Building a computable general equilibrium model on a regional Social Accounting Matrix: the case of Tuscany

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September 2013

Online at https://mpra.ub.uni-muenchen.de/83173/
MPRA Paper No. 83173, posted 8 December 2017 06:22 UTC
BUILDING A COMPUTABLE GENERAL EQUILIBRIUM (CGE) MODEL ON A REGIONAL SAM: THE CASE OF TUSCANY

Stefano ROSIGNOLI¹, Maria GESUALDO²

SUMMARY

This paper aims to provide an empirical framework for building a Computable General Equilibrium (CGE) model from a regional Social Accounting Matrices (SAM). It illustrates the case of TUSCANI, a single-region comparative-static CGE model of the Tuscan economy, built on the regional SAM, which is provided by IRPET (Institute for regional economic planning of Tuscany). The strict linkage between regional SAMs and CGE models is identified. First of all, a detailed overview of the structure of the regional SAM for Tuscany is provided. The circular flow of the economy, so well summarized by the SAM, is the framework at the core of the CGE model, and comes to represents the benchmark equilibrium of TUSCANI.

Because of the inner relationship between the benchmark economy and the SAM data structure, the CGE theoretical model is adapted in order to reflect the SAM structure adopted by IRPET. In particular, together with the main theoretical features of a regional model, TUSCANI incorporates a detail on income generation and income re-distribution for the households institutional sector, by using data as shown in the SAM.

In conclusion, this paper demonstrates the structure of a regional SAM suitable for constructing a computable general equilibrium model’s database, which provides a guideline for future applications.

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1. Introduction

Computable general equilibrium (CGE) models have been widely used in economic policy analysis in recent years. Despite their increasing popularity at national level, the use of CGE models at regional level remains critical. One of the key issues in regional modeling is data availability. This is particularly the case for data intensive models, such as CGE, which are based on regional Social Accounting Matrices (SAMs).

This paper presents a single-region sub-national comparative-static computable general equilibrium model of Tuscany, TUSCANI. Conceptually and empirically, the system of national accounts forms the base for the model. The entire process of income generation and of primary and secondary income distribution described by the SAM, perfectly suits into a computable general equilibrium's framework. A framework built around microeconomic fundamentals, consistent with macro balancing equations, via behavioural equations and constraints. Via a calibration procedure the equivalence between the system of equations in the theoretical model with those in the accounting model is ensured. In particular, the accountability principles expressed in the SAM are the reflection of the Kuhn-Tucker complementarity slackness conditions for the optimal allocation of commodities and factors in the economy, computed in the theoretical model.

This paper lies at the intersection of the SAM and regional CGE literatures and borrows insights from both. On the CGE side, the approach used expands on earlier works by the Centre of Policy Studies (CoPS), in particular on ORANI, the national CGE model of the Australian economy (see Dixon et al., 1997 and Horridge, 2003, for a full model description). The earlier work of Corong and Horridge (2012) also presents a CGE model built on a SAM.

Because of the inner relationship between the theoretical and data structures, a point of departure from previous works is required to reflect the structure of the IRPET’s SAM. A national model such as ORANI, in fact, is usually built on an Input-Output table, which does not show the interrelationships between value added and expenditure. On the contrary, TUSCANI is built on the regional SAM for 2008, which captures the entire circular flow of the economy, production-income-expenditure, at the macro level.

In term of policy analysis, the CGE model tracks and explains the behaviour of macroeconomic variables, capturing both positive gross multiplier and negative displacement effects from exogenous stimuli, thanks to short run supply constraints and less than instantaneous adjustment response in prices. It is a suitable tool for regional policy analysis, capable of investigating the regional impacts of national policies and shocks, and the regional effects of regional policies; as well as to identify winners and losers from a particular policy.

With a properly designed model, policy makers can evaluate alternative courses of action at the local level, which can mitigate the harmful effects or enhance the benefits of the national
policy. The structure of the paper is as follows. Section 2 describes the SAM for Tuscany built by IRPET, followed by a brief description of criticalities in dealing with the regional variables. Section 3 is devoted to explain the CGE structure. Concluding remarks are in section 4.

2. Structure of IRPET regional SAMs

2.1 Circular flows of the economic system

According to the System of National Accounts (SNA93), a social accounting matrix (SAM) is defined as a means of presenting the SNA accounts in a matrix which elaborates the linkages between a supply and use table and institutional sector accounts. In other words, a SAM provides a comprehensive representation of the transactions and transfers between economic agents in the system during an accounting period (Pyatt and Round, 1985; Reinert and Roland-Holst, 1997; Round 2003), describing the process of production, consumption and of income generation and distribution. It is represented by a square matrix - with rows showing receipts and columns outlays, and where each cell records a transaction between agents - dominated by a primary balance: for each account, total revenue (row total) should be equal to total expenditure (column total).

The rigorous system of accounts which characterized the SAM provides a reliable base to investigate the linkages in the economy. Even more, because of its accounting consistency and comprehensiveness in recording data, the SAM provides a direct input on which to create a macroeconomic database suitable to computable general equilibrium (CGE) models and it serves as a useful bridge between a macro framework and a more detailed description of markets and institutions. The SAM represents the economic process by highlighting its circularity. A diagram of the circular flows of goods and services, factor of productions and income between institutional sectors (households, governments and enterprises, rest of the world) is given in figure 1.

Figure 1: Schematic summary of the flows between sectors of an economic system
According to this scheme, the first exchange relationship in the market occurs between enterprises and households: the first sell goods and services to the second at market price, and pay income in exchange for the factors of production, capital and labour. The government mitigates the excesses of income distribution by taxation and other transfers. The system is an open one, with the foreign sector represented by the rest of the world (ROW). Even more, figure 2 diagrams the flows in the economy in terms of accounts.

Figure 2: Flows in the economic system in terms of accounts

The construction of a SAM relies on a variety of sources, each one having different nature and purposes, and each one helps to re-built the core structure of an economy. It is obvious that a system like this requires an effort of collecting and processing data very intense. Although it is usually set up in a standard framework, there is a flexibility in the degree of disaggregation and in the emphasis placed on different parts of the economic system.

2.2 Blocks of a regional SAM

The IRPET regional SAM consists of 157 rows and columns (Figure 3). The production account is disaggregated by 37 industries, according to the classification Ateco07 (corresponding to the ESA95 NACE Rev.2); and by 54 commodities, according to the ESA95 CPA 2008 classification. The institutional accounts, are split into 17 institutional sectors: Consumer households (10 groups) by percentiles of gross income; Producer households,
which include all unincorporated enterprises that produce for both the market and their own final uses; Non-financial and financial corporations; Non-profit institutions serving households; General government, which includes central and local government units, as well as social security funds, the rest of Italy, the rest of the world.

**Figure 3: Blocks of the IRPET regional SAMs**

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A detail specification of the single matrix is given below.

**Intermediate demand (block 1,2 and size 54x37):** Purchase of goods and services that are used as inputs in production processes by industry.

**Final consumption expenditure of the household sector (block 1,6 and size 54x12):** Consumption of goods and services by household residents, classified, along the column, in division 12 of the Classification of Individual Consumption by Purpose (COICOP).

**Final consumption expenditure by general government and by non-profit institutions serving households (NPISH)s (block 1,7 and size 54x11):** Consumption of goods and services by general government, classified according the Classification of the Functions of Governments (COFOG); and consumption by NPISHs.

**Exports to the rest of Italy and to the rest of the world (block 1,10 and size 54x2):** Exports of goods and services from residents to non-residents.
Investment demand (block 1,12 and size 54x7): gross (net) fixed capital formation, demanded by seven institutional sectors.

Supply (block 2,1 and size 37x54): Supply of goods, presented as columns, by different industries listed in the rows.

Value added (block 3,2 and size 5x30): Value added at basic prices (in the row) paid by industries (in the column). In detail, value added consists of: compensation of employees, net mixed income, net operating system and capital depreciation.

Income from the rest of Italy and the rest of the world (block 3,13 and size 5x2): Value added, as define above, produced in other Italian regions or abroad and paid to resident institutional sectors.

Net indirect taxes on products and on production (block 4,2 6 7 10 12 and size 2x37 12 11 2 7): Net indirect commodity taxes levied on intermediate and final consumption of goods and services.

Passive financial capital incomes (block 5,9 and size 5x17): Income payments by institutional sectors. In particular, five types of payments are recorded, namely: interests on the public debt; other interests; dividends; mixed-income from producer households to consumer households and income from other sources.

Income use and distribution are recorded respectively in blocks 6.9 and 9.5. Along the rows, 17 receiving institutional sectors are listed.

Use of income by households (block 6,9 and size 12x17): Consumption of goods and services classified according COGOF final expenditure functions (in the row) by households disaggregated by percentiles of income (in the column).

Expenditure of non-residents at home (block 6,11 and size 12x2): Final expenditure classified according to COICOP by non-residents at home (in the column).

Use of income by general government and NPHISs (block 7.6 and size 11x17): Government expenditure classified according to COFOG along the row, and allocated to users along the column. In particular, the “divisible” component of the government expenditure is allocated to households (mainly health, recreation, education and social protection), and the “indivisible” one directly to the general government sector (General Services, Defense, Public Order and Security, Economic Affairs, Environmental Protection Agency, Housing and land use). An additional row contains the consumption of NPHIS allocated to households.

Lending / net borrowing (block 8,12 and size 1X7): Budget balance of institutional sectors (by column), computed as the difference between total revenue and total expenditures. Particularly informative for the general government sector.

Income distribution (block 9.3 and size 17x5): Payment of factors income, which are split into five different components along the columns, to holders of the factors of production.
Net indirect taxes (block 9,4 and size 17x2): Indirect taxes on products (including VAT and taxes on imports) and indirect taxes on production, collected by European, central and local public administrations.

Financial capital income (block 9,5 and size 17x5): Financial income earned by institutional sectors as holders of shares and/or bonds. Read along the columns, the block has 5 types of income such as: interest on the public debt; other interest; dividends and income from other financial assets; mixed-income transferred from family businesses to consumers; and income from other sources.

Income re-distribution (block 9,9 and size 17x17): Transfers between institutional sectors, recorded by payers in the column and recipients in the row. Among the main items of transfers there are direct taxes, social contributions, pensions, bonuses and insurance claims.

Transfers from rest of Italy and from the rest of the world (block 9,13 and size 17x13): Income from the rest of the world, e.g. pensions from abroad, foreign insurance premiums paid by foreigners to Italian insurance or insurance claims paid by foreign insurance to regional sectors, International aid and transfers between households.

Imports from the rest of Italy and from the rest of the world (block 10,1 and size 2x59): Purchasers of goods and services (in the columns) by residents from the rest of Italy or the rest of the world (in the rows).

Regional and world trade balance (block 10,8 and size 2x1): Difference between exports and imports with the rest of Italy and with the rest of the world.

Regional and international tourism balance (block 11,8 and size 2x1): difference between the expenditure of goods and services of foreign tourists in the region and expenditure of residents outside the region.

Expenditure of residents in other regions and in the rest of the world (block 6,7 and size 12x2): Consumption of household residents in the region (divided by deciles of income and spanned by columns) in the other rest of Italy or the world, as shown in the row.

Gross saving (block 12,9 and size 7x17): Disposable income not consumed by institutional sectors (in the columns). It contains the depreciation of capital for each institutional sector. Henceforth, households are no longer divided by percentiles of income.

Capital transfers (block 12,12 and size 7x7): Capital transfers between resident institutional sectors, with in the columns and receipt in the rows. Examples of capital transfers are inheritance tax, tax on extraordinary properties, financial capital and building amnesties, donations of property.

Capital transfers from rest of Italy and from rest of the world (block 12,13 and size 7x2): Inward capital flows such as direct investment, capital taxes, capital donations or other type of capital transfers from non-residents.
Primary income from the rest of Italy and from the rest of the world (block 13,3 and size 2x5): Portion of primary income (value added components) produced by internal industries and paid to institutional sectors that are outside the region.

Net indirect taxes to rest of Italy and rest of the world (block 13,4 and size 2x2): Because of difficulties of imputation those direct to other regions are set equal to 0. Those paid to the rest of the world are mainly taxes (net of subsidies) paid by firms to European Union.

Financial Income to the rest of Italy and to the rest of the world (block 13,5 and size 2x5): Financial income paid by resident institutional sectors to other institutional sectors with headquarters in other Italian regions or abroad. Flows of this type are, for example, dividends or bond coupons paid by resident companies to shareholders in rest of Italy or rest of the world, or interests paid by resident borrowers to non-residential lenders.

Regional and Rest of the world income balances (block 13,8 and size 2x1): Difference between the primary income flows and transfers (current and capital) from and to the rest of Italy and the rest of the world.

Current transfers from the rest of Italy and from the rest of the world (block 13,9 and size 2x17): Outward current transfers that the resident institutional sectors (by columns) pay to the rest of Italy and to the rest of the world (by rows). Example are remittances, pensions or foreign taxes, international aids paid by resident institutional sectors.

Capital transfers to the rest of Italy and to the rest of the world (block 13,12 and size 2x7): Outward capital flows such as direct investment capital taxes, capital donations or other type of capital transfers paid by residents.

2.3 Some definition aspects of regional SAMs respect to National one

The regionalization of a Social Accounting Matrix contains some definition problems: many flows that are well defined and estimated at national level are not commonly used at regional scale. Here it is a briefly list of those aggregates for which it is difficult to find a regional estimation: Interregional trade; Private Capital income (profits, dividends, interests) paid by enterprises resident in one region and earned by institutional sectors resident possibly in other regions; Interests on public debt of central government; Transfers (remittances, penalties, taxes, insurances, etc.) between institutional sectors of different regions.

At national scale these flows are registered into the balance of payments, but at regional levels it is very difficult estimate flows of these aggregates. Commonly for all these flows it is make an indirect estimation that is successively corrected using a simultaneous balancing procedure (Stone et al 1942) for the SAM of all Italian regions it is possible improve the quality and guarantee the confidence (with regional accounts) of the estimates of these aggregates at
regional scale. For all the listed aggregate we describe briefly the concept meaning and the estimation method.

Interregional trade is the exchange of goods from a production unit in an origin region to another production unit of a destination region, it is estimated by way of a gravity model applied of each good and an origin destination matrix of order 20 (number of Italian regions).

Private Capital Income (such as profits, dividends, interests) are income flows paid by enterprises (with the headquarter in an origin region to other institutional sectors possibly resident in other regions). These flows are estimated considering the localization of the enterprise’s headquarter (by origin region, type of enterprise, production sector and powered by employees) and the amount of financial assets possessed by institutional sectors resident into the destination region. The value of a single cell flow is obtained assuming independence from flow and regional distance.

Interest on public debt of central government are the amount of interests paid bay central government to institutional sector owner of governments bonds. The origin destination matrix is made considering the distribution of public debt between region (in accordance with ESA95 rules) and the residence of institutional sectors owners of government bonds. The value of a single cell flow is obtained assuming independence from flow and regional distance.

Interregional Transfer flows are all that flow paid by an institutional sector resident in an origin region to another sector possibly resident in other regions paid for several reasons (a penalty tax, a property tax, remittances, etc.). It is very difficult to estimate origin destination matrix of these flow, the way is to estimate the amount of marginal during the simultaneous SAM balancing procedure and to estimate matrix cell assuming independence from flows and regional distance.

3. Single-region CGE model

3.1 Theoretical framework

TUSCANI consists of a system of first order differential equations, represented by the general form: \( F(X)=0 \), where \( F \) is a vector of \( m \) twice differentiable functions of the \( n \) vector \( Z \), which is composed by the model variables. The \( m \) equations represent final demands for commodities, demand for intermediate inputs and primary factors; pricing equations relating commodity prices to costs; and, market clearing equations for primary factors and
commodities. In turn, such equations impose production-function constraints, equate demands and supplies, relate values of production to costs of production. The structure of the model is consistent with the neoclassical theory. The economy is a small and open one, populated by a continuum of measure one of risk neutral agents, who consume all of their disposable income, maximizing their utility function. On the supply side, firms produce output by combining intermediate inputs and factors of production, according to the available Leontief production technology. The small economy is endowed with labour and capital, combined through a CES function. Labour demand, in turn, is split among types, i.e. self-employed and employees, through a CES function. Factors are immobile and must be used for production within the region. Domestic supply of inputs competes with imports, from the rest of Italy and from the rest of the world, according to the Armington assumption (1969). On the commodity supplies, firms maximize profits subject to a CET function, taking prices as given. Profit maximization, in turn, dictates minimizing costs, with inputs demand responsive to prices. Output is exported to the other regions and to the rest of the world, or is consumed in the local market. The economy is on the frontier of the production possibilities, under the assumption of full employment. Perfect competition and free entry drive firms' profits to zero. All markets clear, and all budget constraints are binding.

From industry inputs to commodity outputs, the model allows for multi-production. The structure of production relies on the input-output separability assumption, and on a nested structure. The production sector is composed by j industry, with j equal to 37, according the production account of Tuscany. Industry j’s production takes place by means of a concave, strictly increasing, and differential production function f(z), which is modeled as a Constant-Elasticity-of-Substitution (CES) function. Although all industries share a common production structure, the use of the CES production function allows input proportions to vary between industries. Even more, the choice of a CES is justified by its flexibility. For specific values of its parameters, a CES function can come to represents Leontief or Cobb-Douglas functions.

Focusing on input decisions, firm choose the optimal composite of inputs subject to the production technology available in the economy. In other words, each firm j demand input i (z_{ij}) according to a minimization costs problem:

\[
\min \sum_i w_i z_{ij} \text{ subject to } f(z_{ij}) \geq q_j
\]

where w_i is the market price of input i (with i=1,...,54; or i=local, imported from ROI, imported from the ROW; or i=labour, capital; or i=self-employed, employees), and q_j represents industry j’s activity level (with j=1,...,37). Using the method of Lagrange multipliers, the optimal demand for inputs is found by mean of first-order conditions, which lead to the equality between marginal rate of technical substitution between inputs and their
economic rate of substitution. Because the production function \( f(z) \) is differentiable, the first-order conditions must hold for every input, and given the concavity of the production function, they are necessary and also sufficient for an optimum. In order to compute the competitive equilibrium price and output, the solutions of the optimization problems for each input are combined with market clearing conditions. It follows that, because the demand for inputs is homogenous of degree zero in the price vector, only relative prices are determined in equilibrium; and demand for each input will depend on the overall factor demand and on the relative price. This implies that changes in relative prices will induce substitution in favor of relatively cheapening inputs. From the system of first order differential equations it follows that demand for each input, \( z_i \), is expressed in terms of percentage changes. As a rule:

\[
 z_{ij} - a_{ij} = z_j - \sigma_{ij}(p_{ij} + a_{ij} + \sum_i S_{ij} p_{ij}) \text{ where } \sigma_{ij} \text{ indicates the elasticity of substitution between inputs, } S_{ij} \text{ is the share of good } i \text{ in the total input composite. The same form holds for commodities supply, with the only difference of a positive sign for the elasticity of transformation } \sigma.
\]

On the output supplies, in fact, the mix of produced commodities depends on the relative prices. Given output prices \( p \) and input prices \( w \), a profit maximizing production plan for firm \( j \) solve:

\[
 \max_z \sum_i p_i f(z_{ij}) - w_i z_{ij} \text{ subject to } q_j \leq f(z_{ij})
\]

with \( p_i \) price of produced commodities. The constraint holds because of no waste in production, therefore the solution lies on the frontier. Weierstrass theorem holds and so the equilibrium exists. Because of the concavity of firms' production functions, first-order conditions are both necessary and sufficient for the characterization of optimal commodity supply. To determine equilibrium prices only prices at which one of the markets clears is needed, the other markets will necessarily clear at these prices. This translates into a choice of a numeraire.

Turning to final demand, a representative household face a maximization problem structured in two nests: first she chooses how much of each good to consume; and then, the proportion of imported and domestic goods, through a CES function. The household demand is consistent with the following optimization problem:

\[
 \max_{x_i} U(x_i), \text{ subject to the budget constraint } c = p_i x_i, \text{ with } p \text{ price, } x \text{ quantity and } c \text{ the consumers budget; and to a CES function, } x_i = CES(x_{is}) \text{ with } s=\text{domestic, imported from other regions, imported from the rest of the world. In this model the utility is shaped by a}
\]
Klein-Rubin function, which has the features of non-homotheticity, allowing budget shares to vary in response to changes in relative prices and income. The total household demand results by adding up the subsistence constant demand, and the luxury demand. The subsistence consumption depends on the population and preferences. Luxury demand, in turn, is modeled through a Cobb-Douglas demand function, with luxury spending as a fixed portion of supernumerary income. Inter-regional and foreign demand for Tuscan exports are modeled via a commodity and destination specific constant elasticity demand schedules. Lastly, some balancing equation close the model. Under the assumption that commodities are not freely disposable, firms' outputs are fully consumed by households, and households' endowment of primary factors is in turn fully employed by firms. It follows that for each commodity the quantity produced by firms must equal the sum of the intermediate and final demands, in other words markets clear. Furthermore, the revenue gained by renting out primary factors balances the gross expenditure on the satisfaction of demands. This is the equivalence between aggregate disposable income and the value of utility, which implies the supply-demand balance condition for goods.

3.2 Model database structure

This paper presents a single-region sub-national comparative-static computable general equilibrium model of Tuscany, built on the regional SAM for 2008, provided by IRPET. In terms of methodology, this paper closely follows the theoretical and data structures of ORANI-G, the national CGE model of the Australian economy (see Dixon et al., 1997 and Horridge, 2003, for a full model description). The original ORANI’s structure has been modified in order to account for theoretical features typical of a regional economy (Giesecke, 2011); as well as to reflect the specific data structure of the Tuscan SAM, which represents the model’s database. In a regional economy, inputs of production are sourced from three different sources, namely local, the rest of the country, and the rest of the world; and output is exported to the rest of the region or to the rest of the word. Quite sensible is the setting of the environment for demand and price of the factors of production. Following Giesecke 2011, a dedicated regional labour theory and closure are modeled in order to account for simultaneous stickiness in short run employment and wages. Diverting from the ORANI-G structure, TUSCANI does not explicitly model margins, and has only labour and capital as primary factors. We decide to do not take land into account, because of no data availability. Furthermore, TUSCANI builds new dimensions around the standard core structure, by using the data information contained in the SAM. In particular,
building on block 1.6, described in section 1, households demand is split according to the COICOP expenditure functions; and government demand is further disaggregated according the COFOG classification. Focusing on households demand, the description use of income is included in the model, relying on block 6.9. In detail, summing up over the commodity dimension, the 12 COICOP expenditure functions are, in turn, linked to multiple households split into ten income percentiles. The analysis moves forward, by including the income re-distribution account, via block 9.9. Accordingly, all transfers from institutional sectors to households by income percentile are recorded.

In summary, the model identifies 54 products; 37 sectors, which represent the intermediate demand; 37 investors; multiple households, whose demands for commodities is additionally split according the individual consumption expenditure classification COICOP; a regional government, whose demand for commodities is additionally split according the functions of government's classification COFOG; inter-regional and foreign export demands for local commodities.

The CGE is set out in the SAM and benchmarked to allow rows and columns to equal. Most of the parameters of the system of linear equations are cost and sales shares, whose values are set to be consistent with a balanced SAM. The quantity variables equal the corresponding value in the SAM, and the flows of goods are expressed in terms of units of a single commodity, the so-called numeraire, whose price is taken to be a fixed reference point. The benchmark so computed represents the initial equilibrium of the economy. The system is then closed, shocked according to what policies are being modeled, and solved by conventional numerical integration technique for the endogenous results. The model is static, therefore no time dimension exists. The analysis is then split into short run and long run period, respectively via a short run and a long run closure. Results of a simulation for a counterfactual equilibrium are expressed as percentage changes deviation from the benchmark economy, with no track of the evolution path of the variables.

4. Conclusions

This paper shows and explains the development of a single-region CGE model for Tuscany, based on a regional SAM, which was provided by IRPET. The aim is to contribute to the understanding of how to build a CGE model database from a regional SAM. SAMs in fact provide a resourceful amount of information on the entire flow of the economy, at macro level, in a structure that perfectly suits the CGE modeling approach. Regional policy analysis, which suffers from data availability and that requires dedicated tools, can benefit from further research on this direction.
In term of issues investigated, CGE modeling delivers a detailed method of ex post analysis grounded in a comprehensive theoretical structure. By taking into account resource constraints and price responsiveness, CGE models can evaluate winners and losers from a specific policy. They have been proven to be a flexible approach, capable of investigating both national and regional-specific shocks; and of modeling a wide range of policy issues, such as fiscal, environmental, industrial, trade, as well as public projects or events. Support for regional CGE modeling can be found in several studies, particularly in those dealing with fiscal policy analysis. TUSCANI, in fact, has been built with the purpose of policy analysis, and future research will work in this direction. In particular, in light of the current austerity plan that Italy has implemented in the recent months, the effects of the fiscal multipliers on the economy will be investigated.

To conclude, although the complex and complete data information made available by the SAM, some words of caution are needed. For policy analysis purposes, a key point for a successful model is to provide transparent and easy to interpret results. It follows that a desirable strategy for a CGE modeler is to carefully develop the model’s structure in line with the aim of the analysis. This is achieved in this paper by focusing on the analysis of the household institutional sector, and that proves how helpful can be the marriage of SAMs with CGE models.

5. Bibliography


