motivated by these observations I analyze how the presence of such an uncertainty affects the above results.

I find that the presence of such an uncertainty tends to decrease the negative effect of austerity: Uncertain as to whether higher taxes will be implemented households do not decrease their investment as much as they would otherwise. On the other hand, economic policy uncertainty decreases the benefits of fiscal stimulus: Unsure whether stimulus will be implemented or not households do not expand their investment as much as they would otherwise. Thus, the presence of economic policy uncertainty further strengthen the case for austerity relative to fiscal stimulus.

However, I also find that economic policy uncertainty decreases overall effect that both policies have on the probability of default. This is because agents, uncertain about the final government decisions, do not adjust their expectations about the likelihood of default as much as they do in the absence of economic policy uncertainty, which implies that the amplifying effect of endogenous adjustments in expectations is weak. In the extreme case, when a policy change is unexpected and agents' information is very precise, the multiplier effect is completely missing and government policies cease to have any impact on the probability of default. This last result provides a strong warning against unexpected policy U-turns.

In the final part of the paper, I investigate numerically how the effectiveness of the policies described above depends on the values of the model's main parameters. In addition, I investigate the importance of the endogenous expectations (as captured by the multiplier effect) in driving these adjustments and link their importance to the characteristics of the economy. The numerical results suggest that for reasonable values of parameters an increase in the tax rate tends to decrease while a fiscal stimulus tends to increase the probability of default and that these results are robust to alternative choices of parameters. Thus, both numerical and analytical results indicate that austerity is preferred to stimulus as a way of preventing a debt crisis. As such these results provide a support for the policies adopted by European countries during the recent debt crisis.

Related Literature — The framework developed in the paper unifies two popular approaches to modeling self-fulfilling debt crises: the micro-funded general equilibrium approach of Cole and Kehoe (2000) and the game-theoretic approach of global games as in Corsetti et al. (2006) and Morris and Shin (2006). The key difference between my model and that of Cole and Kehoe (2000) lies in the information structure, which captures the uncertainty surrounding debt crises and which leads to a unique equilibrium in my model. The equilibrium uniqueness follows from global games literature as started by Carlsson and Damme (1993) and Morris and Shin (1998). Corsetti et al. (2006) and Morris and Shin (2006) use reduced-form global game models to study the effectiveness of IMF assistance in preventing a self-fulfilling debt crisis and the moral hazard such assistance creates.[4] In a parallel work, Zabai (2014), uses global games to study how tax and borrowing policies can be used by the government to manage probability of default in a model in the spirit of Calvo (1988). In contrast to the above work, the focus of this paper is on understanding the impact that endogenous expectations and policy uncertainty have on the effectiveness of fiscal policies.

Models of self-fulfilling crises have a long tradition in the literature on sovereign default, beginning with Sachs (1984) and Calvo (1988). Following the debt crisis in Europe, this literature has experienced a revival. Corsetti

[4]See also Zwart (2007) for the signaling effects of IMF policy choices in a global game model of sovereign debt crisis. Morris and Shin (2003) provide an excellent survey of the early global games literature.

and Dedola (2011), Corsetti and Dedola (2016), and Aguiar et al. (2013) investigate how monetary policy can help to avoid a crisis. Lorenzoni and Werning (2013) focus on the role of the interest rate as the main driver of sovereign default. Finally, Cooper (2013) studies the role of debt guarantees as a way to avert a crisis within a federation of countries.

This paper is also related to the literature on sovereign debt in the spirit of Eaton and Gersovitz (1981), which is summarized well in Aguiar and Amador (2014) and Panizza et al. (2009). More recently, this line of research has focused on developing quantitative models of sovereign default that can account for the observed dynamics surrounding the default episodes. (See Aguiara and Gopinath (2006), Arellano (2008), Hatchondo and Martinez (2009), or Mendoza and Yue (2012), and references therein, for more on quantitative models of sovereign default.) Cuadra and Sapriza (2008) study quantitatively the role of political uncertainty. Typically, this strand of literature assumes away the possibility of a belief-driven crisis.

A large body of work, motivated by the recent events in Europe, studies possible policy responses to the recession that accompanied the European debt crisis. Several papers use DSGE models to evaluate the effectiveness of various policies. For example, Eggertsson et al. (2014) study the effects of structural reforms, while Corsetti et al. (2013) investigate the effects of expansionary fiscal policy. My work complements these papers by providing an analysis of austerity and fiscal stimulus in an environment with a self-fulfilling debt crisis and dispersed beliefs.

1 Model

There are two periods, $t = 1, 2$, and three types of agents: a continuum of identical households, a continuum of identical lenders, and the government. The economy is characterized by the average productivity level A, which is distributed according to a normal distribution with mean A_{-1} and standard deviation σ_A - that is $A \sim N\left(A_{-1}, \sigma_A^2\right)$. Here, A_{-1} denotes the past average productivity level in the economy, which all agents know. The current average level of productivity, A, is realized at the beginning of period 1 and is constant across the two periods, but it is initially unobserved by the agents. Instead, households and lenders receive private noisy signals about A; its value is revealed to everyone at the end of period 1.

1.1 Households

There is a continuum of identical households, indexed by $i \in [0, 1]$. Households are risk averse and have preferences given by

$$\sum_{t=1,2} [\log (c_t) + \log (g_t)],$$

where c_t is private consumption and g_t is government spending. Each household initially is endowed with the same amount of capital k_1, and has access to a production function:

$$y_t^i = \widetilde{Z} e^{A_i} f\left(k_t^i\right),$$

where $f(k) = k^\alpha$, $0 < \alpha < 1$. Here, A_i is a household-specific productivity level; $\widetilde{Z}$ is the aggregate productivity level, which depends on the government's default decision; and f is a production function that takes as inputs capital and, implicitly, inelastically supplied labor. The proceeds from production are the only source of income for the household and are taxed at a rate $\tau > 0$. Finally, capital is assumed to fully depreciate each period.[5]

Households receive their idiosyncratic productivity shocks A_i at the beginning of period $t = 1$. The idiosyncratic productivity is constant across time and given by

$$A_i = A + \varepsilon_i,$$

where ε_i is *i.i.d.* across households and is uniformly distributed on $[-\varepsilon, \varepsilon]$, $\varepsilon > 0$. Note that this implies that A is the average level of productivity in the economy, and that knowing A is equivalent to knowing the aggregate output. After the households observe their respective productivity realizations, household i makes its investment decision, that is it choose its capital stock, k_2^i, for period 2. Households make these choices before $\widetilde{Z}$ is determined (and before the actual production takes place). Thus, when making their investment decisions, households face uncertainty regarding

[5] The assumption that capital fully depreciates implies that the households' optimal investment choice is linear in e^{A_i}, which simplifies the subsequent analysis.

their future income.[6] Households are committed to their investment decisions; they cannot adjust them later. The production takes place at the end of period 1, after $\widetilde{Z}$ is determined, at which point the households invest the amount chosen earlier and consume the rest of their income.

Households make no decisions in period 2. They simply use their capital to produce, and they consume all of their after-tax income.

1.2 The Government

The government is benevolent and maximizes households' utility. In each period t, it provides households with public consumption goods, g_t, and finances its expenditure by taxing households' income and (in period 1) by borrowing in the bond market. The government enters period 1 with a legacy debt, B_1, which is due later in this period, and it initially does not observe the average level of productivity in the economy, A.

At the beginning of period 1, the government announces an interest rate $r > 0$ at which it is willing to borrow in the bond market. Once the households and lenders make their choices, the government observes A and decides how much to borrow, B_2; whether to default or not, d_1; and how much of public goods to provide to households, g_1. In period 2, the government repays its debt B_2, if it did not default on it earlier, and provides g_2 to households. The government can default only in period 1, in which case it defaults on all of its debt.[7]

Following the large literature on sovereign default, I assume that default is costly and associated with a drop in aggregate productivity (and, hence, in output) by a factor Z. In particular, when the government defaults, $\widetilde{Z}$ takes a value $Z < 1$, while $\widetilde{Z} = 1$ otherwise. There is also an additional cost of default: If the government issues a positive amount of debt at $t = 1$

[6] This assumption captures two realistic features of an investment process. First, investment takes time and often requires prior planning. Second, investment decisions are made under uncertainty regarding future economic conditions (in this case, uncertainty about $\widetilde{Z}$).

[7] I allow for default in period 1 only, because of an inherent asymmetry between the two periods in the model. Since period 2 is the last period of the model, it is hard to support repayment as an equilibrium outcome in that period — compared to period 1— because in period 2 the government faces much smaller costs of default and lacks the ability to roll over part of its debt.

(i.e., $B_2 > 0$) and then decides to default, it faces a further cost of default equal to ξB_2, $0 < \xi \leq 1$. I interpret ξB_2 as a "litigation cost" associated with the legal battles between bondholders and the government following a default.[8]

1.3 Lenders and the Bond Market

There is a continuum of identical, risk-neutral lenders, indexed by $j \in [0,1]$, each with finite wealth $b > 0$. Lenders choose at $t = 1$ whether to participate in the bond market or invest in a risk-free asset. The net return on the risk-free asset is normalized to 0, while the return from participating in the bond market is endogenous and determined in equilibrium. Lenders do not observe the realization of the average productivity; instead, each lender j observes a private signal x_j about A where

$$x_j = A + v_j, \;\; v_j \sim N\left(0, \sigma_x^2\right),$$

with v_j being *i.i.d.* across lenders and independent of A and ε_i.

Only the government and lenders have access to the bond market. I assume that the government has all the market power in the bond market, and therefore, the government sets an interest rate r at which it is willing to borrow new funds. Taking r as given, lenders decide whether to supply their funds to the bond market, determining the total funds available in the bond market, S. The government then chooses its new borrowing, B_2, where $B_2 \in [0, S]$. After the government raises new funds, the bond market shuts down and lenders invest the funds not borrowed by the government in storage. For each unit of funds lent to the government, lender j receives a gross return of $1 + r$ in period $t = 2$ if the government repays its debt, and nothing otherwise.

The above bond market structure differs substantially from a Walrasian market typically considered in the sovereign debt literature. However, the

[8] Following a default, creditors tend to file a substantial number of lawsuits against a defaulting government. For example, in the case of default by Argentina in 2001, there were over 140 lawsuits filed abroad, including 15 class action lawsuits, in addition to a large number of lawsuits filed in Argentine courts (Panizza et al. (2009)). I interpret ξB_2 as the costs to the government associated with these legal battles. For more discussion of this assumption, see Section 2.1 below.

assumption that the government has all the market power in the bond market and the resulting lack of learning from prices are not unrealistic. Most governments issue debt using sealed-bid auctions and have considerable leeway in choosing the amount of borrowing based on the bids effectively controlling the volume and, to a lesser extent, the price.[9] This auctioning mechanism also means that the price in the primary bond market cannot be used directly to infer any information.

1.4 Timing

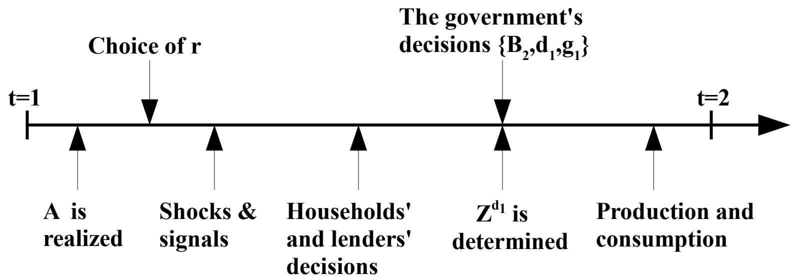


Figure 2: Timeline

The timing of period 1 is summarized in Figure 2. At the beginning of period 1, nature draws the productivity level A, which is initially unobserved by the government as well as by the households and the lenders. Then, based only on the information contained in the prior belief, the government sets an interest rate r, at which it is willing to borrow from the lenders. Once r is announced, households receive their idiosyncratic productivity shocks and lenders observe their private noisy signals about A. Given their productivity shocks, households choose how much they want to invest, while lenders, using their private signals, decide whether to supply their funds in the market. At this point, the government learns the true A, and based on lenders' and households' decisions and the realization of A,

[9] For example, Spanish government provides only a lower and upper bound on the amount of funds accompanied by a note which says that "The announced issuance target is indicative and it may be modified according to market conditions" (for more information see http://www.tesoro.es/en). What this means is that typically if the demand is strong and bids are high the government will decide to issue more debt and at lower interest rate then if the demand is weak and bids are low. Thus effectively the government controls both the volume and to some extent the interest rate on its debt.

it decides how much it will borrow today, B_2, whether to default or not, d_1, and how much of public goods to provide to households, g_1. Once the government borrows its desired amount, the bond market shuts down and the lenders' remaining funds are invested in the risk-free asset. Finally, at the end of the period, production, actual investment, and consumption take place and the average productivity level is revealed to all the agents.

Period 2 is much simpler. At the beginning of the period, production takes place. Then the government collects the taxes, provides public goods, g_2, and, if it did not default earlier, repays its remaining debt. Finally, households consume their after-tax output.

2 Equilibrium Analysis

An equilibrium in the model is defined as follows:

Definition 1 *An equilibrium is a set of government policy functions {r, d_1, g_1, g_2, B_2} a profile of households' consumption and investment choices* $\{c_1, c_2, k_2\}_{i\in[0,1]}$*, a profile of lenders' supply decisions* $\{\beta\}_{j\in[0,1]}$*, such that:*

1. $\{r, d_1, g_1, g_2, B_2\}$ *solves the government's problems at* $t = 1,2$*, taking households' and lenders' decisions as given.*

2. *For every* i*,* $\{c^i_1, c^i_2, k^i_2\}$ *solves household* i*'s problems at* $t = 1,2$*, taking as given the other agents' decisions.*

3. *For every* j*,* β^j *solves lender* j*'s problem, taking as given the other agents' decisions.*

The above definition of an equilibrium is standard, and it requires that all the agents behave optimally in each subgame, taking as given the actions of the others. It also requires that the supply of funds in the bond market be consistent with lenders' supply decisions.

The equilibrium can be computed by backward induction, starting with period 2 and then moving to period 1. The key (and the most difficult step) is to solve simultaneously for the households' investment choices, the lenders' supply decisions, and the government's default decision. In what follows I will focus on equilibria in monotone strategies. This greatly simplifies the task of solving the model and renders the analysis more tractable.

2.1 Additional Assumptions

To simplify the analysis and ensure that the government problem is well-posed, I make the following assumptions (listed below from the least to the most restrictive).[10]

Assumption 1 *The legacy debt is large enough, $B_1 > \overline{B}_1$ for some threshold $\overline{B}_1$.*

Assumption 1 ensures that if the government decides to repay its legacy debt, it will find it optimal to borrow a positive amount. Otherwise, lenders stop playing any role in the model.

Assumption 2 *The wealth of each lenders j is bounded by $\bar{b}$ (i.e., $b < \bar{b}$).*

Assumption 2 simply implies that the total liquidity in the bonds market is finite. This is a typical assumption in the models with risk neutral traders and incomplete information (see e.g. Albagli et al., 2015).[11]

Assumption 3 *$Z > \underline{Z}$, that is, output cost of default is not too large.*

Assumption 3 implies that the output cost of default at time t is bounded from below by $(1 - \underline{Z})\, Y_t$. This implies that the government's optimal unconstrained borrowing, the amount it would like to borrow if it repays the debt, is monotone in A.

Assumption 4 *The "litigation costs" are large (i.e., $\xi \to 1$).*

Assumption 4 implies that the main benefit to the government from defaulting comes from repudiation of the legacy debt, B_1, rather than from defaulting on the new debt, B_2, which seem to be the relevant case empirically. This assumption also ensures that the government's incentive to

[10] For a further discussion of these assumptions see Section E of the Appendix.

[11] For some parameters, this assumption is also needed to ensure that the difference in the value of repaying and defaulting is sufficiently monotone. See Section $A.1.3$ of the Appendix.

default decreases as the supply of funds in the market increases, and is essential for establishing existence of equilibrium.[12]

Given the above assumptions, I now analyze the equilibrium of the model. I compute the equilibrium using backward induction. Note that once the government makes its choices of B_2, d_1, g_1, no agent makes any decision and the equilibrium outcomes are determined. Therefore, I begin the analysis by describing the government's new borrowing, default, and spending decisions in period 1.

2.2 Period $t = 1$: The Government's Decisions

The government decides how much to borrow, whether or not to default, and how much to spend to maximize the households' utility, internalizing how each of these decisions affects consumption, aggregate productivity, and future tax revenues. The government makes these decisions after observing households' investment decisions, the supply of funds in the market, and the average level of productivity in the economy.

Let $\mathbf{k}_2 = \{k_2^i\}_{i\in[0,1]}$, and let $V_1^R(A, \mathbf{k}_2, S)$ be the value to the government of repaying its debt when the average productivity is equal to A, the households' investment profile is $\mathbf{k}_2$, and the supply of funds in the bond market is S. Then $V_1^R(A, \mathbf{k}_2, S)$ is given by

$$V_1^R(A, \mathbf{k}_2, S) = \max_{B_2\in[0,S]} \sum_{t=1,2} \left\{ \int_0^1 \left[\log\left(c_t^{i,R}\right) + \log\left(g_t^R\right)\right] di \right\}$$
$$s.t.\ g_1^R = \tau Y_1^R - B_1 + B_2$$
$$g_2^R = \tau Y_2^R - (1 + r) B_2,$$

where g_t^R is the government spending in period t, Y_t^R is the aggregate output at time t if the government repays the debt. When the government decides to repay its debt, it chooses its new borrowing, B_2, to maximize

[12] Note that a high ξ is needed to ensure that there is a region where the government is exposed to self-fulfilling beliefs. For example in Cole and Kehoe (2000) $\xi = 0$, and as the consequence they can only ensure the existence of such a region at extreme parameter values. A separate issue arises from the fact that in my model lenders and households have incomplete information. As noted by Kletzer (1984) in debt crises models with asymmetric information an equilibrium may not exists. Assumption 4 ensures that this is not an issue.

households' utility subject to the available funds in the market, S, and its budget constraints.

Let $V_1^D(A, \mathbf{k}_2, S)$ be the value associated with defaulting, that is,

$$V_1^D(A, \mathbf{k}_2, S) = \max_{B_2 \in [0,S]} \sum_{t=1,2} \left\{ \int_0^1 \left[\log\left(c_t^{i,D}\right) + \log\left(g_t^D\right) \right] di \right\}$$
$$s.t.\ g_1^D = \tau\left(ZY_1^R\right) + (1-\xi) B_2$$
$$g_2^D = \tau\left(ZY_2^R\right)$$

If the governments defaults, it borrows the maximum possible amount in the market (i.e., $B_2 = S$) and then repudiates all of its debt, and both of these actions tend to increase government spending in period 1. When $\xi \to 1$, this effect of borrowing as much as possible vanishes and the main benefit of default is an increase in the g_1 due to repudiation of the "legacy debt" B_1. The negative effect of defaulting is a drop in aggregate productivity by factor Z.

When deciding whether or not to default, the government compares $V_1^R(A, \mathbf{k}_2, S)$ with $V_1^D(A, \mathbf{k}_2, S)$ and chooses to repay its debt if and only if the value associated with repaying is larger than the value associated with defaulting, that is, if and only if

$$\Delta V(A, \mathbf{k}_2, S) \equiv V_1^R(A, \mathbf{k}_2, S) - V_1^D(A, \mathbf{k}_2, S) \geq 0 \tag{1}$$

2.3 Default Decisions and the Fragility Region

For sufficiently low productivity levels, the government finds it optimal to default regardless of the households' and lenders' actions — when A is low, defaulting leads to an increase in government spending. On the other hand, when the average level of productivity is high, the government always finds it optimal to repay the debt. Intuitively, for high A, defaulting not only leads to a drop in private consumption but also results in less government spending. Accordingly, for each interest rate r, there exist two thresholds, $\underline{A}(r)$ and $\overline{A}(r)$, such that the government always defaults if $A < \underline{A}(r)$ and never defaults if $A > \overline{A}(r)$.

For all $A \in \left[\underline{A}(r), \overline{A}(r)\right)$, the government's default decision depends on the households' and lenders' choices. If the lenders expect default, they invest all their funds in the risk-free asset. In this case, the government

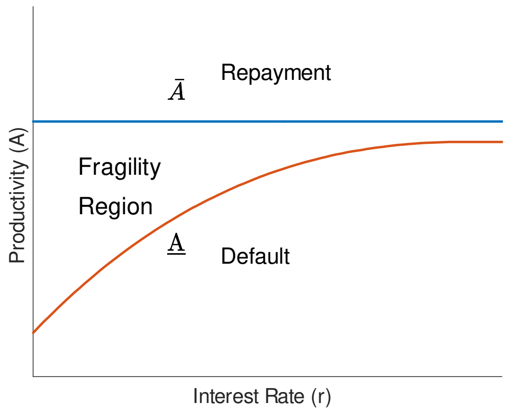


Figure 3: Fragility Region

cannot roll over its debt, and hence repaying B_1 becomes very costly in terms of the forgone utility from government spending. If, on the other hand, the households expect default, they decrease their investment, leading to a drop in the government's revenues (taxes) in the future. This translates into a drop in government expenditure in both periods (since the government smooths out the drop in its revenue across time) and leads to a higher cost of repaying the legacy debt. If $A \in \left[\underline{A}(r), \overline{A}(r)\right)$, these costs are large enough that in response to a shift in households' or lenders' expectations the government finds it optimal to default. Figure 3 depicts the fragility region $\left[\underline{A}(r), \overline{A}(r)\right)$

2.4 Household's Problem

Consider household i with an idiosyncratic productivity shock A_i that must choose how much to invest. This household's problem can be written as

$$\max_{k_2} \ E\left[\sum_{t=1,2} \left[\log(c_t) + \log(g_t)\right] \middle| A_i, \boldsymbol{\sigma}\right]$$
$$s.t. \ c_1 = (1-\tau) Z^{d_1(\boldsymbol{\sigma})} e^{A_i} f(k_1) - k_2$$
$$c_2 = (1-\tau) Z^{d_1(\boldsymbol{\sigma})} e^{A_i} f(k_2)$$
$$\boldsymbol{\sigma} = \{\mathbf{k}_2, \boldsymbol{\beta}, r, d_1, g_1, g_2, B_2\},$$

where $\boldsymbol{\sigma}$ is the strategy profile of all players and the expectations are taken over the government default decisions, $d_1(\boldsymbol{\sigma})$, as well as over the average level of productivity, A. Household i chooses k_2 to maximize its utility subject to the budget constraint, taking $\boldsymbol{\sigma}$ as given. Lemma 1 characterizes households' optimal investment when households believe that the government will always default if the average productivity is less than A^* (i.e., that the government follows a monotone default strategy with threshold A^*).

Lemma 1 *Suppose that the government defaults if and only if $A < A^*$. Then household i's optimal investment is given by*

$$k_2 = (1-\tau)\, e^{A_i} f(k_1)\, \Lambda(A_i; \varepsilon, A^*),$$

where $\Lambda(A_i; \varepsilon, A^*)$ is increasing in the idiosyncratic productivity, A_i, and decreasing in the default threshold, A^*.[13]

2.5 Lender's Problem

Simultaneously with the households' investment choices, the lenders must decide whether to supply their funds to the bond market or to invest their funds in storage. Lenders base their decisions on the prior belief about A and their private signals, x_j. Let $\mathcal{R}(\boldsymbol{\sigma})$ be the government repayment set for a fixed strategy profile $\boldsymbol{\sigma}$. Then the expected payoff to lender j from supplying the funds to the bond market is given by

$$\int_{A \in \mathcal{R}(\boldsymbol{\sigma})} \left(1 + r \min\left\{1, \frac{B_2^{R,u}(A; \boldsymbol{\sigma})}{S(A; \boldsymbol{\beta})}\right\}\right) f(A|x_j)\, dA,$$

where $f(A|x_j)$ is lender j's posterior belief about A, $B_2^{R,u}(A; \boldsymbol{\sigma})$ is the unconstrained desired borrowing by the government in repayment, and $S(A; \boldsymbol{\beta})$ is the supply function implied by the lenders' supply strategy profile $\boldsymbol{\beta}$. Finally, $\min\left\{1, B_2^{R,u}(A; \boldsymbol{\sigma}) / S(A; \boldsymbol{\beta})\right\}$ is the amount that lender j expects to lend to the government given that the average productivity level is A.[14] Lender j supplies his funds to the bond market if and only if the expected return from supplying the funds is higher than 1, the return from investing in storage. The next lemma characterizes Lenders' behavior.

[13] See Section A of the Appendix for the exact definition of $\Lambda(A_i; \varepsilon, A^*)$.

[14] For all $A \notin \mathcal{R}(\boldsymbol{\sigma})$, the government borrows all available funds in the market and then defaults, implying that in this case lender j earns nothing. If $A \in \mathcal{R}(\boldsymbol{\sigma})$, the government would like to borrow $B_2^{R,u}$.

Lemma 2 *Suppose that the government defaults if and only if $A < A^*$. Then an optimal strategy for each lender j is to supply the funds to the bond market if and only if he receives a signal $x_j \geq x^*$. Moreover, x^* is the unique solution to the equation*

$$\int_{A^*}^{\infty} \left(1 + r \min\left\{1, \frac{B_2^{R,u}(A; \boldsymbol{\sigma})}{S(A; \mathbf{x}^*)}\right\}\right) f(A|x^*)\, dA = 1,$$

where $S(A; \mathbf{x}^*)$ is the supply function when all lenders follow this strategy.

2.6 Equilibrium Default Threshold

Above I characterized the optimal behavior of each type of agent. This, in turn, allows me to prove the following proposition, which states that for any interest rate r there exists a unique equilibrium in monotone strategies.

Proposition 1 *There exist $\overline{\varepsilon} > 0$ and $\overline{\sigma}_x > 0$ such that for any interest rate r, any $\varepsilon \in (0, \overline{\varepsilon}]$, and any $\sigma_x \in (0, \overline{\sigma}_x]$, the model has a unique equilibrium in monotone strategies where the following hold:*

1. *The government defaults if and only if $A < A^*(r)$.*[15]

2. *Each lender provides the funds if and only if $x_j \geq x^*(r)$.*

3. *Households' investment rules, k_2, are increasing in A_i.*

The proof of Proposition 1 builds on the insights and results of Athey (1996) and Morris and Shin (2003). The above result is non-trivial for several reasons. First, difficulty comes from the fact that in the model, the global game is played by three different types of agents, each with its own preferences and choice sets. Second, the lenders' payoff function satisfies only a weak single-crossing condition, rather than global strategic complementarities, as in typical global games.[16] Finally, the regime-change

[15] The default threshold $A^*(r)$ depends also on all the parameters of the model such as the tax rate τ, the capital stock k_1, the legacy debt B_1, etc. For notational convenience, I suppress this dependence whenever this does not lead to a confusion.

[16] Applying global games results in a complex environment in which payoff functions satisfy only the weak single-crossing condition, rather than global strategic complementarities, is not without cost. In particular, I need to restrict my attention to monotone strategies. Morris and Shin (2003) discuss why, in general, the single-crossing condition is not enough to prove uniqueness without such a restriction.

condition (i.e., the condition that determines whether default will occur) arises endogenously from the government's optimal behavior — unlike in the typical global games literature, where it is exogenously imposed.

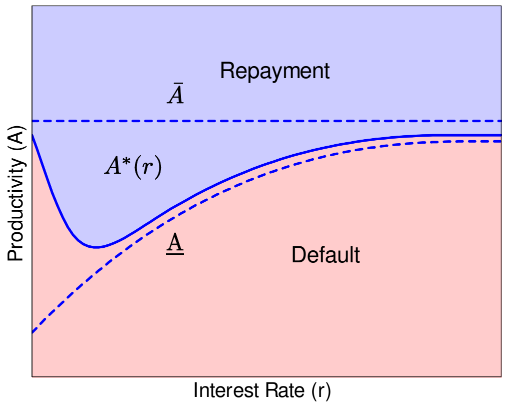


Figure 4: Default Threshold

Figure 3 depicts the equilibrium default threshold A^* as a function of the interest rate r. We see that $A^*(r)$ is a non-monotone function of r. To understand this, note that when the interest rate is low, few lenders supply their funds to the bond market. As a result, the government finds it optimal to default for most productivity values in the "fragility region." As r increases, the supply of funds increases since higher r compensates lenders for exposing themselves to default risk. At the same time, households' investment rules shift upwards since they anticipate that the government will choose to repay the debt for a larger set of productivity levels. This decreases the government's incentives to default and leads to a lower $A^*(r)$. A higher interest rate, however, increases the costs of rolling over the debt, discouraging the government from smoothing debt repayment over time. This tends to decrease the value of repaying debt to the government. For sufficiently high r, this negative effect dominates, implying that $A^*(r)$ becomes an increasing function of r.

It is important to stress that, while the default threshold is unique, the outcome of the model in the fragility region is driven fully by households' and lenders' expectations. For all productivity levels in the fragility region, both repayment and default could be supported as equilibrium outcomes if we had the freedom to choose the lenders' and households' expectations.

However, the households' and lenders' expectations are not free objects. An incomplete-information structure transforms beliefs into equilibrium objects and requires them to be sequentially rational and consistent with agents' strategy profiles. This imposes requirements on the beliefs that are not present in the complete-information game.

2.7 Optimal Choice of r

It remains to characterize the government's optimal choice of interest rate, r. The government chooses the interest rate based on the current and past fundamentals of the economy, $\{B_1, k_1, A_{-1}\}$. The government also knows its future policy functions $\{d_1, g_1, g_2, B_2\}$ and realizes that it can affect consumption, investment, and the supply of funds through its choice of interest rate. To choose the optimal interest rate, the government solves the following problem:

$$W\left(A_{-1}, B_1, \mathbf{k}_1; \boldsymbol{\sigma}\right) = \max_r E\left[\sum_{t=1,2} \int_{i=0}^{1} \left[\log\left(c_t^i\right) + \log\left(g_t\right)\right] di \middle| A_{-1}\right]$$
$$s.t. \text{ optimal policy functions } \{c_1, c_2, d_1, B_2, g_1, g_2\}$$
$$\text{optimal lenders' and households' strategies } \{\boldsymbol{\beta}, \mathbf{k}_2\}.$$

When choosing the interest rate, the government faces the following trade-off: On the one hand, at least initially, a higher r tends to decrease the default threshold. On the other hand, a higher r increases the cost of borrowing at $t = 1$, making it more costly to roll over the maturing debt. Thus, the government weighs the positive effect of a lower default threshold against the increase in the borrowing costs. The above trade-off implies that the government will always set an interest rate on the decreasing portion of the $A^*(r)$-curve.

3 Preventing Self-fulfilling Debt Crises

Having characterized the equilibrium of the model, I now focus on the main questions that motivated this paper: (1) how the government can decrease the ex-ante probability of default (i.e., prevent a debt crisis), and (2) what role endogenous expectations play in determining the effect of government policies on the probability of crises.

I start by considering a case where each policy change is announced in period 1 before the households and lenders make their decisions but after r is set, and that the government is committed to implementing the announced policies. The policy itself is, however, is not implemented until the end of that period. These assumptions are made for simplicity and allow me to focus on the fundamental forces at play in the model while abstracting away from the effects of other factors. I relax these assumptions in the following sections. In Section 4, I analyze what happens if either the policy adjustment is unexpected or if there is uncertainty as to whether the government will implement the announced policy, while in Section F of the Appendix I analyze the case when the policy announcement is made before the interest rate is set. Figure 5 depicts the timing for the policy adjustment considered in this section.

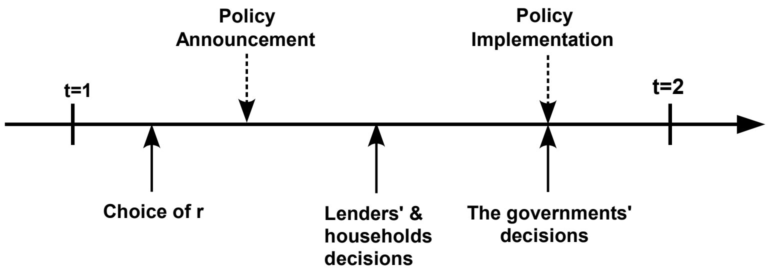


Figure 5: Timing of Policy Adjustments

In order to simplify analysis and make the problem more tractable, I make the following assumption:

Assumption 5 *B_1 is large enough so that for all $A > \underline{A}(0)$ the government's desired borrowing in repayment exceeds the supply of funds in the market.*[17]

Assumption 5 simplifies the problem by eliminating the issue of competition between lenders in the bond market, in which case the lender's problem can be solved in closed form.[18]

[17]Recall from Section 2.3 that $\underline{A}(0)$ is the lower bound for the fragility region when $r = 0$. Thus, it is the productivity level below which the government will always default, regardless of the interest rate and regardless of the households' and lenders' decisions.

[18]While Assumption 5 simplifies the comparative statics analysis, it does not affect

3.1 Equilibrium Effects of Policy Adjustments[19]

Before analyzing specific policies, it is useful to understand the equilibrium forces that are at play when the government adjusts its policy. For this purpose, consider an abstract policy adjustment, captured by a change in a parameter ψ.[20] We would like to understand how a change in ψ affects the ex-ante probability of default which, for a given interest rate r, is given by $\Pr(A < A^*)$. This preliminary abstract analysis has additional advantages: (1) It highlights how dispersed beliefs and endogenous expectations affect the of government policies, and (2) is helps to understand how and when predictions of the model with dispersed beliefs will differ from the predictions of the models where defaults are driven only by fundamentals.

Let A^{**} denote households' and lenders' belief regarding the default threshold (where in equilibrium we have $A^* = A^{**}$ as agents' beliefs have to be correct). We have the following Proposition.

Proposition 2 *The change in default threshold implied by the adjustment in a policy parameter ψ is given by*

$$\frac{dA^*}{d\psi} = \underbrace{\frac{1}{1 - \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} - \int_0^1 \frac{\partial A^*}{\partial k_2^i}\frac{\partial k_2^i}{\partial A^{**}}di}}_{\textit{Multiplier effect } (\boldsymbol{M})} \times \underbrace{\left(\frac{\partial A^*}{\partial \psi} + \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi} + \int_0^1 \frac{\partial A^*}{\partial k_2^i}\frac{\partial k_2^i}{\partial \psi}di\right)}_{\textit{Direct effect } (\boldsymbol{D})} \tag{2}$$

The multiplier effect is always strictly greater than 1 *so that* $sgn\,(dA^*/d\psi) = sgn\,(D)$.

The above Proposition establishes that the effect of an adjustment in any parameter ψ on A^* can be decomposed into the direct effect and the multiplier effect. To understand the intuition behind Equation (2) consider a change in ψ, but keep first households' and lenders' beliefs about A^* constant. Then a change in ψ affects the government's incentive to default,

its underlying logic. In particular, Proposition 2 holds in the same form regardless of whether we impose Assumption 5. For a more detailed discussion of the consequences of this assumption see Section E of the Appendix.

[19] For comparison of predictions based on the baseline model and its version where crises are driven purely by fundamentals see Section C of the Appendix.

[20] For concreteness, one can think of this policy as an increase in taxes, in which case $\psi = \tau$.

by changing the difference between the values of repaying and defaulting on the debt. This effect works through the government's indifference condition; I denote it by $\partial A^*/\partial\psi$, since it corresponds to the partial effect of a change in policy keeping strategies of households and lenders fixed. Moreover, the policy change potentially affects households' and lenders' decision problems, thereby leading households and lenders to adjust their strategies and in turn bringing about a further change in the government's incentive to default (these effects are captured by terms $\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial \psi}$ and $\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi}$, respectively). Thus, the "direct effect" is equal to the change in the default threshold, keeping households' and lenders' expectations fixed.

The households' and lenders' expectations, however, are not fixed. In response to this initial change in the default threshold, the households and lenders adjust their expectations, and thus their strategies, which leads to a further change in A^*, inducing another round of adjustment in the households' and lenders' expectations and so on. Thus, "multiplier effect" capture the change in default threshold driven by the adjustment in households' and lenders' expectations.

Proposition 2 leads to three important implications. First, whether a change in a government policy increases or decreases the probability of default is determined by the "direct effect." Thus, to establish whether a given policy decreases or increases the likelihood of a debt crisis one can focus on understanding how the policy affects the government incentive to default holding agents' beliefs. Second, adjustments in endogenous expectations always amplify the initial impact of any policy adjustments, and thus are key for quantifying the impact that any policy has on the probability of default (see Section 5 for the analysis when this effect is particularly strong). Third, the presence of dispersed beliefs affects the qualitative predictions of the model: Even though the "direct effect" captures intuitive forces that are present in standard models, these forces are distorted by the presence of dispersed information. Intuitively, the direct effect of a given policy depends on the agents' behavior without the policy change as well as their response to a change in a policy, both of which are distorted by the presence of dispersed information (see Section C of the Appendix).

3.2 Overview of Policies

Using the above insights, I now analyze two policy measures that received a lot of attention in policy debates during the recent sovereign debt crisis in Europe: (1) austerity (increase in taxes) and (2) a fiscal stimulus (financed with debt). The European debt crisis generated a lively debate about viability of the above policies for preventing debt crises (see Brunnermeier et al. (2016)). Below, I describe how each of these policies is introduced into the model.

Increase in Taxes In the model, a rise in the tax rate is captured by an increase in τ, the fraction of output that the government takes away from households. Below I consider the case where once adjusted, τ is kept constant across periods and is the same regardless of whether the government defaults. This fits a scenario where the government finds it difficult to change tax laws once they have been enacted (for example because of the lengthy political process it involves). In Section C of the Appendix, I consider the situation where higher τ is implemented only if the government repays the debt, a case that is relevant in the situation where policymakers are willing to increase taxes only to avoid default and once the default occurs they are likely to abandon this idea. The results are similar for the both cases.

Fiscal StimulusI model fiscal stimulus as an increase in the initial capital stock of each household from k_1 to $(1+s)\,k_1$ financed by the government, where s measures the size of the stimulus as a percentage of the initial capital stock. Thus, if the government decides to engage in a stimulus the total output of the economy will increase.[21] I do not explicitly model

[21] This is a simple way to model a fiscal stimulus in the current framework. One should interpret the increase in k_1 not as an increase in physical capital owned by households but rather as an increase in government spending on public goods and services that enhance production (e.g., an increase in expenditure on infrastructure or on the maintenance of the rule of law). An alternative way to model stimulus would be to explicitly allow government spending to enter the production function, that is to write the household production function as $y_t^i = e^{A_i} f\left(k_t^i, h_t\right)$ where h_t captures explicitly the government expenditure that is important for production. However, the qualitative conclusions would remain unchanged.

the government's financing decision. Instead, I assume that to finance a stimulus, the government issues additional debt at the end of the period preceding period 1. I consider separately the case where this additional debt matures at the end of period 1 together with B_1 (short-term debt financing with interest rate $r^{ST} \geq 0$) or in period 2 (long-term debt financing with interest rate $r^{LT} \geq 0$).

3.3 Increase in Taxes

As explained above, to understand the effect of an increase in the tax rate τ on the default threshold, it is enough to focus on its direct effects. A higher tax rate leads to a change in the government's incentives to repay debt equal to

$$\underbrace{Y_1^R\left(u_{g_1}^R - u_{g_1}^D\right) + Y_2^R\left(u_{g_2}^R - u_{g_2}^D\right)}_{\text{Concavity effect}} + \underbrace{Y_1^R\left(1-Z\right)u_{g_1}^D + Y_2^R\left(1-Z\right)u_{g_2}^D}_{\text{Differential increase in tax revenues}}$$
$$\underbrace{-\frac{\alpha}{1-\tau}\tau Y_2^R\left(u_{g_2}^R - Zu_{g_2}^D\right)}_{\text{Investment distortion}}, \tag{3}$$

where $u_{g_t}^R$ and $u_{g_t}^D$ are the marginal utilities from government spending in period t in repayment and default, respectively, and is Y_t^R the total output of the economy in period t in repayment, all evaluated at the threshold productivity level A^*. If the expression in (3) is positive, then the government's incentive to repay its debt increases following an increase in τ.[22]

The expression in (3) tells us that an increase in the tax rate affects the government's default incentives through three channels. First, a higher τ implies higher tax revenues. Since at A^* the government's spending is lower in repayment than in default, the concavity of the utility function implies that a given increase in government spending leads to a greater increase in the value of repaying than in the value of defaulting, thus decreasing the government's default incentive (the "concavity effect"). Second, since the total output is higher in repayment, a given increase in the tax rate

[22]The expression in (3) corresponds to $\frac{\partial}{\partial\tau}\Delta V\left(A^*, k_2, x^*; \psi\right)$. The direct effect is equal to $\frac{\partial}{\partial\tau}\Delta V\left(A^*, k_2, x^*; \psi\right)$ divided by $-\frac{\partial}{\partial A^*}\Delta V\left(A^*, k_2, x^*; \psi\right) < 0$. In particular, the sum of the concavity effect and the differential increase in tax revenues divided by $-\frac{\partial}{\partial A^*}\Delta V\left(A^*, k_2, x^*; \psi\right)$ is equal to $\frac{\partial A^*}{\partial\psi}$, while the expression for investment distortion divided by $-\frac{\partial}{\partial A^*}\Delta V\left(A^*, k_2, x^*; \psi\right)$ corresponds to $\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial\psi}$ in Equation (2).

translates into a greater increase in tax revenues in repayment than in default, further decreasing the government's default incentives (the "differential increase in tax revenues"). The last term captures the negative effect of higher taxes on households' investment decisions, where $\alpha/(1-\tau)$ is the rate at which output decreases with higher taxes and $u^R_{g2} - Zu^D_{g2}$ measures how "painful" this decrease in spending is to households in repayment compared to default (the "investment distortion").

Proposition 3 *There exists $\underline{\tau} > 0$ such that for all $\tau \leq \underline{\tau}$ an increase in taxes decrease the probability of default. Moreover, if $\sigma_x \to 0$ and $r \times b < B_1$ then $\underline{\tau} > 1/(1+\alpha)$.*

The above proposition states that if the initial tax rate is not "too high" (i.e., $\tau \leq \underline{\tau}$) then an increase in the tax rate will decrease the probability of default. This result follows from the observation that the "investment distortion" $\alpha/(1-\tau)$ is a convex function of τ and for high values of τ it dominates the positive effect of higher tax revenues. The second part of Proposition 3 states that if the supply of funds in the bond market (which, when lenders have precise information, is bounded from above by rb) is lower than B_1 then an increase in τ decreases the default threshold for all $\tau \leq 1/(1+\alpha)$. In other words, if the government is unable to roll over all of its debt then an increase in taxes necessarily decreases the probability of default for all $\tau \leq 1/(1+\alpha)$.

How likely is this last condition satisfied in reality? Note that in the model α can be interpreted as the capital share of output, and thus $\alpha \approx 0.33$. The average ratio of government tax revenues to GDP in Eurozone in 2011 was according to Eurostat about 0.4 (translating into $\tau \approx 0.4$ in the model) which implies that the sufficient conditions for austerity to decrease the probability of default during the recent European debt crisis were likely satisfied.

The next result further strengthens the case for austerity. It shows that when the initial expectations about the current economic situations (as captured by A_{-1}) are low then an increase in the tax rate will decrease the probability of default even if τ is already very high.

Corollary 1 *For any $\tau \in (0,1)$ there exists $\underline{A}_{-1}(\tau)$ such that if $A_{-1} <$*

$\underline{A}_{-1}(\tau)$ *then* $dA^*/d\tau < 0$.[23]

While this result might seem surprising at first, it is rather intuitive: When A_{-1} is low then lenders are unwilling the supply the funds to bond market unless they receive very high signals, which implies that the total amount of funds available in the bond market is low. As the consequence, for low enough A_{-1} the government is able to borrow very little and the only way it can repay the debt and avoid default is by increasing its revenues. An increase in τ is one way to achieve this.

3.4 Fiscal Stimulus

Now consider the effect of a fiscal stimulus on the probability of default. A fiscal stimulus leads to a change in government's incentives to repay debt equal to

$$\underbrace{\tau\frac{\partial Y_1^R}{\partial s}\left(u_{g_1}^R - u_{g_1}^D\right) + \tau\frac{\partial Y_2^R}{\partial s}\left(u_{g_2}^R - u_{g_2}^D\right)}_{\text{Concavity effect}} + \underbrace{\left[\frac{\partial Y_1^R}{\partial s}u_{g_1}^D + \frac{\partial Y_2^R}{\partial s}u_{g_2}^D\right]\tau(1-Z)}_{\text{Differential increase in tax revenues}}$$
$$\underbrace{-u_{g_t}^R\left(1+r^{stim}\right)k_1}_{\text{Increase in debt}}, \tag{4}$$

where $r^{stim} \in \left\{r^{ST}, r^{LT}\right\}$ is the interest rate on the debt issued to finance the stimulus, $\partial Y_t^R/\partial s$ is the increase in output in period t resulting from the stimulus, and where $u_{g_t}^R$, $u_{g_t}^D$ and Y_t^R are defined as in Section 3.3.

The expression in (4) tells us that a fiscal stimulus affects the government's default incentive through three channels: (1) the "concavity effect; (2) a differential increase in government tax revenues in repayment and default (both of which were also present in the case of a tax increase); and (3) a negative effect due to an increase in the government's debt burden (equal to $u_{g_1}^R\left(1+r^{ST}\right)k_1$ if the stimulus is financed with short-term debt, or to $u_{g_2}^R\left(1+r^{LT}\right)k_1$ if financed with long-term debt).

Proposition 4 *Consider a stimulus financed with short-term debt. There exists* $\overline{B}_1$ *such that stimulus decreases probability of debt crisis if and only if* $B_1 > \overline{B}_1$. *Moreover,* $\overline{B}_1/k_1 > (1+r^{ST})\frac{1}{\alpha}$.

[23] Recall that A_{-1} denotes the past level of productivity and is equal to the mean of agents' prior belief.

Proposition 4 establishes that stimulus decreases the probability of default if and only if the debt to capital stock ratio is high. The intuition behind this observation is simple: A higher B_1 implies a higher marginal benefit from an increase in output in repayment while a higher k_1 implies a higher cost of increasing capital stock by a given percentage. Proposition 4 provides also a necessary condition for the stimulus to work: The ratio of debt to capital has to be larger than $\frac{1}{\alpha}$.

It is important to stress the even though the above proposition identifies conditions under which fiscal stimulus financed with short term debt can work, these conditions are unlikely to hold in practice. Since α can be interpreted as the capital share of output so that $\alpha \approx 0.33$, the above proposition suggests that in order for a fiscal stimulus financed with short-term debt to work one needs capital to debt ratio in excess of 3. This is unlikely to be the case for most countries. For example, this ratio is less than 1 for Eurozone countries suggests that stimulus was not a valid option for the governments during the recent European debt crisis.[24]

When a stimulus is financed with long term debt the necessary condition for the stimulus to work becomes $\overline{B}_1/k_1 > (1+r^{LT})\frac{1}{\alpha}(u^R_{g_1}/u^R_{g_2})$. Since $u^R_{g_1}/u^R_{g_2} < 1$,[25] as long as r^{LT} is not significantly higher than r^{ST}, the condition under which fiscal stimulus financed with long-term to decrease the probability of default is less stringent compared to the one in the case of short-term debt financing. However, given the discussion, even this condition is unlikely to hold since it would require an implausible large drop in government spending in period 1 compared to period 2.[26]

4 Economic Policy Uncertainty and Its Consequences

Above I considered a situation where a policy change was expected by both households and lenders. In this section, I investigate how the above

[24] The capital-output ratio for most Eurozone countries is above 3 (see Penn World Tables, Feenstra et al. (2015)) while the debt-to-GDP ratio is smaller than 2.

[25] In equilibrium the government expenditure in period 1 is always lower than in period 2 in repayment as the government is unable to smooth debt repayment over time.

[26] Given that for most countries $\frac{1}{\alpha} \approx 3$ and $B_1/k_1 \leq 1$ we would need the government spending in period 2 to be three times higher than in period 1 in order for this condition to be satisfied.

results change if the households and lenders are uncertain as to whether the government will adjust its policies. The analysis is motivated by the observation that often there is a strong disagreement among policymakers regarding the political and economic desirability of given economic policies, thereby giving rise to a substantial policy uncertainty. Indeed, as discussed in the introduction (Figure 1) there was a large spike in such an uncertainty during the European debt crisis.[27] Thus, it is important to understand if and how such uncertainty distorts the effectiveness of austerity and stimulus.

I consider two cases. First, I investigate the model's predictions when a policy change is unexpected by lenders and households. This case describes a situation where either government announcements have no credibility (so that agents do not believe there will be any policy change), or when the government decides to do an unexpected U-turn on its economic policy. Second, I analyze a situation where households and lenders expect that the government will adjust its policy with probability $p \in (0,1)$. Otherwise, there are no changes compared to Section 3.

4.1 Unexpected Policy Adjustment

Proposition 5 *Suppose that a policy change is unexpected. Then*

$$\frac{dA^*}{d\psi} = \frac{\partial A^*}{\partial \psi}.$$

Moreover, $dA^*/d\psi \to 0$ *as* $\varepsilon, \sigma_x \to 0$.

Proposition 5 tells us that when a policy change is unexpected the change in the default threshold is equal to the direct effect the policy has on the government's incentives to default. Since agents expect no policy adjustment, their strategies are unchanged, implying that the multiplier

[27] Policy uncertainty played an important role in Greece, where after winning the unexpected early elections in January 2015 the Syriza-led coalition stopped implementation of reforms, only to suddenly change its mind six months later, but not until after pushing Greece to the verge of default. This issue also played an important role in Italy. In response to the crisis, the Italian parliament formed a technocratic government, with Mario Monti as prime minister, to implement a package of structural reforms. Lacking political support the government was less successful than expected in passing the reforms.

effect and the part of the direct effect that operates through households' and lenders' choices are absent. Moreover, in the limit an unexpected policy change becomes completely ineffective as the direct effect converges to 0.[28]

This last result provides a strong warning against unexpected policy U-turns so that agents are not surprised by the government actions. It also worth emphasizing that the same logic applies to policy announcements that are viewed by agents as not credible, and hence governments should strive to communicate its policy plans not only in advance but also in a credible manner.

Corollary 2 *Suppose that a policy change is unexpected and that $\varepsilon, \sigma_x > 0$.*

1. *An increase in the tax rate τ always decreases the government's incentives to default.*

2. *A fiscal stimulus financed with short-term debt decreases the government's incentives to default if and only if*

$$\varsigma_{ST}^{unexp} = \frac{\alpha\left(B_1 - B_2\right)}{\tau Y_1 - B_1 + B_2} - \frac{\left(1 + r^{ST}\right) k_1}{\tau Y_1 - B_1 + B_2} > 0$$

while in case of the long-term debt financing the relevant condition is

$$\varsigma_{LT}^{unexp} > \frac{\alpha\left(B_1 - B_2\right)}{\tau Y_1 - B_1 + B_2} - \frac{\left(1 + r^{LT}\right) k_1}{\tau Y_2 - \left(1 + r\right) B_2} > 0$$

The above corollary implies that, as long as $\varepsilon, \sigma_x > 0$, an unexpected increase in tax rate always leads to decrease in the probability of default. This is because the negative effect of higher taxes on households' investment choices is now absent (no investment distortion). On the other hand, a fiscal stimulus, if unexpected, leads only to an expansion of output in period 1;

[28]To understand this consider lender j who can observe A. Lender j would lend to the government if and only if $A > A^{**}$, where A^{**} corresponds to households' and lenders' beliefs about default threshold. Thus, lender j will not respond to any policy change unless it also leads to a change in A^{**}, that is it leads to a change in beliefs of other agents. But since a policy change in unexpected agents' beliefs are fixed and A^{**} is unchanged implying that lender j does not adjust his behavior following the policy change. While in the model lenders cannot observe true A, as $\sigma_x \to 0$ the uncertainty about A disappears and we converge to the case described above. Similar logic applies to the behavior of households.

households keep their investment strategies constant as they do not expect any change in the economy. As a consequence, a fiscal stimulus is now more likely increase the probability of default than before. It follows that if the government lacks credibility or if it suddenly decides to act, austerity is a better option than stimulus. However, it should be kept in mind that, in light of Proposition 5, the overall effect of these policies on the probability of default will be rather small, especially when households' and lenders' private information is precise.

4.2 Uncertainty about Reforms

In this section I consider a case where agents expect the government to implement a given reform with probability $p \in (0,1)$. Let $dA^*/d\psi\,(p)$ denote the total change in the default threshold when the agents expect the policy to be implemented with probability p and the government does implement the announced policy. It can be shown that in this case we have:[29]

$$\frac{dA^*}{d\psi}(p) = p\frac{dA^*}{d\psi}(1) + (1-p)\frac{\partial A^*}{\partial \psi} \tag{5}$$

Thus, a change in the default threshold is a weighted average of the change in the default threshold when there is no uncertainty ($dA^*/d\psi\,(1)$) and when the policy change is unexpected ($\partial A^*/\partial\psi$). Intuitively, when agents expect that the policy will be implemented with probability p, their response to the prospect of the policy adjustment is proportionately less than in the case of no economic policy uncertainty. This results in an adjustment of the default threshold equal to $p\frac{dA^*}{d\psi}(1)$. On the other hand, with probability $1-p$ households and lenders do not expect the adjustment, in which case if the policy adjustment happens it is driven by the direct change in the government's default incentive (and hence the adjustment in A^* is equal to the change in the default threshold when the policy adjustment is unexpected).

[29] For more details behind derivations of Equation 5 see Section D of the Appendix. It is worth stressing that derivations of this decompositions are non-trivial and that the fact that such a linear decomposition holds for the default threshold is surprising as the model itself is highly non-linear.

Proposition 6 *Suppose that agents attach probability $p \in (0,1)$ to the announced policy being implemented.*

1. *Then an increase in τ decreases probability of default for a wider range of initial conditions than in the case of no uncertainty ($p = 1$), that is*

$$\frac{dA^*}{d\tau}(1) < 0 \implies \frac{dA^*}{d\tau}(p) < 0 \text{ but not vice verse}$$

2. *Then a fiscal stimulus decreases probability of default for a more limited range of initial conditions than in the case of no uncertainty ($p = 1$), that is*

$$\frac{dA^*}{ds}(p) < 0 \implies \frac{dA^*}{ds}(p) < 0 \text{ but not vice verse}$$

3. *If ε and σ_x are small then* $\left|\frac{dA^*}{d\psi}(p)\right| < \left|\frac{dA^*}{d\psi}(1)\right|$

Proposition 6 shows that the conclusion obtained in the case of unexpected policy changes extend to the case when policies are implemented with positive probability. In particular, Part 1 establishes that in the presence of uncertainty as to whether the government will implement announced policies an increase in taxes is an effective way to decrease the likelihood of a crisis for a wider range of initial conditions. The intuition behind this result is the same as before: Uncertain as to whether higher taxes will be implemented households do not decrease their investment as much as they would otherwise. Similarly, Part 2 establishes that in the presence of such an uncertainty the range of conditions under which fiscal stimulus decrease the likelihood of a crisis shrinks. Thus, the presence of policy uncertainty strengthens the appeal of austerity compared to stimulus. However, as shown in Part 3, in both cases economic policy uncertainty decreases the overall effect both policies have on the default threshold.

Thus, Proposition 6 leads to two conclusions. First, economic policy uncertainty is undesirable as it decreases the overall effectiveness of government policies. Second, in the presence of economic policy uncertainty austerity becomes relatively more preferred option compared to stimulus.

5 Numerical Analysis and The Role of the Multiplier Effect

Above I analyzed analytically how fiscal stimulus and increase in taxes affect the government incentives to default and how these effects depends on the degree of economic policy uncertainty. In this section I complement the above analytical results with numerical a investigation. In particular, I investigate numerically: (1) whether for reasonable parameter values the government policies considered above tend to decrease or increase the probability of default, and (2) when is the effect of expectations particularly important (i.e., when is the multiplier effect large).

5.1 The Multiplier Effect and the Role of Beliefs

Since the multiplier effect captures the role of beliefs, we should expect that the multiplier effect plays an important role if changes in households' and lenders' beliefs have a relatively strong impact on the value to the government of repaying its debt and defaulting on its debt. Below, I argue that households' and lenders' beliefs have a strong impact on the government's decisions when households tend to invest a high fraction of their income and the government desired borrowing is high.

Households' expectations are important if the difference between an investment of a pessimistic household and an optimistic household (holding productivity level constant) is large since then an adjustment in households expectations will lead to a large change in the total output, and hence in tax revenues. Since this difference is equal to

$$k_2^R - k_2^D = (1 - Z)(1 - \tau) e^{A_i} k_1^{\alpha} \frac{\alpha}{1 + \alpha}$$

one should expect that households' beliefs play an important role when $k_2^R - k_2^D$ is large, which is the case when τ, Z are low and α, k_1 are high.

Lenders' beliefs affect the government default decision by determining how much the government can borrow. However, if the government's desired borrowing is low then the quantity of funds supplied to the market matters relatively little since the government would not want to borrow much anyway. Therefore, one should expect that the role of lenders' expectations is large when the government's desired borrowing is high. From the government's problem it follows that the government's desired borrowing is equal to

$$B_2^{R,u}(A) = \frac{(1+r)\,B_1 + \tau Y_2^R(A) - (1+r)\,\tau Y_1^R(A)}{2\,(1+r)}$$

where $Y_t^R(A)$ is the aggregate output at time t if the government repays its debt when the average productivity is A. The desired borrowing tends to be high when τ is low (a high τ decreases investment, and hence decreases Y_2), k_1 is low and α is high (since then Y_2 is relatively high compared to Y_1) or B_1 is high.

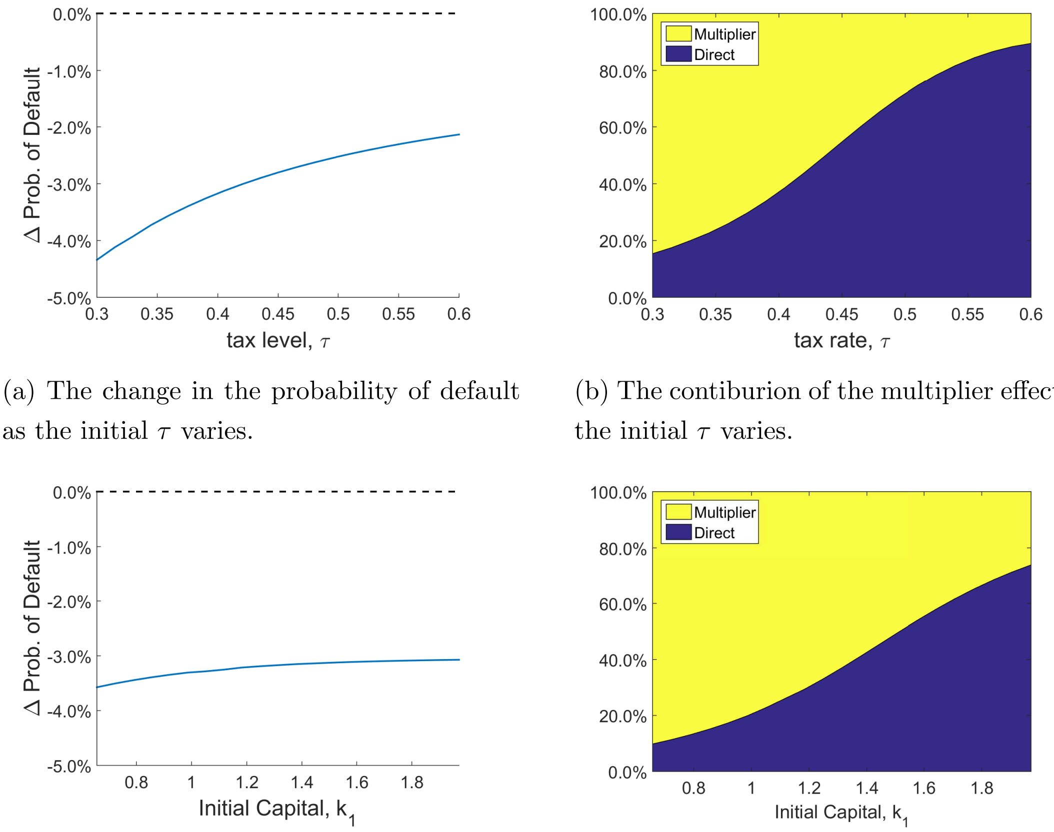
5.2 Numerical Analysis

The next goal is to understand: (1) whether for reasonable parameter choice an increase in tax and fiscal stimulus tend to decrease or increase the probability of default, and (2) how important is the multiplier effect in driving these results.

I choose a reference set of parameters in a way that the model resembles the GIIPS economies (i.e., Greece, Ireland, Italy, Portugal, and Spain) at the onset of the European debt crisis in 2008. I then vary key parameters from this reference point, one at a time, to see how the effectiveness of the government policies and the importance of the multiplier effect varies with the parameters.[30] To make results comparable across different parameter values, following each change in a parameter of the model, I adjust the mean of the prior belief so that the ex-ante probability of default, before

[30] From the perspective of the analysis, the most important parameters are τ, the tax rate; Z, the output costs of default; k_1, the initial the capital stock; and α, the capital share of output, since these parameters determine directly the costs and benefits of both policies considered above. I set $\tau = 0.4$, the average ratio of governments' tax revenue to GDP in the Eurozone in 2011 as reported by Eurostat, and $Z = 0.92$, implying that in the case of a debt crisis, output declines by 8% (the observed output decline in Greece after it defaulted in 2010). I choose $k_1 = 1.31$ to match the average growth of the net capital stock of 2% in the GIIPS economies in the run-up to the crisis (period 2004-2008), and $\alpha = 0.4$ (see Arpaia et al. (2009)). The information parameters are $\sigma_x = 1/20$, $\varepsilon = \sqrt{3}\sigma_x$, and $\sigma = 1/12$. Mean of prior, A_{-1}, is set to imply a 10% probability of default. The initial debt is $B_1 = 1$, and the total wealth of the lenders is four times the maturing debt, implying the ratio $b/B_1 = 4$, which is twice the average bid-to-cover ratio in the debt auctions in Germany and Italy as reported in Beetsma et al. (2013).

a new policy is implemented, is equal to 10%. For space considerations, I report below only results where I vary the tax rate τ and the initial level of capital k_1. Additional results can be found in Section G of the Appendix.



(a) The change in the probability of default as the initial τ varies.

(b) The contiburion of the multiplier effect as the initial τ varies.

(c) The change in the probability of default as the initial k_1 varies.

(d) The contiburion of the multiplier effect as the initial k_1 varies.

Figure 6: The effect of a 1% increase in the tax rate.

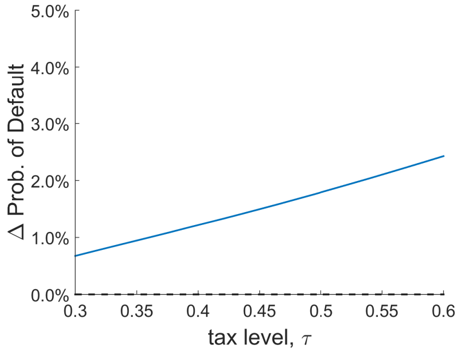
Increase in the tax rate I consider first the effect of a 1% increase in taxes for different initial values of the tax rate τ and the capital stock k_1. Panel A of Figure 6 shows how the effect of this policy varies with the initial tax level while Panel B depicts how much of the change in the default threshold is driven by the multiplier effect. We see that an increase in the tax rate has a larger positive effect when initially taxes are low. This is because at low τ the distortive effect of a tax increase is small while the multiplier effect is large. Panel B shows that the relative importance of the multiplier effect decreases as τ increases: When the initial tax rate is low then the majority of the adjustment in the default threshold A^* is driven by the adjustment in households' and lenders' beliefs, but as initial τ increases the importance of beliefs decreases. This is in line with the intuition provided in Section 5.1.

Panels C and D of Figure 6 depict the corresponding results of a 1% increase in the tax rate τ for different values of k_1. We see that varying the initial level of capital has relatively little effect on the efficacy of an increase in taxes. However, we see that the initial level of capital stock does affect the importance of the multiplier effect with multiplier effect being stronger for low values of k_1. To understand why this is the case note that, as explained in Section 5.1, as k_1 increases the importance of the households' beliefs tends to increase while the importance of the lenders' beliefs tends to decrease. For the parameters considered here the latter effect dominates (as the difference between k_2^R and k_2^D is relatively small) and the importance of the multiplier effect declines as k_1 increases.

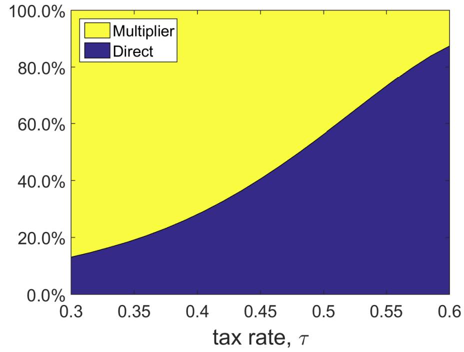
Fiscal Stimulus Next, I report the effects of a fiscal stimulus for different values of the initial tax rate τ and capital stock k_1. I consider a fiscal stimulus wit size equal to 1% of the initial capital stock and financed with short-term debt (with $r^{ST} = 0$).[31] Panels A and C of Figure 7 show that engaging in fiscal stimulus when a crisis is likely is not a good idea as fiscal stimulus tend to increase the probability of default. Moreover, we see that this negative effect is stronger when initial tax rate is high (since at higher τ households invest less leading to a lower positive effect of stimulus on the future output) and when k_1 is high (since then the marginal value of extra unit of capital is low while the cost of such a policy is high). Moving our attention to Panels B and D we observe that, as in the case of an increase in τ, the multiplier effect is an important driver of the adjustment in the probability of default when k_1 or τ are relatively low and its role diminishes as k_1 and τ increase.

Summary The above results indicate that an increase in the tax rate is an effective policy for decreasing probability of default for a wide range of parameters while the opposite is true for a fiscal stimulus. They also

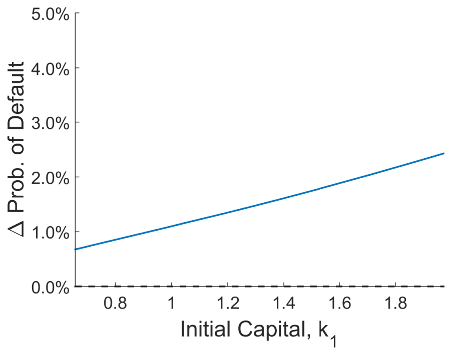
[31] The results for a fiscal stimulus financed with long-term debt are similar.



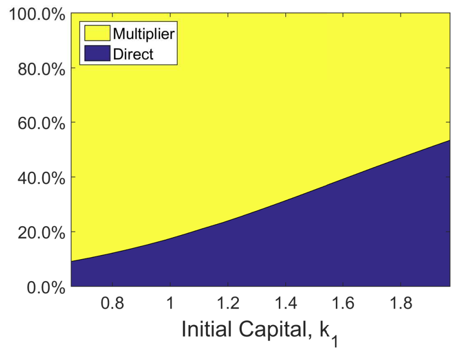
(a) The change in the probability of default as the initial τ varies.



(b) The contiburion of the multiplier effect as the initial τ varies.



(c) The change in the probability of default as the initial k_1 varies.



(d) The contiburion of the multiplier effect as the initial k_1 varies.

Figure 7: The effect of a 1% stimulus.

support the intuition provided above that endogenous adjustments in expectations play an important role in determining the total change in the default threshold A^*.

6 Conclusions

In this paper, I investigated how a government can prevent a self-fulfilling debt crisis. To answer this question I developed a model of self-fulfilling sovereign default with endogenous expectations and dispersed information. I then used this model to how fiscal policies, such as an increase in taxes or fiscal stimulus, affect the probability of a crisis and how these effects are perturbed by the presence of endogenous expectations and dispersed beliefs. I showed that typically austerity policies tend to decrease the probability of default while fiscal stimulus tends to increase the probability of default. I also found that endogenous expectations tend to amplify the effects of these policies. Finally, I studied how uncertainty about government economic policies changes the effect of government policies and found that such uncertainty further makes an increase in taxes more attractive options than fiscal stimulus, but in general it decreases the total impact those policies have on the economy.

The findings of this paper contribute to the debate whether the government that faces a looming debt crisis should engage in austerity or fiscal stimulus that took place during European debt crisis, and provide support for the choice of austerity. My results suggest that the austerity is particularly preferable to fiscal stimulus in an environment where there is high uncertainty about future economic policies, as often is the case during debt crises. Thus, the results provide support for the policies adopted during European debt crises while suggesting that they would have been substantially more effective in the absence of policy uncertainty that accompanied their implementation.

A few words of caution are needed regarding the interpretation of the results. First, the paper abstracts from analyzing interactions between actions of an international lender of last resort (such as ECB) and domestic government policies. While important, such a question is beyond the scope of the current paper. Second, in this paper I analyzed a situation when the government finds itself at a spot where a debt crisis is looming. Indeed, the main question this paper addresses is how to avoid a debt crisis when such crisis is likely in the near future. For that purposes, that fact that the model presented above is two-period is a minor issue. However, the fact that the model is not dynamic becomes key when trying to answer questions regarding medium-term policies. A question of particular importance is what should the government do to avoid facing another debt crisis in the future once the debt crisis has been averted today. This remains an important question for the future research.

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Preventing Self-fulfilling Debt Crises: Appendix (For Online Publication)

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This appendix contains the proofs of the results that have been stated in the paper and is divided into six sections. In Section A I solve the main model. This section contains the proofs of Lemma 1 and Lemma 2, and the main uniqueness result (Proposition 1). Section B contains derivations of the direct and multiplier effects and the proofs of Propositions 2 to 4 and Corollary 1 from the paper. Section C includes additional results that have been omitted from the paper but may be of interest to a reader. In particular, it includes a comparison between predictions based on the baseline model and its version where crises are driven purely by poor fundamentals and the analysis of an increase in τ when it is implemented only in repayment. Section D contains brief derivations of the total change in the default threshold when the agents expect the policy to be implemented with probability p, i.e., $dA^*/d\psi(p)$, as well as proofs of Propositions 5 and 6, and Corollary 2. In Section E I briefly discuss how the results would change if Assumption 5 was not imposed. Section F contains a discussion of the effect of an adjustment in the interest rate on the effects of policy changes while Section G contains several technical claims invoked in proofs throughout the Appendix. Finally, Section H contains further numerical results that have not been reported in the paper.[1]

A Global Game model

A.1 Uniqueness Result

Proposition A *There exist $\bar{\varepsilon} > 0$ and $\bar{\sigma}_x > 0$ such that for all $\varepsilon \in (0, \bar{\varepsilon}]$ and all $\sigma_x \in (0, \bar{\sigma}_x]$ the model has a unique equilibrium in monotone strategies.*

To prove the above result, I first characterize the optimal households' and lenders' strategies in response to a monotone default strategy by the government. Then I show that in response to these households' and lenders' strategies the government indeed finds it optimal to follow a monotone default strategy. Finally, I show that there exists a unique fixed-point of this argument. Before proceeding any further I introduce notation that will be useful when analyzing the model.

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[1] The solution to the complete information version of the model, and detailed derivations of the multiplier and direct effects when agents are uncertain whether announced policies will be implemented, can be found in the "Additional Results" document available on the author's website (http://economics.ubc.ca/faculty-and-staff/michal-szkup/).

Notation 1 *I will use the following notation throughout the Appendix:*

1. *A^* denotes the default threshold used by the government.*

2. *A^{**} denotes the default threshold expected by the households and lenders.*

A.1.1 Households

Suppose that households expect the government to repay its debt if and only if $A \geq A^{**}$. Household i's optimal investment then solves the household's problem specified in Section 2.4. Each household receives a productivity shock A_i, where $A_i = A + \varepsilon_i$ and $\varepsilon_i \in [-\varepsilon, \varepsilon]$.

If $A_i > A^{**} + \varepsilon$, then household i expects no default; in that case,

$$k_2(A_i) = (1-\tau) e^{A_i} f(k_1) \frac{\alpha}{1+\alpha}.$$[2]

If household i receives productivity $A_i < A^{**} - \varepsilon$, then household i believes that the government will always default and

$$k_2(A_i) = (1-\tau) e^{A_i} f(k_1) \frac{\alpha Z}{1+\alpha}.$$

Finally, in the case when $A_i \in (A^{**} - \varepsilon, A^{**} + \varepsilon)$ the household is uncertain as to whether the government will default. In that case,

$$k_2(A_i) = (1-\tau) e^{A_i} f(k_1) \Lambda(A_i; \varepsilon, A^{**})$$

where

$$\begin{aligned}\Lambda(A_i; \varepsilon, A^{**}) \quad = \quad & \frac{\alpha(1+Z) + P(A^{**}|A_i) + Z(1 - P(A^{**}|A_i))}{2(1+\alpha)} \\ & - \frac{\sqrt{[\alpha(1+Z) + P(A^{**}|A_i) + Z(1 - P(A^{**}|A_i))]^2 - 4\alpha Z(1+\alpha)}}{2(1+\alpha)}\end{aligned}$$

and $P(A^{**}|A_i) \equiv \Pr(A < A^{**}|A_i)$. It is straightforward to show that $\Lambda(A_i; \varepsilon, A^{**})$ is increasing in A_i and decreasing in A^{**}. This establishes Lemma 1 in the paper.

Next, I perform a change of variables $\kappa = \frac{\varepsilon_i}{\varepsilon}$, where $\varepsilon_i \in [-\varepsilon, \varepsilon]$ so that $\kappa \in [-1, 1]$. This change of variables turns out to be useful for computing the output in the limiting case as $\varepsilon \to 0$, and in general, when analyzing the effect of changes in ε. Define

$$\Lambda(A + \kappa\varepsilon; \kappa, A^{**}) \equiv \begin{cases} \frac{\alpha}{(1+\alpha)} & \text{when } A_i = A + \kappa\varepsilon > A^{**} + \varepsilon \\ \Lambda(A_i; \varepsilon, A^{**}) & \text{when } A_i = A + \kappa\varepsilon \in (A^{**} - \varepsilon, A^{**} + \varepsilon) \\ \frac{\alpha Z}{(1+\alpha)} & \text{when } A_i = A + \kappa\varepsilon < A^{**} - \varepsilon \end{cases}$$

In what follows I will denote the optimal choice of capital as $k_2^*(A, \kappa, A^{**})$ to emphasize its dependence on A, κ and household's belief about the default threshold A^{**}.

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[2] It is here that the assumption of full depreciation of households' capital simplifies the model. When the capital depreciates fully each period, the optimal choice of capital is linear. As we will see below, this will make the government's default condition near linear in e^A.

A.1.2 Lenders[3]

Denote by $p_x = 1/\sigma_x^2$ and $p_A = 1/\sigma_A^2$ the precisions of the lenders' private signals and the prior, respectively. As usual, it is more convenient to work with precisions rather than standard deviations or variances.

Let $u(1, A; x^{**}, A^{**})$ be the expected payoff to lender j from lending to the government when the average productivity is equal to A, the government uses a threshold strategy with cutoff A^{**}, and the other lenders use monotone strategies with cutoff x^{**}. Similarly, denote by $u(0, A; x^{**}, A^{**})$ the payoff to lender j from investing in the risk-free asset. Then

$$\begin{aligned} u(1, A; x^{**}, A^{**}) &= \begin{cases} 1 + r \min\left\{\frac{B_2^{R,u}(A)}{S(A;x^{**})}, 1\right\} & \text{if } A \geq A^{**} \\ 0 & \text{otherwise} \end{cases} \\ u(0, A; x^{**}, A^{**}) &= 1 \end{aligned}$$

Define $\Delta u(A; x^{**}, A^{**}) \equiv u(1, A; x^{**}, A^{**}) - u(0, A; x^{**}, A^{**})$.

It is immediate to see that for any pair (A^{**}, x^{**}), and regardless of the government's desired borrowing function $B_2^{R,u}$, the function $\Delta u(A; x^{**}, A^{**})$ satisfies a weak single crossing property in A.[4] Moreover, it is well-known that a family of normal density functions parameterized by x_j

$$\left\{ (p_x + p_A)^{1/2} \phi\left(\frac{A - \frac{p_x x_j + p_A A_{-1}}{p_x + p_A}}{(p_x + p_A)^{-1/2}} \right) \right\}_{x_j \in \mathbb{R}}$$

satisfies the strict monotone likelihood ratio (MLR) property, implying that the above density function is strictly log-supermodular in (A, x_j) (see Athey, 1996). By Theorem 3.2 in Athey (1996),

$$\Delta U(x_j; x^*, A^{**}) \equiv \int_{A^{**}}^{\infty} \Delta u(A; x^{**}, A^{**}) (p_x + p_A)^{1/2} \phi\left(\frac{A - \frac{p_x x_j + p_A A_{-1}}{p_x + p_A}}{(p_x + p_A)^{-1/2}} \right) dA$$

satisfies the strict single-crossing property in A^{**}. Thus, in response to monotone strategies by the government and the other lenders, lender j finds it optimal to follow a monotone strategy.

Consider $\Delta U(x^*; x^*, A^{**})$, the expected utility difference from supplying the funds to the market versus not supplying them, evaluated at x^* and let $L(A^{**}, x^*) \equiv \Delta U(x^*; x^*, A^{**})$. I want to show that for each A^{**} there exists unique x^* such that $L(A^{**}, x^*) = 0$. First note that $\Delta u(A; x^*, A^{**})$ as defined above is increasing in x^*. This is because $S(A; x^*) = b\left(1 - \Phi\left(\frac{x^* - A}{p_x^{-1/2}}\right)\right)$ is decreasing in x^*. Moreover, for all $A \geq A^{**}$ $B_2^{R,u}(A)$ is differentiable in A and therefore $\Delta u(A; x^*, A^{**})$ is piecewise continuous. Second, note that the product of $\Delta u(A; x^*, A^{**})$ and $(p_x + p_A)^{1/2} \phi\left(\frac{A - \frac{p_x x^* + p_A A_{-1}}{p_x + p_A}}{(p_x + p_A)^{-1/2}} \right)$ is different than 0, at least for all $A < A^{**}$. Then, by Theorem 3.4 in Athey (1996) it follows that $L(A^{**}, x^*)$ satisfies a strict single-crossing condition in x^*. This proves Lemma 2 in the text.

[3] In this section I make use of two results established in Athey (1996). The first of the results, Theorem 3.2 in Athey (1996), establishes that if g satisfies the weak single-crossing property, and if k is strictly log-supermodular and $k(s, \theta)$ has constant support in θ, then $G(\theta) \equiv \int_S g(s) k(s; \theta) ds$ satisfies the strict single-crossing property in θ. Theorem 3.4 in Athey (1996) extends this conclusion to the case where g also depends on θ under the additional assumption of piecewise continuity of g .

[4] A function $f(x)$, where $f\colon \mathbb{R} \to \mathbb{R}$, satisfies a weak single-crossing property in x if for all $x_H > x_L$, $f(x_L) > 0$ implies $f(x_H) \geq 0$.

A.1.3 The Government's Monotone Default Strategy

Suppose that the households follow investment strategies as characterized above and the lenders use monotone strategies with a common threshold x^*. I show that $\Delta V(A, \mathbf{k}_2^*, S)$ is strictly increasing in A.

Define $\mathbf{k}_2^*(A, A^{**}) \equiv \{k_2(A, \kappa, A^{**})\}_{\kappa \in [-1,1]}$, that is, $\mathbf{k}_2^*(A)$ denotes the households' investment choices when the average productivity is equal to A and when all households expect that the default threshold is A^{**}. Note that if the lenders follow monotone strategies, then $S = b\left[1 - \Phi\left(\frac{x^* - A}{\sigma_x}\right)\right]$. Thus, with a slight abuse of notation I will write $\Delta V(A, \mathbf{k}_2^*(A, A^{**}), S)$ as $\Delta V(A; \mathbf{k}_2^*(A, A^{**}), x^*)$. Finally, let $B_2^{R,u}$ denote the government optimal unconstrained borrowing.

Using the definition of $\Delta V(A, \mathbf{k}_2^*(A, A^{**}), x^*)$, substituting for $\mathbf{k}_2^*(A)$ the expression found in Section $A.1.1$ and rearranging, we get

$$\begin{aligned}\Delta V(A, \mathbf{k}_2^*(A, A^{**}), x^*) \quad &= \quad \int_{-1}^{1} \frac{1}{2} \log\left(\frac{1 - \Lambda(A + \kappa\varepsilon, \kappa, A^{**})}{Z - \Lambda(A + \kappa\varepsilon, \kappa, A^{**})}\right) d\kappa + \log\left(\frac{\tau Y_1^R - B_1 + B_2^{R^*}}{\tau Z Y_1^R + (1-\xi) B_2^{D^*}}\right) \\ &\quad + \log\left(\frac{1}{Z}\right) + \log\left(\frac{\tau Y_2^R - (1+r) B_2^{R^*}}{Z\tau Y_2^R}\right),\end{aligned}$$

where

$$B_2^{R^*} = \begin{cases} B_2^{R,u}(A) & \text{if } B_2^{R,u} \leq S(A, x^*) \\ S(A, x^*) & \text{if } B_2^{R,u} > S(A, x^*) \end{cases}.$$

Differentiating with respect to A, simplifying, and taking the limit as $\xi \to 1$, we get

$$\frac{\partial \Delta V(A; \mathbf{k}_2^*(A, A^{**}), x^*, A^*)}{\partial A} \geq \frac{B_1 - B_2^{R^*}}{\tau Y_1^R - B_1 + B_2^{R^*}} + \frac{(1+\alpha) B_2^{R^*}(1+r)}{\tau Y_2^R - (1+r) B_2^{R^*}}. \tag{1}$$

where I used the observation that if $B_2^{R^*} = B_2^{R,u}(A)$, then by the optimality of the government borrowing choices the terms containing $\partial B_2^{R^*}/\partial A$ add up to 0, while otherwise their sum is strictly positive.

Add the above fractions on the right-hand side of 1. The resulting numerator can be written as

$$2(1+r)\left(B_2^{R^*}\right)^2 - B_2^{R^*}\left(\tau Y_2^R + 2(1+r) B_1 - (1+r)\tau Y_1^R\right) + B_1 \tau Y_2^R.$$

This expression is quadratic in $B_2^{R^*}$. Let $B_2^{R^*,1}(A)$ and $B_2^{R^*,2}(A)$ be its two roots. Whether these roots are real or not depends on the parameters of the model. For all $A \in [\underline{A}, \overline{A}]$, define $\bar{b}(A) = \min\left\{B_2^{R^*,1}(A), B_2^{R^*,2}(A)\right\}$ if the roots are real, and $\bar{b}(A) = \infty$ if they are complex. Let $\bar{b} = \min_{A \in [\underline{A}, \overline{A}]} \bar{b}(A)$. It follows that if $b < \bar{b}$ then the government's best response to monotone strategies is itself monotone. I assume that the lenders' wealth b satisfies this constraint (Assumption 3 in the paper).[5]

[5] One may wonder how restrictive this assumption is. The answer is that it depends on the parameters. However, numerical simulations suggest that unless α or Z is very close to 1 both roots are complex, which means that the bound can be made arbitrarily large (though it has to be finite). In particular, this is the case for the calibration used in the paper.

A.1.4 Uniqueness of Equilibrium

In light of the above results, to establish uniqueness it is enough to show that

$$\Delta V\left(A^*, \mathbf{k}_2^*\left(A^*, A^*\right), x^*\left(A^*\right)\right)$$

is monotone in A^*, where $\mathbf{k}_2^*\left(A^*\right) \equiv \left\{k_2\left(A^*, \kappa, A^*\right)\right\}_{\kappa\in[-1,1]}$ is a vector whose components are the individual households' investment strategies when the households have the correct expectations about the default threshold (i.e., $A^{**} = A^*$), and x^* is the common signal threshold used by the lenders when households and lenders expect the default threshold to be A^*. I denote the optimal lender's threshold by $x^*\left(A^*\right)$, to emphasize that it depends on A^*.

Fix $\eta > 0$, where η is a small positive number. Differentiating $\Delta V\left(A^*; \mathbf{k}_2^*\left(A^*\right), x^*\left(A^*\right)\right)$ with respect to A^* and taking the limit as $\xi \to 1$ we get

$$\begin{aligned}\frac{d\Delta V}{dA^*} &= \int_{-1}^{1} \frac{-\frac{\partial\Lambda}{\partial A^*}\left[Z-\Lambda\right]+\left[1-\Lambda\right]Z\frac{\partial\Lambda}{\partial A^*}}{\left[1-\Lambda\right]\left[Z-\Lambda\right]}d\kappa \\ &\quad + \frac{\frac{dB_2^{R^*}}{dA^*}}{\tau Y_1^R - B_1 + B_2^{R^*}} - \frac{\left(1+r\right)\frac{\partial B_2^{R^*}}{\partial A^*}}{\tau Y_2^R - \left(1+r\right)B_2^{R^*}} \\ &\quad + \frac{B_1 - B_2^{R^*}}{\tau Y_1^R - B_1 + B_2^{R^*}} + \frac{\left(1+\Psi\right)\left(1+r\right)B_2^{R^*}}{\tau Y_2^R - \left(1+r\right)B_2^{R^*}},\end{aligned}$$

where

$$\Psi \equiv \frac{\int_{-1}^{1}\frac{1}{2}\frac{\partial}{\partial A^*}f\left(k_2\left(A^*+\kappa\varepsilon;\varepsilon,A^*\right)\right)d\kappa}{Y_2^R} \to \alpha \text{ as } \varepsilon \to 0.$$

Since

$$\lim_{\varepsilon\to 0}\frac{\partial \Pr\left(A^*|A^*+\kappa\varepsilon\right)}{\partial A^*} \to 0,$$

there exists $\overline{\varepsilon}$ such that for all $\varepsilon \in (0, \overline{\varepsilon}]$ we have

$$\int_{-1}^{1} \frac{-\frac{\partial\Lambda}{\partial A^*}\left[Z-\Lambda\right]+\left[1-\Lambda\right]Z\frac{\partial\Lambda}{\partial A^*}}{\left[1-\Lambda\right]\left[Z-\Lambda\right]}d\kappa < \frac{\eta}{2}$$

Next, since $\frac{\partial S(A^*)}{\partial A^*} > -b\frac{p_A}{p_x^{1/2}}\frac{1}{\sqrt{2\pi}} \to 0$ as $p_x \to \infty$, it follows that there exists a large enough $\overline{p}_x$ such that for all $p_x > \overline{p}_x$ we have

$$\frac{\frac{dB_2^{R^*}}{dA^*}}{\left[\tau Y_1^R - B_1 + B_2^{R^*}\right]} - \frac{\left(1+r\right)\frac{\partial B_2^{R^*}}{\partial A^*}}{\left[\tau Y_2^R - \left(1+r\right)B_2^{R^*}\right]} > -\frac{\eta}{2}.$$ [6]

Finally, following the same argument as in Section $A.1.3$ one can show that there exists $\overline{\overline{b}}\left(\varepsilon\right)$ such that for all $b < \overline{\overline{b}}\left(\varepsilon\right)$ we have

$$\frac{B_1 - B_2^{R^*}}{\tau Y_1^R - B_1 + B_2^{R^*}} + \frac{\left(1+r\right)B_2^{R^*}}{\left[\tau Y_2^R - \left(1+r\right)B_2^{R^*}\right]} > \eta.$$

Therefore, for all ε with $0 < \varepsilon < \overline{\varepsilon}$ and all $p_x > \overline{p}_x$ we have

$$\frac{d\Delta V}{dA^*} > -\frac{\eta}{2} - \frac{\eta}{2} + \eta = 0$$

implying that there exists a unique default threshold A^* that satisfies all the equilibrium conditions.

The above analysis applies to a fixed value of A^*. However, since $A^* \in \left[\underline{A}, \overline{A}\right]$, which is a compact interval, there exists bounds $\overline{\varepsilon}$ and $\overline{p}_x$ which are independent of A^*, such that if $\varepsilon < \overline{\varepsilon}$ and $p_x < \overline{p}_x$, then $d\Delta V/dA^*$ is strictly positive for all $A^* \in \left[\underline{A}, \overline{A}\right]$. This completes the proof.

[6] If $\partial B_2^{R^*}/\partial A^* = \partial B_2^{R,u}/\partial A^*$, then the sum of these terms is 0.

B Policy Analysis: Benchmark Case

This Section of the Appendix contains proofs of all the claims made in Section 3 of the paper.

B.1 Proof of Proposition 2

Let ψ denote a parameter of the model (for concreteness, one can think of the tax rate, in which case $\psi = \tau$). Then, for given r, the equilibrium conditions can be written as

$$I\left(A^* + \kappa\varepsilon, A^{**}, k_2^*\left(\kappa\right), \psi\right) = 0,$$

which is the equilibrium condition for a households with productivity $A^* + \kappa\varepsilon$ and which determines the capital choice for a household with productivity shock $\kappa\varepsilon$;

$$L\left(A^{**}, x^*, \psi\right) = 0,$$

which is the equilibrium condition that describing the lenders' behavior and which determines x^*; and finally,

$$\Delta V\left(A^*, \left\{k_2^*\left(\kappa\right)\right\}_{\kappa\in[-1,1]}, x^*, \psi\right) = 0$$

which is the equilibrium condition that describes the government's default decision and determines A^*.[7]

Note that, for each $\kappa \in [-1, 1]$, the equation $I\left(A^* + \kappa\varepsilon, A^*, k_2^*\left(\kappa\right), \psi\right) = 0$ specifies $k_2^*\left(\kappa\right)$ as a function of household's productivity $A^* + \kappa\varepsilon$, household's belief about the default threshold A^{**}, and the policy parameter ψ. for each $\kappa \in [-1, 1]$. Similarly, the equation $L\left(A^*, x^*, \psi\right) = 0$ determines x^* as a function of the lenders' belief about the default threshold A^{**} and ψ. Without loss of generality, I assume that the households hold the same belief as the lenders in regard to the default threshold. In equilibrium, $A^{**} = A^*$, that is the households and lenders hold correct beliefs about the government's default decision. However, to derive the effect of a change in the households' and lenders' beliefs on the default threshold, we have to differentiate between the belief about the threshold held by the households and lenders and the actual default threshold, where the latter is defined as the level of productivity at which the government defaults.

(Derivations of the multiplier and the direct effect) To compute the equilibrium change in A^* due to a change in ψ, I compute the total derivatives of the expressions on the both sides of equilibrium conditions and solve the resulting linear system of equations for $dA^*/d\psi$:

$$I_1\left(\kappa\right)\frac{dA^*}{d\psi} + I_2\left(\kappa\right)\frac{dA^{**}}{d\psi} + I_3\left(\kappa\right)\frac{dk_2^*\left(\kappa\right)}{d\psi} + I_4\left(\kappa\right) = 0 \tag{2}$$

$$L_1\frac{dA^{**}}{d\psi} + L_2\frac{dx^*}{d\psi} + L_3 = 0 \tag{3}$$

$$\Delta V_1\frac{dA^*}{d\psi} + \int_{-1}^{1}\frac{1}{2}\Delta V_2\left(\kappa\right)\frac{dk_2^*\left(\kappa\right)}{d\psi}d\kappa + \Delta V_3\frac{dx^*}{d\psi} + \Delta V_4 = 0 \tag{4}$$

where I_n is the partial derivative of $I\left(A^* + \kappa\varepsilon, A^{**}, k_2^*\left(\kappa\right), \psi\right)$ with respect to its nth argument and similarly for L_n and ΔV_n. $dA^{**}/d\psi$ is the total change in agents' beliefs regarding the government

[7]Note that this condition implicitly assumes that the government's borrowing and spending decisions are optimal. In other words, $\Delta V = 0$ determines the productivity default threshold, given that the government behaves optimally in the case when it repays its debt as well as in the case when it chooses to default.

default threshold implied by a change in ψ. In equilibrium, $dA^{**}/d\psi = dA^{*}/d\psi$, but for now it is important to keep the distinction between the two objects.

Solving for $dx^{*}/d\psi$ and $dk_2^{*}/d\psi$ using Equations (3) and (2) we get

$$\begin{aligned}
\frac{dx^{*}}{d\psi} &= -\frac{L_1}{L_2}\frac{dA^{**}}{d\psi} - \frac{L_3}{L_2} \\
\frac{dk_2^{*}(\kappa)}{d\psi} &= -\frac{I_1(\kappa)}{I_3(\kappa)}\frac{dA^{*}}{d\psi} - \frac{I_2(\kappa)}{I_3(\kappa)}\frac{dA^{**}}{d\psi} - \frac{I_4(\kappa)}{I_3(\kappa)}
\end{aligned}$$

or, recognizing that $\partial x^{*}/\partial A^{**} = -L_1/L_2$, $\partial k_2^{*}(\kappa)/\partial A^{*} = -I_1(\kappa)/I_3(\kappa)$, $\partial k_2^{*}(\kappa)/\partial A^{**} = -I_2(\kappa)/I_3(\kappa)$, and $\partial k_2^{*}(\kappa)/\partial\psi = -I_4(\kappa)/I_3(\kappa)$:

$$\begin{aligned}
\frac{dx^{*}}{d\psi} &= \frac{\partial x^{*}}{\partial A^{**}}\frac{dA^{**}}{d\psi} + \frac{\partial x^{*}}{\partial\psi} \\
\frac{dk_2^{*}(\kappa)}{d\psi} &= \frac{\partial k_2^{*}(\kappa)}{\partial A^{*}}\frac{dA^{*}}{d\psi} + \frac{\partial k_2^{*}(\kappa)}{\partial A^{**}}\frac{dA^{**}}{d\psi} + \frac{\partial k_2^{*}(\kappa)}{\partial\psi}
\end{aligned}$$

Substituting the above expressions into Equation (4) and rearranging, we get

$$\begin{aligned}
\left[\Delta V_1 + \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^{*}(\kappa)}{\partial A^{*}}d\kappa\right]\frac{dA^{*}}{d\psi} &= \qquad (5)\\
-\int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\left[\frac{\partial k_2^{*}(\kappa)}{\partial A^{**}}\frac{dA^{**}}{d\psi} + \frac{\partial k_2^{*}(\kappa)}{\partial\psi}\right]d\kappa &- \Delta V_3\left[\frac{\partial x^{*}}{\partial A^{**}}\frac{dA^{**}}{d\psi} + \frac{\partial x^{*}}{\partial\psi}\right] - \Delta V_4,
\end{aligned}$$

where $\left[\Delta V_1 + \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^{*}(\kappa)}{\partial A^{*}}d\kappa\right]$ captures the effect of an increase in the productivity on the government's incentives to default.

At this point it is key to differentiate between a change in the households' investments due to a change in the households' strategies and a change in the households' investments due to merely a change in productivity holding households' strategies fixed. Recall that an individual household's investment strategy is a function that maps the individual productivity into an investment choice, that is it is a map $k_2^{*}: A_i \to \mathbb{R}$. Thus, a change in the household's strategy is defined as a shift in this mapping, that is a change in k_2^{*} for each A_i. On the other hand, holding household strategies constant, a change in A_i also affects household i's investments: It is simply a movement along the curve $k_2: A_i \to \mathbb{R}$. Thus, the term $\Delta V_1 + \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^{*}(\kappa)}{\partial A^{*}}d\kappa$ captures the effect of a change in the productivity on the government's incentives to default holding households' and lenders' strategies constant.

Using the above observation, divide Equation (5) by $\Delta V_1 + \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^{*}(\kappa)}{\partial A^{*}}d\kappa$ to obtain

$$\begin{aligned}
\frac{dA^{*}}{d\psi} = &\frac{-\int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^{*}(\kappa)}{\partial\psi}d\kappa}{\Delta V_1 + \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^{*}(\kappa)}{\partial A^{*}}d\kappa} - \frac{\Delta V_3\frac{\partial x^{*}}{\partial\psi}}{\Delta V_1 + \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^{*}(\kappa)}{\partial A^{*}}} - \frac{\Delta V_4}{\Delta V_1 + \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^{*}(\kappa)}{\partial A^{*}}} \\
&-\frac{\int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^{*}(\kappa)}{\partial A^{**}}d\kappa}{\Delta V_1 - \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^{*}(\kappa)}{\partial A^{*}}}\frac{dA^{**}}{d\psi} - \frac{\Delta V_3\frac{\partial x^{*}}{\partial A^{**}}}{\Delta V_1 - \int_{-1}^{1}\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^{*}(\kappa)}{\partial A^{*}}}\frac{dA^{**}}{d\psi}
\end{aligned}$$

The first three terms capture the direct effects of a change in ψ on the equilibrium strategies of the households', the lenders' and the government, respectively, holding households' and lenders' beliefs about the default threshold constant (i.e., holding A^{**} constant). The two remaining terms capture the effect of a change in ψ has on the the households' and lenders' beliefs. In particular, note that

$$\frac{\partial A^*}{\partial \psi} = -\frac{\Delta V_4}{\Delta V_1 + \int\limits_{-1}^{1} \Delta V_2 \frac{\partial k_2^*(\kappa)}{\partial A^*} d\kappa},$$

that is, the third term captures the partial effect of a change in ψ on the government's default incentives holding households' and lenders' strategies and beliefs constant. Similarly,

$$\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} = \frac{\Delta V_3}{\Delta V_1 + \int\limits_{-1}^{1} \Delta V_2 \frac{\partial k_2^*(\kappa)}{\partial A^*}}\frac{\partial x^*}{\partial A^{**}}$$

and, slightly abusing notation,

$$\int\limits_{-1}^{1} \frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial A^{**}} d\kappa = -\int\limits_{-1}^{1} \frac{\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^{**}} d\kappa}{\Delta V_1 + \int_{-1}^{1} \frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^*}},$$

where this term captures the effect of a change in the households' beliefs on the government's incentives to default. In a similar fashion,

$$\int\limits_{-1}^{1} \frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial \psi} d\kappa = -\int\limits_{-1}^{1} \frac{\frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial \psi} d\kappa}{\Delta V_1 + \int_{-1}^{1} \frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^*} d\kappa},$$

where $\int_{-1}^{1} \frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial \psi} d\kappa$ captures the effect of a change in the households' strategies caused by a change in ψ holding the households' beliefs about the default threshold, A^{**}, constant.

Using the above notation, we obtain

$$\frac{dA^*}{d\psi} = \int\limits_{-1}^{1} \frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial \psi} d\kappa + \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi} + \frac{\partial A^*}{\partial \psi} + \int\limits_{-1}^{1} \frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial A^{**}} d\kappa \frac{\partial A^{**}}{\partial \psi} + \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}\frac{\partial A^{**}}{\partial \psi}$$

In equilibrium, $A^{**} = A^*$, and so it has to be the case that $\partial A^{**}/\partial\psi = dA^*/d\psi$. Thus, after rearranging,

$$\frac{dA^*}{d\psi} = \frac{\frac{\partial A^*}{\partial \psi} + \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi} + \int\limits_{-1}^{1} \frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial \psi} d\kappa}{1 - \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^*} - \int\limits_{-1}^{1} \frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial A^{**}} d\kappa} \tag{6}$$

Finally, note that $\int_{-1}^{1} \frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial \psi} d\kappa$ corresponds simply to $\int_0^1 \frac{\partial A^*}{\partial k_2^{i,*}}\frac{\partial k_2^{i,*}}{\partial \psi} di$, while the term $\int_{-1}^{1} \frac{1}{2}\frac{\partial A^*}{\partial k_2^*(\kappa)}\frac{\partial k_2^*(\kappa)}{\partial A^{**}} d\kappa$ corresponds to $\int_0^1 \frac{1}{2}\frac{\partial A^*}{\partial k_2^{i,*}}\frac{\partial k_2^{i,*}}{\partial A^{**}} di$. Thus, we obtain

$$\frac{dA^*}{d\psi} = \frac{\frac{\partial A^*}{\partial \psi} + \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi} + \int_0^1 \frac{\partial A^*}{\partial k_2^{i,*}}\frac{\partial k_2^{i,*}}{\partial \psi} di}{1 - \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^*} - \int_0^1 \frac{\partial A^*}{\partial k_2^{i,*}}\frac{\partial k_2^{i,*}}{\partial A^{**}} di},$$

which corresponds to Equation (2) in the paper.

(Establishing that M>1) Recall from the proof of uniqueness that the government default condition, after taking into account the dual role of A^* as the average value of productivity in the economy and the default threshold, is strictly increasing in A^*. Thus,

$$\Delta V_1 + \int_{-1}^{1} \frac{1}{2} \Delta V_2 (\kappa) \frac{\partial k_2^* (\kappa)}{\partial A^*} d\kappa + \int_{-1}^{1} \frac{1}{2} \Delta V_2 (\kappa) \left. \frac{\partial k_2^* (\kappa)}{\partial A^{**}} \right|_{A^{**}=A^*} d\kappa + \Delta V_3 \left. \frac{\partial x^*}{\partial A^{**}} \right|_{A^{**}=A^*} > 0,$$

where the third and fourth terms capture the effect of a change in the households' and lenders' beliefs, respectively. Dividing the above expression by $\Delta V_1 + \int_{-1}^{1} \frac{1}{2} \Delta V_2 (\kappa) \frac{\partial k_2^*(\kappa)}{\partial A^*} d\kappa$ establishes the non-negativity of the multiplier effect.

Under Assumption 4 we have $\frac{B_2^{R,u}(A)}{S(A;x^{**})} = 1$ for all A, and hence it can be shown that $x^* = \frac{p_x + p_A}{p_x} A^{**} - \frac{p_A}{p_x} A_{-1} + \frac{\sqrt{p_x + p_A}}{p_x} \Phi^{-1} \left(\frac{1}{1+r} \right)$, implying that $\frac{\partial A^*}{\partial x^*} \frac{\partial x^*}{\partial A^{**}} > 0$. Similarly, it is straightforward to show that $\partial k_2^* / \partial A^{**} < 0$. Since a higher investment by all households decreases the government's incentives to default ($\int_{-1}^{1} \frac{1}{2} \partial A^* / \partial k_2^* (\kappa) d\kappa < 0$), we have $\int_{-1}^{1} \frac{1}{2} \frac{\partial A^*}{\partial k_2^*(\kappa)} \frac{\partial k_2^*(\kappa)}{\partial A^{**}} d\kappa > 0$. It follows that the denominator of the multiplier effect is less than 1, so that the multiplier effect is greater than 1.

B.2 Policies

The default threshold is determined by the following condition:

$$\begin{aligned} 0 \quad = \quad \Delta V (A^*, \mathbf{k}_2^*, x^*) = & \int_{-1}^{1} \log (c_1^R) d\kappa + \log \left(\tau Y_1^R - B_1 + B_2^{R^*} \right) \\ & + \int_{-1}^{1} \log (c_2^R) d\kappa + \log \left(\tau Y_2^R - (1+r) B_2^{R^*} \right) \\ & - \int_{-1}^{1} \log (c_1^D) d\kappa - \log (\tau Z Y_1^R) \\ & - \int_{-1}^{1} \log (c_2^D) d\kappa - \log (\tau Z Y_2^R), \end{aligned} \tag{7}$$

where c_t^R and c_t^D are the consumption in period t in repayment and default, respectively, Y_t^R is the total output of the economy in period t, and $B_2^{R^*}$ is the equilibrium borrowing by the government, all evaluated at the threshold productivity A^*. Before proceeding further, note that $\int \log \left(\frac{c_t^R}{c_t^D} \right) d\kappa$ is independent of τ and k_1 for $t = 1, 2$, and thus policy change will affect the government's incentive to default only through its effect on government spending in repayment and in default.[8] Equation (7) plays a key role in establishing Propositions 3, 4 and 5.[9]

B.2.1 Proof of Proposition 3 and Corollary 1

Differentiate $\Delta V (A^*, \mathbf{k}_2^*, x^*)$ with respect to τ to obtain

$$u_{g_1}^R Y_1^R + u_{g_2}^R Y_2^R + u_{g_2}^R \tau \frac{\partial Y_2^R}{\partial \tau} - u_{g_1}^D Z Y_1^R + u_{g_2}^D Y_2^R + u_{g_2}^D Z \tau \frac{\partial Y_2^R}{\partial \tau},$$

where $u_{g_t}^R$ and $u_{g_t}^D$ are the marginal utility from government spending in period t in repayment and default, respectively, and Y_1^R the total output of the economy in period t in repayment, all evaluated

[8] This is because $c_2^D = Z c_2^R$, $c_1^D = Z (1-\tau) e^{A_i} f(k_1) - k_2$, $c_1^R = (1-\tau) e^{A_i} f(k_1) - k_2$, and k_2 is linear in $f(k_1)$ and τ.

[9] Equations (3) and (4) can be computed directly from Equation 7.

at A^*.[10] Given households' investment choices, $\partial Y_2^R/\partial\tau = \frac{\alpha}{1-\tau}$. Thus, rearranging the terms in the above expression, we obtain

$$\underbrace{Y_1^R(1-Z)\,u_{g_1}^D + Y_2^R(1-Z)\,u_{g_1}^D}_{\text{Differential increase in tax revenues}} + \underbrace{Y_1^R\left[u_{g_1}^R - u_{g_1}^D\right] + Y_2^R\left[u_{g_2}^R - u_{g_2}^D\right]}_{\text{Concavity effect}} - \underbrace{\frac{\partial Y_2^R}{\partial\tau}\tau\left[u_{g_2}^R - Zu_{g_2}^D\right]}_{\text{Investment distortion}}$$

which corresponds to the expression (3) in the paper.

By noting that $u_{g_t}^D = 1/\left(Z\tau Y_t^R\right)$, $u_{g_1}^R = 1/(\tau Y_1 - B_1 + B_2)$ and $u_{g_2}^R = 1/(\tau Y_2 - (1+r)B_2)$ one can write the above condition as

$$\frac{(B_1 - B_2)}{\tau Y_1 - B_1 + B_2} + \frac{(1+r)B_2}{\tau Y_2 - (1+r)B_2} - \frac{\alpha\tau}{1-\tau}\frac{(1+r)B_2}{\tau Y_2 - (1+r)B_2} \tag{8}$$

The first part of Proposition 3 follows from the observation that, according to the proof of equilibrium uniqueness, $\frac{(B_1-B_2)}{\tau Y_1 - B_1 + B_2} + \frac{(1+r)B_2}{\tau Y_2 - (1+r)B_2}$ is bounded away from 0 while $\frac{\alpha\tau}{1-\tau} \to 0$ as $\tau \to 0$. The second part of the Proposition 3 follows from the observation that $\lim_{\tau_x\to\infty} S = b\left[1 - \Phi\left(\frac{x^* - A^*}{\sigma_x}\right)\right] = b\frac{r}{1+r}$. Thus, if $B_1 > rb$ then the first term is Equation 8 is positive. It follows that as long as $1 > \frac{\alpha\tau}{1-\tau}$ then an increase in τ will decrease the probability of default. Rearranging this inequality we arrive at the inequality stated in the text.

The proof of Corollary follows from the observation that $\lim_{A_{-1}\to-\infty} S = 0$ in which case 8 becomes $\frac{B_1}{\tau Y_1 - B_1} > 0$.

B.2.2 Proof of Proposition 4

The proof of Proposition 4 is similar to the proof of Proposition 3. I consider only a stimulus financed with short-term debt. The case of a stimulus financed with long-term debt is analogous.

Note first that when the government engages in a fiscal stimulus financed with short-term debt that matures at the end of period 1, government spending in repayment in period 1 becomes $\tau Y_1^R - B_1 + B_2^R - \left(1 + r^{ST}\right) sk_1$, where sk_1 is the size of stimulus. The positive effect of such a stimulus is that it leads to expansion of output. Differentiating both sides of the government indifference condition with respect to s, and rearranging, we get

$$\underbrace{\frac{\partial Y_1}{\partial s}\tau(1-Z)\,u_{g_1}^D + \frac{\partial Y_2}{\partial s}\tau(1-Z)\,u_{g_2}^D}_{\text{Differential increase in tax revenues}} + \underbrace{\tau\frac{\partial Y_1}{\partial s}\left[u_{g_1}^R - u_{g_1}^D\right] + \tau\frac{\partial Y_2}{\partial s}\left[u_{g_2}^R - u_{g_2}^D\right]}_{\text{Concavity effect}} - \underbrace{\left(1 + r^{ST}\right)k_1 u_{g_1}^R}_{\text{Increase in debt}}$$

When the government engages in a stimulus, $k_2^i = (1-\tau)\,e^{A_i} f\left(k_1(1+s)\right)\Lambda\left(A_i;\varepsilon, A^*\right)$, and thus

$$\frac{\partial Y_1}{\partial s} = \frac{\alpha}{1+s}Y_1^R \quad\text{and}\quad \frac{\partial Y_2}{\partial s} = \frac{\alpha^2}{1+s}Y_2^R$$

Following similar steps as in the proof of Proposition 3 one simplify the above equation for the effect of stimulus to

$$\frac{\alpha(B_1 - B_2)}{\tau Y_1^R - B_1 + B_2} + \frac{\alpha^2 B_2}{\tau Y_2^R - (1+r)B_2} - \frac{\left(1 + r^{ST}\right)k_1}{\tau Y_1^R - B_1 + B_2} \tag{9}$$

[10] There is no effect of a change of τ on B_2^R, the equilibrium level of borrowing, since under Assumption 4, $B_2^R = S(A, x^*)$ and $\partial S(A, x^*)/\partial\tau = 0$. If Assumption 4 were relaxed there would be an additional term capturing the potential impact of a change in taxes on government borrowing in equilibrium (via the competition effect among lenders).

To establish the proposition note first that $u_{g_1}^R \geq u_{g_2}^R$ (with equality only if the government can borrow the unconstrained optimal amount) and, thus

$$\alpha (B_1 - B_2) u_{g_1}^R + \alpha^2 (1+r) B_2 u_{g_2}^R - \left(1 + r^{ST}\right) k_1 u_{g_1}^R < \left[\alpha B_1 - \left(1 + r^{ST}\right) k_1\right] u_{g_1}^R$$

where the right-most expression is negative as long as $B_1/k_1 < \left(1 + r^{ST}\right)(1/\alpha)$. Moreover,

$$\alpha (B_1 - B_2) u_{g_1}^R + \alpha^2 (1+r) B_2 u_{g_2}^R - \left(1 + r^{ST}\right) k_1 u_{g_1}^R > u_{g_1}^R \left[\alpha (B_1 - B_2) - \left(1 + r^{ST}\right) k_1\right]$$

where the last term is positive for sufficiently high B_1 (as $B_2 < b \in \mathbb{R}$). Thus, for sufficiently high B_1 stimulus increases government's incentives to repay its debt. Finally, by continuity of expression in 9 we know that there exists B_1 such that this expression is equal to 0 and hence $\partial A^*/\partial s = 0$. It is easy to see that at such B_1 the derivative of expression in 9 is positive which implies that there exists unique $\overline{B}_1$ such that if $B_1 < \overline{B}_1$ then stimulus decreases government's incentives to repay its debt while the opposite is true when $B_1 > \overline{B}_1$. Finally note that since $\alpha < 1$ and $\tau Y_1^R - B_1 + B_2 < \tau Y_2^R - (1+r) B_2$ it follows that expression in 9 is necessarily negative if $\alpha B_1 < \left(1 + r^{ST}\right) k_1$. This establishes the proposition.

When the stimulus is financed with long-term debt then the last term of the expression in 9 becomes $\left(1 + r^{LT}\right) k_1 / \left(\tau Y_2^R - (1+r) B_2\right)$. It follows that in this case expression 9 is necessarily negative when $\alpha B_1 u_{g_1}^R < \left(1 + r^{ST}\right) k_1 u_{g_2}^R$

C Additional Results

C.1 Predictions when debt crisis are driven by fundamentals

One may wonder how the predictions of the model with dispersed information and endogenous expectations differ from predictions of the model were default is driven purely by fundamentals. To answer this question, I consider the model of Section 1, but allow agents to observe A and coordinate their beliefs on repayment equilibrium whenever A belongs to the "fragility region." In this case, the government defaults only when fundamentals are poor enough, which happens when $A < \underline{A}$ (i.e., below the lower bound of the fragility region). I refer to this version of the model as "the model with fundamental crises only." The question is then whether the government policies considered above have the same effect on the threshold $\underline{A}$ as they have on the threshold A^*.

There are two forces that lead to potentially different predictions based on the model with self-fulfilling crises and dispersed information compared to the model with fundamental crises only. First, since $A^* > \underline{A}$ it follows that the government revenues are higher at the default threshold in the model with self-fulfilling beliefs and dispersed information. This tends to decrease the benefit of policies that expand government income, such as stimulus or increase in taxes. Second, under dispersed information, the government is unable to roll over its maturing debt as those lenders who receive low signals decide not to supply their funds to the government. As the consequence, in the model with dispersed information if the government repays its debt then its expenditure is substantially lower in period 1 than in period 2. This in turn implies that policies which result in a larger increase in government's revenues in period 1 than in period 2 (such as fiscal stimulus) or policies whose negative effect fall in period 2 (such as an increase in taxes) tend to decrease the government's incentives to default by more under dispersed information. The next proposition states the conditions under which the latter effect dominates and hence government policies tend to be more effective under dispersed information.

Proposition B *Let $\psi \in \{\tau, s\}$ and keep other parameters of the model fixed. Then there exists $\bar{A}_{-1}$ such that for all $A_{-1} < \bar{A}_{-1}$ we have*

$$d\underline{A}/d\psi < 0 \implies dA^*/d\psi < 0$$

but not vice versa.

The above proposition follows from the observation that when the past level of productivity is low, that is low A_{-1}, then the supply of funds in the bond market is low, holding everything else constant. Thus, when A_{-1} is sufficiently low and if austerity or stimulus decreases probability of default according to the model with only fundamental crises then it also does so according to the model with dispersed information and self-fulfilling crises, but not vice versa. Indeed, if $B_1/k_1 > 1/\alpha$ then the two models will provide opposite predictions as according to the model with self-fulfilling crises and dispersed beliefs stimulus will decrease probability of default while according to the model with only fundamental crises stimulus will increase probability of default. A similar observation applies to an increase in taxes when τ is already high. Thus, there are situations when predictions of the two models will substantially differ not only quantitatively but also qualitatively.

Proposition B follows the following two results. The first of the two results provide a general conditions under which we have the two models provide different predictions. the second result derives the sufficient conditions under which we have $\frac{\partial \underline{A}}{\partial \tau} < 0 \Longrightarrow \frac{\partial A^*}{\partial \tau} < 0$ or $\frac{\partial \underline{A}}{\partial s} < 0 \Longrightarrow \frac{\partial A^*}{\partial s}$.

Lemma C.1 *Let A^* and $\underline{A}$ be the default thresholds in the model with self-fulfilling crises and dispersed information and in the model with only fundamental crises, respectively.*

1. *Consider an increase in taxes. For each A^* there exists $\overline{B}^{\tau} > 0$ such that*

   (a) *If $B_2^*(A^*) < \overline{B}^{\tau}$ then $\frac{\partial \underline{A}}{\partial \tau} < 0 \Longrightarrow \frac{\partial A^*}{\partial \tau} < 0$.*

   (b) *If $B_2^*(A^*) = \overline{B}^{\tau}$ then $\frac{\partial \underline{A}}{\partial \tau} < 0 \Longleftrightarrow \frac{\partial A^*}{\partial \tau} < 0$.*

   (c) *If $B_2^*(A^*) > \overline{B}^{\tau}$ then $\frac{\partial \underline{A}}{\partial \tau} < 0 \Longleftarrow \frac{\partial A^*}{\partial \tau} > 0$.*

2. *Consider a fiscal stimulus (financed either by short-term or long-term debt). For each A^* there exists $\overline{B}^{s} > 0$ such that*

   (a) *If $B_2^*(A^*) < \overline{B}^{s}$ then $\frac{\partial \underline{A}}{\partial s} < 0 \Longrightarrow \frac{\partial A^*}{\partial s} < 0$.*

   (b) *If $B_2^*(A^*) = \overline{B}^{s}$ then $\frac{\partial \underline{A}}{\partial s} < 0 \Longleftrightarrow \frac{\partial \underline{A}}{\partial s} < 0$.*

   (c) *If $B_2^*(A^*) > \overline{B}^{s}$ then $\frac{\partial \underline{A}}{\partial s} < 0 \Longleftarrow \frac{\partial A^*}{\partial s} > 0$.*

Proof. I only prove the first part of the proposition since the proof of part 2 is analogous. First consider the effect of a higher tax rate when crises occur for all $A < \underline{A}$ only (i.e., fundamentals driven crises). Then, $\partial \underline{A}/\partial \tau < 0$ if and only if

$$\frac{\left(B_1 - B_2^{R,u}\right)}{\tau Y_1 - B_1 + B_2^{R,u}} + \frac{(1+r)\, B_2^{R,u}}{\tau Y_2 - (1+r)\, B_2^{R,u}} - \frac{\alpha\tau}{1-\tau} \frac{(1+r)\, B_2^{R,u}}{\tau Y_2 - (1+r)\, B_2^{R,u}}$$

where $B_2^{R,u}$ is the unconstrained optimal borrowing by the government, which satisfies

$$\frac{1}{\tau Y_1 - B_1 + B_2^{R,u}} = \frac{(1+r)}{\tau Y_2 - (1+r) B_2^{R,u}}$$

Therefore, the condition for $\partial \underline{A}/\partial \tau < 0$ can be simplified to

$$B_1 - \frac{\alpha\tau}{1-\tau} B_2^{R,u}(\underline{A}) > 0 \tag{10}$$

Next, recall from the Proposition 3 that $\partial A^*/\partial \tau < 0$ if and only if

$$B_1 - B_2^*(A^*) + \frac{(1+r) u_2^g(A^*)}{u_1^g(A^*)} B_2^*(A^*) \left[1 - \frac{\alpha\tau}{1-\tau}\right] > 0 \tag{11}$$

where $B_2^*(A^*)$ is the equilibrium government borrowing at $A = A^*$ and $u_t^g(A^*)$ is the marginal utility of government spending at time $t = \{1, 2\}$. To establish the first part of the proposition I need to show that there exists $\overline{B}^\tau > 0$ such that

$$\frac{\alpha\tau}{1-\tau} B_2^{R,u}(\underline{A}) > B_2 - \frac{(1+r) u_2^g(A^*)}{u_1^g(A^*)} B_2 \left[1 - \frac{\alpha\tau}{1-\tau}\right] \tag{12}$$

if and only if $B_2 < \overline{B}^\tau$.

Towards this goal note that if $B_2 = 0$ then the inequality in Equation (12) is satisfied. Next, recall that government's desired borrowing is increasing in A, and thus $B_2^{R,u}(\underline{A}) < B_2^{R,u}(A^*)$. Moreover, if the government can borrow desired amount then $(1+r) u_2^g(A^*) = u_1^g(A^*)$. Hence, at $B_2 = B_2^{R,u}(A^*)$ then the inequality in Equation (12) is reversed. By continuity of the RHS of Equation (12) it follows that there exists $\overline{B}^\tau > 0$ such that

$$\frac{\alpha\tau}{1-\tau} B_2^{R,u}(\underline{A}) = \overline{B}^\tau - \frac{(1+r) u_2^g(A^*)}{u_1^g(A^*)} \overline{B}^\tau \left[1 - \frac{\alpha\tau}{1-\tau}\right]$$

I now argue that such $\overline{B}^\tau$ is unique.

First, note that

$$\begin{aligned} &\frac{\partial}{\partial B_2}\left(B_2(A^*) - \frac{(1+r) u_2^g}{u_1^g} B_2(A^*) \left[1 - \frac{\alpha\tau}{1-\tau}\right]\right) \\ = \; &\underbrace{1 - \frac{(1+r) u_2^g}{u_1^g}\left[1 - \frac{\alpha\tau}{1-\tau}\right]}_{>0} - (1+r)(u_2^g)\left[\frac{(1+r) u_2^g}{u_1^g} + 1\right] B_2(A^*) \left[1 - \frac{\alpha\tau}{1-\tau}\right] \end{aligned}$$

where we used the observation that

$$\frac{\partial u_1^g}{\partial B_2} = \frac{-1}{\left[\tau e^A f(k_1) - B_1 + B_2\right]^2} = (u_1^g)^2 \quad \text{and} \quad \frac{\partial u_2^g}{\partial B_2} = \frac{(1+r)}{\left[\tau e^A f(k_2) - (1+r) B_2\right]^2} = (1+r)(u_2^g)^2$$

If $1 - \alpha\tau/(1-\tau) \leq 0$ the above derivative is positive and the claim follows. Thus, in what follows I suppose that $1 - \alpha\tau/(1-\tau) > 0$.

Note that

$$\frac{\partial}{\partial B_2} \frac{(1+r) u_2^g}{u_1^g} > 0,$$

and thus

$$\begin{aligned}
&\frac{\partial}{\partial\left(B_2\right)^2}\left(B_2-\frac{(1+r)\,u_2^g}{u_1^g}B_2\left[1-\frac{\alpha\tau}{1-\tau}\right]\right)\\
=&\quad 1-\frac{(1+r)\,u_2^g}{u_1^g}\left[1-\frac{\alpha\tau}{1-\tau}\right]-(1+r)\,u_2^g\left[\frac{(1+r)\,u_2^g}{u_1^g}+1\right]B_2\left[1-\frac{\alpha\tau}{1-\tau}\right]\\
=&\quad -\left[1-\frac{\alpha\tau}{1-\tau}\right]\frac{\partial}{\partial B_2}\left(\frac{(1+r)\,u_2^g}{u_1^g}\right)-(1+r)\,u_2^g\left[\frac{(1+r)\,u_2^g}{u_1^g}+1\right]\left[1-\frac{\alpha\tau}{1-\tau}\right]\\
&\quad -(1+r)\,u_2^g B_2\left[1-\frac{\alpha\tau}{1-\tau}\right]\frac{\partial}{\partial B_2}\left(\frac{(1+r)\,u_2^g}{u_1^g}\right)\\
<&\quad 0
\end{aligned}$$

it follows that

$$B_2-\frac{(1+r)\,u_2^g}{u_1^g}B_2\left[1-\frac{\alpha\tau}{1-\tau}\right]$$

is concave in B_2. Together with observations that at $B_2=0$ we have

$$\frac{\partial}{\partial B_2}\left(B_2-\frac{(1+r)\,u_2^g}{u_1^g}B_2\left[1-\frac{\alpha\tau}{1-\tau}\right]\right)>0$$

and at $B_2=B_2^{R,u}\left(A^*\right)$ we have

$$B_2^{R,u}\left(A^*\right)-\frac{(1+r)\,u_2^g}{u_1^g}B_2^{R,u}\left(A^*\right)\left[1-\frac{\alpha\tau}{1-\tau}\right]=\frac{\alpha\tau}{1-\tau}B_2^{R,u}\left(A^*\right)\geq B_2^*\left(\underline{A}\right)\frac{\alpha\tau}{1-\tau}$$

the concavity of $B_2\left(A^*\right)-\frac{(1+r)u_2^g}{u_1^g}B_2\left(A^*\right)\left[1-\frac{\alpha\tau}{1-\tau}\right]$ implies that there exists a unique value of $\overline{B}^\tau$ such that

$$\overline{B}^\tau-\frac{(1+r)\,u_2^g}{u_1^g}\overline{B}^\tau\left[1-\frac{\alpha\tau}{1-\tau}\right]=B_2^*\left(\underline{A}\right)\frac{\alpha\tau}{1-\tau}$$

This establishes the result for the case when $1-\alpha\tau/\left(1-\tau\right)>0$. ■

C.2 Higher tax rate in repayment only

Consider now the case when the government implements an increase in taxes only in the case it repays the debt. Let τ^R denote the tax rate in repayment and τ^D the tax rate in default where initially $\tau^R=\tau^D=\tau$. An increase in the tax rate only in repayment is captured by an increase in τ^R holding τ^D constant.

An increase in τ^R can be analyzed the same way as an increase in τ considered above. A higher τ^R leads to a change in the government's incentives to repay the debt equal to

$$\underbrace{Y_1^R u_{g_1}^R+Y_2^R u_{g_2}^R}_{\substack{\text{Increase in the government's}\\ \text{revenue in repayment}}}\quad\underbrace{-\int_{i=0}^{1}\left(u_{c_1}^R\frac{\partial c_1^R}{\partial\tau^R}+u_{c_2}^R\frac{\partial c_2^R}{\partial\tau^R}-u_{c_1}^D\frac{\partial c_1^D}{\partial\tau^R}-u_{c_2}^D\frac{\partial c_2^D}{\partial\tau^R}\right)di}_{\substack{\text{Differential decrease}\\ \text{in private consumption}}}-\underbrace{\tau^R\frac{\partial Y_2^R}{\partial\tau^R}\left(u_{g_2}^R-Zu_{g_2}^D\right)}_{\text{Investment distortion}}\tag{13}$$

where, $u_{g_t}^R$ and $u_{g_t}^D$ are the marginal utilities from the government spending in period t in repayment and default, respectively, $u_{c_t}^R$ and c_t^R are household i's marginal utility from the private consumption and private consumption at time t in repayment, and Y_t^R is the total output of the economy in period t in repayment, all evaluated at the threshold A^*. If the expression in (13) is positive, then the government's incentives to repay its debt increase following an increase in τ^R.

There are three noticeable differences compared to the case when the tax rate is increased in both repayment and default. First, a higher τ^R increases government tax revenues only in repayment, which tends to increase the government's incentives to repay the debt more than in the earlier case. On the other hand, a higher τ^R decreases the government's incentives to repay by decreasing private consumption in repayment by more than in default (private consumption in default is affected indirectly through the change in households' investment strategies). Finally, the investment distortion effect, while still present, is now smaller since the households are uncertain whether the announced tax increase will be implemented at the time they make their investment decisions.

While a choice whether to increase the tax rate only in repayment or both in repayment and in default is most likely determined by the political constraints, it is of interest to compare the effect of increasing τ^R against increasing the tax rate in both repayment and default. The following proposition establishes that an increase only in τ^R leads to a larger increase in the government's incentives to repay then an increase in both τ^R and τ^D when initial tax rate is low while the opposite is true when the initial tax rate is high.

Proposition C *Let* $\frac{\partial \Delta V}{\partial \tau^R}$ *and* $\frac{\partial \Delta V}{\partial \tau}$ *denote the effect on the government incentives of increasing the tax rate only in repayment and both in repayment in default, respectively. Then there exists* $\underline{\underline{\tau}}$ *and* $\overline{\overline{\tau}}$*, with* $0 < \underline{\underline{\tau}} < \overline{\overline{\tau}} < 1$ *such that*

1. *If* $\tau > \overline{\overline{\tau}}$ *then* $\frac{\partial \Delta V}{\partial \tau^R} < \frac{\partial \Delta V}{\partial \tau}$.
2. *If* $\tau < \underline{\underline{\tau}}$ *then* $\frac{\partial \Delta V}{\partial \tau^R} > \frac{\partial \Delta V}{\partial \tau}$.

To understand this result note that when the tax rate is initially low then households' private consumption is relatively high while government spending is relatively low. Thus, the negative effect of higher τ^R on the utility from the private consumption in repayment is small while the the positive effect of higher τ^D on the utility from the government spending would be high. It follows that at if initially both $\tau^R = \tau^D = \tau$ where τ is low then increasing only τ^R has larger effect on the government incentives to repay than increasing both τ^R and τ^D at the same time; the opposite is true when the initial tax level is low.

Proof of Proposition C. Let τ^R denote the tax rate in repayment and τ^D denote the tax rate in default. When $\tau^R \neq \tau^D$ then solving problem of a household with productivity $A_i = A^* + \kappa\varepsilon$ we get

$$k_2\left(A^* + \kappa\varepsilon, \kappa, \varepsilon; \tau^R, \tau^D\right) = e^{A+\kappa\varepsilon} f\left(k_1\right) \Lambda\left(A^* + \kappa\varepsilon, \kappa, \varepsilon; \tau^R, \tau^D\right)$$

where

$$\begin{aligned}
\Lambda\left(A^* + \kappa\varepsilon, \kappa, \varepsilon; \tau^R, \tau^D\right) &\equiv \frac{\lambda\left(A^* + \kappa\varepsilon, \kappa, \varepsilon\right) - \sqrt{\lambda\left(A^* + \kappa\varepsilon, \kappa, \varepsilon\right)^2 - 4\alpha Z\left(1+\alpha\right)\left(1-\tau^R\right)\left(1-\tau^D\right)}}{2\left(1+\alpha\right)} \\
\lambda\left(A^* + \kappa\varepsilon, \kappa, \varepsilon\right) &\equiv \left(P\left(\varepsilon\right) + \alpha\right)\left(1-\tau^R\right) + Z\left(1 - P\left(\varepsilon\right) + \alpha\right)\left(1-\tau^D\right)
\end{aligned}$$

Note that if $\tau^R = \tau^D$ the expression for k_2 becomes identical to the expression reported in Section A.1.1 of this Appendix. Moreover, it can be shown that

$$\frac{\partial k_2\left(A^* + \kappa\varepsilon, \kappa, \varepsilon; \tau^R, \tau^D\right)}{\partial \tau^R} \in \left[-\frac{1}{1-\tau} k_1, 0\right]$$

as κ varies from -1 to 1.[11] The above discussion implies that

$$\frac{\partial Y_2}{\partial \tau^R} = \frac{\partial}{\partial \tau^R} \int\limits_{\kappa=-1}^{1} k_2 \left(A^* + \kappa\varepsilon, \kappa, A^*\right)^{\alpha} d\kappa < \frac{\alpha}{1-\tau} \int\limits_{\kappa=-1}^{1} k_2 \left(A^* + \kappa\varepsilon, \kappa, A^*\right)^{\alpha} d\kappa$$

and hence, as remarked in the text, the distortionary effect of higher taxes is lower when the higher tax is implemented only in repayment.

Next differentiating ΔV (as defined in Section A.1.3) with respect to τ^R, imposing that initially $\tau^R = \tau^R = \tau$, and simplifying, we obtain

$$\begin{aligned}
\frac{\partial \Delta V}{\partial \tau^R} \quad = \quad & \underbrace{-\int\limits_{\kappa=-1}^{1} \frac{1}{(1-\tau)\left(1-\Lambda(\kappa)\right)} d\kappa - \int\limits_{\kappa=-1}^{1} \frac{\frac{\partial k_2}{\partial \tau^R}}{(1-\tau) e^A f(k_1)\left(1-\Lambda(\kappa)\right)} \left[1 - \frac{1}{Z}\right] d\kappa - \frac{1}{1-\tau}}_{\text{The effect of a higher } \tau^R \text{ on the private consumption in repayment versus default}} \\
& + \underbrace{\frac{Y_1}{\tau Y_1 - B_1 + B_2} + \frac{Y_2}{Y_2 - (1+r) B_2}}_{\text{An increase in government tax revenues in repayment}} \\
& + \underbrace{\frac{\tau \frac{\partial Y_2}{\partial \tau^R}}{\tau Y_2 - (1+r) B_2} - \frac{\frac{\partial Y_2}{\partial \tau^R}}{Y_2}}_{\text{Investment distortion in repayment versus default}}
\end{aligned}$$

Now,

$$\Lambda(\kappa) \in \left[\frac{\alpha Z}{1+\alpha}, \frac{\alpha}{1+a}\right] \text{ and } \frac{\partial Y_2}{\partial \tau^R} \in \left(0, \frac{\alpha}{1-\tau}\right),$$

and therefore,

$$\begin{aligned}
\frac{\partial \Delta V}{\partial \tau^R} \quad < \quad & \frac{Y_1}{\tau Y_1 - B_1 + B_2} - \frac{1}{1-\tau} + \frac{Y_2}{\tau Y_2 - (1+r) B_2} - \frac{1}{1-\tau} \\
& -\tau \frac{\partial Y_2}{\partial \tau^R} \left[\frac{1}{\tau Y_2 - (1+r) B_2} - \frac{1}{\tau Y_2}\right] - \frac{1}{1-\tau} \frac{\alpha Z}{1+\alpha-\alpha Z}
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial \Delta V}{\partial \tau^R} \quad > \quad & \frac{Y_1}{\tau Y_1 - B_1 + B_2} - \frac{1}{1-\tau} + \frac{Y_2}{Y_2 - (1+r) B_2} - \frac{1}{1-\tau} \\
& -\tau \frac{\partial Y_2}{\partial \tau^R} \left[\frac{1}{\tau Y_2 - (1+r) B_2} - \frac{1}{\tau Y_2}\right] - \frac{\alpha}{1-\tau} \frac{1}{Z}
\end{aligned}$$

We obtained the upper bound and lower bound for the effect of an increase in τ^R The result then follows from comparing the upper bound and the lower bound for $\partial \Delta V / \partial \tau^R$ with the expression for $\partial \Delta V / \partial \tau$ derived in the proof of Proposition 3. In particular, one can show that for all low enough τ the lower bound for $\partial \Delta V / \partial \tau^R$ is greater than $\partial \Delta V / \partial \tau$. Similarly, for high enough τ, the upper bound for $\partial \Delta V / \partial \tau^R$ is smaller than $\partial \Delta V / \partial \tau$. ■

[11] When $\kappa = -1$ then $P(\varepsilon) = 1$ which means that these households expect default with probability 1 and as a consequence they assign probability 0 to taxes being increased and leave their investment decisions unchanged. On the other end of the spectrum lie households which received $\kappa = 1$.

D Policy Adjustments under Uncertainty

To derive the change in the default threshold when households and lenders are uncertain as to whether the policy change will be implemented, I start by considering a situation where with probability $(1-p)$ the policy parameter takes value ψ (which I associate with the case when the policy change is not implemented) and with probability p the policy parameter takes value ψ' (which I associate with the new level of the policy parameter if the policy is implemented). I then follow the same steps as in the proof of Proposition 2 to compute the effect of a further change in ψ'. Finally, I impose the condition that initially $\psi' = \psi$. By following these steps, I obtain the effect of an announcement of a change in the policy parameter when such a change will take place with probability p.

Let A^* be the threshold if the policy parameter takes value ψ (i.e., the policy change is not implemented) and $A^{*\prime}$ the policy threshold when the policy parameter takes value ψ' (i.e., the policy change is implemented).[12] Then the equilibrium conditions can be written as

$$(1-p)\, I\left(A^* + \kappa\varepsilon, A^*, k_2^*(\kappa), \psi\right) + pI\left(A^* + \kappa\varepsilon, A^{*\prime}, k_2^*(\kappa), \psi'\right) = 0 \tag{14}$$

$$(1-p)\, I\left(A^{*\prime} + \kappa\varepsilon, A^*, k_2^{*\prime}(\kappa), \psi\right) + pI\left(A^{*\prime} + \kappa\varepsilon, A^{*\prime}, k_2^{*\prime}(\kappa), \psi'\right) = 0 \tag{15}$$

$$(1-p)\, L\left(A^*, x^*, \psi\right) + pL\left(A^{*\prime}, x^*, \psi'\right) = 0 \tag{16}$$

$$\Delta V\left(A^*; \left\{k_2^*(\kappa)\right\}_{\kappa\in[-1,1]}, x^*, A^*, \psi\right) = 0 \tag{17}$$

$$\Delta V\left(A^{*\prime}; \left\{k_2^*(\kappa)\right\}_{\kappa\in[-1,1]}, x^*, A^{*\prime}, \psi'\right) = 0, \tag{18}$$

where $k_2^*(\kappa)$ denotes an individual household's equilibrium investment when that household's productivity is equal to $A^* + \kappa\varepsilon$, while $k_2^{*\prime}$ denotes the individual household's equilibrium investment when that household's productivity is equal $A^{*\prime} + \kappa\varepsilon$.

When households and lenders are uncertain whether an announced policy will be implemented there are additional equilibrium equations compared to the case considered in Section B of this appendix. This is because we need to determine the default threshold both when the policy in implemented and when it is not (the possibility of a policy change also affects the threshold even if in the end the policy is not implemented). In particular, to compute the equilibrium default threshold when the policy parameter takes value ψ, we need both the government's default condition and the household investment decisions evaluated both evaluated at ψ (Equations 14 and 17). Similarly, to compute the equilibrium default threshold when the policy parameter takes value ψ', we need both the government's default condition and the household investment conditions evaluated both evaluated at ψ' (Equations 15 and 18).

To compute the effect of a policy announcement when the policy is expected to be implemented with probability p, one can follow an approach similar to the one in Section B of this appendix, that is consider the total derivatives of both sides of all equilibrium condition with respect to ψ'. Solving the resulting system of equations for $dA^*/d\psi$ and $dA^{*\prime}/d\psi'$ and evaluating all derivatives at $\psi = \psi'$ (since we consider a small policy change from its initial level at ψ) yields the desired result.[13]

[12] For example, if the relevant policy parameter is a tax rate τ and the government contemplates increasing the tax rate to $\tau' > \tau$ then $\psi = \tau$ while $\psi' = \tau'$.

[13] The detailed derivations can be found in the "Additional Results" document available on author's website.

D.1 Proofs of Propositions 5

That $dA^*/d\psi = \partial A^*/\partial\psi$ when a policy change in unexpected is immediate from the discussion of Proposition 5 is the main text. Thus, it remains to show that $\lim_{\sigma_x,\varepsilon\to 0} \partial A^*/\partial\psi = 0$. For simplicity, I consider the case when only $\varepsilon \to 0$.

Note that $\lim_{\varepsilon\to 0} \frac{\partial \Pr(A^{**}|A^*+\kappa\varepsilon)}{\partial A^*}\Big|_{A^{**}=A^*} = \infty$ (see Claim 4 in Section G of this appendix), and thus from expression for $k_2^*(A^*+\kappa\varepsilon,\kappa,A^{**})$ we obtain $\lim_{\varepsilon\to 0} \frac{\partial k_2^*(A^*+\kappa\varepsilon,\kappa,A^{**})}{\partial A^*}\Big|_{A^{**}=A^*} = \infty$.[14] From this it follows that $\Delta V_1 + \int_{-1}^{1} \frac{1}{2}\Delta V_2(\kappa)\frac{\partial k_2^*(\kappa)}{\partial A^*}d\kappa \to \infty$ as $\varepsilon \to 0$. Now, recall from the proof of Proposition 2 that

$$\frac{\partial A^*}{\partial\psi} = -\frac{\Delta V_4}{\Delta V_1 + \int\limits_{-1}^{1} \Delta V_2 \frac{\partial k_2^*(\kappa)}{\partial A^*}d\kappa}$$

$\Delta V_4 = \partial\Delta V/\partial\psi$ is well-defined for each parameter of the model, and hence finite. It follows that

$$\lim_{\varepsilon\to 0}\frac{\partial A^*}{\partial\psi} = 0$$

D.2 Proof of Corollary 2

Corollary 2 follows from Propositions $3-5$.

D.3 Proof of Propositions 6

Proposition 6 follows from Equation (5) in the paper which states that $\frac{dA^*}{d\psi}(p) = p\frac{dA^*}{d\psi}(1)+(1-p)\frac{\partial A^*}{\partial\psi}$, Proposition 5 and Corollary 2.

D.4 Discussion of Assumption 5

The above analysis was conducted under the following assumption:

Assumption 5 *B_1 is large enough so that for all $A > \underline{A}(0)$ the government's desired borrowing in repayment exceeds the supply of funds in the market.*

To determine a bound on B_1, which is assumed implicitly in Assumption 5, assume that interest rate r is less than $\widehat{r}$ for some arbitrarily high $\widehat{r}$. Note that the unconstrained optimal borrowing by the government in repayment is given by

$$B_2^{R,u} = \frac{(1+r)B_1 + \tau Y_2^R - (1+r)\tau Y_1^R}{2(1+r)}.$$

For a fixed $r < \widehat{r}$, a higher B_1 increases B_2, not only directly, but also indirectly by shifting the lower bound of the fragility region, $\underline{A}(r)$, upwards. For sufficiently high $\underline{A}(r)$, we have $Y_2^R(\underline{A}(r)) > Y_1^R(\underline{A}(r))$, where $Y_t^R(A)$ denotes the total output at time t when average productivity is A. Moreover, $\partial Y_2^R/\partial A = (1+\alpha)Y_2^R$ and $\partial Y_1^R/\partial A = Y_1^R$, implying that once $\underline{A}(r)$ is high enough so that $Y_2^R(\underline{A}(r)) > Y_1^R(\underline{A}(r))$, a further increase in $\underline{A}(r)$ leads to an increase in $\tau Y_2^R - (1+r)\tau Y_1^R$, and hence in the desired borrowing. It follows that for a fixed b and a fixed r, there exists a high enough

[14] For the proof of this statement and other statements regarding the limiting behavior of $P(A^{**}|A^*+\kappa\varepsilon)$, see Section F of this appendix.

B_1 such that $B_2^{R,u} > b$. Since $[0, \hat{r}]$ is a compact interval there exists a high enough B_1, call it $\widehat{B}_1$, such that if $B_1 > \widehat{B}_1$ then $B_2^{R,u} > b$ for all $r \in [0, \hat{r}]$.

Assumption 5 simplifies the lender's problem. The difficult part of the lender's problem is the competition effect: Ceteris paribus, a higher supply of funds in the bond market decreases the lenders' expected return from lending. This effect, however, is not present when $B_2^{R,u} > b$, in which case there exists a closed-form solution for x^*. In particular, under Assumption 5, we have

$$x^* = \frac{p_x + p_A}{p_x} A^{**} - \frac{p_A}{p_x} A_{-1} + \frac{\sqrt{p_x + p_A}}{p_x} \Phi^{-1}\left(\frac{1}{1+r}\right).$$ [15]

This in turn substantially simplifies the analysis presented in Sections 3 and 4 of the paper.[16] In Section G of this appendix, I discuss briefly how the result change if Assumption 5 is not imposed in Section F of this Appendix.

E Discussion of Assumptions 1–5

E.1 Assumptions 1–4

To solve the model described in Section 1 of the paper, I imposed Assumptions 1–4 (Section 2.1 in the paper). Assumption 1, which states that $B_1 \geq \underline{B}_1$ is needed to make the problem interesting. It is straightforward to show that the unconstrained optimal borrowing by the government when the interest rate is $r = 0$ is given by

$$B_2^{R,u} = \frac{B_1 + \tau Y_2 - \tau Y_1}{2}$$

If B_1 is low, then the government might have no incentives to borrow in the fragility region (low B_1 means that the fragility region contains low values of productivity A, for which Y_2 tends to be substantially smaller than Y_1). But in this case lenders' expectations stop playing role in the model. By imposing an appropriate lower bound on B_1, I can ensure that the government will always want to borrow in the fragility region.[17]

Assumption 2 imposes a bound on the total wealth of the lenders. This is needed for two reasons. First of all, an individual lender's wealth has to be bounded, since (given the assumption of risk-neutrality) after receiving a good signal he always supplies all his funds to the market. Thus, if lenders had an infinite amount of funds, the government would always be able to borrow funds from the few agents that receive high signals. Second, a bound on b is needed to ensure that $\Delta V\left(A^*, \mathbf{k}_2^*, x^*\right)$ is increasing. The details of establishing the bound on b can be found in sections $A.1.3$ and $A.1.4$ in this appendix.[18]

[15] Derivations of the threshold x^* when there is no competition effect are standard and can be found, for example, in Szkup and Trevino (2015).

[16] Assuming that lenders ignore the competition effect would have the same implications.

[17] The details of deriving a sufficient bound on B_1 can be in "Addtional Results" document on the author's website.

[18] As shown in Section A of this appendix $\partial \Delta V / \partial A$ depends on B_2, the amount that the government can borrow. A bound on b, and hence on B_2, ensures that $\partial \Delta V / \partial A > 0$ for all A in the fragility region and for all possible choices of B_2, that is for all $B_2 \in [0, b]$. As numerical simulations suggest, unless parameters are extreme (Z is close to 1 or α close to 1) this is not an issue. However, analytically this is hard to show and hence I take care of this issue by imposing an appropriate bound on b.

Assumption 3 implies that $B_2^{R,u}$ is increasing in the fragility region. This simplifies the analysis of the lender's problem (when the stronger Assumption 5 is not imposed), and I use it to establish that x^* is increasing in A^*. Under Assumption 3, a lender who observes a higher signal not only believes that default is less likely but also that he will be able to lend more to the government. The details of the derivations of the bound on Z can be found in Section $A.5$ in the "Additional Results" document on the author's website. Numerical simulations suggest that this assumption is not crucial for the model to have a unique equilibrium in monotone strategies.

Finally, Assumption 4 imposes that the "legacy costs" of defaulting are large, that is $\xi \to 1$. This assumption is needed to ensure existence of (1) the fragility region for any parameter values, and (2) existence and uniqueness of equilibrium. When $\xi < 1$ then it is possible that for some parameter values the fragility region does not exists. This is because in this case the government's incentives to default are very strong since not only the government will not have to repay its initial debt B_1, but also it will be able to use some of the new borrowing to increase its spending. This can make government's incentive to default so strong that it will always default for intermediate values of A (i.e., for all $A < \overline{A}$). For example, in Cole and Kehoe (2000) we have $\xi = 0$ and they can only ensure existence of the fragility region when the fraction of output lost in default is close to 1 or when the government does not care much about its spending compared to private consumption. A separate issue is created by the fact that lenders do not observe A. As noted by Kletzer (1984) in debt crises models with asymmetric information there might not exist an equilibrium. Under Assumption 4 this is not the issue, and indeed I construct explicitly an equilibrium and show that it is unique (Section A of this Appendix).

E.2 Policy Analysis without Assumption 5

Assumption 5 is useful, since it simplifies the lender's problem. However, one can obtain a similar decomposition of $dA^*/d\psi$ when Assumption 5 is not imposed.

Without Assumption 5, a change in households' investment strategies will affect the lenders' equilibrium behavior. This is because the government's desired unconstrained borrowing, $B_2^{R,u}$, depends on Y_2, and a change in $B_2^{R,u}$ translate into a change in x^*. Thus, the lenders' indifference condition has to be written as

$$L\left(A^{**}, x^*, \psi, k_2\right) = 0$$

rather than as $L\left(A^{**}, x^*, \psi\right) = 0$. This is the only change compared to the case when Assumption 4 is imposed. Following the same steps, one can show that

$$\frac{dA^*}{d\psi} = \frac{\frac{\partial A^*}{\partial \psi} + \int_0^1 \frac{\partial A^*}{\partial k_2^i}\frac{\partial k_2^i}{\partial \psi}di + \frac{\partial A^*}{\partial x^*}\left[\frac{\partial x^*}{\partial \psi} + \int_0^1 \frac{\partial x^*}{\partial k_2^i}\frac{\partial k_2^i}{\partial \psi}di\right]}{1 - \int_0^1 \frac{\partial A^*}{\partial k_2^i}\frac{\partial k_2^i}{\partial A^{**}}di - \frac{\partial A^*}{\partial x^*}\left[\frac{\partial x^*}{\partial A^{**}} + \int_0^1 \frac{\partial x^*}{\partial k_2^i}\frac{\partial k_2^i}{\partial A^{**}}di\right]}$$

Thus, compared to the case when Assumption 5 holds, there is an additional term in the expression for the direct effect, $\frac{\partial A^*}{\partial x^*}\int\limits_0^1 \frac{\partial x^*}{\partial k_2^i}\frac{\partial k_2^i}{\partial \psi}di$. This is because a change in ψ leads to an adjustment in the households' investment which affects the government's desired borrowing. Without Assumption 5 there is "competition effect": a higher supply of funds to the bond market tends to mean less lending per lender. Thus, a change in the households' investment strategies leads to an adjustment in x^*. Similarly, the multiplier effect has an additional term equal to $\frac{\partial A^*}{\partial x^*}\int\limits_0^1 \frac{\partial x^*}{\partial k_2^i}\frac{\partial k_2^i}{\partial A^{**}}di$, since now a change in households' expectations affects the lenders' behavior through its impact on the government's desired borrowing.

There are two main reasons why in the paper I consider a case when Assumption 5 holds. First of all, Assumption 5 substantially simplifies the subsequent analysis. This is particularly true when considering effects of an increase in taxes and of a fiscal stimulus, or when deriving an expression for $dA^*/d\psi$, since $\int_0^1 \left[\partial x^*/\partial k_2^i\right]\left[\partial k_2^*/\partial \psi di\right]$ is a complicated object and can be computed only implicitly. Second, numerical simulations suggests that the competition effect, which is assumed away when Assumption 5 is imposed, plays only a minor role when determining the desirability of a particular policy.

F The Effect of the Interest Rate on Policy Adjustments

Above I analyzed the case when the policy change takes place after the interest rate has been set, and thus the change in the policy and the resulting change in the default threshold A^* do not affect the interest rate r. In this section I analyze what happens when the policy change is announced before the government chooses the interest rate, in which case we have to take into account how a policy change affects the choice of interest rate and how this change in the interest rate affects the default threshold.

Recall that the government chooses the interest rate to maximize the ex-ante welfare. The optimal interest rate is then the solution to the first-order condition associated with this problem, which can be written as

$$R\left(A^*, k_2, x^*, \psi, r^*\right) = 0$$

Here, we recognize that r^* depends on the government's future decisions, households' investment choices, and lenders' supply decisions. The choice of r^* is also affected by the policy parameters, since ψ affects the gains and costs associated with a higher r.

Following the same approach as in Section $B.1$ of this appendix I find that the total effect of a change in policy ψ on the default threshold is given by

$$\begin{aligned}
\frac{dA^*}{d\psi} &= M_{Total}\left[1 - \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} - \int_{i=0}^{1}\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di\right]\left[\frac{\frac{\partial A^*}{\partial \psi} + \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi} + \int_{i=0}^{1}\frac{\partial A^*}{\partial k_2}\frac{\partial k_2^i}{\partial \psi}di}{1 - \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} - \int_{i=0}^{1}\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di}\right] \\
&\quad + M_{Total}\left[1 - \frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} - \int_{i=0}^{1}\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di\right]\left[\frac{\left(\frac{\partial A^*}{\partial r} + \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial r}\right)\left(\frac{\partial r^*}{\partial \psi} + \frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial \psi} + \int_{i=0}^{1}\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial \psi}di\right)}{1 - \frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} - \int_{i=0}^{1}\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di}\right],
\end{aligned}$$

where M_{Total} is the (total) multiplier effect that is present in the model when r can adjust; is given by

$$M_{Total} = \frac{1}{1 - \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} - \int_{i=0}^{1}\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di - \frac{\left(\frac{\partial A^*}{\partial r} + \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial r}\right)}{1 - \frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial r}}\left(\frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} + \int_{i=0}^{1}\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di\right)}.[19]$$

To understand the above expression, note first that $\left[1 - \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} - \int_{i=0}^{1}\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}\right]^{-1}$ is the multiplier effect in the case when we hold the interest constant, and $\left[1 - \frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} - \int_{i=0}^{1}\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}\right]^{-1}$ is the

[19] The above expression can be derived by following the same steps as in Section $B.1$.

multiplier effect in the case when the government's default decision is affected by the change in households beliefs only through an implied adjustment in the interest rate. Then the first term in the expression for $dA^*/d\psi$ captures the change in the default threshold implied by a change in the policy holding the interest rate constant (the expression in the square brackets) weighted by the relative importance of the "partial" multiplier effect (i.e., multiplier effect when r is kept constant as in Section 3 of the paper) compared to the total multiplier effect, M_{Total}. This effect is familiar from the earlier analysis. The second term captures the total change in the default threshold implied by the adjustment in r^*. Here, $\left(\frac{\partial A^*}{\partial r}+\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial r}\right)\left(\frac{\partial r^*}{\partial \psi}+\frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial \psi}+\int_{i=0}^{1}\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial \psi}di\right)$ captures the effect that an adjustment in ψ has on r^* (and hence on A^*) holding households' and lenders' expectations constant: A change in ψ leads to a change in r^*, which then affects A^*. This effect is then reinforced by the associated multiplier effect that results from the initial adjustment in r^* and is adjusted by the relative importance of its partial multiplier effect.

How does an adjustment in r^* alter the effectiveness of various government policies compared to the case when r^* is constant? While it is difficult to answer this question analytically, intuition suggests that an adjustment in r^* tends to decrease the magnitude of the change in A^* implied by ψ as long as the default threshold A^* is lower than A_{-1}, the prior of the mean belief about A. To understand this, note that a decrease in A^* decreases the benefit of a higher r (since a lower A^* means that a further decrease in A^* due to the choice of higher r translates into a lower decrease in the probability of default) and increases the cost of a higher r (since a fall in A^* implies that the government has to incur the cost of a higher r for a larger set of productivity values). The opposite is true when A^* increases. This suggests that a policy change that leads to a decrease in A^* is accompanied by a decrease r^*, which decreases the positive effect of the policy adjustment. On the other hand, a policy change that leads to an increase in A^* is accompanied by an increase in r^*, which tends to partially offset the negative effect that such a policy has on the probability of default.

G Auxiliary Results

In this section I provide proofs of several results that have been invoked throughout this appendix. First, I show that $\partial x^*/\partial A^* < \frac{p_x+p_A}{p_x}$. Then I compute limits of several expressions as $\varepsilon, \sigma_x \to 0$ and which where used in the proof of Proposition 2.

Lemma 2 *The derivative of x^* with respect to A^* is bounded from above by $\frac{\sigma_x^2+\sigma_A^2}{\sigma_A^2}$.*

Proof. Applying the implicit function theorem to the lenders' indifference condition, we get

$$\frac{dx^*}{dA^*}=-\frac{(-1)\left(1+r\min\left\{1,\frac{B_2^u(A^*)}{S(A^*,x^*)}\right\}\right)f\left(A^*|x^*\right)}{\frac{\partial}{\partial x^*}\left[\int\limits_{A^*}^{\infty}\left(1+r\min\left\{1,\frac{B_2^u(A)}{S(A.x^*)}\right\}\right)f\left(A|x^*\right)dA\right]},$$

where $f(A|x)$ is the conditional density of A given lender j observed signal $x_j=x^*$. Define $\mathcal{A}^u=\left\{A\geq A^*|B_2^{R,u}(A)<S(A)\right\}$ and $\mathcal{A}^c=\left\{A\geq A^*|B_2^{R,u}(A)\geq S(A)\right\}$, and note that $B_2^{R,u}(A)$ and $S(A)$ intersect at most finitely many times. Without loss of generality, I assume that $B_2^{R,u}(A)$ and $S(A)$ intersect at least once (otherwise, the result follows immediately). Then we can write $\mathcal{A}^u$ and $\mathcal{A}^c$ as $\mathcal{A}^u=\cup_{i=1}^{N_u}\left[A_{i_0}^u,A_{i_1}^u\right]$ and $\mathcal{A}^c=\cup_{i=1}^{N_c}\left(A_{i_0}^c,A_{i_1}^c\right)$, where $N_u,N_c\in\mathbb{N}$, $\left\{A_{i_0}^u\right\}_{i=1}^{N_u}$ are the values of the productivity at which $B_2^{R,u}(A)$ intersects $S(A)$ from above and $\left\{A_{i_1}^u\right\}_{i=1}^{N_u}$ are the values of

productivity at which $B_2^{R,u}(A)$ intersects $S(A)$ from below.[20] With these definitions, we can write the above derivative as

$$\frac{dx^*}{dA^*} = \frac{\left(1 + r\min\left\{1, \frac{BB_2^{R,u}(A)}{S(A^*,x^*)}\right\}\right) f(A^*|x^*)}{\sum \frac{\partial}{\partial x^*} \int_{A_{i_0}^c}^{A_{i_1}^c} (1+r) f(A|x^*)\, dA + \sum \frac{\partial}{\partial x^*} \int_{A_{i_0}^u}^{A_{i_1}^u} \left\{1 + r\frac{B_2^{R,u}(A)}{S(A,x^*)} f(A|x^*)\right\} dA}$$

Consider the case where at $A = A^*$ we have $B_2^u(A^*) \geq S(A^*, x^*)$. Then the denominator becomes:

$$\begin{aligned}
& \sum_{i=1}^{N_u} \int_{A_{i_0}^c}^{A_{i_1}^c} \frac{\partial}{\partial x^*} (1+r) f(A|x^*)\, dA + \sum_{i=1}^{N_c} \int_{A_{i_0}^u}^{A_{i_1}^u} \frac{\partial}{\partial x^*} \left\{1 + r\frac{B_2^{R,u}(A)}{S(A,x^*)} f(A|x^*)\right\} dA \\
= & \int_{A^*}^{\infty} \frac{\partial}{\partial x^*} (1+r) f(A|x^*)\, dA + \sum_{i=1}^{N_c} \int_{A_{i_0}^u}^{A_{i_1}^u} \frac{\partial}{\partial x^*} \left\{r\left(\frac{B_2^{R,u}(A)}{S(A,x^*)} - 1\right) f(A|x^*)\right\} dA
\end{aligned}$$

It remains to show that the second of the above terms is positive. Intuitively, that is what we expect, since a higher x^* makes high values of A more likely and $B_2^u(A)$ is increasing in A. The remainder of this proof is devoted to establishing it analytically.

The idea of the next few steps is to change differentiation with respect to x^* with the differentiation with respect to A. First, note that, since $f(A|x^*) = (p_A + p_x)^{1/2} \phi\left(\frac{A - \frac{p_x x^* + p_A A_{-1}}{p_x + p_A}}{(p_A+p_x)^{-1/2}}\right)$, we have

$$\int_{A^*}^{\infty} \frac{\partial}{\partial x^*} (1+r) f(A|x^*)\, dA = -\frac{p_x}{p_x + p_A} \int_{A^*}^{\infty} \frac{\partial}{\partial A} (1+r) f(A|x^*)\, dA = \frac{p_x}{p_x + p_A} (1+r) f(A^*|x^*)$$

Next, let $H(A, x^*) = \left(\frac{B_2^u(A)}{S(A,x^*)} - 1\right) f(A|x^*)$. Then,

$$\frac{\partial}{\partial x^*} H(A,x^*) = -\frac{p_x}{p_x + p_A}\frac{\partial}{\partial A} H(A,x^*) + \frac{\partial B_2^{R,u}(A)}{\partial A}\frac{1}{S(A,x^*)} f(A|x^*) - \frac{p_A}{p_x + p_A}\frac{B_2^{R,u}(A)}{S(A,x^*)}\frac{\frac{\partial}{\partial x^*} S(A,x^*)}{S(A,x^*)},$$

where, since $\partial B_2^{R,u}(A)/\partial A > 0$ and $\frac{\partial}{\partial x^*} S(A,x^*) < 0$, the last two terms are strictly positive.[21] Moreover, note that for $i = 1, ..., N_c$ we have $H\left(A_{i_1}^u, x^*\right) = H\left(A_{i_0}^u, x^*\right) = 0$. Therefore,

$$\begin{aligned}
& \sum_{i=1}^{N_c} \int_{A_{i_0}^u}^{A_{i_1}^u} \frac{\partial}{\partial x^*} \left\{r\left(\frac{B_2^{R,u}(A)}{S(A,x^*)} - 1\right) f(A|x^*)\right\} dA \\
> & \sum_{i=1}^{N_c} \int_{A_{i_0}^u}^{A_{i_1}^u} -\frac{p_x}{p_x + p_A}\frac{\partial}{\partial A} H(A,x^*)\, dA \\
= & -\frac{p_x}{p_x + p_A} \sum_{i=1}^{N_c} \left[H\left(A_{i_1}^u, x^*\right) - H\left(A_{i_0}^u, x^*\right)\right] = 0
\end{aligned}$$

[20] If at A^* we have $S(A,x^*) > B_2^{R,u}(A)$, then $A_{1_0}^u = A^*$, $A_{1_1}^u = A_{1_0}^c$, $A_{i_1}^c = A_{2_0}^u$, and so on. If at A^* we have $S(A,x^*) < B_2^{R,u}(A)$ then $A_{1_0}^c = A^*$, $A_{1_1}^c = A_{1_0}^u$, $A_{i_1}^u = A_{2_0}^c$, and so on.

[21] The second and third terms "correct" for the fact that

$$\frac{\partial}{\partial x^*} H(A,x^*) \neq -\frac{p_x}{p_x + p_A}\frac{\partial}{\partial A} H(A,x^*)$$

This establishes the claim for the conclusion of the Lemma when at $A = A^*$ we have $B_2^u(A^*) \geq S(A^*, x^*)$. The case when $B_2^u(A^*) < S(A^*, x^*)$ is established in an analogous way. ■

The next claim has been used in Section $A.1.4$ to establish uniqueness of equilibrium in monotone strategies.

Claim 3 $\lim_{\varepsilon\to 0} \frac{\partial \Pr(A^*|A^*+\kappa\varepsilon)}{\partial A^*} = 0$

Proof. Note that

$$\begin{aligned}
&\frac{\partial \Pr(A^*|A^*+\kappa\varepsilon)}{\partial A^*} \\
= \quad & \frac{\frac{1}{\sigma_A}\phi\left(\frac{A^*-A_{-1}}{\sigma_A}\right) - \frac{1}{\sigma_A}\phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)} + \\
& + \frac{\left[\frac{1}{\sigma_A}\phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right) - \frac{1}{\sigma_A}\phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)\right]\left[\Phi\left(\frac{A^*-A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)\right]}{\left[\Phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)\right]^2}
\end{aligned}$$

Taking the limit as $\varepsilon \to 0$ and using l'Hôpital's rule one can show that the first term converges to $\frac{A^*-A_{-1}}{\sigma_A}\frac{(1-\kappa)}{2}$ while the second term converges to $-\frac{A^*-A_{-1}}{\sigma_A}\frac{(1-\kappa)}{2}$. It follows that $\lim_{\varepsilon\to 0}\frac{\partial \Pr(A^*|A^*+\kappa\varepsilon)}{\partial A^*} = 0$.

■

The next two claims have been used in the proof of Proposition 2.

Claim 4 $\lim_{\varepsilon\to 0} \left.\frac{\partial \Pr(A^{**}|A^*+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^*} = \infty$ *and* $\lim_{\varepsilon\to 0} \frac{\left.\frac{\partial \Pr(A^{**}|A^*+\widehat{\kappa}\varepsilon)}{\partial A^*}\right|_{A^{**}=A^*}}{\left.\frac{\partial \Pr(A^{**}|A^*+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^*}} = -1$

Proof. If $A^{**} \in (A^* - (1-\kappa)\varepsilon, A^* + (1+\kappa)\varepsilon)$, then

$$\Pr(A^{**}|A^*+\kappa\varepsilon) = \frac{\Phi\left(\frac{A^{**}-A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}$$

Differentiating with respect to A^{**}, we get

$$\frac{\partial \Pr(A^{**}|A^*+\kappa\varepsilon)}{\partial A^{**}} = \frac{\phi\left(\frac{A^{**}-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}$$

Taking the limit as $\varepsilon \to 0$ at $A^* = A^{**}$, we get

$$\lim_{\varepsilon\to 0}\frac{\partial \Pr(A^{**}|A^{**}+\kappa\varepsilon)}{\partial A^{**}} = \infty$$

Next, consider

$$\frac{\left.\frac{\partial \Pr(A^{**}|A^*+\widehat{\kappa}\varepsilon)}{\partial A^*}\right|_{A^{**}=A^*}}{\left.\frac{\partial \Pr(A^{**}|A^*+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^*}} = -\frac{\frac{\frac{1}{\sigma_A}\phi\left(\frac{A^*-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^*+(1+\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^*-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)}}{\frac{\frac{1}{\sigma_A}\phi\left(\frac{A^*-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}} - \frac{\frac{\left[\Phi\left(\frac{A^*-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^*-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)\right]\left[\frac{1}{\sigma_A}\phi\left(\frac{A^*+(1+\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)-\frac{1}{\sigma_A}\phi\left(\frac{A^*-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)\right]}{\left[\Phi\left(\frac{A^*+(1+\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^*-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)\right]^2}}{\frac{\frac{1}{\sigma_A}\phi\left(\frac{A^*-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}}$$

Using l'Hôpital's rule, one can establish that

$$\lim_{\varepsilon\to 0} -\frac{\dfrac{\frac{1}{\sigma_A}\phi\left(\frac{A^*-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^*+(1+\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^*-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)}}{\dfrac{\phi\left(\frac{A^*-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}} = -1$$

Similarly, using l'Hôpital's rule, one can show that the second term converges to 0. ∎

Claim 5 $\lim_{\sigma_x\to 0} \frac{\partial}{\partial A}\, S\left(A,x^*\right)\big|_{A=A^*} = \infty$

Proof. Note that

$$S\left(A,x^*\right) = b\left[1-\Phi\left(\frac{x^*-A}{\sigma_x}\right)\right]$$

Taking the derivative with respect to A, we get

$$\frac{\partial S\left(A,x^*\right)}{\partial A} = \frac{1}{\sigma_x}\phi\left(\frac{x^*-A}{\sigma_x}\right).$$

Under Assumption 4, we have $x^* = \frac{\sigma_x^2+\sigma_A^2}{\sigma_A^2}A^* - \frac{\sigma_x^2}{\sigma_A^2}A_{-1} + \sigma_x^2\sqrt{\frac{1}{\sigma_x^2}+\frac{1}{\sigma_A^2}}\Phi^{-1}\left(\frac{1}{1+r}\right)$, and thus $\lim_{\sigma_x\to 0}\frac{x^*-A}{\sigma_x} = \phi\left(\Phi^{-1}\left(\frac{1}{1+r}\right)\right)$. The Claim follows immediately from this observation. ∎

Claim 6 *We have* $dA^*/dA_{-1} < 0$.

Proof. In light of Proposition 2 it is enough to consider the direct effect of increasing A_{-1}. By inspection, we see that A_{-1} does not directly affect the government incentives to default. Next, recall that household i's investment choice is given by

$$k_2\left(A_i\right) = \left(1-\tau\right)e^{A_i}f\left(k_1\right)\Lambda\left(A_i;\varepsilon,A^{**}\right)$$

where

$$\begin{aligned}\Lambda\left(A_i;\varepsilon,A^{**}\right) &= \frac{\alpha\left(1+Z\right)+P\left(A^{**}|A_i\right)+Z\left(1-P\left(A^{**}|A_i\right)\right)}{2\left(1+\alpha\right)} \\ &\quad -\frac{\sqrt{\left[\alpha\left(1+Z\right)+P\left(A^{**}|A_i\right)+Z\left(1-P\left(A^{**}|A_i\right)\right)\right]^2-4\alpha Z\left(1+\alpha\right)}}{2\left(1+\alpha\right)}\end{aligned}$$

Here, A_{-1} affects $P\left(A^{**}|A_i\right)$ where $P\left(A^{**}|A_i\right) \equiv \Pr\left(A<A^{**}|A_{-1},A_i\right)$. Since $\Lambda\left(A_i;\varepsilon,A^{**}\right)$ is decreasing in $P\left(A^{**}|A_i\right)$ and $P\left(A^{**}|A_i\right)$ is decreasing in A_{-1} it follows that $\partial\Lambda\left(A_i;\varepsilon,A^{**}\right)/\partial A_{-1}$ is increasing in A_{-1}. Thus, it follows that an increase in A_{-1} leads to a higher investment by households. Since a higher investment strictly decreases governments' incentives to default we have

$$\int_{i=0}^{1}\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial\psi}di < 0$$

Next, consider lenders. Recall that the signal threshold above which lenders supply their funds to the bond market is defined implicitly by

$$\int_{A^{**}}^{\infty}\Delta u\left(A;x^*,A^{**}\right)\left(\tau_x+\tau\right)^{1/2}\phi\left(\frac{A-\frac{\tau_x x^*+\tau A_{-1}}{\tau_x+\tau}}{\left(\tau_x+\tau\right)^{-1/2}}\right)dA = 0$$

where A^{**} denote expected default threshold by the lenders. Then

$$\frac{\partial x^*}{\partial A_{-1}} = -\frac{\tau}{\tau_x} < 0$$

Thus, an increase in A_{-1} leads to a decrease in x^* implying that lenders supply more funds to the bond market for any given A. Since a higher supply of funds weakly decreases the government's default incentives we have

$$\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi} \leq 0.^{22}.$$

The result follows then from Proposition 2. ■

H Numerical Examples: Further Results

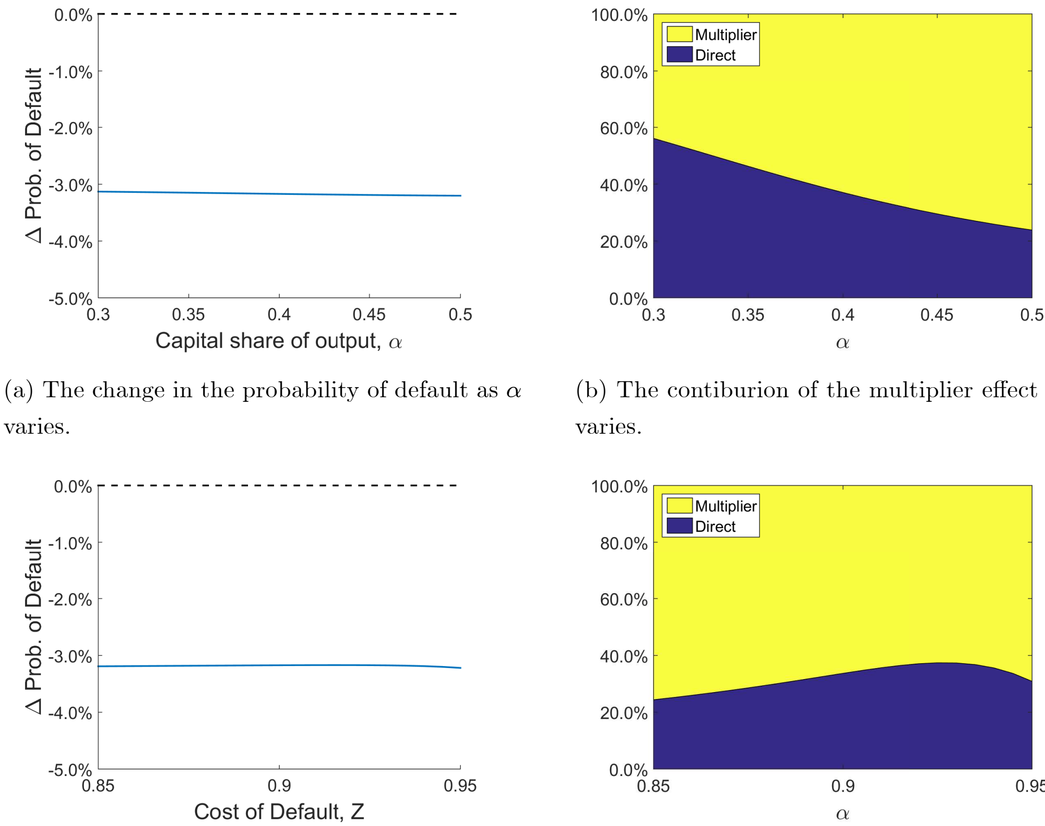
In this section I report additional numerical results. In particular, I investigate how the effects of adjustments in taxes and of a fiscal stimulus on the ex-ante probability of default and the importance of the multiplier effect depend on α and Z.

Increase in the tax rate Figure 1 reports the results for the case of adjustment in the tax rate τ. Panels A and C show that the effectiveness of a 1% increase in the tax rate does not depend on the values of α and Z and such a policy remains an attractive option to the government if the government's goal is to decrease probability of a debt crisis.

Panels B and D show how the relative importance of the multiplier and direct effect in driving the effects of an increase in taxes changes as we vary α and Z. As predicted in Section 5 the importance of the multiplier effect increases as α increases and tends to decrease as Z increases (though in the latter case the role of the multiplier effect tends to increase for large values of Z).

These results show that the conclusion regarding the effectiveness of an increase in tax rate reported in the main paper is robust. They also support the intuition regarding the importance of the multiplier effect provided in Section 5.

Fiscal Stimulus Figure 1 reports the results for the case of fiscal stimulus financed with short-term debt. Panel A shows that while at higher α the increase in the probability of default following a stimulus is lower this effect is small (the increase in probability of default falls from 1.58% when $\alpha = 0.3$ to 1.43% when $\alpha = 0.5$). Panel C show that varying Z has almost no effect on the effectiveness of a fiscal stimulus (the increase in the probability of default is equal to 1.59% when $Z = 0.85$ to 1.54% when $Z = 0.95$). It might be somewhat surprising that varying α has such a modest effect on the results. The reason why the effect of higher α is so modest can be deduced from expression 9 reported in Section B of this Appendix. On the one hand, it is true that a higher α tends to increase the direct effect since it increases the sum of the two first terms (which capture the concavity effect and the differential increase in tax revenues). On the other hand, a higher α also increases the desired borrowing by the government since it increases Y_2 relative to Y_1 which tends to increase the B_2. Higher desired borrowing also means that the interest rate that the government sets before it decides on its further policies also increases. This further increases supply of funds, and hence B_2. The increase in the amount the government borrows tends to decrease expression 9 counter-acting the positive effect described above.



(a) The change in the probability of default as α varies.

(b) The contiburion of the multiplier effect as α varies.

(c) The change in the probability of default as Z varies.

(d) The contiburion of the multiplier effect as Z varies.

Figure 1: The effect of a 1% increase in τ as α and Z vary.

Panels B and D show how the relative importance of the multiplier effect varies with α and Z. The importance of the multiplier effect increases as α increases and tends to decrease as Z increases (though in the latter case the role of the multiplier effect tends to increase for large values of Z). Note that these results are qualitatively and quantitatively very similar to the results reported above for the increase in the tax rate. This strongly indicates that the results concerning the relative importance of the multiplier effect are robust across different government policies.

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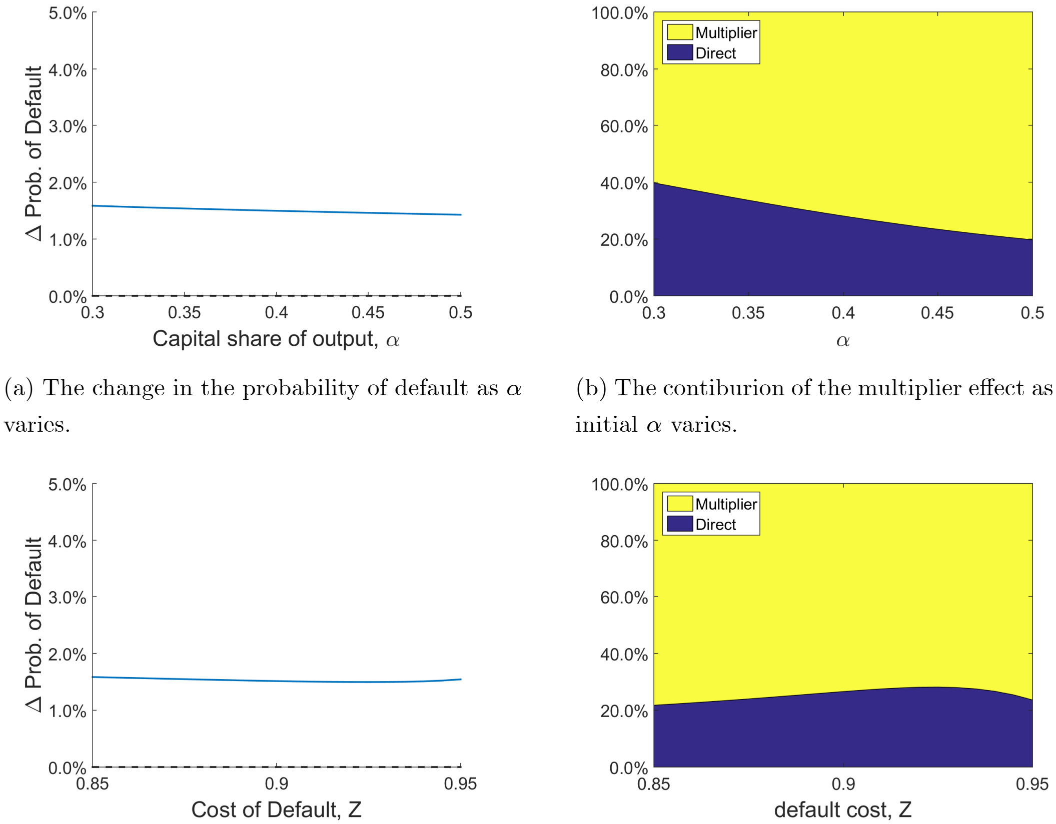

(a) The change in the probability of default as α varies.

(b) The contiburion of the multiplier effect as the initial α varies.

(c) The change in the probability of default as Z varies.

(d) The contiburion of the multiplier effect as Z varies.

Figure 2: The effect of a 1% stimulus as α and Z vary.

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