Dealing with Misspecification in DSGE Models: A Survey

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Abstract

Dynamic Stochastic General Equilibrium (DSGE) models are the main tool used in Academia and in Central Banks to evaluate the business cycle for policy and forecasting analyses. Despite the recent advances in improving the fit of DSGE models to the data, the misspecification issue still remains. The aim of this survey is to shed light on the different forms of misspecification in DSGE modeling and how the researcher can identify the sources. In addition, some remedies to face with misspecification are discussed.

JEL CODES: C11, C15, C32

KEYWORDS: DSGE Models, Misspecification, Estimation, Bayesian Estimation

"Essentially, all models are wrong, but some are useful"

George Box and Norman Draper in "Empirical Model-Building and Response Surfaces" (1987, pag.424)

"A well-defined statistical model is one whose underlying assumptions are valid for the data chosen"

Aris Spanos in "The Simultaneous-Equations Model Revisited: Statistical Adequacy and Identification" (1990, pag.89)

1 Introduction

Dynamic Stochastic General Equilibrium (DSGE) models are the workhorse of modern macroeconomists in both Academia and policymaker institutions, such as Central Banks (Kydland and Prescott, 1982 and Rotemberg and Woodford, 1997). Introduced to satisfy the Lucas critique (Lucas, 1976), with respect to structural macroeconometrics models, DSGE describe the business cycle using micro-economic foundations. Thanks to these features, these models are particularly suited for policy evaluations and for forecasting

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analysis of the key economic indicators (output, inflation, and short-term interest rate), as illustrated in the works of Smets and Wouters (2003, 2004, and 2007), Del Negro and Schorfheide (2004), Christiano, Eichenbaum, and Evans (2005), and Adolfson, Laséen, Lindé, and Villani (2008) among others.

Despite these recent advances in improving the fit of DSGE models to the data, the misspecification issue still remains. A growing number of papers have discussed the important role of misspecification in DSGE models and how we can identify the sources of misspecification. In particular, the recent Great Recession has brought new importance to investigate about the structural economic model's difficulties in explaining the data to make policy evaluation and for forecasting purposes.

This survey aims to shed light on this issue and reply to the following questions:

What does the misspecification mean in a DSGE model? How can we identify the sources of misspecification? What are the main approaches to face with misspecification?

The review starts examining the standard New Keynesian DSGE model à la Smets and Wouters (2007) which is the benchmark model to explain the business cycle behavior in the current DSGE modeling literature. After that, we discuss the different forms of misspecification the researcher can face in using the benchmark model to make policy evaluation and forecasting analysis. So far, the literature does not propose a unique definition of misspecification. We can distinguish four forms of misspecification: a) Misspecification in the State-Space Representation, b) Misspecification in Parameters, c) Misspecification in the Assumptions of the DSGE model, and d) Misspecification in Computational Methods. Furthermore, each form of misspecification includes different aspects. Misspecification in the State-Space Representation relies on three problems: Misspecification and Model Features, Misspecification and Shocks, and Misspecification of the Statistical Representation. Instead, Misspecification in Parameters refers to the role of the parameter stability and the identification in DSGE misspecification. Meanwhile, Misspecification in the Assumptions refers to the hypothesis of rational expectations and of linearity. Last but not least, Misspecification in Computational Methods refers to estimation of the posterior in the DSGE modeling.

The review also delineates the approaches to detect the sources of misspecification, focusing on Monti (2015), Inonue, Kuo, and Rossi (2017), Canova and Matthes (2017), and Den Haan and Dreschel (2017). After that, we discuss about the econometric methods used to deal with the DSGE misspecifications. In particular, we focus on the (additive and hierarchical) hybrid DSGE models as shown in Schorfheide (2013).

The remainder of the paper is organized as follows. Section 2 illustrates the medium scale DSGE model à la Smets and Wouters (2007). Section 3 discusses the different forms of misspecification. Section 4 overviews the methodologies to detect the sources of misspecification. Section 5 shows how the researcher can deal with DSGE misspecification using hybrid models. Section 6 summarizes the findings and provides concluding remarks.

2 Medium Scale Model: Smets and Wouters (2007)

The Smets and Wouters (2007) model is a medium scale model which features sticky nominal price and wage contracts, habit formation, variable capital utilization and investment adjustment costs. The closed economy consists of households, labor unions, labor packers, a productive sector, and a monetary policy authority. Households consume, accumulate government bonds and supply labor. A labor union differentiates labor and sets wages in a monopolistically competitive setup. Competitive labor packers buy labor services from the union, package and sell them to intermediate goods firms. Output is produced in several steps, including a monopolistically competitive sector with producers facing price rigidities. The monetary policy authority sets the short-term interest rate according to a Taylor rule. As described in Smets and Wouters (2007) and Bekiros and Paccagnini (2014), the model is represented by the following equations.

The demand side of the economy is composed by consumption (c_t), investment (i_t), capital utilization (z_t), and government spending $\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \sigma_g \eta_t^g + \rho_{ga} \eta_t^a$ which is assumed to be exogenous.

The total output (y_t) is represented by:

$$y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g, \tag{1}$$

where c_y is the steady-state share of consumption in output and equals $(1 - g_y - i_y)$, where g_y and i_y are respectively the steady-state exogenous spending-output ratio and investment-output. Instead, $z_y = R_*^k k_y$, where R_*^k is the steady-state rental rate of capital, and k_y is the steady-state capital-output ratio.

The consumption Euler equation evolves as:

$$\begin{aligned} c_t \quad = \quad & \frac{\lambda/\gamma}{1+\lambda/\gamma} c_{t-1} + \left(1 - \frac{\lambda/\gamma}{1+\lambda/\gamma}\right) E_t c_{t+1} + \\ & \frac{(\sigma_c - 1)\left(W_*^h L_*/C_*\right)}{\sigma_c\left(1+\lambda/\gamma\right)} \left(l_t - E_t l_{t+1}\right) - \\ & \frac{(1-\lambda/\gamma)}{\sigma_c\left(1+\lambda/\gamma\right)} (r_t - E_t \pi_{t+1} + \varepsilon_t^b), \end{aligned} \tag{2}$$

where l_t is the hours worked, r_t is the nominal interest rate, and π_t is the rate of inflation. If the degree of habits is zero ($\lambda = 0$) and $\sigma_c = 1$, Equation (2) reduces to the standard forward looking consumption Euler equation. The disturbance is assumed to follow a first-order autoregressive process with an IID-Normal error term: $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$.

The linearized investment equation is as follows:

$$
\begin{aligned}
i_t \;=\; & \frac{1}{1+\beta\gamma^{(1-\sigma_c)}} i_{t-1} + \left(1 - \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}\right) E_t i_{t+1} + \\
& \frac{1}{(1+\beta\gamma^{(1-\sigma_c)})\gamma^2\varphi} q_t + \varepsilon_t^i,
\end{aligned} \tag{3}
$$

where i_t denotes the investment and q_t is the real value of existing capital stock (Tobin's Q). φ is the steady-state elasticity of the capital adjustment cost function, and β is the discount factor applied by households. The investment-specific technology process follows a first-order autoregressive process with an IID-Normal error term: $\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i$.

The arbitrage equation for the value of capital is given by:

$$
\begin{aligned}
q_t \;=\; & \beta\gamma^{-\sigma}(1-\delta)E_t q_{t+1} + (1-\beta\gamma^{-\sigma}(1-\delta))E_t r_{t+1}^k \\
& -(r_t - E_t\pi_{t+1} + \varepsilon_t^b),
\end{aligned} \tag{4}
$$

where $r_t^k = -(k_t - l_t) + w_t$ denotes the real rental rate of capital which is negatively related to the capital-labour ratio and positively to the real wage.

On the supply side of the economy, the aggregate production function is defined as:

$$
y_t = \phi_p(\alpha k_t^s + (1-\alpha)l_t + \varepsilon_t^a), \tag{5}
$$

where ϕ_p and α are respectively one plus the share of fixed costs in production and the share of capital in production. The total factor productivity follows a first-order autoregressive process: $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$.

k_t^s represents capital services which is a linear function of lagged installed capital (k_{t-1}) and the degree of capital utilization, $k_t^s = k_{t-1} + z_t$. Capital utilization is proportional to the real rental rate of capital, $z_t = \frac{1-\Psi}{\Psi} r_t^k$, where Ψ is a positive function of the elasticity of the capital utilization adjustment cost function and normalized from zero (in equilibrium the rental rate on capital is constant) to one (the utilization of capital is constant).

The accumulation process of installed capital is simply described as:

$$
k_t = \frac{1-\delta}{\gamma} k_{t-1} + \frac{\gamma - 1 + \delta}{\gamma} i_t + \left(1 - \frac{(1-\delta)}{\gamma}\left(1 + \beta\gamma^{(1-\sigma_c)}\gamma^2\varphi\right)\right)\varepsilon_t^i, \tag{6}
$$

Monopolistic competition within the production sector and Calvo-pricing constraints gives the New-Keynesian Phillips curve for inflation:

$$\begin{aligned}\pi_t &= \frac{\iota_p}{1+\beta\gamma^{(1-\sigma_c)}\iota_p}\pi_{t-1} + \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}\iota_p}E_t\pi_{t+1} \\ &\quad - \frac{1}{1+\beta\gamma^{(1-\sigma_c)}\iota_p}\frac{1-\beta\gamma^{(1-\sigma_c)}\xi_p(1-\xi_p)}{\left(\xi_p(\phi_p-1)\varepsilon_p+1\right)}\mu_t^p + \varepsilon_t^p,\end{aligned} \tag{7}$$

where $\mu_t^p = \alpha(k_t^s - l_t) - w_t$ is the marginal cost of production and the price mark-up disturbance follows an ARMA(1,1)[1] process $\varepsilon_t^p = \rho_p\varepsilon_{t-1}^p + \eta_t^p - \mu_p\eta_{t-1}^p$, where η_t^p is an IID-Normal price mark-up shock. If the degree of indexation to past inflation is zero, $\iota_p = 0$, the Equation (7) becomes a standard forward-looking Phillips curve. The speed of adjustment depends on the degree of price stickness (ξ_p), the curvature of the Kimball goods market aggregator (ε_p), and the steady-state mark-up which is related in equilibrium to $(\phi_p - 1)$, the share of fixed costs in production.

Monopolistic competition in the labour market also produces a similar wage New-Keynesian Phillips curve:

$$\begin{aligned}w_t &= \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}w_{t-1} + \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}}\left(E_t w_{t+1} - E_t\pi_{t+1}\right) \\ &\quad - \frac{1+\beta\gamma^{(1-\sigma_c)}\iota_w}{1+\beta\gamma^{(1-\sigma_c)}}\pi_t + \frac{\iota_w}{1+\beta\gamma^{(1-\sigma_c)}}\pi_{t-1} \\ &\quad - \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}\frac{1-\beta\gamma^{(1-\sigma_c)}\xi_w(1-\xi_w)}{(\xi_w(\phi_w-1)\varepsilon_w+1)}\mu_t^w + \varepsilon_t^w,\end{aligned} \tag{8}$$

where $\mu_t^w = w_t - \sigma_l l_t + \frac{1}{1-\lambda/\gamma}\left(c_t - \lambda/\gamma c_{t-1}\right)$ is the households' marginal benefit of supplying an extra unit of labour service and the wage mark-up shock is an ARMA(1,1)[2] process, $\varepsilon_t^w = \rho_w\varepsilon_{t-1}^w + \eta_t^w - \mu_w\eta_{t-1}^w$,where η_t^w is an IID-Normal error term. If the degree of indexation to past inflation is zero, $\iota_w = 0$, the Equation (8) does not depend on lagged inflation. The speed of adjustment depends on the degree of wage stickness (ξ_w), the curvature of the Kimball labour market aggregator (ε_w), and the steady-state labour market mark-up $(\phi_w - 1)$.

The model is closed by the Taylor rule:

$$r_t = \rho r_{t-1} + (1-\rho)\left[r_\pi\pi_t + r_Y(y_t - y_t^p)\right] + r_{\Delta y}\left[(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)\right] + \varepsilon_t^r, \tag{9}$$

where y_t^p is the flexible price level of output and $\varepsilon_t^r = \rho_r\varepsilon_{t-1}^r + \eta_t^r$ follows a first-order autoregressive process with an IID-Normal error term.

[1] The MA(1) term is included to capture the high-frequency fluctuations in inflation.
[2] The MA(1) term is included to capture the high-frequency fluctuations in wages.

Equations (1) to (9) determine 14 endogenous variables: y_t, c_t, i_t, q_t, k_t^s, k_t, z_t, r_t^k, μ_t^p, μ_t^w, π_t, w_t, l_t and r_t. The stochastic behaviour of the system of linear rational expectations equations is driven by 7 exogenous disturbances: total factor productivity (ε_t^a), investment-specific technology (ε_t^i), risk-premium (ε_t^b), exogenous spending (ε_t^g), price mark-up (ε_t^p), wage mark-up (ε_t^w), and monetary policy shock (ε_t^r).

The model can be solved by applying the algorithm proposed by Sims (2002). As discussed in Chib and Ramamurthy (2010), the vector of states has a 53 dimensional, given the sticky price-wage and flexible price-wage settings (in asterisks):

$\tilde{Z}_t = (y_t,\ k_t^s,\ l_t,\ r_t^k,\ w_t,\ \pi_t,\ \mu_t^p,\ c_t,\ r_t,\ z_t,\ q_t,\ i_t,\ k_t,\ \mu_t^w,\ E_t\pi_{t+1},\ E_tc_{t+1},\ E_tl_{t+1},\ E_tq_{t+1},\ E_tr_{t+1}^k,\ E_ti_{t+1},\ E_tw_{t+1},\ y_{t-1},\ c_{t-1},\ i_{t-1},\ w_{t-1},\ u_t^a,\ u_t^b,\ u_t^g,\ u_t^i,\ u_t^r,\ u_t^p,\ u_t^w,\ \varepsilon_t^p,\ \varepsilon_t^w,\ y_t^*,\ k_t^{s*},\ l_t^*,\ r_t^{k*},\ w_t^*,\ \pi_t^*,\ \mu_t^{p*},\ c_t^*,\ r_t^*,\ z_t^*,\ q_t^*,\ i_t^*,\ k_t^*,\ \mu_t^{w*},\ E_tc_{t+1}^*,\ E_tl_{t+1}^*,\ E_tq_{t+1}^*,\ E_tr_{t+1}^{k*},\ E_ti_{t+1}^*,\ y_{t-1}^*)$.

The vector of innovations:

$\epsilon_t = \left(\varepsilon_t^a,\ \varepsilon_t^i,\ \varepsilon_t^b,\ \varepsilon_t^g,\ \varepsilon_t^p,\ \varepsilon_t^w,\ \varepsilon_t^r\right)$

and the vector of the endogenous rational expectations errors:

$\eta_t = (\pi_t - E_{t-1}\pi_t,\ c_t - E_{t-1}c_t,\ l_t - E_{t-1}l_t,\ q_t - E_{t-1}q_t,\ r_t^k - E_{t-1}r_t^k,\ i_t - E_{t-1}i_t,\ w_t - E_{t-1}w_t,\ c_t^* - E_{t-1}c_t^*,\ l_t^* - E_{t-1}l_t^*,\ q_t^* - E_{t-1}q_t^*,\ r_t^{k*} - E_{t-1}r_t^{k*},\ i_t^* - E_{t-1}i_t^*)$.

Therefore the previous set of Equations, (1) - (9), can be recasted into a set of matrices $(\Gamma_0, \Gamma_1, C, \Psi, \Pi)$ accordingly to the definition of the vectors $\tilde{Z}_t$ and ϵ_t :

$$\Gamma_0\tilde{Z}_t = C + \Gamma_1\tilde{Z}_{t-1} + \Psi\epsilon_t + \Pi\eta_t. \tag{10}$$

The following transition equation is the solution written as policy function:

$$\tilde{Z}_t = T(\theta)\,\tilde{Z}_{t-1} + R(\theta)\,\epsilon_t, \tag{11}$$

and in order to provide the mapping between the observable data and those computed as deviations from the steady state of the model we set the following measurement equations as:

$$
Y_t = \begin{bmatrix} \Delta \ln y_t \\ \Delta \ln c_t \\ \Delta \ln i_t \\ \Delta \ln w_t \\ \ln l_t \\ \Delta \ln P_t \\ \ln R_t^a \end{bmatrix} = \begin{bmatrix} \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \bar{l} \\ \overline{\pi} \\ \overline{r} \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ l_t \\ \pi_t \\ r_t \end{bmatrix},
$$

where ln denotes 100 times log and $\Delta \ln$ refers to the log difference. $\overline{\gamma} = 100(\gamma - 1)$ is the common quarterly trend growth rate to real GDP, consumption, investment, and wages. Instead, $\overline{\pi} = 100(\Pi_* - 1)$ is quarterly steady-state inflation rate, $\overline{r} = 4 * 100(\beta^{-1}\gamma^{\sigma_c}\Pi_* - 1)$ is the steady-state nominal interest rate, and $\bar{l}$ is the steady-state hours worked, which is normalized to be equal to zero.

We can write the following equation:

$$
Y_t = \Lambda_0 (\theta) + \Lambda_1 (\theta) \tilde{Z}_t + v_t, \tag{12}
$$

where $Y_t = (\Delta \ln y_t, \Delta \ln c_t, \Delta \ln i_t, \Delta \ln w_t, \ln l_t, \Delta \ln P_t, \ln R_t^a)'$, $v_t = 0$ and Λ_0 and Λ_1 are defined accordingly. The matrices T, R, Λ_0 and Λ_1 are in function of the structural parameters in the model.

The linearized state-space representation of the Smets and Wouters (2007) (but in general of any DSGE model with no-time-varying parameters (θ)) is as follows:

$$
\begin{aligned}
Y_t &= \Lambda_0 (\theta) + \Lambda_1 (\theta) \tilde{Z}_t + v_t, \qquad (13) \\
\tilde{Z}_t &= T (\theta) \tilde{Z}_{t-1} + R (\theta) \epsilon_t \ ,
\end{aligned}
$$

where Y_t is a vector of $(k \times 1)$ observables, such as aggregate output, inflation, and interest rates. This vector represents the measurement equation. Instead, the vector $\tilde{Z}_t$ $(n \times 1)$ contains the unobserved exogenous shock processes and the potentially unobserved endogenous state variables of the model. The model specification is completed by setting the initial state vector $\tilde{Z}_0$ and making distributional assumptions for the vector of innovations ϵ_t $(E[\epsilon_t] = 0, E\left[\epsilon_t \epsilon_t^{'}\right] = I$ and $E[\epsilon_t \epsilon_{t-j}] = 0$ for $j \neq 0)$. v_t is the measurement error.

Over the last few years, Bayesian estimation of DSGE models has become very popular. As discussed by An and Schorfheide (2007), the popularity depends on three important features. First, the Bayesian estimation is system-based and fits the solved DSGE model to a vector of aggregate time series, as opposed to the Generalized Method of Moments (GMM) which is based on equilibrium relationships, such as the Euler equation for the consumption or the monetary policy rule. Second, Bayesian estimation is based on the likelihood function generated by the DSGE model rather than the discrepancy between DSGE responses and VAR impulse responses. Third, prior distributions can be used to incorporate additional information into the parameter estimation.

On a theoretical level, the Bayesian estimation takes the observed data as given, and treats the parameters of the model as random variables. In general terms, the estimation procedure involves solving the linear rational expectations model described above and the solution is written in Equation (13). After that, the Kalman Filter is applied to develop the likelihood function. Prior distributions are important to estimate DSGE models. According to An and Schorfheide (2007), priors might downweigh regions of the parameter space that are at odds with observations which are not contained in the estimation sample. Priors could add curvature to a likelihood function that is (nearly) flat for some parameters, given a strong influence to the shape of the posterior distribution. Posterior distribution of the structural parameters is formed by combining the likelihood function of the data with a prior density, which contains information about the model parameters obtained from the other sources (microeconometrics, calibration, and cross-country evidence), thus allowing to extend the relevant data beyond the time series which are used as observables. Numerical methods such as Monte-Carlo Markov-Chain (MCMC) are used to characterize the posterior with respect to the model parameters.[3]

3 Forms of Misspecification

The statistical representation in Equation (13) could be misspecified. As stated in Fernández-Villaverde, Rubio-Ramirez, and Schorfheide (2016), model misspecification can be interpreted as a violation of the cross-coefficient restrictions embodied in the mapping from the DSGE model parameters θ into the system of matrices of the state-space representation. Canova and Matthes (2017) and Canova (2017) discusses about the misspecification referring not only about the state-space representation but adding issues about parameters in the misspecified model. So far, in the DSGE modeling literature there is not a unique definition of misspecification. This survey sheds light distinguishing four forms of misspecification: a) Misspecification in the State-Space Representation, b) Misspecification in Parameters, c) Misspecification in the Assumptions of the DSGE model, and d) Misspecification in Computational Methods.

[3]See Smets and Wouters (2003, 2007), An and Schorfheide (2007), and Herbst and Schorfheide (2015) for more details on Bayesian estimation of DSGE models.

The first form of misspecification(a)) refers to the definition described in Fernández-Villaverde, Rubio-Ramirez, and Schorfheide (2016), adding the second form of misspecification (b)), we can refer to the general definition of misspecification discussed in Canova and Matthes (2017) and Canova (2017).

Moreover, each form of misspecification includes different aspects. Misspecification in the State-Space Representation relies on three problems: Misspecification and Model Features, Misspecification and Shocks, and Misspecification of the statistical representation. Instead, Misspecification in Parameters refers to the role of the parameter stability and the identification in DSGE misspecification. Meanwhile, Misspecification in the Assumptions refers to the hypothesis of rational expectations and of linearity. Last but not least, Misspecification in Computational Methods refers to estimation of the posterior in the DSGE modeling. Table 1 summarizes the misspecification aspects referring to the Smets and Wouters (2007) model.

3.1 Misspecification in the State-Space Representation

1) Misspecification and Model Features. DSGE models, as any other models, are a stylized picture of reality. Small scale DSGE and medium scale DSGE are often used for policy analysis and forecasting comparisons. Some models could be misspecified since they do not include relevant variables.

For example, Smets and Wouters (2007) ignore financial and housing markets which are relevant variables to explain the shocks dynamics, in particular during crisis periods. Furthermore, fiscal sector and labor market are stylized and not properly modelled in the Smets and Wouters (2007). Recently, these medium scale models have been criticized for their limitation to explain the Great Recession[4]. Del Negro and Schorfheide (2013) is an example of improvement of the fit of structural economic framework during the crisis, including inflation expectations, financial frictions, and interest rate spreads.

In addition, there are several model features made by researchers to simplify the theoretical model, such as constant real interest rate, quadratic preferences in consumption, homogeneous agents, and exogenous labour income. But some of these elements could be relaxed to improve the matching between the theory and the data. For example, several models show interesting results using heterogeneous agents (see Colander, Howitt, Kirman, Leijonhufvud and Mehrling, 2008, Den Haan, 2010, Massaro, 2013, McKay and Reis, 2016, and Kaplan, Moll, and Violante, 2017 among others). In terms of consumers, DSGE models could incorporate a fraction of Non-Ricardian households who do not hold any wealth and entirely consume their disposable labor income in each period (the Limited Asset Market Participation hypothesis) (as discussed in theoretical framework in Galí et al., 2004 and Bilbiie, 2008; and recently in empirical analysis in Albonico, Paccagnini, and Tirelli, 2016 and 2017).

[4] Christiano, Eichenbaum, and Trabandt (2017) review the state of DSGE models before the financial crisis and how the role of these models changed after the crisis in the policy analysis.

2) Misspecification and Shocks. Usually, in a DSGE model the number of observable variables matches the number of shocks to face with the non-singularity condition. The Smets and Wouters (2007) model features seven observable variables which match seven exogenous disturbances: total factor productivity (ε_t^a), investment-specific technology (ε_t^i), risk-premium (ε_t^b), exogenous spending (ε_t^g), price mark-up (ε_t^p), wage mark-up (ε_t^w), and monetary policy shock (ε_t^r). All shocks are modelled as white noise with exception price and wage mark-ups which are modelled as ARMA processes.

Adding shocks has been always a common practice to improve the connection of the theoretical DSGE with the observed data and there are several interesting contributions. Firstly, Sargent (1989) and Ireland (2004) introduce serial correlated errors in measurement equations of the state-space representation of the model. Recently, Canova, Ferroni, and Matthes (2014) propose two methods to choose the variables to be used in the estimation of the structural parameters of a DSGE model which suffers from singularity. The first method allows to select the vector of observables which optimizes the parameter identification; the second one allows to select the vector which minimizes the gap between the singular and non-singular model. Meanwhile, Ferroni, Grassi, and Léon-Ledesma (2017) focus on the difference between "primal" and "no primal", called "non-existent" shocks. Typically DSGE models are estimated assuming the existence of certain "primal" or structural shocks which drive the business cycle. Ferroni, Grassi, and Léon-Ledesma (2017) analyze the consequences of estimating shocks which are "non-existent" and they propose a rigorous method to select the structural or primal shocks driving macroeconomic uncertainty. They provide evidence how forcing the existence of "non-existent" shocks generates a downward bias in the estimated internal persistence of the DSGE model. They evidence how the researcher can avoid or reduce these distorsions by allowing the covariance matrix of the structural shocks to be rank deficient. To avoid the downward bias, they propose to use normal or exponential priors (which include zero) for standard deviations together with measurement error to avoid stochastic singularity. At the same time, Meyer-Gohde and Neuhoff (2015) discuss the importance of stochastic shocks in the misspecification of DSGE models relying on an ARMA set-up for them. They propose a Bayesian approach to estimate the order as well as the parameters of generalized ARMA representations of exogenous driving forces within the DSGE. To make this generalization, they adopt the Reversible Jump Markov Chain Monte Carlo (RJMCMC) methodology introduced by Green (1995).

3) Misspecification of the Statistical Representation. Several research studies[5] have challenged the validity of a Vector Autoregressive (VAR) or a Structural VAR (SVAR) as main tool for estimating and studying the transmission mechanisms of macroeconomic shocks. In particular, linearization of DSGE with first-order approximation made linear time series models such as VARs suitable for evaluating DSGE model

[5] Ravenna (2007), Liu and Theodoridis (2012), Giacomini (2013), Franchi and Vidotto (2013), Pagan and Robinson (2016), Ramey (2016), Morris (2016 and 2017), Forni, Gambetti, and Sala (2017) and Poskitt and Yao (2017) among others.

restrictions.

First, DSGE and VARs models can be related in an indirect inference or minimum-distance set-up in which we assume that the DSGE model provides a realistic probabilistic representation of the data. Hence, the researcher chooses the DSGE model parameters such that VAR coefficients or impulse response functions realized from the actual data match those obtained from the DSGE model-simulated data as closely as possible. The magnitude of the minimized discrepancy provides a measure of fit. This approach, of using the VAR as an auxiliary model, was firstly discussed in Smith (1993) and Cogley and Nason (1994).

Second, following Schorfheide (2000) who discusses the idea that the DSGE model is considered (potentially) misspecified, Del Negro and Schorfheide (2004) and Del Negro, Schorfheide, Smets, and Wouters (2007a) introduce the DSGE-VAR, providing a hybrid model to combine the information derived from the prior of the theoretical model with the time series properties through the VAR representation which approximates the DSGE model. Chari, Kehoe, and McGrattan (2008) examine a stylized business cycle model and find that the impulse response function (IRF) computed from a finite-order VAR yields a poor characterization of the true responses. Alike, in a study about the real business cycle (RBC) models, Erceg, Guerrieri, and Gust (2005) evidence that the error associated with using a finite-order VAR model can be large and attribute this to small-sample error. In contrast, Ravenna (2007) discusses how a finite-order SVAR model can lead to inaccurate estimates of the true IRFs but points out that this may not be a small-sample problem. Moreover, Ravenna (2007) shows that the error derives from two separate sources: a "truncation bias" and an "identification bias". Recently, Poskitt and Yao (2017) provide a detailed theoretical examination of the loss incurred when approximating a VAR(∞) process by a finite lag VAR(p) model. They name them: "estimation error" (the difference between the estimated VAR(p) and its theoretical counterpart) and "approximation error" (the difference between the theoretical minimum mean squared error VAR(p) approximation and the true VAR(∞) process).

We have to point out that many DSGE models have a solution which is not compatible with a finite VAR representation, but they should be represented by a VARMA model as discussed in Giacomini (2013), Franchi and Vidotto (2013), Pagan and Robinson (2016), and Morris (2016 and 2017) among others.

The Smets and Wouters (2007) has the price and wage mark-up shocks which are ARMA processes, hence, this model is represented by a VARMA and the solution does not involve a finite order VAR.

The DSGE model validation cannot rely on the VAR since the true statistical representation of a DSGE model is not always a finite order VAR. Hence, using the traditional modeling approach, we have several weakness such as statistical misspecification, non-identification of deep parameters (of the optimizing model), weak forecasting evaluation, and potentially misleading Impulse Response Functions (IRFs) as shown in Poudyal and Spanos (2016) among others. This problem is known as "non-fundamentalness" or "non-invertibility" of the moving average representation implied by the model. When such representation is not invertible, a VAR representation in terms of all of the structural shocks does not exist [6]. Fernández-Villaverde, Rubio-Ramirez, Sargent and Watson (2007) derive a condition for the validity of VAR methods, related to the state-space representation of the macroeconomy. The condition, known as the "Poor Man's Condition", implies fundamentalness of the corresponding moving average representation and the possibility of recovering all of the structural shocks from a VAR.

Poudyal and Spanos (2016) contribute the literature presenting a rigorous statistical analysis to evaluate the validity of the implicit statistical model. The failure of these tests evidences how the Normal VAR representation is statistically misspecified. They propose a Student's t VAR model to overcome the problem of the weak model validation. The Student's t VAR model is also useful to identify the deep structural parameters, and hence to improve the forecasting performance and the policy analysis through IRFs.

However, if the true statistical representation for a DSGE model is the VARMA, the natural counterpart should the VARMA model. As stated in Morris (2016), VARMA representations of DSGE models are currently not widely utilized[7].

3.2 Misspecification in Parameters

1) Misspecification and Parameter Instabilities. The Smets and Wouters (2007) model features parameters without instabilities, but DSGE empirical literature has discussed alternative approaches to deal with the possible problem of parameters instabilities[8].With Markov-switching DSGE framework, the researcher models and estimates the regime change in some of the key parameters (Bianchi, 2013, Foerster, Rubio-Ramirez, Waggoner, and Zha, 2016, Eo and Kim, 2016). Similar to Markov-switching, Waggoner and Zha (2012) estimate a Markov-switching mixture of two models: a DSGE model and a Bayesian VAR. They find that the Markov-switching mixture model dominates both models and improves the fit. This interesting approach is introduced to deal with misspecification issues. A practical way to introduce the parameter instabilities in a DSGE model, in particular in a forecasting exercise is shown in Kolasa and Rubaszek (2015). They observe that central banks are used to re-estimate DSGE models only occasionally but this practice might affect the forecasting performance. Hence, they investigate how frequently models

[6] Lippi and Reichlin (1993), Alessi, Barigozzi, and Capasso (2011), Sims (2012), Liu and Theodoridis (2012), Leeper, Walker and Yang (2013), Beaudry, Fève, Guay, and Portier (2015), Forni and Gambetti (2016), Soccorsi (2016), Chen, Choi and Escanciano (2017), Forni, Gambetti, Lippi and Sala (2017a) and (2017b), and Forni, Gambetti, and Sala (2017) among others.

[7] An exception to this, Kascha and Mertens (2009) propose an interesting application about business cycle estimating a VARMA model.

[8] Fernández-Villaverde, Guerrón-Quintana, and Rubio-Ramirez (2010), Inoue and Rossi, (2011), Caldara, Fernández-Villaverde, Rubio-Ramirez, and Yao (2012), Castelnuovo (2012), Bekiros and Paccagnini (2013) Hurtado (2014), Galvao, Giraitis, Kapetanios, and Petrova (2016), Bekiros, Cardani, Paccagnini, and Villa (2016) among others.

should be re-estimated so that the accuracy of forecasts they generate may be unaffected. Even if they show the advantage of updating the model parameters for calculating density forecasting, updating the model parameters only once a year does not lead to a significant deterioration in the accuracy of point forecasts.

2) Misspecification vs Identification. Identification investigates whether a parameter vector θ is identifiable based on a sample Y. Parameters must be "identified" to obtain meaningful results of estimation. Using limited-information methods, Canova and Sala (2009) show that many structural parameters in stylized New-Keynesian DSGE models are not identified. Most of the literature has focused on local identification, Beyer and Farmer (2004), Canova and Sala (2009), Iskrev (2010), Komunjer and Ng (2011), Qu and Tkachenko (2012), Dufour, Khalaf, and Kichian (2013) since it is easy to verify.

In particular, Iskrev (2010) and Komunjer and Ng (2011) develop necessary and sufficient rank conditions for assessing identifiability of DSGE model parameters. In addition, Iskrev (2010) applies this method to the Smets and Wouters (2007) model showing how this model does not face with the rank condition. This problem of lack of identifiability of two curvature parameters for the goods and labor markets, and the Calvo wage and price parameters. As suggested by Guerrón-Quintana, Inonue, and Kilian (2013) and Beltran and Draper (2016), even weakly identified DSGE models are an issue for the researcher, in particular for valid inference. Recently, Kocieki and Kolasa (2013), Qu and Tkachenko (2016) and Naghi (2017) propose different methodologies to check for global identification. However, if the model's parameters are not identified, any solutions about misspecification are useless.

3.3 Misspecification in the Assumptions

1) Misspecification of the Expectations. Assuming rational expectations implies assuming that agents know the data generating process and form their expectations consistently.

In the literature, Learning is the first attempt to deviate from rational expectations. Adaptive learning in a Bayesian estimation of a DSGE model was mainly discussed by Milani (2007 and 2012 for a survey)[9]. This econometric approach allows joint estimation of the main learning rule coefficient (called the "constant gain"), together with the structural parameters of a small scale DSGE model. Furthermore, Slobodyan and Wouters (2012a) extend the adaptive learning in the Smets and Wouters model. Meanwhile, Slobodyan and Wouters (2012b) contribute to the literature proposing ad hoc update by Kalman filter to avoid the potential arbitrariness the researcher could face using the constant gain.

As second attempt, Angelini and Fanelli (2016) propose a statistical state-space model for the data, ignoring adaptive learning approach as in Milani, (2007) and Slobodyan and Wouters (2012a). Angelini

[9] The concept of learning in macroeconomics models is already discussed in Evans and Honkapohja (1999, 2001); Branch and Evans (2006).

and Fanelli (2016) show the existence of two types of restrictions on the model's reduced form solution: a) parametric nonlinear cross-equation restrictions (CER) that map the structural to the reduced form parameters and b) constraints on the lag order and correlation structure of the variables. Parametric nonlinear cross-equation restrictions are the traditional metric for evaluation of forward-looking models and rational expectations (RE) (Hansen and Sargent, 1980 and 1981, and Hansen, 2014). Constraints about the lag order are implicit, as Angelini and Fanelli (2016) evidence, and very often researchers are not aware of their importance estimating DSGE models. They introduce a "pseudo-structural" model that combines the structural information of the DSGE model with the data features. In this pseudo-structural format, Angelini and Fanelli (2016) specify the Euler Equation augmented by a given number of additional lags of the variables to fill the gap between the dimension of the state vector of the structural model and the dimension of the state vector of the statistical model.

2) Misspecification of the Linearity Approximation. Most of the estimated DSGE models are linearized around a steady state since a linear state-space representation together with the assumption of normality of exogenous shocks allows the researcher to estimate the likelihood using the Kalman Filter. However, DSGE models are often highly non-linear models and linearization is a simple way to deal with this problem. Fernández-Villaverde and Rubio-Ramirez (2005) and Fernández-Villaverde, Rubio-Ramirez, and Santos (2006) evidence that the level of likelihood and parameter estimates based on a linearized model can be significantly different from those based on its original nonlinear model. As discussed by Hirose and Sunakawa (2016), one of the main reason of using linear instead of nonlinear estimation is given by high computational costs for the estimation of nonlinear models. To evaluate the likelihood function in a nonlinear framework, the researcher needs to rely on a nonlinear solution method and a particle filter, both of which require iterative procedures, and their computational procedure grows rapidly with an increase in the dimensionality of problems. For this purpose, Hirose and Sunakawa (2016) investigate about the possible parameter bias when we adopt linear solution instead of nonlinear one. For many of DSGE models, for example, both standard stochastic growth and New Keynesian model, built to explain pre-Great Recession business cycle fluctuations, the endogenous nonlinearities are small and only matter for the calculation of asset prices and welfare comparisons as discussed in Arouba, Bocola, and Schorfheide (2017). However, recently the literature has presented models with explicit nonlinearities such as stochastic volatility (e.g., Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramirez, 2015; Fernández-Villaverde, Guerrón-Quintana, and Rubio-Ramirez, 2015; Justiniano and Primiceri, 2008, and Diebold, Schorfheide, and Shin, 2017), an effective lower bound on nominal interest rates (e.g., Ngo, 2014; Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramirez, 2015; Gavin, Keen, Richter, and Throckmorton, 2015; Maliar and Maliar, 2015; Braun, Korber, and Waki, 2016; Nakata, 2016 and 2017; Gust, Herbst, Lopez-Salido, Smith, 2017; Aruoba, Bocola, and Schorfheide, 2017; and Basu and Bundick, 2017), or financial frictions (e.g., Brunnermeier and Sannikov, 2014; Gertler, Kiyotaki, and Queralto, 2012; He and Krishnamurthy, 2015, and Bocola, 2016). We need to state that this growing research field got advantages from recent computational advances in DSGE models solutions with nonlinearity as explained in Maliar and Maliar (2014), Fernández-Villaverde, Rubio-Ramirez, and Schorfheide (2016), and Gust, Herbst, Lopez-Salido, and Smith (2017). In particular, Arouba, Bocola, and Schorfheide (2017) build several time series models that mimic nonlinearities of DSGE models and these models are used as a benchmark to evaluate nonlinear DSGEs.

3.4 Misspecification in Computational Methods

Misspecification in Posterior Estimation. Recent developments in Bayesian computations have helped the researcher to improve the quality of the estimation of DSGE models (see for example, Fernández-Villaverde and Rubio-Ramirez, 2004; Lubik and Schorfheide, 2004; Smets and Wouters, 2003 and 2007; An and Schorfheide, 2007; Canova, 2007; Karagedikli, Matheson, Smith, and Vahey, 2010; DeJong and Dave, 2011; Del Negro and Schorfheide, 2011; Herbst and Schorfheide, 2016, and Fernández-Villaverde, Rubio-Ramirez, and Schorfheide, 2016). In the Bayesian approach, we define a prior distribution for parameters of the model which combines with the maximum likelihood driven by the data. MonteCarlo Markov Chain simulation methods are the machinery for sampling the posterior distribution of the parameters (Chib and Greenberg, 1995 and Chib, 2001). As pointed by Chib and Ramamurthy (2009), the traditional approach is to sample the posterior distribution by what is formally known as a single block random-walk Metropolis Hastings (MH) algorithm (RW-MH). In the RW-MH algorithm, the parameters are sampled in a single block by drawing a proposal from a random walk process. This proposal value is then accepted as the next draw according to the corresponding MH probability of move (which in this case is essentially the ratio of the posterior density at the proposed value and the posterior density at the current value); if the proposed value is rejected, the current value is retained as the new value of the Markov Chain. This approach is easy and quick. But when the posterior distribution is irregular, the RW-MH algorithm is not straightforward. As already demonstrated by An and Schorfheide (2007), Chib and Ramamurthy (2010) discuss how in a multi-modal problem, the effect of the initial value in the algorithm may not wear off in realistic sampling time. Another issue about the RW-MH algorithm is that the variance of the increment in the random walk proposal can be difficult to set, especially in higher-dimensional problems, and the sampler performance can be severely comprised by a poor choice of it. With too small a variance the search process can be extremely slow, whereas with a large variance there can be many rejections and the same value can be repeated many times in the chain. Hence, the sampled sequence tends to exhibit high serial correlations and slow convergence to the posterior distribution. To solve all these issues, Chib and Ramamurthy (2010) propose new MCMC schemes for the estimation of DSGE models using Bayesian approach. They combine the efficiency of tailored proposals (Chib and Greenberg, 1994) with a flexible blocking strategy that virtually eliminates pre-run tuning. In their approach, called the Tailored Randomized Block MH or TaRB-MH algorithm, the parameters of the model are clustered at every iteration into a random number of blocks. Then each block is sequentially updated through an MH step in which the proposal density is tailored to mimic closely the target density of that block. Chib and Ramamurthy (2010) apply their procedure to the Smets and Wouters (2007) model and the An and Schorfheide bimodal problem showing an improvement in the quality of posterior calculation.

Smets-Wouters (2007)	
Misspecification in the State-Space Representation	
Model Features	No financial variables, No housing sector, No heterogenous agents, No Non-Ricardian Consumers
Shocks	ARMA for price and wage mark-up shocks
Statistical Representation	VARMA
Misspecification in Parameters	
Paramenter Instability	Time-varying and instable parameters no modeled
Identification	Identification problems
Misspecification in the Assumptions	
Expectations	Rational Expectation
Linear Approximation	Linear Approximation
Misspecification in Computational Methods	
Posterior Estimation	Random-Walk Metropolis-Hastings

Table 1: Misspecification issues in the Smets and Wouters (2007)

4 Detecting the Sources of Misspecification

The current literature provides evidence of several attempts to detect the sources of misspecification. In particular, to distinguish possible misspecifications in the state-space representation. Originally, Sargent (1989) and Ireland (2004) introduce errors in measurement equations of the state-space representation of the model to assess whether the model is misspecified. After that, Del Negro and Schorfheide (2004, 2007) develop a framework for Bayesian estimation of possibly misspecified DSGE models by using DSGE-model implied parameters as priors for vector autoregressive (VAR) models. This methodology allows for model misspecification and produces the posterior distribution of structural parameters. Moreover, Corradi and Swanson (2007) introduce new tools for comparing the empirical joint distribution of historical time series with the empirical distribution of simulated time series based on structural macroeconomic models. They detect whether the whole distribution of a macroeconomic model is correctly specified. Del Negro and Schorfheide

(2009) and Curdia and Reis (2010) have proposed to investigate about the sources of the misspefication by allowing a more flexible and general correlation structure for the shocks and analyzing which interactions among the disturbances are preferred by the data.

Monti (2015) and Inonue, Kuo, and Rossi (2017), contemporaneously, explore the sources of misspecification proposing two different approaches.

On one side, Monti (2015) proposes to model the states of the DSGE and auxiliary variables jointly, imposing the restrictions implied by the DSGE as priors, and then verify how much weight is given to the priors in the estimation. Hence, using the Granger-causality test [10] on some auxiliary variables, the researcher can verify if the driving processes of the model are assumed to be exogenous in the DSGE, hence there is some form of misspecification[11]. An illustrative example is proposed using Justiniano, Primiceri, and Tambalotti (2010) and Galí, Smets, and Wouters (2012) medium DSGE models.

On the other side, Inonue, Kuo, and Rossi (2017) propose an empirical approach to detect misspecification in structural models, such as DSGE models, assessing which parts of the model are troubled by the misspecification and how qualitatively impact it is. This method formalizes the common practice of adding shocks in the model, and potential misspecification is identified using forecast error variance decomposition (FEVD) and marginal likelihood analyses. In details, they consider two kinds of exogenous processes. The first kind of exogenous processes are structural shocks of the model. The second ones, called "margins" in Inonue, Kuo, and Rossi (2017), are not structural and they are used as check for model misspecification. They incorporate misspecification in the model by including these margins, or independent disturbances in the equilibrium conditions of the model. After the estimation of these disturbances, the researcher is able to identify the source of the misspecification and the behaviour over time. Inonue, Kuo, and Rossi (2017) illustrate an example using the medium scale DSGE in Justiniano, Primiceri, and Tambalotti (2010), showing how asset and labor markets are the main source of misspecification. This methodology helps the researcher to detect different types of misspecifications: exogeneity of the margins, over-parametrization, and non-nesting misspecification.

Last but not least, Canova and Matthes (2017) and Den Haan and Dreschel (2017), contribute to the literature proposing two different methodologies which are not only able to detect the sources but they aim to reduce the misspecification in DSGE models.

Canova and Matthes (2017) propose the composite likelihood, which combines the likelihood of distinct

[10] The idea of using Granger Causality test is borrowed from Evans (1992) who investigates about the exogeneity of productivity shocks in Real Business Cycle (RBC) models, using a bivariate-Granger causality test between the productivity shock implied by an RBC model and a wide number of relevant macro variables.

[11] The approach presented in Monti (2015) is close to the method illustrated by Giannone and Reichlin (2006) to empirically investigate if the shocks recovered from the estimates of a structural VAR are truly structural, which is possible only if the shocks are fundamental. The non-fundalmentalness, as described in Giannone and Reichlin (2006), can be identified by testing whether the VAR is (weakly) exogenous with respect to potentially relevant additional blocks of variables.

misspecified structural or underspecified statistical models, to detect misspecification in the state-space of the DSGE model using a simple diagnostic analysis[12]. This approach is not only able to investigate about the sources of misspecification, but it improves the estimation, and solves the computational, and the inferential problems in misspecified models. The composite likelihood is helpful: 1) to increase the robustness of parameter estimates and to decrease the degree of misspecification for each individual model; 2) to ameliorate population and sample identification problems, 3) to solve singularity issues, 4) to combine information coming from different sources, frequencies and levels of aggregation, and 5) to improve estimation, computational and inferential problems in misspecified DSGE.

Den Haan and Dreschel (2017) show how the Smets and Wouters (2007) model is misspecified using a simple diagnostic test in a MonteCarlo experiment. To reduce the misspecification degree, they suggest to add structural disturbances (called Structural Agnostic Disturbances (SADs)) which are part of the system and propagate as other disturbances are propagated.

5 Dealing with Misspecifications

As discussed in Section 4, there are several proposals to investigate whether a DSGE is misspecified. In this Section, we focus on the solution of the model misspecification, in the state-space representation, illustrating the use of hybrid models. As surveyed in Schorfheide (2013), hybrid models are empirical models that relax DSGE model restrictions which provide a complete analysis of the data law of motion and better capture the dynamics properties of the theoretical model. Following the definitions proposed by Paccagnini (2011) and Schorfheide (2013), we show two different approaches: the Additive Hybrid and the Hierarchical Hybrid Models.

Table 2 compares and summarizes the different hybrid models, showing the estimation method, their main contribution, and Google Scholar citations (November 2017 updated).

5.1 Additive Hybrid Models

The additive hybrid model augments the state-space model Equations (13) with a latent process z_t:

[12] Originally, the composite likelihood is built combining marginal or conditional likelihoods of the true Data Generation Proess to improve computation with intractable or complicated integrals, due to the presence of latent variables (see Besag, 1974 and Lindsay, 1988 for more details).

$$
\begin{aligned}
Y_t &= \Lambda_0(\theta) + \Lambda_1(\theta)\tilde{Z}_t + \Lambda_z z_t, && (14)\\
\tilde{Z}_t &= T(\theta)\tilde{Z}_{t-1} + R(\theta)\epsilon_t \ , \\
z_t &= \Gamma_1 z_{t-1} + \Gamma_\eta \eta_t. && (15)
\end{aligned}
$$

The process z_t is the measurement error represented by an autoregressive process. In such way, we fill the gap between the theory and the data relying on the dynamic structure of this error. There are several examples of additive hybrid models: the DSGE-AR (Sargent, 1989, Altug, 1989), the DSGE-VAR à l' Ireland (2004), the DSGE-DFM (Boivin and Giannoni, 2006), the DSGE with non-modelled variables (Schorfheide, Sill, and Kryshko, 2010), and the Augmented DSGE for Trends (Canova, 2014).

5.1.1 The DSGE-AR method

The first additive hybrid model was introduced by Sargent (1989) and Altug (1989). They propose to solve DSGE model misspecification by augmenting the model with (possibly serial correlated) unobservable errors as described in Equation (14). This methodology combines the DSGE model with an AR model for the measurement residuals.

In detail, a matrix Γ_1 governs the persistence of the residuals; the covariance matrix, $E_t\eta_t\eta_t' = V$, is uncorrelated. In this specification the ϵ_t 's generate the comovements between the observables, whereas the elements of z_t pick up idionsyncratic dynamics which are not explained by the structural part of the hybrid model. However, if we set Λ_0, Λ_1 and Λ_z to zero, the DSGE model components can be used to describe the fluctuations of Y_t around a deterministic trend path, ignoring the common trend restrictions of the structural model. For instance, Smets and Wouters (2003) estimate their model using this pattern with a two-step procedure. In the first step, the deterministic trends are extracted from the data; in the second step, the DSGE model is estimated using linear detrended observations.

Sargent (1989) and Altug (1989) assume that the measurement errors are uncorrelated with the data generated by the model, hence the matrices Γ_1 and V are diagonal and the residuals are uncorrelated across variables:

$$
\Gamma_1 = \begin{bmatrix} \gamma_y & 0 & 0 \\ 0 & \gamma_c & 0 \\ 0 & 0 & \gamma_l \end{bmatrix}
$$

$$
V = \begin{bmatrix} v_y^2 & 0 & 0 \\ 0 & v_c^2 & 0 \\ 0 & 0 & v_l^2 \end{bmatrix}.
$$

5.1.2 The DSGE-VAR à l' Ireland

Ireland (2004) propose a general and multivariate framework for measurement errors, allowing the residuals to follow an unconstrained, first-order vector autoregression. This approach has the main advantage of imposing no restrictions on the cross-correlation of the measurement errors, allowing it to capture all the movements and co-movements in the data not explained by the DSGE model. The matrices Γ_1 and V are given by:

$$
\Gamma_1 = \begin{bmatrix} \gamma_y & \gamma_{yc} & \gamma_{yl} \\ \gamma_{cy} & \gamma_c & \gamma_{cl} \\ \gamma_{ly} & \gamma_{lc} & \gamma_l \end{bmatrix}
$$

$$
V = \begin{bmatrix} v_y^2 & v_{yc} & v_{yl} \\ v_{cy} & v_c^2 & v_{cl} \\ v_{ly} & v_{lc} & v_l^2 \end{bmatrix}.
$$

This framework is more flexible and general in the treatment of measurement errors, but some empirical evidence (such as Fernández-de-Córdoba and Torres, 2011) shows the forecast performance of the traditional DSGE-AR outperforms the DSGE-VAR à l'Ireland. Malley and Woitek (2010) propose an extension, allowing for a vector autoregressive moving average (VARMA) process to describe the movements and co-movements of the model's errors not explained by the basic RBC model.

5.1.3 The DSGE-DFM

In macroeconomics, the researchers have access to large cross-sections of aggregate variables that include measures of sectorial economic activities and prices as well as financial variables. Hybrid models can also be implemented to connect DSGE models with aggregate variables which are not explicit in the model. Using these additional variables in the estimation potentially sharpens inference about latent state variables:

$$
\begin{aligned}
Y_t &= \Lambda_0(\theta) + \Lambda_1(\theta)\,\tilde{Z}_t + z_{y,t}, &(16)\\
\tilde{Z}_t &= T(\theta)\,\tilde{Z}_{t-1} + R(\theta)\,\epsilon_t, &(17)\\
x_t &= \Lambda_0 + \Lambda_1 t + \Lambda_s s_t + z_{x,t}, &(18)
\end{aligned}
$$

where Y_t is the vector of the observable variables that are described by the DSGE model and x_t is a large vector of non-modelled variables.

Since the structure of this model resembles that of a dynamic factor model (DFM), e.g. Sargent and Sims (1977), Geweke (1977), and Stock and Watson (1989), Schorfheide (2013) refers to the system (16) to (18) as an example of a combination of DSGE and DFM (Boivin and Giannoni, 2006). Roughly speaking, the vector of factors is given by the state variables associated with the DGSE model. The processes $z_{y,t}$ and $z_{x,t}$ are uncorrelated across series and model idiosyncratic but potentially serially correlated movements (or measurement errors) in the observables. Moreover, Equation (17) links the variables x_t to the DSGE model. This relation generates comovements between the Y_t' s and the x_t' s and allows the computation of impulse responses to the structural shocks ϵ_t.

5.1.4 DSGE with non-modelled variables

Schorfheide, Sill, and Kryskho (2010) develop a method of generating a DSGE model-based forecast for variables that do not explicitly appear in the model (non-core variables). They consider the following representation:

$$
\begin{aligned}
Y_t &= \Lambda_0(\theta) + \Lambda_1(\theta)\,\varsigma_t, &(19)\\
\tilde{Z}_t &= T(\theta)\,\tilde{Z}_{t-1} + R(\theta)\,\epsilon_t,
\end{aligned}
$$

where Eq (19) is the measurement equation, where $\varsigma_t = [\tilde{Z}_t', \tilde{Z}_{t-1}', M_s'(\theta)]'$ includes the state variables of the model ($\tilde{Z}_t$), the lagged variables for the growth rates, $\tilde{Z}_{t-1}' M_s'(\theta)$[13]. To this state-space representation, we add an auxiliary regression:

[13] In Schorfheide, Sill, and Kryskho (2010), they assume the lagged values of output, consumption, investment, and real wages. These variables are part of the set of the endogenous state variables, in which we have capital and interest rate.

$$z_t = \Lambda_0 + \widehat{\tilde{Z}}'_{t|t}\Lambda_s + \xi_t,$$

where the $\widehat{\tilde{Z}}'_{t|t}$ is derived by the Kalman Filter to obtain estimates of the latent state variables, based on the DSGE model parameter estimates. ξ_t is a variable-specific noise process, $\xi_t = \rho\xi_{t-1} + \eta_t$ and $\eta_t \sim N(0, \sigma^2_\eta)$.

This augmented state-space can be interpreted as a factor model. The factors are given by the state variables of the DSGE model, while the measurement equation associated with the DSGE model describes the way in which the core macroeconomic variables load on factors, and the auxiliary regression describes the way in which additional (non-core) macroeconomic variables load on the factors. This representation is a simplified version of the DSGE-DFM since the DSGE with non-modelled variables do not attempt to estimate the DSGE model and the auxiliary regression simultaneously.

5.1.5 The Augmented DSGE for Trends

One of the most discussed problem in using a DSGE model for estimation is its inability to capture the long-run features of the data. Canova (2014) proposes a way to correct these problems using the following hybrid model:

$$\begin{aligned}
Y_t &= \Lambda_0(\theta) + \Lambda_1(\theta)\tilde{Z}_t + \Lambda_z z_t \qquad (20)\\
\tilde{Z}_t &= T(\theta)\tilde{Z}_{t-1} + R(\theta)\epsilon_t,\\
z_t &= \Gamma_1 z_{t-1} + \Gamma_2 \overline{z}_{t-1} + \Gamma_\eta \eta_t,\\
\overline{z}_t &= \overline{z}_{t-1} + v_t.
\end{aligned}$$

Depending on the restrictions imposed on the variances of η_t and ν_t, the process z_t is integrated of order one or two and can generate a variety of stochastic trend dynamics.

5.2 Hierarchical Hybrid Models

The second class of hybrid models used for estimating the DSGE model is the hierarchical hybrid.

Consider the following modification of the additive hybrid model:

$$
\begin{aligned}
Y_t &= \Lambda_0(\theta) + \Lambda_1(\theta)\tilde{Z}_t + \upsilon_t, \\
\tilde{Z}_t &= \Gamma_1(\theta)\tilde{Z}_{t-1} + \Gamma_\epsilon(\theta)\epsilon_t,
\end{aligned}
\tag{21}
$$

where

$$
\begin{aligned}
\Lambda_i &= \Psi_i(\theta) + \eta_i^\Psi, \quad i = 0, 1 \\
\Gamma_i &= \Phi_i(\theta) + \eta_i^\Phi, \quad i = 1, \epsilon.
\end{aligned}
\tag{22}
$$

In this setup, $\Psi_i(\theta)$ and $\Phi_i(\theta)$ are interpreted as restrictions on the unrestricted state-space matrices Λ_i and Γ_i; instead, the disturbances, η_i^Ψ and η_i^Φ can capture deviations from the restriction functions $\Psi_i(\theta)$ and $\Phi_i(\theta)$. This kind of hybrid model is related to Bayesian econometrics, since the stochastic restrictions (22) correspond to a prior distribution of the unrestricted state-space matrices conditional on the DSGE model parameters θ.

In the literature, there are three examples of hierarchical hybrid models: the DSGE-VAR (Del Negro and Schorfheide, 2004), the DSGE-FAVAR (Consolo, Favero, and Paccagnini, 2009), and the Augmented (B)VAR (Fernández-de-Córdoba and Torres, 2011).

5.2.1 The DSGE-VAR

Based on the work of Ingram and Whiteman (1994), the DSGE-VAR approach proposed by Del Negro and Schorfheide (2004) uses the DSGE model to generate prior distributions for the VAR. The starting point for the estimation is the unrestricted VAR of order p:

$$
\mathbf{Y}_t = \mathbf{\Phi}_0 + \mathbf{\Phi}_1 \mathbf{Y}_{t-1} + ... + \mathbf{\Phi}_p \mathbf{Y}_{t-p} + \mathbf{u}_t. \tag{23}
$$

The companion form is:

$$
Y = X\Phi + U, \tag{24}
$$

Y is a $(T \times n)$ matrix with rows Y_t', X is a $(T \times k)$ matrix ($k = 1 + np, p =$ number of lags) with rows $X_t' = [1, Y_{t-1}', ..., Y_{t-p}']$, U is a $(T \times n)$ matrix with rows u_t' and Φ is a $(k \times n) = [\Phi_0, \Phi_1, ..., \Phi_p]'$.

The one-step-ahead forecast errors u_t have a multivariate normal distribution $N(0, \Sigma_u)$ conditional on past observations of Y.

The log-likelihood function of the data is written as a function of Φ and Σ_u:

$$L(Y|\Phi,\Sigma_u) \propto |\Sigma_u|^{-\frac{T}{2}} \exp\left\{-\frac{1}{2} tr\left[\Sigma_u^{-1}\left(Y'Y - \Phi'X'Y - Y'X\Phi + \Phi'X'X\Phi\right)\right]\right\}. \tag{25}$$

Meanwhile, the prior distribution for the VAR parameters proposed by Del Negro and Schorfheide (2004) is based on the statistical representation of the DSGE model given by the VAR approximation.

Let Γ^*_{xx}, Γ^*_{yy}, Γ^*_{xy} and Γ^*_{yx} be the theoretical second-order moments of the variables Y and X implied by the DSGE model, where:

$$\begin{aligned} \Phi^*(\theta) &= \Gamma^{*-1}_{xx}(\theta)\,\Gamma^*_{xy}(\theta) \qquad (26)\\ \Sigma^*(\theta) &= \Gamma^*_{yy}(\theta) - \Gamma^*_{yx}(\theta)\,\Gamma^{*-1}_{xx}(\theta)\,\Gamma^*_{xy}(\theta). \end{aligned}$$

The moments are the "dummy observation priors" (Theil and Goldberg, 1961, and Ingram and Whiteman, 1994) implemented in the hybrid model. These vectors can be interpreted as the probability limits of the coefficients in a VAR estimated on the artificial observations generated by the DSGE model.

Conditional on the vector of structural parameters in the DSGE model θ, the prior distributions for the VAR parameters $p(\Phi,\Sigma_u|\theta)$ are of the Inverse-Wishart (IW) and Normal forms:

$$\begin{aligned} \Sigma_u\,|\theta &\sim IW\left((\lambda T\Sigma^*_u(\theta), \lambda T - k, n\right) \qquad (27)\\ \mathbf{\Phi}\,|\Sigma_u,\theta &\sim N\left(\mathbf{\Phi}^*(\theta), \Sigma_u \otimes (\lambda T\Gamma_{XX}(\theta))^{-1}\right), \end{aligned}$$

where the parameter λ controls the degree of model misspecification with respect to the VAR: for small values of λ the discrepancy between the VAR and the DSGE-VAR is large and a sizeable distance is generated between the unrestricted VAR and DSGE estimators. On the contrary, large values of λ correspond to small model misspecification and for $\lambda = \infty$ beliefs about DSGE misspecification degenerate to a point mass at zero. Bayesian estimation could be interpreted as estimation based on a sample in which data are augmented by a hypothetical sample in which observations are generated by the DSGE model, the "dummy observation priors". Within this framework, λ determines the length of the hypothetical sample.

The posterior distributions of the VAR parameters have also the Inverse-Wishart and Normal forms. Given the prior distribution, posterior distributions are derived by the Bayes theorem:

$$\Sigma_u | \theta, Y \sim IW\left((\lambda+1) T \hat{\Sigma}_{u,b}(\theta), (\lambda+1) T - k, n \right) \tag{28}$$

$$\mathbf{\Phi} | \Sigma_u, \theta, Y \sim N\left(\hat{\mathbf{\Phi}}_b(\theta), \Sigma_u \otimes [\lambda T \Gamma_{XX}(\theta) + \mathbf{X}'\mathbf{X}]^{-1} \right) \tag{29}$$

$$\begin{aligned} \hat{\mathbf{\Phi}}_b(\theta) &= (\lambda T \Gamma_{XX}(\theta) + \mathbf{X}'\mathbf{X})^{-1} (\lambda T \Gamma_{XY}(\theta) + \mathbf{X}'\mathbf{Y}) \\ \hat{\Sigma}_{u,b}(\theta) &= \frac{1}{(\lambda+1) T} \left[(\lambda T \Gamma_{YY}(\theta) + \mathbf{Y}'\mathbf{Y}) - (\lambda T \Gamma_{XY}(\theta) + \mathbf{X}'\mathbf{Y}) \hat{\mathbf{\Phi}}_b(\theta) \right], \end{aligned}$$

where the matrices $\hat{\mathbf{\Phi}}_b(\theta)$ and $\hat{\Sigma}_{u,b}(\theta)$ have the interpretation of maximum likelihood estimates of the VAR parameters based on the combined sample of actual observations and artificial observations generated by the DSGE. Equations (28) and (29) show that the smaller λ is, the closer the estimates are to the OLS estimates of an unrestricted VAR. Instead, the higher λ is, the closer the VAR estimates will be tilted towards the parameters in the VAR approximation of the DSGE model ($\hat{\mathbf{\Phi}}_b(\theta)$ and $\hat{\Sigma}_{u,b}(\theta)$).

To obtain a non-degenerate prior density (27), which is a necessary condition for the existence of a well-defined Inverse-Wishart distribution and for computing meaningful marginal likelihoods, λ has to be greater than λ_{MIN}, such that: $\lambda_{MIN} \geq \frac{n+k}{T}; k = 1 + p \times n$, where $p =$ lags and $n =$ endogenous variables. Consequently, the optimal lambda must be greater than or equal to the minimum lambda $\left(\widehat{\lambda} \geq \lambda_{MIN}\right)$.

The DSGE-VAR tool allows the researcher to draw posterior inferences about the DSGE model parameters θ. Del Negro and Schorfheide (2004) provide evidence that the posterior estimate of θ has the interpretation of a minimum-distance estimator, where the discrepancy between the OLS estimates of the unrestricted VAR parameters and the VAR representation of the DSGE model is a sort of distance function. The estimated posterior of parameter vector θ depends on the hyperparameter λ. When $\lambda \to 0$, in the posterior the parameters are not informative, so the DSGE model is of no use in explaining the data. Unfortunately, the posteriors (29) and (28) do not have a closed form and we need a numerical method to solve the problem. The posterior simulator used by Del Negro and Schorfheide (2004) is the Markov Chain Monte Carlo Method and the implemented algorithm is the Metropolis-Hastings acceptance method. This procedure generates a Markov Chain from the posterior distribution of θ and this Markov Chain is used for Monte Carlo simulations. See Del Negro and Schorfheide (2004) for more details.

The optimal λ is given by maximizing the log of the marginal data density:

$$\widehat{\lambda} = \underset{\lambda \geqslant \lambda_{MIN}}{\arg\max} \ln p(Y|\lambda).$$

According to the optimal lambda $\left(\widehat{\lambda}\right)$, a corresponding optimal mixture model is chosen. This hybrid model is called DSGE-VAR$\left(\widehat{\lambda}\right)$ and $\widehat{\lambda}$ is the weight of the priors. It can also be interpreted as the restriction of the theoretical model on the actual data.

Unfortunately, Del Negro and Schorfheide (2004) do not propose any statistical tool to verify the power of their procedure. Moreover, Del Negro, Schorfheide, Smets and Wouters (2007b) explain "*...the goal of our article is not to develop a classical test of the hypothesis that the DSGE model restrictions are satisfied; instead, we stress the Bayesian interpretation of the marginal likelihood function of* $p(\lambda|Y)$, *which does not require any cutoff or critical values. ...* ".

Several recent papers apply the DSGE-VAR to detect possible misspecifications in DSGE models and evidence how this econometric tool is a useful forecasting combination model which improves the prediction ability of DSGE model with the powerful time series analysis through VAR (see Adolfson, Laséen, Lindé, and Villani, 2008; Ghent, 2009; Kolasa, Rubaszek, and Skryzpczynski, 2012; Consolo, Favero, and Paccagnini, 2009; Lees, Matheson and Smith, 2011; Bekiros and Paccagnini, 2013, 2014, 2015, and 2016; Gupta and Steinbach, 2013; Bhattacharjee and Gelain, 2017, among others).

5.2.2 The DSGE-FAVAR

In the DSGE-FAVAR (Consolo, Favero, and Paccagnini, 2009), the statistical representation is a Factor Augmented VAR instead of a VAR model. A FAVAR benchmark for the evaluation of the previous DSGE model will take the following specification:

$$\begin{pmatrix} \mathbf{Y}_t \\ \mathbf{F}_t \end{pmatrix} = \begin{bmatrix} \mathbf{\Phi}_{11}(L) & \mathbf{\Phi}_{12}(L) \\ \mathbf{\Phi}_{21}(L) & \mathbf{\Phi}_{22}(L) \end{bmatrix} \begin{pmatrix} \mathbf{Y}_{t-1} \\ \mathbf{F}_{t-1} \end{pmatrix} + \begin{pmatrix} \mathbf{u}_t^Z \\ \mathbf{u}_t^F \end{pmatrix}, \tag{30}$$

where Y_t are the observable variables included in the DSGE model and F_t is a small vector of unobserved factors extracted from a large data-set of macroeconomic time series, which capture additional economic information relevant to modelling the dynamics of Y_t. The system reduces to the standard VAR used to evaluate DSGE models if $\Phi_{12}(L) = 0$.

Importantly, and differently from Boivin and Giannoni (2006), the FAVAR is not interpreted as the reduced form of a DSGE model. In fact, in this case the restrictions implied by the DSGE model on a general FAVAR are very difficult to trace and model evaluation becomes even more difficult to implement. A very tightly parameterized theory model can have a very highly parameterized reduced form if one is prepared to accept that the relevant theoretical concepts in the model are a combination of many macroeconomic and financial variables. The remaining part of the procedure is implemented in the same way as the DSGE-VAR.

5.2.3 The Augmented (B)VAR

The Augmented (B)VAR (Fernández-de-Córdoba and Torres, 2011) is a combination of the unrestricted VAR with the DSGE model and is conducted by increasing the size of the VAR representation. In this methodology, x_t is a vector of observable economic variables assumed to drive the dynamics of the economy. The structural approach assumes that DSGE models contain additional economic information, not fully captured by x_t. The additional information is summarized by using a vector of unobserved variables z_t. Fernández-de-Córdoba and Torres (2011) explain that these non-observed variables can be total factor productivity, marginal productivity, or any other information given by the economic model, but they do not belong to the observed variable set.

The joint dynamics of (x_t, z_t) are given by the following transition equation:

$$\begin{bmatrix} x_t \\ z_t \end{bmatrix} = \Phi(L) \begin{bmatrix} x_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^x \\ \varepsilon_t^z \end{bmatrix}.$$

This system cannot be estimated directly since z_t are non-observed, but z_t can be obtained using the DSGE model to create a new variable Z_t, which is used to expand the size of the VAR. It is possible to construct a VAR with the following specification:

$$\begin{bmatrix} x_t \\ Z_t \end{bmatrix} = \begin{bmatrix} \phi_{11}(L) & \phi_{12}(L) \\ \phi_{21}(L) & \phi_{22}(L) \end{bmatrix} \begin{bmatrix} x_{t-1} \\ Z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^x \\ \varepsilon_t^z \end{bmatrix},$$

where x_t are the macroeconomic data that the DSGE model seeks to explain and Z_t is a vector derived from the DSGE model. If the model specification is correct, the relation between x_t and Z_t should then capture additional economic information relevant to modelling the dynamics of x_t. A standard unrestricted VAR implies that $\phi_{12}(L) = 0$.

	ESTIMATION	MAIN CONTRIBUTION for DSGE	CITATIONS (NOVEMBER 2017)
ADDITIVE			
DSGE-AR Altug (1989), Sargent (1989)	Maximum Likelihood	Introduce a univariate measurement (AR)	Altug (1989): 321; Sargent (1989): 419
DSGE-VAR à l'Ireland Ireland (2004)	Maximum Likelihood	Introduce a multivariate measurement (VAR)	502
DSGE-DFM Boivin and Giannoni (2006)	Bayesian	Latent variables added to state-space	263
DSGE with non-modelled variables Schorfheide, Sill, and Kryshko (2010)	Bayesian	Auxiliary regressions like measurement equations in a DFM linking non-core variables to state-space of DSGE	56
DSGE for trends Canova (2012)	Maximum Likelihood Bayesian	De-trend equation for variables added to state space	40
HIERARCHICAL			
DSGE-VAR Del Negro and Schorfheide (2004)	Bayesian	VAR representation added by artificial data from DSGE	543
DSGE-FAVAR Consolo, Favero, and Paccagnini (2009)	Bayesian	FAVAR representation added by artificial data from DSGE	47
Augmented (B) VAR Fernandéz-de-Cordoba and Torres (2011)	Bayesian	Non-observed variables from DSGE added to state space	12

Table 2: Comparison

6 Concluding Remarks

Dynamic Stochastic General Equilibrium (DSGE) models are the main tool used in Academia and in Central Banks to evaluate the business cycle for policy and forecasting analyses. Despite the recent advances in improving the fit of DSGE models to the data, misspecification issue still remains. This survey shed light on the sources and the remedies to face with misspecified DSGE models. We distinguish four forms of misspecification: a) Misspecification in the State-Space Representation, b) Misspecification in Parameters, c) Misspecification in the Assumptions of the DSGE model, and d) Misspecification in Computational Methods. We discuss several attempts to identify the sources of misspecification, in particular about the State-Space representation, such as Monti (2015) and Inonue, Kuo, and Rossi (2017). Meanwhile, Canova and Matthes (2017) and Den Haan and Dreschel (2017) contribute to the literature proposing two different methodologies which are not only able to detect the sources but they aim to reduce the degree of misspecification.

In addition, Additive Hybrid and Hierarchical Hybrid models are illustrated as remedies to face with misspecified DSGE models.

References

[1] Adolfson, Malin, Stefan Laséen, Jesper Lindé, and Mattias Villani (2008): "Evaluating an Estimated New Keynesian Small Open Economy Model", *Journal of Economic Dynamics and Control* Elsevier, vol. 32(8), pages 2690-2721.

[2] Albonico, Alice, Alessia Paccagnini, and Patrizio Tirelli (2016): "In Search of the Euro-Area Fiscal Stance", *Journal of Empirical Economics*, Elsevier, vol. 39(PB), pages 254-264.

[3] Albonico, Alice, Alessia Paccagnini, and Patrizio Tirelli (2017): "Great Recession, Slow Recovery, and Muted Fiscal Policies in the US", *Journal of Economic Dynamics and Control*, Elsevier, vol. 81(C), pages 140-161.

[4] Alessi, Lucia, Matteo Barigozzi, and Marco Capasso (2011): "Nonfundamentalness in structural econometric models: a review", *International Statistical Review*, 79 (1). 16-47. ISSN 0306-7734.

[5] Altug, Sumro (1989): "Time-to-Build and Aggregate Fluctuations: Some New Evidence", *International Economic Review*, 30(4), pp. 889-920.

[6] An, Sungbae and Frank Schorfheide (2007): "Bayesian Analysis of DSGE Models", *Econometric Reviews*, Taylor and Francis Journals, vol. 26(2-4), pages 113-172.

[7] Angelini, Giovanni and Luca Fanelli (2016): "Misspecification and Expectations Correction in New Keynesian DSGE Models", *Oxford Bulletin of Economics and Statistics*, 78 (5), 623-649.

[8] Aruoba, Boragan, Luigi Bocola, and Frank Schorfheide (2017): "Assessing DSGE model nonlinearities", *Journal of Economic Dynamics and Control,* Volume 83, October, Pages 34-54.

[9] Bhattacharjee, Arnab and Paolo Gelain (2017): " How much does the FOMC care about model misspecification?", Manuscript.

[10] Basu, Susanto and Brent Bundick (2017): "Uncertainty Shocks in a Model of Effective Demand", *Econometrica*, Volume 85, Issue 3, May, Pages 937–958.

[11] Beaudry, Paul, Patrick Fève, Alain Guay, and Franck Portier (2015): "When is nonfundamentalness in VAR a real problem? An application to news shocks", NBER Working Paper No. 21466.

[12] Bekiros, Stelios and Alessia Paccagnini (2013): "On the Predictability of Time-Varying VAR and DSGE Models", *Empirical Economics*, Springer, vol. 45(1), pages 635-664, August.

[13] Bekiros, Stelios and Alessia Paccagnini (2014): "Bayesian Forecasting with a Small and Medium Scale Factor-Augmented Vector Autoregressive DSGE Model", *Computational Statistics & Data Analysis*, Elsevier, vol. 71(C), pages 298-323.

[14] Bekiros, Stelios and Alessia Paccagnini (2015): "Macroprudential Policy and Forecasting using Hybrid DSGE Models with Financial Frictions and state-space Markov-Switching TV-VARs", *Macroeconomic Dynamics*, Cambridge University Press, vol. 19(07), pages 1565-1592, October.

[15] Bekiros, Stelios and Alessia Paccagnini (2016): "Policy-Oriented Macroeconomic Analysis and Forecasting with DSGE and SVAR Models", *Journal of Forecasting*, John Wiley & Sons, Ltd., vol. 35(7), pages 613-632, November.

[16] Bekiros, Stelios, Roberta Cardani, Alessia Paccagnini, and Stefania Villa (2016): "Dealing with Financial Instability under a DSGE Modeling Approach with Banking Intermediation: A Predictability Analysis versus TVP-VARs", *Journal of Financial Stability*, Elsevier, vol. 26(C), pages 216-227.

[17] Beltran, Daniel O. and David Draper (2016): "Estimating Dynamic Macroeconomic Models: How Informative Are the Data?, *International Finance Discussion Papers* 1175, Board of Governors of the Federal Reserve System.

[18] Besag, Julian (1974): "Spatial interaction and the statistical analysis of lattice systems", *Journal of the Royal Statistical Society, Series B (Methodological)* 36, 192-236.

[19] Beyer, Andreas and Roger E.A. Farmer (2004): "On the indeterminacy of new Keynesian economics", *ECB Working Paper* 323.

[20] Bianchi, Francesco (2013): "Regime switches, agents' beliefs, and post-world war II US macroeconomic dynamics", *The Review of Economic Studies*, 80(2):463–490.

[21] Bilbiie, Florian O. (2008): "Limited asset market participation, monetary policy and (inverted) aggregate demand logic", *Journal of Economic Theory* 140, 162–196.

[22] Bocola, Luigi (2016): "The pass-through of sovereign risk", *Journal of Political Economy* 124 (4), 879–926.

[23] Boivin, Jean and Marc P. Giannoni (2006): "DSGE Models in a Data-Rich Environment", *NBER Working Papers* 12772.

[24] Box, George and Norman Draper (1987): "Empirical Model-Building and Response Surface", *Wiley Series in Probability and Statistics*.

[25] Branch, William and George Evans (2006): "Intrinsic heterogeneity in expectation formation", *Journal of Economic Theory*, 2006, vol. 127, issue 1, 264-295.

[26] Braun, Toni, Lena Korber, and Yuichiro Waki (2016): "Some Unpleasant Properties of Log-linearized Solutions when the Nominal Rate is Zero", *Journal of Monetary Economics*, 84, December, p. 216-232.

[27] Brunnermeier, Markus K. and Yuliy Sannikov (2014): "A macroeconomic model with a financial sector", *The American Economic Review* 104 (2), 379–421 .

[28] Caldara, Dario, Jesus Fernández-Villaverde, Juan F. Rubio-Ramírez, and Wen Yao (2012): "Computing DSGE models with recursive preferences and stochastic volatility", *Review of Economic Dynamics*, 15(2):188–206.

[29] Canova, Fabio (2007): "Methods for Applied Macroeconomic Research ", Princeton University Press, Princeton.

[30] Canova, Fabio and Luca Sala (2009): "Back to Square One: Identification Issues in DSGE Models", *Journal of Monetary Economics*, 56, 431-449.

[31] Canova, Fabio (2014): "Bridging Cyclical DSGE Models and the Raw Data," *Journal of Monetary Economics*, Volume 67, October, Pages 1-15.

[32] Canova, Fabio, Filippo Ferroni, and Christian Matthes (2014): "Choosing The Variables To Estimate Singular DSGE Models," *Journal of Applied Econometrics*, John Wiley & Sons, Ltd., vol. 29(7), pages 1099-1117, November.

[33] Canova, Fabio (2017): "Dealing with misspecification in macroeconometrics", Invited Lecture in Workshop on Time-Varying Uncertainty in Macro (https://sites.google.com/site/uncertaintyinmacroworkshop/home).

[34] Canova, Fabio and Christian Matthes (2017): "A composite likelihood approach for dynamic structural models", Manuscript.

[35] Castelnuovo, Efrem (2012): "Fitting U.S. Trend Inflation: A Rolling-Window Approach", In Balke, N., Canova, F., Milani, F., and Wynne, M., editors, *Advances in Econometrics: DSGE Models in Macroeconomics - Estimation, Evaluation, and New Developments*, pages 201–252.

[36] Chari, V.V., Patrick Kehoe, and Ellen R. McGrattan (2008): "Are Structural VARs with Long-Run Restrictions Useful in Developing Business Cycle Theory?", *Journal of Monetary Economics*, Elsevier, vol. 55(8), pages 1337-1352, November.

[37] Chen, Bin, Jinho Choi, and Juan Carlos Escanciano (2017): "Testing for fundamental vector moving average representation", *Quantitative Economics*, Vol. 8, pages 149-180.

[38] Chib, Siddhartha (2001): "Markov chain Monte Carlo methods: Computation and inference", *Handbook of Econometrics* 5, 3569 3649.

[39] Chib, Siddhartha and Edward Greenberg (1994): "Bayes inference in regression models with ARMA(p,q) errors", *Journal of Econometrics* 64, 183 206.

[40] Chib, Siddhartha and Edward Greenberg (1995): "Understanding the Metropolis Hastings algorithm", *American Statistician* 49, 327 335.

[41] Chib, Siddhartha and Srikanth Ramamurthy (2010): "Tailored randomized block MCMC methods with application to DSGE models", *Journal of Econometrics*, Elsevier, vol. 155(1), pages 19-38, March.

[42] Christiano, Lawrence J., Martin Eichenbaum and Charles Evans (2005): "Nominal Rigidities and the Dynamic effects of a Shock to Monetary Policy", *Journal of Political Economy* 113, 1-45.

[43] Christiano, Lawrence J., Martin Eichenbaum, and Mathias Trabandt (2017): "On DSGE Models", Manuscript.

[44] Cogley, Timothy F and James M. Nason (1994): "Testing the Implications of Long-Run Neutrality for Monetary Business Cycle Models", *Journal of Applied Econometrics*, vol. 9(S), pages S37-70, Suppl. De.

[45] Colander, David, Peter Howitt, Alan Kirman, Axel Leijonhufvud, and Perry Mehrling (2008): "Beyond DSGE Models: Toward an Empirically Based Macroeconomics", *The American Economic Review* 98, no. 2: 236-40.

[46] Consolo, Agostino, Carlo A. Favero, and Alessia Paccagnini (2009): "On the Statistical Identification of DSGE Models", *Journal of Econometrics,* 150, 99-115.

[47] Corradi, Valentina and Norman Swanson (2007): "Evaluation of dynamic stochastic general equilibrium models based on distributional comparison of simulated and historical data", *Journal of Econometrics* 136, 699-723.

[48] DeJong, David N. and Chetan Dave (2011): " Structural Macroeconometrics", Second Edition, Princeton University Press, Princeton.

[49] Del Negro, Marco and Frank Schorfheide (2004): " Priors from General equilibrium Models for VARs", *International Economic Review*, 45, 643-673.

[50] Del Negro, Marco, Frank Schorfheide, Frank Smets and Raf Wouters (2007): "On the Fit of New-Keynesian Models", *Journal of Business, Economics and Statistics, 25,2, 124-162.*

[51] Del Negro, Marco, Frank Schorfheide, Frank Smets and Raf Wouters (2007b): "On the Fit of New-Keynesian Models: rejoinder", *Journal of Business, Economics and Statistics, 25,2, 159-162.*

[52] Del Negro, Marco and Frank Schorfheide (2009): "Monetary policy analysis with potentially misspecified models", *The American Economic Review* 99, 1415-50.

[53] Del Negro, Marco and Frank Schorfheide (2011): "Bayesian macroeconometrics". In: Geweke, J., Koop, G., van Dijk, H. (Eds.), The Oxford *Handbook of Bayesian Econometrics*. Oxford University Press, pp. 293–389.

[54] Del Negro, Marco and Frank Schorfheide (2013): "DSGE Model-Based Forecasting, Del Negro, Marco, and Schorfheide Frank. In: G. Elliott and A. Timmermann (eds.), *Handbook of Economic Forecasting,* Volume 2, Part A, p.57 – 140, Elsevier, New York.

[55] Den Haan, Wouter J. (2010): "Comparison of solutions to the incomplete markets model with aggregate uncertainty," *Journal of Economic Dynamics and Control*, Elsevier, vol. 34(1), pages 4-27, January.

[56] Den Haan, Wouter J. and Thomas Drechsel (2017). "Agnostic structural disturbances in DGSE models: detecting and reducing misspecification", Manuscript.

[57] Diebold, Francis, Frank Schorfheide, and Michul Shin (2017): "Real-time forecast evaluation of DSGE models with stochastic volatility", *Journal of Econometrics*, Volume 201, Issue 2, December, Pages 322-332.

[58] Dufour, Jean-Marie, Lynda Khalaf, and Maral Kichian (2013): "Identification-robust analysis of DSGE and structural macroeconomic models", *Journal of Monetary Economics* 60, 340-350.

[59] Eo, Yunjong and Chang-Jin Kim (2016): "Markov-Switching Models with Evolving Regime-Specific Parameters: Are Postwar Booms or Recessions All Alike?", *The Review of Economics and Statistics*, Volume 98, Issue 5, December, pages 940-949.

[60] Erceg, Christopher J, Luca Guerrieri, and Christopher Gust (2005): "Can Long-Run restrictions Identify Technology Shocks?", *Journal of the European Economic Association*, MIT Press, vol. 3(6), pages 1237-1278, December.

[61] Evans, Charles L. (1992): "Productivity shocks and business cycles", *Journal of Monetary Economics*, 29(2), 191-208.

[62] Evans, George W. and Seppo Honkapohja (1999): "Learning dynamics", Handbook of Macroeconomics,in: J. B. Taylor and M. Woodford (ed.), *Handbook of Macroeconomics*, edition 1, volume 1, chapter 7, pages 449-542, Amsterdam: Elsevier.

[63] Evans, George W. and Seppo Honkapohja (2001): "Learning and Expectations in Economics", Princeton University Press, Princeton, NJ.

[64] Fernández-de-Córdoba, Gonzalo, and José L. Torres (2010): "Forecasting the Spanish Economy with a DSGE Model: An Augmented VAR Approach", *SERIEs-Journal of the Spanish Economic Association,* Volume 2, Pages 379-399.

[65] Fernández-Villaverde, Jesus and Juan F. Rubio-Ramirez (2004): "Comparing dynamic equilibrium models to data: a Bayesian approach", *Journal of Econometrics*, Elsevier, vol. 123(1), pages 153-187, November.

[66] Fernández-Villaverde, Jesus and Juan F. Rubio-Ramirez (2005): "Estimating Dynamic Equilibrium Economies: Linear Versus Nonlinear Likelihood", *Journal of Applied Econometrics*, 20(7), 891-910.

[67] Fernández-Villaverde, Jesus, Juan F. Rubio-Ramirez, and Manuel S. Santos (2006): "Convergence Properties of the Likelihood of Computed Dynamic Models", *Econometrica*, Econometric Society, vol. 74(1), pages 93-119, 01.

[68] Fernández-Villaverde, Jesus, Juan F. Rubio-Ramirez, Thomas J. Sargent, and Mark W. Watson (2007): "ABCs (and Ds) of understanding VARs, *The American Economic Review* 97, pp. 1021-1026.

[69] Fernández-Villaverde, Jesus, Pablo Guerrón-Quintana, and Juan F. Rubio-Ramírez (2010): "Fortune or virtue: Time-variant volatilities versus parameter drifting in U.S. data", *NBER Working Papers*, 15928.

[70] Fernández-Villaverde, Jesus, Grey Gordon, Pablo A. Guerrón-Quintana, and Juan F. Rubio-Ramírez (2015): "Nonlinear adventures at the zero lower bound", *Journal of Economic Dynamics and Control,* 57, 182–204 .

[71] Fernández-Villaverde, Jesus, Pablo A. Guerrón-Quintana, and Juan F. Rubio-Ramírez (2015): "Estimating dynamic equilibrium models with stochastic volatility", *Journal of Econometrics* 185, 216–229.

[72] Fernández-Villaverde, Jesus, Juan F. Rubio-Ramirez, and Frank Schorfheide (2016): "Solution and estimation methods for DSGE models". In John Taylor and Harald Uhlig, editors, *Handbook of Macroeconomics*, volume 2A, pages 527-724. Elsevier, Amsterdam.

[73] Ferroni, Filippo, Stefano Grassi, and Miguel A. Léon-Ledesma (2017): "Selecting Primal Innovations in DSGE models", Federal Reserve Bank of Chicago, WP 2017-20.

[74] Forni, Mario, Luca Gambetti, Marco Lippi, and Luca Sala (2017a): "Noise Bubbles", *The Economic Journal*, Vol. 127, issue 604, pages 1940-1976.

[75] Forni, Mario, Luca Gambetti, Marco Lippi, and Luca Sala (2017b): "Noisy News in Business Cycles", *American Economic Journal: Macroeconomics*, Vol. 9, October, pages 122-152.

[76] Forni, Mario, Luca Gambetti, and Luca Sala (2017): "Reassessing Structural VARs: Beyond the ABCs (and Ds), Manuscript.

[77] Foerster, Andrew T., Juan F. Rubio-Ramirez, Daniel F. Waggoner, and Thao A. Zha (2016): "Perturbation methods for Markov-switching DSGE models", *Quantitative Economics*, 7:637–669.

[78] Franchi, Massimo and Anna Vidotto (2013): "A check for finite order VAR representations of DSGE models", *Economics Letters*, pages 100-103, Volume 120, Issue 1.

[79] Galí, Jordi, David López-Salido, and Javier Vallés (2004): "Rule-of-thumb consumers and the design of interest rate rules", *Journal of Money, Credit and Banking* 36, 739–764.

[80] Galí, Jordi, Frank Smets and Raf Wouters (2012): "Unemployment in an Estimated New Keynesian Model", *NBER Macroeconomics Annual*, University of Chicago, Press, vol. 26(1), pages 329 - 360.

[81] Galvao, Ana B., Liudas Giraitis, George Kapetanios, and Katerina Petrova (2016): "A time varying DSGE model with financial frictions", *Journal of Empirical Finance*, 38:690–716.

[82] Gavin, William T., Benjamin D. Keen, Alexander Richter, and Nathaniel Throckmorton (2015): "The Zero Lower Bound, the Dual Mandate, and Unconventional Dynamics", *Journal of Economic Dynamics and Control*, 55, 14-38.

[83] Gertler, Mark , Nobuhiro Kiyotaki, and Albert Queralto (2012): "Financial crisis, bank risk exposure and government financial policy", *Journal Monetary Economics* 59, S12–34.

[84] Geweke, John (1977): "The Dynamic Factor Analysis of Economic Time Series", in: D.J. Aigner and A.S. Goldberg (eds.), *Latent Variables in Socio-Economic Models*, North-Holland, Amsterdam.

[85] Ghent, Andra (2009): "Comparing DSGE-VAR Forecasting Models: How Big are the Differences?", *Journal of Economic Dynamics and Control*, 2009, 33:4, pp. 864-882.

[86] Giacomini, Raffaella (2013): "The relationship between VAR and DSGE models", in *Advances in Econometrics*, vol. 32, VAR models in macroeconomics: new developments and applications: Essays in honor of Christopher A. Sims.

[87] Giannone, Domenico and Lucrezia Reichlin (2006): "Does information help recovering structural shocks from past observations?", *Journal of the European Economic Association*, 4(2-3), 455-465.

[88] Green, Peter J. (1995): "Reversible Jump Markov Chain Monte Carlo", *Biometrika*, 82, 711–732.

[89] Guerron-Quintana, Pablo, Atsushi Inoue, and Lutz Kilian (2013): "Frequentist inference in weakly identified dynamic stochastic general equilibrium models", *Quantitative Economics* 4 (2), 197-229.

[90] Gupta, Rangan and Rudi Steinbach (2013): "A DSGE-VAR model for forecasting key South African macroeconomic variables", *Economic Modelling*, Elsevier, vol. 33(C), pages 19-33.

[91] Gust, Christopher, Edward Herbst, David López-Salido, and Matthew E. Smith (2017): "The Empirical Implications of the Interest-Rate Lower Bound", *The American Economic Review*, 107(7): 1971-2006.

[92] Hansen, Lars P. and Thomas J. Sargent (1980): "Formulating and estimating dynamic linear rational expectations models", *Journal of Economic Dynamics and Control*, Vol. 2, pp. 7–46.

[93] Hansen, Lars P (2014): "Nobel Lecture: Uncertainty outside and inside economic models", *Journal of Political Economy*, Vol. 122, pp. 945–987.

[94] Hansen, Lars P. and Thomas J. Sargent (1981): "Linear rational expectations models for dynamically interrelated variables", in Lucas Jr, R. E. and Sargent T. J. (Eds.), *Rational Expectations and Econometric Practise*, University of Minnesota Press, Minnesota, MN, pp. 127–156.

[95] He, Zhiguo and Arvind Krishnamurthy (2015): "A Macroeconomic Framework for Quantifying Systemic Risk", Booth School of Business. Manuscript.

[96] Herbst, Edward and Frank Schorfheide (2015): "Bayesian Estimation of DSGE Models", Princeton University Press, Princeton.

[97] Hirose, Yasuo and Takeki Sunakawa (2016): "Parameter Bias in an Estimated DSGE Model: Does Nonlinearity Matter ?", *JSPS Grants-in-Aid for Scientific Research (S), Working Paper Series*, No. 081.

[98] Hurtado, Samuel (2014): "DSGE Models and the Lucas Critique", *Economic Modelling*, 44(S1):S12 –S19.

[99] Ingram, Beth, and Charles Whiteman (1994): "Supplanting the Minnesota Prior - Forecasting Macroeconomics Time Series using Real Business Cycle Model Priors", *Journal of Monetary Economics,* 34, pp 497-510.

[100] Inoue, Atsushi and Barbara Rossi (2011). Identifying the sources of instabilities in macroeconomic fluctuations", *Review of Economics and Statistics*, 93(4):1186–1204.

[101] Inonue, Atsushi, Chang-Hung Kuo, and Barbara Rossi (2017): "Identifying the Sources of Model Misspecification", Manuscript.

[102] Ireland, Peter (2004): "A Method for Taking Models to the Data", *Journal of Economic Dynamics and Control*, 28, pp 1205-1226.

[103] Iskrev, Nikolay (2010): "Local identification in DSGE models", *Journal of Monetary Economics,* 57 (2), 189-202.

[104] Justiniano, Alejandro and Giorgio Primiceri (2008): "The time-varying volatility of macroeconomic fluctuations", *The American Economic Review,* 98 (3), 604–641.

[105] Justiniano, Alejandro, Giorgio Primiceri, and Andrea Tambalotti (2010) "Investment Shocks and Business Cycles", *Journal of Monetary Economics*, 57(2),132-145.

[106] Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante (2017): "Monetary policy according to HANK", Manuscript.

[107] Karagedikli, Ozer, Troy Matheson, Christie Smith, and Shaun Vahey (2010): "RBCs AND DSGEs: The Computational Approach to Business Cycle Theory and Evidence", *Journal of Economic Surveys*, 24, issue 1, p. 113-136.

[108] Kascha, Christian and Karel Mertens (2009): " Business Cycle Analysis and VARMA Models", *Journal of Economic Dynamics and Control,* Volume 33, Issue 2, February, Pages 267-282.

[109] Kociecki, Andrzej and Marcin Kolasa (2013): "Global Identification of Linearized DSGE Models", *Working Paper 170*, National Bank of Poland.

[110] Kolasa, Marcin, Michal Rubaszek, and Pawel Skrzypczynski (2012): "Putting the New Keynesian DSGE Model to the Real-Time Forecasting Test", *Journal of Money, Credit and Banking*, Volume 44, Issue 7 October, Pages 1301–1324.

[111] Kolasa, Marcin and Michal Rubaszek (2015): "How Frequently Should We Reestimate DSGE Models?," *International Journal of Central Banking*, International Journal of Central Banking, vol. 11(4), pages 279-305, December.

[112] Komunjer, Ivana and Serena Ng (2011): "Dynamic identification of dynamic stochastic general equilibrium models", *Econometrica* 79 (6), 1995-2032.

[113] Kydland, Finn E. and Edward C. Prescott (1982): "Time to Build and Aggregate Fluctuations", *Econometrica*, 50, 1345-70.

[114] Lees, Kirdan, Troy Matheson and Christie Smith (2011): "Open Economy DSGE-VAR Forecasting and Policy Analysis: Head to Head with the RBNZ Published Forecasts", *International Journal of Forecasting*, Elsevier, vol. 27(2), pages 512-528, April.

[115] Leeper, Eric M., Todd B. Walker, and Shu-Chun Susan Yang (2013): "Fiscal foresight and information flows", *Econometrica* 81, pp. 1115-1145.

[116] Lindsay, Bruce G. (1988): "Composite Likelihood Methods", *Contemporary Mathematics*, Volume 80.

[117] Lippi, Marco and Lucrezia Reichlin (1993): "The dynamic effects of aggregate demand and supply disturbances: comment", *The American Economic Review,* 83, pp.644-652.

[118] Liu, Philip and Konstantinos Theodoridis (2012): "DSGE Model Restrictions for Structural VAR Identification", *International Journal of Central Banking*, vol. 8(4), pages 61-95.

[119] Lubik, Thomas A. and Frank Schorfheide (2004): "Testing for Indeterminacy: An Application to U.S. Monetary Policy", *The American Economic Review*, American Economic Association, vol. 94(1), pages 190-217, March.

[120] Lucas, Robert (1976): "Econometric policy evaluation: A critique", *Carnegie-Rochester Conference Series on Public Policy*, 1(1):19-46.

[121] Maliar, Lilia and Serguei Maliar (2015): "Merging simulation and projection approaches to solve high-dimensional problems with an application to a new Keynesian model", *Quantitative Economics*, Volume 6, Issue 1, March, Pages 1–47.

[122] Malley, Jim and Ulrich Woitek (2010) "Technology shocks and aggregate fluctuations in an estimated hybrid RBC model", *Journal of Economic Dynamics and Control*, Elsevier, vol. 34(7), pages 1214-1232, July.

[123] Massaro, Domenico (2013): "Heterogeneous Expectations in Monetary DSGE Models", *Journal of Economic Dynamics and Control*, 37, pp. 680 - 692.

[124] McKay, Alisdair and Ricardo Reis (2016): "The Role of Automatic Stabilizers in the U.S. Business Cycle", *Econometrica*, 84 (1), 141–194.

[125] Meyer-Gohde, Alexander and Daniel Neuhoff (2015): "Generalized Exogenous Processes in DSGE: A Bayesian Approach," *SFB 649 Discussion Papers* SFB649DP2015-014, Sonderforschungsbereich 649, Humboldt University, Berlin, Germany.

[126] Milani, Fabio (2007): "Expectations, Learning and Macroeconomic Persistence", *Journal of Monetary Economics*, Volume 54, Issue 7, October 2007, Pages 2065-2082.

[127] Milani, Fabio (2012): "The Modeling of Expectations in Empirical DSGE Models: a Survey", *Advances in Econometrics*, Volume 28, DSGE Models in Macroeconomics: Estimation, Evaluation, and New Developments.

[128] Monti, Francesca (2015): "Can a data-rich environment help identify the sources of model misspecification?", *Discussion Papers 1505*, Centre for Macroeconomics (CFM).

[129] Morris, Stephen D. (2016): "VARMA representation of DSGE models", *Economics Letters*, pages 30-33, Vol. 138.

[130] Morris, Stephen D. (2017): "DSGE pileups", *Journal of Economic Dynamics and Control*, pages 56-86 Vol. 74.

[131] Nakata, Taisuke (2016): "Optimal Fiscal and Monetary Policy with Occasionally Binding Zero Bound Constraints", *Journal of Economic Dynamics and Control*, vol. 73, pp. 220–240.

[132] Nakata, Taisuke (2017): "Uncertainty at the Zero Lower Bound", *American Economic Journal: Macroeconomics*, vol. 9, no. 3, pp. 186-221.

[133] Naghi, Andrea A. (2017): "Global Identification Failure in DSGE Models and its Impact on Forecasting", manuscript.

[134] Ngo, Phuong V. (2014): "Optimal Discretionary Monetary Policy in a Micro-Founded Model with a Zero Lower Bound on Nominal Interest Rate", *Journal of Economic Dynamics and Control*, 45,44-65.

[135] Paccagnini, Alessia (2011): " DSGE Model Evaluation and Hybrid Models: A Comparison", *EUI Working Papers*, Max Weber Programme MWP011.

[136] Pagan, Adrian and Tim Robinson (2016): "Investigating the Relationship Between DSGE and SVAR Models", NCER Working Paper N. 112, May.

[137] Poskitt, D. S. and Wenying Yao (2017): "Vector Autoregressions and Macroeconomic Modeling: An Error Taxonomy", *Journal of Business & Economic Statistics*, 35:3, 407-419.

[138] Poudyal and Spanos (2016): "Model Validation and DSGE Modeling", Manuscript.

[139] Qu, Zhongjun and Denis Tkachenko (2012): "Identification and frequency domain quasi-maximum likelihood estimation of linearized dynamic stochastic general equilibrium models", *Quantitative Economics,* Econometric Society, vol. 3(1), pages 95-132, 03.

[140] Qu, Zhongjun and Denis Tkachenko (2016): "Global Identification in DSGE Models Allowing for Indeterminacy", *The Review of Economic Studies*, Volume 84, Issue 3, 1 July 2017, Pages 1306–1345.

[141] Ramey, Valerie A. (2016): "Macroeconomic Shocks andTheirPropagation", *Handbook of Macroeconomics*, 2, 71–162.

[142] Ravenna, Federico (2007): "Vector Autoregressions and Reduced Form Representations of DSGE models", *Journal of Monetary Economics*, 54, 7, 2048-2064.

[143] Rotemberg, Julio and Michael Woodford (1997): "An optimization-based econometric framework for the evaluation of monetary policy". In *NBER Macroeconomics Annual 1997*, Volume 12, NBER Chapters, pp. 297-361.National Bureau of Economic Research, Inc.

[144] Sargent, Thomas J. and Christopher A. Sims (1977): "Business Cycle Modeling without Pretending

to Have Too Much A-Priori Economic Theory", in C. Sims et al. (eds), *New Method in Business Cycle Research*, Federal Reserve Bank of Minneapolis, Minneapolis.

[145] Sargent, Thomas J. (1989): "Two Models of Measurements and the Investment Accelerator", *Journal of Political Economy*, University of Chicago Press, vol. 97(2), pages 251-87, April.

[146] Schorfheide, Frank (2000): "Loss function-based evaluation of DSGE models", *Journal of Applied Econometrics*, John Wiley & Sons, Ltd., vol. 15(6), pages 645-670.

[147] Schorfheide, Frank, Keith Sill and Maxym Kryshko (2010): "DSGE Model-Based Forecasting of Non-Modelled Variables", *International Journal of Forecasting*, Elsevier, vol. 26(2), pages 348-373.

[148] Schorfheide, Frank (2013): " Estimation and Evaluation of DSGE Models: Progress and Challenges", In: D. Acemoglu, M. Arellano, and E. Deckel (eds.), *Advances in Economics and Econometrics*, Volume 3, Number 51, p.184–230, Cambridge University Press, Cambridge.

[149] Sims, Christopher A. (2002): "Solving Linear Rational Expectations Models", *Computational Economics*, 20 (1-2), 1-20.

[150] Sims, Eric R. (2012): "News, non-invertibility, and structural VARs", *Advances in Econometrics* 28, pp. 81-136.

[151] Slobodyan, Sergey and Raf Wouters (2012a): "Learning in an estimated medium-scale DSGE model", *Journal of Economic Dynamics and Control*, Elsevier, vol. 36(1), pages 26-46.

[152] Slobodyan, Sergey and Raf Wouters (2012b): "Learning in a Medium-Scale DSGE Model with Expectations Based on Small Forecasting Models", *American Economic Journal: Macroeconomics*, American Economic Association, vol. 4(2), pages 65-101, April.

[153] Smets, Frank and Raf Wouters (2003): "An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area", *Journal of the European Economic Association*, 1, 1123-75.

[154] Smets, Frank and Raf Wouters (2004): "Forecasting with a Bayesian DSGE Model: An Application to the Euro Area", *Journal of Common Market Studies*, Wiley Blackwell, vol. 42(4), pages 841-867, November.

[155] Smets, Frank and Raf Wouters (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach", *American Economic Review*, American Economic Association, vol. 97(3), pages 586-606, June.

[156] Smith, Anthony A Jr. (1993): "Estimating nonlinear time-series models using simulated vector autoregressions", *Journal of Applied Econometrics*, 8, S63–S84.

[157] Soccorsi, Stefano (2016): "Measuring nonfundamentalness for structural VARs", *Journal of Economic Dynamics and Control*, Vol. 71, October, Pages 86-101.

[158] Spanos, Aris (1990): "The Simultaneous-Equations Model Revisited: Statistical Adequacy and Identification", *Journal of Econometrics, 44, 87-105.*

[159] Theil, Henry and Arthur S. Goldberg (1961): "On Pure and Mixed Estimation in Economics", *International Economic Review*, 2, pp 65-78.

[160] Waggoner, Daniel and Thao Zha (2012): "Confronting model misspecification in macroeconomics", *Journal of Econometrics*, 171(2):167–184.