



Munich Personal RePEc Archive

# **Optimal Production Tax and Privatization Policies under an Endogenous Market Structure**

Cato, Susumu and Matsumura, Toshihiro

Institute of Social Science, The University of Tokyo, Institute of  
Social Science, The University of Tokyo

20 December 2017

Online at <https://mpra.ub.uni-muenchen.de/82893/>  
MPRA Paper No. 82893, posted 23 Nov 2017 16:17 UTC

# Optimal Production Tax and Privatization Policies under an Endogenous Market Structure\*

Susumu Cato<sup>†</sup>

Institute of Social Science, The University of Tokyo

and

Toshihiro Matsumura<sup>‡</sup>

Institute of Social Science, The University of Tokyo

November 23, 2017

## Abstract

We investigate the optimal tax and privatization policies in a mixed oligopoly in which a state-owned public firm competes against private firms in a free-entry market. First, we investigate the domestic private firm case. The optimal tax rate is strictly positive except for the full privatization and full nationalization cases, and the relationship between the optimal tax rate and degree of privatization is inverted U-shaped. Further, the optimal degree of privatization is decreasing in the tax rate. Next, we investigate the foreign private firm case and find that the two policies are mutually independent.

**JEL classification numbers:** H42, H44, L13

**Key words:** industrial policy, privatization, free entry, unit tax-subsidy, foreign ownership

---

\*This work was supported by JSPS KAKENHI (15K03347).

<sup>†</sup> Corresponding author: Susumu Cato, Institute of Social Science, The University of Tokyo, 7-3-1, Hongo, Bunkyo-ku, Tokyo 113-0033, Japan. Phone:+81-042-677-2312. Fax:+81-042-677-2298. E-mail:susumu.cato@gmail.com

<sup>‡</sup> Institute of Social Science, University of Tokyo, 7-3-1, Hongo, Bunkyo-ku, Tokyo 113-0033, Japan. E-mail: matsumur@iss.u-tokyo.ac.jp

# 1 Introduction

Tax-subsidy policies are widely observed as industry policies in many industries (Itoh *et al.*, 1991). In particular, these policies prevail in typical mixed oligopolies such as the banking, energy, automobile, telecommunications, and transportation industries, and are intensively discussed in the literature (Mujumdar and Pal, 1998). If the government can choose the tax or subsidy rate without incurring any political cost, this is an effective and efficient policy instrument in such industries. Indeed, White (1996) investigated a mixed oligopoly in which a state-owned public firm competes against private firms. He showed that the first-best outcome is achieved both before and after the privatization of the public firm by adopting the same optimal tax-subsidy policy as long as the public firm is as efficient as the private firms. His result implies that privatization does not matter (i.e. the privatization neutrality theorem), and thus, there is no room for discussing how privatization affects the optimal tax-subsidy policy.

Cato and Matsumura (2013), however, showed that the privatization neutrality theorem does not hold in free-entry markets, and thus it is worth discussing how privatization affects the optimal tax-subsidy policy in these markets. Indeed, the literature on mixed oligopolies has intensively investigated free-entry markets.<sup>1</sup> The reason for this is associated with recent deregulation and liberalization, which have weakened entry restrictions in mixed oligopolies significantly. As a result, private enterprises have newly entered such mixed oligopolies as the above-mentioned banking, energy, telecommunications, and transportation industries.

Based on the foregoing, this study analyses (i) how tax-subsidy policy affects the optimal privatization policy and (ii) how the optimal privatization policy affects tax-subsidy policy. In other words,

---

<sup>1</sup>By using a monopolistic competition framework, Anderson *et al.* (1997) showed that privatization may improve welfare when private competitors are domestic, and Matsumura *et al.* (2009) showed that privatization is more likely to improve welfare when private enterprises are foreign. Matsumura and Kanda (2005) adopted the partial privatization approach formulated by Matsumura (1998) and showed that the optimal degree of privatization is zero when private competitors are domestic, while Cato and Matsumura (2012) showed that it is strictly positive when they are foreign and that this is increasing in the foreign ownership share in private firms. Chen (2017) revisited this problem by introducing the cost-reducing effect of privatization. Fujiwara (2007) discussed the relationship between the degree of product differentiation and optimal degree of privatization. Cato and Matsumura (2015) discussed the relationship between optimal trade and privatization policies.

we investigate the relationship between privatization policy and industrial policy. First, we investigate how the degree of privatization affects the optimal tax-subsidy policy in a free-entry market with domestic private firms. We find that the optimal tax rate is zero before and after the full privatization of the public firm. Then, we naturally expect that the optimal tax rate is zero regardless of the degree of privatization. However, we show that the relationship between the optimal tax rate and degree of privatization is inverted U-shaped, which implies that the optimal tax rate is strictly positive except for the two polar cases (i.e. the full privatization and full nationalization cases). This finding reveals the possible risk of restricting the analysis to these two polar cases.

Two underlying assumptions are crucial here. First, we adopt the partial privatization approach of Matsumura (1998) in our model (i.e. there exists a degree of privatization). As discussed above, the optimal tax rate is then zero both for full privatization and for full nationalization. Therefore, the optimal tax rate seems to be independent of privatization policy if we do not allow partial privatization and instead focus on the two polar cases. Thus, introducing partial privatization is crucial. Second, we assume that the degree of privatization is given exogenously for some political reason. This assumption might correspond to the situation in which the adjustment of privatization is slow, while the government can choose any tax rate flexibly. If the government can choose both the tax-subsidy rate and the degree of privatization without incurring any political costs, the optimal tax rate is again zero to minimize the tax distortion. Therefore, this assumption, which is natural in the presence of lobbying or influencing activities for privatization policies, is also crucial .

Next, we investigate how industrial policy affects the optimal privatization policy in a free-entry market with domestic private firms. This situation corresponds to the case where barriers to choosing the optimal tax rate exist. Industrial policy might not be optimal and might be given exogenously because of the lobbying activities of firms and industrial associations for tax policies or the government's budget constraint and fiscal concerns. We find that the optimal privatization rate is increasing in the tax rate. Thus, stronger industrial policy leads to stronger privatization policy.

Our observations indicate that the two policies are mutually dependent. If the government has a

department for industrial policy and a department for privatization policy, it is important to create an interaction between them, even if a cost of doing so exists or the other department cannot choose the optimal policy. However, our interdependency result depends on the assumption that the private firms are domestic. We find that when the private firms are foreign, the two policies are independent. That is, the optimal tax rate is not dependent on the degree of privatization and the degree of privatization is not dependent on the tax rate. This finding suggests that a change in economic environment (such as ownership structure in private firms) that affects the strategic relationship between firms can affect their basic principles of optimal policies.<sup>2</sup> Thus, policy-makers should take account of such a change in the economic environment.

The rest of this paper is organized as follows. Section 2 introduces our basic model with domestic private firms. Section 3 presents our main results. Section 4 extends our model to the case with foreign private firms. Section 5 concludes.

## 2 Basic Model

Firms produce homogeneous goods and engage in Cournot competition. The inverse demand function is assumed to be  $f(X) = A - X$  ( $A$  is a positive real number and  $X$  is total output). Here, market demand  $A$  is assumed to be sufficiently large. We consider  $N + 1$  firms. Firm 0 is a partially state-owned public firm, while the other firms  $i$  ( $i = 1, 2, \dots, N$ ) are private. Let  $\alpha \in [0, 1]$  be the degree of privatization of firm 0.

All firms have the same cost function  $g(x_i) = cx_i^2/2 + K$ , where  $x_i \geq 0$  is firm  $i$ 's output level,  $c$  and  $K$  are positive real numbers, and  $K$  is the entry cost of each private firm. The government levies a simple unit production tax  $\tau$  (if  $\tau$  is negative, the tax becomes a production subsidy). Each firm  $i$ 's profit is given by

$$\Pi_i = f(X)x_i - g(x_i) - \tau x_i \quad (i = 0, 1, \dots, N),$$

---

<sup>2</sup>If there is no public firm, the presence of foreign firms does not affect the strategic relationship between them. The public firm cares about welfare, which includes the producer surplus of domestic firms. Thus, the presence of foreign firms changes the objective of the public firm. In other words, the strategic relationship changes by introducing foreign firms.

where  $X = \sum_{i=1}^N x_i$ . Tax revenue  $R$  is  $R = \tau X$ .

Economic welfare  $E$  consists of the sum of the consumer surplus, firms' profits, and tax revenue as follows :

$$\begin{aligned} E &= \int_0^X f(x)dx - f(X)X + \sum_{i=0}^N \Pi_i + R \\ &= \int_0^X f(x)dx - \sum_{i=0}^N (g(x_i) + K). \end{aligned} \quad (1)$$

Each private firm maximizes its profit. Firm 0's objective is the weighted average of  $\Pi_0$  and  $E$ :

$$\alpha \Pi_0 + (1 - \alpha)E.$$

The timing of the game is as follows. In the first stage, the government chooses production tax  $\tau$  (Model 1) or the degree  $\alpha$  of privatization (Model 2) to maximize  $E$ . In the second stage, private firms enter after observing the government's decision. Given the number of entering private firms, all firms engage in Cournot competition.

### 3 Results

#### 3.1 Second- and third-stage subgames

First, we present the common analysis in Models 1 and 2. In the third stage, given  $N$ ,  $\alpha$ , and  $\tau$ , each firm chooses its output independently. Firm 0's first-order condition is given by

$$f(X) + \alpha f'(X)x_0 - g'(x_0) - \alpha\tau = 0,$$

where  $X = \sum_{i=0}^N x_i$ . The private firm's first-order condition is given by

$$f(X) + f'(X)x_i - g'(x_i) - \tau = 0.$$

We focus on the symmetric equilibrium at which all private firms produce the same output ( $x_1 = x_2 = \dots = x_N$ ).

In the second stage, given  $\alpha$  and  $\tau$ , private firms enter the market when their profits are nonnegative. Hence, the zero-profit condition holds:

$$f(X)x_1 - g(x_1) - \tau x_1 = 0.$$

By solving the above equilibrium conditions (in the second and third stages), the following equilibrium outcomes are derived:

$$x_0^*(\tau, \alpha) = \frac{1+c}{\alpha+c} \sqrt{\frac{2K}{2+c}} + \frac{1-\alpha}{\alpha+c} \tau, \quad (2)$$

$$x_1^*(\tau, \alpha) = \sqrt{\frac{2K}{2+c}}, \quad (3)$$

$$X^*(\tau, \alpha) = A - \tau - (1+c) \sqrt{\frac{2K}{2+c}}, \quad (4)$$

$$N^*(\tau, \alpha) = \frac{X^*(\tau, \alpha) - x_0^*(\tau, \alpha)}{x_1^*(\tau, \alpha)}. \quad (5)$$

Since  $A$  is sufficiently large,  $X^*$  is positive. We here use superscript  $*$  to denote the equilibrium values.

From (2)–(5), we obtain the following result.

**Lemma 1.** (i)  $\partial x_0^*/\partial \alpha < 0$ . (ii)  $\partial x_0^*/\partial \tau \geq 0$  and the equality holds only if  $\alpha = 1$ . (iii)  $\partial x_0^*/\partial \tau$  is decreasing in  $\alpha$ . (iv)  $\partial x_1^*/\partial \alpha = \partial x_1^*/\partial \tau = 0$ . (v)  $\partial N^*/\partial \alpha > 0$ ,  $\partial N^*/\partial \tau < 0$ .

Cato and Matsumura (2012) showed that  $x_0^*$  is decreasing in  $\alpha$ ,  $N^*$  is decreasing in  $\alpha$ , and  $x_1^*$  is independent of  $\alpha$ , when  $\tau = 0$ . The same principle can apply to the case with nonzero  $\tau$ .

An increase in  $\tau$  increases the marginal cost of each private firm and reduces the production of each private firm (the cost effect); at the same time, an increase in  $\tau$  raises the equilibrium price, which stimulates the production of each private firm (the price effect). These two effects are cancelled out, and thus  $x_1^*$  does not depend on  $\tau$ . However, a higher price caused by a higher tax rate reduces total output, and thus reduces the number of entering firms. For the welfare-maximizing public firm, the tax is not its real cost (because the tax is an income transfer from the public firm to the government), and thus the cost effect is weaker unless  $\alpha = 1$ . Therefore, the cost effect is dominated by the price effect, and thus  $x_0$  is increasing in  $\tau$  (Lemma 1(ii)). However, the difference of the cost effect between

firm 0 and each private firm is smaller when  $\alpha$  is larger, and thus the output expansion effect of firm 0 by the tax weakens when  $\alpha$  is larger (Lemma 1(iii)).

Equilibrium welfare  $E^*(\tau, \alpha)$  is given as follows:

$$E^*(\tau, \alpha) = \int_0^{X^*(\tau, \alpha)} f(x)dx - [g(x_0^*(\tau, \alpha)) + K] - N^*(\tau, \alpha)[g(x_0^*(\tau, \alpha)) + K].$$

### 3.2 Model 1 (endogenous $\tau$ )

We consider the optimal tax policy. In the first stage, the government maximizes  $E^*$  with respect to  $\tau$ . Define  $\tau^B$  as follows:

$$\tau^B(\alpha) := \arg \max_{\tau} E^*(\tau, \alpha).$$

By solving this problem, we obtain the optimal production tax:

$$\tau^B = \sqrt{\frac{2K}{2+c}} \frac{(1-\alpha)\alpha}{\alpha^2+c}.$$

Note that the optimal tax rate is zero under full privatization and full nationalization. These two extreme cases are proved by Cato and Matsumura (2013). One might guess that it is zero for any degree of partial privatization because partial privatization is a combination of full privatization and full nationalization. However, it is strictly positive in between these extremes. We present a nonmonotonic property between the optimal tax rate and degree of privatization.

**Proposition 1.**  $\tau^B(\alpha)$  is increasing in  $\alpha$  for  $\alpha < \hat{\alpha}$  and decreasing in  $\tau$  for  $\alpha > \hat{\alpha}$ , where

$$\hat{\alpha} = \sqrt{c^2 + c} - c.$$

**Proof.** By differentiating  $\tau^B$  with respect to  $\alpha$ , we get the following:

$$\frac{d\tau^B}{d\alpha} = \sqrt{\frac{2K}{2+c}} \frac{c - 2c\alpha - \alpha^2}{(\alpha^2 + c)^2}.$$

$d\tau^B/d\alpha$  is positive when  $\alpha < \hat{\alpha}$  and negative when  $\alpha > \hat{\alpha}$ . ■



Note that  $0 < \sqrt{c^2 + c} - c < 1$  for any  $c$ . Therefore, the inverse U-shape is a robust observation with respect to  $c$ .

We explain the intuition behind Proposition 1. Because the producer surplus of the private firms is zero, welfare is the sum of the consumer surplus, tax revenue, and public firm's profit. The tax-subsidy distorts consumption and production, and thus the optimal tax rate is zero if  $x_0$  is given exogenously. As Matsumura and Kanda (2005) showed,  $\alpha = 0$  yields the optimal  $x_0$  given  $\tau = 0$ . Under these conditions,  $\tau = 0$  is optimal when  $\alpha = 0$ . However, a marginal increase in  $\alpha$  reduces  $x_0$  (Lemma 1(i)), which reduces the resulting profit of firm 0. An increase in  $\tau$  stimulates the production of firm 0, thus increasing firm 0's profit and improving welfare. Therefore, the optimal tax rate becomes positive. Moreover, an increase in  $\alpha$  makes firm 0 less aggressive, and thus the government adjusts  $\tau$  to stimulate firm 0's production further. However, when  $\alpha$  reaches a critical point, the loss of the tax distortion caused by the higher tax rate dominates the welfare gain of the further production of firm 0. Note that  $\partial x_0^*/\partial \tau$  is decreasing in  $\alpha$ . After that, a further increase in  $\alpha$  reduces the optimal tax rate because a higher tax rate is less likely to stimulate the production of firm 0. When  $\alpha = 1$ , the production expansion effect of firm 0 caused by the higher tax rate disappears and the optimal tax rate becomes zero to minimize the tax distortion.

We now consider the welfare implications. Each component of economic welfare under the optimal tax rate is defined as follows:

$$CS^B(\alpha) = (X^*(\tau^B(\alpha), \alpha))^2/2;$$

$$PS^B(\alpha) = (A - X^*(\tau^B(\alpha), \alpha) - \tau^B(\alpha))x_0^*(\tau^B(\alpha), \alpha) - c(x_0^*(\tau^B(\alpha), \alpha))^2/2;$$

$$R^B(\alpha) = \tau^B(\alpha)X^*(\tau^B(\alpha), \alpha).$$

Let  $E^B = CS^B + PS^B + R^B$ .

First, we consider the change in the consumer surplus. By differentiating  $CS^B$  with respect to  $\alpha$ ,

we obtain the following:

$$\frac{dCS^B}{d\alpha} = -\frac{(c - 2c\alpha - \alpha^2) \left[ A(\alpha^2 + c) - (\alpha + c + \alpha^2c + c^2) \sqrt{\frac{2K}{2+c}} \right]}{(\alpha^2 + c)^3} \sqrt{\frac{2K}{2+c}}.$$

Since  $A$  is sufficiently large, the sign of this is dependent only on  $(c - 2c\alpha - \alpha^2)$ . This implies that we observe a U-shape. That is,  $CS^B$  is decreasing in  $\alpha$  for  $\alpha < \hat{\alpha}$ , while  $CS^B$  is increasing in  $\alpha$  for  $\alpha > \hat{\alpha}$ . This result is intuitive. The consumer surplus is increasing in total output, which is decreasing in the tax level. Thus, the consumer surplus is negatively associated with the tax. Since the tax is an inverted U-shape with respect to  $\alpha$ , the consumer surplus is U-shaped.

Second, we consider the change in the producer surplus. By differentiating  $CS^B$  with respect to  $\alpha$ , we obtain the following:

$$\frac{dPS^B}{d\alpha} = -\frac{2K\alpha(\alpha + c)(c + 2\alpha - \alpha^2)}{(2 + c)(\alpha^2 + c)^3}.$$

Since  $(c + 2\alpha - \alpha^2)$  is always positive for  $\alpha \in [0, 1]$ ,  $PS^B$  is always decreasing in  $\alpha$ .

Third, we consider the change in tax revenue:

$$\frac{dR^B}{d\alpha} = \frac{(c - 2c\alpha - \alpha^2) \left[ A(\alpha^2 + c) - (2\alpha + \alpha^2(c - 1) + c(c + 1)) \sqrt{\frac{2K}{2+c}} \right]}{(\alpha^2 + c)^3} \sqrt{\frac{2K}{2+c}}.$$

The inverted U-shape holds for  $R^B$ . That is,  $R^B$  is increasing in  $\alpha$  for  $\alpha < \hat{\alpha}$ , while  $R^B$  is decreasing in  $\alpha$  for  $\alpha > \hat{\alpha}$ . Therefore, the nonmonotonic relationship holds for the consumer surplus and tax revenue.

Finally, we consider the total effect of partial privatization on economic welfare:

$$\frac{dE^B}{d\alpha} = \frac{dCS^B}{d\alpha} + \frac{dPS^B}{d\alpha} + \frac{dR^B}{d\alpha} = -\frac{2K\alpha(1 + c)}{(2 + c)(\alpha^2 + c)^2}.$$

Thus,  $E^B$  is decreasing in  $\alpha$  if  $\alpha > 0$  (and  $dE^B/d\alpha = 0$  if  $\alpha = 0$ ). Therefore, we get a downward sloping curve for total welfare.

### 3.3 Model 2 (endogenous $\alpha$ )

Next, we consider the optimal degree of privatization, which is defined as  $\alpha^B$ :

$$\alpha^B(\tau) := \arg \max_{\alpha} \hat{E}(\tau, \alpha).$$

By solving this problem, we have the optimal degree of privatization:

$$\alpha^B(\tau) = \frac{tc(2+c)(\sqrt{2K} + t\sqrt{2+c})}{2(1+c)K\sqrt{2+c} + t(2+5c+2c^2)\sqrt{2K} + t^2c(2+c)\sqrt{2+c}}. \quad (6)$$

**Proposition 2.**  $\alpha^B(\tau)$  is increasing in  $\tau$ . Moreover,  $\alpha^B(0) = 0$  and  $\alpha^B(\tau) < 1$  for all  $\tau$ .

**Proof.** By differentiating  $\alpha^B$  with respect to  $\tau$ , we get the following:

$$\frac{d\alpha^B}{d\tau} = \frac{c(c^2 + 3c + 2)[2K\sqrt{(2K)(2+c)} + 4t(2+c)K + t^2(2+c)\sqrt{(2K)(2+c)}]}{[2(1+c)K\sqrt{2+c} + t(2+5c+2c^2)\sqrt{2K} + t^2c(2+c)\sqrt{2+c}]^2}.$$

From (6), it follows that  $\alpha^B(0) = 0$ . Moreover,  $\alpha^B(\tau) < 1$  because

$$tc(2+c)(\sqrt{2K} + t\sqrt{2+c}) < 2(1+c)K\sqrt{2+c} + t(2+5c+2c^2)\sqrt{2K} + t^2c(2+c)\sqrt{2+c}. \quad \blacksquare$$

We now provide a brief analysis of welfare. Total output is not dependent on  $\alpha$ , and thus we find that  $CS^B(\tau)$  is decreasing in  $\tau$ . When  $c = 1$  (the general case is complicated),

$$\frac{dE^B(\tau)}{d\tau} = -\tau < 0 \text{ for } \tau > 0.$$

For most parameters, we can see that  $PS^B(\tau)$  is decreasing in  $\tau$  and  $R^B(\tau)$  is increasing in  $\tau$ . The positive effect on tax revenue is not large, and thus it is dominated by the negative effects on the consumer surplus and producer surplus.

Finally, let us consider the case where the government adjusts both tax and privatization levels. The optimal degree of privatization and tax rate in such an ideal situation is as follows:

$$\alpha^{BB} = 0 \text{ and } \tau^{BB} = 0.$$

That is, a combination of full nationalization and no production tax maximizes economic welfare. Nonzero tax distorts the consumption/production decision and reduces welfare given the output of firm 0. As Lee *et al.* (2017) showed, the marginal cost pricing of firm 0 is optimal for welfare when  $\tau = 0$ . In the case of domestic private firms,  $\alpha = 0$  equalizes the public firm's marginal cost to the price and this is best for welfare. These lead to the above result.

## 4 Foreign Firms

In the previous section, we showed that tax-subsidy and privatization policies are interdependent. In this section, we show that this result does not hold if the private firms are owned by foreign investors.<sup>3</sup>

We assume the private firms to be foreign-owned. In this case, the profit of each private firm is not counted in the economic welfare of the home country. Thus, economic welfare is as follows:

$$E = \int_0^X f(z)dz - f(X)X + \Pi_0 + \tau \sum_{i=0}^n x_i. \quad (7)$$

The objective of firm 0 is

$$\alpha\Pi_0 + (1 - \alpha)E = \Pi_0 + (1 - \alpha)\left[\int_0^X f(z)dz - f(X)X + \tau \sum_{i=0}^n x_i\right].$$

Since the objective of firm 0 is  $\alpha\Pi_0 + (1 - \alpha)E$ , the first-order condition of firm 0 is as follows:

$$f(X) - (1 - \alpha)f'(X)X + f'(X)x_0 - g'(x_i) - \alpha\tau = 0.$$

We assume that the government sells the ownership of firm 0 to domestic investors. The other two conditions are the same as before (the first-order condition of private firms and zero-profit condition).

By solving the above equilibrium conditions (in the second and third stages), the equilibrium outcomes are derived:

$$x_0^*(\tau, \alpha) = \frac{1 - \alpha}{1 + c}A + \alpha\sqrt{\frac{2K}{2 + c}}, \quad (8)$$

$$x_1^*(\tau, \alpha) = \sqrt{\frac{2K}{2 + c}}, \quad (9)$$

$$X^*(\tau, \alpha) = A - \tau - (1 + c)\sqrt{\frac{2K}{2 + c}}, \quad (10)$$

$$N^*(\tau, \alpha) = \frac{X^*(\tau, \alpha) - x_0^*(\tau, \alpha)}{x_1^*(\tau, \alpha)}. \quad (11)$$

From (8)–(11), we obtain the following result.

---

<sup>3</sup>Whether the private firm is domestic or foreign often yields contrasting results in the literature on mixed oligopolies. See Corneo and Jeanne (1994), Fjell and Pal (1996), Pal and White (1998), Bárcena-Ruiz and Garzón (2005a, 2005b), Heywood and Ye (2009), and Lin and Matsumura (2012).

**Lemma 2.** (i)  $\partial x_0^*/\partial\alpha < 0$ . (ii)  $\partial x_0^*/\partial\tau = 0$ . (iii)  $\partial x_1^*/\partial\alpha = \partial x_1^*/\partial\tau = 0$ . (iv)  $\partial N^*/\partial\alpha > 0$ ,  $\partial N^*/\partial\tau < 0$ .

Lemma 2 is the same as Lemma 1 except for Lemma 2(ii). Lemma 1(ii) states that firm 0's output is increasing in  $\tau$  when private firms are domestic, and Lemma 2(ii) states that this is independent of  $\tau$  when private firms are foreign. We explain the intuition behind this result. Suppose that private firms are domestic. An increase in  $\tau$  reduces the total output of private firms and thus raises the price if  $x_0$  remains unchanged. Because the social marginal cost of firm 0 does not depend on  $\tau$  given  $x_0$ , an increase in  $\tau$  increases the price–(social) cost margin in firm 0, which increases  $x_0$  (the margin effect).

However, if private firms are foreign, an additional effect exists. An increase in  $x_0$  reduces the price and thus reduces the outflow of the surplus to foreign investors. This effect is weaker when the total output of private firms is smaller (the outflow-restricting effect). An increase in  $\tau$  reduces the total output of private firms, which strengthens the margin effect and weakens the outflow-restricting effect. These two effects are cancelled out, and thus  $\tau$  does not affect  $x_0^*$ .

We then derive the optimal tax rate and optimal degree of privatization by adopting the same procedure as in the previous section.

**Proposition 3.**

$$\tau^B(\alpha) = 0 \text{ and } \alpha^B(\tau) = \frac{Ac(2+c) - (1+c)^2\sqrt{2(2+c)K}}{Ac(2+c) - c(1+c)\sqrt{2(2+c)K}} \in (0, 1).$$

The optimal tax rate is always zero independent of the degree of privatization. The degree of privatization is positive at some value between zero and one, and it is independent of the tax rate. These results suggest that the two policies are independent.

The main point of this result is that we do not observe a nonmonotonic relationship between the optimal tax rate and degree of privatization. That is, when private firms are domestic, the optimal tax rate increases until a certain degree of privatization and then decreases. The U-shape disappears when private firm are foreign. The driving force of this result is Lemma 2(ii). In contrast to the

domestic private firm case, the tax rate does not affect the behaviour of the public firm. Therefore, the government chooses  $\tau = 0$  to minimize the tax distortion. As discussed in the previous section, the marginal cost pricing of the public firm is best for welfare. Because the tax rate does not affect the behaviour of the public firm (only  $\alpha$  does), the optimal  $\alpha$  that induces the marginal cost pricing of firm 0 is independent of  $\tau$ .

## 5 Concluding Remarks

In this study, we investigate the relationship between privatization and industrial policy. We find that regardless of whether private firms are domestic and foreign, the optimal tax rate is zero in both the full nationalization and the full privatization cases. However, the optimal tax rate is strictly positive except for these two cases if private firms are domestic. Our result suggests the possible risk of restricting the analysis to these two polar cases and highlights the importance of partial privatization.

A comparison with the case with foreign firms shows that our interdependency result is dependent on the ownership of private firms. Since the recent trend of entry deregulation not only enhances the entry of domestic firms but also allows foreign firms to penetrate markets, it is natural to consider the presence of foreign firms. However, this study focuses on the two extreme cases: (i) the case where all private firms are domestic and (ii) the case where all private firms are foreign. We can consider the intermediate case: foreign inventors own some share  $\theta(\in [0, 1])$  of private firms and domestic welfare includes  $(1 - \theta)(\text{profits of private firms})$ . In this case, we show that the main implications are between these two extreme cases (i.e. when  $\theta = 0$  and  $\theta = 1$ ). Moreover, the interdependency of the two policies weakens as  $\theta$  increases.

In this study, we assume that the policies are implemented before the entry of private firms. However, as Lee *et al.* (2017) showed, the timing of such policies may affect policymaking in mixed oligopolies. Investigating this topic is left to future research.

## References

- Anderson, Simon P., André de Palma, and Jacques-François Thisse (1997) ‘Privatization and efficiency in a differentiated industry.’ *European Economic Review* 41(9), 1635–1654.
- Bárcena-Ruiz, Juan Carlos and Maria Begona Garzón (2005a) ‘Economic integration and privatization under diseconomies of scale.’ *European Journal of Political Economy* 21(1), 247–67.
- Bárcena-Ruiz, Juan Carlos and Maria Begona Garzón (2005b) ‘International trade and strategic privatization.’ *Review of Development Economics* 9(4), 502–13.
- Cato, Susumu and Toshihiro Matsumura (2012) ‘Long-run effects of foreign penetration on privatization policies.’ *Journal of Institutional and Theoretical Economics* 168(3), 444–454.
- Cato, Susumu and Toshihiro Matsumura (2013) ‘Long-run effects of tax policies in a mixed market.’ *FinanzArchiv* 69(2), 215–240.
- Cato, Susumu and Toshihiro Matsumura (2015) ‘Optimal privatisation and trade policies with endogenous market structure.’ *Economic Record* 91(294), 309–323.
- Chen, Tai-Liang (2017) ‘Privatization and efficiency: a mixed oligopoly approach.’ *Journal of Economics* 120(3), 251–268.
- Corneo, Giacomo and Olivier Jeanne (1994) ‘Oligopole mixte dans un marche commun.’ *Annales d’Economie et de Statistique* 33, 73–90.
- Fujiwara, Kenji (2007) ‘Partial privatization in a differentiated mixed oligopoly.’ *Journal of Economics* 92(1), 51–65.
- Fjell, Kenneth and Debashis Pal (1996) ‘A mixed oligopoly in the presence of foreign private firms.’ *Canadian Journal of Economics* 29(3), 737–743.

- Heywood, John S. and Guangliang Ye (2009) 'Mixed oligopoly and spatial price discrimination with foreign firms.' *Regional Science and Urban Economics* 39(5), 592–601
- Itoh, Motoshige, Kazuharu Kiyono, Masahiro Okuno-Fujiwara, and Kotaro Suzumura (1991) *Economic Analysis of Industrial Policy* Academic Press, Inc. San Diego, California
- Lee, Sang-Ho, Toshihiro Matsumura, and Susumu Sato (2017) 'An analysis of entry-then-privatization model: welfare and policy implications.'  
forthcoming in *Journal of Economics*. <https://doi.org/10.1007/s00712-017-0559-z>
- Lin, Ming Hsin and Toshihiro Matsumura (2012) 'Presence of foreign investors in privatized firms and privatization policy.' *Journal of Economics* 107(1), 71–80.
- Matsumura, Toshihiro (1998) 'Partial privatization in mixed duopoly.' *Journal of Public Economics* 70(3), 473–483.
- Matsumura, Toshihiro and Osamu Kanda (2005) 'Mixed oligopoly at free entry markets.' *Journal of Economics* 84(1), 27–48.
- Matsumura Toshihiro, Noriaki Matsushima, and Ikuo Ishibashi (2009) 'Privatization and entries of foreign enterprises in a differentiated industry.' *Journal of Economics* 98(3), 203–219.
- Mujumdar, Sudesh and Pal Debashis (1998) 'Effects of indirect taxation in a mixed oligopoly.' *Economics Letters* 58(2), 199–204.
- Pal, Debashis and Mark D. White (1998) 'Mixed oligopoly, privatization, and strategic trade policy.' *Southern Economic Journal* 65(2), 264–281.
- White, Mark D. (1996) 'Mixed oligopoly, privatization and subsidization.' *Economics Letters* 53(2), 189–195.