A new theory of seigniorage and optimal inflation

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A new theory of seigniorage and optimal inflation[1]

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Abstract

Central banks like the Bank of England or the Bundesbank have highlighted recently that the supply of currency is achieved not by means of printing and spending but by means of credit. This clarification raises further issues. This article addresses the issue of seigniorage and optimal inflation.

So far approaches to seigniorage and optimal inflation are still based on the assumption of a currency which is printed and spend by a central authority. From this perspective central banks' inflation targets and optimal inflation targets are at odds with those suggested by economic theory. The so-called Friedman-rule, the common core of optimal inflation theory, determines optimal inflation via the (opportunity) cost of producing currency. This basic approach is amended by "external effects", e.g. the impact of monetary non-neutrality or wage rigidities and so on. However, even under consideration of external effects there remains a significant gap between actual inflation targets and optimal rates as suggested by theory.

The supply by means of credit, however, involves "costs of production" which do not appear in Friedman's case: losses from borrower defaults. Incorporating expected losses into economic theory contributes significantly in aligning central banks' optima with economic theory and provides a new theory of seigniorage for a credit currency.

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1 A new theory of seigniorage and optimal inflation

Central banks' inflation targets are all strictly positive. The lowest are around 2%. Most central banks define a range somewhere between 2% and 8%. Only a few countries have (temporarily) departed from this range, aiming at higher rates, like Argentina, Belarus, Malawi, Ukraine in 2016 (see table 1).

Some economists, like Phelps (1973) and his followers, have supported "higher" inflation rates on the basis of utility based concepts. Krugman (1997) for instance remarks, "one of the dirty little secrets of economic analysis is that even though inflation is universally regarded as a terrible scourge, efforts to measure its costs come up with embarrassingly small numbers." Blanchard "derives" an optimal 4% target (Blanchard, Dell'Ariccia, and Mauro 2010), and Rogoff even proposed up to 6% (Rogoff 2013).

Table 1: Central banks' inflation targets

Country	Mean Target
Euro Area, Switzerland	<2.00%
Canada, Czech Republic, Israel, Japan, New Zealand, Peru, South Korea, Sweden, United Kingdom, USA, West African States	2.00%
Australia, Iceland, Norway, Romania, Poland, Thailand	2.50%
Samoa, China	3.00%
Albania, Armenia, Chile, Colombia, Costa Rica, Dominican Republic, Guatemala, Hungary, Indonesia, Mexico, Samoa, Serbia, Phillipines, Russia	4.00%
Paraguay, Botswana, Brazil, South Africa	4,50%
Georgia, Kenia, Moldova, Uganda, Uruguay, Turkey	5,00%
Jamaica, Mozambique, Azerbaijan	5,50%
Bangladesh, Pakistan	6,00%
Kazakhstan, Kyrgyzstan, Mongolia, Vietnam, Zambia	7.00%
Nigeria	7,50%
Ghana, India	8,00%
Belarus, Ukraine	12.00%
Argentina, Malawi	14.20%
Reference: Bundesbank Data, Siklos 2008 and Central Bank News 2016	

More theoretically oriented approaches, however, come up with significantly lower numbers. Admittedly, there is no generally accepted approach to determine optimal inflation. But there is a common core. In general analyses start from the Friedman-rule and "add" further external effects, i.e. the impact of monetary non-neutrality, incomplete taxation, or wage rigidities. The difficulty for monetary policy posed by a lower bound of zero on the nominal interest rate has received some special attention in the latest publications, which identify a trade-off between the inflation objective and macroeconomic stability (see Coenen, Orphanides, and Wieland 2004 or Reifschneider and Williams 2000).

Stephanie Schmitt-Grohe's and Martin Uribe's contribution to the third volume of the *Handbook of Monetary Economics* (Schmitt-Grohe and Uribe 2011) provides a good overview. They determine the cost optimal inflation according to the Friedman rule and then add the different external effects. For all of them they derive an optimal inflation close to or below zero. Hence they conclude that from a theoretical point of view the gap between the actual inflation targets of central banks and the theoretically optimal inflation target cannot be explained even if various external effects are considered (see Schmitt-Grohe and Uribe 2011, p. 715).

Table 2: Optimal inflation as suggested by theory

Assumption	Optimal rate of inflation
Friedman rule	-î
+ distortionary taxes	<0%
+ untaxed income	<0%
+ foreign demand for domestic currency	<2%
+ price stickiness	≤0%
+ size and Elasticity of Money Demand	<0%
+ zero lower bound	<-0,4%
+ downward nominal rigidities	<0,35%
+ sticky nonquality-adjusted prices	=0%
+ sticky quality-adjusted prices	=k%
Reference: Schmitt-Grohe and Uribe 2011	

One of the few exceptions is a high foreign demand for domestic currency. The incentive to tax foreign holdings of domestic currency explains inflation targets up to 2%, an argument which applies to countries whose currencies circulate widely outside of their borders. The United States or the Euro area would qualify as such examples. However, these countries are those with the lowest inflation targets (see Schmitt-Grohe and Uribe 2010, p. 32). In other words, the actual target rates of central banks are considerably above the rate suggested by economic theory.

> "The two leading sources of monetary nonneutrality in modern models of the monetary transmission mechanism—the demand for money and sluggish price adjustment—jointly predict optimal inflation targets of at most zero percent per year. Additional reasons frequently put forward in explaining the desirability of inflation targets of the magnitude observed in the real world—including incomplete taxation, the zero lower bound on nominal interest rates, downward rigidity in nominal wages, and a quality bias in measured inflation—are shown to deliver optimal rates of inflation insignificantly above zero" (Schmitt-Grohe and Uribe 2010, p. 65).

As a result there is a gap between actual inflation targets as pursued by central banks optimal rate of inflation ($\Delta ZERO - TARGET = \pi^{*}_{CB}$) and theoretical estimates of optimal inflation targets. This gap may be estimated either by the difference between inflation targets and negative real interest rates ($\Delta R - TARGET = \hat{\imath}_B + \pi^*_{CB}$) or by the difference between inflation targets and zero (π^*_{CB}). The difference between target rates and zero are directly observable as stated in table (1). The difficulty with real interest rates is their determination. Combining World Bank Data on real interest rates with the IMF World Economic Outlook yields the following figure (see Figure 1).

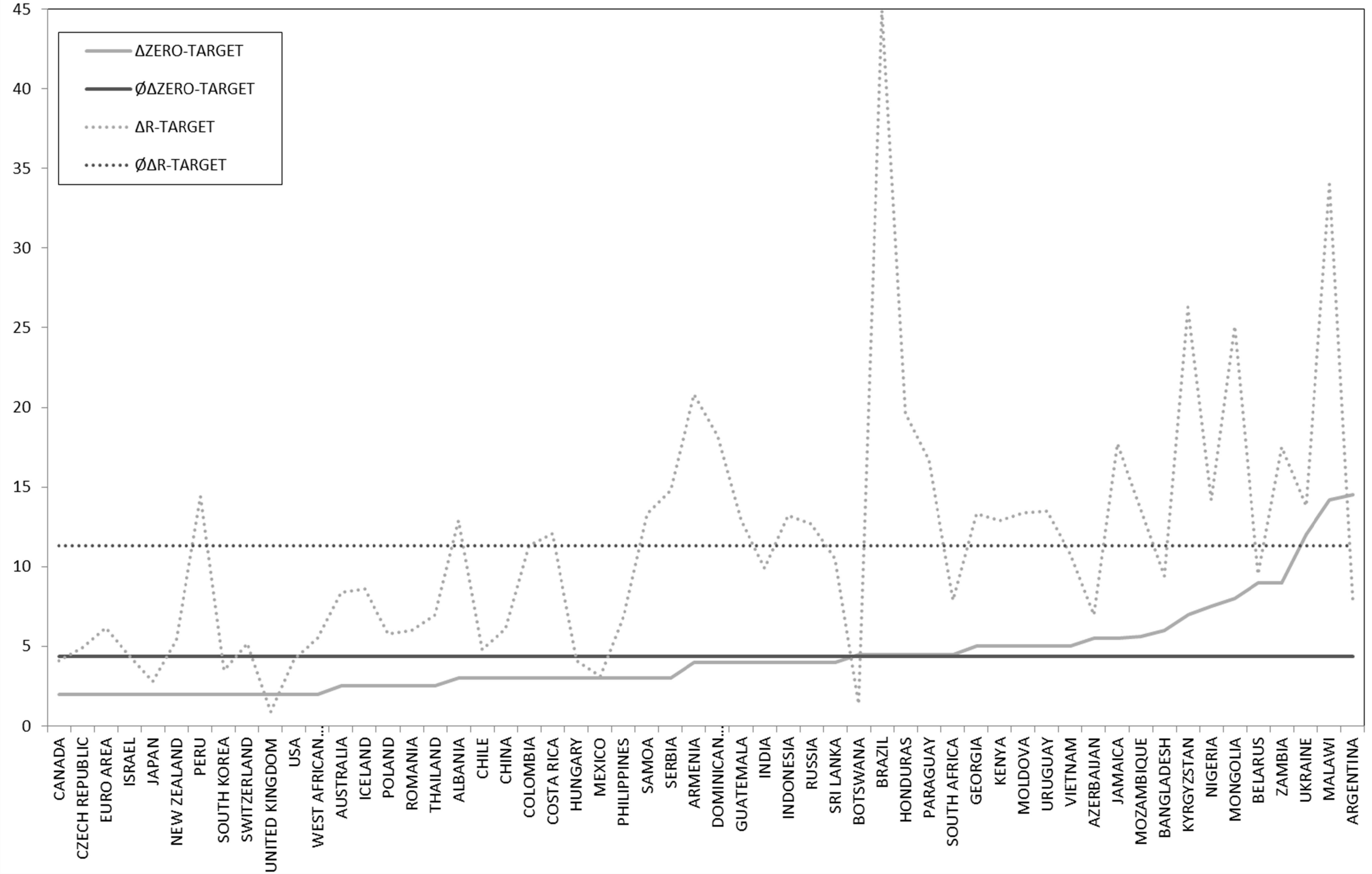
The average difference is about 4,4 percentage points with respect to zero and about 11(!) percentage points with respect to the real interest rate. Hence inflation targets are on average 4,4 or 11 percentage points above optimal rates as suggested by the Friedman rule for a fiat currency. The actual nominal inflation target are quite heterogeneous and range from close but below 2% (European Central Bank and Swiss National Bank) to values above 10% (Central Bank of Argentina, the Reserve Bank of Malawi or the National Bank of Ukraine).

As shown above, the perceived gap between targeted inflation rates and theoretical optima exists irrespective of how it is calculated. Nominal targets diverge significantly from zero or real interest rates. While central bankers are known to apply economic theory and to rely upon it they have reaffirmed their inflation targets. Leading central bankers like Weidman or Yellen reaffirmed their targets (see Weidmann 2015 and Yellen 2015). Nonetheless central bankers take the theoretical results serious. Even though a theoretical estimate of an optimal inflation target is regarded as “a rough approximation”, it “likewise seems crucial to making good policy in the next few years” (see Bernanke 2004, p. 166). If this is supposed to be consistent central bankers have to adjust their target inflation rates or economists would have to adjust their theoretical results. In this essay I argue in favour of the latter. I argue that targeted rates can be aligned with theory by reassessing the underlying Friedman rule.

2 The Friedman-rule in a very simple model

The common core of modern approaches to determine the optimal inflation is the Friedman-rule. In earlier works Friedman had argued for a constant rate of currency growth. Friedman argued that anticipated inflation is safer to unanticipated inflation (due to the risk of an upward inflationary pressure in the latter). A government should make its monetary policy transparent and stick to it. A fixed-growth rule would assist the formation of a constant level of prices, which was regarded as a worthwhile goal in itself. This view was revised in his essay on the *Optimum Quantity of Money*. Here Friedman challenged the goal of a constant level of prices and embeds the problem in a theoretical framework (see Friedman 1969, pp. 47-48). The supply of commodities is optimal if the price paid in the market equals the (marginal) cost of production. Friedman's argument is that what is true for commodities must be true for the supply of currency.

Figure 1: Difference between inflation targets and zero (solid line) and real interest rates (dotted line) in percentage points in 2016



The orientation on cost of production can be justified, as national legal tender fulfils the properties of private goods. Currency is rivalrous, i.e. ownership of one prevents ownership of another, and it is excludable, i.e. owners can exercise their property rights. The government is asked to maintain a functioning currency regime, a common unit of account, and the currency's purchasing ability, but there is no reason why a currency units' price should deviate from its cost of production. With respect to optimal seigniorage and optimal inflation Friedman concluded – in a long tradition – that seigniorage is optimal if it is cost-covering. A view Friedman retained in later publications (see Issing 2011, p. 258).

For a commodity currency this approach is straight forward. The purchasing power of the issued coins must be equal to the price of the contained currency-commodity, e.g. gold, plus charging the additional cost for minting. The government must offer to coin gold deliveries freely, charging a fee for minting. The difference between the cost of production of a coin and those of the gold contained in it would then, in the end, be determined solely by the cost of minting. Fiscal seigniorage accordingly is zero. For negligible costs of "minting" there should be no difference between the cost for buying and selling the currency-commodity.

Friedman assumes a fiat currency. For him this means a paper based currency supplied by discrete choices on the part of government. The government uses the printing press to finance its spending. He assumes that the costs of production of such fiat currency units are negligible. The price of the currency should by the same logic be zero. If this were interpreted in the way it is interpreted for a commodity currency the government would have to offer to print currency from paper deliveries until the value of the currency approaches zero; a questionable proposal.

For negligible cost of production, purchasing power cannot be adapted to the former. Friedman suggests interpreting the money rate of interest (i_B) as the currency's "price", drawing a line from the opportunity cost to the price. Consequently he demanded a nominal interest rate of zero.

> "Our final rule for the optimum quantity of money is that it will be attained by a rate of price deflation that makes the nominal rate of interest equal to zero" (Friedman 1969, p. 34).

The argument which can be constructed is the following: If the cost of production of a commodity is zero, then its price should be zero. A price of zero should correspond to a rental price of zero. If the price is zero why should anyone pay for renting the commodity (Friedman explicitly compares money to other goods in his essay). Even though the purchasing power of money is not zero he demands that the supply of currency be adapted to its demand, such that the money rate of interest for a fiat currency is zero.[2] This rate is then judged as the optimal rate of interest (i_B^{F*})[3]. The optimal

2 I must say that I find this logic far from convincing. While I agree that cost of production determine prices in the long run, it makes no sense to draw conclusions from cost of production which are in absolute contrast to prices. A positive price has to correspond to a positive rental price. It is furthermore shown that a more straight forward

rate of inflation (π_B^{F*}) follows directly. For a zero interest rate it must – according to the simple Fisher equation – be equal to the negative "real" interest rate ($\hat{\imath}_B$):

$$i_B^{F*} = 0 = \pi_B^{F*} + \hat{\imath}_B \tag{1}$$
$$\text{or} \quad \pi_B^{F*} = -\hat{\imath}_B$$

This is the so called Friedman rule. It is usually not translated into a currency growth rule in the literature. Yet there is no difficulty in doing so. According to the quantity equation, currency growth equals the change in liquidity demand (g_k), plus inflation, plus the growth in trade (g_q). If inflation is equal to the negative real interest rate one obtains:

$$g_B^{F*} = -\hat{\imath}_B + g_k + g_q \tag{2}$$

Using the Cambridge Equation, there is a simple way to reformulate this result. The Cambridge Equation states that the rate of economic growth (g_Y) (which is therefore assumed to be identical to the growth in trade (g_q)[4] equals the real rate of interest ($\hat{\imath}_B$) times the rate of savings out of profits (s_P) for constant capacity utilization (g_u). For a saving rate from profits of one ($s_P = 1$) and constant capacity utilization ($g_u = 0$) the growth rate equals the real rate of interest, the so-called golden rule:[5]

$$\hat{\imath}_B = g_k \; for \; s_P = 1 \; and \; g_q = 0. \tag{3}$$

If the real rate of interest corresponds to the rate of economic growth (g_Y) then the optimal currency growth rate is, as suggested by Friedman, given by changes in liquidity preference:

$$g_B^{F*} = g_k, \tag{4}$$
$$\text{where} \quad \pi_B^{F*} = -\hat{\imath}_B = -g_Y.$$

Thus the optimal real currency growth is zero except for changes in liquidity demand; thus for changes in the currency demand function. Whenever the demand for currency rises it pushes the interest rate up, and hence the government is supposed to issue more currency to reduce the nominal

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interpretation can be given. If cost of production are positive (no matter how insignificant) a direct cost equal price logic can be applied (see Reich 2017).

3 "B" refers to the underlying legal tender currency or base money and the letter "F" refers to assumed supply mechanism of a fiat currency.

4 In many presentations it is assumed that the volume of trade is linear with respect to the national income (see Gebauer 2004, pp. 154-155 or Issing 2011, pp. 31-32). Replacing the quantity of transactions with National Income is rather problematic. The quantity of transactions might be different from the national income, and national income is usually a weighted measure, thus a price weighted index, of the total volume of transactions. Transforming the vector **q** into a scalar is highly problematic since it assumes the neutrality of money. It should therefore always be kept in mind that in fact it is the volume of transactions that is meant, and that National Income is only a rough approximation.

5 Instead of assuming constant capacity utilization, a variant of the Cambridge Equation could be used, where the growth rate equals the real rate of interest times the rate of savings from profits plus the change in capacity utilization. It states that growth is determined by the retained earnings from profits plus changes in capacity utilization. If the golden rule does not apply, hence if capacity utilization changes (i.e. $g_u \neq 0$), for example because a monetary expansion induces growth by a rise in capacity utilization, Friedman's result will not hold. A growth in currency supply which induces non-inflationary economic growth by rising capacity utilization leaves the real rate of interest unaffected. Consequently, the optimal currency growth rate is higher for economies below full employment.

rate of interest. For a stable currency demand function the currency growth rate should be zero. The optimal rate of inflation is the negative real interest rate or – following the Cambridge equation – the negative growth rate of the economy.

A simple model suffices to illustrate the practical implications of the Friedman rule. Following the standard approach it is assumed that the demand for currency B^D can be expressed as:

$$B^D = k_B^Q(\pi_B^F)\boldsymbol{p}_B\,\boldsymbol{Y}. \tag{5}$$

The demand for currency is often determined by empirically estimated demand functions. Theoretically the demand depends according to the quantity equation on liquidity demand (k_B^Q) times the level of transactions, output or income ($\boldsymbol{p}_B\mathbf{Y}$).[6] In real terms, the demand for real currency, i.e. divided by prices, one gets:

$$\boldsymbol{p}_B^{-1}B^D = k_B^Q(\pi_B^F)\,\boldsymbol{Y}. \tag{1}$$

The supply of currency (B^S) is determined by the supply of fiat currency (B_F^S). The supply of fiat currency depends on the discrete choice of the central government, as the government chooses to print and spend a certain amount of currency (B_F^G). The real currency supply hence depends on the amount of government purchases financed by means of the printing press ($\boldsymbol{p}_B\boldsymbol{A}_G^M$). In real terms the supply of currency ($\hat{B}$=$\boldsymbol{p}_B^{-1}B$) is given as:

$$\hat{B}^S = \hat{B}_F^S = \hat{B}_F^G = \boldsymbol{A}_G^M. \tag{2}$$

If the central government follows the Friedman rule it needs to adapt its currency supply to the demand for currency such that the rate of inflation is equal to the negative real interest rate, or if that is equivalent equal to then negative growth rate of the economy. As a side note, the revenue from seigniorage can be positive even though the optimal level of inflation might be negative (see Reich 2017).

$$\hat{B}_F^{S*} = \boldsymbol{A}_G^{M*} = k_B^Q(\pi_B^{F*})\boldsymbol{Y}. \tag{3}$$

The amount of real resources finance by means of the printing press is optimal if it is in proportion to the gross domestic product, such that the corresponding rate of inflation is equal to the negative growth rate of the gross domestic product.

The following figure (2) pictures a currency supply of this sort which adapts to currency demand. If currency demand changes (dotted curve) the supply has to be adapted (dotted curve), such that the interest rate does not rise permanently.

The conflict between theory and central banks' practice which was shown to exist in the first section is due to this analysis. It is the peculiar perspective introduced by Friedman and the Friedman-rule which builds the theoretical basis for this perception.

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6 Referring to real currency demand is tricky. $\boldsymbol{p}_B^{-1}$ is usually referred to as the level of prices. That means it is the price of currency p_B in terms of the vector of all prices $\mathbf{p}$, or inversely, the level of prices in terms of currency. Except for a one good world, the level of prices is therefore a vector of prices or a certain aggregated index.

There are different ways to resolve this perceived gap. One way is to stick to the Friedman rule and to justify central banks' actual inflation targets is by assuming higher costs of production. The higher these cost the lower the perceived gap. And indeed, central bankers have stressed the costs of supplying and maintaining the currency: “resource costs, associated functions, competitiveness, inventory costs, volume of regular (liquidity) traders, extent of exposure to informed insiders and risk of being caught by an unobservable shift in underlying value” (Goodhart 1989, p. 10).

Figure 2: Optimal fiat currency supply

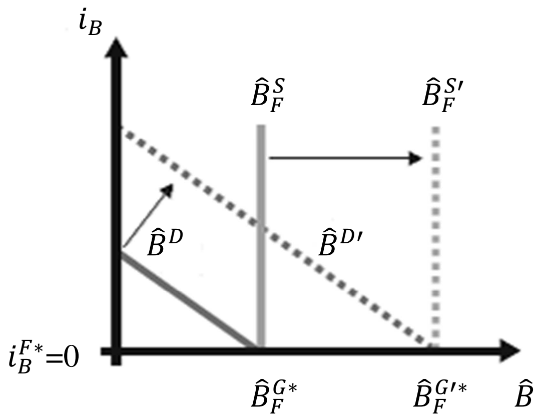


Illustration based on this research

There is however a second argument which has so far not been developed which shall be put forward here. Friedman assumes a fiat currency which is issued by printing and spending on behalf of a central government. Hence he assumes that the currency is printed and distributed by the famous parable of a helicopter. Leading central banks have however stressed more recently that the supply of currency is not organized in such a way by the treasury. Instead the Bank of England (2014) or Bundesbank (2017) have stressed that that modern currency supply is organized by crediting private banks’ accounts held at the central bank. This endogenous theory of a credit currency which is for a considerable amount of time applied in several macroeconomic models is still not applied if it comes to seigniorage and the theory of optimal inflation. This is a path chosen in this essay. Friedman’s approach is amended not regarding the assumptions on the unit cost of producing paper currency or coin. Instead his approach is applied to a currency which is issued by means of credit by a central bank. It will be shown in the next section that taking the actual currency supply mechanism into account changes the optimal inflation as suggested by economic theory in a way which renders the theoretical result much closer to economic reality.

3 Optimal inflation and modern central banking

In a credit currency framework the responsible authority, usually a public central bank, determines interest rates at which the currency can be borrowed, and maintains these rates by accommodating the corresponding demand. An increasing number of economists have put forward analyses assuming a refinancing central bank. There are early contributors, like Wicksell (1898 or 1922), Keynes in his Treatise (1930 [1971]), Hahn (1930), Hawtrey (1962), Davidson (1972), Black (1987), or Moore (1991). Recently many more contributions around this topic have been made. A (not always homogeneous) shift in assumptions, or simply the assumption of a credit currency can be found in the work of New Keynesians (Woodford 2003), Post-Keynesians (Arestis and Sawyer 2008, Lavoie and Godley 2012), other economists (see Binswanger 2013, Gebauer 2004, Nautz 2000), and Central Banks (Bank of Canada 2010, Bundesbank 2010, European Central Bank 2011, Bank of England 2014, and Bundesbank 2017). Surprisingly, these authors either omit the issue of seigniorage and so provide no analysis of it, or revert to the standard assumption of a fiat currency. Hence, regarding optimal seigniorage and optimal inflation there is no shift in perception. The gap between economic theory and central bank policy – it will be argued – is due to the fact that this theoretical shift has not yet been completely digested by economists.

This is surprising in so far as empirical research of the money supply and its revenue already suggested that the actual monetary system has progressed to a credit currency regime. The most valuable empirical contribution to the changed institutional framework was provided during the introductory phase of the Euro. A small group of German authors resurrected the issue of seigniorage in connection with the creation of the European Monetary Union (see Gros 1989, 1990, 1996, 1998, Klopfleisch 2000, Lange 1995, p. 25, Neumann 1992 and 1996, Rosl 2002, pp. 37-42, Sinn and Feist 2000, Wesche and Weidmann 1995, p. 27). Trying to estimate the gains and losses from seigniorage for the individual national states these authors note that the traditional approach of a fiat currency is inappropriate to the modern institutional framework. Instead of dropping the assumption of a fiat currency right away they assume that seigniorage is earned as a mix of the revenue from printing and spending currency and in form of printing and spending it on private interest bearing assets by a central bank, thus interest payments to the central bank. The institutional perspective and the issue of the optimal rate of inflation is of minor interest to the pursued empirical estimations by the above authors. As these authors therefore do not question the institutional framework the approach was initially labelled *extended* monetary seigniorage (Neumann 1992) and later *total* seigniorage (Neumann 1996, p. 107). From an institutional perspective, the recognition of the seigniorage revenue from interest payments to the central bank is however the first important step into the direction of acknowledging an institutional change.

In a credit currency regime seigniorage stems from interest payments made by those demanding credit currency from the monetary authority. The monetary authorities supplying the currency receive the monetary seigniorage and, after allowing for certain costs, transfer the net revenue, the fiscal seigniorage, to the government (the treasury).

For a credit currency, the supply of currency depends on the interest rates which the central bank both wishes to sustain, and is able to. Usually a central bank distinguishes a main (i_B^{CB}) and a marginal refinancing rate (i_B^{CB-}) and a deposit facility (i_B^{CB+}) for balances beyond the minimum reserves.

The marginal refinancing rate is usually a mark-up on the main refinancing rate. At this rate central banks provide in principle unlimited amounts of currency (against security). For simplicity, the main and marginal refinancing rate may be referred to as “the central bank rate”. Positive balances on central banks’ accounts are usually paid a deposit rate close to the main refinancing rate for balances due to minimum reserve requirements. The central bank deposit rate constitutes the lower and the central bank lending rate the upper bound of the currency or money rate of interest (i_B). Depending on central bank policy and alternative currency supplies, the currency rate of interest may vary between these rates. If there is no alternative currency supply the currency rate of interest will be close to the central bank's main refinancing rate. Hence, by controlling the central bank rate and the deposit rate the central bank may more or less precisely determine the currency rate of interest. Put the other way around and slightly simplified, the supply of credit currency follows endogenously from the currency rate target pursued by the central bank (Bundesbank 2017).

With respect to seigniorage and optimal inflation the main difference between a printing-and-spending fiat currency on the one hand and a lending-against-interest credit currency is the risk of default. Lending the currency and demanding interest for this loan involves the risk that the borrower defaults during this time. The difference is obvious with respect to seigniorage. For a fiat currency the seigniorage is completely and finally realized in the moment of issuance. Paying by means of printing and spending makes the government realize the seigniorage revenue completely in the moment of issuance. Lending a credit currency means to realize seigniorage partially, i.e. over time as interest payments while lending the whole amount. Hence, while the seigniorage revenue from fiat currency cannot be negative this is possible for a credit currency (see Reich 2017).

In other words, what is usually overlooked is that the supply of credit currency involves a default risk, the risk of bankruptcy. In another context Stiglitz and Greenwald highlight that the probability of bankruptcy is an often omitted variable. Reflecting on macroeconomics as taught before the publication of their book they note:

> “There was a single, crucial variable that was omitted from the analysis: the probability of

bankruptcy, the variable which we have argued, is at the center of all monetary analysis. If everyone always repaid their loans, then there would be little role for financial institutions. Credit would be a trivial matter. It was understandable, perhaps, for economists who had been trained in macro-economics a quarter of a century ago to have failed to pay attention to that variable. Even today, the term 'bankruptcy' does not appear in the indices of most macro-economics textbooks. Yet, for policy makers the mistake is unforgivable" (Stiglitz and Greenwald 2003, p. 265).

This applies also to the theory of seigniorage (see Reich 2017) and optimal inflation as will be shown.

The lending operations of every bank are connected to risks, in particular default risk. That is the risk that a borrower goes bankrupt. Whatever the quality of the securities, every private bank demands interest, partly to cover the risks incurred, and partly to cover other costs. The same is true for central banks and central bank rates. Central banks face the risk that their debtors will default, and hence they may be confronted with losses from their credit currency supply (Φ_B^K). Indeed, central banks demand good security. However, only the loss given default, not the risk of default, can be reduced by the central bank demanding good security prior to issuing currency. As a result, the risk of default and the loss given default, that is expected losses, have to be taken into account in the calculation of optimal seigniorage and optimal inflation.

Friedman argued that the price of currency should match its cost of production and furthermore he assumed that the costs of production of units of currency are negligible. Hence the Friedman rule cannot be transferred to a credit currency straight forward but needs to be reinterpreted. One may, for reasons of comparison, keep the assumption of negligible costs of production of the actual currency. However, due to the default risk the cost of production are not zero. Otherwise one misses an important category of costs arising for a credit currency: credit risk and the losses from a default, i.e. expected losses.[7]

If the expected loss on a unit of currency is constant over all issued currency units, then the first derivative of the "risk function", i.e. marginal risk, is a constant:

$$\frac{\mathrm{d}\Phi_B^K(B_K)}{dB_K} = \Phi_B^{K\prime}(B_K) = \varphi_{B_K}. \tag{4}$$

This assumes that expected losses do not depend on the size of the stock of issued currency. In a simplified example, where the central bank lends a certain sum to the public with an expected loss of 1 % per unit of currency per annum of credit granted, then the central bank would be required to demand a central bank rate of at least 1 %. If the central bank demands a lower rate it will, on average, run a deficit which the government has to cover by other means (for instance, through additional taxes).

The optimal seigniorage has to be such that it permits the cost of production, maintenance cost, *and*

7 Expected losses are the product of the probability of default and the loss given default.

expected losses to be covered. The same applies to the optimal bank rate, and hence the established currency rate of interest. As a result, equation (1) becomes:

$$i_B^{K*} = \varphi_{B_K} \tag{5}$$

Following the simple Fisher equation the optimal rate of inflation hence becomes:

$$\varphi_{B_K} = \pi_B^{K*} + \hat{\imath}_B \tag{6}$$

$$\text{or} \quad \pi_B^{K*} = \varphi_{B_K} - \hat{\imath}_B.$$

As a result, Friedman's claim of a zero interest rate does not apply for a credit currency, even if cost of production and maintenance costs are neglected. The central bank rate charged has to cover expected losses. The cost-optimal seigniorage rule has therefore to be adapted for a credit currency even if other costs are – as suggested by Friedman – neglected. For the credit currency optimality requires a central bank rate (i_B^{K*}) to exceed the optimal rate for a fiat currency as suggested by the Friedman rule (i_B^{F*}) by a markup to cover expected losses:

$$i_B^{K*} = i_B^{F*} + \varphi_{B_K}. \tag{7}$$

Similarly the optimal rate of inflation for a credit currency exceeds the optimal inflation for a fiat currency regime by the allowance for expected losses.

$$\pi_B^{K*} = \pi_B^{F*} + \varphi_{B_K}. \tag{8}$$

Hence, even if all other costs are neglected a positive central bank rate target can be required for a credit currency if expected losses are positive. Depending on expected losses and the real interest rate, a positive inflation target can be justified according to equation (10) if expected losses exceed the real interest rate or according the Cambridge equation the growth rate. In general it may be said that, with a credit currency, the targeted rate of inflation and the currency rate of interest have to be higher than in a fiat currency in order to cover expected losses from the supply of credit currency.

The simple model for a fiat currency of section (2) can easily be adapted to a credit currency. Currency demand is assumed to remain the same, as given in equation (5). The supply of currency is however not exogenously determined by the discrete choice of the government but by the actual demand at a given currency rate of interest. The supply is therefore expressed as a function depending on the interest rate.

The central bank determines its targeted currency rate of interest and supplies the currency necessary to defend this rate. Assuming that it is capable of achieving this rate the currency rate of interest enters directly into the supply function.

In the simplest case the supply function is a horizontal, depending on the currency rate of interest:

$$B^S = B_K^S(i_B). \tag{9}$$

In a slightly more complex version the deposit and marginal lending facility can be added, still assuming that the central bank's main refinancing rate is equal to currency rate of interest. This yields a “z” shaped supply curve.

$$B^S = B_K^S\left(i_B^{\mathrm{CB}+}, i_B^{CB}, i_B^{\mathrm{CB}-}\right) \tag{10}$$

If the central bank aims at the optimal rate of inflation as determined in section (3) it sets the main, deposit and marginal lending rates such that the currency rate of interest is equal to the percentage of expected losses as stated in equation (11) and such that the rate of inflation is equal to the difference between expected losses and the real interest rate. This applies in both cases, the very simple (13) and the simple supply function (14). The optimal supply is therefore:

$$B^{S*} = B_K^{S*}(i_B^{K*}) = k_B^Q(\pi_B^{K*})\, Y. \tag{11}$$

The following figure (3) pictures an endogenous credit currency supply. The dotted line shows a change in the supply due to a rise in expected losses per unit of currency.

Figure 3: Optimal credit currency supply

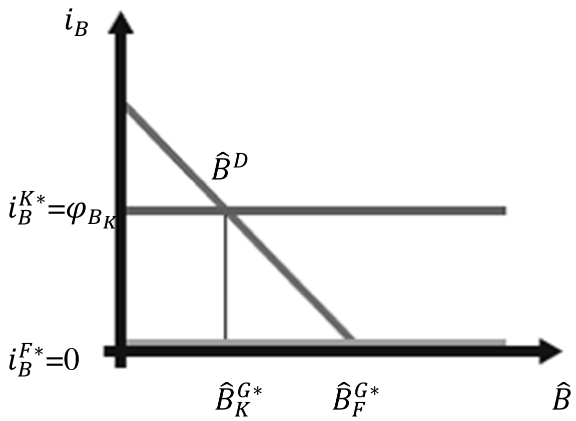


Illustration based on this research

The optimal currency rate of interest and the optimal rate of inflation depend on expected losses per unit of currency lent. Depending on estimates of these losses the optimal currency rate and the optimal inflation rate for a credit currency exceed the rates for a fiat currency. Any judgement on central banks’ targets hence crucially depends on the underlying assumptions regarding the currency supply. The perceived gap between central banks inflation targets and optimal inflation for a fiat currency are due to the fact that currency is supplied by means of credit. Hence the assumption of a fiat currency is not applicable and as a result the respective interest and inflation optima for a fiat currency do not apply to the supply of credit currency of modern central banks. The perceived gap is therefore rather due to an outstanding update on economic assumptions regarding the supply of currency. The gap should disappear or decrease substantially if expected losses are taken into account. The next section corrects actual inflation targets by a rough estimate of expected losses.

4 Loss corrected inflation targets

The purpose of this section is to correct the perceived difference between actual inflation targets and optimal inflation target. To repeat, according to the Friedman rule inflation should be equal to the negative real interest rate for a fiat currency or at least be zero if further effects are considered. It was shown above that inflation targets are on average 4,4 percentage points above an optimum of zero or 7 percentage points above real interest rates.

Section 1 demonstrated that expected losses are not part of this consideration. And it was shown in section 3 that expected losses should be considered for currency which is supplied by means of granting credit. The purpose of this section is to provide an estimate of expected losses, to revise theoretical optima, and to compare these to central banks' inflation targets.

The approach taken here is to estimate expected losses for central banks by the respective countries' government bond risk. Central banks accept national government bonds as security or deal in government securities. Hence their operations depend either way on the solvency of their respective central government. And therefore the expected loss of a central bank will be estimated by the expected loss on respective government bonds. This, it is argued, applies in both cases where central banks issue currency by means of buying such securities or by means of granting credit to private companies (i.e. banks) against government bonds as security.

This raises the problem of estimating the expected loss from government bonds. For purposes of simplicity the expected losses on government bonds are inferred from the average country rating of Moody's, S&P, and Fitch. Average country ratings are then translated into Basel weights. Ratings are classified in groups of different risk weights according to Basel rules. These risk weights here are used to roughly estimate expected losses. Countries with an average A+ to A- rating are attributed a 20% weight. Against such collateral 20%*8% of equity have to be held. This is interpreted such that on average 20%*8% might be lost by holding this asset; hence an expected loss of 1,6%. This yields the respective expected loss per unit of currency (φ) lent by a central bank (see Table 3).

This is admittedly a very rough estimate of expected losses. If it is taken into account the former results change. Correcting inflation targets yields a considerably lower gap as usually perceived. This applies for the zero target as well as for the real interest rate target.

Inflation targets (π^*_{CB}) are compared first to the zero target and a zero target corrected for expected losses. The difference between targets and optima is given by the nominal target itself and the difference to the corrected target by the difference between inflation targets ($\Delta ZERO - TARGET = \pi^*_{CB}$)and expected losses ($\Delta Phi - TARGET = \pi^*_{CB} - \varphi_{B_K}$). The difference between targets and optima with respect to real interest rates is given by the difference between negative real interest rates and central bank targets ($\Delta\mathrm{R}-\mathrm{TARGET}=\hat{\imath}_B+\pi^*_{CB}$) and for corrected optima by the difference between negative real interest rates, inflation targets and expected losses ($\Delta R-Phi-TARGET=\hat{\imath}_B+\pi^*_{CB}-\varphi_{B_K}$).

Table 3: Country rating, respective risk weight and loss estimate

Country Rating	Basel risk weight	φ(B)
AAA to AA-	0%	0,00%
A+ to A-	20%	1,60%
BBB+ to BBB-	50%	4,00%
BB+ to B-	100%	8,00%
below B-	150%	12,00%
without rating	100%	8,00%
Reference: http://countryeconomy.com/ratings		

Taking the risk of government bonds into account changes the result dramatically. On average the target rate minus expected losses is negative. Hence, deducting expected losses inflation due not ensure a positive return on the supply of currency. The average difference between optima and target rates changes from 4,4 percentage points to (on average) minus 1,2 percentage points. Individual rates as compared to expected losses are much more heterogeneous. Nominal target rates varied between 2% and 14,2%. Correcting for expected losses yields corrected targets between -9% and 6,5%. To avoid a netting of positive and negative deviations the average of absolute differences can be estimated. In this case the average difference changes from former 4,4 percentage points to 2,8 percentage points.

The picture is different if real interest rates are taken as optima. Central bank targets differed on average 11,3 percentage points from real interest rates. Taking expected losses into account yields corrected optima from which inflation targets differ on average by only 5,7 percentage points. Furthermore there is almost no undershooting. The average of absolute differences is 6,3 percentage points. That means the average difference is lower due to individually lower differences between corrected optima and target rates, not because of a netting between positive and negative deviations from optima.

In other words, taking expected losses into account shrinks the perceived gap between central banks' targeted inflation rates and those rates suggested by economic theory. In comparison to the zero target the average difference shrinks to 63%. In comparison to the real interest rate target the average difference shrinks to 55,7%.

Figure 2: Difference between inflation targets and zero (solid line) and corrected optima (dotted line) in 2016

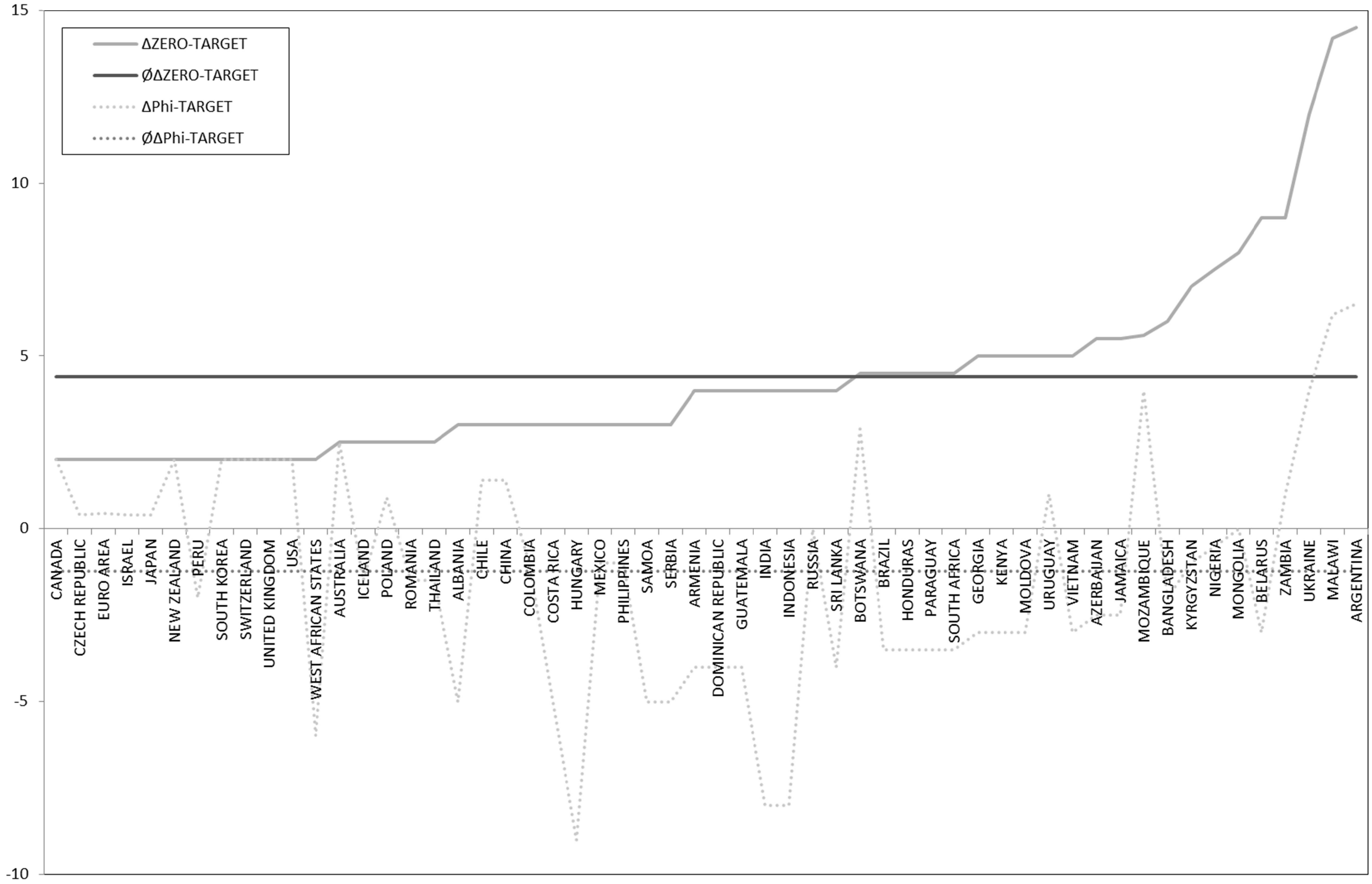
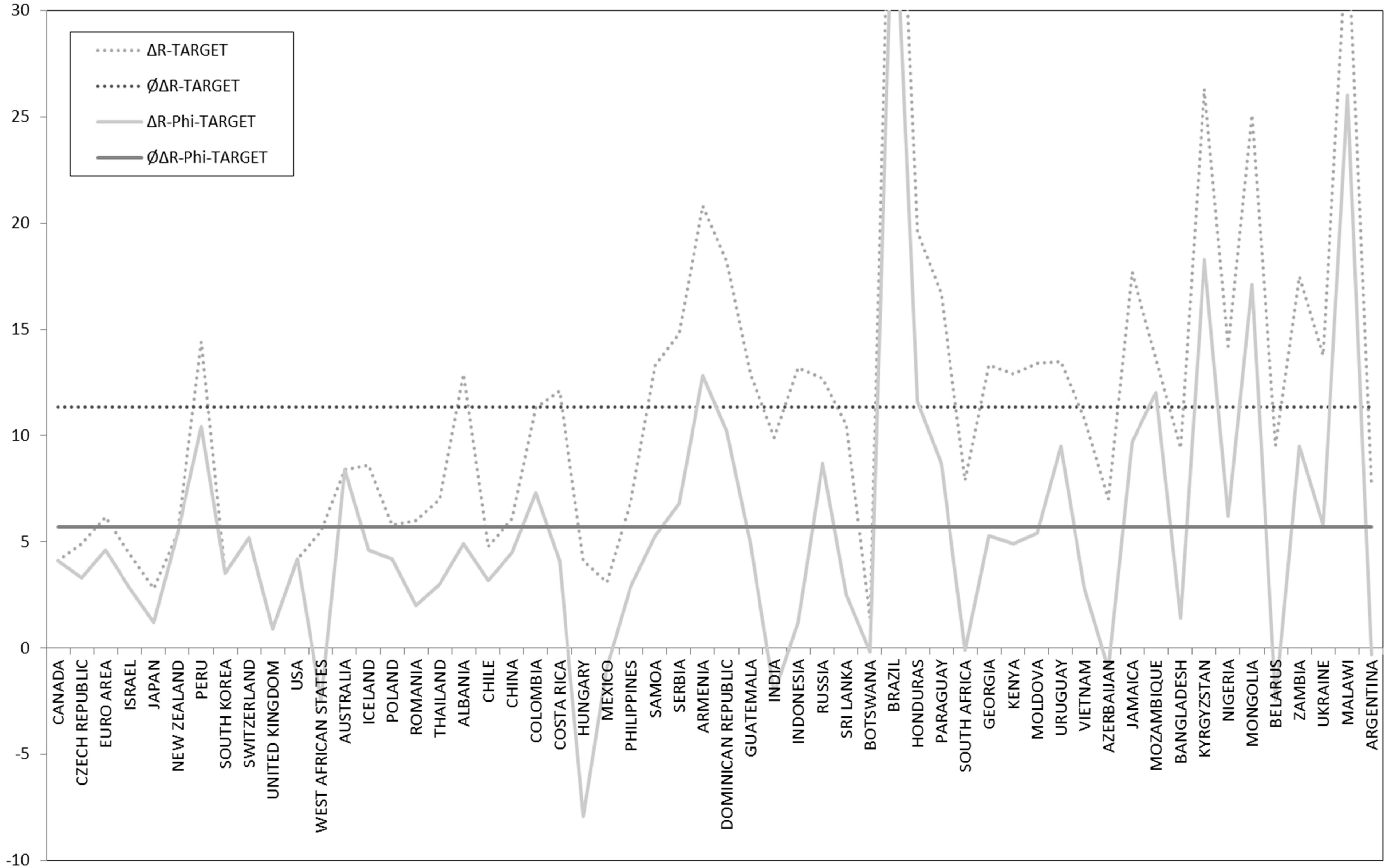


Figure 3: Difference between inflation targets and real interest rates (solid line) and corrected optima (dotted line) in 2016



5 Conclusion

Economists criticize central banks for their inflation targets. In respect to economic theory the perused targets are judged as to high. Interpreting high inflation targets with high seigniorage income this suggests that central banks support the treasury and the government's budget. It was argued however that the economic theory which is used for this comparison assumes a specific currency supply mechanism. It assumes that fiat currency is issued by printing and spending on behalf of the treasury. In such a supply mechanism there is no risk. Seigniorage is realized in the moment of issuance by acquiring real resources.

In contrast to this assumption the modern currency supply is largely organized by means of lending currency. Currency is lent to private entities, in particular banks. Banks receive the full amount of currency, but the central bank does not receive equivalent real resources. Instead it realizes its seigniorage over time by means of interest payments. Thus, while the bank may spend the full amount borrowed right away the central bank needs to await its receipts from interest payments. In the worst case the central bank acquire losses if banks cannot repay their loans. While these loans are usually secured by means of government bonds, central banks might still incur losses if a bank defaults and if the government defaults on its security. Hence there is a small risk, identified with the default risk on government bonds, that a central bank may incur losses from its currency supply. If these expected losses are taken into account the optimal, that is cost covering, rate of inflation is higher than suggested by Friedman. For a credit currency optimal inflation and a central bank's targeted interest rates exceed the rates as suggested by Friedman by expected losses. It was shown that the perceived gap between central bank's target rates and corrected optima is considerably smaller.

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