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Abstract. We propose a general equilibrium model to study the spatial inequality of consumers and firms within a city. Our mechanics rely on Dixit and Stiglitz monopolistic competition framework. The firms and consumers are continuously distributed across a two-dimensional space, there are iceberg-type costs both for goods shipping and workers commuting (hence firms have variable marginal costs based on their location). Our main interest is in the equilibrium spatial distribution of wealth. We construct a model that is both tractable and general enough to stand the test of real city empirics. We provide some theoretical statements, but mostly the results of numerical simulations with the real Moscow data.

Keywords: spatial distribution, linear city, circular city, monopolistic competition

1 Introduction

We propose a general equilibrium model to study the spatial inequality of consumers and firms within a city. Our mechanics rely on Dixit and Stiglitz monopolistic competition framework [1]. The firms and consumers are continuously distributed across a two-dimensional space, there are iceberg-type costs (as in Krugman's models of trade[2]) both for goods shipping and workers commuting (hence firms have variable marginal costs based on their location, we borrow some of Melitz's ideas here [6]). Our main interest is in the equilibrium spatial distribution of wealth. We construct a model that is both tractable and general enough to stand the test of real city empirics. We provide some theoretical statements, but mostly the results of numerical simulations with the real

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Moscow data. Our model is somewhat similar to [7], however, we rely on a different framework, adjusting the ideas from the international trade models to the scale of a city.

Our theoretical modelling process consists of two parts. First, we developed a one-dimensional model of the linear city on the $[0, 1]$ interval. This model is an adaptation of the Dixit-Stiglitz-Krugman model of trade to the case of spatial distribution of firms and consumers within the city. Although it is far from reflecting the real city structure, it is easier to solve and interpret the results. Second, we demonstrate how the unidimensional model can be upgraded to the empirics-friendly two-dimensional setup.

2 One-dimensional City

Consider a city with workers distributed uniformly on $[0, 1]$ and firms distributed uniformly on $\left[\frac{1}{2} - \frac{N}{2}; \frac{1}{2} + \frac{N}{2}\right]$, where N is an endogenous parameter for the total number of the firms within the city. Each firm offers a single variety of a composite differentiated good. We assume that the demand for that goods is not only due to workers spending their wages, but also due to firm owners spending their profits, so the consumers (both "firms" and workers) are distributed on $\left[\min\left\{0, \frac{1}{2} - \frac{N}{2}\right\}; \max\left\{1, \frac{1}{2} + \frac{N}{2}\right\}\right]$, which is hereinafter denoted by χ.

Consumer located at point x (or just consumer x) has an endowment of $e(x)$. Firm located at point y offers its own variety of composite differentiated good at the price $p(y)$. So here the product differentiation coincides with spatial differentiation of firms — there is just one firm standing on the head of a pin. Each consumers preferences are given by a CES-type utility function, thus they exhibit the preference for variety. By solving the consumer's problem we obtain the demand $q(x, y)$ — the amount of good that consumer x wants to buy from firm y.

Our approach to firm's production differs from the classical one in the structure of marginal costs. We assume that a firm has an equal probability of hiring any worker in the city — firms can't discriminate workers by their location, thus there is a uniform equilibrium wage w. So in our model workers do not choose place, where they want to work and wage, that they want to earn - place, where they will work chooses randomly and wage is the same for all workers. Workers commuting costs are paid by the firms, so that distant commuting results in low productivity. Thus we have the following formula for labor requirements to produce a unit of good for a firm located at point y:

$$c(y) = a \int_0^1 (1 + \theta\rho(x, y))dx,$$

where $\rho(x, y)$ is a distance from point x to point y. Firms also have to pay a fixed cost of $fc(y)w$.

The equilibrium in our model is characterized by the following conditions:

1. Full employment: all the labor supplied is used in production. Let $C(y)$ be the total labor used by a firm located at y, so that we have:

$$\int_{\frac{1-N}{2}}^{\frac{1+N}{2}} C(y)dy = 1.$$

2. Free entry: firms located on the borders of the city have zero profits. Let $Q(y)$ be the overall amount of goods sold by firm y, therefore we have:

$$\begin{aligned}&(p\left({}^{N-1}/{}_{2}\right) - wc\left({}^{N-1}/{}_{2}\right))\, Q\left({}^{N-1}/{}_{2}\right) - wc(y)f = \\ &\qquad (p\left({}^{N+1}/{}_{2}\right) - wc\left({}^{N+1}/{}_{2}\right))\, Q\left({}^{N+1}/{}_{2}\right) - wc(y)f = 0.\end{aligned}$$

3. Finally, the budget balance: the total expenditure at some point equals the sum of worker's wage and firm's profits — of course, if they are located there. Thus:

$$e(y) = \begin{cases} 0, & y \notin [0;1] \cup \left[\frac{1}{2} - \frac{N}{2}; \frac{1}{2} + \frac{N}{2}\right], \\ w, & y \in [0;1] \setminus \left[\frac{1}{2} - \frac{N}{2}; \frac{1}{2} + \frac{N}{2}\right], \\ (p\left(y\right) - wc\left(y\right))\, Q\left(y\right) - wc(y)f, & y \in \left[\frac{1}{2} - \frac{N}{2}; \frac{1}{2} + \frac{N}{2}\right] \setminus [0;1], \\ w + (p\left(y\right) - wc\left(y\right))\, Q\left(y\right) - wc(y)f, & y \in [0;1] \cap \left[\frac{1}{2} - \frac{N}{2}; \frac{1}{2} + \frac{N}{2}\right]. \end{cases}$$

3 Results for One-dimensional City

We are able to prove the equilibrium existence result for the model, also we do some natural comparative statics. However, for illustrative purposes we created a MATLAB program to find the equilibrium given all the exogenous parameters and illustrate all the comparative statics of interest. For example, for the following values $f = 0.2,\ a = 1,\ \sigma = 2,\ \tau = 0.1,\ \theta = 0.1, \mathrm{L} = 0.1$ the total number of firms is 2.25 and the corresponding interval is $[-0.625, 1.625]$.

For these values of parameters we can look at the changes in the profit of firms $\pi(y)$ and the optimal number of firms N according to some changes in exogenous parameters. For all of the following graphs horizontal axis shows the interval for firms and vertical axis shows the profits value. Blue lines here in after depict small values of the changing parameter, yellow lines — highest values.

For instance, figure 1 illustrates that when the fixed cost for the firms f rises, the optimal number of firms in the city falls, but the profit level of existing firms $\pi(y)$ in some times increases, but then falls with the rise of fixed costs. This is happening because when the fixed costs rise for sufficiently low number of firms, the effect from rising costs exceeds the one from "killing" competitors, so the profit falls.

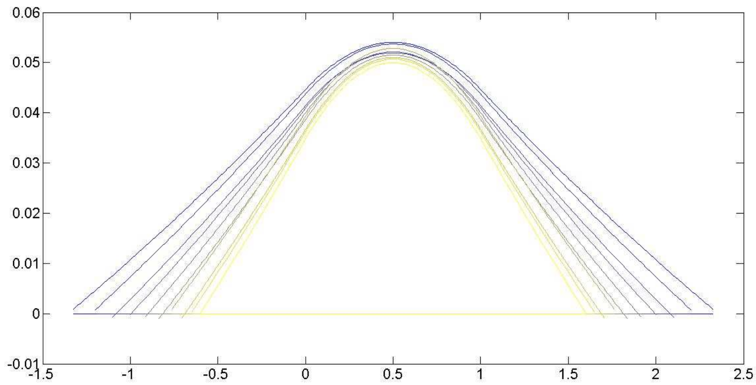
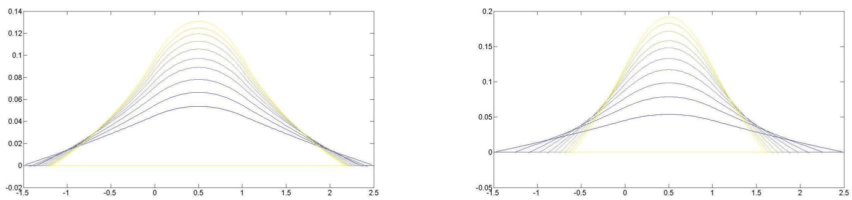


Fig. 1. Firm profits as f changes from 0.1 to 0.2.

As we can see from figure 2, increase in the sensitivity to changes in distance for the transportation of goods (τ) and workers (θ) affect the profit of firms differently. Although the number of firms N decreases only slightly with the rise of both τ and θ, it causes a significant increase in profit, which is 1.5 times greater for θ.



τ θ

Fig. 2. Firm profits as iceberg costs coefficient changes from 0.1 to 1.

Figure 3 shows us, that increase in elasticity of substitution (σ) leads to decrease in number of firms and in their profit. It happens because in this case preference for variety also decreases, so consumers prefer to buy goods from neighbours and the farthest firms lose their profit.

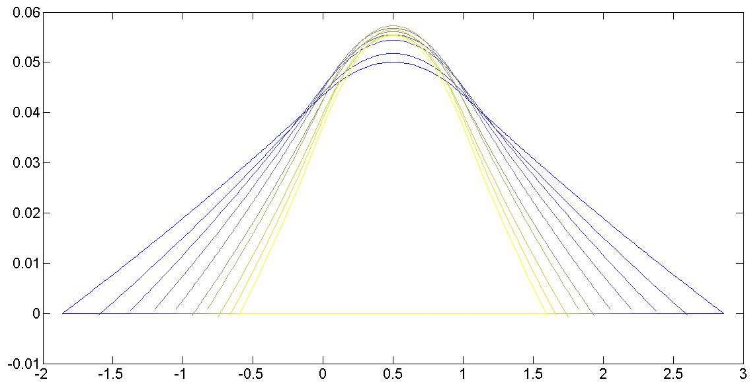


Fig. 3. Firm profits as σ changes from 1.5 to 3.5.

As we can see from figure 4, increase in number of workers gives us decrease in profit for firms. It happens because bigger part of all money in the city goes to wages for workers instead of going to profit for firms (total amount of money in the city is constant and equals to 1)

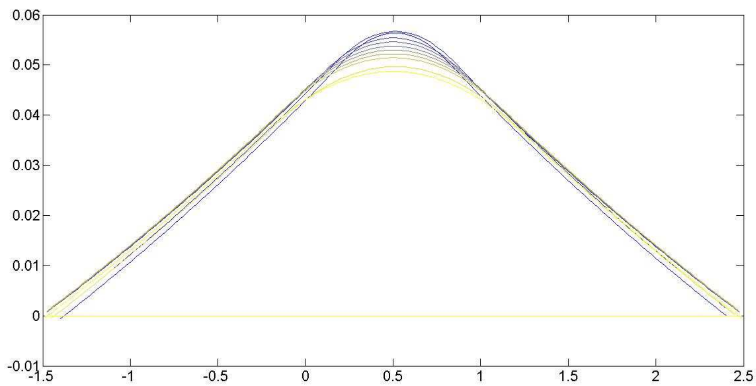


Fig. 4. Firm profits as L changes from 0.5 to 1.5.

4 Two-dimensional City

The generalisation of the model to the 2 dimensional case is straightforward — one just needs to replace the unit interval with some compact set in R^2 and the Euclidean distance with some other metric that reflects the structure of transportation within the city. The assumption that each point can host just one firm cam be relaxed with some more general capacity constraint, but then we will have also take into account that several varieties might be produced in a single point, instead of just one — technically, it adds just one more dimension to our model.

To adjust the model to the empirical estimation we assume that city is divided into M districts, residents are distributed among districts and so do the firms; each district has its own number of workers and a capacity constraint for firms. Also, firms may be located on the radial highways spanning outside from the borders of the city — we need this assumption to be able balance the number of firms with free-entry condition. All the calculations are rather similar to the continuous model, however, we use sums across districts to approximate the integrals across space.

The main difference lies in the form of Melitz-inspired cutoff level condition. We assume that firms, as they enter the market, first fill all the vacant offices inside the city, from best to worst, and then the remaining ones locate along the highways. So, if we have an exogenous value of the maximum number of firms in each district N_j, then we can write the cutoff level condition for each of the highways in linear form as in the one-dimensional model.

5 Finding real data for two-dimensional city

In our model we need to get some statistics about the city, such as distribution of workers, distribution of firms, distances between objects in the city. We get real data about Moscow, making detalization for districts. Distribution of workers we get from official statistics about population in Moscow. Distribution of firms we get by extrapolation of data from one non-official source, and distances between districts we estimated, using specially designed algorithm.

The results of model will be distribution of profit among all firms in city, value of wage and number of firms, which in model means length of highways. To compare these results with real situation, we also get statistics about profit for firms in all districts, using data about tax on profit from tax service.

6 Results for Two-dimensional City

Our next goal was to find out the values of exogenous parameters (elasticity of substitution, for example) such that the results of the model closer to real data.

We were unable to run the classical OLS, instead, we varied our exogenous parameters to make ratio between total profit of firms and total income of workers equal to the real ratio in Moscow. It happens for a set of parameters, so, after that we tried to match the length of "occupied" highways and maximise the Spearman coefficient that shows correlation between lists of districts, sorted by profit for one firm, from model and from real data.

In the end, we get that in all sets f and σ are inversely dependent, τ and θ are close to 1, Spearman coefficient is close to 0.62 and length of highways are at most 25 km.

Also our results can be illustrated by figure 5, where districts are marked with blue, if profit for conventional unit of firms there is bigger than income for conventional unit of workers, else district is marked with yellow.

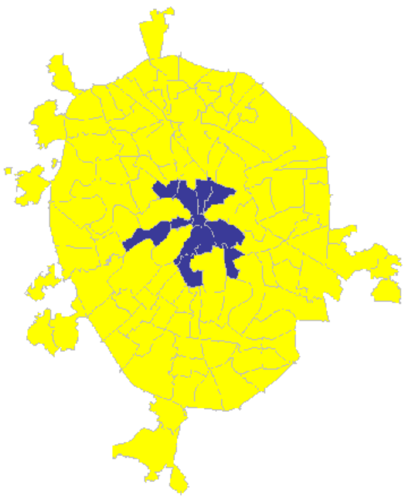


Fig. 5. Comparing profits of firms and income of workers

Fig. 6. Length of the highways

In figure 6 we illustrate length of highways that we get from the model.

And in figure 7 we compare distribution of profit for one firm between results from model and real data.

Model Real data

Fig. 7. Distribution of profit for firms

Conclusion

To summarize, we see our main contribution in developing a model that is both simple enough to be tractable and scalable enough to allow estimations using real city data. The key feature is our ability to incorporate real city travel time costs into a classical new economic geography mathematical framework. This model can be used to address a variety of questions, from estimating economic advantages of a particular location to evaluating the best directions of a long-term city development. Though in this paper we mainly demonstrate the results of model simulations, our efforts lead way to testing the model against a reality of a modern city.

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