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Tsoulfidis, Lefteris and Papageorgiou, Aris

University of Macedonia, University of Macedonia

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The Recurrence of Long Cycles: Theories, Stylized Facts and Figures

Lefteris Tsoulfidis* and Aris Papageorgiou*

Corresponding Author
Professor Lefteris Tsoulfidis
Department of Economics,
University of Macedonia,
Thessaloniki Greece
Tel.: 30 2310 891788
Email: Lnt@uom.edu.gr

ABSTRACT

Basic innovations and their diffusion, the expansion or contraction of the level of economic activity and the volume of international trade, rising sovereign debts and their defaults, conflicts and the outbreak of wars, are some of the major phenomena appearing during the downswing or upswing phases of long cycles. In this article, we examine the extent to which these phenomena constitute stylized facts of the different phases of long cycles which recur quite regularly in the turbulent economic history of capitalism. The main argument of this paper is that the evolution of long cycles is a result of the long-run movement of profitability. During the downswing of a long cycle, falling profitability induces innovation investment and the associated with it 'creative destruction' of the capital stock that eventually set the stage for the upswing phase of a new long cycle.

JEL classifications: B14, B24, E11, E32

Key Words: Long Cycles, Innovations, Profit rate

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1. Introduction

The long cycle is a type of economic fluctuation with a duration ranging approximately from 40 to 50 years. According to the most renowned proponent of its empirical existence, the Russian economist Nikolai Kondratiev, the long cycle commenced its motion with the dawn of modern industrial capitalism during the last quarter of the eighteenth century. The long cycle consists of two phases, its upswing phase lasting just about as much as its downswing phase. This cyclical phenomenon began to be seriously studied in the first decades of the twentieth century and the reason behind this timing is that contemporaries did not fail to observe that what has been known by the economic historians as the ‘Great Depression’ of the late nineteenth century, which lasted approximately from the early 1870s until the mid-1890s, had given its place during the mid to late-1890s to a new and vigorous economic expansion.

The long-cycle phenomenon initially occupied the interest of mainly, what would be today designated as, ‘heterodox’ economists of the early 20th century who challenged the widely accepted view that the so-called ‘industrial cycle’, with a duration ranging from 7 to 11 years, was the sole cycle characterizing capitalist economies and argued that such a cycle was only part of a longer cyclical movement that deserved to be studied on its own terms. The most important among these pioneering economists was Nikolai Kondratiev who during the 1920s made a series of contributions shedding additional light on the phenomenon under investigation and also offered, for the first time, a statistical examination of some relevant economic variables to support his thesis.¹ It is for this reason that, during the 1930s, Joseph Schumpeter, perhaps the second most important proponent of the long cycle before the Second World War, gave the name ‘Kondratiev cycle’ to this type of economic fluctuation.

The remainder of the paper is structured as follows: Section 2 refers to idealized long cycles and extends Kondratiev’s periodization to the present. Section 3 grapples with the major stylized facts of long cycles and provides additional evidence for their existence. Section 4 discusses the details of the introduction of basic innovations in the economy and their effects. Section 5 argues that the movement of long-run profitability, and in particular the evolution of the real mass of profits, is the principal determinant of the long-cycle rhythm and its associated stylized facts. Section 6 summarizes and makes some concluding remarks about future research efforts. An Appendix presents and discusses the forms of the logistic curve used in this paper.

2. Idealized Long Cycles

The motivation to study long cycles lies in that they allow economic history to be conceptualized in such a way that different periods are linked and compared within a single theoretical framework. Thus, one would get a better sense of the current economic phase and

¹ Kondratiev’s first major article was written in Russian in 1925 detailing the phenomenon of long cycles and was expanded in 1926 to include a tentative explanation of this type of economic fluctuation. The first full translation of the expanded article in English was published in Kondratiev (1984) while a second one followed in Kondratiev (1998) as part of an English edition of Kondratiev’s works. The other two pioneering economists who conducted original and very important research on long cycles, prior to Kondratiev, were two Dutchmen, Jacob van Gelderen (1913) and Sam de Wolff (1924), their key works being translated into English for the first time in 1996 and 1999 respectively.
its prospects if he was able to compare it with other periods of the past that belonged to the same long-cycle phase. From such a comparison it could become possible to derive more definitive conclusions about the results of the policies pursued. In addition, the study of long cycles could help pinpoint the determinants of long-term economic performance, as well as improve the accuracy of long-term forecasts.

Table 1 below presents an idealized periodization of the long-cycle rhythm. The chronologies up to the upper turning point of the third long cycle are based on Kondratiev's periodization of the mid-1920s. We extend then the long-cycle periodization to cover the downswing phase of the third long cycle, the fourth long cycle and, finally, (the currently underway) fifth long cycle. It is important to point out that, in such an attempt to periodization, caution should be applied because not all advanced economies turn from a particular long phase to another simultaneously and the same holds true for the movement of important economic variables. It is for this reason that Kondratiev, studying mainly the economies of the UK, USA and France provided a range of five to seven years for the turning 'points' in his periodization of long cycles (Kondratiev 1998, p. 36). The turning points of each cycle must then be seen as attempts to an approximation and not as rigid points in time at which all relevant variables change their route. The periodization in Table 1 is more representative of the US and UK economies which pretty much move together and can be thought of as approximating the trends of the World economy. Each cycle in Table 1 is divided into its upswing phase of prosperity and its downswing phase of stagnation and every such phase is given a 'title' according to the findings of economic historians of these particular periods.

Table 1: Idealized Long Cycles

<table>
<thead>
<tr>
<th>Long Cycle</th>
<th>Period</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Long Cycle</td>
<td>1790-1845</td>
<td>Prosperity (The Industrial Revolution)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stagnation</td>
</tr>
<tr>
<td>2nd Long Cycle</td>
<td>1845-1896</td>
<td>Prosperity (The 'Victorian' Golden Age)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stagnation (The Great Depression of the Later 19th Century)</td>
</tr>
<tr>
<td>3rd Long Cycle</td>
<td>1896-1940(5)</td>
<td>Prosperity (The 'Belle Époque')</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stagnation (The Great Depression of the 1930s)</td>
</tr>
<tr>
<td>4th Long Cycle</td>
<td>1940(5)-1982</td>
<td>Prosperity (The Golden Age)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stagnation (The Great Stagflation)</td>
</tr>
<tr>
<td>5th Long Cycle</td>
<td>1982-2022</td>
<td>Prosperity (The Information Revolution)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stagnation (The Great Recession)</td>
</tr>
</tbody>
</table>
The first long cycle, coinciding with the so-called Industrial Revolution and the absorption of its effects by British society, is the one for which the least empirical proof has been provided in the long-cycle literature but we should bear in mind that the further one goes into the past the more difficult it becomes to collect reliable economic data. For the next three long cycles, there is a broad agreement among adherents of the long-cycle research programme concerning their duration and their turning points. So, the second cycle commences sometime in the mid to late-1840s and lasts until about the mid-1890s with a peak in the early 1870s while the third long cycle starts in the mid-1890s has a peak at some time following the end of WWI and then a downswing that lasts until the late-1930s. The fourth cycle begins with or after WWII and lasts until the early to mid-1980s. Its upper turning point is located at some time in the mid to late-1960s and forms the end of a period known as the 'golden age of accumulation'. Finally, we have the fifth long cycle which started in the early to mid-1980s and has not ran its full course yet, though its midpoint, for reasons which we shall address shortly, seems to be located in the first years of the 21st century.

3. Stylized facts of long cycles

Long cycles are associated with certain recurrent and systematically appearing phenomena which include the procyclical character of a specific index of prices that turns out to be very helpful in delineating the phases of the long cycle even in the absence of other information and the countercyclicality of basic innovations which are introduced mainly during the downswing phase of the long cycle and whose diffusion during the long upswing gives eventually rise to technological revolutions. We may also add the growth rate of the real GDP to the extent that this is available, the likelihood of sovereign defaults which, as expected, is higher during the downswing of the long cycles and also the social unrests and wars whose probability of occurrence increases during the upswing of the long cycle. Other phenomena associated with the long cycles include the procyclical character of the growth rates of the volume of international trade and of world industrial production. From the countercyclical variables, the unemployment rate is perhaps the major indicator of the stage of the economy. As a rule of thumb, an unemployment rate at the trough of the cycle above the 10 percent benchmark (in the US and UK economies, at least) suggests a depressionary situation.

The major variable that indicated the existence of long cycles, for the pioneering economists who studied the phenomenon, including Kondratiev, was the wholesale price level. While it may be hard for a contemporary observer to comprehend, the fact is that the economic history of modern mechanized capitalism teaches us that for a very long period that extended from the dawn of the Industrial Revolution during the 1780s and up to the Great Depression of the 1930s long phases of rising price levels alternated with long phases of falling price levels (Kondratiev 1998, p. 159). This means that, for about 150 years, long periods of deflation were a recurring phenomenon. Rising prices were associated with rising profit margins whereas falling prices were regarded as adverse for profitability and consequently for a healthy capitalist economy (Hobsbawm 1999, pp. 53-54). Thus, despite the fact that before WWII there were no detailed and reliable national accounts data, economists of that period based their empirical studies mainly on the evolution of the price level. Yet
while the recurrence of long deflationary periods was a feature of capitalism before WWII, what we observe after WWII, in advanced capitalist countries at least, is an almost uninterrupted rise in the price level, i.e. continuous inflation. The question at hand then is to what extent does the long cycle in the price level become extinct in the post-WWII period.

Figure 1 below depicts the wholesale price index of the United Kingdom and the United States ‘normalized’ (i.e. divided) by the price index of gold over a period ranging from 1791 to 2015.2 The long fluctuations of the price level, for these two leading capitalist economies, are visible for the pre-WWII period. But the striking element in this graph is that whereas the long cycle would be non-existent if one looked at the wholesale price level (for the post-WWII period) by itself, this type of fluctuation now becomes clearly visible. There is a turning point in the postwar price cycle for both countries that takes place at some time in the latter half of the 1960s and the downswing continues up until the early 1980s. Then the fifth price cycle emerges with a turning point in the early years of the 21st century and a downswing movement ever since.

We can approximate with reasonable accuracy the timing of the depressionary periods in the history of capitalism occurring during the downswings of this normalized (by the price of gold) price cycle. We can see for instance the Great Depression of the last quarter of the 19th century and the Great Depression of the 1930s occurring during the down phases of the second and the third cycle respectively, while the same holds true for the Great Stagflation of the 1970s and early 1980s that took place during the downswing of the fourth golden price cycle. Also, it is clearly visible from the graph that the first Great Recession of the 20th century that began in 2007-2008 occurred during the downswing of the fifth cycle which has not run its full course yet. Figure 1 by depicting the normalized price index of the two leading capitalist economic powers of the past two and one-quarter centuries strongly suggests that gold continues to constitute that money form which is the most secure store of value in capitalism. The idea is that as soon as it becomes difficult for the economic system to keep up its normal functions the demand for gold increases with the result that the normalized index starts a downward course. However one should be careful not to attribute to the normalized price index any causal effect regarding the long cycle. The normalized index is just an indicator (a barometer so to speak) of the phase of the long cycle not an explanation for it.

2 Kondratiev in an appendix to his paper listed his wholesale price data on the UK, the USA and France in terms of gold (Kondratiev 1998, pp. 190-192). Thus the idea of expressing the price level in terms of gold was there from the very beginning but was not noticed by the long-cycle researchers that came after Kondratiev. The idea of ‘normalized prices’ and their usefulness in long-cycle periodization was resurfaced by Shaikh who brought attention to this concept in his lecture notes during the early 1980s and elaborated it further in his book (2016, pp. 62-66 and 184-188).
A similar picture, regarding price movements in the long-cycle framework, can also be obtained from the evolution of the real stock-price index. Figure 2 below presents a quite comprehensive stock-price index, the S&P 500, divided by the wholesale price index. In effect, such a normalized price index shows the purchasing power of the stocks traded in the stock market relative to the average price of goods. From the late 1920s onwards the index seems to accord with the long-cycle periodization. Thus, we clearly distinguish the depressionary periods of the 1930s, the 1970s-early 1980s and the Great Recession. It is important to point out that a very similar periodization would be obtained in the evolution of the stock market price index had we normalized it by the price index of gold instead of the wholesale deflator.

3 The wholesale price index for both countries is from Jastram (2009, Tables AE1 and AE2) up to 2007; it is extended by the following producer price indices provided by the Federal Reserve Economic Data for the UK and the USA: https://fred.stlouisfed.org/series/WPPIUKA and https://fred.stlouisfed.org/series/PPIACO. The price index for gold is from Officer and Williamson (2016).
Having identified the five long cycles in the normalized wholesale price index and, partly at least, in the normalized S&P 500 price index, our efforts now concentrate to single out phenomena associated with the long cycles which appear quite regularly and whose manifestation and consequence need further exploration. We start with a major, we believe, phenomenon which for Schumpeter (1939) is primarily responsible for this long cycle-like evolution of capitalism. If the normalized (wholesale or stock market) prices are characterized by their procyclical nature, then the introduction of basic innovations follows for very good reasons a countercyclical behaviour. Kondratiev (1998, pp. 38-40) had already in the 1920s pointed out that prior to the beginning of the long upswing or, at the latest, at its very beginning there appears a great deal of very important technical innovations in advanced capitalist economies, providing also examples of such innovations. However, unlike many of his contemporaries, Kondratiev (1998 [1935]) endogenized the introduction of innovations, as well as most of the other phenomena he examined, to the rhythm of the long cycle seeking economic interpretations for their appearance and not taking them as conjunctural events. Schumpeter (1939) expanded on this suggestion by Kondratiev is his *Business Cycles*.

Table 2 below summarizes the major phenomena associated with the long cycles, that is, for each of the five long cycles that we managed to collect data, we form the relevant percentage in the upswing and downswing stages. Thus besides the percentage of the basic innovations introduced in each phase of the long cycle, we display the growth rates of the volume of international trade, of industrial production and of the real US GDP, as well as the

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percentage of wars and the percentage of sovereign debt defaults taking place during the upswing and downswing phases of each long cycle.

Table 2: Major phenomena of long cycles

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<tbody>
<tr>
<td>Basic Innovations % share</td>
<td>27</td>
<td>73</td>
<td>36</td>
<td>64</td>
<td>30</td>
</tr>
<tr>
<td>Volume of World Trade (Growth Rate)</td>
<td>4.62</td>
<td>3.00</td>
<td>3.75</td>
<td>2.55</td>
<td>8.49</td>
</tr>
<tr>
<td>World Industrial Production (Growth Rate)</td>
<td>4.42</td>
<td>3.05</td>
<td>4.47</td>
<td>3.19</td>
<td>6.25</td>
</tr>
<tr>
<td>Wars %</td>
<td>59</td>
<td>41</td>
<td>58</td>
<td>42</td>
<td>68</td>
</tr>
<tr>
<td>Sovereign Defaults %</td>
<td>14</td>
<td>86</td>
<td>44</td>
<td>56</td>
<td>36</td>
</tr>
<tr>
<td>GDP (growth rate, %) USA</td>
<td>4.78</td>
<td>3.80</td>
<td>4.67</td>
<td>4.02</td>
<td>3.21</td>
</tr>
</tbody>
</table>

In Table 2 we observe the timing of the introduction of basic innovations which in the downswing of the long cycle outnumber those in the upswing. This is only a quantitative comparison without taking into account the significance of each of the basic innovations and the extent to which they characterize a whole epoch. The reason for this timing might be that during the upward phase of the long cycle, when profitability is rising, businesses have no compelling reasons to risk their good standing by introducing innovations of this type, that is, radical innovations. By contrast, in the downturn of the long cycle when profitability is

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5 See Section 4 about the treatment and sources of basic innovations.
6 Sources: World Trade (1850-2014) – Bank of England (2015), Table 18: “Trade Volumes and Prices”. World Industrial Production (1850-1986) – Kuczynski (1980) for the period 1850-1976, MacAvoy (1988, p. 11) for the period 1976-1986. There is no data about the growth rates of world trade for the periods 1914-1920 and 1939-1949, that is for the time periods related to the two World Wars. For comparison reasons then, we choose for the world industrial production series the same periodization as for the world trade series, for the time range 1850-1982 during which the two series overlap.
7 Source of data: https://en.wikipedia.org/wiki/List_of_wars_1945%E2%80%9389#cite_note-2
8 Source of data: https://en.wikipedia.org/wiki/List_of_sovereign_debt_crises
stagnant or falling and the prospects are bleak challenging the survival of the enterprise the pressure to innovate is at its highest. Capitalists facing, on the one hand, the abyss of default and on the other hand, their possible survival through their innovative activity, are more prone to "choose" the innovation path.

The growth rates of the volume of world trade tend to be by far larger during the upswing phase of the long cycle than during the downswing phase. Thus the upswing of the long cycle tends to be related to an intensified globalizing evolution of the capitalist system. It is important to note that unlike GDP and other national accounts data the volume of world trade data are available from the mid-19th century. Similarly, the growth rate of world industrial production, which is another very important indicator of the health of the world economy, moves pro-(long-)cyclically as well.

Kondratiev further argued that the long upswings are characterized by more warfare and social unrest than the long downswings and he explained this through the creation of new markets during the upswing that intensify geopolitical competitions between the major economies. In Table 2, we observe that the percentage of wars during the upswing of the long cycles is by far higher than during the downswing.

The last two variables of Table 2 were not in the list of phenomena described by Kondratiev probably for the following reasons. The national income accounts were constructed mostly in the post-WWII period and the prewar estimates are assembled retroactively. Sovereign defaults were many before WWI; however, they were not interesting from an economic analysis perspective for they were attributed mainly to irresponsible government borrowing and spending and not to the operation of systematic economic dynamics which might become amenable to abstract theorization. Besides, there were no national income accounts data that would enable such theorization. Only in the recent decades, did public debt and its size relative to GDP along with other related variables become subject to intensive studies. And in these studies based on the accumulation of a lot of historical and descriptive factual material, it becomes increasingly evident that sovereign defaults, although they may occur at any phase of the long cycle, have a much greater likelihood, and therefore frequency, of occurrence during periods of stagnation rather than during periods of prosperity. As we argue in Section 5 a falling rate of profit leads, past a point, to a stagnating mass of real profits which discourages investment. Financial institutions then in order to recover the money they have lent out are interested in increasing the level of the economy's output, something which can be achieved through higher investment spending. As a consequence, financial institutions are prone to reduce their interest rates to make borrowing even more attractive and thus initiate investment spending. However, the lower interest rates force financial institutions to lend out much higher amounts of money in order to acquire the same amount of interest revenues and thus pay much less consideration to the fundamentals, both their own and their borrowers'. Under these circumstances, the lower interest rate encourages governments to increase borrowing, not necessarily for meaningful long-term investment projects but mainly for immediate consumption purposes and thus build a debt bubble along with other bubbles developed in the real estate and the financial markets. The burst of one of these bubbles is capable of triggering the burst of the other ones especially
in economies with weak fundamentals and also with governments eager to give in to populist demands.

The last of our reported important economic variables, that is, the growth rate of the GDP of the US economy pretty much follows the long-cycle rhythm; this is especially true for the years after WWII. Although we have data for the pre-WWII years these are constructed retroactively and therefore are less reliable than those after WWII when data began to be collected systematically. We should also point out that, for the post-war years, the growth rate of the world GDP (for its estimates see Tsoulfidis 2015, pp. 155-157) follows the same rhythm as the USA GDP which we present in Table 2.

4. The Swarms of Innovations

According to Schumpeter, the most important feature of capitalism is its capacity to ‘produce’ innovations. Innovations tend to appear in ‘clusters’ and their subsequent diffusion and absorption result in the rejuvenation of the economy through the acceleration of its growth rate followed by the decelerating and finally the maturity stages in the cyclical growth of the economy. The length of the cycle depends on the type of innovation cluster. Less important innovations that come in clusters tend to produce the 7-to-11-year industrial cycle while clusters of important innovations tend to produce the Kondratiev cycle (Schumpeter 1939, pp. 169-172). In his economic-historical discussion of Kondratiev cycles, Schumpeter argues that the far-reaching innovations that form the motive power of each long upswing tend to appear in clusters during the downswing of the previous Kondratiev cycle, thus essentially adopting Kondratiev’s point of view (Schumpeter 1939, pp. 254-255).

The issue of the relationship between important innovations and the long cycle was revived during the late 1970s and early 1980s, amidst the downswing of the third long cycle, by the appearance of Gerhard Mensch’s Stalemate in Technology (English translation in 1979, German original in 1975). Mensch collected data on so-called basic innovations which he defined as those innovations that create whole new economic sectors or that constitute radical improvements to already established practices and thus completely rejuvenate the relevant economic sectors (Mensch 1979, pp. xvii-xviii and p. 123). Though his initial research purpose was not concerned with the issue of long cycles what he eventually found out was an increased frequency of basic innovations occurring during the 1880s and the 1930s which are decades connected with severe depressions taking place during the downswings of the normalized long cycle (Mensch 1979, p. 130). From this premise, Mensch proceeded to revive the ‘old’ Kondratiev-Schumpeter thesis by arguing that the most relevant timing for the introduction of basic innovations is during a long downswing. The rationale for this argument is that the low profitability and the economic stagnation that characterize the long downswing, especially during its later stages, stimulate competition and make imperative the introduction of new products and new production techniques that would increase the

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10 It is important to emphasize at this juncture that the term ‘innovation’ refers to the commercialization of an invention which has to await its economic application until it is expected that it will become profitable to enter production. Subject to the actual fulfilment of this expectation then, the diffusion process of the innovation is initiated.
profitability of the innovating companies and that would assist the system as a whole to
overcome its stalemate. These basic innovations tend to appear in clusters in specific and
related economic sectors that later become the leading sectors of the economy during the long
upswing. The long upswing is characterized by the diffusion of these basic innovations across
the whole of the economic spectrum, while the type of innovation that tends to take place
during the upswing period is mainly improvement innovations, though of course, the number
of basic innovations does not fall to zero. The long upswing is then characterized by the
diffusion of a technological revolution that is based on radical innovations that encompass
new types of products, the energy sector, the transportation sector and the communications
sector.

In order to test whether there is any empirical basis in the Kondratiev-Schumpeter-
Mensch thesis, that is, that basic innovations tend to be introduced in advanced capitalist
economies during the long downswing we will use the equation of logistic growth.\textsuperscript{11} To
proceed then to an empirical test via logistic fitting, we have collected data from the three
relevant sources in the long-cycle literature that provide lists of annual introductions of basic
innovations. We thus have ended up with creating a database of very important innovations
that were introduced annually from 1850 to 1970, thus covering a period of two and a half
long cycles. The three sources that we used are the basic-innovations samples by Mensch
(1979, pp. 124-128), by Haustein and Neuwirth (1982) and by van Duijn (1983, pp. 176-
179).\textsuperscript{12} For the (many) cases where the same basic innovation is covered in all three samples
we use the following method: (a) if a basic innovation appears in all three samples and there
is disagreement about the year of introduction that concerns only one source then we use the
introduction year mentioned by the other two sources, (b) if an innovation appears in only two
of the samples then we use the source with the earlier introduction year, (c) if an innovation
appears in all three samples but with a different year of introduction in each sample then: (i) if
at least two of our sources provide introduction years that are relatively close to each other we
choose the earlier year, (ii) if the three years given are relatively far from each other we
choose the earlier year.

Our super-sample can be regarded as a rather quite comprehensive, we believe, list of
basic innovations that covers about 120 years of capitalism’s economic history.\textsuperscript{13} Compiling

\textsuperscript{11} See the Appendix for a discussion of the types of logistic curve used in this article and their
properties.

\textsuperscript{12} The last innovation in the Mensch sample occurs in 1955. We follow Kleinknecht (1990) and
expand the Mensch data up to the late 1960s as suggested by Clark \textit{et al.} (1984). Kleinknecht merges
the three samples of basic innovations to perform one-sided \( t \)-tests and check whether the number of
occurrence of basic innovations differs between time periods. The other instance where a combination
of these samples is used in the long-cycle literature is, as far as we know, in Silverberg and Verspagen
samples to perform Poisson regression tests and check whether innovation clusters are randomly
distributed in time.

\textsuperscript{13} That is, from the 1850s until the early 1970s. Even though the first item (‘spinning machine’) in the
super-sample list of our basic innovations appears in 1764, nevertheless up and until the first half of the
19th century the total number of documented basic innovations is relatively small with zero
introductions of basic innovations being observed for quite a few years. Furthermore, as Mahdavi
comments: “Before 1850, research and development were done sporadically, on a small scale and
usually without any definite commercial purpose. Most innovations were the result of individual trial
this super-sample is just the first step. We must then form the appropriate time series out of it in order to test the logistic fit. This is so because if we just take the time series of the annual introduction of basic innovations by itself then we would just end up with fluctuations which would depict no (logistic) trend. The form of time series that will be used for logistic fitting then would be the series of cumulative basic innovations.14

The salient feature of the logistic curve is that up to its midpoint the growth rates of the variable that is being fitted are rising while past that point they are falling. According to the Kondratiev-Schumpeter-Mensch thesis, we would expect the introduction of new innovations to be increasing during the downswing of a long cycle (Schumpeter's "swarms of innovations"), reaching a maximum, and to be decreasing during its upswing. As we have seen the time series of normalized prices can be interpreted as an index of the economic conjuncture, with the long periods of prosperity and stagnation being connected with the (long) rising and falling phases of this particular index. For this reason, the time period which we select in order to test the fitting of a logistic function has as its starting point the onset of the downswing of one long cycle of normalized prices and as its endpoint the end of the upswing of the next long cycle of normalized prices. This is because we would be expecting the rate of introduction of basic innovations to be increasing during the downswing of one long cycle and to be decreasing during the upswing of the next long cycle. So, if it could be possible to fit a logistic curve to a time series of basic innovations then we would expect the inflection point of this curve, which would signify the time point where the examined series would depict its maximum rate of growth, to be located at the end of the down phase of the cycle of normalized prices or, at least, a little after the beginning of the (next) cycle’s upswing.

Figures 3 and 4 below present the fitting of logistic curves to the time series of cumulative basic innovations for the periods 1872-1920 and 1920-1970 and Table 3 summarizes the results of the econometric analysis. The results suggest that the introduction of basic innovations has a phase difference of about half a cycle relative to the fluctuation of the normalized price index, that is, it tends to move counter-cyclically. The time period taken for the first logistic process to cover the range between the 10% level and the 90% level of its asymptote is 37 years while the midpoint (inflection point) of the logistic fit takes place in 1891. This means that the maximum rate of introduction of basic innovations in capitalism, for the 1872-1920 period, occurred in 1891, that is, during the downswing and a few years before the lower turning point of the 2nd long cycle in the normalized price index.

14 The first, and to the best of our knowledge the only one thus far, that had the idea of fitting logistic curves to time series of cumulative innovations was Cesare Marchetti (1980). Dividing Mensch’s sample into 3 time segments he obtained fits of 3 separate logistic curves with midpoints at 1828, 1880 and 1937. Even though the Mensch sample has some obvious issues of comprehensiveness (for example it contains no basic innovations at all for the period 1911-1921), Marchetti’s results were certainly encouraging towards a direction of research that would apply logistic-curve fittings to more inclusive series of innovations. However, this did not happen, while Marchetti himself, in later works of his, insisted in presenting the results of his 1980 paper without attempting to work with other samples of basic innovations that had become available in the relevant long-cycle literature since.
Regarding the 1920 to 1970 period, which covers the second half of the third long cycle and the first half of the fourth one, the logistic fit is again excellent. The mid-point of the logistic curve, that is, the year that saw the maximum rate of introduction of basic innovations after a long period of an increasing trend in this particular rate, is estimated to be the year 1945. This means that the long postwar boom witnessed a falling rate of introduction of basic innovations. Thus, once again the predictions of long-cycle theory concerning basic innovations are corroborated by empirical evidence.

**Figure 3: Cumulative Basic Innovations, 1872-1920**

![Cumulative Basic Innovations, 1872-1920](image1)

**Figure 4: Cumulative Basic Innovations, 1920-1970**

![Cumulative Basic Innovations, 1920-1970](image2)
Table 3: Parameters of Logistic Curves for the Cumulative Series of Basic Innovations

<table>
<thead>
<tr>
<th>Long Counter-Cycles</th>
<th>$K$ Upper Asymptote</th>
<th>$a$</th>
<th>$b$</th>
<th>$t_m = \left( \frac{b - K}{a} \right)$ Inflection point</th>
<th>$\Delta t$ (10%-90%)</th>
<th>$R^2$-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>1872-1920</td>
<td>94.289 (42.8)</td>
<td>0.119 (16.7)</td>
<td>-2.316 (21.6)</td>
<td>Year 1891.5 (2963.33)</td>
<td>37.01 years (16.71)</td>
<td>97.9%</td>
</tr>
<tr>
<td>1920-1970</td>
<td>123.73 (36.3)</td>
<td>0.098 (20.42)</td>
<td>-2.508 (32.35)</td>
<td>Year 1945.47 (2484.05)</td>
<td>44.63 years (20.42)</td>
<td>98.7%</td>
</tr>
</tbody>
</table>

It is important to note that we also tested logistic fits for weighted series of innovations. In creating the times series of our super-sample each basic innovation was considered as important as any other regardless of whether it was included in only one of our three sources or in two of them or in all three. If we regard then an innovation that is included in all three of our sources as ‘more important’ than an innovation that is included in only two of the sources, and in turn if we regard an innovation included in two sources as more important than one included in just one source then we should, perhaps, weight the basic innovations of our super-sample to check whether there would be any changes to the results of Table 3 as far as the logistic fits in the unweighted cumulative series are concerned. We set a weight of 3 then to a basic innovation that appears in all three samples, a weight of 2 to a basic innovation that appears in only two samples, whereas each innovation that appears in just one sample gets a weight of 1. The results of the regressions (not presented here) were once again very good and do not present any significant differences from the unweighted series of basic innovations.

In short, the logistic fit of the series of cumulative basic innovations, whether weighted or unweighted, is supportive of the hypothesis of the swarm-like appearance of innovations during the last years of the depressionary stage of the cycle, as we found by counting innovations in Table 2 but also much more precisely through the logistic fits in Figures 3 and 4. A question that arises then is whether innovations cause the long cycle or whether the downturn of the long cycle creates the fertile environment for the innovations to "grow". We grapple with this question in the next section.

5. Explaining the Long Cycles

The causa causans of the long cycles and the associated with them phenomena can be found in the long-term movement of the rate of profit, that is, the principal economic variable accepted as such by most if not all major economists of the past (Smith, Ricardo, Marx,

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15 Estimations were performed with the logistic equation $Y(t) = K/[1 + e^{-4.39(t-t_m)/\Delta t}]$ and its equivalent form $Y(t) = K/(1 + e^{-4t-P})$. The absolute values of t-statistics are in parentheses and all of them indicate zero p-values. For more about the logistic curves, the meaning and the derivation of the critical points see the Appendix.

16 This is a procedure suggested by Kleinknecht (1990).
Walras, Keynes, Schumpeter, among many others). There is also agreement among major economists about the long run falling tendency of the rate of profit which in its simple but adequate definition is the rate of net profits over the fixed capital stock. From all the above major economists though, it was only Marx that managed to develop a quite precise theory according to which the falling rate of profit past a point leads to a stagnating mass of real net profits which discourage investment spending and therefore no new jobs are created while the capital stock is underutilized and thus unemployment rises. The lack of new investment and the rising unemployment define the state of depression.

The rate of profit defined as net profits, \( s \), over the invested capital \( K \) can be decomposed into two terms, the ratio of \( s \) over net value added \( Y \) which is the share of profits in value added and the ratio of \( Y \) over \( K \) which is the inverse of the capital-output ratio and which could also be interpreted as the maximum rate of profit \( R \), that is, the rate of profit in the extreme case where wages are equal to zero. Thus we may write \( r = (s/Y)(Y/K) \). The overall movement of the rate of profit will depend then on the net result between these two forces. Marx’s argument is that the maximum rate of profit will tend to fall in the long run due to the inherent tendency of the system towards the capitalization of production, i.e. towards a rising capital-output ratio. Marx expects this capitalization of production to come about due to the effects of competition which takes place by the mechanization of production and by the introduction of new techniques and new products as each capitalist strives to expand his market share at the expense of his competitors as a condition for survival in the battle of competition. The profit share on the other hand will depend on the class struggle regarding the distribution of income between capitalists and workers. Yet, while the profit share has an obvious limit on the effect that it can exert on the overall movement of the rate of profit since it cannot exceed one, the output-capital ratio has no such limit. So, Marx’s point is that the net effect of these two ratios, namely the maximum rate of profit and the profit share, on the actual rate of profit will eventually be a negative one and so the actual rate of profit will be expected to fall in the long run.\(^{17}\)

The overall movement of the rate of profit will depend then on the movement of these two variables; in particular, Marx’s argument is that the capital-output ratio will tend to rise in the long-run; the profit to wage ratio, on the other hand, will have a limited effect on the rate of profit due to the inelastic nature of the profit-wage ratio with respect to the rate of profit. Somewhat similarly to the profit-wage ratio, the profit share has a strictly defined limit on the effect that it can exert on the overall movement of the rate of profit since it cannot exceed one, while the output-capital ratio has no such limit. Since capitalization characterizes the nature of the capitalist system, as it has been repeatedly argued and testified empirically, it follows that the rising capital-output ratio or what is the same thing the falling maximum rate of profit shapes, in the long run, the movement of the actual rate of profit. It is important to stress at this point that a falling rate of profit in and of itself does not lead to a stagnant mass of real profits and it is possible to be accompanied even by rising capital accumulation. For example, Marx (1981, p.349) notes: “A fall in the rate of profit and accelerated accumulation are simply different expressions of the same process, in so far as both express the

\(^{17}\) If we had chosen to decompose the rate of profit into a profit-wage (or rate of surplus value) ratio term and a capital-output ratio term then it can be proved that the elasticity of the profit-wage ratio with respect to the profit rate is smaller than that of the capital-output ratio (see Tsoulfidis 2017).
development of productiveness”. Only if the rate of profit falls for a protracted period of time will the mass of profits stagnate and display a growth rate equal to zero which discourages investment, since more investment spending does not change the profit picture of the economy. We can derive this tipping point in the mass of profits starting with the rate of profit \( r \) defined as the ratio of total net profits \( s \) to stock of capital \( K \):

\[
r = \frac{s}{K} \quad \text{or} \quad s = rK
\]

By taking first differences and by dividing by \( \Delta K \neq 0 \), we obtain:

\[
\frac{\Delta s}{\Delta K} = r + K \frac{\Delta r}{\Delta K}
\]

By factoring out the rate of profit, the above can be rewritten:

\[
\frac{\Delta s}{\Delta K} = r \left(1 + \frac{\Delta r}{\Delta K} \frac{K}{r}\right)
\]

The term \( \Delta s/\Delta K \) indicates the change in the mass of real net profits caused by changes in investment \( (l = \Delta K) \). The point of ‘absolute overaccumulation’ according to Marx is reached when \( \Delta s/\Delta K = 0 \) (Marx 1981, pp. 359-360) and for this to occur it must be that either \( r = 0 \) (the trivial case) or the term in parenthesis must be equal to zero and this may occur if and only if the elasticity of the rate of profit with respect to the capital stock is equal to \(-1\), that is if \( \frac{\Delta r}{\Delta K} \frac{K}{r} = -1 \), a condition that presupposes a persistently falling rate of profit (for further discussions see Shaikh 1992; Papageorgiou and Tsoulfidis 2006 and Tsoulfidis and Tsaliki 2014).

When the economy reaches this point, new investment fails to generate any new profit and crisis ensues. It should also be noted that for Marx this effect of a falling rate of profit on the mass of profit takes place after a long period: "The law [of the falling rate of profit] operates therefore simply as a tendency, whose effect is decisive only under certain particular circumstances and over long periods" (Marx 1981, p. 346).

We will test the tendency of the rate of profit to fall as well as the presence of the point of absolute overaccumulation using data for the US economy over a period ranging from the 1840s down to the present. Finally, we will test the extent to which we could identify long cycles in the mass of real net profit by fitting to them a series of logistic curves in an effort to ascertain whether the turning points of the logistic curves are more or less the same to those identified in the normalized price index and the periodization displayed in Table 1.

Figure 5 depicts the rate of profit for the US economy over a period ranging from 1840 to 1939. This period spans approximately two long cycles, one from the mid-1840s to the mid-1890s and another one from the mid-1890s to the mid-1930s. Evidently the rate of profit does exhibit a falling tendency over this time period which covers two long cycles in the normalized price index. Moreover the rate of profit falls during both the prosperity and the stagnation phases of the long cycle.
Table 4 decomposes the movement of the rate of profit into its two main components discussed above, i.e. the output-capital ratio (‘maximum rate of profit’) and the profit share, and depicts the average annual growth rates of all three variables, for the two periods during which there occurred long cycles related to the mass of profits. The results corroborate Marx’s prediction that the capitalization of production, expressed as a falling output-capital ratio, will dominate, in the long run, the effect that the distribution of income might have on the movement of the actual rate of profit.

Table 4: Average Annual Growth Rates of the Rate of Profit and its Components, 1844-1934

<table>
<thead>
<tr>
<th>Period</th>
<th>Maximum rate of profit</th>
<th>Share of profits</th>
<th>Rate of profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1844-1896</td>
<td>-3.10%</td>
<td>0.34%</td>
<td>-2.76%</td>
</tr>
<tr>
<td>Average annual growth rate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1896-1934</td>
<td>-0.99%</td>
<td>0.43%</td>
<td>-0.56%</td>
</tr>
<tr>
<td>Average annual growth rate</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In order to test for the existence of long cycles during that time period, we attempt to fit logistic curves to the mass of profits. The cycle in this case would concern the growth rates of the mass of real profits. In effect we test for two long cycles over the 1844-1934 period, based on the periodization derived by the normalized price index. Figures 6 and 7 then depict the logistic fits to the mass of real profits for the period 1844 to 1896 and for the period 1896 to 1934, respectively. Table 5 summarizes the results of the logistic fits. The fits are pretty good.

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18 The data come from the unpublished dissertation of Malloy (1994) who estimates profits by subtracting from the GNP the sum of depreciation and total wages all expressed in constant 1929 prices (Malloy 1994, p. 199). The economy-wide rate of profit is then estimated as the ratio of the so-derived profits over capital stock, which includes producer durables and structures (Malloy 1994, p. 135) in 1929 prices.
and the estimated parameters are all highly significant. The mass of profits of the USA economy seems to follow two long cycles, in terms of the evolution of its growth rates, almost concurrent with the corresponding long cycles in the normalized price index. The two logistic fits present points of inflection in 1871 and 1914, years which are very much in accordance with the long-cycle periodization. Furthermore, we should note here that we selected the year 1934 as the last year of the logistic evolution of our real profits for reasons that have to do with the exceptional turbulence characterizing the 1930s; more specifically, after reaching an unprecedented trough during the worst years of the Great Depression, real profits increased abruptly by 31.6% during the 1934-1936 period and then they stagnated again up until 1938. It seems not unreasonable that the stagnation in profits would continue but the resumption of large government expenditures, the preparations for War and the War itself halted the continuation of the logistic evolution of profits. It is important to note that once the logistic curve reaches the 90% of its asymptote then it may be thought of as nearly complete. Concerning then the year 1934 our estimations show that the logistic process had completed 89.3% of its growth (relative to its asymptote). In this sense we can, perhaps, claim, without exaggerating, that although this particular logistic fit is probably the least satisfactory among those presented in this paper, it captures nevertheless a great deal of the developments that took place during the turbulent third long cycle.

Figure 6: Real Mass of Profits (billions of $1929), 1844-1896, Logistic Fit
Figure 7: Real Mass of Profits (billions of $1929), 1896-1934, Logistic Fit

Table 5: Mass of real profits, parameters of the logistic curve, 1844-1896 and 1896-1934

<table>
<thead>
<tr>
<th>Long Cycles</th>
<th>A (Lower Asymptote)</th>
<th>K (Upper Asymptote)</th>
<th>α</th>
<th>b</th>
<th>$t_m = \left( -\frac{b}{a \cdot K} \right)$ inflection point</th>
<th>$\Delta t$ (10%-90%)</th>
<th>$R$-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>1844-1896</td>
<td>1.096 (3.03)</td>
<td>10.184 (24.49)</td>
<td>0.118</td>
<td>-3.201</td>
<td>1871.153 (1875.19)</td>
<td>36.48 (7.12)</td>
<td>96.9%</td>
</tr>
<tr>
<td>1896-1934</td>
<td>10.041 (fixed value)</td>
<td>31.62 (16.66)</td>
<td>0.1067</td>
<td>-1.98</td>
<td>1914.534 (890.397)</td>
<td>40.27 (7.01)</td>
<td>92.39%</td>
</tr>
</tbody>
</table>

We repeat the same exercise for the post-WWII period, this time covering the corporate sector of the US economy. Figure 8 below depicts the average corporate rate of profit spanning a period from 1946 to 2016, covering thus the fourth long cycle and (the greater part of) the fifth long cycle which has not ran its full course yet. The falling tendency of the rate of profit is again quite clear. Figure 9 depicts the fit of a logistic curve to the mass of profits for the 1946-1982 period, Table 6 presents the average annual growth rates of the rate of profit and its two components and Table 7 summarizes the econometric results of the logistic fit.

We find that the midpoint of the logistic, that is, the point of alternation from increasing to decreasing growth rates in the mass of real profits occurs in the year 1965. Though we do not show them here, we derived very similar results for the corporate non-financial sector of the US economy. These results, however, do not extend to the mass of profits of the corporate financial sector. The reason probably is that as profitability in the productive sector of the

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19 Estimations for the second long cycle (1844-1896) were performed with the logistic equation $Y(t) = A + (K - A)/\left[1 + e^{-4.39(t-t_m)/\Delta t}\right]$ and its equivalent form $Y(t) = A + (K - A)/(1 + e^{-4t/a-b})$. The absolute values of t-statistics are in parentheses and all of them indicate zero p-values. It might be noticed in passing that for the fitting of the logistic curve in the third long cycle (1896-1934) the parameter $A$ was not found to be statistically significant; since the logistic curve of real profits does not start from zero, we fixed this parameter to equal the first observation of our data series with $A = $10.041 billion dollars.
economy stagnates eventually during the long downswing, capital seeks a safer haven in the financial sector tending to increase the profits of the latter.

So it appears again that a secularly falling rate of profit was accompanied by a long cycle in the growth rate of the mass of profits that took place over the 1946-1982 period at approximately the same time range as the long cycle in the normalized price index of the USA. Furthermore the fall in the rate of profit was dominated by a rising capital-output ratio while the effect of the profit share was rather minimal.

Figure 8. Corporate Rate of Profit, USA

<table>
<thead>
<tr>
<th>1946-1982</th>
<th>Maximum rate of profit</th>
<th>Share of profits</th>
<th>Rate of profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average annual growth rate</td>
<td>-1.86%</td>
<td>-0.18%</td>
<td>-2.04%</td>
</tr>
</tbody>
</table>

Observing the rate of profit over the 1946-2016 period as a whole we note that from the late 1940s up until the mid-1960s the rate of profit is at a relatively high level from which it then falls reaching a trough in 1982 as the stagflation crisis reached its apex. From then onwards, for reasons that have to do also with the policies pursued during the so-called neoliberal period, the rate of profit does not exhibit such a clear trend though it seems that up to 2016 each successive profit-rate cycle depicts a lower peak than the previous one. The year 2001 is

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20 Sources: The mass of profits is equal to current corporate net operating surplus (BEA 2016, NIPA - Table 1.14, Line 8) adjusted for net interest paid, as explained in Shaikh (2016, pp.841-842). The adjusted net operating surplus is thus “conceptually similar to the financial accounting concept of earnings before interest and taxes” (Shaikh 2016, p. 841). The capital stock used is the current gross capital stock for the corporate sector of the US economy calculated in the manner derived by Shaikh (2016, p. 821).
associated with a slowdown in the level of economic activity and the year 2007 with the onset of the 'Great Recession'. The relatively low level of the rate of profit during the neoliberal period renders the US economy rather crisis-prone as can be deduced also from the relatively low GDP growth rates of the US economy during that period in Table 1. Moreover, pretty much like the rate of profit, the growth rates of GDP of the US economy (Table 1) display a falling long-term tendency over the whole trajectory of US capitalism, which becomes much more pronounced during the post-WWII period.

Figure 9. Real Mass of NOS (billions of $2009), 1946-1982, Logistic Fit

Our attention now turns to the fifth long cycle which is still underway. Figure 10 presents a logistic fit for the period 1982-2016. The estimations of the parameters, which are highly significant (with the partial exception of the lower asymptote), are displayed in Table 7. The logistic is projected up until 2028. This attempt to a forecast should be viewed with extreme caution for two main reasons: first because the long cyclical movement is by no means complete and second because the Bureau of Economic Analysis revises each year the data of the last years of the National Income and Product Accounts tables. Nevertheless, with what we have at hand we see a midpoint of the logistic in 2007. The year at which a logistic curve reaches 90% of its asymptote can be viewed as a reasonable time threshold after which the end of the long cycle is approaching. Our parameter values suggest that this 90% threshold will be reached in the year 2027 (to give a better sense of the movement of the economy we extended the fitted data up until the year 2028). Again, though not presented here, we derived similar results for the non-financial mass of profits of the corporate sector for the same time period, with the midpoint reached in 2007 and the 90%-of-the-asymptote point reached in 2025).

Sources: The current mass of profits (derived as indicated in footnote 20) is deflated by the implicit price index of gross fixed nonresidential investment (BEA 2016, NIPA - Table 1.1.9, Line 9). The same procedure is followed for the data of Figure 10.
Summary and conclusions

In this article, we endeavoured to show the existence of long cycles in a number of crucial variables and made an effort to explain these long cycles through the movement of the rate of profit and the associated with it mass of real profits. More specifically, during the upswing of the long cycle we expect to find a rising growth rate in the mass of real profits and usually, at the beginning of the upswing, also a rising rate of profit. Eventually, the rate of profit will assume again its downward tendency, while the long upswing still lasts, due to the effect of a rising capital-output ratio. The upswing will also be characterized by rising investment spending and growing output. Furthermore a general climate of optimism will prevail among

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22 Estimations for the mass of profits of the fourth long cycle (1946-1982) were performed with the same logistic equation as for the second (1844-1896) long cycle (see footnote 19 and the Appendix). The absolute values of the t-statistics are in parentheses and all of them indicate zero p-values. Estimations for the still uncompleted fifth long cycle (1982-2016) were performed again with the same equation. The absolute values of the t-statistics are in parentheses. The p-value of the lower asymptote, A, is 0.101. The p-values of the remaining parameters are equal to zero (at the second decimal level).
businesspeople while among economists many will perhaps share the view that depressions are a thing of the past. For instance, during the 1960s and again during the 1990s, that is, during periods of a long upswing, even great economists downplayed the possibility of any serious cyclical downturn occurring again.

Eventually the falling rate of profit will make its influence felt on the rate of investment and the growth rate of the mass of profit will begin to display a declining tendency, signalling the start of the long downswing. At some time in the downswing the point of absolute over-accumulation will also be reached, the mass of profits will become stagnant and a general crisis will ensue. The banking sector will reduce interest rates to try and generate new demand and therefore will tend to create bubbles. The subsequent bursting of these bubbles will devalue capital giving rise, eventually, to a boost to the rate of profit. But even more important in this respect is the introduction of basic innovations into the system that will tend to devalue elements of the capital stock therefore giving a positive boost to the profit rate. The rate of profit will also eventually be positively affected by the lower wages as a result of the rising unemployment. Eventually the mass of profit will rise and the diffusion of the basic innovations across the whole economy will lead to a new techno-economic configuration that will characterize the upswing of the next long cycle.

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Appendix: Logistic Curve

The logistic curve is given by the equation

$$Y(t) = \frac{K}{1 + e^{-\alpha t - b}}$$  \hspace{1cm} (1)

Where $K, \alpha$ and $b$ are the three (positive) parameters of the logistic function and $t$ is time, the independent variable of the logistic function. $K$ is the (upper) asymptote of the curve, $\alpha$ is known in the relevant literature as the ‘growth rate parameter’ and $b$ is a parameter that is related to the position of the curve with regard to time, i.e. the horizontal axis.

By taking the second derivative with respect to time of equation (1) and setting it equal to zero we find that the inflection point occurs at $t_m = -b/\alpha$. Substituting this $t$ value in (1) we get $Y = K/2$. Thus the midpoint and the inflection point coincide. Substituting $t_m$ for $b$ in equation (1) we get then the expression:

$$Y(t) = \frac{K}{1 + e^{-\alpha (t-t_m)}}$$  \hspace{1cm} (2)

So, before the $(-b/\alpha, K/2)$ point the logistic curve is characterized by increasing growth rates and past that point by decreasing growth rates. This means that a possibly successful fit of a logistic curve to a particular variable, characterizes concurrently not only the trend of this variable but also the cyclicality of its growth rates.

Equation (2) expresses the logistic process in terms of the parameters $K$, $t_m$ and $\alpha$. It would be perhaps more meaningful to replace the parameter $\alpha$ with another parameter which we would express with $\Delta t$. This parameter represents the time period (for example in years) that the fitted variable needs in order to grow from 10% to 90% of its asymptote value. Thus $\Delta t$ is a parameter that measures the speed of completion of the logistic growth process (Marchetti 1991, p. 13 and Grubler 1990, pp. 14-15). We define then $\Delta t$ as

$$\Delta t = t_{90} - t_{10}$$  \hspace{1cm} (3)

We have seen that the value of $t$ at half the completion of the logistic process (i.e. at 50%) is $t_{50} = -b/\alpha$. In order to find the values of $t_{90}$ and $t_{10}$ we substitute $Y(t) = 0.9 \cdot K$ and $Y(t) = 0.1 \cdot K$ in equation (1) respectively. After some manipulation we get

$$\Delta t = \frac{\ln (81)}{\alpha} = \frac{4.39444915...}{\alpha}$$

Inserting the $\Delta t$ term then in (2) we get

$$Y(t) = \frac{K}{1 + e^{-4.39(t-t_m)/\Delta t}}$$  \hspace{1cm} (4)

We have thus expressed the logistic growth process with the following three parameters: $K$ which is the asymptote of the process, $t_m$ which is the point in time at which the process gets at 50% of its completion and simultaneously achieves its maximum growth rate, and $\Delta t$ which expresses the speed of the process and which is inversely related to $\alpha$, i.e. to the growth rate in the special case where the growth process is exponential and unconstrained.

Equation (4) is the equation that we use to fit the logistic curve to our basic-innovation series. However, while the typical logistic curve in mathematics or biology has zero as the
initial value of the variable under examination, most variables in economics to which a logistic curve is to be fit have starting values that are positive and usually distant from zero. In order then to take this information into account, we use the following logistic curve:

\[ Y(t) = A + \frac{K - A}{1 + e^{-at-b}} \]

Which in terms of the \( t_m \) and \( \Delta t \) parameters can be written as:

\[ Y(t) = A + \frac{K-A}{1+e^{-\Delta t(t-t_m)/\Delta t}} \]  \hspace{1cm} (5)

Where \( A \) is the lower asymptote of the logistic function \( Y(t) \). Equation (4) then is a special case of equation (5) for \( A = 0 \). The presence of the lower asymptote, \( A \), assists us in determining the period during which we wish to examine the presence of long cycles, in the time series of a particular variable. Of course, it does not create these cycles; the testing of the possible existence of these cycles will be based on the statistical significance of the estimated parameters. The midpoint and inflection point of (5) take place then at \( \left(-\frac{b}{a}, \frac{K-A}{2}\right) \), which again pinpoints the time (year) at which the fitted variable depicts its maximum rate of growth. We should also point out that in the case of the 4-parameter logistic \( \Delta t \) does not measure anymore the time period that \( Y(t) \) needs in order to go from 10\% to 90\% of its estimated upper asymptote, \( K \), but the time needed by \( Y(t) - A \) to cover the 10\% to 90\% range of \( K - A \), where \( A \) is the estimated lower asymptote.

The equations for the second, fourth and fifth long cycle in the mass of profits have been estimated with the formula of equation (5). For the estimation of the third long cycle in the mass of profits, parameter \( A \) was not found to be statistically significant; in this case we used in place of \( A \), the initial value of our data series, which for the beginning of the third long cycle in 1896 was $10.041 billion dollars (in 1929 prices).