On The "Scientificity" Of Microeconomics: Individual Demand, And Exchange-value Determination

C-Rene Dominique

Laval University

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C-René Dominique

SUMMARY: This note examines how the concept of utility has led neo-classical economists astray. It first briefly reviews the thoughts of the early pioneers who have engaged these economists on the utility trail. It next scrutinizes the requirements imposed on the preference set of the consumer in view of extracting a utility function having anticipative properties. Then it shows how set theory can solve the dynamic exchange process and value determination without any need for a utility function.

KEYWORDS: Utility, Preference, Well-Ordered Sets, Ordinal Space, Binary Relation, Order-isomorphism.

1- INRODUCTION

A conventional definition of "microeconomics" asserts that it is the "study of the behavior of individuals and firms in decision regarding the allocation of scare resources". One could find other equally or more appropriate definitions such as "the study of the process of exchange" or "a search for a 'metric' of value", etc. For, these definitions encapsulate '*totus in toto'* the reasoning behind the choice of the paths that the early scholars of physio-mathematics followed to associate microeconomics to concepts such as "utility", "value", and "needs". Today, the result of that effort appears scanty, while the utility trail is perceived as not only superfluous but somewhat damaging, for it has led neo-classical economists directly to a no- men's land, where they remain trapped.

The concept of utility may have originated mainly in the works of the physiocrats and their followers such as Condillac (1714-1780), Turgot (1727-1781), Condorcet (1743-1794), among others. For example, Condillac introduced the psychological basis of value and anticipated marginal utility, but his ideas were not followed by his contemporaries. Turgot for his part was more interested in the measurement of economic phenomena as a basis for rational administrations. And Condorcet emphasized social mathematics in view of constructing a science of society that would have objective value. While it is true to say that the physiocrats, in general, ignored the notion of 'exchange-value' in favor of 'use-value', as determined by the cost of production, but it is no exaggeration to point out that they are initially responsible for engaging neo-classical economists on the utility trail.

Their immediate successors, known as the forerunners of the marginalists, followed them in the same trail in using the concept of utility, but cast in the analysis of the margin, to analyze specific questions. For example, Jeremy Bentham (1748-1832) gave the analysis of the margin a definite expression and associate it with "pleasure". Jules Dupuit (1804-1866) used it to justify *price discrimination*. Augustin Cournot (1801-1877) and Heinrich von Thünen (1780-1850) developed the concept of *marginal productivity*. Bernoulli (1706-1782) made use of the concept of *marginal increment of incom*e, etc. Yet, these early developments were not fully appreciated before the mid-19th century because both physiocrats and forerunners, deep down, continued to associate the concept of utility to the inherent characteristic of commodities. And I believe that it is the reason why they were unable to distinguish 'value-in-use' and 'value-in-exchange', also known as the diamond-water paradox enunciated by Adam Smith.

Concepts of total and marginal utility received a more complete characterization from William Gossen (1810-1854). He brought back Condillac's and Bentham's idea, but added that the utility function must be concave; then as an economic agent acquires additional units of a particular good, each additional unit yields continuously diminishing "pleasure" up to a point of satiation. From then onward the act of consumption came to be associated with pleasure (sic). But even after Gossen's addition, the concept of the margin was still not fully appreciated as a general tool of analysis until the early 1850s and beyond. That is, until the analyses of Jevons (1957), Léon Walras (1874a, 1874b), and Carl Menger (1870). These marginalists independently formulated a theory of exchange value based on the principle of diminishing marginal utility as opposed to the cost of production. Thus, all three accepted Bentham's definition of a good as an object which brings pleasure. All three emphasized circumstances of things arising out of their relationship to an agent's need rather than intrinsic characteristics of goods. For Walras, limitation begets rareté, and rareté begets value, even though he knew fully well that utility was not measurable. Nevertheless, he had reasoned that rareté is the cause of value in exchange, and until his death in 1912 he remained convinced that one day science would find a way to measure rareté as an absolute magnitude.

Walras' method was nevertheless severely criticized by scientists and mathematicians on the grounds that desire and needs were not susceptible to exact measurements. To counteract such criticisms, Pareto (1848-1923) explored the possibility that consumers' behavior might be better examined without resorting to the notion that utility was a cardinally 'measurable magnitude'. He then proposed the notion of indifference curves as an alternative for determining the allocation of income. John Hicks (1946) followed up on that development and is today credited with the so-called indifference map together with the curious notion of the marginal rate of substitution between pairs of goods. In essence, the contribution of Hicks is that individual demand curve could be derived from the indifference map and the constraint of the consumer's budget. This means that the demand curve is not the same as declining marginal utility, which now appears as non- essential.

All these early developments, in one form or another, constitute what is known today as the "Subjective Theory of Value" which asserts that a commodity's or a service's value is none other than the subjective value assigned to it by his consumer. That theory therefore rejects all notions of labor content and inherent properties that the commodity might have, and solves the diamond-water paradox. Yet today the notion of utility is still surreptitiously associated with pleasure, while the realization that marginal utility constantly falls until it is equal to the price of an item at least supports the belief that individual demand curves are downward sloping. But, as will be argued later, Hicks' demand curve is arrived at through a questionable roundabout procedure.

On a deeper level, these beliefs are pregnant with pathologies relative to the determination of exchange value as discussed in Sonnenschein (1973, 1974) and Mantel (1974). It suffices to consider the indifference map of Hicks and the so-called *price-consumption curve* (the *loci* of tangencies of indifference curves and price lines) out of which the demand curve is derived. To observe a point on that curve one must know prices. But prices are known only in equilibrium. Hence, a consumer is unable to derive another point, for there is no more price change in equilibrium. This means that only a single point of the demand can be observed during a given market period (Dominique, 2017). The same situation arises in the so-called Lagrange constrained utility maximization concept, which skips individual demand all together to differentiate a utility function so as to move directly to a constrained solution even though the so-called utility function remains unobservable.

Today, modern economists live with a contradiction. On the one hand, they accept that value is determined by the market in equilibrium; that is, declining marginal utility is not the same as individual demand. On the other, subjective utility is maximized and inserted into the method of mechanics so as to determine value. Thus, after travelling for almost two hundred years on the subjective utility trail, they are still unable to provide an unambiguous metric for value, and still they remain steadfastly attached to the maximization of an elusive utility function. In the end, the subjective utility trail produces nothing of value except lots of irony from mathematicians and scientists.

Erroneous conclusions such as *linear demand curves*, *differentiation* and *maximization of utility*, *consumption equals pleasure*, etc., could have been avoided had the early pioneers chosen instead a trail that led to scientific achievements. In that context, it is fair to say that the forerunners and marginalists may be forgiven for having lived before the advent of more appropriate tools of analysis. But the same cannot be said about their modern followers; i. e., Hicks, Samuelson, Debreu, among others, who could have oriented the profession toward set theory. For in the mid-19th century there was a renaissance in logic. George Cantor (1845-1918) had taken the idea of set to a higher level, and scholars such as Frege (1848-1925), Russel (1872-1970) and Whitehead (1841-1947) had completed the foundation of mathematical logic, which now stands as a corner stone of mathematics. Gotlob Frege in particular had by then demonstrated that one could use his formal system to resolve theoretical mathematical statements in terms of simpler logical notions. As already stressed above, many pathologies could have been avoided. Instead, the individual demand curve (sic) remains miss-specified and unobservable, while the association of consumption and pleasure carries negative consequences, such as *insatiability of needs, addiction, rising consumer's debt level, waste and environmental degradation*.

The purpose of this note is to show how naïve set theory would have been a better tool to analyze the process of exchange and value determination. But beforehand, we examine some other utility-related requirements of the modern version of microeconomics; hoping that will underline the need to turn a page.

2- SOME BIZARRE REQUIREMENTS

Modern economists argue that there exists a universal consumption set C, and $X \subseteq C$ represents a basket of goods selected by a given consumer. Then $X = \{x_1, x_2, \ldots, x_n\}$ (with the x's as elements) represents consumers' preference. Since preference does not have a 'metric', economists agree since the 1930s that X is an ordinal space, equipped with an order R. They next posit that R must be *complete*, *reflexive*, *continuous*, *transitive*, *monotone* and *convex*. But this set of requirements imposed on R appears both stringent and somewhat redundant. Recall that X is an ordinal space equipped with a relation of order and equivalence. To say that X must be complete means that $\forall x_1, x_2, x_3 \varepsilon X$, either $x_1 \precsim x_2 \vee x_2 \precsim x_1$. Thus, if any two elements $x \varepsilon X$ are comparable, then X must be a well-ordered set. If the ordering is strict, then R is automatically endowed with the properties of *irreflexivity*, *antisymmetry* and *transitivity*. If, on the other hand, the order is non-strict, then R is *reflexive*, *antisymmetric* and *transitive*.

As we will make more explicit in a moment, to be well-ordered a set must have a smallest element. Since there is no zero utility nor zero preference, both the set X and U (utility) are not well-ordered just like the real set (0, 1] is not.

We will then assume that consumers with asymmetric and incomplete information sets and facing new and product differentiation cannot possibly well-order their preference sets. It is therefore reasonable to assume that X is a partially ordered set or a *poset.* In that case, the pair (X, $\precsim$) satisfies reflexivity, antisymmetry, and transitivity; while the pair (X, $\backsim$) satisfies reflexivity, symmetry, and transitivity. Thus if X is a poset, as it is reasonable to suppose, it is rather redundant for the modern version to require reflexivity and transitivity.

Regarding the requirement of convexity, it is understood that if $x_1 \succ x_2$, then $x_2 \leq [\lambda\ x_1 + (1\text{-}\ \lambda)\ x_2]$, where $\lambda\ \varepsilon\ (0, 1)$. I understand that this demand is to insure that the consumer will prefer more to less, but as we will make clear below, no multiplication is defined in ordinal space. There is no doubt that the average consumer prefers more to less, but this cannot be a necessary condition since it might not apply to some. Characteristics such as selfishness, monotonicity, etc. are subsumable in the consumer's behavior. Indeed, the role of the scientist is to observe instead of imposing. In this sense, it is rather incongruous for scientists to impose characteristics on unobservable structure.

Beside stringent demands and redundancies, there are other incongruities to be discussed below after we give formal definitions of the terms used in this study.

2-1 Definition of Terms

Terms and symbols used in the language of set theory vary with authors. For tractability, therefore, we begin by defining the terms used in this study:

The Smallest Element: Let the pair (X, $\precsim$) be a poset. Then an element $x_1\ \varepsilon$ X is the smallest element in X if $x_1 \precsim x_2$, $\forall x_2, x_3, \ldots, x_n\ \varepsilon$ X.

A Binary relation R: A Binary relation R on a set X is: R $\subseteq$ X x X.

A Well-Ordered Set: The poset X is well-ordered if every non-empty subset of X contains a smallest element.

A Partially Ordered Set (poset): A relation ($\precsim$) on a set X is a poset if it is reflexive, antisymmetric and transitive.

The Inverse of a Binary relation: If R is a partial order on a set X, then the inverse R^{-1} is a partial order on X.

An Ordinal Space: is a set X = $\{x_1, x_2, \ldots, x_n\}$ of distinct elements equipped with a relation of order and equivalence[1].

Isomorphism: Let (X, $\precsim_x$) and (Y, $\precsim_y$) be two posets that are isomorphically related. Then, there exists a one-to-one function f from X to Y such that $\precsim_x$ y iff f (x) $\precsim_y$ f (y), $\forall x\ \varepsilon$ X.

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[1] Mathematical operations such as multiplication, addition, subtraction, differentiation, etc. are not defined on ordinal spaces. For a source, see Barsilai (2013).

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Order Isomorphism: Let X and Y be two sets. An order isomorphism between X and Y preserves the largest, smallest, maximal, minimal elements, if they exist. Further, if X does not have a smallest element then the sets are not completely order-isomorphic[2].

3- THE PROCESS OF EXCHANGE

With these definitions in mind, we can now examine other aspects of the modern version of neo-classical economics.

The modern version at times emphasizes two models. Namely:

$$f : X \to \mathbb{R}_+ \quad (1)$$

$$f : X \to U,.\text{and } g : U \to \mathbb{R}_+, \quad (2)$$

where, as before, X is an ordinal space, U is the utility index, another ordinal space, and $\mathbb{R}$ is some real set. It should be noted first that if U is an ordinal space, then model (1) is in fact $f: X \to X = I_x$, the identity function, which implies that the ordinal space X is simply 'cardinalized'. Hence model (2) can be rewritten as:

$$h = (g \circ I_X) : X \to \mathbb{R}_+, \quad (3)$$

where h is the composition of g and f.

Students of economics are taught that if preference satisfies certain conditions (see above), then there exists a utility function f such that $x_1 \prec x_2$ implies $f(x_1) \prec f(x_2)$ without stating how that f is related to X. In (3), we find no such utility function, for as it can be seen in (3), h is no more a utility function nor a preference function. h is a monotone bijection with the ordinal space X as its domain and $\mathbb{R}_+$ as its co-domain, or a mapping from a poset X to another $\mathbb{R}$. If both are ordered antisymmetrically, then they are isomorphically related[3]. However, the idea behind the mapping is to move X to a real set on which mathematical operations are defined. It would not make any sense to map X into a real set that is not pertinent to the problem of exchange.

As shown in Dominique (2017), the only set of reals that will do the trick is the set of budget shares $\alpha = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$. Equation (3) can then be written as:

$$h = (g \circ f): X \to \alpha \quad (4)$$

That real set offers various clues as to how to solve the exchange problem. Its elements appear in every pertinent equation of the system. To see how, we first consider how α enters the equilibrium equations of exchange with m

2 If the sets X and Y are isomorphically related, then $f: X \to Y$ is an injection; $f^{-1}: Y \to X$ is a surjection. And both f and f^{-1} are strictly increasing.

3 There are many proofs in the literature. For more, see: Warner (1965), Karolyi (2016), Simovici, *et al.* (2014), Roitman, (2013). See also: Wikibooks.org/Wiki/abstract-Algebra/Group-Theory/Homomorphism; retrieved on July 9, 2017.

consumers indexed by i and n goods indexed by j, and using superscript and subscript below to avoid double summations. Within a small e > 0 radius, the equilibrium equation perceived by i is:

$$\alpha^i_j = (p_j \,.\, x^i_j\,(p)) \,/\, B^i\,(p_j), \qquad (5)$$

where x^i_j (p) stands for good j purchased by consumer i, p is the market price, and B^i (p) stands for the budget of i. It is clear from (5) that price is a target variable, for if it is known then both quantity and budget will be known as well.

Equation (4) is written in terms of X and α, both are open sets, i. e., without smallest elements. Hence, the graph of h is not defined in the neighborhood of the origin. We will examine the consequence of that below. For now, it suffices to say that, as $\alpha \,\varepsilon\, \mathbb{R}$, it is proper for economists to perform mathematical operations in it. Indeed, many such operations can be performed. For example,

Operation 1: Operation 1 yields the nature of an individual demand curve as,

$$x^i_j = \alpha^i_j\, B^i \,/\, p_j. \qquad (6)$$

Thus, near the equilibrium point, the individual demand curve is a *rectangular hyperbola*; its *instantaneous price elasticity* lies between -1 and 0; and both are derived from real values. If supply is fixed, we have:

Operation 2: The excess demand of good j is:

$$p_j\, .x^i_j - \alpha^i_j\, B^i \lesseqgtr \xi_j, \qquad (7)$$

where ξ is the excess demand of good j perceived by i. The summation over all i's and j's yields another mapping **M** such that:

Operation 3: The mapping **M** solves the exchange problem as:

$$\mathbf{M}\!: \xi\,(p) \rightarrow \mathbf{p}. \qquad (8)$$

M is in fact the monotone bijection performed by all consumers. That mapping is derived in detail in Dominique (2017), where it is shown that every one of its elements is a function of α**.** The following conclusions can then be drawn:

i) due to monotonicity, **M** contains a free variable, hence its rank is (n – 1);

ii) ii) $\mathbf{M}^{-1}$ exists, **M** therefore is a monotone bijection;

iii) iv) if |.| stands for the cardinality of a set, then over the proper range $|X| = |\alpha|$;Type equation here. order is preserved as:

$\sum_i^m \alpha^i_j > \sum^m_i \alpha^I_{J+1} >,\ldots,> \sum_I^m \alpha^i_{j+(n-1)} \rightarrow p_j > p_{j+1} >,\ldots,> p_{j+(n-1)}$, hence **M** is an order-isomorphism:

iv) price is the metric of value, and:

v) individual demand is relatively inelastic, then there is a tendency for prices to increase in the absence of a concomitant increase in supply.

The *free variable* in **M** is what Walras identified as the numéraire. It remains to be seen after further studies, but *it would appear that a mapping from an open ordinal space to another set leaves a variable free due the monotonicity of the mapping h*[4].

It should also be noted that only one point on the individual demand curve is in fact observed. Therefore, the problem of aggregating individual demand curves analyzed by Sonnenschein (1973, 1974) and Mantel (1974) simply does not arise since the operation addition is mathematically defined. Of course, one could use the concept of price elasticity to reconstruct an *ex-post* demand curve, but what would be the use of that?

In the end, this approach reveals that the dynamic process of exchange or the determination of value in economics reduces to consumers' search for the mapping h, which incidentally is not observable except in equilibrium. Further, the approach subsumes the concept of revealed preference proposed by Samuelson (1938) and also shows that the criticisms addressed to revealed preference theory, namely whether consumers' preference scale remains constant over time or not (see Wong, 1978) are not pertinent.

CONCLUSION

The present approach based on naïve set theory is conform to a scientific "démarche" because it is based on observables. The neo-classical approach, on the other hand, is based on 'cardinalizing' an ordinal space while imposing unrealistic demand on individual preference in an attempt to generate a utility function with a convex hypograph. It fails dramatically to reach its objectives. Whereas the present approach based on naïve set theory uses real values to derive a proper individual and community demand curves. It also solves the market equilibrium as a stable sink, while establishing an unambiguous monotone metric for value.

In sum, the utility trail requiring that mathematical operations be performed on ordinal spaces where they are not defined is not only a violation of mathematical rules, but it produces negative consequences. It fails to emphasize the reflexivity of markets. More importantly, it associates consumption with pleasure; as a result, we now have millions of individuals addicted to shopping. Moreover, the concept of maximization of an unobservable utility function (associated with pleasure) might be a bonanza for advertisers, but it is scientifically unjustified and it produces indebtedness, waste, and environmental degradation.

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[4] A similar process occurs in Political Science where the citizens map their preference of candidates (an ordinal space) into ballots (a real set) on which all kinds of mathematical operations can be performed.

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