Firm Size, Bank Size, and Financial Development

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Abstract

Financial intermediation facilitates economic development by providing entrepreneurs with external finance. The relative costs of financing depend on the relative efficiency of the financial sector and the sector using financial intermediation services, the real sector. These costs determine the occupational choices and the set of active firms in the financial and real sectors. A model of firm-size distributions in the financial and real sectors results. This model is calibrated to match facts about the U.S. economy, such as the interest-rate spread and the establishment-size distributions in the financial and real sectors. It is then used to evaluate the importance of the relative technological progress in the financial and real sectors for the dynamics of the average establishment size in the financial sector. The model accounts for 58% of the reduction in the average establishment size in the U.S. financial sector over 1986–2006 and for a 4.5 persons per establishment decline in the average size of the financial sector establishment in Taiwan over 1971–2011.

Keywords: economic development; financial development; technological progress; firm-size distributions; interest-rate spreads.

JEL Classification Numbers: E13; O11; O16; O41.

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1 Introduction

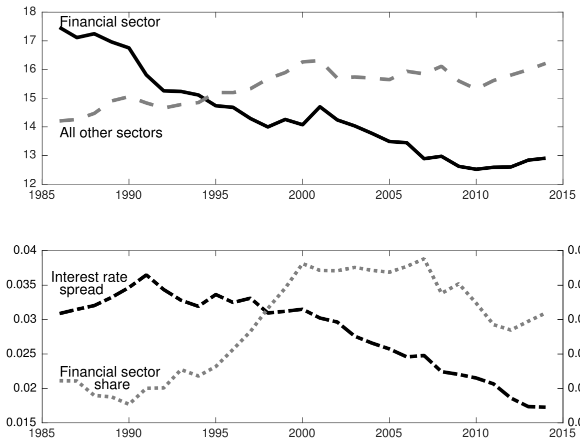
The importance of financial development for economic development has been widely discussed in the recent economic literature.[1] Most of the studies model the financial sector as consisting of competitive firms or introduce financial frictions as a borrowing constraint without explicitly considering the problem that financial intermediary solves when deciding on the allocation of funds.[2] As a result, the characteristics and the dynamics of financial sector establishments have not enjoyed much attention of researchers, meaning that a number of potential channels that influence the evolution of the financial sector and, consequently, economic development, remain obscure.

This study is motivated by four observations from the U.S. data (see Figure 1). First, the average size of the financial establishment measured by the number of persons engaged per establishment has been steadily decreasing over the last three decades. Second, the financial sector share of output has been growing over time. Third, the interest-rate spread has been slightly declining over the same time period. These three observations together suggest that the financial sector has become more efficient as compared to the other sectors of the economy, the fact that has been noted elsewhere, in particular, in Greenwood, Sanchez and Wang (2010) and Greenwood, Sanchez and Wang (2013). The fourth observation is about the average establishment size in all sectors other than finance: it has been increasing over the last three decades (see Figure 1). Thus, over time, the financial sector has been characterized by diminishing labor inputs and growing output, while all other sectors, on average, have been increasing

[1]For examples of early models connecting financial development and economic growth, see Greenwood and Jovanovic (1990); Bencivenga and Smith (1991); and King and Levine (1993b). King and Levine (1993a) and Levine, Loayza, and Beck (2000) are examples of early empirical assessments of financial development– economic growth causality links. Several channels through which the financial development influences economic development have been emphasized; for example, see Greenwood, Sanchez, and Wang (2010) and (2013) on the role of information costs; Erosa (2001), Antunes, Cavalcanti, and Villamil (2008), Amaral and Quintin (2010), and D'Erasmo and Boedo (2012) on the importance of limited enforcement and intermediation costs; and Chiu, Meh, and Wright (2017) on the role of intermediation in efficiency and innovation.

[2]One exception is Laeven, Levine, and Michalopoulos (2015) who model economic growth as an outcome of continuous innovations by profit-maximizing entrepreneurs and financiers. They do not consider the distribution of financial establishments though.

Figure 1: Average establishment size in the financial and other sectors, interest-rate spread, and the finance share in the U.S.



Note: The top panel presents the average establishment size measured by the number of persons engaged in the financial sector (black line) and in the real sector (grey dashed line). The bottom panel presents the interest-rate spread (black dashed line, values on the left y-axis) and the financial share of output (grey dotted line, values on the right y-axis). Data sources are described in the Appendix.

their labor inputs.

I explain these facts using a model that connects the relative efficiency of the financial sector and that of all other sectors, summarized as the real sector, to the establishment-size distribution in the financial and real sectors. The model explicitly considers the problem the profit-maximizing financiers (bankers) face when deciding on the amount of intermediated funds, along with the problem of the profit-maximizing entrepreneurs (producers) that borrow funds to produce final goods. A relatively faster growth in the financial sector technology results in more efficient and more competitive financial services, allowing to produce the same amount of loans with fewer persons employed and under a lower interest-rate spread. If the real sector advances relatively faster than the financial sector, it becomes more competitive, allowing the financial sector to extract higher rents and increase the interest-rate spread. Thus, the relative speed of technological progress in the financial and real sectors defines the dynamics of the firm size in each of these sectors.

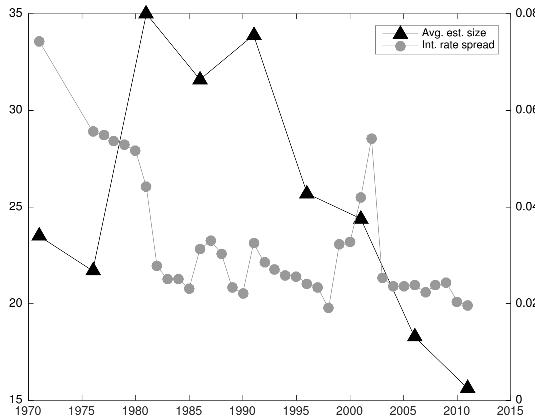
In the model, the production function in each of the sectors is characterized by decreasing returns to scale due to the managerial span-of-control (as in Lucas, 1978). The producers operating in the real sector convert capital and labor into the final good using the existing technology and their entrepreneurial skills. The bankers use households' saving deposits and labor as inputs in the production of financial services with output being the loans combined with monitoring of borrowers to prevent shirking on repayment. I consider the financial and real sector-specific technological progress processes as given and evaluate their impact on the decisions of the producers and bankers to start their own business rather than being hired as workers on the labor market. The speed of technological progress in each of the sectors determines the relative prices of deposits, loans, and final goods and, jointly with individual managerial abilities, defines the occupational choices of the potential producers and bankers and the level of output.

I estimate the parameters of the model by matching empirical moments about establishment-size distributions in the U.S. financial and real sectors in 1986. The model is then used to study the dynamics of the average establishment size in the financial sector and in the rest of industries (the real sector) in the U.S., as functions of the unobserved technological progress in these sectors. I capture the unobserved technological progress processes in the two sectors by mapping them into the observed values of interest-rate spread and real gross domestic product (GDP) per capita (as has been done in Greenwood, Sanchez and Wang, 2013). The model accounts for 58% of the reduction in the establishment size in the U.S. financial sector over 1986–2006. The predictions of the model are also consistent with the data when the model is extended to the open economy where the interest rate is taken as given (or, in other words, partial rather than general equilibrium setup is considered). In the open economy framework, the model is able to account for a rising financial sector share of output.

The availability of the data makes an application of the model to the U.S. very straightforward. One possible criticism of such an application is that the U.S. has been traditionally considered as an economy growing along the balanced growth path, therefore, one could question the significance of the trends observed in Figure 1. An interesting quantitative exercise would be to evaluate the predictions of the model in developing countries. One obstacle with applying the model to such countries is that the long time series of the establishment-size distribution by sector is not generally available.

I attempt to overcome this obstacle for Taiwan, for which a number of Industry Census data points on the establishment size by industry sector is available. The Industry

Figure 2: Average establishment size in the financial sector and the interest-rate spread in Taiwan



Note: The graph presents the average establishment size measured by the number of persons engaged in the financial sector (black line with triangles, values on the left y-axis) and the interest-rate spread (grey line with circles, values on the right y-axis). The triangles and circles represent available data points. Data sources are described in the Appendix.

Census data for Taiwan suggests that the average establishment size in Taiwanese financial sector decreased from 23.51 to 17.73 persons engaged per establishment, comparing years 1971 and 2006 (though the negative trend is not as definite as for the U.S.). At the same time, Taiwan experienced a sharp reduction in the interest-rate spread, from 7.44% in 1971 to 2.40% in 2006 (this fact has been documented and explored in Greenwood, Sanchez and Wang, 2013). The relevant data available for Taiwan is depicted in Figure 2. I apply the calibrated model to Taiwanese data and ask how much of the change in the average establishment size the model can match given the change in the interest-rate spread and the real GDP per capita in Taiwan, comparing years 1971 and 2006. The model predicts a fall in the average establishment size by 4.5 persons engaged, quite close to the number observed in the data.

The importance of financial development for external financing, occupational choice, and firm size has been empirically evaluated by Rajan and Zingales (1998) and Beck, Demirgüç-Kunt and Maksimovic (2006), among others, and clarified in the models by Barseghyan and DiCecio (2011); Greenwood, Sanchez and Wang (2010, 2013); Arellano, Bai, and Zhang (2012); Cooley and Quadrini (2001); Cabral and Mata (2003); Clementi and Hopenhayn (2006); Albuquerque and Hopenhayn (2004); and Buera, Kaboski, and

Shin, (2011, 2015), among many others. This paper follows the conventional approach employed by the literature to connect the financial development and economic growth by exploiting the consequences of the external finance provision for occupational choices and for the dynamics of establishment distribution. However, rather than concentrating on the establishment size in the economy overall, I discuss the dynamics of establishment size in the financial and all other sectors, and take the differences in these dynamics as signals of unequal relative efficiency in these sectors. The model suggests that the relative efficiency has impact on relative prices and on occupational choices, and therefore, the distinction between financial and other sectors establishment distributions can be useful for policy design and welfare considerations.

The rest of the paper is organized as follows. Section 2 presents a model which incorporates the profit-maximizing producers and bankers in a general equilibrium framework with exogenous sector-specific technological progress. Section 3 discusses the intuition behind the predictions of the baseline general equilibrium model and its partial equilibrium version. Section 4 provides quantitative analysis of the model and evaluates its relevance for the U.S. and Taiwanese data. Section 5 concludes.

2 The Model

The model economy consists of overlapping generations of heterogeneous agents. Each generation lives for two periods and is composed of two groups of agents, each group being of measure one. The first group represents "potential producers," the individuals who are able to run a firm that produces output of the final good. The second group consists of "potential bankers," the individuals who are able to run a financial intermediary institution.[3] The individuals from both groups can be hired as workers in either the productive or financial sector. The groups do not have common members.

All agents are born with zero assets, and work only in the first period of their life. They may save out of their first-period income to consume during the second period of their life. The only source of heterogeneity across agents within a group is their

[3]The assumption that producers and bankers belong to different groups is not crucial and is made to simplify the interpretation. All results hold if there is only group composed of potential bankers and producers.

ability to be a successful entrepreneur: to run a firm in the final goods sector if the individual belongs to group one, or to run an intermediary institution if the individual belongs to group two. The distribution of abilities in each group is time-invariant and characterized by cumulative distribution function $F(z)$ and probability density function $f(z)$ for $z \in [\underline{z}, \bar{z}]$.

The final good is assumed to be perfectly storable and transformable into capital at zero cost.

At the beginning of the first period of their life, the individuals decide whether to become an entrepreneur in their group or to be hired in the labor market as a worker. Those who decide to become entrepreneurs have a span of control to operate a decreasing returns-to-scale technology and choose the optimal amount of capital and labor inputs to hire, given expectations about the output that they can produce. The output is uncertain, with the probability of success depending on the ability of the entrepreneur. The members of the two groups interact in the competitive markets. The "bankers" intermediate transfers of capital from savers to "producers," and all entrepreneurs hire labor. The financial intermediaries arise to mitigate information asymmetries and to screen entrepreneurs—the tasks the savers cannot perform. All agents receive their income and decide on savings at the end of the first period of their life.

The problem of the individuals in each group, the role of abilities, and the markets are described in more detail below.

2.1 The problem of a "potential producer"

Each individual from the group of potential producers decides whether to run a firm and produce output in the form of final goods, or to be hired as a worker in the labor market. The decision is made based on the expected payoffs of these occupational choices.

The technology that a potential producer can operate has the following form:

$$A(k^a l^{1-a})^q, \tag{1}$$

where k and l are capital and labor hired by the entrepreneur; $a \in (0, 1)$ is the capital share; $q \in (0, 1)$ is the span of control parameter; and $A > 0$ represents the real sector's state of technology.

Given that entrepreneurs start life with zero assets, they have to borrow capital to run their firms. The borrowing is complicated by two factors: the ultimate success of the entrepreneur's project is uncertain, and the entrepreneur can hide the final outcome of his production project.

The probability of success of the project $\pi(z)$ is increasing and concave in the ability of the potential producer z, that is, $\pi_z > 0$ and $\pi_{zz} < 0$. With probability $1 - \pi(z)$ the project is unsuccessful in the sense that no output is produced.

The financial intermediaries (individuals from the second group who chose to become entrepreneurs) are able to identify the ability of the producers, and thus, to estimate $\pi(z)$ correctly. They issue loans at the risk-adjusted competitive interest rate $r_e/\pi(z)$.

The producer borrows capital from the financial intermediaries and hires labor at a competitive expected wage w, before he knows if his project is successful. Once the firm's inputs are employed, a random draw from uniform distribution on [0,1] determines if the project is successful. The entrepreneur can hide the successful realization of his project with probability $1 - P$, which depends on the level of financial development in the economy and will be defined below. If the project is successful, the entrepreneur produces final goods according to the technology (1) and repays the loan conditional on successful monitoring by the intermediaries. If the project is unsuccessful, the entrepreneur announces bankruptcy and does not repay the loan to the financial intermediaries. For simplicity, the liquidation value of the bankrupt firm is zero.

The maximization problem of the producer is the following:

$$\max_{k,l} \; E\Pi_e = \pi(z) \left[(k^a l^{1-a})^q A - \frac{r_e P}{\pi(z)} k \right] - wl. \tag{2}$$

The solution to this problem characterizes optimal capital and labor inputs as functions of producer's ability. If the project is successful, the entrepreneur hires labor according to the labor demand:

$$l(z) = L_e \pi(z)^{\frac{1}{1-q}}, \tag{3}$$

where

$$L_e = \left(\frac{w^{1-aq} r_e^{aq} P^{aq}}{qA(1-a)^{1-aq} a^{aq}} \right)^{\frac{1}{q-1}}. \tag{4}$$

The optimal capital is proportional to labor and given by

$$k(z) = \frac{aw}{(1-a)r_e P} l(z). \tag{5}$$

The expected profits of the potential entrepreneur with ability z can be expressed as follows:

$$E\Pi_e(z) = \pi(z)^{\frac{1}{1-q}} \frac{wL_e}{(1-a)} \left(\frac{1}{q} - 1 \right). \tag{6}$$

Each potential producer decides whether to undertake an entrepreneurial project with expected payoff $E\Pi_e(z)$ or to become a worker with expected payoff w. Given that the expected profits are monotone increasing in ability, there is a threshold ability z_e^* above which all potential producers undertake an entrepreneurial project, and below which all potential producers become workers. This threshold can be found from the following equation:

$$w = E\Pi_e(z_e^*). \tag{7}$$

Given z_e^*, the total expected profits of all operating producers are given by:

$$TotalE\Pi_e = \frac{L_e w}{(1-a)} \left(\frac{1}{q} - 1 \right) \int_{z_e^*}^{\bar{z}} \pi(z)^{\frac{1}{1-q}} f(z) d(z). \tag{8}$$

The potential producers with ability lower than z_e^* choose to become workers, so that the total supply of labor from the group of potential producers is given by $\int_{\underline{z}}^{z_e^*} f(z)dz$.

Given the occupational choices, the average size in terms of labor of the real sector firm is given by the ratio of the labor demand to the measure of operative firms:

$$Avg.Size_{real} = \frac{\int_{z_e^*}^{\bar{z}} l(z) f(z) dz}{\int_{z_e^*}^{\bar{z}} f(z) dz}. \tag{9}$$

2.2 The problem of a "potential banker"

Each individual from the group of potential bankers decides whether to run a financial intermediary institution or to be hired as a worker in the labor market. The decision is made based on the expected payoffs of these occupational choices. If the potential banker runs a financial intermediary institution, he can make profits by intermediating the funds from savers to borrowers, the operating producers. The bankers buy deposits, d, on the deposits market at a competitive deposit interest rate, r_b, and sell loans to the producers at the competitive loan interest rate as defined above. Each potential banker can operate a common to the financial sector technology, which allows him to correctly identify the ability of the borrower to run a firm and to monitor borrowers to reduce the probability $1-P$ of their hiding the successful realizations of projects.

Monitoring requires labor input; therefore the bankers also hire workers in the labor market. The success of the monitoring depends positively on the banker's ability, z, and labor input, x, and depends negatively on the volume of intermediated funds, d. In particular (similar to Greenwood, Sanchez, and Wang, 2010), the probability of successful monitoring, P, is given by:

$$P = 1 - \frac{1}{(\frac{zTx^{\gamma}}{d})^{\psi}},\ Tx^{\gamma}/d > 1/z,\ \psi, \gamma \in (0,1), \tag{10}$$

where $T > 0$ represents the financial sector's state of technology. The inequality implies that the technology-augmented labor effort for monitoring, adjusted for the amount of resources monitored, must exceed the inverse of the ability of the banker to insure a positive probability of successful monitoring.

The maximization problem of the banker is the following:

$$\begin{aligned} \max_{d,x}\ E\Pi_b &= \left(1 - \frac{1}{(\frac{zTx^{\gamma}}{d})^{\psi}}\right) r_e d - r_b d - wx, \\ s.t. &: \ Tx^{\gamma}/d > 1/z. \end{aligned} \tag{11}$$

The solution to this problem characterizes optimal deposits and labor inputs as functions of the banker's ability. In particular, the banker hires labor according to the labor demand:

$$x(z) = L_b z^{\frac{1}{1-\gamma}}, \tag{12}$$

where

$$L_b = \left(\frac{w r_e^{1/\psi} (1+\psi)^{1/\psi+1}}{\psi\gamma T (r_e - r_b)^{1/\psi+1}} \right)^{\frac{1}{\gamma-1}}. \tag{13}$$

The optimal amount of deposits is proportional to labor demand and defined as follows:

$$d(z) = \frac{(1+\psi)w}{\psi\gamma(r_e - r_b)} x(z). \tag{14}$$

The expected profits of the potential banker can be expressed as follows:

$$E\Pi_b(z) = z^{\frac{1}{1-\gamma}} L_b w \left(\frac{1}{\gamma} - 1\right). \tag{15}$$

Each potential banker decides whether to run an intermediary institution with expected payoff $E\Pi_b(z)$ or to become a worker with expected payoff w. There is a threshold ability z_b^* above which all potential bankers run a financial intermediary institution, and below which all potential bankers become workers. This threshold can be found from the equation:

$$w = E\Pi_b(z_b^*). \tag{16}$$

Given the occupational choices, the average size in terms of labor of the financial sector firm is given by the ratio of the labor demand to the measure of operative firms:

$$Avg.Size_{financial} = \frac{\int_{z_b^*}^{\bar{z}} x(z)f(z)dz}{\int_{z_b^*}^{\bar{z}} f(z)dz}. \tag{17}$$

The total expected profits of all operating bankers are given by:

$$TotalE\Pi_b = L_b w \left(\frac{1}{\gamma} - 1\right) \int_{z_b^*}^{\bar{z}} z^{\frac{1}{1-\gamma}} f(z)dz. \tag{18}$$

The potential bankers with ability lower than z_b^* choose to become workers, so that the total supply of labor in the labor market from the group of potential bankers is given by $\int_{\underline{z}}^{z_b^*} f(z)dz$.

Substituting the expressions for labor and deposits demand by a financial intermediary, I obtain that the probability of success that each operating banker faces in equilibrium depends only on the prices of capital:

$$P = 1 - \frac{1}{(\frac{zTx^\gamma}{d})^\psi} = \frac{\psi r_e + r_b}{r_e(1+\psi)}. \tag{19}$$

For positive interest-rate spread, $r_e - r_b$, P is bounded between zero and one.

Therefore, at optimum, the probability of successful monitoring is the same across all active financial intermediaries. The intermediaries with less ability to monitor borrowers will optimally choose to intermediate fewer funds.

The common probability of successful monitoring makes all financial intermediaries identical from the point of view of both savers and borrowers. The set of active financial intermediaries represents a homogeneous financial system that accepts deposits and issues loans, performing screening and monitoring along the way. Depositors can invest in, and producers can borrow from, several financial intermediaries within a period.

2.3 The saving decision and assumptions

At the end of the first period of their life, all individuals in the economy decide whether to save some of their income for the second period of life. Given that there is a possibility that an individual earns zero income in the first period of life (as a worker in an unsuccessful firm or as an unsuccessful entrepreneur), I simplify the discussion by assuming that the agents are risk-neutral and save all of their income if the expected rate of return is positive. Thus, the total supply of funds in the deposit market at a given period of time is given by the sum of total realized profits (in expectation equal to the total expected profits) of all the workers, bankers, and producers, operating during the preceding period of time.

The total expected output in this economy is given by the sum of all profits plus the interest on total savings, or by the total expected output in the real sector:

$$EY = A\left(\frac{a(1+\psi)w}{(1-a)(\psi r_e + r_b)}\right)^{aq} L_e^q \int_{z_e^*}^{\bar{z}} \pi(z)^{\frac{q}{1-q}} f(z) d(z). \tag{20}$$

To facilitate the characterization of equilibrium in this economy, several assumptions must be imposed. The first assumption imposes a particular distribution of abilities. The second assumption ensures that, given the assumed distribution, all expected profits are finite and positive, and the equilibrium can be computed analytically. The third assumption will be useful in the discussion of the uniqueness of equilibrium in the model economy.

Assumption 1: Abilities in each group follow Pareto distribution of the following form: $F(z) = 1 - z^{-v}$, $f(z) = vz^{-v-1}$, where $v > 0$, $\underline{z} = 1, \bar{z} = \infty$.

Assumption 2: Assume the function governing the success of the production project is given by the following expression: $\pi(z) = (1 - z^{-v})^{\mu}$, where $\mu > 0$; moreover, $1/(1-\gamma) - v < 0$.

Assumption 3: To ensure the existence of the equilibrium, assume that the following conditions hold: $1/\psi + 1 - 1/v - 1/(1-aq) > 0$, $(1-aq)^2(v-1) - va\gamma q > 0$, $\frac{(1-aq)(1-a)}{a\gamma}(1/\psi + 1 - 1/v) - aq > 1$.

2.4 The equilibrium

In equilibrium, all markets clear. In particular, at the edge of every two periods the total amount of deposits collected by the newly born financial intermediaries is equal to the total realized profits of all agents who earned labor or entrepreneurial income and are moving to the second period of their life:

$$DepositsMarket: \int_{z_b^*}^{\bar{z}} d(z)f(z)dz = Total\Pi_e + Total\Pi_b + Total\Pi_w. \tag{21}$$

During each period, the loans market clear: the total amount of loans offered by financial intermediaries is equal to the total demand for capital by the producers,

$$LoansMarket: \int_{z_e^*}^{\bar{z}} k(z)f(z)dz = \int_{z_b^*}^{\bar{z}} d(z)f(z)dz. \tag{22}$$

The labor market clears: the total demand for labor by producers and financial intermediaries is equal to the total supply of labor,

$$LaborMarket: \int_{z_e^*}^{\bar{z}} l(z)f(z)dz + \int_{z_b^*}^{\bar{z}} x(z)f(z)dz = \int_{\underline{z}}^{z_e^*} f(z)dz + \int_{\underline{z}}^{z_b^*} f(z)dz. \tag{23}$$

The market-clearing conditions define the prices r_b, r_e, and w. More formally, a competitive equilibrium is defined as follows.

Definition: *A competitive equilibrium given A and T is described by the thresholds z_e^*, z_b^*, allocations $\{k(z), l(z)\}_{z_e^*}^{\bar{z}}$, $\{d(z), x(z)\}_{z_b^*}^{\bar{z}}$, wages w, and interest rates r_e, r_b, such that,*

- given w, r_e, r_b, all agents maximize their utility by choosing their occupation and savings;

- given w, r_e, r_b, all producers and bankers maximize their profits; and

- the markets for capital, labor, and deposits clear.

It is possible to show that there exists a unique equilibrium with positive interest rates in the described economy.

Proposition 1: Given A, for all $T > \frac{(\psi v - \psi + \gamma v)^{1/\psi}\left[\frac{qAva\gamma}{v-1} + qA(\frac{1}{q} - a)\right]^{\frac{1}{1-aq}}\left(\frac{\frac{1}{q}-1}{1-a}\right)^{\frac{1-q}{1-aq}}}{(v(1-\gamma)-1)^{-\frac{1}{v}}(\gamma v)^{1/\psi+1}\left[\frac{a\gamma}{(1-a)(v-1)}\right]^{1-1/v}\left(\frac{1}{\gamma}-1\right)^{1-\gamma}}$, the equilibrium with positive interest rates and positive interest-rate spread exists and is unique.

(All proofs are in the Appendix).

In what follows, only the equilibrium with positive interest rates and positive interest-rate spread will be analyzed. Given that there exists a solution for the economy at a given point in time, it is possible to construct a growth path using this solution. Depending on the relative pace of technological progress in the real and financial sectors, the growth path may be balanced or unbalanced. The relative growth rates of the real and financial sectors' technologies A and T define the evolution of the economy over time: the thresholds z_e^*, z_b^*, the distribution of firms in each of the sectors, the prices r_e, r_b, w, and the level of efficiency of the financial system P.

Balanced growth

Proposition 2: Let T grow at rate g and A grow at rate $(1+g)^{1-aq}-1$. There exists a balanced growth path where the wages, output, capital demand, and deposits all grow at rate g. The thresholds z_e^* and z_b^*, labor demand and supply and interest rates remain constant.

This result is similar to the conclusion of Greenwood, Sanchez, and Wang (2010) that a balanced development of the real and financial sectors does not make the financial sector more efficient. The probability of catching the firm that misrepresents its earnings is constant over time. The number of active firms in both sectors does not change over time.

Unbalanced growth

An unbalanced growth occurs whenever technology in either sector outpaces the balanced growth of the other sector's technology. Intuitively, faster technological progress in the financial sector makes it relatively more efficient in comparison to the real sector (P increases). The relative cost of monitoring producers drops, leading to relatively higher competition for deposits, higher interest rates for deposits, lower interest rates for loans, and crowding out of the least efficient financial intermediaries. At the same time, a greater supply and lower price of funds makes borrowing affordable to less efficient producers. Therefore, technological progress in the financial sector leads to worsening of the pool of borrowers from the real sector.

The opposite occurs when the real sector's technology grows relatively faster than the financial sector's technology: the demand for loans increases, increasing their price and crowding out the least efficient producers. High demand for loans makes financial intermediation profitable for less efficient financial intermediaries, and reduces the relative efficiency of the financial system.

This intuition is formalized in the following proposition.

Proposition 3: In equilibrium, given A (T), z_e^* decreases (increases) and z_b^* increases (decreases) with a rise in T (A); labor demand in the real sector increases (decreases) in T (A); labor demand in the financial sector decreases (increases) in T (A); the interest rate on deposits increases (decreases) and the interest rate spread shrinks (expands) with a rise in T (A).

Corollary 1: Let A grow at rate $(1+g)^{1-aq}-1$, and T grow at rate $g' < (>)\ g$ and T, g' being such that the equilibrium exists. The threshold z_e^* increases (decreases) over time, the threshold z_b^* decreases (increases), and the interest-rate spread increases (decreases) over time.

Whenever one sector's technology growth outpaces that of the other sector's, the economy experiences unbalanced growth, with an increasing number of entrepreneurs in the slower growing sector and a decreasing number of entrepreneurs in the faster growing sector.

3 Discussion

The model has a number of features that makes it easily applicable to the data for the evaluations of the relative efficiency of the financial and other sectors and its implications for the establishment-size distributions.

First, the unbalanced growth path has straightforward implications regarding the establishment size in the model. Note that the average size of the establishment and the total number of establishments are defined by the same parameter, the threshold z_e^* or z_b^*, in the real and financial sector, respectively. A lower value of z_e^* (z_b^*) implies that there are more establishments operating in the real (financial) sector and more labor demanded by that sector. In terms of relative efficiency, a lower value of z_e^* (z_b^*) implies that the real (financial) sector is less efficient, and this is reflected in a slower growing sector-specific technology.

Second, note that the technology of the financial sector T includes the factors that make the financial monitoring more efficient, and as formulated, is incomparable with the Solow residual, commonly reported as an estimate of the sector technology. That is, T represents an unobserved technological progress. Given the predictions of the model, this unobserved process can be estimated given the observable variables, such as the interest-rate spread and the output in the economy.

Third, the model generates testable predictions regarding the dynamics of the financial sector share of output, wages, and interest rates on deposits and loans. The baseline model is a general equilibrium model where the interest rate on deposits is determined on the loanable funds market and depends on the amount of savings supplied. As constructed, such a model predicts that the capital to output ratio should decline and the interest rate on deposits should increase when the financial sector becomes more efficient. This happens, because the improvement in the financial sector technology releases labor and increases competition in the labor market, leading to a fall in wages. This implies that the total savings as a share of output decrease. Given that the total savings equal the total intermediated funds in this general equilibrium closed economy, the share of the financial sector in output decreases as the financial sector becomes more efficient compared to the real sector. This is not what is observed in reality, where the share of financial sector has been increasing over time (see Figure 1).

This inconsistency can be solved if the model is considered in a partial equilibrium, with the interest rate on deposits taken as given by the bankers. The partial equilibrium version can be viewed as an extension to the open economy, where the amount of intermediated funds is not restricted by the domestic savings.

4 Quantitative Analysis

I take the model to the data to evaluate its relevance in explaining the relationship between the relative efficiency of the financial sector, the interest-rate spread, and the dynamics of the establishment size in the financial sector. In particular, I ask how much of the decline in the average number of persons engaged per establishment in the U.S. financial sector, depicted in Figure 1, can be explained by the change in the relative efficiency of the financial and real sectors, as reflected in the dynamics of the interest-rate spread and the real GDP per capita. As a baseline period for quantitative analysis, I choose the time period from 1986 to 2006, during which the interest-rate spread was declining. In addition, I evaluate the predictions of the model before and after this period, to capture the episodes of the rising interest-rate spread (before 1986) and of the recession following the financial crisis (after 2006). Furthermore, I use the calibrated model to evaluate its prediction regarding the decline in the Taiwaneese financial sector establishment size, depicted in Figure 2.

4.1 Calibration

The model is defined by a set of parameters which can be grouped into those characterizing the distribution of abilities (v), the real sector production function (these parameters include α, q, and μ), those characterizing the financial sector production function (these parameters include γ and ψ), and the parameters characterizing the relative efficiency of the financial and real sectors (T and A, respectively). Some of the parameters characterizing the real sector are standard and thus can be chosen on the basis of a priori information. They are given conventional values. In particular, capital's share of income, α, is chosen to be 0.35 (as in Greenwood, Sanchez and Wang, 2013). The span of control parameter q is set to 0.80 (as in Gruner et al., 2008). In addition, I impose an upper bound on the maximum ability and set $\bar{z}$=500 for the financial sector and $\bar{z}$=10000 for the real sector.[4]

The remaining parameters do not have standard numerical counterparts in the literature and, therefore, must be estimated.[5] These parameters are chosen to minimize the distance between a set of facts characterizing the U.S. data in 1986 and the model's predictions for these facts. This is done subject to a constraint that requires that the selected values for A and T must generate the observed level of real GDP per capita

[4]The restriction on the maximum ability in the financial sector is important for the dynamics of the average establishment size; increasing $\bar{z}$ makes the financial sector establishment size less responsible to the changes in relative technology. I choose not to calibrate $\bar{z}$ because its calibration leads to the corner solutions where there is only one operating banker with ability $\bar{z}$.

[5]Greenwood, Sanchez and Wang (2013) estimate the parameters governing the intermediary's monitoring technology ψ and γ in a setup with a competitive financial sector. I obtain very similar estimates for these parameters.

and the interest-rate spread in 1986.[6]

Similar to Greenwood, Sanchez, and Wang (2013), I choose the parameters to minimize the distance between seven points characterizing the distribution of employment by establishment size (the seven points are corresponding to the first 60, 75, 87, 95, 98, 99.3, and 99.7% of the total number of establishments), in the model and in the data. Given that I distinguish between the financial and real sectors in the model, I fit the model to the data on the distribution of establishments in each of these sectors. In addition, I target the average establishment size, measured by the average employment per establishment, in the financial sector.[7] Thus, I select 4+2 parameters on the basis of 7+7+1+2 data targets. Table 1 summarizes all parameter values and the targets. Figure 3 presents the establishment-size distributions in the financial sector and all other sectors, summarized as the real sector, in the 1986 U.S. data and in the model. The model generates the financial sector distribution quite close to that observed in the data, and the fit is quite robust to changes in the model parameters. The model-generated distribution of the real sector establishment size is determined by the value of parameter μ; the model fits well the lower tail for the distribution but underestimates the share of employment by the largest establishments.

4.2 Results for the U.S.

Given the parameters, I evaluate the predictions of the baseline model regarding the average size of the financial sector establishment given the interest-rate spread and the real GDP per capita. The results are presented in Column 2 of Table 2, which compares the average size of the financial sector establishment in the data and in the model. The observation for 1986 is used as a target, therefore the values for the data and the

[6]Fitting the model to the data from any other year within 1986-2007 period yields very similar results.

[7]The model cannot simultaneously reproduce the average employment in the real sector and the establishment-size distribution in the real sector (partially because the values for the parameters a and q are standard and fixed). Therefore, the former is not a target in the calibration. Targeting the average size of establishment in the real sector rather than its distribution results in a much smaller μ, around 3.93, but does not affect the resulting statistics about the characteristics of the financial sector.

Table 1: Parameters.

Parameter	How obtained
$\alpha = 0.35$	Literature
$q = 0.80$	
$v = 2.104$	Calibrated to fit
$\mu = 1313$	est. size distributions
$\gamma = 0.594$	and avg. size of financial
$\psi = 0.895$	sector establishment
A	To match output per capita
T	To match interest-rate spread

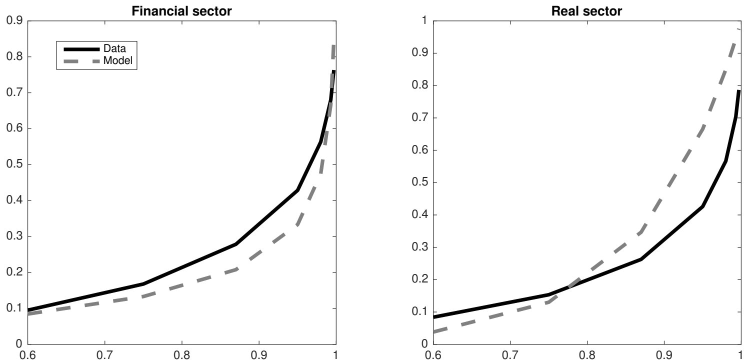
model are the same. The model predicts a decline in the average financial establishment size during the period 1986 to 2006 and further decline during 2006–2014, consistently with the data. The model also predicts a smaller average size of the financial sector establishment in 1977 compared to that in 1986, consistent with the data.

The changes in the establishment size predicted by the model are smaller than the changes observed in the data, though. The reason is that, in the model, the elasticity of the demand for labor and, therefore, the firm size, are determined by the same variables as the average firm size and its distribution. The model parameters that replicate the average firm size and its distribution are such that the response to the changes in interest-rate spread are relatively small.[8]

The penultimate row of Table 2 reports the change in the average establishment size in all sectors other than financial, summarized as the real sector, over the entire time period considered, in the data and in the model. The model predicts an increase in the average size of the real sector establishment, consistent with the data, but underestimates the

[8]Even if one of the calibration targets includes the change in the establishment size for a given time period, the admissible parameter values do not generate a sufficiently large change, that is, cannot fit such a target.

Figure 3: Establishment-size distributions in the financial and other sectors, model and the U.S. data, 1986



percentage of the increase observed in the data.[9] The last row of Table 2 reports the change in the financial sector share of output over the entire time period considered. The financial sector share increased by 69.62% comparing 1977 and 2011; the baseline model which is the general equilibrium model predicts a decline in the financial share of output, by 0.222%. As discussed above, the general equilibrium forces the capital share of output to decline as a consequence of an increased efficiency of the financial sector (caused by a faster technological progress in this sector relative to the technological progress in the real sector).

Next, I evaluate the performance of a partial equilibrium version of the economy with the same values of parameters as those used for the general equilibrium version. I fix the interest rate on deposits at the value obtained from the model matched to the U.S. data in 1986 and take it as given in the computations for all other time periods. The results for the average establishment size are presented in Column 3 of Table 2. These results are very similar to those obtained for the general equilibrium (baseline) model. Nevertheless, the partial equilibrium version of the model delivers wages and the financial sector share of output growing over time, consistent with the data. A model-predicted increase in the financial sector share of output over the time period considered is 2.064%.

The results from columns 2 and 3 of Table 2 suggest that there is no much gain in

[9]Even if the real sector establishment size is used as the main target in the calibration, the dynamics of the technological processes A and T, as dictated by the interest-rate spread and the real GDP level, results in a very small change in the real sector establishment, less that 1%.

Table 2: The moments in the data and in the model, the U.S.

Years	Data	Model	
		general eq.	partial eq.
	(1)	(2)	(3)
	avg. est. size, financial sector		
1977	13.51	16.38	16.41
1986	17.47	17.47	17.47
2006	13.45	16.68	16.71
2011	12.59	16.09	16.15
2014	12.91	15.50	15.58
	change in avg. est. size, real sector		
1977–2014	15.63%	0.191%	0.177%
	change in financial sector output share		
1977–2014	69.62%	-0.222%	2.064%

terms of the model's explanatory power when using a general equilibrium version rather than a partial equilibrium version of the model. Moreover, the partial equilibrium model is able to account for the existence of an upward trend in the financial sector share of output (although it significantly underestimates the trend), besides accounting for the downward trend in the financial sector average establishment size.

4.3 Results for Taiwan

Finally, I evaluate the predictions of the model for Taiwan, a country which experienced an unbalanced growth with a sharp decline in the interest-rate spread during the period from 1970s to 2006 and for which some data on the establishment-size distribution by sector is available. In particular, the information on the average establishment size by sector is available for every fifth year starting from 1971 and ending in 2011 (the complete information that would allow to recover the establishment distribution is available for every fifth year starting from 1996). Figure 2 shows that during the period from 1971 to 1976 there was a significant decline in the interest rate accompanied by the decline in the average size of the financial sector establishment. After a shape rise in 1981, the average financial sector establishment size continued to decline until 2011.

Table 3: The moments in the data and in the model, Taiwan

Years	Data	Model	
		general eq.	partial eq.
	(1)	(2)	(3)
	avg. est. size, financial sector		
1971	23.51	20.80	20.80
1976	21.74	19.65	19.77
1986	31.61	17.52	17.78
2006	17.73	16.58	16.87
2011	15.65	15.93	16.24
	change in avg. est. size, real sector		
1971–2011	-23.92%	2.017%	1.946%
	change in financial sector output share		
1971–2011	59.84%	-2.322%	7.361%

I use the parameter values calibrated for the U.S., as reported in Table 1, and ask how much of the change in the average size of the financial sector establishment in Taiwan can be explained by the model, given the Taiwanese data on the interest-rate spread and real GDP per capita. The model does not account for a rise in the establishment size during 1981–1991, but it accounts for a significant fraction of the decline in the establishment size when comparing the 1971 and 2011 data points. The predictions of the baseline (general equilibrium) model are reported in Column 2 of Table 3. The average establishment size in the Taiwanese financial sector decreased from 23.51 persons engaged per establishment in 1971 to 17.73 persons engaged in 2006. The model predicts a change from 20.80 to 16.58 persons engaged per establishment, given the interest-rate spreads and the real GDP per capita for these years.

The last two rows of Table 3 report the changes in the average size of the real sector establishment and the financial sector share of output, in the data and in the model. Qualitatively, the predictions of the model for Taiwan are similar to those for the U.S. Differently from the U.S., the average size of the real sector establishment in Taiwan decreased together with the average size of the financial sector establishment. The model cannot account for this fact (which could be caused, for example, by industrial improvements through the factors other than those summarized in A, the technology in the model).

The partial equilibrium version of the model can be viewed as more relevant for Taiwan, a small open economy. I re-estimate the model keeping the interest rate on deposits fixed and varying the interest rate on loans to clear the market for loans. The results are reported in Column 3 of Table 3. Similar to the results for the U.S., the partial and general equilibrium versions of the model deliver very similar predictions regarding the financial sector establishment size. The partial equilibrium version of the model, differently from the general equilibrium version, generates an increase in the financial sector share of output, but not as significant as the increase observed in the data (see the last row of Table 3).

The quantitative analysis suggests that the model is able to account for a significant fraction of the decline in the average size of the financial sector establishment over the last three decades in the U.S. and Taiwan. The general and partial equilibrium versions of the model deliver very similar predictions regarding the establishment-size distribution, but only the partial equilibrium version is able to generate an increase in the financial sector share of output. For Taiwan, there is a number of observations in the data that cannot be explained by the model. I attribute these observations to the nature of the unbalanced growth path experienced by Taiwanese economy during the time period considered.

5 Conclusions

It has been generally recognized that financial development is important for economic development. This paper evaluates the importance of the relative speed of the financial and real sector development, captured by the speed of technological progress in each of these sectors. The model that incorporates the decision-making by the profit-maximizing producers and bankers is developed and calibrated to the U.S. economy. The model replicates a significant fraction of the decline in the average size of the financial sector establishment and attributes this decline to the increased relative efficiency of the financial sector.

This paper's findings have an important implication: the unobserved sector-specific technological progress can be estimated from the observed firm-size distributions. The trends in the dynamics of sector-specific firm-size distributions can signal incommensurate sector-specific technological progress processes.

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Appendix

Data Sources

Establishment-size distribution: I look at the distribution of establishment by size measured as the number of person engaged by the establishment. For the U.S., I rely on the data from County Business Patterns from the U.S. Census Bureau. This data with detailed decomposition by sector is available for years 1986–2014 at: https://www.census.gov/data. The "Business Dynamics Statistics" section of the census webpage offers a summary of establishment characteristics by sector for years 1977–2014; however, the resulting distributions are much less skewed, and I refer to the County Business Patterns data instead to make the results comparable with the study by Greenwood, Sanchez, and Wang (2013) and with the corresponding data for Taiwan. For 1977, I estimate the average establishment size in the real and financial sector using the regression results of the County Business Patterns data on the Business Dynamics Statistics data. For Taiwan, I use Industry and Service Census data from the National Statistics of Republic of China (Taiwan), available at: http://eng.stat.gov.tw. The summary data is available for years 1971, 1976, 1981, 1986, 1991, 1996, 2001, 2006, and 2011. The complete data on the size distribution of establishment by sector in Taiwan is available starting from 1996.

Interest-rate spread: I compute the interest-rate spread for the U.S. as suggested by Mehra et al. (2009) and used by Greenwood, Sanchez and Wang (2013). I strictly follow the rules described by Mehra et al. (2009) and use the data sources suggested by these authors. The resulting interest-rate spread path is slightly different from the path reported by Greenwood, Sanchez and Wang (2013) which might be due to the adjustment in data measurements that occurred over time (I use the NIPA Tables last revised on August 3, 2017 and the Financial Accounts of the United States issued in 2017). For Taiwan, the interest-rate spread is computed as in Lu (2013), using the data from the Central Bank of the Republic of China (Taiwan), available at: https://www.cbc.gov.tw/ct.asp?xItem=30010&CtNode=517&mp=2.

Real GDP per capita: I use the real GDP divided by population, both series taken from the Penn World Tables (Feenstra et al., 2015).

Financial Industry Share: This variable is computed using the industry output data from the U.S. Bureau of Economic Analysis website. For Taiwan, the financial industry share is computed using the industry output from the National Statistics of Republic of China (Taiwan).

Proofs

Lemma 1. Under Assumption 1, the equations characterizing the economy simplify as follows.

i) The thresholds z_e^* *and* z_b^* *are as follows:*

$$z_e^* = \left(1 - \left(\frac{L_e(\frac{1}{q}-1)}{1-a}\right)^{\frac{q-1}{\mu}}\right)^{-1/v}, \tag{24}$$

$$z_b^* = L_b^{\gamma-1}\left(\frac{1}{\gamma}-1\right)^{\gamma-1}. \tag{25}$$

ii) The labor demand by the producers and financiers is given by the following equations:

$$L(z_e^*) = \int_{z_e^*}^{\bar{z}} l(z)f(z)dz = \frac{L_e(1-q)}{(1+\mu-q)}\left(F(\bar{z})^{\frac{1+\mu-q}{1-q}} - F(z_e^*)^{\frac{1+\mu-q}{1-q}}\right) =$$
$$\frac{(1-a)q}{(1+\mu-q)}\left(F(z_e^*)^{\frac{1}{q-1}} - F(z_e^*)^{\frac{\mu-q}{1-q}}\right), \tag{26}$$

where the last line follows from the fact that $L_e = \pi(z_e^*)^{\frac{1}{q-1}}\frac{(1-a)q}{1-q}$.

$$X(z_b^*) = \int_{z_b^*}^{\bar{z}} x(z)f(z)dz = \frac{L_b v(1-\gamma)}{1-v(1-\gamma)}\left(\bar{z}^{\frac{1-v(1-\gamma)}{1-\gamma}} - z_b^{*\frac{1-v(1-\gamma)}{1-\gamma}}\right) = \frac{v\gamma}{v(1-\gamma)-1}z_b^{*-v}. \tag{27}$$

iii) The market-clearing conditions simplify as follows:

$$\textit{Deposits Market}: \frac{(1+\psi)X(z_b^*)}{\psi\gamma(r_e-r_b)} = \mathbf{\Pi}(z_e^*, z_b^*), \tag{28}$$

$$\textit{Loans Market}: \frac{aL(z_e^*)}{(1-a)(\psi r_e + r_b)} = \frac{X(z_b^*)}{\psi\gamma(r_e-r_b)}, \tag{29}$$

$$\textit{Labor market}: L(z_e^*) + X(z_b^*) = W(z_e^*, z_b^*), \tag{30}$$

where

$$\mathbf{\Pi}(z_e^*, z_b^*) = \frac{\frac{1}{q}-1}{(1-a)}L(z_e^*) + \left(\frac{1}{\gamma}-1\right)X(z_b^*) + W(z_e^*, z_b^*). \tag{31}$$

Proof of Proposition 1. First it will be shown that the equilibrium exists; second, that it is unique.

Express the interest rates from equations (28), (29) defining the equilibrium conditions:

$$r_e = \frac{\frac{aL(z_e^*)}{(1-a)} + \frac{X(z_b^*)}{\psi\gamma}}{\mathbf{\Pi}(z_e^*, z_b^*)}, r_b = \frac{\frac{aL(z_e^*)}{(1-a)} - \frac{X(z_b^*)}{\gamma}}{\mathbf{\Pi}(z_e^*, z_b^*)}. \tag{32}$$

Using the expressions for interest rates and the expressions for thresholds (24) and (25) together with (4) and (13), the equilibrium conditions can be simplified as follows:

$$L(z_e^*) + X(z_b^*) = 2 - {z_e^*}^{-v} - {z_b^*}^{-v}, \tag{33}$$

$$\pi(z_e^*) = \frac{w^{1-aq}\left(\frac{L(z_e^*)}{\mathbf{\Pi}(z_e^*, z_b^*)}\right)^{aq}}{qA(1-a)} \left(\frac{\frac{1}{q}-1}{1-a}\right)^{q-1}, \tag{34}$$

$$z_b^* = \frac{w\left(\frac{a\psi\gamma L(z_e^*)}{1-a} + X(z_b^*)\right)^{1/\psi} \mathbf{\Pi}(z_e^*, z_b^*)}{TX(z_b^*)^{1/\psi+1}} \left(\frac{1}{\gamma}-1\right)^{\gamma-1} \tag{35}$$

The last expression for z_b^* defines wage as a function of z_e^*, z_b^*:

$$w = z_b^* / \left(\frac{\left(\frac{a\psi\gamma L(z_e^*)}{1-a} + X(z_b^*)\right)^{1/\psi} \mathbf{\Pi}(z_e^*, z_b^*)}{TX(z_b^*)^{1/\psi+1}}\right) \left(\frac{1}{\gamma}-1\right)^{1-\gamma}, \tag{36}$$

The labor market clearing condition (33) gives the solution for z_b^* as a function of z_e^*:

$$z_b^* = \left(\frac{v-1}{v(1-\gamma)-1}\right)^{1/v} \left[2 - {z_e^*}^{-v} - L(z_e^*)\right]^{-1/v}. \tag{37}$$

Plugging w from (36) and z_b^* from (37) into (34) which defines $\pi(z_e^*)$, the equilibrium conditions can be summarized in one equation in terms of z_e^*:

$$G(z_e^*) = \pi(z_e^*) - \frac{\bar{C}\left(2 - {z_e^*}^{-v} - L(z_e^*)\right)^{(1-aq)(1+\frac{1}{\psi}-\frac{1}{v})} L(z_e^*)^{aq}}{\left(\left[\frac{a\psi\gamma}{1-a} - \frac{\gamma v}{v-1}\right] L(z_e^*) + \frac{\gamma v[2-{z_e^*}^{-v}]}{v-1}\right)^{\frac{1-aq}{\psi}} \left(\frac{v[2-{z_e^*}^{-v}]}{v-1} + \left[\frac{\frac{1}{q}-1}{1-a} - \frac{1}{v-1}\right] L(z_e^*)\right)} = 0, \tag{38}$$

where

$$\bar{C} = T^{1-aq}\left(\frac{v-1}{v(1-\gamma)-1}\right)^{\frac{1-aq}{v}} \left(\frac{\gamma v}{v-1}\right)^{(1-aq)(\frac{1}{\psi}+1)} \left(\frac{1-\gamma}{\gamma}\right)^{(1-\gamma)(1-aq)} \frac{\left(\frac{1}{q}-1\right)^{q-1}}{qA(1-a)^q}.$$

The equations (32), (36), and (37) uniquely define prices r_e, r_b, w and the threshold z_b^*, given the solution for z_e^* from (38).

The interval in which equilibrium with positive interest rates and positive interest-rate spread can exist is given by the following:

$$r_e - r_b \geq 0, r_b \geq 0,$$

or

$$X(z_b^*) \geq 0, a\gamma L(z_e^*) - (1-a)X(z_b^*) \geq 0.$$

Note that $X(z_b^*) = \frac{\gamma v}{v-1}(2 - z_e^{*-v} - L(z_e^*))$. The expressions for r_e, r_b imply the following restrictions on the range of z_e^*:

$$2 - z_e^{*-v} - L(z_e^*) \geq 0, \tag{39}$$

$$2 - z_e^{*-v} - L(z_e^*) \leq a\gamma L(z_e^*)/(1-a), \tag{40}$$

or

$$L(z_e^*) \leq 2 - z_e^{*-v} \leq \left(\frac{a\gamma}{1-a} + 1\right) L(z_e^*).$$

Denote the minimum and maximum positive real solutions to the equalities corresponding to (39) and (40) as z_1 and z_2.[10] The function $G(z_e^*)$ is continuous on $[z_1, z_2]$.

Consider the value of $G(z_e^*)$ on the borders z_1 and z_2:

1. $L(z_e^*) = 2 - z_e^{*-v}$ Then the equation defining equilibrium (38) becomes

$$G(z_e^*|r_e - r_b = 0) = \pi(z_e^*) - 0 > 0.$$

2. $2 - z_e^{*-v} = \left(\frac{a\gamma}{1-a} + 1\right) L(z_e^*)$. Then the equation defining equilibrium (38) becomes:

$$G(z_e^*|r_b = 0) = \pi(z_e^*) - \frac{\bar{C}\left(\frac{a\gamma}{1-a}\right)^{(1-aq)(1/\psi+1-1/v)}}{\left(\frac{a\gamma(\psi v - \psi + \gamma v)}{(1-a)(v-1)}\right)^{(1-aq)/\psi}\left(\frac{v}{v-1}\frac{a\gamma}{1-a} + 1 + \frac{\frac{1}{q}-1}{1-a}\right)} L(z_e^*)^{(aq-1)/v}.$$

[10]It is easy to show that such solutions exist. Consider the closed interval starting at $z_e^* = 1$ and ending at some very large value, say $z_e^* = 10000$. The value of (39) and (40) at $z_e^* = 1$ is positive; at $z_e^* = 10000$, it is negative; both equalities define continuous functions on [1,10000]. By the Intermediate Value theorem, there are real $z1, z2 \in [1, 10000]$ that solve the inequalities corresponding to (39) and (40).

Denote the product of all constant terms as C:

$$G(z_e^*|r_b = 0) = \pi(z_e^*) - CL(z_e^*)^{(aq-1)/v}.$$

Note that, in equilibrium the following inequality must hold ($aq < 1$):

$$-CL(z_e^*)^{(aq-1)/v} \leq -C\left(2 - z_e^{*-v}\right)^{(aq-1)/v},$$

So that

$$\pi(z_e^*) - CL(z_e^*)^{(aq-1)/v} \leq \pi(z_e^*) - C\left(2 - z_e^{*-v}\right)^{(aq-1)/v}.$$

Given that $2 - z_e^{*-v}$ is bounded between 1 and 2,

$$2^{(aq-1)/v} \leq \left(2 - z_e^{*-v}\right)^{(aq-1)/v} \leq 1,$$

$$G(z_e^*|r_b = 0) \leq \pi(z_e^*) - C\left(2 - z_e^{*-v}\right)^{(aq-1)/v} \leq \pi(z_e^*) - C \leq 1 - C \leq 0,$$

given that T is such that $C > 1$.

Therefore, the function $G(z_e^*)$ alternates in sign on the interval $[z_1, z_2]$. According to the Intermediate Value theorem, there is a value of $z_e^* \in [z_1, z_2]$ such that $G(z_e^*) = 0$. So, the equilibrium with positive interest rates and positive interest-rate spread exists.

To show that the equilibrium is unique, consider the derivative of the function $G(z_e^*)$ with respect to z_e^*, and conclude that this derivative is always negative on the interval $[z_1, z_2]$ under conditions specified in Assumption 3, so that the function is strictly monotone on that interval. ■

Proof of Proposition 2. Let A grow at rate g^{1-aq}, and T grow at rate g. If there exist a solution, there exist w, r_e, r_b that clear the markets. Conjecture that along a balanced growth path wages, w, grow at rate g, and interest rates, r_e, r_b, are constant. Then L_e and L_b are constant. From (24) and (25) it means that the thresholds z_e^* and z_b^* are constant. Therefore, given (5) and (4) $d(z)$ grows at rate g. This implies that probability $P(z)$ is constant.

Given that L_e, L_b, z_e^* and z_b^* are constant over time, labor demand functions $l(z)$, $x(z)$ are constant over time. From (5) and (4), the capital demand is growing at rate g, same rate as capital supply. Finally, output given by (20) is proportional to wages and grows at rate g. Therefore, the conjectured solution for the rates of growth of w, r_e, r_b was correct. ■

Proof of Proposition 3. Consider $G(z_e^*) = 0$. From the proof of Proposition 1, $dG(z_e^*)/dz_e^* < 0$ on the interval where the equilibrium can exist. Moreover, $dG(z_e^*)/dT < 0, dG(z_e^*)/dA > 0$. By the Implicit Function theorem, $dz_e^*/dT < 0$, $dz_e^*/dA > 0$. From (37), $dz_b^*/dz_e^* < 0$, so z_e^* decreases (increases) and z_b^* increases (decreases) with a rise in T (A) given A (T). From (26) and (27), $dL(z_e^*)/dz_e^* = \frac{(1-a)q}{(1+\mu-q)}(\frac{1}{q-1}\pi(z_e^*)^{\frac{1}{q-1}-1} + \frac{\mu-q}{q-1}\pi(z_e^*)^{\frac{\mu-q}{1-q}-1})\pi_z(z_e^*) < 0$, $dX(z_b^*)/dz_e^* = \frac{\gamma v}{v-1}\left[v z_e^{*-v-1} - dL(z_e^*)/dz_e^*\right] > 0$, so labor demand in the real (financial) sector is increasing (decreasing) in T (A). Simplifying (31), $\mathbf{\Pi}(z_e^*, z_b^*) = \Pi(z_e^*) = \left[\frac{\frac{1}{q}-1}{1-a} - \frac{1}{v-1}\right] L(z_e^*) + \frac{v}{v-1}(1 - z_e^{*-v}) + \frac{v}{v-1}$. Taking derivative of the interest rates spread with respect to z_e^*, $\frac{d(r_e - r_b)}{dz_e^*} = \frac{1+\psi}{\psi\gamma\mathbf{\Pi}(z_e^*, z_b^*)^2}\left(X_{z_e^*}\Pi - \Pi_{z_e^*}X\right) > 0$ Therefore, $d\left(r_e - r_b\right)/dT < 0$, $d\left(r_e - r_b\right)/dA > 0$. Similar, taking derivative of the interest rates on deposits r_b, defined in (32), with respect to z_e^*, $\frac{dr_b}{dz_e^*} < 0$. Therefore, $dr_b/dT > 0$, $dr_b/dA < 0$. ■