The Cyclicity as Evolution Form of Economic Activities

UNGUREANU, Laura

Spiru Haret University, Faculty of Finnancial Management Accounting Craiova

17 April 2008

The Cyclicity as Evolution Form of Economic Activities

Assistant Professor PhD. Laura Ungureanu
Spiru Haret University

Abstract: The persistent of cycles was remark even to the 19th century economists and the rigorous theory of fluctuation or bussines cycle are take form past century. In the analyses dynamics macroeconomic area is can observe a big variety of method and techniques for research fluctuates from economy and financial date. A complex way for evidence the economic cycles is to determine limits cycles for the dynamical system which model the economic phenomenon.
Key words: cyclic evolution, dynamical modeling, nonliniarity.

Jel Classification: C53, E37

1. Economic Fluctuations and Economic Trend

Uncertainty is a crucial characteristic of the present economy, based on private property and individual decisions of economic agents. Extension of economic coverage area during the recent years and especially the acceleration of economic growth rhythm under progress of scientific and technical led to increase of these oscillations and fluctuations accompanying this economy.

The economic dynamics is vacillating and irregular over time and space because the economic growth factors have a non-linear and irregular evolution. Some evolutions and changes of economic activity conditions and outputs are accidental and other take place regularly, coming into a cycle movement, as sequence on some time interval, of alternating expansion and contraction phases. On the ground of this cyclical evolution, economic reproduction and growth may face with crisis status or phenomena. So the contemporary national economies have a vacillating evolution, with greater or smaller deviations from the general trend. In the literature this specific dynamics is known as *cyclical fluctuation,* fluctuation of business or business cycles.

Cyclicity as a form of movement of economic phenomena is featured under two main aspects [2]:
a) sequence and repeatability over time of some economic status, named cycle steps, that are generally the same from a cycle to another, during every step, economic condition and its aggregate performances – rhythm of growing of the domestic gross income, industrial and agricultural production, level labor employment, evolution of economic effectiveness, dynamics of living standard and life quality - have some characteristics and record significant fluctuation from a step to another;
b) Phase prepare, in their linkage, the requirements leading to qualitative changes of economic growth, providing their continuity and economic progress as well.

An economic cycle or a business cycle is featured by simultaneous growing of the most economic activities, followed by a decrease of these levels, after which the expansion phase of the next cycle occurs. So, the cycles may be defined as fluctuations around a magnitude that is a mean of economic growth during a given period of time.

In order to emphasize the cyclical fluctuations, chronological series of data regarding some indicators as gross and net domestic product, volume of retail, level of bank interests, employment rate etc. are processed.

In the field of review of macroeconomic dynamics and business cycles, a large variety of approaching methods and types for studying the fluctuations in economy and financial data can by observed today. These approaches have been inspired by a series of schools of thought as the kynesist, monetarist or that of reasonable expectations and they have been named as models of kynesian, expectation or real business cycles.

Of course, there is a long list of economic cycle theory. The persistence of cycles have been observed first by the XIX century economists, and rigorous theories on fluctuations and business cycles started to raise during the forth and fifth decade of the last century, through the contributions of Kalecki (1937), Frisch (1933), Kaldor (1940), Hicks (1950), Samuelson (1939) and Goodwin (1951). Still then two possible prospective in review of business cycle arose their development either by mechanisms generating oscillation (non-linear endogen cycles), or by random shocks over a fundamentally stable economic system (Slutsky-Frisch stochastic cycles).

During the recent years it can be seen an increasing interest of the scientific community for the review of non-linear systems, in this sense, a theory of endogen cycles is already outlining [1]. The research of such systems, started with the studies conducted by the researchers in the mathematics and natural science field, led to development of some new fundamental concepts and methods. Event hat their application within economic science is at the beginning, some remarkable results of large importance for economists have been already achieved. There are a number of economic fields and contexts where the non-linear techniques are very useful, as the behavior of capital market and exchange rate [see 6], the matters of external debt, economic depressions, hyperinflation and back risk, estimation of yearly unemployment rate, capital or labor evolution [see 8].

The study of business cycles is very large, uses macroeconomy terms and has a large applicability into economic growth, monetary economy, inflation study, financial instability and gradual adjustment of prices.

2. Simulation and dynamic models

As we already discussed, starting with the years 30 a proliferation of formal methods using variants as investment accelerator and consumption-multiplicator type can be seen. A more general class of models is those based on the adjusted capital principle (or „flexible accelerator"): current investments are equal to the ratio between the desired, expected and real capital. The expected stock varies directly with the output. Net investments depend directly on output and reversely on initial capital stock. The dynamics of these models is determined by non-linearities. Non-linearities are used in the theory of business cycles ant they were systematically explored by the recent literature in the field. In this way, the analytic methods of the bifurcation theory or of the catastrophe theory in reviewing of economic fluctuations, crisis, depressions and fast recovering have been applied.

Another model, more complex, of outlining the business cycles is that of determination of the limit cycles appropriate to a dynamic system simulation the studied economic phenomenon.

The model, as scientific knowledge tool, is used by numerous theoretical and practical disciplines. The knowledge achieved from work with models and trial to apply them may develop valuable designs related to a problem and types of decision to be made. Simple knowledge of decision areas may be a major progress in many situations. Further, by using models, the economists may recognise the variables that may be controlled in order to influence the system performance, the relevant costs and their dimensions and also the correlation between costs and variables, the options for important costs included (achieving of a new location etc). Riatt appreciated that the "models developed in economy, generally, are took over in an extensive measure from real world than those developed in physics. The models regarding the system behaviour or reasonable ways of approaching the matter allow us a fundamental understanding need for diagnosis that remains a practical and pragmatic process".

The economic sciences, the models are used in all diversity of existing types. But, during the last decades, the trends to use pre-eminently, in these disciplines, the mathematic type models, due to their special ability to condensate rigorously the essential and also to their ability to be programmed by means of computational techniques, making up together a investigational scientific tool of an unknown power until now, a prodigious "extension" of human mind, are more and more outlined.

The mathematic application developed in many directions, over time, the most important being: model of approaching the decisions in management through simulation and elaboration of standard powerful models and methods able to solve some classes of decisions well determined and that are commonly met.

Simulation of an economic process is a scientific mean to detect the determinant factors arising within that phenomenon. In order to set the importance of these factors for the process considered, it is necessary to implement the model built of the most important factors, so that, whenever possible, their possible quantification allowing the mathematical treatment is provided.

Starting from the idea that every model is based on real data and parameters it is obvious that reliable data allowing a convenient representation of reality through model become necessary. In that manner, it is identified, when the case may be, the **cyclical** or periodic **aspect** of the studied phenomenon, the time horizon to which is refers included.

Occurrence of some global, planetary problems, influencing the growth process, of national, area or global level development as depletion of some non-recoverable natural sources, population structure, damaging of natural environment, technical development, food problem, excessive urbanization, economic sub-development, economic policy led to investigation of many theoretic problems and especially practical in the field of evolutional economy.

From econometric point of view the classical models, base on continuity, linearity and stability have been proved inadequate to be able to represent economic phenomena and processes of a high complexity level. Researchers are obliged to follow up these processes in a **dynamic way**, study the quantitative changes appearing among the economic variables involved and also the results achieved by their mean. Besides other characteristics, the mathematic models allow introduction of an isomorphism between the real economic system and the ideal one, represented by model. By their means, the approaching of instable behaviors of different non/linear economic system becomes possible, underlining that, in fact, linearity and stability are particular cases of economic evolution.

Simulation, consists in fact in building a representation of a variable fidelity degree of the real world or of a part of it. Understanding of phenomenon or of the approached reality segment, comprehensive knowledge and also the action of the reviewed phenomenon ground the reason to make appeal to such representations. The mathematic language is the most frequently used. The use of mathematic modelling helps to ground a decision in effective condition, giving the opportunity to think better and faster without distorting the reality.

Non-linearity of evolution of an enough large number of physics, biology, ecology and economy phenomena led to development of some modern sciences, sciences that try to approach, conceptualize and then use another aspect of reality, more fluctuant, more dynamic. These sciences are the result of integration of some model, theories and techniques of solving the systems of non-linear differential equations, of a prospective change of which, sometimes, arise new starting points in trying to better understand the studied phenomena. If models are adequate, then from knowing their solutions we can deduct the behavior of modeled phenomena. Even it is said that every non-linear model has its own theory, they have common, and unifying features, the behavior, as much as strange may be, of their solutions having correspondence in the aspect of the modeled phenomenon. The fact that this behavior has not been indicated yet, is duet o the complexity of the non-linear problems whose systematic review started few decades before.

Dynamic simulation is based on the fact that the running of a system is represented by knowledge of interactions between information flows, orders, human resources and material resources etc. A dynamic model surprises the behavior of complex systems showing how their structure determines the path, behavior over time, respectively.

Occurrence of non/linear dynamic theories enabled understanding and development of some processes and methods that bring us closer to phenomenon of reality. Development of singularity theory and bifurcation theory completed the numerous means we dispose to review and represent dynamics more and more complex, giving the opportunity to review some systems that were hard, if not impossible, to be approached through traditional methods. The review of non/linear dynamics is of maximum interest because the economic systems are above all non-linear systems. Most of them contain multiple discontinuities and incorporate an inherent instability being permanently subject of shock actions and external and domestic perturbations.

For smaller values of some parameters especially large changes of variables can take place, so bifurcations entering the considered system on other paths may be produced.

The bifurcation theory has the advantage of a very well elaborated mathematic device, reviewing both existence and stability of equilibrium situations, because a solution of instable equilibrium cannot be observed in reality.

For example, the simulation of the relation between economic development and pollution may be achieved through equation for determination of equilibrium status with keeping of environment quality, effect of policies over the equilibrium point, influence of anti-polluting equipment in establishing the maximum productive level, equilibrium status under dynamic conditions.

In practice, a series of techniques and methods is used including macroeconomic and international studies, sphere studies, prognosis, cost-benefit reviews etc.

The dynamic systems having as subject the study of determinist processes have known an extraordinary development, with applications in sundry fields as physics, chemistry, engineering and economy. A process is a determinist one if both its subsequent behavior and the past are uniquely determined by its present condition; the dynamic system is the mathematic model of a determinist process.

3. **Cyclical Evolution in a Non-linear Model of Economic Dynamics**

Generally, the models of economic growth explain the abundance of products and income through the combined contribution of two production factors: capital (K) and work (L). Dynamic or the rhythm of this growth depends on one hand on the capital and investments accumulation ratio, so on K growth, and on the other hand on the technical progress (that enable to reduce the L weight as a result of productivity increase). From the multitude of existing methods we shall pay attention to one of them for which we shall study the response of the economic system (as it is described by model) to the variations of the model data, and parameters. By means of data structures, transitions diagrams for status indication all specific operations for every type of article and corresponding class are built.

From the models governing the evolution of economic process, we shall consider a model consisting in a Cauchy's problem for a system of two ordinary differential equations of first order [n real field. It describes the evolution of company capital and of labour involved.

Either K_t capital at t time and L_t volume of labour (number of employed persons) then the company has a turnover y_t given by production function $y_t = F(K_t, L_t)$.

The capital evolution is function of company development policy, through the quota from revenues intended for investments $(1-\delta_t)\pi_t$, where π_t is the net profit achieved during year *t*, profit that can be allocated for development in whole or in part and namely the part remained after coverage of dividends to the company shareholders, through a quota of δ_t. Therefore, $\delta_t\pi_t$ is the dividend mass and $(1-\delta_t)\pi_t$ is the volume remained for investments. Considering the capital depreciation with the mean factor μ_t and the revenues achieved from liquidation of redeemed assets at the recovery cost λ_t the mathematic model of a company development is achieved, the basic equation of capital evolution [1]:

$$\dot{K}(t) = (1-\delta_t)\pi_t - \mu_t(1-\lambda_t)K_t$$

Either γ_t growing rhythm of capital expressed in percentage. Because $\pi_t = \gamma_t y_t$ it results

$$\dot{K}(t) = \gamma_t(1-\delta_t)F(K_t, L_t) - \mu_t(1-\lambda_t)K_t$$

We suppose that the variation of labour is

$$\dot{L}(t) = \alpha_1 K_t + \alpha_2 L_t - \alpha_0$$

And the company is featured by a production function with the form Cobb-Douglas, $y_t = AK^{\alpha}L^{\beta}$. If we consider the production growth greater than the growth of other factors, as a result of effectiveness increase (fact characteristic for the most dynamic economic fields), so production with increasing physic yield ($\alpha + \beta > 1$) to study the particular situation $y_t = AK^2L$.

In these conditions we achieve the equation system>

$$\begin{cases} \dot{K} = A\gamma_t(1-\delta_t)K^2L - \mu_t(1-\lambda_t)K, \\ \dot{L} = \alpha_1 K + \alpha_2 L - \alpha_0. \end{cases} \qquad (1)$$

Within this system K and $L : \mathrm{R} \to \mathrm{R}$ are unknown functions depending on independent variable t (time).

We make the simplifying method regarding the ratio constant from (1) and we note: $a = A\gamma_t(1-\delta_t),\ b = -\mu_t(1-\lambda_t)$.

Under the conditions mentioned above, the evolution of the company capital is governed by the Cauchy's problem $K(0) = K_0$, $L(0) = L_0$ for the system

$$\begin{cases} \dot{K} = aK^2L + bK, \\ \dot{L} = \alpha_1 K + \alpha_2 L - \alpha_0. \end{cases} \qquad (2)$$

With change of variable $x = \beta_1 K,\ y = \beta_2 L$, where $\beta_1, \beta_2 \neq 0$ and $\beta_1 = \alpha_1/\alpha_0,\ \beta_2 = 1/\alpha_0$, if $\alpha_0 \neq 0$ and $\alpha_1 \neq 0$, (2) becomes

$$\begin{cases} \dot{x} = cx^2 y + bx, \\ \dot{y} = x + \alpha_2 y - 1, \end{cases} \qquad (3)$$

where $c = a\alpha_0^2 / \alpha_1$. In this way, only three parameters remained in equation *b*, *c* and α_2. This reduction has as economic consequence emphasizing of some expressions, functions of primary economic parameters, that occur in capital and labour evolution in a company. So, the same value of a new parameter corresponds to an extended diversity of values of the former economic parameters, forming as such classes of equivalent economic situations.

4. System Dynamics

If $\alpha_2 bc \neq 0$, then the dynamic generated by (3) is extremely complex [7]. The equilibrium point $\bar{u} = (\bar{x}, \bar{y})$ is the solution of the algebraic system

$$c\bar{x}^2 \bar{y} + b\bar{x} = 0,\ \bar{x} + \alpha_2 \bar{y} - 1 = 0$$

The system linear element around the point $\bar{u}$ is:

$$\begin{cases} \dot{X} = (2c\bar{x}\bar{y} + b)X + c\bar{x}^2 Y \\ \dot{Y} = X + \alpha_2 Y \end{cases} \qquad (4)$$

And the own values s_1, s_2 are the solutions of the characteristic equation $s^2 - (\alpha_2 + b + 2c\bar{x}\bar{y})s + \alpha_2 (b + 2c\bar{x}\bar{y}) - c\bar{x}^2 = 0$

If $\operatorname{Re} s_1$, $\operatorname{Re} s_2 \neq 0$ the $\bar{u}$ equilibrium is hyperbolic and if $\operatorname{Re} s_1 = 0$ or/and $\operatorname{Re} s_2 = 0$ we say that the equilibrium is non/hyperbolic.

For hyperbolic equilibriums the Hartman-Grobman theorem asserts that, locally, the phase portrait of a liniarezed system (4) is topologically equivalent to phase portrait of the non-linear system (3). Of course, the linear case is much easier to treat and when the equilibrium is hyperbolic we can consider the problem solved. But it stays to study the non-hyperbolic equilibriums.

For the case of nonhyperbolic equilibriums there is not a theorem analogue to that of Hartman-Grobman, usually the phase portrait not being equivalent topologically. Therefore, the non-linear case should be treated separately, using the normal formulas or the theorem of central varieties.

For the case $\alpha_2 bc = 0$ we find that in the plan $b = 0$ (without axis and origin) there is the point of non-hyperbolic equilibrium $\bar{u}_1 = (0, 1/\alpha_2)$. In this case $s_1 = 0$, $s_2 = \alpha_2$.

Further, we will treat this case and we meet a situation less encountered: the centre from the linear case is kept and the case of the non-linear also. As consequence, **the evolution of the capital dynamics is cyclical.**

For $\alpha_2 = b = 0$, $c < 0$, (3) becomes

$$\begin{cases} \dot{x} = cx^2 y \\ \dot{y} = x - 1 \end{cases} \qquad (5)$$

Lemma 1. *The system (5) is invariant at transformation* $(t, y) \to (-t, -y)$.

Within the space (c, x, y) *the chart of static bifurcation is right line* $x = 1,\ y = 0$ *without the point* $(0,1,0)$.

Corollary 2. *If (5) has a sector of path its symmetrical to the Ox axis is also a sector of path. Further, on the sectors of path with y > 0 the time arrow is reversed to that on the sectors y < 0.*

Corollary 3. *If a path of (5) crosses the Ox axis in two points, then it is closed (i.e. is the centre).*

Linear system. The system (5) has an unique equilibrium $\bar{u}_1 = (1,0)$. The linear line around it is

$$\begin{cases} \dot{X} = cY \\ \dot{Y} = X \end{cases} \quad (6)$$

and enables the origin as solely equilibrium point. The own corresponding values are $s_{1,2} = \pm i\sqrt{-c}$, namely $\bar{u}_1$ is the center for the linear system (6), the phase paths being ellipses centered in this point and of implicit equation $X^2 - cY^2 = X_0^2 - cY_0^2$. The parametric equations may be achieved through direct integration. The phase portrait for (6) is given in fig.1a).

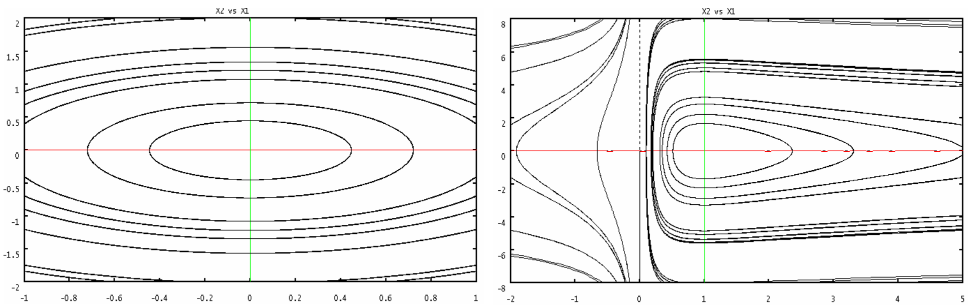

Figure no 1 - Phase portrait for a) (6) and b) (5)

Non-linear System In this case also, the explicit form of paths may be determined, using o first integral. Therefore, the straight line x = 0 is invariant, on it the dynamics being given by $y = -t + y_0$. This is a path for every initial point with the form $(0, y_0)$. For $x \neq 0$ the path through (x_0, y_0) has explicit form.

$$y = \begin{cases} \sqrt{2(\ln|x| + 1/x)/c + \Phi(x_0, y_0)},\ for\, y_0 \geq 0, \\ -\sqrt{2(\ln|x| + 1/x)/c + \Phi(x_0, y_0)},\ for\ y_0 < 0 \end{cases} \quad (7)$$

where $\Phi(x_0, y_0) = y_0^2 - 2(\ln|x| + 1/x)/c$, and $\Phi(x, y) = \begin{cases} y^2 - 2(\ln|x| + 1/x)/c,\ x \neq 0 \\ y^2, x = 0 \end{cases}$ is a first integral of a (5). In expressing y we included also the information that $y_0 = 0$ involves $x_0 = 1$ because (1, 0) is the equilibrium point. Even explicit, the complicated form of paths does not enable us to assert the existence of closed paths and if they are or not limit cycles (so, further, isolated into the path space). For this reason, three sentences are demonstrated, showing that for c < 0 any path passing through a point (x_0, y_0) is closed:

Proposition 1. *Curve* $g(x, y) = \frac{2}{c}(1 - \frac{1}{x} - \ln x) + y^2 = 0$ *is an invariant to the general dynamic by (5).*

Proof. Through direct calculation we have $\frac{dg}{dt} = \frac{dg}{dx}\frac{dx}{dt} + \frac{dg}{dy}\frac{dy}{dt} = \frac{2}{c}\left(\frac{1}{x^2} - \frac{1}{x}\right)cx^2 y + 2y(x-1) = 0$, from where comes the Sentence 1. It could be deduced that $dg/dt = 0$ and for this that $g(x, y) = \Phi(x, y) + 2/c$, where Φ is primary integral.

Proposition 2. *If* $c < 0$ *and* $x_0 < 0$ *then the paths of (5) cross the Ox axis in a single point.*

Proof. For $y_0 > 0$ the expression under radical from formula (7) is an ascending function of x and namely, increase from $-\infty$, for $x = -\infty$, at y_0^2, in x_0. As it is strictly monotone it results that there is a single value for x for which $y = 0$. The case $y_0 < 0$ is analogue.

Proposition 3. *If* $c < 0$ *and* $x_0 > 0$ *then the paths of (5) are closed curves.*

Proof. For $y_0 > 0$ the function under radical from formula (7) increases.

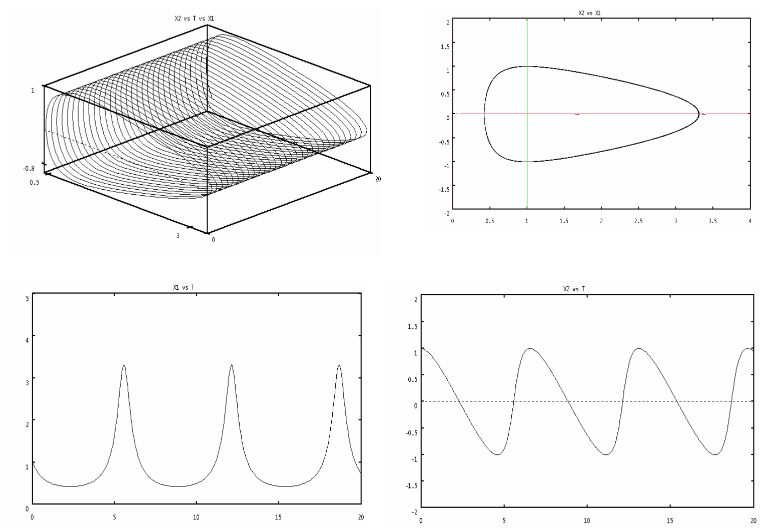

Figure no 2 - The Behavior of non-linear system

for $x_0 < 1$ and decreases for $x_0 > 1$. In x_0 has a point of maximum equal to $g(x_0, y_0) = \frac{2}{c}\left(1 - \frac{1}{x_0} - \ln x_0\right) + y_0^2 > y_0^2$, even that the function under radical crosses the axis Ox in two points. The case $y_0 < 0$ is analogue.

Corollary. *For $c < 0$ the non-linear system (5) has a centre in point (1, 0).*

We showed that, locally, the parametric portrait around $\overline{u}_1 = (1,0)$ for the non-linear system is topologically equivalent to that around the origin for the linearezed system, even that this equilibrium is not hyperbolic. In [8] we demonstrate that this equilibrium point is not neither of Hopf type nor of Bautin type and that the possible non/resonant terms that could lead to the topological non-equivalence between the linear and non-linear case are strictly of a degree greater than six.

5. Conclusions

The phases of business cycle are: crisis, depression, retaking and boom of economic activity.

The properly crisis is a sudden phenomenon, violent, disturbating, that is construed through a rude comedown, company bankruptcy and dismissing of labour. Crisis is a temporary breach bur sometimes violent and deep of an ascendant economic evolution, of the equilibrium between production and consumption, following up after the maximum point of increase in a cycle and it is expressed through a reversion of the trend making the balance of economy from increase to depression.

Localization of phase between the two phases of the disturbating cycle, should not lead to its association and confusion with the period of depression, coming after, under no circumstances. A difference should be made between crisis and depression. Depression is opposed to the violent, unexpected, disturbing character of crisis, as a slow period of restructuring, adjusting of the productive system, during which, negative phenomena as: production and wages decrease, increase of unemployment rate.

Depression is the phase coming after crisis and it is featured by stagnation of economic activity and even further decrease of production, increase of unemployment rate. It takes place a shortage of current production volume, severe measure to reduce costs are adopted of which the first is renewal of active work means on technical innovation basis.

Retaking or speeding up of economic activities is in a close connection with renewal of fixed assets and especially of its active parts, that leading to outrunning of the lowest point of that cycle. Speeding up of investment process aiming both development of new We shown that, locally, the parametric portrait around and renewal of those existing ones, feeds the demand for production means and labour, reducing the unemployment rate.

Expansion and boom are featured by credit abundance and a general increase of incomes. In such environment, more and more favourable, businesses become prosperous, with prospective for consolidation. A new investment process is launched, the existing production means are refurbished and new ones are developed. The continuous process started by increase of demand for consumption goods is the determinant factor for production increase and employment rate.

In conclusion, the economic forecast has a theoretic and practical aim by use of some modern, performing methods of review and quantification, able to see the core of economic phenomena and processes reviewed and to evaluate in a realistic way, their trends and evolution in the future, they being influenced by numerous factors that are in inter-conditioning relationships.

Indeed, our calculations above, confirm the theoretic forecast from previous section, construed with methods of bifurcation theory.

From economic point of view, it can be seen the variation of capital K and labor L over time, starting from the initial significant data corresponding to some points in the parameter space. Therefore, there are situation when the system considered enable a periodic solution appropriate to a cyclical economic evolution. Negative phenomena as production shortage and increase of unemployment rate and also the positive ones, featured by refurbishment of production capacities that could start the growth of demand for consumption goods and determination of employment level, can be relieved. Therefore, there is a theoretical and practical aim only by using modern and performing methods in reviewing the non-linear dynamic systems, able to catch the core of economic phenomena and processes, and asses in a realistic way their future dimensions and trends of evolution.

References

[1] Dobrescu, E, Albu, L-L. – *The Durable Development in Romania – models for medium and long term*, Expert Publishing House, Bucharest, 2005.

[2] Dobrotă, N. (coordinator) – *Economics Politics*, A.S.E , Publishing House, Bucharest, 1992.

[3] Gandolfo, G. – *Economic dynamics*, Springer, Berlin, 1996.

[4] Oprescu, Gh.-*Matematics for economics*, “România de Mâine” Publishing House, Bucharest, 1996

[5] Tu, P.N.V.- *Dynamical Systems*, Springer, 1994.

[6] Ştefănescu, L. – *Managementul operațiunilor cu produse financiare derivate asistat de calculator*, Editura Didactică şi Pedagogică, Bucureşti, 2005.

[7]Ungureanu, L.- *Cyclic Evolution of the Capital of a Firm*, Economic Computation and Economic Cybernetics Studies and Research, no. 4/2005.

[8]Ungureanu, L. –*Stability and bifurcation in two mathematicals models for economic dynamics*, Piteşti University Press, 2004.