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Optimal degree of privatization in a mixed oligopoly with multiple public enterprises

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Abstract

I discuss the optimal degree of privatization in a mixed oligopoly in which multiple public enterprises exist. I find that the optimal degree of privatization is increasing in the number of private firms n and independent of the number of public firms m . These results suggest that no matter how many public firms exist, an increase in the number of private firms would increase the optimal degree of privatization as long as all public firms are partially privatized at the same degree.

1. Introduction

The world has witnessed an increasing of privatization of state-owned public enterprises since the 1980s. Still, public firms, semi-public (partially privatized) firms and private firms coexist in many markets. Mixed oligopolies involving both private and public enterprises have drawn an increasing interest.¹ The privatization of public enterprises remains an important research topic in the literature on mixed oligopolies. De Fraja and Delbono (1989) showed that social welfare can be higher when a public firm is a profit maximizer rather than a welfare maximizer, which suggests that privatization of public enterprises can lead to a higher welfare. Most works in this field assumed that there is only one public firm. However, there are many economies with more than one public firm in the real world, for example, there are almost 400 public and semi-public banks and 3 state-owned internet service provider enterprises in China. It is not unusual that several public enterprises be privatized at the same time. Matsumura and Shimizu (2010) discussed privatization in a mixed oligopoly with multiple public enterprises and investigated the relationship between the number of public firms and welfare. They showed that the relationship can be U-shaped (i.e., welfare is locally maximized when all firms are privatized.) However, they neglected the possibility of partial privatization.

In the literature on mixed oligopolies where only one public firm exists, the approach of Matsumura (1998) is often used by many papers, and they investigated the optimal degree of privatization. Lin and Matsumura (2012) and Matsumura and Okamura (2015) found that the optimal degree of privatization is increasing in the number of private firms and decreasing in the foreign ownership share in private firms. When private competitors are domestic in free-entry markets, Matsumura and Kanda (2005) found that the optimal degree of privatization is zero, while Cato and Matsumura (2012) showed that the optimal degree of privatization is strictly positive when there are foreign private competitors and Cato and Matsumura (2012) found that the optimal degree of privatization increases with the foreign ownership share in private firms. In addition, even in free-entry markets, under the circumstance where privatization improves production efficiency, Chen (2017) showed that the optimal degree of privatization is positive. Fujiwara (2007) investigated the relationship between the degree of product differentiation and optimal degree of privatization, and found the relationship can be nonmonotonic (monotonic) in a non-free-entry (free-entry) market. Cato and Matsumura (2015) discussed the relationship between optimal trade and privatization policies, showing that a higher tariff rate reduces the optimal degree of privatization in free-entry markets. Lee *et al.* (2017) showed that the optimal degree of privatization depends on the timing of privatization. Heywood *et al.* (2017) investigated how asymmetric information on demand conditions affects the optimal degree of privatization. However, these papers did not investigate the case with multiple public firms.

In this paper, following Matsumura and Shimizu (2010), I adopt a standard model of De Fraja and Delbono (1989) except for the number of public firms. There are m public firms and n private firms. Matsumura and Shimizu (2010) assumed that each of public firm is

¹This interest in mixed oligopolies derives from their importance to Europe, Canadian, and Japanese economies. There are still some examples of mixed oligopolies such as the packing and overnight-delivery industries despite the less significance in the USA. Analyses of mixed oligopolies date back to at least Merrill and Schneider (1966). The literature in this field has become richer and more diverse only recently.

either fully privatized or remains as a pure public firm, and showed that a welfare gain of privatization of one public firm depends on the proportion of public firms $m/(n+m)$ as well as the total number of public and private firms $(n+m)$. I adopt the partial privatization approach of Matsumura (1998) and investigate the relationship between the optimal degree of privatization of each public firm and the number of public firms m and the number of private firms n . From the result of Matsumura and Shimizu (2010), I naturally expect that the optimal degree of privatization depends on both n and m . I showed that the optimal degree of privatization is increasing in the number of private firms n and independent of the number of public firms m . The latter result is in sharp contrast to that of Matsumura and Shimizu (2010). My result suggests that optimal privatization policy does not depend on the number of public firms as long as partial privatization policy is adopted and all public firms are partially privatized at the same degree.¹

The remainder of this paper is organized as follows. Section 2 presents the model, and Section 3 shows the results.

2. Model

I formulate a mixed oligopoly model with m state-owned public firms (firm 1, 2, ..., m) and n private firms (firm $m+1, m+2, \dots, m+n$). All firms produce perfectly substitutable commodities for which the market demand function is given by $p(Q) = a - Q$, where a is a positive constant, p is the price and Q is the total output. Firm i 's cost function is given by $c_i(q_i) = (k/2)(q_i)^2$, where q_i is the output quantity of firm i ($i = 1, 2, \dots, m, m+1, m+2, \dots, m+n$) and k is a positive constant.

Social welfare W is the sum of consumer surplus and firms' profits, and is given by

$$W = \int_0^Q p(q) dq - pQ + \sum_{i=1}^{m+n} \Pi_i = \int_0^Q p(q) dq - \sum_{i=1}^{m+n} c_i(q_i) \quad (1)$$

where Π_i ($i = 1, 2, \dots, m, m+1, m+2, \dots, m+n$) are firm i 's profit and Q is the total output given by

$$Q \equiv \sum_{i=1}^{m+n} q_i. \quad (2)$$

State-owned public firms' payoff U_i is given by

$$U_i = (1 - \alpha)W + \alpha \Pi_i \quad (i = 1, 2, \dots, m), \quad (3)$$

where $\alpha \in [0, 1]$ represents the degree of privatization.

The game runs as follows. In the first stage, the government chooses α . In the second stage, after observing α , firms compete in a Cournot fashion. I restrict my attention to the symmetric equilibrium in which $q_1 = q_2 = \dots = q_m$ and $q_{m+1} = q_{m+2} = \dots = q_{m+n}$.²

¹Haraguchi and Matsumura (2016) also discussed a case with multiple public firms and discussed endogenous competition structure which is originally discussed by Singh and Vives (1984) in a private duopoly. They also showed that the equilibrium competition structure depends on both m and n . However, they did not discuss partial privatization.

²I can show that the unique equilibrium is symmetric.

3. Equilibrium analysis and results

I adopt subgame perfection as the equilibrium concept, and solve the game by backward induction. First, I investigate the second-stage competition. The first-order conditions of payoff/profit maximization for the public/private firms, respectively are given by

$$\begin{aligned}\frac{\partial U_i}{\partial q_i} = 0 &\Leftrightarrow a - Q - (k + \alpha)q_i = 0 & i = 1, 2, \dots, m, \\ \frac{\partial \Pi_j}{\partial q_j} = 0 &\Leftrightarrow a - Q - (k + 1)q_j = 0 & j = m + 1, m + 2, \dots, m + n.\end{aligned}$$

The second-order conditions are satisfied. I can show that the equilibrium is symmetric, i.e. $q_1 = q_2 = \dots = q_m$ and $q_{m+1} = q_{m+2} = \dots = q_{m+n}$ in equilibrium. Let q^{s*} denotes the equilibrium output of each state-owned public firm and q^{pr*} denotes the equilibrium output of each private firm. Solving the first-order conditions, I obtain the equilibrium outputs

$$q^{s*} = \frac{(k + 1) a}{(k + \alpha) (k + 1 + n) + (k + 1) m}, \quad (4)$$

$$q^{pr*} = \frac{(k + \alpha) a}{(k + \alpha) (k + 1 + n) + (k + 1) m}. \quad (5)$$

I obtain the following as the equilibrium total output, price and welfare, respectively:

$$Q^* = \frac{[(k + 1) m + (k + \alpha) n] a}{(k + \alpha) (k + 1 + n) + (k + 1) m}, \quad (6)$$

$$p^* = \frac{(k + \alpha) (k + 1) a}{(k + \alpha) (k + 1 + n) + (k + 1) m}, \quad (7)$$

$$W^*(\alpha) = \left(1 - \frac{(k + 1)^2 km + (k + \alpha)^2 [(k + 1)^2 + kn]}{[(k + \alpha) (k + 1 + n) + (k + 1) m]^2} \right) \frac{a^2}{2}. \quad (8)$$

I consider the effects of partial privatization of all public firms. I investigate the relationship between W^* (equilibrium social welfare) and α (degree of privatization). The government chooses α to maximize W . The first-order condition is

$$\frac{\partial W^*(\alpha)}{\partial \alpha} = \frac{a^2 m (k + 1) \{ k (k + 1 + n) (k + 1) - (k + \alpha) [(k + 1)^2 + kn] \}}{[(k + \alpha) (k + 1 + n) + (k + 1) m]^3} = 0.$$

The second-order condition is satisfied. Solving this yields

$$\alpha^* = \frac{kn}{(k + 1)^2 + kn}. \quad (9)$$

Lemma 1 $q^{s*} \geq q^{pr*}$, and the equality holds if and only if $\alpha = 1$.

Proof Since $(k + \alpha)(k + 1 + n) + (k + 1)m > 0$ and $\alpha \in [0, 1]$, from (4) and (5), I obtain

$$q^{s*} - q^{pr*} = \frac{(1 - \alpha)a}{(k + \alpha)(k + 1 + n) + (k + 1)m} \geq 0,$$

and the equality holds only if $\alpha = 1$. ■

Lemma 1 is a common result in the literature. The public firm is more aggressive than private firms, unless it is fully privatized, because public firm would be concerned with social welfare, which includes consumer surplus. Thus, it produces aggressively in order to increase consumer surplus.

Proposition 1 $\alpha \in (0, 1)$.

Proof Since n and k are both positive, α^* is positive. From (9), let the denominator minus the numerator, we have $(k + 1)^2 + kn - kn = (k + 1)^2 > 0$. Since the denominator is always larger than the numerator, I obtain $\alpha^* < 1$. ■

Proposition 1 is also a common result in the literature. This is shown in the case with $m = n = 1$ in Matsumura (1998) and shown in the case with $m = 1$ and $n \geq 1$ in Matsumura and Kanda (2005). A marginal decrease of α from $\alpha = 0$ induces production substitution from the public firms to the private firms and it improves welfare because the marginal cost of the public firm is higher than that of each private firm.

I now present the main result.

Proposition 2 (i) α^* does not depend on m . (ii) α^* is increasing in n .

Proof As (9) shows that the only elements that affect α^* are the positive constant k in cost function and the number of private firms n . Thus, the number of public firms m does not affect the optimal degree of privatization α^* . Proposition 2 (i) shows that no matter how many public firms exist, the optimal degree of privatization would be unaffected as long as all public firms are partially privatized at the same degree.

Proof From (9), I obtain

$$\frac{\partial \alpha^*}{\partial n} = \frac{k(k + 1)^2}{[(k + 1)^2 + kn]^2} > 0. \quad \blacksquare$$

Proposition 2 (ii) is similar to the findings of existing studies (i.e., social welfare would be improved when there are more private firms). De Fraja and Delbono (1989) showed that social welfare could be improved more by privatization when the number of private firms is larger, and Matsumura and Shimizu (2010) showed the same result even in the model with multiple public firms. Lin and Matsumura (2012) and Matsumura and Okamura (2015) found that the optimal degree of privatization increases with the number of private firms n in various contexts in their model in the context of single public firm.

However, Proposition 2 (i) is in sharp contrast to the result of Matsumura and Shimizu (2010) that welfare gain depends on both m and n . As Matsumura and Okamura (2015) showed, privatization has two effects, total output effect and production substitution effect. Privatization reduces the total output and thus worsens social welfare (total production

effect). At the same time privatization induces production substitution, and it can improve social welfare. An increase in the number of private firms n can mitigate the total output effect, and thus, privatization more likely improves social welfare when there are more private firms.

In Matsumura and Shimizu (2010), the production substitution caused by the privatization would be from the public firm being privatized to private firms and to remaining public firms. The former production substitution improves welfare, while the latter production substitution reduces welfare, and thus, the number of the public firm also affects the welfare gain of the privatization. In contrast, in my model, an increase in α induces production substitution of all public firms to private firms, and the welfare-reducing production substitution does not exist. This is why my model and that of Matsumura and Shimizu (2010) yield contrasting result.

4. Concluding remarks

In this study, I discuss the optimal degree of privatization in a mixed oligopoly with multiple public enterprises. I find that the optimal degree of privatization is increasing in the number of private firms n and independent of the number of public firms m . However, this result may not hold when the public firms are not privatized at the same degree. My result suggests that no matter how many public firms exist, an increase in the number of private firms would increase the optimal degree of privatization as long as all public firms are partially privatized at the same degree.

In this study, I assume that the degree of privatization does not affect enterprises' production costs. However, privatization can possibly improve firms' production efficiency in some cases, which then affects the resulting welfare. It is quite difficult to add this effect into my model for the relationship between the improvement of production efficiency and the degree of privatization might not be linear. Besides, the situation that multiple public firms can be privatized at different degree is also very intriguing. These tasks remain for future research.

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