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# Coordination Costs, Market Size, and the Choice of Technology

Haiwen Zhou

## Abstract

Impact of coordination costs and market size on a firm's choice of technology is studied in a general equilibrium model in which firms engage in oligopolistic competition. A firm establishes an organizational hierarchy to coordinate its production. First, it is shown that an increase in market size leads a firm to choose a more specialized technology. Second, surprisingly, a robust result is that an increase in the level of coordination efficiency leads a firm to choose a less specialized technology.

**Keywords:** Division of labor, coordination efficiency, technology choice, hierarchy, market size

**JEL Classification:** L13, D43, O14

## 1. Introduction

The enormous benefit from specialization is well recognized since the work of Adam Smith. When firms adopt more specialized technologies, they need to make organizational changes to coordinate new technologies.<sup>1</sup> In this paper, we study how coordination costs and market size affect a firm's choice of the degree of specialization of its technology in a general equilibrium model in which firms engage in oligopolistic competition. There is a continuum of technologies with distinct levels of fixed and marginal costs of production. A more specialized technology has a higher fixed cost but a lower marginal cost of production. A firm establishes a hierarchy to coordinate its production. Individuals at higher levels of the hierarchy coordinate activities of those at lower levels. The total number of tiers in the hierarchy is called the height of the hierarchy. The span of control in the hierarchy captures the level of coordination efficiency. That is, if a superior can coordinate a higher number of direct subordinates, the level of coordination efficiency increases.

We show that a more specialized technology is associated with a taller hierarchy. The reasoning is as follows. A more specialized technology with a higher level of fixed cost is more

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<sup>1</sup> Historically, higher degrees of increasing returns in military technologies led to significant organizational changes in China and Europe. In China, during the Spring-Autumn and the Warring States periods, the adoption of the county system was an organizational response to higher degrees of increasing returns in military technologies. In Europe, military technologies changed the conduct of war (Parker, 1996). The city-states and city-empires lost out to national states when mass armies recruited from a state's own population became essential to successful warfare (Tilly, 1992).

profitable when the level of output is higher, because the higher level of fixed cost can be spread to a higher level of output. To produce a higher level of output, even though the marginal cost for each unit of output decreases, the total number of workers needed for production increases. To coordinate a higher number of workers, a firm establishes a taller hierarchy.

Interestingly, we show that an increase in the level of coordination efficiency leads a firm to choose a less rather than a more specialized technology. The reason is that an increase in the level of coordination efficiency increases a firm's level of total profit, but decreases the marginal benefit of choosing a more specialized technology. It is the marginal benefit rather than total profit that determines a firm's choice of the degree of specialization.

This paper is related to the literature on hierarchies. In a seminal paper, Williamson (1967) has studied the wage structure in a corporate hierarchy. Calvo and Wellisz (1978) have shown that the hierarchical loss of control limitation of the firm size depends on the nature of the supervision process. Qian (1994) has studied a model of hierarchy in which the number of tiers, the span of control, and the wage structure are all optimally determined. Like the case when the effort of a person is either zero or one in Qian, individuals in the hierarchy receive the same wage rate in this model. Garicano (2000) have used a team production approach to study hierarchies. Those models provide very detailed studies of hierarchies. One significant difference between this paper and the above ones is that this paper studies how market structure affects the hierarchy of a firm and the choice of technology in a general equilibrium model while the above papers studies hierarchies in isolation. As shown later after Proposition 3, this general equilibrium approach is useful to address how coordination efficiency affects a firm's choice of the degree of specialization when we need to determine whether the direct effect of a decrease in marginal benefit or the indirect effect of an increase in output dominates. When a firm produces a higher level of output, it needs more labor. If we used a partial equilibrium approach without labor market, we would not be able to address whether a firm's output expansion would be consistent with the clearance of the labor market. With a general equilibrium approach used here, the labor market also clears.

This paper is also related to the literature on the division of labor and a firm's specialization. Since Smith (1776), the division of labor has been studied by various authors such as Stigler (1951), Rosen (1978), Kim (1989), Yang and Borland (1991), Becker and Murphy (1992), and Zhou (2004).<sup>2</sup> Stigler (1951) has emphasized the role of the extent of the market in affecting a firm's

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<sup>2</sup> See Yang and Ng (1998) for a survey on the literature on the division of labor.

degree of specialization. Rosen (1978) has studied a model in which workers differ in their skills and the division of labor is determined by a worker's comparative advantage. Kim (1989) has presented a model in which a worker makes investment decisions on the depth and breadth of her skills. He shows that a worker's human capital will be more specialized if market size is larger. Yang and Borland (1991) have addressed the implication of learning by doing on growth. In their model, an individual is a consumer-producer. An increase in the division of labor means an increase in the proportion of output that is sold to other consumer-producers. This paper is most closely related to Becker and Murphy (1993) and Zhou (2004). Becker and Murphy (1992) have stressed the role of coordination costs in determining the division of labor. There are some significant differences between this paper and Becker and Murphy (1992). First, we provide a detailed specification of coordination costs. Second, in their model, there is no fixed cost of production and firms engage in perfect competition. In this model, with fixed costs of production and the existence of increasing returns, firms engage in oligopolistic competition. Zhou (2004) has demonstrated the mutual dependence between a firm's degree of specialization and the extent of the market in a general equilibrium model. One crucial difference between this paper and Zhou (2004) is that coordination costs are not considered in Zhou (2004).

The plan of the paper is as follows. Section 2 specifies the organizational hierarchy to coordinate a firm's production. Section 3 sets up the model. First, we address a firm's choices of technology and organizational hierarchy when the number of firms is exogenously given. Second, we reexamine a firm's choices when the number of firms is endogenously determined by the zero-profit condition. Section 4 discusses some possible generalizations and extensions of the model and concludes.

## **2. Organizational hierarchy to coordinate production**

Modern production could be complicated and firms frequently establish sophisticated organizations to coordinate their activities (Chandler, 1990).<sup>3</sup> In this section, we specify the organizational hierarchy to coordinate a firm's production.

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<sup>3</sup> As discussed in Chandler (1990), during the Second Industrial Revolution, firms established large management teams to coordinate large-scale production and distribution. Firms with first-mover advantages in exploiting increasing returns in management, production, and distribution established dominant positions in their industries.

There is a continuum of goods indexed by a number  $\varpi \in [0,1]$ . Goods are independent from each other. As discussed in Neary (2003), the purpose of the introduction of a continuum of goods rather than only one good is to eliminate a firm's market power in the labor market.<sup>4</sup> Otherwise, we can view there is only one good in this model. Goods are symmetric in the sense that they have the same costs of production and enter a consumer's utility function in the same way. For a firm producing good  $\varpi$ , this firm's level of output is  $x(\varpi)$ . We usually write  $x(\varpi)$  just as  $x$  when there is no confusion from not indexing goods. If each unit of output requires  $\beta$  units of workers ( $\beta$  later will depend on a firm's technology), then there are  $\beta x$  workers engaging in direct production.

The firm establishes a corporate hierarchy to coordinate  $\beta x$  workers. In this model, the sole purpose of the hierarchy is to coordinate production. The total number of tiers in the hierarchy is called the height of the hierarchy. Following Qian (1994), tiers of the hierarchy are denoted by subscript  $t$  when counted from the top to the bottom. The number of employers in tier  $t$  is  $e_t$ , with  $e_0 = 1$ . Workers engaging in direct production are at the level  $T$ . Because all workers engaging in production need to be coordinated,  $e_T = \beta x$ . We may interpret the coordination process as follows. Individuals in the hierarchy send signals to their superiors. A superior has the maximum capacity to process  $s$  signals, where  $s$  is a positive constant larger than one. The number of employees in tier  $t$  who are subordinates of a common superior is the span of control in tier  $t-1$ , and is equal to  $s$ .

Because the span of control is constant in each tier (Williamson, 1967), the number of individuals in tier  $t$  is equal to the product of the number of individuals in tier  $t-1$  and the span of control:  $e_t = e_{t-1}s$ . For simplicity, we restrict our attention to the situation that the number of tiers is a continuous rather than a discrete variable (Qian, 1994). A continuous approximation of  $e_t = e_{t-1}s$  is

$$\dot{e}_t = e_t \log s. \quad (1)$$

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<sup>4</sup> When there is only one good and it is produced by a small number of firms, firms will have market power in both goods market and labor market. With a continuum of goods and each good is produced by a small number of firms, a firm still has market power in the goods market but no market power in the labor market because there is an infinite number of firms demanding labor.

Integration of equation (1) yields  $e_t = b e^{t \ln s}$ , where  $b$  is a constant to be determined. Using the boundary condition  $e_0 = 1$ , we get  $b = 1$ . Thus,  $e_t = e^{t \ln s}$ . With  $e_T = \beta x$ , the total number of tiers in the hierarchy is  $T = \ln(\beta x) / \ln s$ . Thus, the total number of employees in the hierarchy is

$$\int_0^T e_t dt = \int_0^{\ln(\beta x) / \ln s} e^{t \ln s} dt = \frac{\beta x - 1}{\ln s}.$$

The costs of coordination arise from the wages paid to persons in the coordinating hierarchy. If each person in the hierarchy receives a wage rate of  $w$ , coordination costs are  $\frac{(\beta x - 1)w}{\ln s}$ . Thus, coordination costs increase when the number of workers to be coordinated ( $\beta x$ ) increases. Also, coordination costs decrease with  $s$ . That is, if the span of control in the hierarchy increases, other things equal, the costs of coordination decrease. In this sense, an increase in the magnitude of  $s$  is an increase in the level of organizational efficiency.

One alternative to the above derivation of coordination costs is to specify a general function of coordination costs, with the level of coordination efficiency and output as arguments. However, adopting a general function of coordination costs leads to many ambiguous results without clear economic intuitions. Thus, it is not tried here.

### 3. The model

Labor is the only factor of production. Population size is  $L$ , and each individual supplies one unit of labor inelastically. The wage rate is  $W$ . The price of good  $\varpi$  is  $p(\varpi)$ . If a consumer's consumption of good  $\varpi$  is  $c(\varpi)$ , her utility function is specified as  $\int_0^1 \ln c(\varpi) d\varpi$ . A consumer's budget constraint is  $\int_0^1 p(\varpi) c(\varpi) d\varpi = I$ , where  $I$  is a consumer's income. When firms earn zero profits, a consumer's income will be equal to the wage rate; when firms earn positive profits, a consumer's income may also include profit income.<sup>5</sup> A consumer takes the prices of goods and the wage rate as given, and chooses her quantities of consumption of the goods to maximize her utility. Her utility maximization leads to the result that the absolute value of a consumer's elasticity of demand for a good is one.

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<sup>5</sup> Since preferences are assumed to be homothetic, the distribution of firm ownership will not affect the total demand for a good.

To produce a good, a firm incurs three types of costs: coordination costs, fixed costs, and marginal costs. With coordination costs specified in Section 2, we now specify the fixed costs and marginal costs of production. Similar to Zhou (2014a, 2014b), to produce a good, we assume that there is a continuum of technologies indexed by a positive number  $n$ . A higher value of  $n$  indicates a more specialized technology. For technology  $n$ , the fixed costs in terms of labor units are  $f(n)$  and the corresponding marginal costs in terms of labor units are  $\beta(n)$ .

Modern production is associated with the extensive use of machines. Machines are fixed costs of production. The usage of machines decreases the unit labor requirement. To capture the substitution between fixed and marginal costs of production, we assume that  $f'(n) > 0$  and  $\beta'(n) < 0$ .<sup>6</sup> That is, a more specialized technology has higher fixed costs but lower marginal costs of production. Young (1928) illustrates that some technologies are more specialized and suitable for larger scale production, while some others are less specialized and are suitable for smaller production needs.

“It would be wasteful to make a hammer to drive a single nail: it would be better to use whatever awkward implement lies conveniently at hand. It would be wasteful to furnish a factory with an elaborate equipment of specially constructed jigs, gauges, lathes, drills, presses and conveyors to build a hundred automobiles; it would be better to rely mostly upon tools and machines of standard types, so as to make a relatively larger use of directly-applied and a relatively smaller use of indirectly-applied labor. Mr. Ford’s methods would be absurdly uneconomical if his output were very small, and would be unprofitable even if his output were what many other manufacturers of automobiles would call large.” (Young, 1928, p. 530)

Here we provide some examples of the choice of the degree of specialization of technologies. First, container ports are more specialized than traditional ports. Compared with traditional terminals, container terminals are ten times costlier to build and can handle volumes of trade more than twenty times higher (Levinson, 2006). Second, the movement of some goods requires specialized vessels such as oil tanks (Stopford, 2009). Oil tanks are specialized to

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<sup>6</sup> We also assume that  $f''(n) \geq 0$  and  $\beta''(n) \geq 0$ . That is, when firms adopt more advanced technologies, fixed costs increase at a nondecreasing rate and marginal costs decrease at a nonincreasing rate.

transport oil and may not be convenient in transporting other goods. High volume of trade makes it profitable to adopt oil tanks.

For a firm producing good  $\varpi$  with output  $x(\varpi)$ , its total revenue is  $p(\varpi)x(\varpi)$ . Its fixed production costs are  $f(\varpi)w$ , marginal production costs are  $\beta(\varpi)x(\varpi)w$ , and coordination costs are  $\frac{[\beta(\varpi)x(\varpi)-1]w}{\ln s}$ .<sup>7</sup> Thus, total costs for a firm are  $\left(f + \beta x + \frac{\beta x - 1}{\ln s}\right)w$ . As a result, a firm's profit as the difference between total revenue and total costs is

$$\pi = px - \left(f + \beta x + \frac{\beta x - 1}{\ln s}\right)w. \quad (2)$$

For good  $\varpi$ , there are  $m(\varpi)$  identical firms producing it. Firms producing the same good engage in Cournot competition. In a Cournot-Nash equilibrium, a firm takes the wage rate and output of other firms producing the same good as given, and chooses its levels of output and technology to maximize its profit.<sup>8</sup> A firm's optimal choice of output requires that marginal revenue equals marginal cost:  $p\left(1 + \frac{\partial p}{\partial x} \frac{x}{p}\right) = \left(1 + \frac{1}{\ln s}\right)\beta w$ . Remembering that utility maximization of a consumer leads to a unitary elasticity of demand for a good. Combination of this result of utility maximization with the above condition for a firm's optimal choice of output shows that a firm's price is determined by the number of firms in the industry and its marginal cost:<sup>9</sup>

$$p\left(1 - \frac{1}{m}\right) = \left(1 + \frac{1}{\ln s}\right)\beta w. \quad (3)$$

A firm's optimal choice of technology yields the following first order condition with respect to  $n$ :

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<sup>7</sup> Here the ratio between non-production workers (workers in the hierarchy) and production workers is an increasing function of output.

<sup>8</sup> In this model, when a firm chooses its technology, it takes the output of other firms producing the same good as given. This assumption that firms do not internalize the strategic effect on rivals is consistent with the "open loop" approach in the R&D literature with oligopoly such as Vives (2008). Vives has addressed the impact of the degree of competition on R&D spending under both the open loop and the closed loop (in which the strategic interaction effect is considered) approaches. He demonstrates that the incorporation of the strategic interaction has an ambiguous impact on comparative static results under free entry. In this paper, we do not focus on a firm's strategic choice of technology and we are mainly interested in how coordination efficiency affects a firm's technology choice. For simplicity, we adopt the open loop approach.

<sup>9</sup> For a detailed illustration of the derivation of this type of result, see Zhou (2015, p. 673).



$$-f' - \frac{x\beta'(1 + \ln s)}{\ln s} = 0. \quad (4)$$

The second order condition requires that

$$-f'' - \frac{x\beta''(1 + \ln s)}{\ln s} < 0. \quad (5)$$

With  $f''(n) \geq 0$  and  $\beta''(n) \geq 0$  in footnote 6, the second order condition is always satisfied. This second order condition is used later to sign comparative statics.

For the labor market, demand for labor from a firm is  $f + \beta x + \frac{\beta x - 1}{\ln s}$  and total demand for labor from firms is  $\int_0^1 m \left( f + \beta x + \frac{\beta x - 1}{\ln s} \right) d\varpi$ . Supply of labor is  $L$ . The clearance of the labor market requires

$$\int_0^1 m \left( f + \beta x + \frac{\beta x - 1}{\ln s} \right) d\varpi = L. \quad (6)$$

To close the model, we need to determine the profit of a firm. Depending on whether the number of firms is exogenously given or endogenously determined, a firm may earn a positive profit or a zero profit. In the following, we study the two scenarios in turn.

### 3.1. Exogenous number of firms

In this subsection, we study firms' choices of their degrees of specialization of technologies when the number of firms is exogenously given.<sup>10</sup>

For the goods market, the value of output produced by one firm is  $p x$  and the total value of output produced by all firms is  $\int_0^1 m p x d\varpi$ . Total profits for all firms are  $\int_0^1 m \left( p x - \left( f + \beta x + \frac{\beta x - 1}{\ln s} \right) w \right) d\varpi$  and labor income is  $w L$ . Thus, the total demand for goods is  $\int_0^1 m \left( p x - \left( f + \beta x + \frac{\beta x - 1}{\ln s} \right) w \right) d\varpi + w L$ . Goods market clearance requires

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<sup>10</sup> One justification of this assumption is that governments in developing countries may use licenses to restrict the number of firms in strategic industries or incumbent firms may have patents to prevent other firms from entering their industries.

$$\int_0^1 p m x d\varpi = \int_0^1 m \left( p x - \left( f + \beta x + \frac{\beta x - 1}{\ln s} \right) w \right) d\varpi + wL. \quad (7)$$

In a symmetric equilibrium, a consumer purchases the same amount of each good. Also, there is the same number of firms producing each good and all goods have the same price and output. Since the measure of goods is one and all goods are symmetric, we drop the integration operator. When the number of firms is exogenously given, equations (3), (4), (6), and (7) form a system of four equations defining four variables  $p$ ,  $x$ ,  $n$ , and  $w$  as functions of exogenous parameters. An equilibrium with an exogenously given number of firms is a tuple  $(p, x, n, w)$  satisfying equations (3), (4), (6), and (7). For the rest of the paper, the price of a good is normalized to one:  $p \equiv 1$ .

When equations (3), (4), and (6) are satisfied, equation (7) is automatically satisfied. That is, one equation is redundant. With Walras' law in mind, this redundancy is not surprising. Equations (3), (4), and (6) form the following system of three equations defining three variables  $n$ ,  $x$  and  $w$  as functions of exogenous parameters.

$$\Phi_1 \equiv \frac{\beta(1+\ln s)w}{\ln s} - \left(1 - \frac{1}{m}\right) = 0, \quad (8a)$$

$$\Phi_2 \equiv -f' - \left(\frac{1+\ln s}{\ln s}\right)\beta' x = 0, \quad (8b)$$

$$\Phi_3 \equiv m \left( f + \beta x + \frac{\beta x - 1}{\ln s} \right) - L = 0. \quad (8c)$$

Partial differentiation of equations  $\Phi_1$ ,  $\Phi_2$ , and  $\Phi_3$  with respect to  $x$ ,  $n$ ,  $w$ ,  $m$ ,  $s$ , and  $L$  yields

$$\begin{pmatrix} \frac{\partial \Phi_1}{\partial n} & 0 & \frac{\partial \Phi_1}{\partial w} \\ \frac{\partial \Phi_2}{\partial n} & \frac{\partial \Phi_2}{\partial x} & 0 \\ 0 & \frac{\partial \Phi_3}{\partial x} & 0 \end{pmatrix} \begin{pmatrix} dn \\ dx \\ dw \end{pmatrix} = - \begin{pmatrix} \frac{\partial \Phi_1}{\partial m} \\ 0 \\ \frac{\partial \Phi_3}{\partial m} \end{pmatrix} dm - \begin{pmatrix} \frac{\partial \Phi_1}{\partial s} \\ \frac{\partial \Phi_2}{\partial s} \\ \frac{\partial \Phi_3}{\partial s} \end{pmatrix} ds - \begin{pmatrix} 0 \\ 0 \\ \frac{\partial \Phi_3}{\partial L} \end{pmatrix} dL. \quad (9)$$

Let  $\Delta$  denote the determinant of the coefficient matrix of endogenous variables of (9):  
 $\Delta \equiv \frac{\partial \Phi_1}{\partial w} \frac{\partial \Phi_2}{\partial n} \frac{\partial \Phi_3}{\partial x}$ . Since  $\frac{\partial \Phi_1}{\partial w} > 0$ ,  $\frac{\partial \Phi_2}{\partial n} < 0$ , and  $\frac{\partial \Phi_3}{\partial x} > 0$ , it is clear that  $\Delta < 0$ . With  $\Delta$  nonsingular, a unique equilibrium exists.

When population size increases, market size increases. The following proposition studies the impact of a change in market size on a firm's choice of the degree of specialization.

Proposition 1: When market size increases, the wage rate increases, a firm produces a higher level of output and chooses a more specialized technology, and the height of its organizational hierarchy increases.

Proof: An application of Cramer's rule on the system (9) yields

$$\frac{dw}{dL} = -\frac{\partial \Phi_1}{\partial n} \frac{\partial \Phi_2}{\partial x} \frac{\partial \Phi_3}{\partial L} / \Delta > 0,$$

$$\frac{dx}{dL} = -\frac{\partial \Phi_1}{\partial w} \frac{\partial \Phi_2}{\partial n} \frac{\partial \Phi_3}{\partial L} / \Delta > 0,$$

$$\frac{dn}{dL} = \frac{\partial \Phi_1}{\partial w} \frac{\partial \Phi_2}{\partial x} \frac{\partial \Phi_3}{\partial L} / \Delta > 0.$$

With  $T = \ln(\beta x) / \ln s$ , the height of the organizational hierarchy  $T$  is positively correlated with  $\beta x$ . From equation (4), a firm's output is

$$x = \frac{-f' \ln s}{\beta'(1 + \ln s)}. \quad (10)$$

Thus,  $\beta x = \frac{-\beta f' \ln s}{\beta'(1 + \ln s)}$ . Differentiation of  $\beta x$  with respect to  $n$  yields

$$\frac{d(\beta x)}{dn} = -\frac{\ln s}{(1 + \ln s)} \left( \frac{(\beta' f' + \beta f'') \beta' - \beta f' \beta''}{(\beta')^2} \right).$$

Plugging the value of  $x$  from equation (10) into equation (5), it can be shown that  $f'' \beta' - f' \beta'' < 0$ . Thus,  $d(\beta x) / dn > 0$ . ■

The intuition behind Proposition 1 is as follows. An increase in market size has two effects. First, demand for a good increases. This leads to the adoption of a more specialized technology.

With a more specialized technology, the marginal cost of production for each unit of output decreases. This decreases the number of workers needed to produce a unit of output. Second, because the level of output increases with the degree of specialization, the latter effect tends to increase the number of workers employed by a firm. Overall the latter effect dominates and the total number of workers employed by a firm increases with the level of specialization. To coordinate a higher number of workers, a firm establishes a taller hierarchy. Thus, a more specialized technology is associated with a taller hierarchy.

For empirical research on the importance of market size on the division of labor, Baumgardner (1988) has shown that doctors are more specialized in larger cities. Garicano and Hubbard (2005, 2007) have demonstrated that the share of lawyers working in field-specialized firms increases with the size of the market.

While restricting entry can increase a firm's market power and decrease efficiencies, the following proposition studying the impact of a change in the number of firms provides a reason to restrict entry in some cases.

Proposition 2: A decrease in the number of firms induces a firm to produce a higher level of output and choose a more specialized technology, and industry output decreases. The impact on the wage rate is ambiguous.

Proof: An application of Cramer's rule on the system (9) yields

$$\begin{aligned}\frac{dx}{dm} &= -\frac{\partial\Phi_1}{\partial w} \frac{\partial\Phi_2}{\partial n} \frac{\partial\Phi_3}{\partial m} / \Delta = -\frac{\partial\Phi_3 / \partial m}{\partial\Phi_3 / \partial x} < 0, \\ \frac{dn}{dm} &= \frac{\partial\Phi_1}{\partial w} \frac{\partial\Phi_2}{\partial x} \frac{\partial\Phi_3}{\partial m} / \Delta < 0, \\ \frac{dw}{dm} &= \left( -\frac{\partial\Phi_1}{\partial n} \frac{\partial\Phi_2}{\partial x} \frac{\partial\Phi_3}{\partial m} - \frac{\partial\Phi_1}{\partial m} \frac{\partial\Phi_2}{\partial n} \frac{\partial\Phi_3}{\partial x} \right) / \Delta.\end{aligned}$$

Industry output is  $mx$ . We have

$$\frac{d(mx)}{dm} = x + m \frac{dx}{dm} = x - m \frac{\partial\Phi_3 / \partial m}{\partial\Phi_3 / \partial x} = x - \frac{L}{m\beta} \frac{\ln s}{(1 + \ln s)} = \frac{\left( f - \frac{1}{\ln s} \right) \ln s}{\beta(1 + \ln s)}.$$

Since a firm's fixed cost cannot be negative,  $f - \frac{1}{\ln s} > 0$ . Thus  $d(mx)/dm > 0$ .

Because the sign of  $-\frac{\partial\Phi_1}{\partial n}\frac{\partial\Phi_2}{\partial x}\frac{\partial\Phi_3}{\partial m}-\frac{\partial\Phi_1}{\partial m}\frac{\partial\Phi_2}{\partial n}\frac{\partial\Phi_3}{\partial x}$  is ambiguous, the sign of  $\frac{dw}{dm}$  is ambiguous. ■

Proposition 2 shows that a lower degree of competition (as measured by the number of firms) encourages firms to adopt more specialized technologies. To understand Proposition 2, when the number of firms decreases, each firm produces a higher level of output. Thus, a firm adopts a more specialized technology. The reason is that the higher fixed costs associated with a more specialized technology can be spread to a higher level of output.

Social welfare can be measured by the sum of utilities of consumers and profits of firms. Will a decrease in the number of firms increase or decrease social welfare? Since a decrease in the number of firms increases profits while the impact on the wage rate is ambiguous, overall the impact on social welfare is ambiguous.

Industrial policies of Japan and South Korea have attracted much attention. Scholars have argued that governments in Japan and South Korea used licenses to prevent too many firms from entering strategic industries. The restriction of entry would decrease the degree of competition in an industry and a lower degree of competition could harm the level of economic efficiency. Then why did government try to restrict entry? In Proposition 2, a decrease in the number of manufacturing firms leads to a higher level of output for each firm and the adoption of more advanced technologies in the manufacturing sector. More advanced technologies lead to a lower marginal cost of production. Chang (2003) has argued that in many industries the inefficiency losses from failing to achieve the minimum efficient scales of production dominate the inefficiency losses from monopoly pricing. Thus, Proposition 2 provides a rationale for the practice of restricting entry in Japan and South Korea: the existence of too many firms in an industry could harm the adoption of increasing returns technologies and thus lead to a lower level of overall economic efficiency.

Technological progresses such as the invention of telephones and telegraphs and the usage of railways can increase the level of organizational efficiency. Improvements in institutions such as a better legal environment can also increase the level of organizational efficiency. By applying envelop theorem on equation (2), we get  $\frac{d\pi}{ds} = \frac{\partial\pi}{\partial s} = \frac{s(\beta x - 1)}{(\ln s)^2} w > 0$ : an increase in the level of

coordination efficiency always increases a firm's profit. Thus, an increase in the level of coordination efficiency benefits a firm. But will a higher level of coordination efficiency always lead a firm to adopt a more specialized technology? The following proposition addresses this question.

**Proposition 3:** When the number of firms is exogenously given, an increase in the level of coordination efficiency leads a firm to produce a higher level of output and choose a less specialized technology.

**Proof:** Partial differentiation of (8a)-(8c) yields

$$\frac{\partial \Phi_2}{\partial x} \frac{\partial \Phi_3}{\partial s} - \frac{\partial \Phi_2}{\partial s} \frac{\partial \Phi_3}{\partial x} = -\frac{m\beta'(1+\ln s)}{s(\ln s)^3}.$$

An application of Cramer's rule on the system (9) yields

$$\frac{dx}{ds} = -\frac{\partial \Phi_1}{\partial w} \frac{\partial \Phi_2}{\partial n} \frac{\partial \Phi_3}{\partial s} / \Delta > 0,$$

$$\frac{dn}{ds} = \frac{\partial \Phi_1}{\partial w} \left( \frac{\partial \Phi_2}{\partial x} \frac{\partial \Phi_3}{\partial s} - \frac{\partial \Phi_2}{\partial s} \frac{\partial \Phi_3}{\partial x} \right) / \Delta < 0. \blacksquare$$

The intuition behind Proposition 3 is as follows. From equation (4), when a firm chooses its degree of specialization, it compares the marginal cost and marginal benefit from adopting a more specialized technology. The marginal cost of a more specialized technology comes from increased fixed costs. The marginal benefit of a more specialized technology is that coordination costs and marginal costs of production decrease. When the level of coordination efficiency increases, there are two effects. First, the direct effect is that the saving on coordination costs decreases. That is, the marginal benefit of a more specialized technology decreases. This will decrease a firm's incentive to adopt a more specialized technology. Second, the indirect effect is that a firm's level of output increases. This will increase a firm's incentive to adopt a more specialized technology. The two effects work in opposite directions and the direct effect dominates the indirect one. Thus, an increase in the level of coordination efficiency leads a firm to choose a less specialized technology.

The following table summarizes comparative statics for an exogenous number of firms. A positive (negative) sign indicates a positive (negative) relationship between two variables. A question mark indicates the relationship is ambiguous.

Table 1: Comparative statics for an exogenous number of firms

	Market size	Number of firms	Coordination efficiency
Wage rate: $w$	+	?	?
Output: $x$	+	-	+
Technology: $n$	+	-	-
Hierarchy height: $T$	+	-	+

### 3.2. Endogenous number of firms

In some cases, the level of fixed costs determines the number of firms in an industry.<sup>11</sup> In this section, we study a firm's choice of the degree of specialization when the zero-profit condition determines the number of firms.<sup>12</sup>

The zero-profit condition for a firm requires

$$\pi = px - \left( f + \beta x + \frac{\beta x - 1}{\ln s} \right) w = 0. \quad (11)$$

The clearance of goods market requires

$$\int_0^1 pmx d\varpi = wL. \quad (12)$$

Equations (3), (4), and (6) are still valid when the zero-profit condition determines the number of firms endogenously. Together with equations (11) and (12), those equations form a system of five equations defining five variables  $m$ ,  $p$ ,  $x$ ,  $n$ , and  $w$  as functions of exogenous parameters. An equilibrium in which the number of firms is endogenously determined is a tuple  $(m, p, x, n, w)$  satisfying equations (3), (4), (6), (11), and (12).<sup>13</sup>

<sup>11</sup> For some examples of models in which firms engage in Cournot competition and earn zero profits, see Dasgupta and Stiglitz (1980), Zhang (2007), and Chen and Shieh (2011).

<sup>12</sup> To facilitate analysis, the number of firms in this model is a real number rather than restricted to be an integer number. With a real number of firms, a firm may make a profit exactly equaling zero.

<sup>13</sup> When equations (3), (4), (6), and (11) are satisfied, equation (12) is automatically satisfied. That is, one equation is redundant. With Walras' law in mind, this redundancy is not surprising.

To conduct comparative statics, the above system of five equations defining the equilibrium in which the number of firms is endogenously determined is reduced to the following system of three equations defining three variables  $x$ ,  $n$ , and  $w$  as functions of exogenous parameters.<sup>14</sup>

$$\Gamma_1 \equiv x - \left( f + \beta x + \frac{\beta x - 1}{\ln s} \right) w = 0, \quad (13a)$$

$$\Gamma_2 \equiv -f' - \left( \frac{1 + \ln s}{\ln s} \right) \beta' x = 0, \quad (13b)$$

$$\Gamma_3 \equiv x \left( 1 - \frac{\beta w (1 + \ln s)}{\ln s} \right) - wL = 0. \quad (13c)$$

Partial differentiation of equations  $\Gamma_1$ ,  $\Gamma_2$ , and  $\Gamma_3$  with respect to  $x$ ,  $n$ ,  $w$ ,  $s$ , and  $L$  yields

$$\begin{pmatrix} \frac{\partial \Gamma_1}{\partial x} & 0 & \frac{\partial \Gamma_1}{\partial w} \\ \frac{\partial \Gamma_2}{\partial x} & \frac{\partial \Gamma_2}{\partial n} & 0 \\ \frac{\partial \Gamma_3}{\partial x} & \frac{\partial \Gamma_3}{\partial n} & \frac{\partial \Gamma_3}{\partial w} \end{pmatrix} \begin{pmatrix} dx \\ dn \\ dw \end{pmatrix} = - \begin{pmatrix} \frac{\partial \Gamma_1}{\partial s} \\ \frac{\partial \Gamma_2}{\partial s} \\ \frac{\partial \Gamma_3}{\partial s} \end{pmatrix} ds - \begin{pmatrix} 0 \\ 0 \\ \frac{\partial \Gamma_3}{\partial L} \end{pmatrix} dL. \quad (14)$$

Let  $\Delta_\Gamma$  denote the determinant of the coefficient matrix of endogenous variables of the system (14). Stability of (14) requires that  $\Delta_\Gamma < 0$ .<sup>15</sup>

The following proposition revisits the impact of an increase in the level of coordination efficiency on a firm's choice of the degree of specialization when the zero-profit condition determines the number of firms endogenously. Together with Proposition 3, the two propositions show that the result that an increase in coordination efficiency leads firms to choose less specialized technologies is robust regardless of whether the number of firms is exogenously given or endogenously determined.

<sup>14</sup> Equations (13a)-(13c) are derived as follows. First, equation (13a) comes from equation (11). Second, equation (13b) comes from equation (4). Third, equation (13c) is derived from equations (3) and (12).

<sup>15</sup> See Samuelson (1983, chap. 9) for a justification of this type of assumption on stability.



Proposition 4: When the number of firms is endogenously determined, an increase in the level of coordination efficiency leads a firm to choose a less specialized technology.

Proof: Partial differentiation of equations (13a)-(13c) yields

$$\begin{aligned} & \frac{\partial \Gamma_1}{\partial w} \frac{\partial \Gamma_2}{\partial s} \frac{\partial \Gamma_3}{\partial x} + \frac{\partial \Gamma_1}{\partial s} \frac{\partial \Gamma_2}{\partial x} \frac{\partial \Gamma_3}{\partial w} - \frac{\partial \Gamma_1}{\partial x} \frac{\partial \Gamma_2}{\partial s} \frac{\partial \Gamma_3}{\partial w} - \frac{\partial \Gamma_1}{\partial w} \frac{\partial \Gamma_2}{\partial x} \frac{\partial \Gamma_3}{\partial s} \\ &= -\frac{\beta' x}{s(\ln s)^2} \left( f + \beta x + \frac{\beta x - 1}{\ln s} \right) + \frac{\beta'}{s(\ln s)^2} \left( x - \frac{w(1 + \ln s)}{\ln s} \right) \left( L + \frac{x\beta(1 + \ln s)}{\ln s} \right) \\ &= -\frac{\beta'}{s(\ln s)^2} \left( \frac{\beta x w (1 + \ln s)^2}{(\ln s)^2} + \frac{wL(1 + \ln s)}{\ln s} \right) > 0. \end{aligned}$$

An application of Cramer's rule on the system (14) yields

$$\frac{dn}{ds} = \left( \frac{\partial \Gamma_1}{\partial w} \frac{\partial \Gamma_2}{\partial s} \frac{\partial \Gamma_3}{\partial x} + \frac{\partial \Gamma_1}{\partial s} \frac{\partial \Gamma_2}{\partial x} \frac{\partial \Gamma_3}{\partial w} - \frac{\partial \Gamma_1}{\partial x} \frac{\partial \Gamma_2}{\partial s} \frac{\partial \Gamma_3}{\partial w} - \frac{\partial \Gamma_1}{\partial w} \frac{\partial \Gamma_2}{\partial x} \frac{\partial \Gamma_3}{\partial s} \right) / \Delta_\Gamma.$$

With  $\Delta_\Gamma < 0$ ,  $dn/ds < 0$ . ■

What is the impact of an increase in coordination efficiency on social welfare? Since firms earn profits of zero, utilities of consumers measure social welfare. A consumer's utility increases when the wage-price ratio increases. When the level of coordination efficiency increases, since consumers are identical and the wage-price ratio increases, social welfare increases.

Like the proof of Proposition 4, it can be shown that the level of output, the degree of specialization, and the wage rate increase with the size of the population when the number of firms is endogenously determined.

The following table summarizes comparative statics for an endogenously determined number of firms.

Table 2: Comparative statics for an endogenous number of firms

	Market size	Coordination efficiency
Wage rate: w	+	+
Output: x	+	?
Technology: n	+	-
Hierarchy height: T	+	?

## 5. Conclusion

In this paper, we have studied how the level of coordination efficiency affects a firm's choice of the degree of specialization of its technology in a general equilibrium model. We have established the following results. First, an increase in market size leads a firm to choose a more specialized technology. Second, an increase in the level of organizational efficiency leads a firm to choose a less rather than more specialized technology.

There are some possible generalizations and extensions of the model. First, in this model, we take the level of coordination efficiency as exogenously given. Studying how factors such as institutions affect the level of coordination efficiency and thus the choice of technology will be an interesting avenue for future research. Second, in this model, we assume technologies with different degrees of specialization are always available. Embedding a firm's choice of technology into a dynamic model with endogenous development of new technologies will be valuable.

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