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13 October 2017

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This version: 13 October 2017

Abstract

In this paper, we argue that systemic risk should be understood from two different perspectives, the homogeneity of portfolios (or called asset homogeneity) and the contagion mechanism. The homogenization of portfolios held by different financial institutions increases the positive correlations among them and therefore the probability of simultaneous collapses of a considerable part of the network, which are prerequisites and amplifiers of contagion**.** We first theoretically analyze the influence of asset homogeneity on the initial risk, fragility and systemic risk of the network. Based on the theoretical predictions, we perform simulations on regular networks and Poisson random networks to illustrate the effects of portfolio homogeneity on systemic risk. It is shown that the relationship between asset homogeneity and systemic risk is not always positively related. When the network contagion is weak, then a high asset homogeneity will lead to a high systemic risk. However, if the network contagion is considerably strong, the systemic risk is quite likely to be negative related to the asset homogeneity, so that a high homogeneity will produce a low systemic risk. Moreover, networks with strong contagion and low asset homogeneity tend to have the greatest systemic risk. Results from logistic regression analysis further clarify the relationships between systemic risk and asset homogeneity.

JEL classification: G38; D85; G01

***Keywords*:** Financial network; Portfolio homogenization; Contagion; Systemic risk

1. Introduction

Along with the continuous development of financial markets, the relationships between various institutions become more and more complex. Many financial institutions are connected with each other by holding assets, ownership, and credit-debt relationships, which constitute a large financial network. The financial institutions referred to here can be governments, central banks, investment banks, and companies, etc. The mutual interdependencies among these institutions promote the occurrence of economic crisis, such as the Great Depression in 1930, the Asian financial crisis in 1997 and the international financial crisis triggered by the subprime crisis of the United States in 2008. A major contributor to the financial crisis is systemic risk, widely defined as the possibility that an event at the institution level triggers severe instability or collapse of an entire industry or the economy. Moreover, the financial network structure plays an important role in generating systemic risk (Acemoglu et al., 2015). It is imperative to improve our understandings of the relationship between the network structure and systemic risk, which would help regulating and designing the network of financial institutions (Schweitzer et al., 2009).

This paper investigates the effects of the homogenization of portfolios held by institutions on systemic risk in a financial network. The applications of network theory to systemic risk analysis is not new (see Diamond and Dybvig, 1983, Allen and Gale, 2000, Freixas et al., 2000, Gai and Kapadia, 2010, Acemoglu et al. 2012, Elliott et al., 2014). Most works focus on the mechanism of contagions with cascades of failure based on the connectivity arising from the inter-institutional lending, share cross-holdings or overlapping portfolios. The inter-institutional lending drives the problems of counterparty and roll-over risk, which have been extensively studied (Gai and Kapadia, 2010, Staum, 2013). The basic framework for the share cross-holdings was developed by (Brioschi et al., 1989) and (Fedenia et al., 1994). In the seminal paper, (Allen and Gale, 2000) firstly present the contagion channel of interbank cross holdings of deposits and show how the network structure matters. Gai and Kapadia (2010) develop a standard model of epidemics in which the network is characterized by its degree distribution and observe that cascades can be non-monotonic in connectivity (Gai and Kapadia, 2010). Elliott et al. (2014) develop a general cross-holdings model to show that integration (increasing dependence of each organization on its counterparties) and diversification (each organization interacting with a larger number of counterparties) have different, non-monotonic effects on the extent of cascades (Elliott et al., 2014).

While all these works emphasize contagions as the basic mechanism of financial crisis and the key root of systemic risk, they do not distinguish two different channels that amplify systemic risk due to connectivity: the financial contagions, which stress sequential collapses of financial institutions induced by the preceding collapses, and the portfolio homogenization, which increases the correlations between the returns of financial institutions and thus the likelihood of their simultaneous failures. Both financial contagions and portfolio homogenization could be the byproducts of financial networks, but they play different roles for systemic risk. The former highlights that the healthy nodes in the network could fail under the influence of poor performances of others, whereas the latter is directed to the property that different nodes are exposed to similar external risk factors. To avoid financial contagion, one needs to prevent the first failure from occurring or to cut off the contagion channels. Both measures are futile in the presence of portfolio homogeneity since all the nodes have already been assaulted by the same external risks. By the metaphor of epidemics, "portfolio homogenization" corresponds to the situation where too much people are exposed to the same external source of infection. Under this situation, we should try to control the infection source to prevent more contagions. Moreover, portfolio homogenization facilitates the course of failures contagions because it increases the probability of a considerable number of companies going bankrupt, which then increases the possibility to infect more.

The main conclusion of this paper is that, the individual behavior of risk aversion and asset diversification induce portfolio homogenization which, in turn, increases systemic risk when the network contagion is not strong. The well-known two-fund separation theorem shows that different financial institutions tend to hold similar portfolios of risky assets, and henceforth portfolio homogeneity (also known as asset homogeneity) prevails. Under very general assumptions, a simple model is proposed to show that the homogenization of portfolios held by different institutions will increase the positive correlations among them and therefore increase the probability of simultaneous collapses of some considerable part of the network, and result in a high systemic risk, defined as the probability of cascades of failures of all nodes in the network. We show that when all companies have the same investment proportions for different original assets, the companies' portfolios are completely homogeneous, and the probability that all individuals collapse at the same time is the largest. This failure is evaluated at the initial moment and named the initial risk, which is unrelated to contagion consideration. In practice, the connection between financial network nodes is complex and it is unlikely that all the companies homogenizing their portfolios completely. Despite the possibility of all the institutions failing at the same time, directly from portfolio homogenization, is quite small, a fair number of initial failures may lead to further widespread failures and eventually produce the failures of the whole network, provided that a group of companies are more likely to fail under similar conditions due to portfolio homogenization.

Another main conclusion of this paper is that if the network contagion is strong, asset homogeneity is likely to reduce systemic risk. An increase in the correlation of the portfolios among different companies greatly reduces the probability that at least one company becomes bankrupt. If a network is highly contagious, a single company's failure may be able to produce the risk of bankruptcy to all companies through the contagion mechanism. Under such circumstances, low homogeneity of assets can lead to high systemic risk. In general, when the network is less contagious, asset homogeneity can be seen as a prerequisite and an extension of infection. If contagion is strong, then we have the opposite conclusion. Based on the theoretical analysis, we compare the effects of asset homogeneity on systemic risk under different contagion schemes with numerical simulations on networks with different structures. The simulation results are consistent with the theoretical predictions. In addition, the results indicate that, for any given value of asset homogeneity, systemic risk is positively related to contagions of network; which is congruent with the previous literature.

Finally, logistic regression is applied to the simulation data from large-scale randomly generated networks to further examine the relationship between systemic risk and asset homogeneity. The results show that the relationship between systemic risk and asset homogeneity is dependent upon the network contagion. When the contagion is week, systemic risk exhibits a positive correlation with asset homogeneity. As contagion increases, systemic risk is reduced and may become negatively associated with asset homogeneity. Furthermore, a significant inverted-U relationship between systemic risk and asset is found. The regression analysis therefore provides a comprehensive explanation to the results obtained from the simulated observations.

There are several research works related to the current paper. The issues of overlapping portfolios have been considered in (Nier et al., 2007, May and Arinaminpathy, 2010, Beale et al., 2011, Caccioli et al., 2014). For example, Caccioli et al. (2014) develop a measure to compute the stability of financial networks under contagion due to overlapping portfolios and examine the circumstances under which systemic instability is likely to occur. All these papers focus on financial contagion due to overlapping portfolios such that, a bank has to liquidate all his assets when bankrupt, the asset values are reduced by a simple market impact function. Instead, we focus on portfolio homogenization and the systemic risk originates from the collapse of the financial institutions itself and how does portfolio homogenization in conjunction with contagion mechanism affect the systemic risk of the network . Our methodology and results are different from any that we are aware of and our work is a complementary to existing works.

The structure of the paper is as follows: Section 2 describes the model and analyses the initial risk and fragility of the network. Theoretical analysis of a simple network is reported in Section 3 to highlight the dependence of systemic risk on asset homogeneity. In Section 4, the dependence of systemic risk on asset homogeneity is investigated with simulated observations from different network structures. Section 5 performs logistic regressions on the simulated data to analyze the relationship between systemic risk and asset homogeneity. Finally, the main results are summarized in Section 7.

2. Financial Network Model

2.1. The basic framework

We consider a financial network containing n companies and m primitive risky assets. The network is bipartite, as shown in Figure 1. A link between a company and an asset in the network represents the company makes an investment in the asset. The price of asset j is denoted by $P_j (j=1,\cdots,m)$. For convenience, the total amount of each asset is standardized to be 1. Thus, P_j is also the total value of asset j in the market. Let $A_{ij} (0 \le A_{ij} \le 1)$ be the share of the value of asset j held by company i and let A denote the matrix whose (i, j) -th entry is equal to A_{ij}. The total value of company i is

$$Y_i = \sum_{j=1}^{m} A_{ij} P_j ,$$

Pooling together all the companies, we have

$$Y = AP, \tag{1}$$

where $Y = (Y_1, Y_2, \cdots, Y_n)^T$ and $P = (P_1, P_2, \cdots, P_m)^T$. The superscript T denotes transpose operation. We name matrix A the investment matrix. This portfolio model can be extended to the case of inter-institutional lending and share cross-holdings. For the case of share cross-holding, $m=n$ and Y is exactly the market value. Note that, at the market equilibrium, the demand and supply of assets must equal to each other to clear the market. Therefore, $\sum_{i=1}^{m} A_{ij} = 1$.

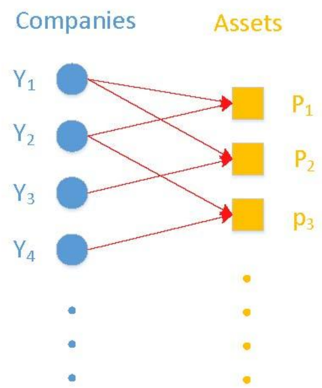

Figure 1: Graphical representation of a bipartite network of companies and assets. Companies are denoted by blue circles, assets by yellow squares. A link connecting a company to an asset is established when the asset is included in the company's portfolio.

In the current paper, the contagion is induced by fire sales. Specifically, whenever a company does not satisfy the solvency condition, its portfolio will undergo a fire sale, i.e. all its assets are immediately liquidated. The fire sale causes the price of the assets in the bank's portfolio to drop. This further causes a loss for other companies that invest in the same assets because we consider a mark-to-market accounting framework in which all companies update the value of their portfolios according to the most recent observed prices.

Under this contagion mechanism, we consider two types of network: a network with the same degree of company nodes (regular network) and a random network with Poisson distributions for company nodes. The degree of a company node refers to the number of assets that it invests in. As long as $A_{ij} \neq 0$, there is a link between i-th company and j-th asset, i.e., the i-th company invests in the j-th asset. Thus, the number of non-zero elements in the i-th row of investment matrix A is the degree of the i-th company node. In a regular network, the degree of every company node is the same; that is, all companies invest in the same number of assets. In the random network, for each company-asset pair a link is drawn with the same probability. We consider a network with a sufficiently large number of nodes ($n = 1000$) with different average degree of company nodes, from very small to very large.

We adopt the following assumptions. The prices of the m assets $P_1, P_2, \cdots, P_m$ can be describes by an m-dimensional normal random variable such that each price has the same expectation and the same variance.[①] We normalize the variance to be one. The covariance matrix $Cov(P)$ is simply denoted by C whose (i, j)-th entry is C_{ij}. Each company has the initial endowment: $E(Y_i) = \sum_{j=1}^{m} A_{ij} E(P_j) = E(P_j) \sum_{j=1}^{m} A_{ij}$. Moreover, when the value of a company's assets is lower than τ ($\tau \geq 0$) times its initial endowment, the company will face bankruptcy; τ is called the collapse threshold.

Financial contagion in our model takes place as follows. At the initial moment, all the asset prices $P_1, P_2, \cdots, P_m$ are randomly selected and we calculate the value of each company to determine whether or not it will collapse. A bankrupt company will liquidate its assets

① The joint distribution of the asset prices affects the value of the systemic risk, but not the qualitative conclusions obtained in this paper.

immediately. As a consequence, each asset involved in the fire sales will be devaluated such that the price decreases from P_j tp $P_j f(P_j)$ where $f(P_j)$ is market impact function. Following Caccioli et al. (2014), we adopt the functional form: $f(s_j^t) = e^{-\alpha s_j^t}$, where s_j^t is the fraction of asset j being liquidated by bankrupt companies up to time t. The parameter α measures the devaluation of an asset caused by the collapse of a company, further reflecting the infectious ability of the network[②]. In (Caccioli et al., 2014), α is chosen to be 1.0536 such that the price drops by 10% when 10% of the asset is liquidated. We will consider various choices of α in our study.

2.2. Characterization of asset homogeneity and systemic risk

Two-fund separation theorem (Huang and Litzenberger, 1988) shows different investors would choose the same portfolio. In reality, different companies may hold different but similar portfolios. The degree of similarity among company portfolios has important implications for systemic risk of a financial network. Systemic risk, defined as the probability of failure of all companies ultimately after a series of risky contagions in the financial network (Lopomo and Wegner, 2014), has been extensively studied (Allen and Gale, 2000, Gai and Kapadia, 2010, Elliott et al., 2014). Differences in company portfolios are caused by different asset allocations and differences in assets. We will first suggest a measure of asset homogeneity. Specifically, we sum up all the elements of correlation matrix of Y ($Cov(Y)$) and divide it by n^2 to represent the asset homogeneity. The measure is denoted by SC/n^2. The larger the asset homogeneity is, the more likely the asset returns will all co-move. Portfolio homogeneity describes the similarity of company portfolios, which is determined by

[②] The market impact function can take other forms such as a linear function in the fraction of asset liquidated. Nonetheless, the results remain qualitatively valid.

asset homogeneity and asset allocation in the company portfolio. In this paper, we focus on asset homogeneity and adopt other parameters to characterize differences in asset allocation.

We assume the asset prices are equi-correlated such that the correlation matrix can be written as follows:

$$C=\begin{pmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & \cdots & \rho \\ \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \cdots & 1 \end{pmatrix}_{n\times n} .$$

where ρ is positive. That is, all pairs of asset returns are positively correlated and have the same correlation coefficient[③]. For applications of equi-correlated random variables in economics, see (Elton and Gruber, 1973, Ledoit and Wolf, 2004, Engle and Kelly, 2012). Particularly, Elton and Gruber (1973) find that equi-correlation assumption reduces estimation noises and provide superior portfolio allocations. In the following, we first study the nature of the network for the special case where all assets are fully homogeneous. Then we return to the general case to investigate the relationship between systemic risk and asset homogeneity.

2.3. Initial risk and fragility

In this section, we analyze two properties of the network: initial risk and the fragility of the network. Initial risk is defined as the probability that all the companies fail at the initial moment. Based upon the previous definition of threshold collapse, a company goes bankrupt when its asset value is less than τ times its initial endowment. This probability is denoted as $p(Y_i < \tau E(Y_i))$. The probability that all companies fail at the same time is

[③] Since the homogeneity of assets is captures in positive correlations, all the assets we consider here are positively correlated. Consequently, there is no hedging market.

$p(Y_1 < \tau E(Y_1), \cdots, Y_n < \tau E(Y_n))$. Furthermore, the fragility of the network is defined as the probability of at least one company going bankrupt (Lopomo and Wegner, 2014) which equals to 1 minus the probability that no company is failing and can be written as $1 - p(Y_1 \geq \tau E(Y_1), \cdots, Y_n \geq \tau E(Y_n))$.

Theorem 1: When all the companies allocate the same percentages to different original assets, the company portfolios are completely homogeneous and the initial risk reaches its maximum value whereas the probability of at least one company going bankrupt obtains its minimal.

Proof: Firstly, we prove that

$$\begin{aligned} & p(Y_1 \leq \tau E(Y_1), \cdots, Y_n \leq \tau E(Y_n)) \\ & \leq p(Y_1 + \cdots + Y_n \leq \tau E(Y_1) + \cdots + \tau E(Y_n)) \\ & = p(P_1 + \cdots + P_m \leq \tau E(P_1) + \cdots + \tau E(P_m)) \end{aligned} \tag{2}$$

When the investments of all the companies have the following form:

$$\begin{aligned} Y_1 \quad & = v_1(P_1 + P_2 + \cdots + P_m), \\ & \vdots \\ Y_n \quad & = v_n(P_1 + P_2 + \cdots + P_m), \end{aligned}$$

with $v_1 + v_2 + \cdots + v_n = 1 (v_i \geq 0)$, (2) is satisfied at the equality; thus, the probability that all companies fail at the same time will reach its maximum value $p(P_1 + \cdots + P_m \leq \tau E(P_1) + \cdots + \tau E(P_m))$.

Similarly, the probability of at least one company failing is:

$$\begin{aligned} 1 - p(Y_1 \geq \tau E(Y_1), \cdots, Y_n \geq \tau E(Y_n)) & \\ \geq 1 - p(Y_1 + \cdots + Y_n \geq \tau E(Y_1) + \cdots + \tau E(Y_n)) & \\ = 1 - p(P_1 + \cdots + P_m \geq \tau E(P_1) + \cdots + \tau E(P_m)) & \end{aligned} \tag{3}$$

When the investments of all the companies have the following form:

$$
\begin{aligned}
Y_1 &= v_1(P_1 + P_2 + \cdots + P_m), \\
&\vdots \\
Y_n &= v_n(P_1 + P_2 + \cdots + P_m),
\end{aligned}
\tag{4}
$$

with $v_1 + v_2 + \cdots + v_n = 1 (v_i \geq 0)$, (3) is satisfied at the equality and, hence, the probability that at least one company going bankrupt reach its minimum value.

Note that, while the possibility that all financial institutions fail at the same time as a direct result of portfolio homogenization is quite small for a large-scale network, the possibility that a considerable number of nodes in the network fail simultaneously remains fairly high. The failure of many companies exhibits stronger contagion than the failure of one or just a few companies, leading to a cascade of failures. That is, portfolio homogenization tends to produce high systemic risk. Nevertheless, if the network is highly contagious, then greater homogeneity of assets may be preferred in terms of lower systemic risk. Low homogeneity of the network implies a great probability of several company collapses at the beginning which may lead to global cascade of failure due to high contagion in the network. A more in-depth theoretical analysis on this issue is presented in the next section.

3. Theoretical analysis of a simple network

We examine the network with 3 company nodes and 3 original assets (see Figure 2). The original asset prices, $P_1, P_2,$ and P_3 are assumed to have the two point distribution $\{P_L, P_H\}$; the probabilities of P_L and P_H are $1-p$ and p ($0 < p < 1$), respectively. For simplicity, P_L and P_H are set to be 0 and 1, respectively. Thus, the expectation of the original asset price is p, and the variance is $p(1-p)$. As mentioned above, we assume that the correlation coefficients of any pair of the three primitive assets are the same, and can take any value in the interval [0,1]. That is, the correlation coefficient matrix:

$$C = \begin{pmatrix} 1 & \rho & \rho \\ \rho & 1 & \rho \\ \rho & \rho & 1 \end{pmatrix}_{3\times 3},$$

and $\rho \in [0,1]$.

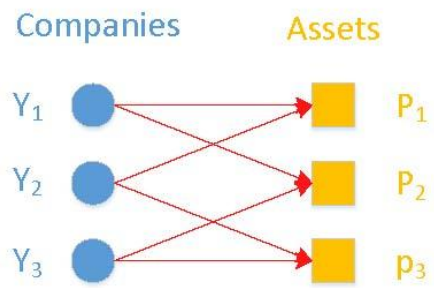

Figure 2: A bipartite network formed by 3 companies and 3 original assets.

For this network, we examine three special cases such that the degree of each company node is 1, 2, and 3 respectively in each case. The investment matrix is assumed to be symmetric as follows:

$$A_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, A_2 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}, A_3 = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix},$$

For the investment matrix A_1,

$$\begin{cases} Y_1 & = P_1, \\ Y_2 & = P_2, \\ Y_3 & = P_3, \end{cases}$$

The correlation of Y_1, Y_2 and Y_3 is exactly the same as that of P_1, P_2 and P_3, and there is no contagion in the network. The initial risk is just systemic risk. An increase in asset homogeneity ineases the systemic risk of the network as well. When the investment matrix is A_3, $Y_1 = Y_2 = Y_3 = \frac{1}{3}(P_1 + P_2 + P_3)$. No matter what the correlation of P_1, P_2 and P_3 is, Y_1, Y_2 and Y_3 are exactly the same random variable. All the company nodes in the network will either stay safe or bankrupt simultaneously. Consequently, the initial risk is systemic risk. Finally, when the investment matrix is A_2,

$$\begin{cases} Y_1 & = \frac{1}{2}(P_1 + P_2), \\ Y_2 & = \frac{1}{2}(P_1 + P_3), \\ Y_3 & = \frac{1}{2}(P_2 + P_3), \end{cases}$$

This scenario requires extensive discussions. First, note that the values of P_1, P_2 and P_3 can be categorized into the following four cases:

$$\begin{aligned} K_0 &= \{(0,0,0)^T\}, \\ K_1 &= \{(1,0,0)^T, (0,1,0)^T, (0,0,1)^T\}, \\ K_2 &= \{(1,1,0,)^T, (0,1,1)^T, (1,0,1)^T\}, \\ K_3 &= \{(1,1,1)^T\}, \end{aligned}$$

Next, we will compare how the systemic risk changes as the correlation of P_1, P_2 and P_3 changes.

3.1. Extreme correlations between original assets

We consider two extreme cases: the correlation of any pair of P_1, P_2, P_3 is 1 (all assets are perfectly correlated) and P_1, P_2, P_3 are mutually independent. When the correlation of any pair of P_1, P_2, P_3 is 1, we have

Case	K_0	K_3
Probability	$1-p$	p

When P_1, P_2, P_3 are mutually independent, we have

Case	K_0	K_1	K_2	K_3
Probability	$(1-p)^3$	$3p(1-p)^2$	$3p^2(1-p)$	p^3

Given the probability of occurrence, we examine whether all the companies will go bankrupt in each case. In case K_0, $Y = A_2 P = (0,0,0)^T$ and all companies go bankrupt, whereas in case K_3, $Y = A_2 P = (1,1,1)^T$ and all companies are safe. For the two remaining cases, under K_1, $Y = A_2 P = (1/2, 1/2, 0)^T$ and under K_2, $Y = A_2 P = (1, 1/2, 1/2)^T$. Herein the bankruptcy condition is determined by the comparison between $1/2$ and τp.

Suppose that $\frac{1}{2} \geq \tau p$, then the company will not fail when $Y_i = \frac{1}{2}$. For the case of K_2, no company will fail. For K_1, there is one company bankrupt at the initial time. The prices of primitive assets invested by the company are both 0. After liquidating of these assets, their prices are still 0. As a result, the remaining two companies will not be affected by the failing company. Thus, only case K_0 will encounter systemic risk. It is obvious that the network has a higher systemic risk when the correlation of any pair of P_1, P_2, P_3 is 1.

If $\frac{1}{2} < \tau p$, the company will fail when $Y_i = \frac{1}{2}$. For the case of K_1, all the companies will fail. For the case of K_2, there are two companies bankrupt at the initial time. If the network is highly contagious, the third company will eventually fail as well. Actually, the value of the third company is $e^{-0.5\alpha}$. The condition for the third company to fail is $e^{-0.5\alpha} < \tau p$, or $\alpha > -2log(\tau p)$. When this condition is satisfied, for cases K_0, K_1 and K_2 all companies will fail. The systemic risk is then $1-p$ when the correlation of any pair of P_1, P_2, P_3 is 1 and $1-p^3$ when P_1, P_2, P_3 are mutually independent. As $1-p^3 > 1-p$, the network has a higher systemic risk when P_1, P_2, P_3 are mutually independent.

On the other hand, if the network contagion is weak, i.e., $\alpha \le -2log(\tau p)$, then all companies fail only for cases K_0 and K_1. Because $1-p > (1-p)^3 + 3p(1-p)^2$ if $p > \frac{1}{2}$, the network has a higher systemic risk when the correlation of any pair of P_1, P_2, P_3 is 1. We can summarize the conditions for each case to encounter systemic risk as shown in the following table:

Case	K_0	K_1	K_2	K_3
Condition for systemic risk	always	$\tau > \frac{1}{2p}$	$\tau > \frac{1}{2p}$ and $\alpha > -2\log(\tau p)$	never

From the above analysis, we conclude that, only when K_0, K_1 and K_2 all encounter systemic risk, it is possible for the network to have a higher systemic risk when P_1, P_2, P_3 are mutually independent. The conditions for such situation are $\tau > \frac{1}{2p}$ and $\alpha > -2log(\tau p)$. Namely, the parameters τ and α are both large enough to ensure strong contagion in the network. Overall, when the network contagion is weak, a large asset homogeneity brings in a large systemic risk; but when the network is highly contagious, on the contrary, asset homogeneity is likely to make the systemic risk small.

3.2. General correlation between original assets

We now turn to the general case where the correlations of P_1, P_2 and P_3 cake take any value between 0 and 1. Because the asset price can take only one of two values, there are eight possible outcomes. These outcomes and their occurrence probabilities are shown in the following table:

Value	(0, 0, 0)	(0, 0, 1)	(0, 1, 0)	(0, 1, 1)	(1, 0, 0)	(1, 0, 1)	(1, 1, 0)	(1, 1, 1)
Probability	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8

Note that the correlation of any pair of P_1, P_2, P_3 is ρ, we have

$$Cov(P_i, P_j) = \rho = \frac{E(P_i - EP_i)(P_j - EP_j)}{\sqrt{Var(P_i)}\sqrt{Var(P_j)}} = \frac{E(P_iP_j) - (EP_i)(EP_j)}{\sqrt{p-p^2}\sqrt{p-p^2}} = \frac{E(P_iP_j) - p^2}{p-p^2}$$

where $i, j = 1,2,3$ and $i \neq j$. Since $E(P_1P_2) = x_7 + x_8, E(P_2P_3) = x_4 + x_8, E(P_1P_3) = x_6 + x_8$, we have the following equations:

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 1 - p, \\ x_5 + x_6 + x_7 + x_8 = p, \\ x_1 + x_2 + x_5 + x_6 = 1 - p, \\ x_1 + x_3 + x_5 + x_7 = 1 - p, \\ x_7 + x_8 = \rho p(1-p) + p^2, \\ x_4 + x_8 = \rho p(1-p) + p^2, \\ x_6 + x_8 = \rho p(1-p) + p^2, \end{cases} \Rightarrow \begin{cases} k = \rho p(1-p) + p^2, \\ x_1 = 1 - 3p + 3k - x_8, \\ x_2 = x_3 = x_5 = p - 2k + x_8, \\ x_4 = x_6 = x_7 = k - x_8, \end{cases}$$

We further assume that ρ and x_8 have a linear relationship. The results from the previous section show that the line must go through the two points, $(0, p^3)$ and $(1, p)$. Therefore, the line can be expressed as:

$$x_8 = (p - p^3)\rho + p^3.$$

The probabilities of K_0, K_1, K_2 and K_3 can be written as follows:

Case	Probability
K_0	$\rho(2p - 3p^2 + p^3) + 1 - 3p + 3p^2 - p^3$
K_1	$\rho(6p^2 - 3p - 3p^3) + 3p - 6p^2 + 3p^3$
K_2	$\rho(3p^3 - 3p^2) + 3p^2 - 3p^3$
K_3	$\rho(p - p^3) + p^3$

By simple calculation, we find that, as ρ increases, the occurrence probabilities of K_0 and K_3 will both increase whereas the occurrence probabilities of K_1 and K_2 will both decrease. That is, an increase in asset correlation promotes the probability that P_1, P_2, P_3 simultaneously take upon the same value. Moreover, if $p < \frac{1}{2}$, it can be shown that, as ρ increases, the sum of the probability of K_0 and the probability of K_1 will decrease, whereas the conclusion is reversed when $p > \frac{1}{2}$. The sum of the probability of K_0 and the probability of K_2 will always decrease as ρ increases. We obtain

Case	K_0	K_1	K_2	K_3
$\rho\uparrow$	Probability $\uparrow$	Probability $\downarrow$	Probability $\downarrow$	Probability $\uparrow$

and

Case	K_0	K_0+K_1	K_0+K_1+K_2
$\rho\uparrow$	Probability $\uparrow$	Probability $\uparrow$ if $p > \frac{1}{2}$ Probability $\downarrow$ if $p < \frac{1}{2}$	Probability $\downarrow$

In order to determine the response of the systemic risk to changes in the correlation between P_1, P_2 and P_3, we need to determine the location of the dividing line that separates the case where all companies fail from the case where at least one company survives. It is obvious that in the case of K_0, systemic risk always prevails and in the case of K_3 systemic risk never occurs. There are only three locating possibilities for the dividing line. (1) If the dividing line is located between K_0 and K_1, the increase of the correlation increases the systemic risk. (2) If the dividing line lies between K_1 and K_2, the increase of the correlation reduces the systemic risk decreasing when $p < \frac{1}{2}$, and increases systemic risk when $p > \frac{1}{2}$. (3) If the dividing line falls between K_2 and K_3, then the increase of the correlation of P_1, P_2 and P_3 reduces the systemic risk.

Overall, for the investment matrix A_2, we obtain the following theorem.

Theorem: *Let SR denote the systemic risk of the network.*

1. *When $\tau \le \frac{1}{2p}$, only case K_0 encounters systemic risk. The occurrence probability of K_0 is SR; hence, $\frac{\partial SR}{\partial \rho} > 0$;*

2. *When $\tau > \frac{1}{2p}$ and $\alpha \le -2log(\tau p)$,both cases K_0 and K_1 encounter systemic risk. Furthermore, $\tau > \frac{1}{2p}$ implies $p > \frac{1}{2}$. Hence $\frac{\partial SR}{\partial \rho} > 0$;*

3. *When $\tau > \frac{1}{2p}$ and $\alpha > -2log(\tau p)$, cases K_0, K_1 and K_2 all encounter systemic risk . Hence, $\frac{\partial SR}{\partial \rho} < 0$.*

The location of the dividing line is determined by the company's bankruptcy threshold parameter τ and the parameter α in the market impact function. The larger the two parameters, the stronger the network contagion is, and the closer the dividing line is to K_3. The systemic risk is likely to decrease when the correlation of the original assets increases. Therefore, only when the network contagion is weak, the systemic risk is positively related to asset homogeneity; otherwise, systemic risk is negatively related to asset homogeneity. This conclusion is extended to large-scale networks and further verified by numerical simulations in the following section

4. Dependence of systemic risk on asset homogeneity

With the insights derived from the simple model in the previous section, we now investigate the relationship between asset homogeneity and systemic risk for a general network taking into account contagion effect as well. To be specific, a large number of networks and corresponding randomly distributed assets are randomly generated to simulate the collapse cascade of the companies. We then measure the percentage of all random networks in which all the companies fail in the end.

4.1. Simulation algorithm

We present the numerical simulation algorithm adopted in this paper. For an arbitrarily chosen network and an arbitrarily chosen distribution of original assets, we follow the chain of events caused by the initial shock. Specifically, after the price of each asset $P_1, P_2, \cdots, P_m$ is randomly determined at time $t=0$, at each future time $t=1,2,\cdots$, the value of each company is calculated and compared with the threshold of collapse. If the value of a company is lower than its collapse threshold, the company will become bankrupt. The portfolios of bankrupted companies are liquidated immediately and the asset prices change accordingly. The new asset prices are computed from the impact functions. Then the values of all surviving companies will be recalculated at the next time. The dynamics continue until when there is no new bankruptcy occurring between two consecutive times. In details, the algorithm consists of the following steps.

1. Randomly generate a network, i.e., the investment matrix A. First, the degree d_i of all the companies, i.e., the number of nonzero elements in each row of A is determined. Then, randomly select $m-d_i$ elements in the i-th row of A and set them to 0. Each of the other d_i elements is randomly drawn from the uniform distribution over [0,1]. Finally, each element of the resulting matrix is divided by the sum of all elements in its column. The resulting matrix is the invest matrix A. For a regular network, d_i is the same for all the

companies and randomly generated from the set $\{1, 2, \cdots, n\}$ with equal probability. For a random network, d_i is generated from a Poisson distribution where the mean is randomly selected from $\{1, 2, \cdots, n\}$ with equal probability.

2. At $t = 0$, randomly generate the correlation coefficient among all the assets and a set of prices of all the assets that subject to the normal distribution $N(3,1)$. If the asset price is negative, then it is reset to be 0.
3. Calculate the values of all the companies, and compare the values to their threshold of bankruptcy (initial endowment) to determine whether there is a company failure. If there is, then enter step 4; otherwise, the program terminates.
4. Obtain the collection of failed companies D_t at time t, and liquidate the original assets held by these companies.
5. Recalculate the prices of all the original assets and the values of all the surviving companies according to the market impact function.
6. If there are new companies going to fail, return to step 4 and time $t+1$; otherwise, $D_{t+1} = D_t$ and the program terminates.

4.2. Simulation results

We first study the network with the same degree of company nodes (regular network) and $m = n$. The correlation matrix, equals to the covariance coefficient matrix since the variance of each asset P_j $(j = 1, \cdots, m)$ is assumed to be 1, is as follows:

$$C = \begin{pmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & \cdots & \rho \\ \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \cdots & 1 \end{pmatrix}_{n \times n} .$$

where ρ is randomly selected from the set $\{0, 0.1, 0.2, \cdots, 0.9, 1\}$ with equal probability. Upon applying the algorithm given in the previous section with $\alpha = 1.0536$ and $\tau = 0.75$, we generate a large number of observations from which we obtain the relationship between systemic risk and the correlation coefficient of assets. For the sake of graphic presentation, we use different transformation to represent the homogeneity of the assets (instead of SC/n^2) in the figures.

Figure 3 is obtained by simulating 10^5 random networks with $n = m = 4$. The degree of company nodes is 1,2,3,4 for Figure 3 (A)-(D), respectively. Note that, there are some individual points in the figure with large deviations from the normal trend. This is largely caused by the sampling errors and will reappear in other subsequent figures as well. We can ignore these data points as they do not affect our conclusions. From Figure 3, we find that, regardless of the degree of the company node, systemic risk increases with the increase in the correlation coefficient. Moreover, when the degree of the company nodes is small, systemic risk is greatly affected by the correlation coefficient. As the degree increases, systemic risk is less affected by the correlation coefficient.

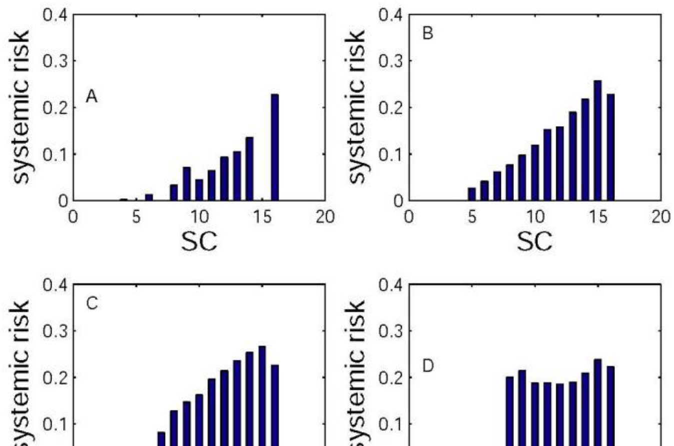

Figure 3: A total of 10^5 regular networks are randomly generated with $n = m = 4$. From (A)-(D), the degree of each company node is 1,2,3, and 4, respectively. Each subgraph shows the relationship between the systemic risk and the correlation coefficient of Y. The horizontal coordinate SC represents rounding the summation of all the elements of correlation coefficient matrix of Y. The vertical coordinate represents the percentage of the total networks in which all companies fail in the end.

Figure 4 is obtained by simulating 10^6 random networks with $n = m = 10$. The degree of company nodes is $1, 2, \cdots, 10$ for Figure 4 (A)-(J), respectively. Furthermore, we represent all the results in Figure 4 by a three-dimensional graph in Figure 5. From these figures, we find that, starting from a small number, as the degree of network nodes increases, systemic risk will increase significantly. However, there is a turning point at which if the degree is increased further, systemic risk will decline slightly. Thus, the most unstable network is the network whose degree is neither too small nor too large. In addition, accompanied with the increase in the degree of network nodes, systemic risk becomes more and more independent of the correlation of Y, or there may even be a slightly negative relationship between systemic risk and the correlation coefficient. However, as the degree of network nodes further increases, the positive relationship is recovered.

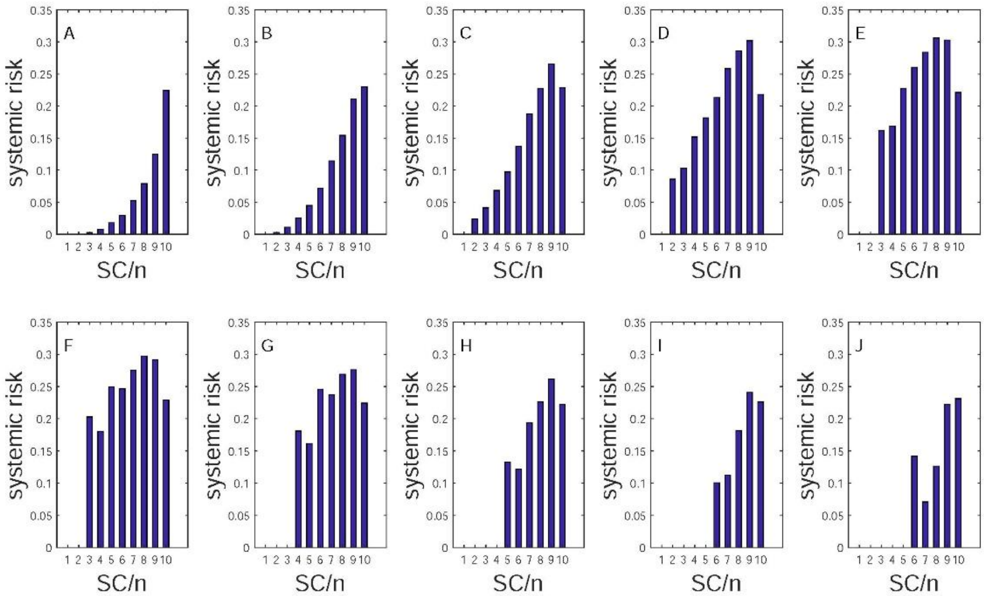

Figure 4: A total of 10^6 regular networks are randomly generated with $n = m = 10$. From (A)-(J), the degree of each company node is $1, 2, \cdots, 10$, respectively. Each subgraph shows the relationship between the systemic risk and the sum of correlation coefficients of Y.

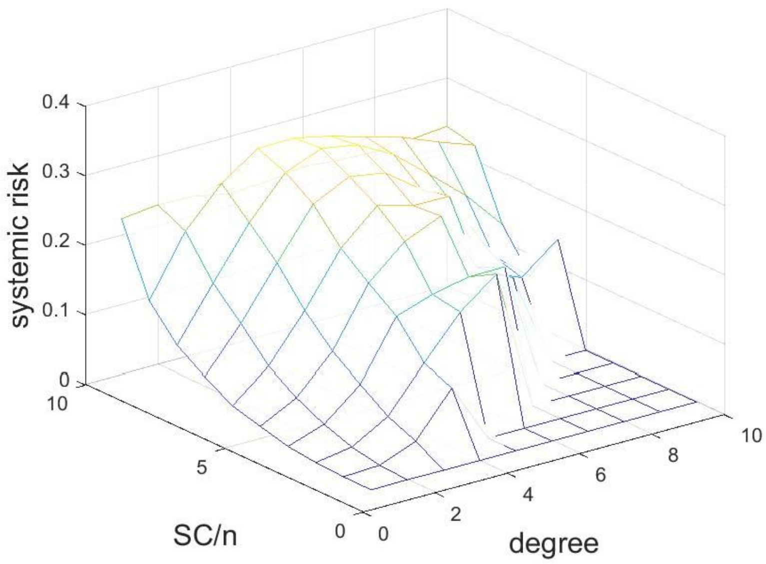

Figure 5: The graph is drawn by integrating all the results in Figure 4. The vertical coordinate is the systemic risk. The two coordinates of the horizontal plane represent asset homogeneity (SC/n) and the degree of company nodes.

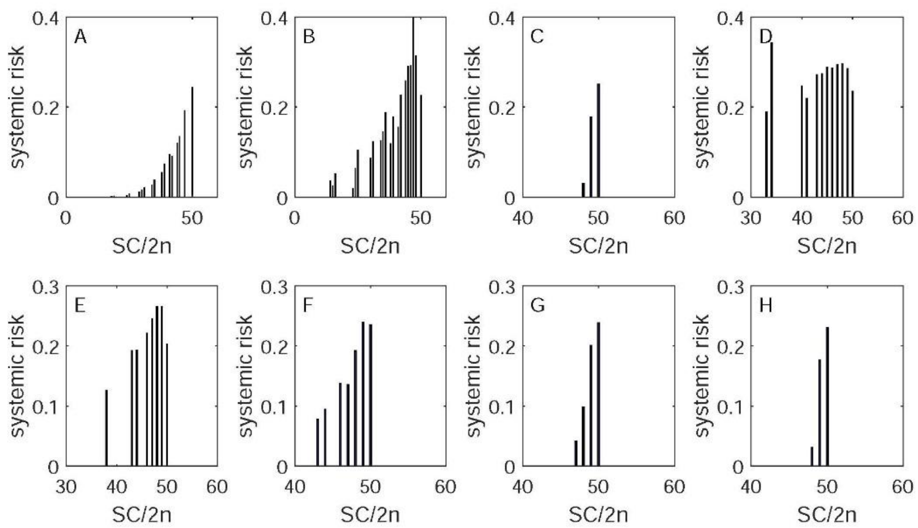

Figure 6: A total of 10^6 regular networks are randomly generated with $n = m = 100$. From (A)-(H), the degree of each company node is 3, 5, 10, 20, 30, 50, 80, and 100, respectively. Each subgraph shows the relationship between systemic risk and asset homogeneity.

Finally, we study large networks with the number of company nodes and asset nodes $n = m = 100$ and $n = m = 1000$. The results are shown in Figure 6 and Figure 7, respectively. We also investigate the Poisson distribution network with 1000 nodes with the average degree of nodes increasing gradually. The results are shown in Figure 8. All these figures present the same properties highlighted in Figure 4.

To sum up, Figures 3-8 show that the influence of asset homogeneity on the systemic risk is dependent on the degree of company nodes in the network. When the degree is small, the connectivity of the network is low. Thus, the systemic risk is roughly equivalent to the initial risk, and the contagion has little effect if any. Furthermore, a larger correlation of Y leads to a higher probability that a considerable number of companies failing simultaneously. Thus, systemic risk is proportional to the correlation of Y when the degree is small. As the degree increases, the network connectivity is strengthened such that a small number of initial collapses are likely to infect the entire network. The systemic risk is not so much affected by the correlation of Y; it may even be negatively affected by the correlation. However, when the degree of network nodes is sufficiently large, the systemic risk and the correlation of Y will be positively related once again. As a remark, for a given degree of network, there is an inverted U relationship between the systemic risk and the correlation coefficient, and the inverted U type is more distinguished when the degree is moderate.

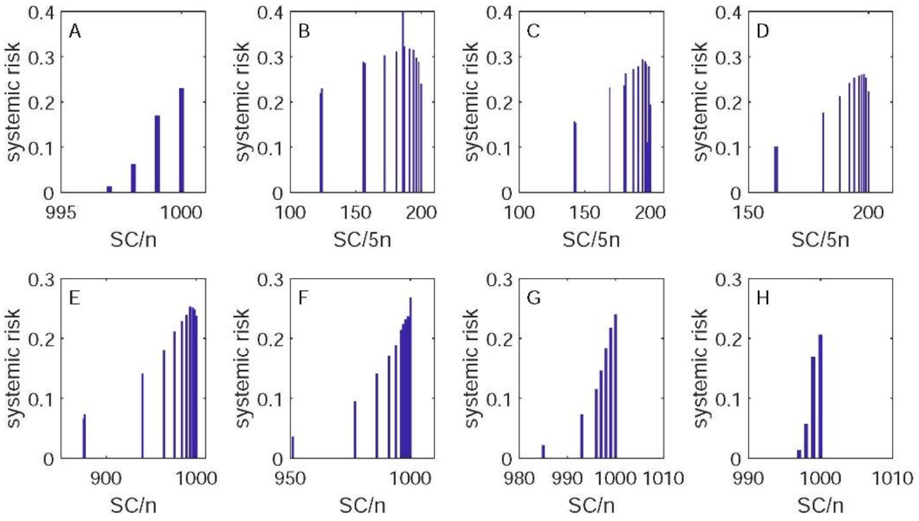

Figure 7: A total of 10^6 regular networks are randomly generated with $n = m = 1000$. From (A)-(H), the degree of each company node is 10, 20, 30, 50, 80, 200, 500, and 1000 respectively. Each subgraph shows the relationship between systemic risk and asset homogeneity.

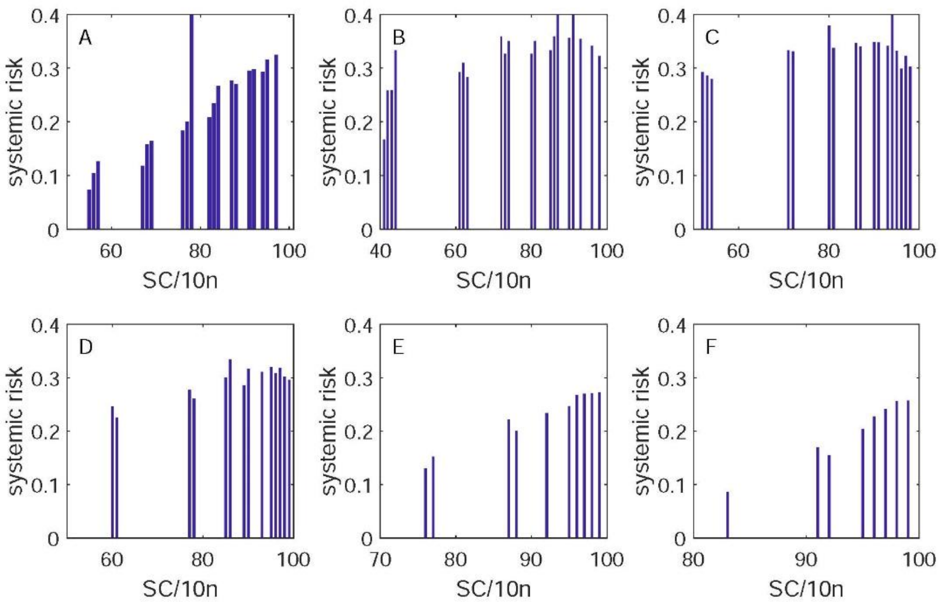

Figure 8: A total of 10^6 Poisson distribution networks are randomly generated with $n = m = 1000$. From (A)-(F), the average degree of the nodes is 8, 10, 15, 20, 40, and 60 respectively. Each subgraph shows the relationship between systemic risk and asset homogeneity.

4.2.1 Effects of contagion parameters

We have characterized the relationship between the systemic risk of network and asset homogeneity allowing the average degree of the network to vary. Gai and Kapadia (2010), points out the average degree of the network plays a key role on network contagions. There are two other important parameters that can control the effects of contagions: α and τ. For a given network, when α increases or τ increases, the network will become more contagious, and vice versa. This aspect, however, has not been studied by the previous work. We expect these two parameters to have great impacts on our results, both qualitatively and quantitatively. Therefore, we conduct simulations to re-examine the relationship between the systemic risk and by the asset homogeneity under different contagion schemes (i.e., allowing both α and τ to change as well). First, we discuss the numerical simulation results generated from different compositions of d, α, τ. Then, we will apply the methodology of (Caccioli et al., 2014) to characterize contagions and analyze the influence of contagion on the relationship between asset homogeneity and the systemic risk.

Figure 9 and Figure 10 consider the regular networks with $n = m = 10$ and the Poisson distribution networks with $n = m = 1000$, respectively. Since the results of these two figures are consistent, we only discuss the results shown in Figure 9. In Figure 9, α is either 0.5, 1.0536, or 5, and τ is either 0.5, 0.75, or 0.9. From left subgraphs to right subgraphs, α is 0.5, 1.0536 and 5, respectively. From top subgraphs to bottom subgraphs, τ is 0.5, 0.75 and 0.9, respectively. A larger value of α indicates that, the bankruptcy of a company will cause a larger devaluation of the assets contained in its portfolio. A larger value of τ signals the company is more vulnerable to the fluctuations in its market value and is easier to collapse. Clearly, the increase in of α or τ will make the network more contagious, and vice versa. The middle subgraph (E) corresponds to the case that $\alpha = 1.0536$ and $\tau = 0.75$, whose result was given in Figure 4.

There is a big difference between subgraph (E) and the other subgraphs. For the case of $\alpha = 0.5$ and $\tau = 0.5$, which is a very weak contagion, systemic risk and asset homogeneity are significantly positively correlated, regardless of the average degree of network. In the case of $\alpha = 5$ and $\tau = 0.9$ where a very high contagion prevails, a completely different pattern prevails.

When the average degree of the network is small, the systemic risk maintains a positive relationship with asset homogeneity. As the average degree of the network increases, the systemic risk may become negatively correlated with asset homogeneity. However, when the average degree of the network is sufficiently large, the negative relationship between the systemic risk and asset homogeneity is weakened. This result is consistent with our theoretical analysis which suggests, a low asset homogeneity may be accompanied by a high systemic risk for a highly contagious network.

In all the subgraphs with a small average degree, the systemic risk displays a significant and positive dependence on asset homogeneity. The reason is that, a small average degree implies a low contagion and the systemic risk of the network is equivalent to the initial risk. A comparison across all the figures shows that the network with a large α and a large τ has the greatest systemic risk. On the contrary, the network with a small α and a small τ has very small systemic risk. To sum up, the most stable network prevails when α and τ are both small; moreover, the systemic risk can be further reduced by reducing asset homogeneity. When α and τ are relatively large, the average degree of network must be very small to contain the systemic risk of the network by reducing the asset homogeneity. If the average degree of network is not small, then controlling asset homogeneity does not help reducing the systemic risk at all.

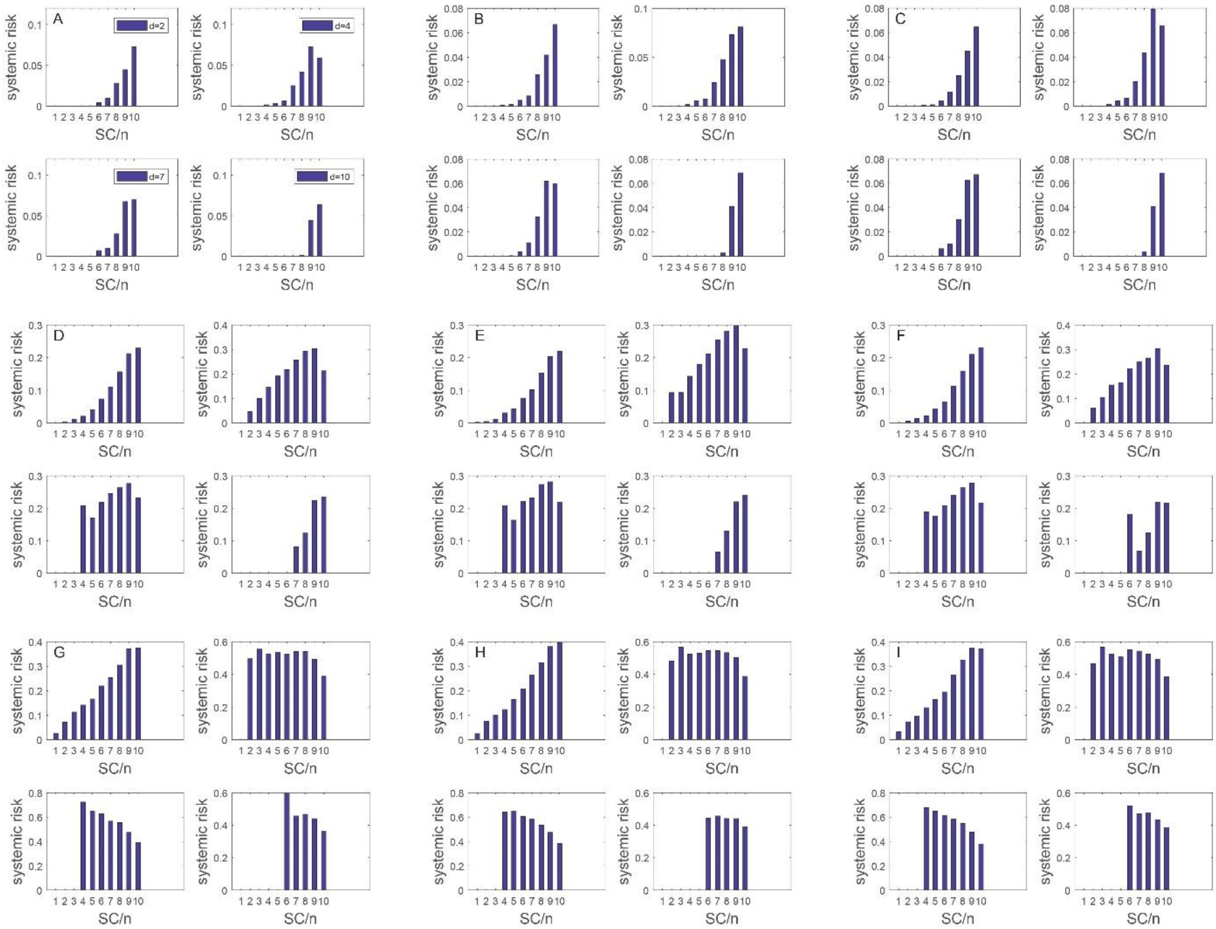

Figure 9: The regular networks are all randomly generated with $n = m = 10$. The horizontal coordinate SC/n represents rounding the summation of all the elements of the correlation matrix of Y divided by n. The vertical coordinate represents the systemic risk. Each subgraph is marked with A, B, $\cdots$, I. For each subgraph, the average degree of the network is (from left to right and from top to bottom) 2, 4, 7 and 10 respectively, which has been show in the subgraph (A). From left subgraphs to right subgraphs, α is 0.5, 1.0536 and 5 , respectively. From top subgraphs to bottom subgraphs, τ is 0.5, 0.75 and 0.9, respectively. The subgraph (E) is the same as that presented in Figure 4.

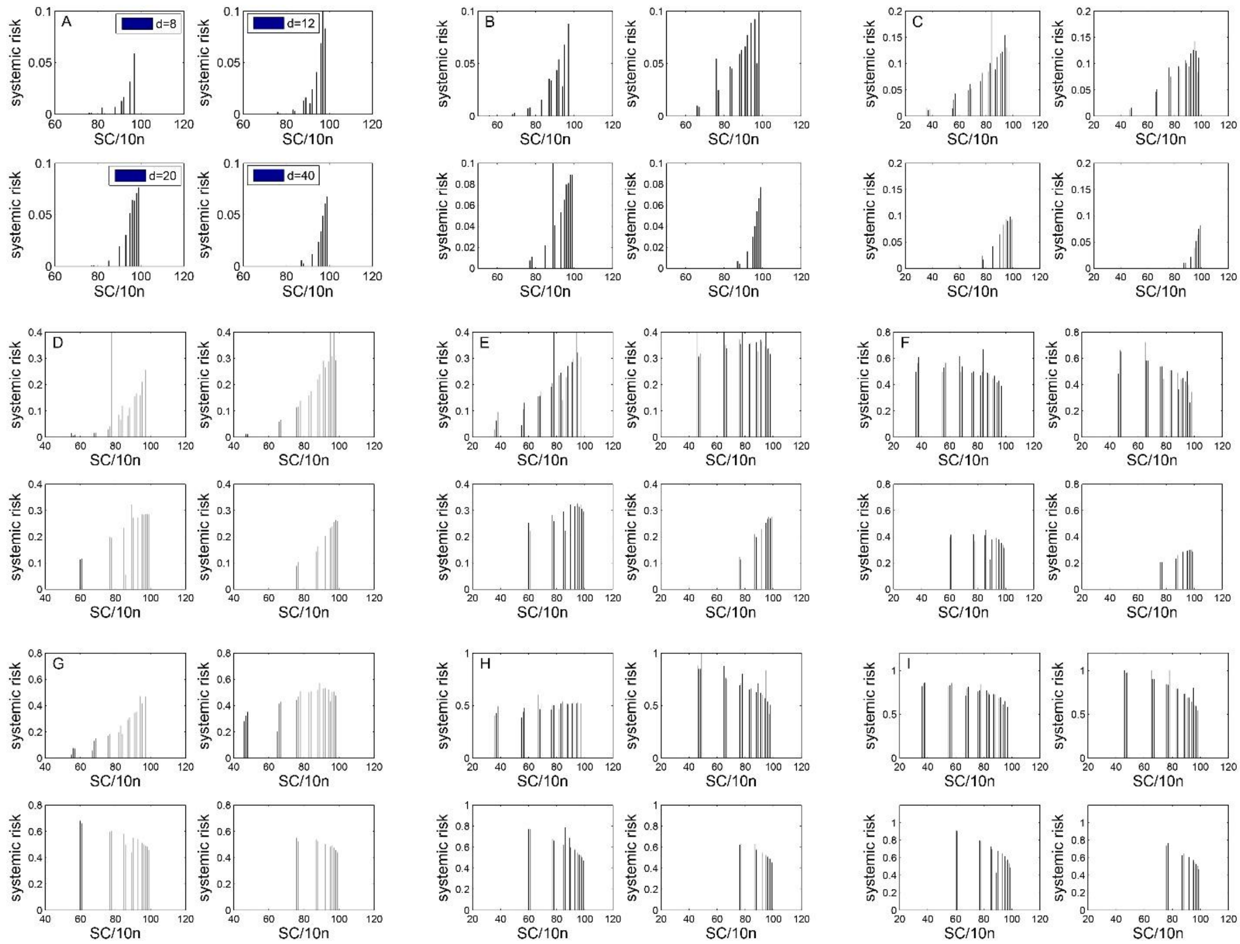

Figure 10: The Poisson distribution networks are all randomly generated with $n = m = 1000$. The horizontal coordinate SC/10n represents rounding the summation of all the elements of the correlation matrix of Y divided by 10n. The vertical coordinate represents the systemic risk. Each subgraph is marked with A, B, $\cdots$, I. For each subgraph, the average degree of the network is (from left to right and from top to bottom) 8, 12, 20 and 40 respectively, which has been show in the subgraph (A). From left subgraphs to right subgraphs, α is 0.5, 1.0536 and 5 , respectively. From top subgraphs to bottom subgraphs, τ is 0.5, 0.75 and 0.9, respectively. The subgraph (E) is the same as that presented in Figure 8.

4.2.2 Results based on comprehensive analysis of contagions

In this section, we consider the contagion index constructed by (Caccioli et al., 2014). To be specific, Caccioli et al. (2014) propose a matrix N whose element N_{hk} represents the expected number of companies of degree h failing because of the failure of a company of degree k ($h,k=1,2,,\cdots,m$). The expression of N_{hk} is given in Appendix A. The contagion of the system can be estimated by computing the largest eigenvalue (λ) of N. For the regular networks, we simulate the dependence of the systemic risk on both asset homogeneity and contagion. The results are displayed in Figure 11. The results for Poisson distribution networks are similar.

From Figure 11, it is shown that, when the contagion of the network is low, the systemic risk is positively related to asset homogeneity. When the contagion of the network is high, the systemic risk has a strongly negative correlation with asset homogeneity. Thus, a network has the greatest systemic risk when the contagion is highest and the asset homogeneity is smallest at the same time. However, high contagion implies that the network has a high degree of connectivity, and hence the asset homogeneity cannot be too low. As a consequence, the case that a network has highest contagion and lowest homogeneity at the same time hardly exists. Indeed, there is no such case in Figure 11. The greatest systemic risk will occur in the network that has fairly high contagion and fairly low homogeneity of companies, but the contagion is not the highest possible and asset homogeneity is not the lowest possible. Finally, we can see from Figure 11 that no matter what the value the of asset homogeneity takes up, the systemic risk is always positively related to the contagion parameter λ, which is consistent with the previous analysis (Caccioli et al., 2014) and further supports the validity of our simulation results.

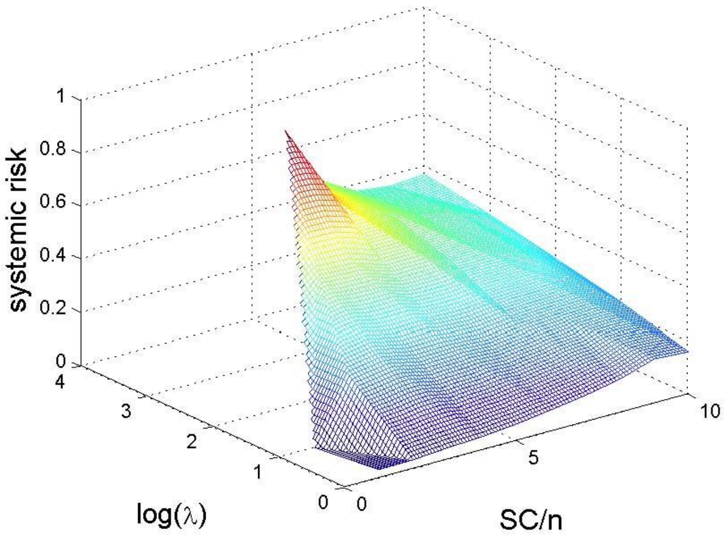

Figure 11: The graph is drawn after randomly generating 10^6 networks (with the same degree of company nodes) with $m = n = 10$. The vertical coordinate is the systemic risk. The two coordinates of the horizontal plane represent asset homogeneity and contagion index.

5. Regression analysis

5.1. Data and variables

For regression analysis discussed in this section, the data are obtained from the simulation results of 10^6 randomly generated regular networks with $n = m = 10$ (The regression results for Poisson degree distribution random networks with $n = m = 1000$ are presented in Appendix B). The degree of companies (d) is randomly generated from $\{1, 2, \cdots, n\}$. Whenever a network (investment matrix A) of a certain degree is randomly generated, the prices of the original assets ($P_1, P_2, \cdots, P_m$) are also randomly generated, as well as parameters α and τ. The values of α and τ, assumed to come from a set of reasonable ranges, can play a significant role in the systemic risk of the network. Given d, α and τ, the contagions parameter λ can be calculated. We apply the algorithm described in Section 4.1 and determine whether all companies fail for each random network. If all the companies fail in the end, we set the index R to be 1; otherwise, R equals to 0. The systematic risk (for each type of network) is calculated as the ratio of the number of networks where R equals to 1 to the total number of networks. In the simulation process, there are cases that the sum of a column of the investment matrix A is 0. After removing these data, we have a total of 788512 observations. The variables of the regression model are described in

Table 1 and their corresponding descriptive statistics are given in

Table 2.

Table 1: Definition and range of main variables

variables	variable descriptions	range of values
R	Are all companies bankrupt? (Yes=1,No=0)	$\{0,1\}$
SC/n^2	asset homogeneity	$[1/n, 1]$
d	average degree of company nodes	$[1, n]$
τ	bankruptcy threshold	{0.5, 0.6, 0.7, 0.8, 0.9}
α	asset devaluation speed	{0.5, 1, 1.5, 2, 2.5}
λ	Contagion index	$[0, +\infty]$

Table 2: Descriptive statistics of main variables ($n = m = 10$)

variable	mean	standard deviation	minimum	maximum
R	0.21960	0.4140	0	1
SC/n^2	0.7907	0.1808	0.1	1
d	6.4738	2.3458	1	10
τ	0.7002	0.1415	0.5	0.9
α	1.5000	0.7074	0.5	2.5
λ	9.011	16.751	0	90.841

5.2. Regression design and results

Since the dependent variable R is a binary variable with a value of 0 or 1, we apply the logistic regression model with the following specification:

$$E(R) = f\Big(c_0 + c_1(SC/n^2) + c_2 d + c_3\tau + c_4\alpha + c_{11}(SC/n^2)^2 + c_{22}d^2 + c_{33}\tau^2 + c_{44}\alpha^2 \\ + c_{12}(SC/n^2)d + c_{13}(SC/n^2)\tau + c_{14}(SC/n^2)\alpha + c_5 log(\lambda) + c_{15}(SC/n^2)\lambda\Big),$$

where

$$f(x) = \frac{exp(x)}{(1+exp(x))}.$$

Our focus is the effect of homogeneity variables SC/n^2 on the systemic risk. Thus, the interaction terms between d, τ, α and λ are not included in the model. In order to validate our predictions in the previous section, we gradually added the interaction terms and the square terms of the explanatory variables into the regression model. The results of regression model which does not include λ are shown in Table 3 whereas the results of regression model which includes λ are displayed in Table 4. In both tables, the numbers inside the brackets are standard errors. All the coefficients are statistically significant at the 1% level.

Firstly, we discuss the results in Table 3. Model 1 is a linear model and the estimation results show that systemic risk is positively related to SC/n^2, τ and α, but negatively related to d. Next, we introduce model 2 which incorporate the cross terms $(SC/n^2)\tau$ and $(SC/n^2)\alpha$ to model 1 to investigate how the dependence of systemic risk on SC/n^2 changes with τ and α. The results show that the effect of (SC/n^2) on R is ($13.026 - 12.948\tau - 1.257\alpha$) which is negatively dependent on the value of τ and α. When τ and α are small, the effect of (SC/n^2) is positive and systemic risk will exhibit a positive correlation with the SC/n^2. When τ and α are sufficiently large, the effect of

(SC/n^2) is negative. In such cases, systemic risk will be negatively related to SC/n^2. These regression results explain our theoretical predictions and simulation results in Sections 3 and 4.

In order to examine how the dependence of systemic risk on SC/n^2 changes with d, we further add the cross term $(SC/n^2)d$ to obtain model 3. The result indicates the degree of company nodes significantly reduces the impacts of asset homogeneity on systemic risk. On the other hand, our numerical simulation suggests low and high degrees of company nodes have opposite influence. Thus, we divide our samples into two subsamples. Subsample I contains observations with the degree less than 6 (a total of 293191 data points) and Subsample II includes observations with the degree no less than 6 (a total of 495321 data points). Logistic regressions were performed for the two subsamples respectively. The results are presented in Table 5 and 6. The regression coefficient of $(SC/n^2)d$ is negative for Subsample I and the positive for Subsample II. That is, when the degree is small, an increase in the degree will reduce the positive correlation between systemic risk and SC/n^2. On the other hand, when the degree is large, an increase in degree will increase the positive correlation between systemic risk and SC/n^2. As a consequence, in the presence of moderate network connectivity, i.e., when the average degree is neither too large nor too small, the systemic risk is most likely negatively correlated with asset homogeneity, which is congruent with the simulation results displayed in Figure 9.

Model 4 and model 5 add the square terms d^2 and $(SC/n^2)^2$ successively. In both models, the coefficient of d^2 is statistically significantly negative. Thus, consistent with the previous literature, the relationship between systemic risk and the degree of network is captured by an inverted U curve. The negative correlation of $(SC/n^2)^2$ in model 5 suggests that the relationship between systemic risk and asset homogeneity also appeared to be of the inverted-U shape. Moreover, the vertex of the inverted U curve changes with τ and α. If τ and α are small, the vertex appears at a large value of SC/n^2 and systemic risk exhibits a positive correlation with SC/n^2 when SC/n^2 is valued in $[1/n,1]$. As increase τ and α increase, the vertex gradually moves toward the axis $SC/n=0$. In such cases, systemic risk first increases and then decrease as SC/n^2 increases from $1/n$ to 1. When τ and α are sufficiently large, the vertex corresponds to a negative value of SC/n^2 which means that, as long as $SC/n^2>0$, systemic risk is always negatively related to SC/n^2. Note that the inverted U type relationship between systemic risk and SC/n^2 is embedded in our simulation results and depicted in Figure 4 through Figure 8.

The squared terms τ^2 and α^2 are now added to construct model 6. The regression coefficients of these two terms are both negative. Therefore, the dependence of systemic risk on τ or α is of inverted U type as well. The magnitude for the coefficients of τ and α are much greater than those of τ^2 and α^2. As a result, the values of τ and α at the vertices would be very large. The graphs of systemic risk with respect to τ and α would appear to be the left half of a parabolic curve that open downward.

At last, we analyze the results of regression models incorporating the contagion parameter, λ. In model 1 of Table 4, the regression coefficient of λ is positive. In model 2, the effect of λ on systemic risk is $(0.158-0.149(SC/n^2))$, which is larger than 0 since $SC/n^2\leq1$. Thus, the systemic risk and contagion parameters λ are positively related. On the other hand, the negative coefficient of $(SC/n^2)\lambda$ in model 2 implies that the dependence of systemic risk on asset homogeneity SC/n^2 is affected by λ. If λ is small, systemic risk will exhibit a positive correlation with SC/n^2. As λ increases, the systemic risk will gradually decrease and eventually become negatively related to SC/n^2. These results are consistent with the simulation results presented in Figure 11.

We successively add d, τ, α in model 3, and then the cross terms $(SC/n^2)d$, $(SC/n^2)\tau$, $(SC/n^2)\alpha$ in model 4, and finally the square terms d^2, $(SC/n^2)^2$, τ^2 and α^2 in model 5. The estimation results indicate that additions of the variables d, τ, α would disperse the effect of λ on the systemic risk. This is not unexpected since λ is indeed a function of d, τ, α. In summary, the regression results present a clearer view of the relationship between systemic risk and asset homogeneity, and the relationship is influenced by network contagion.

Table 3: Regression results of regular networks with $n=m=10$

Variable	model 1	model 2	model 3	model 4	model 5	model 6
Constant	-7.686 (0.025)	-17.779 (0.129)	-20.121 (0.144)	-20.560 (0.145)	-21.904 (0.158)	-24.126 (0.208)
SC/n^2	0.801 (0.018)	13.026 (0.151)	15.857 (0.168)	14.859 (0.170)	18.737 (0.228)	19.192 (0.232)
d	-0.022 (0.001)	-0.021 (0.01)	0.323 (0.008)	0.613 (0.010)	0.506 (0.010)	0.505 (0.010)
τ	7.592 (0.025)	18.269 (0.148)	19.006 (0.152)	18.982 (0.152)	19.815 (0.160)	24.693 (0.372)
α	0.201 (0.004)	1.225 (0.022)	1.269 (0.022)	1.270 (0.022)	1.316 (0.023)	1.779 (0.032)
$(SC/n^2)\tau$		-12.948 (0.173)	-13.815 (0.178)	-13.746 (0.178)	-14.742 (0.187)	-15.225 (0.193)
$(SC/n^2)\alpha$		-1.257 (0.026)	-1.309 (0.027)	-1.308 (0.027)	-1.364 (0.027)	-1.382 (0.028)
$(SC/n^2)d$			-0.411 (0.009)	-0.236 (0.010)	-0.086 (0.011)	-0.086 (0.011)
d^2				-0.034 (0.001)	-0.035 (0.001)	-0.035 (0.001)
$(SC/n^2)^2$					-2.720 (0.100)	-2.740 (0.100)
τ^2						-3.049 (0.208)
α^2						-0.149 (0.007)

Table 4: Regression results (λ included)

Variable	model 1	model 2	model 3	model 4	model 5
Constant	-1.801 (0.013)	-2.829 (0.017)	-8.565 (0.035)	-18.238 (0.185)	-22.709 (0.251)
SC/n^2	0.263 (0.016)	1.481 (0.021)	1.945 (0.024)	13.649 (0.216)	17.483 (0.282)
λ	0.030 (0.000)	0.158 (0.001)	0.093 (0.001)	0.028 (0.002)	0.021 (0.002)
$(SC/n^2)\lambda$		-0.149 (0.001)	-0.110 (0.001)	-0.032 (0.002)	-0.023 (0.002)
d			-0.029 (0.002)	0.226 (0.010)	0.428 (0.012)
τ			7.579 (0.032)	17.303 (0.185)	23.634 (0.411)
α			0.192 (0.005)	1.054 (0.026)	1.596 (0.036)
$(SC/n^2)\tau$				-11.812 (0.217)	-13.770 (0.238)
$(SC/n^2)\alpha$				-1.055 (0.031)	-1.200 (0.032)
$(SC/n^2)d$				-0.300 (0.012)	-0.003 (0.014)
d^2					-0.035 (0.001)
$(SC/n^2)^2$					-2.675 (0.100)
τ^2					-3.224 (0.223)
α^2					-0.143 (0.007)

Table 5: Regression results ($d \leq 5$)

Variable	model 1	model 2
Constant	-22.706 (0.202)	-29.602 (0.324)
SC/n^2	18.166 (0.245)	23.264 (0.332)
d	1.233 (0.021)	2.308 (0.051)
τ	17.727 (0.189)	25.361 (0.594)
α	1.491 (0.029)	2.407 (0.049)
$(SC/n^2)\tau$	-12.863 (0.231)	-14.688 (0.257)
$(SC/n^2)\alpha$	-1.440 (0.036)	-1.584 (0.038)
$(SC/n^2)d$	-1.249 (0.027)	-1.149 (0.029)
d^2		-0.152 (0.006)
$(SC/n^2)^2$		-2.870 (0.118)
τ^2		-4.218 (0.351)
α^2		-0.263 (0.012)

Table 6: Regression results ($d \geq 6$)

Variable	model 1	model 2
Constant	-18.036 (0.279)	-22.179 (0.369)
SC/n^2	13.801 (0.320)	20.423 (0.441)
d	-0.540 (0.023)	-0.674 (0.040)
τ	24.661 (0.268)	30.206 (0.520)
α	0.785 (0.039)	1.096 (0.049)
$(SC/n^2)\tau$	-19.877 (0.306)	-21.909 (0.332)
$(SC/n^2)\alpha$	-0.812 (0.045)	-0.849 (0.046)
$(SC/n^2)d$	0.503 (0.026)	0.826 (0.030)
d^2		-0.009 (0.002)
$(SC/n^2)^2$		-4.810 (0.209)
τ^2		-2.578 (0.261)
α^2		-0.093 (0.009)

6. Conclusions

In this article, we propose a general portfolio model that can capture the relationships among firms, cross shareholdings, and more general portfolio strategies to investigate the effects of portfolio homogenization of companies (or called asset homogeneity) on systemic risk in financial networks. There are two different channels that may amplify systemic risk because of connectivity. One is the financial contagions, which stress the sequential collapse of financial organizations induced by the preceding collapses. The other is the portfolios homogenization, which increases the correlation between the returns of financial organizations and then the likelihood of their simultaneous failure. The existing works mainly emphasize contagions as the basic mechanism of financial crisis while, to our knowledge, the effect of portfolio homogenization on the systemic risk has not yet been extensively examined. Portfolio homogenization might facilitate the course of financial contagions because it increases the probability that a considerable number of companies going bankrupt, enhancing the possibility of further spread. As two-fund separation theorem suggests that the investment portfolios among different institutions have convergent characteristics, analyzing the effect of asset homogeneity on systemic risk offers important practical guidelines.

Asset homogeneity reflects the correlation of portfolios among different companies. The greater the correlation, the stronger the asset homogeneity is. We sum up all the elements in the correlation matrix of the asset returns to character asset homogeneity. We first theoretically analyze the relationship between the initial risk, fragility and asset homogeneity. The homogenization of portfolios increases the positive correlations among financial institutions, leading to an increase in the initial risk and systemic risk of the network. Thus, the systemic risk is positively related to the asset homogeneity. The network in which all companies' portfolios are completely homogeneous has the highest systemic risk. However, for the network with low homogeneity of assets, the probability that at least one company collapses is greater than that of the network with high homogeneity of assets. If the network contagion is strong, the collapse of one node is likely to spread the risk to the entire network. Therefore, to the contrary, low asset homogeneity may quite likely produce higher systemic risk.

Based on the results from the theoretical analysis, we perform simulations to illustrate the influences of asset homogeneity on systemic risk under different contagion schemes. We study two different network structures: the regular network and Poisson random network. For either type of network structure, the effect of asset homogeneity on systemic risk is closely related to the contagion, which is consistent with our theoretical prediction. Systemic risk is significantly and positively related to asset homogeneity when the network contagion is weak. As the network contagion increases, the systemic risk becomes inversely related to asset homogeneity. Therefore, networks with fairly strong contagion and fairly low asset homogeneity tend to have the greatest systemic risk.

We further analyze the data obtained from simulations of large-scale randomly generated networks by logistic regressions. The regression results show that the effect of asset homogeneity on systemic risk is closely dependent upon the network contagion. Weak contagion of network induces the systemic risk to exhibit a positive correlation with asset homogeneity. As contagion increases, the correlation gradually decreases and may even become negative. Moreover, systemic risk is quite likely to have an inverted U type relationship with asset homogeneity and the location of the vertex is related to the contagion of the network. Overall, the regression results offer comprehensive explanations to the patterns prevailed in the simulation data.

In summary, we have investigated how the systemic risk is affected by asset homogeneity and how this effect varies for networks with different contagion capacity. In the analysis, during the contagion stage, we assume all the surviving companies retain the initial portfolio with no asset reallocation. To allow asset reallocation, we need to specify the decision-makings of the companies and introduce more dynamic changes into the model. Nevertheless, we expect our conclusions to be qualitatively unchanged. On the other hand, we also plan to analyze real-world financial networks and determine to what extent the past financial crises were attributed to the mechanism of portfolio homogenization and to the mechanism of contagions.

Acknowledgments

This work was supported by the major project of the National Social Science Foundation of China (Grant No.16ZDA008), and by the China Postdoctoral Science Foundation (Grant No. 2015M581050 and 2016T90072).

Appendix A. Explicit calculation of the matrix N

In order to understand how the systemic risk of network depends on asset homogeneity under different contagion schemes, we refer to (Caccioli et al., 2014) and let

$$N_{hk} = N_h \sum_{a=1}^{m} P(h,k \mid a) F(h \mid k,a)$$

where N_h is the number of companies of degree h, $P(h,k \mid a)$ is the probability that a given company of degree h and a given company of degree k share (i.e., are both connected to) a given asset a, $F(h \mid k,a)$ is the probability that a company of degree h fails given that it is connected to a failed company of degree k through asset a.

For the Poisson random networks with Poisson degree distributions for both banks and assets, we set the average degree of all the companies to be μ_c and the average degree of all the assets to be μ_a. Thus, the number of companies of degree h is $N_h = np_c(h)$ where

$$p_c(h) = \frac{e^{-\mu_c}\mu_c^h}{h!}$$

is the probability that a company has degree h. A given company of degree h is connected to a given asset of degree l_a (i.e, the number of companies investing in this asset) with probability

$hl_a/(\mu_c n)$, where $\mu_c n$ is the total number of edges in the network. The probability that a failed company of degree k is also connected to the same asset a is $h(k-1)l_a(l_a-1)/(\mu_c n)^2$, where the factor of $k-1$ comes from the fact that one of the k edges of the failed company is already connected to the asset that caused its failure. Thus, we have

$$P(h,k\,|\,a)=\frac{hl_a(k-1)(l_a-1)}{\mu_c^2 n^2}.$$

Now, we compute the probability $F(h\,|\,k,a)$. When a fraction s_a of an asset is sold, the asset price is reduced to $(1-f(s_a))$ of its original value. Thus, the condition for a company of degree k to fail is

$$(1-f(s_a))>1-\tau,\quad \text{i.e.}\quad f(s_a)<\tau.$$

In (Caccioli et al., 2014), s_a is calculated as follows. Let $v(a)$ denote the set of companies investing in asset a, the fraction of a that is liquidated is

$$s_a=\frac{1/k}{\sum\limits_{m\in v(a)}1/k_m}$$

where k_m denotes the degree of company m. This formula is obtained under the simplifying assumption that each company has the same investment proportion for different assets that it invests in, i.e., the nonzero elements of each row of matrix A have the same value. Besides, the investment matrix A in (Caccioli et al., 2014) has the property $\sum_{j=1}^{m}A_{ij}=1$. This further deduces that the non-zero elements in the i-th row of matrix A are all $1/h$ if the degree of the i-th company is h. Following Caccioli et al., 2014, in order to derive the conditional probability, we adopted a similar approach by assuming that the proportion of each asset held by different companies that invest in it is the same. This simplifying assumption does not affect the outcome since we have confirmed it by simulation. Thus, combining the properties of $\sum_{i=1}^{m}A_{ij}=1$, we deduce that the non-zero elements in the

j-th column of matrix A are all $1/k$ if the degree of the j-th asset is k, which gives

$$F(h|k,l)=\Theta\left\{(1-f(1/l))-(1-\tau)\right\}$$

where l denotes the degree of asset that a given company of degree h and a given company of degree k are both connected to. Θ is the Heaviside step function, $\Theta(x)=1$ if $x>0$ and zero otherwise. Furthermore, it is easy to get

$$P(h,k\,|\,l)=P(h,k\,|\,a)M_l=\frac{hl_a(k-1)(l_a-1)}{\mu_c^2n^2}mp_a(l)$$

where $l_a=l$ and $M_l=mp_a(l)$ is the number of assets of degree l and

$$p_a(l)=\frac{e^{-\mu_a}\mu_a^l}{l!}.$$

Thus, expression of N_{hk} can be transform to be:

$$\begin{aligned}N_{hk}&=N_h\sum_{l=1}^{n}P(h,k\,|\,l)F(h\,|\,k,l)\\&=n\frac{e^{-\mu_c}\mu_c^h}{h!}\sum_{l=1}^{n}\frac{hl(k-1)(l-1)}{\mu_c^2n^2}\frac{e^{-\mu_a}\mu_a^l}{l!}\Theta\left\{(1-f(1/l))-(1-\tau)\right\}\\&=\frac{e^{-\mu_c}\mu_c^h}{h!}\frac{h(k-1)}{\mu_c^2n}\sum_{l=1}^{n}\frac{e^{-\mu_a}\mu_a^l}{l!}l(l-1)\Theta\left\{(1-f(1/l))-(1-\tau)\right\}.\end{aligned}$$

For the networks in which all the companies have the same degree k, the matrix N will reduce to the scalar quantity:

$$N=(k-1)k(n/m)\frac{\Gamma(l^*-1,k(n/m))}{\Gamma(l^*-1)}$$

where

$$l^*=-\frac{\alpha}{log(\tau)},$$

$$\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}$$

is the gamma function, and

$$\Gamma(x,z)=\int_z^{\infty} t^{x-1}e^{-t}$$

is the incomplete Gamma function.

Appendix B. The regression results for Poisson degree distribution random networks

The data are obtained from the simulations of 10^6 randomly generated networks of poisson degree distributions for company nodes with $n=m=1000$. For any large-scale network with a lot of nodes, when the node degree is moderate, the network will become fully connected and the contagion will become very weak. This character is called phase change of network (Jackson, 2008). As an example, Caccioli et al. (2014) consider only networks with average company degree from 0 to 12 for networks with 10^4 company nodes. Thus, it is imperative not to choose a large degree of companies. In our simulation, d is randomly generated from $\{1,2,\cdots,20\}$. In the simulation process, there will be cases that the sum of a column of the investment matrix A is 0. After removing these data, there are 650993 observations left.

The regression results for models not including λ are shown in Table B. 1, and the results for models including λ are shown in Table B. 2. The statistical significant levels of all coefficients are less than 0.001, except for λ and $(SC/n^2)\lambda$ in model 4 of Table B. 2. All the regression coefficients in Table B. 1: Regression results of random networks with $n=m=1000$, are consistent with those of the low dimensional network (see Table 3 in the main text). But the regression coefficients in model 4 and model 5 are slightly different from those in Table 4, due to the additions of λ and $(SC/n^2)\lambda$. The results suggest, for a large-scale network, the introduction of variables d, τ, α in the model has a greater influence on the role of λ as compared to the low dimensional network.

Table B. 1: Regression results of random networks with $n = m = 1000$

Variable	model 1	model 2	model 3	model 4	model 5	model 6
Constant	-8.746	-27.852	-30.799	-32.553	-33.050	-36.747
	(0.027)	(0.155)	(0.175)	(0.179)	(0.183)	(0.238)
SC/n^2	0.082	22.702	26.194	25.851	27.032	27.708
	(0.011)	(0.171)	(0.194)	(0.459)	(0.212)	(0.216)
d	0.018	0.016	0.165	0.459	0.465	0.465
	(0.001)	(0.001)	(0.003)	(0.007)	(0.007)	(0.007)
τ	9.226	29.940	30.965	30.978	31.320	39.104
	(0.027)	(0.170)	(0.175)	(0.175)	(0.178)	(0.413)
α	0.468	2.263	2.333	2.337	2.351	3.324
	(0.005)	(0.021)	(0.021)	(0.021)	(0.021)	(0.032)
$(SC/n^2)\tau$		-24.542	-25.666	-25.637	-26.053	-26.876
		(0.189)	(0.195)	(0.195)	(0.198)	(0.205)
$(SC/n^2)\alpha$		-2.138	-2.215	-2.217	-2.234	-2.234
		(0.024)	(0.024)	(0.024)	(0.024)	(0.024)
$(SC/n^2)d$			-0.181	-0.158	-0.161	-0.161
			(0.004)	(0.004)	(0.004)	(0.004)
d^2				-0.011	-0.011	-0.011
				(0.000)	(0.000)	(0.000)
$(SC/n^2)^2$					-0.707	-0.687
					(0.048)	(0.048)
τ^2						-4.822
						(0.230)
α^2						-0.321
						(0.008)

Table B. 2: Regression results (λ included)

Variable	model 1	model 2	model 3	model 4	model 5
Constant	-1.248 (0.008)	-1.863 (0.010)	-9.191 (0.032)	-30.852 (0.205)	-37.340 (0.274)
SC/n^2	0.080 (0.010)	0.809 (0.012)	0.705 (0.013)	26.060 (0.227)	28.420 (0.262)
λ	0.015 (0.000)	0.080 (0.001)	0.038 (0.000)	0.000④ (0.001)	-0.003 (0.001)
$(SC/n^2)\lambda$		-0.074 (0.001)	-0.044 (0.001)	-0.001⑤ (0.001)	0.003 (0.001)
d			0.024 (0.001)	0.165 (0.003)	0.464 (0.007)
τ			9.058 (0.033)	31.019 (0.214)	39.742 (0.463)
α			0.452 (0.005)	2.334 (0.025)	3.399 (0.036)
$(SC/n^2)\tau$				-25.514 (0.236)	-27.600 (0.254)
$(SC/n^2)\alpha$				-2.185 (0.028)	-2.313 (0.029)
$(SC/n^2)d$				-0.180 (0.004)	-0.160 (0.004)
d^2					-0.011 (0.000)
$(SC/n^2)^2$					-0.751 (0.050)
τ^2					-4.810 (0.254)
α^2					-0.323 (0.008)

④ the statistical significance is 0.939

⑤ the statistical significance is 0.075

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