# An Overlapping-Generations Model of Firm Heterogeneity in Economic Development

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Abstract

We study firm heterogeneity in economic development in an overlapping-generations general equilibrium model in which manufacturing firms engage in oligopolistic competition. Individuals differ in their productivities in the manufacturing sector and choose to become entrepreneurs or workers. The model is surprisingly tractable. In the steady state, an increase in the entry barrier in the manufacturing sector or an increase in the percentage of income spent on the agricultural good decreases the wage rate, but the level of output in the manufacturing sector does not necessarily decrease. An increase in the degree of patience of an individual increases the steady state wage rate and the capital stock. Even with increasing returns in manufacturing and constant returns in agriculture, neither the wage rate nor the output level in the manufacturing sector may increase with the size of the population.

**Keywords:** Firm heterogeneity, overlapping-generations model, oligopolistic competition, career choice, economic development

**JEL Classification Number:** D43, L13, O10

## 1. Introduction

With the seminal contribution of Melitz (2003), firm heterogeneity has been studied extensively in the field of international trade. For developing countries, due to more segmented markets and poorer communication and transportation facilities, firm heterogeneity as shown in aspects such as differences in technologies is an even more salient issue. For example, Banerjee and Duflo (2005) show that technologies used by Indian firms vary significantly.[1]

In this paper, we study firm heterogeneity in economic development in an overlapping-generations general equilibrium model in which manufacturing firms engage in oligopolistic competition. Several factors herald the prevalence of oligopolistic competition in developing countries. Entry is restricted in many industries through barriers such as burdensome licensing

[1] For additional evidence on firm heterogeneity, Rosenberg and Birdzell (1986) show that water power continued to be used long after the introduction of the steam engine (p. 154) and the power loom and handloom coexisted for more than fifty years (p. 160). Chandler (1990, Chap. 7) shows that at the beginning of the twentieth century, British firms in various capital-intensive industries used technologies systematically less efficient than those used by their counterparts in the United States.

process and complex financing arrangements. Weak financial system and capital markets may also make entry more difficult. Trade regimes are often highly protective, which limits foreign competition. In addition, rent-seeking through erecting barriers to entry is often less controllable in developing countries due to weaker legal systems.

In this model, by incurring fixed costs through acquisition of capital, an individual can operate a manufacturing firm. A higher level of fixed costs indicates an increase in the level of entry barriers. Firm heterogeneity is shown as different marginal costs for firms in the manufacturing sector. Following Lucas (1978), individuals differ in their productivities in the manufacturing sector. A manufacturing firm managed by a more productive entrepreneur will have a lower marginal cost of production. For an entrepreneur managing a firm, the return to this person includes the profit of this firm. If an individual chooses to become a worker, the return to this person is just the wage rate regardless of this person's entrepreneurial skill. Because the return to an entrepreneur is an increasing function of the level of productivity, individuals with lower marginal costs are more likely to choose to become entrepreneurs.

While this is an overlapping-generations general equilibrium model with firms engaging in oligopolistic competition, surprisingly analytical results are available. First, we show that in the steady state an increase in the level of entry barriers leads to a decrease in wage rate and the critical level of productivity to become an entrepreneur is more demanding[2]. Interestingly, total output in the manufacturing sector does not necessarily decrease. The reason is that a decrease in the wage rate reduces marginal costs and induces manufacturing firms to produce more. By incorporating effects from changes in the wage rate and the critical level of productivity to become an entrepreneur, this model shows the value of a general equilibrium model in incorporating multiple effects from a parameter change. If the output expansion effect was not incorporated, it could lead to the conclusion that an increase in the level of entry barriers always reduces total output in the manufacturing sector.

Second, we show that in the steady state an increase in population may neither increase the wage rate nor total output in the manufacturing sector. With increasing returns in the manufacturing sector and constant returns in the agricultural sector, an increase in population

[2] Nicoletti and Scarpetta (2005), Griffith and Macartney (2007), and Fiori *et al.* (2008) demonstrate the beneficial effects on employment and real wages of labor market deregulation and reduction of entry barriers in product market. Jean and Nicoletti (2004) find higher wage premium in industries where legislation limits competition.

increases the wage rate in a one-period model such as Zhou (2010a). In this overlapping-generations model, the amount of capital is endogenously determined by saving. For the wage rate to increase, the amount of capital needs to increase so that the measure of firms is higher and demand for labor is higher. The amount of saving is an increasing function of the sum of wage income and profits. If an increase in population increases the wage rate, a higher wage rate decreases profits, and overall total saving and thus capital stock in the next period may not increase. Without more capital, the wage increase is not sustainable.

This paper is related to research on firm heterogeneity, oligopoly, and overlapping-generations models. First, for the literature on firm heterogeneity, Montagna (1995) and Melitz (2003) have addressed the impact of international trade in models featuring heterogeneous firms engaging in monopolistic competition.[3] For models on firm heterogeneity with oligopolistic competition, Lahiri and Ono (2004) have shown that helping less efficient firms may reduce social welfare. Qiu and Zhou (2007) have examined incentives for firms to merge when they engage in oligopolistic competition. They demonstrate that firm heterogeneity is a necessary condition for mergers to occur. Zhou (2010a) studies the impact of firm heterogeneity on international trade when firms engage in oligopolistic competition. Zhou (2010b) studies how the size distribution of firms is determined in an open economy. One important difference between the above papers and this one is that this is an overlapping-generations model with the amount of capital endogenously determined by saving. This feature of endogenous determination of capital is consistent with the accumulation of capital in economic development. Second, for the literature on overlapping-generations model, Eaton (1987) and Drazen and Eckstein (1988) have studied models in which firms engage in perfect competition. Zhou and Zhou (2016) have studied a model in which firms engage in oligopolistic competition. Firm heterogeneity is not addressed in these models. Third, for the literature on oligopolistic competition, Neary (2003) has studied a general equilibrium model of oligopoly. One important difference between his paper and this one is that firm heterogeneity is not studied in his model.

[3] Shiferaw (2007) has investigated market selection and industry dynamics in Ethiopia. He shows that markets are effective in selecting efficient firms. When the degree of exposure to international competition increases, less efficient firms are more likely to exit. Reallocation of resources played a significant role for industry-level productivity growth.

The plan of the paper is as follows. Section 2 sets up the model and establishes equilibrium conditions. Section 3 studies properties of the steady state equilibrium. Section 4 discusses some possible generalizations of the model and concludes.

## 2. The model

We study an overlapping-generations model in a closed economy. There are two types of goods: the agricultural good and the manufactured good. In this model, subscripts are used to denote time periods and superscripts are used to denote sectors of production.

An individual lives for two periods: young and old. In each period, $L$ young individuals will be born and $L$ old ones will die. The size of the population is $2L$ and does not change over time.[4] The agricultural good is used for consumption only. It is produced by using labor only, with a technology featuring constant returns to scale. The number of individuals employed in the agricultural sector is $L_t^a$. Without further loss of generality, we assume that each worker in the agricultural sector produces one unit of the agricultural good. The production of the manufactured good uses capital as fixed costs and labor as marginal costs of production. The manufactured good can be used either for consumption or for investment (Drazen and Eckstein, 1988). Each unit of the manufactured good can produce one unit of capital. Thus, the price of capital is equal to that of the manufactured good. The initial amount of capital stock in this economy is $K_0$. For simplicity, we assume that capital does not depreciate (Eaton, 1987).[5]

An individual supplies one unit of labor only when she is young. In each period, an individual derives utility from the consumption of the two goods and her utility is assumed to be separable in the two periods. A consumer's discount factor is $\rho$. For an individual born in period $t$, her consumption of the agricultural good is $c_{t,1}^a$ and consumption of the manufactured good is $c_{t,1}^m$ when she is young. Her consumption of the agricultural good is $c_{t+1,2}^a$ and consumption of the manufactured good is $c_{t+1,2}^m$ when she is old. For $\theta \in (0,1)$, the utility function of an individual born in period $t$ is specified as

$$U(c_{t,1}^a,\ c_{t,1}^m,\ c_{t+1,2}^a,\ c_{t+1,2}^m) = \theta \ln c_{t,1}^a + (1-\theta)\ln c_{t,1}^m + \rho\theta \ln c_{t+1,2}^a + \rho(1-\theta)\ln c_{t+1,2}^m . \qquad (1)$$

[4] For a model of economic development with endogenous population, see Zhang (2002).
[5] One alternative interpretation of the assumption that capital does not depreciate is that output is net of depreciation cost (Solow, 1956).

The interest rate in period $t$ is $r_t$. The price of the manufactured good in period $t$ is $p_t^m$, and the price of the agricultural good is $p_t^a$. An individual's income in the first period is $I_t$, which comes from wage income and profit (if this person chooses to become an entrepreneur).[6] She faces the following budget constraint:

$$p_t^a c_{t,1}^a + p_t^m c_{t,1}^m + \frac{p_{t+1}^a c_{t+1,2}^a}{1+r_t} + \frac{p_{t+1}^m c_{t+1,2}^m}{1+r_t} = I_t . \tag{2}$$

We assume that individuals have perfect foresight in this model. Since there is no uncertainty, a consumer knows prices in the two periods of consumption. A consumer chooses the amounts of consumption in the two periods $c_{t,1}^a$, $c_{t,1}^m$, $c_{t+1,2}^a$, and $c_{t+1,2}^m$ to maximize utility (1), subject to budget constraint (2). Utility maximization yields

$$c_{t,1}^a = \frac{\theta}{(1+\rho)p_t^a} I_t , \tag{3}$$

$$c_{t,1}^m = \frac{1-\theta}{(1+\rho)p_t^m} I_t , \tag{4}$$

$$c_{t+1,2}^a = \frac{\rho(1+r_t)\theta}{(1+\rho)p_{t+1}^a} I_t , \tag{5}$$

$$c_{t+1,2}^m = \frac{\rho(1+r_t)(1-\theta)}{(1+\rho)p_{t+1}^m} I_t . \tag{6}$$

From equations (4) and (6), the absolute value of this individual's elasticity of demand for the manufactured good in each period is one. From equations (3) and (4), the ratio between a consumer's spending on the agricultural good and that on the manufactured good in each period is $\theta/(1-\theta)$. A consumer's total spending in the first period is

$$p_t^a c_{t,1}^a + p_t^m c_{t,1}^m = \frac{1}{1+\rho} I_t . \tag{7}$$

The amount of saving for a young individual is $s_t$. From equations (2) and (7), the amount of saving can be expressed as

$$s_t = \frac{\rho}{1+\rho} I_t . \tag{8}$$

[6] With the homothetic utility function assumed in this paper, the distribution of income will not affect aggregate demand of the two goods.

A young individual may be employed either in the agricultural sector or the manufacturing sector. If an individual is employed in the manufacturing sector, she may choose to become a worker or an entrepreneur. If a young individual becomes a worker, she receives the wage rate regardless of her entrepreneurial productivity. If she chooses to become an entrepreneur, in addition to wage income, she receives the firm's profit as return. Entrepreneurship is an important factor in affecting the performance of a developing economy (Hirschman, 1958). We assume that individuals differ in their productivities as entrepreneurs (Lucas, 1978). A young individual's productivity $\beta$ is distributed on an interval $[\underline{\beta}, \overline{\beta}]$ with density function $g$, which is assumed to be continuous. A firm managed by an entrepreneur with productivity $\beta$ can produce the manufactured good at a constant marginal cost of $\beta$ units of labor. To operate a firm, an entrepreneur needs to rent $z$ units of capital. Capital is the fixed cost of production for a manufacturing firm. Regardless of the sizes of firms, the fixed costs for all manufacturing firms are the same.[7]

From the law of demand, the price of the manufactured good is a decreasing function of total output in the manufacturing sector $Q_t$: $\partial p_t^m(Q_t)/\partial Q_t < 0$. For a manufacturing firm operated by an entrepreneur with marginal cost $\beta$, its level of output is $q_t(\beta)$ and revenue is $p_t^m q_t$. When the wage rate is $w_t$, this firm's labor cost is $\beta q_t w_t$. In addition, its rental cost of capital is $r_t z$. Like Drazen and Eckstein (1988), this firm's profit is the difference between total revenue and costs of hiring labor and renting capital:

$$p_t^m(Q_t)q_t - \beta w_t q_t - r_t z \, . \tag{9}$$

Similar to Neary (2003), we assume that manufacturing firms engage in oligopolistic competition. The importance of oligopoly as a type of market structure in a modern society is well recognized in the literature. Modern production is associated with the adoption of machines. The existence of fixed costs of production such as machines leads to increasing returns in production. There are also increasing returns in management and distribution. With the existence of internal increasing returns, there is a tendency for an industry to be monopolized. However, countries such as the United States have anti-trust laws that can be used to prevent monopoly from happening.

[7] This specification is similar to Holmes and Schmitz (1990) in which the fixed cost of setting up a firm is the foregone wage income regardless of the size of the firm. One difference between their model and this one is that firms engage in perfect competition in their model while manufacturing firms engage in oligopolistic competition in this model.

Thus, many industries are characterized by oligopolistic competition (Chandler, 1990). The importance of oligopolistic competition is highlighted by Pindyck and Rubinfeld (2005, p. 441) who argue that oligopoly is a prevalent form of market structure. They also provide examples of oligopolistic industries: automobiles, steel, aluminum, petrochemicals, electrical equipment, and computers.

Specifically, we assume that manufacturing firms engage in Cournot competition. A manufacturing firm takes the wage rate and output of other manufacturing firms as given and chooses its output optimally to maximize profit. From (9), the first order condition for a manufacturing firm's optimal choice of output is

$$p_t^m(Q_t) + q_t \frac{\partial p_t^m}{\partial q_t} - \beta w_t = 0 . \tag{10}$$

For the market for the manufactured good in period $t$, demand is the sum of consumption and investment. Consumption comes from individuals born in periods $t-1$ and $t$. For an individual born in period $t-1$, the demand is $c_{t,2}^m$. Integrating over all possible levels of income, total demand of the manufactured good from individuals born in period $t-1$ is $\int c_{t,2}^m dI$.[8] For an individual born in period $t$, the demand is $c_{t,1}^m$. Integrating over all possible levels of income, total demand of the manufactured good from individuals born in period $t$ is $\int c_{t,1}^m dI$. The amount of investment is $K_{t+1} - K_t$. Thus, total demand for the manufactured good in period $t$ is $K_{t+1} - K_t + \int c_{t,2}^m dI + \int c_{t,1}^m dI$. Total supply of the manufactured good in period $t$ is $Q_t$. The clearance of the market for the manufactured good requires that

$$K_{t+1} - K_t + \int c_{t,2}^m dI + \int c_{t,1}^m dI = Q_t . \tag{11}$$

The amount of net investment is zero in a steady state. Keeping in mind that a consumer has a unitary elasticity of demand for the manufactured good ($-\frac{\partial c_{t,2}^m}{\partial p_t^m}\frac{p_t^m}{c_{t,2}^m} = 1$ and $-\frac{\partial c_{t,1}^m}{\partial p_t^m}\frac{p_t^m}{c_{t,1}^m} = 1$) and a firm takes the output of other firms as given when it chooses its level of output in a Cournot

[8] The lowest possible income is wage income, and the highest possible income is the sum of wage income and profit from the entrepreneur with the lowest marginal cost of production.

competition, partial differentiation of equation (11) yields $\frac{\partial p_t^m}{\partial q_t} = -\frac{p_t^m}{Q_t - (K_{t+1} - K_t)}$. Plugging this result into the condition for a manufacturing firm's optimal choice of output (10) leads to

$$p_t^m \left( 1 - \frac{q_t}{Q_t - (K_{t+1} - K_t)} \right) - w_t \beta = 0 \,. \tag{12}$$

From equation (12), a firm's output can be expressed as

$$q_t = \frac{[Q_t - (K_{t+1} - K_t)](p_t^m - \beta w_t)}{p_t^m} \,. \tag{13}$$

From equation (13), for the same price level, the level of output for a firm with a lower marginal cost is higher than that for a firm with a higher marginal cost. Plugging equation (13) into equation (9), the profit for a firm can be expressed as

$$\pi_t = \frac{[Q_t - (K_{t+1} - K_t)](p_t^m - \beta w_t)^2}{p_t^m} - r_t z \,. \tag{14}$$

From (14), since $\pi_t$ increases when $\beta$ decreases, an individual with a lower marginal cost of production has a higher profit. Thus, it is more profitable for an individual with a lower marginal cost to become an entrepreneur. In addition, from (14), since $\pi_t$ is a convex function of $\beta$, the return to productivity is a convex function of the level of marginal cost. This is similar to the result of Rosen (1981) in which an individual marginally better than another one in productivity can earn a much higher return.

For a young individual who is indifferent between being an entrepreneur and a worker, let her productivity or marginal cost be denoted by $\beta^*$. In equilibrium, individuals with productivities in the interval $[\underline{\beta}, \beta^*]$ become entrepreneurs and individuals with productivities in the interval $[\beta^*, \overline{\beta}]$ become workers (Lucas, 1978). From equation (14), the cutoff level of productivity $\beta^*$ is defined by[9]

[9] In this paper, an individual chosen to become an entrepreneur can still be hired by providing labor in production. The reason for this specification is as follows. Since capital is the fixed cost of production for an individual being an entrepreneur, the wage cost would have become the second type of fixed cost of production if an entrepreneur did not provide labor. Two kinds of fixed costs would make the analysis more complicated. For simplicity, we assume that an entrepreneur also provides one unit of labor. This can be understood as follows: an entrepreneur uses her own labor first before hiring additional workers.

$$\frac{[Q_t-(K_{t+1}-K_t)](p_t^m-\beta^* w_t)^2}{p_t^m}-r_t z=0\,. \tag{15}$$

For the market for labor, labor demand is the sum of demand from the agricultural sector $L_t^a$ and the demand from the manufacturing sector. A manufacturing firm with a marginal cost of $\beta$ demands $\beta q_t$ or $\frac{\beta[Q_t-(K_{t+1}-K_t)]}{p_t^m}(p_t^m-\beta w_t)$ units of labor and labor demand from the manufacturing sector is $L\int_{\underline{\beta}}^{\beta^*}\frac{\beta[Q_t-(K_{t+1}-K_t)](p_t^m-\beta w_t)}{p_t^m}gd\beta$. Thus, total labor demand in period $t$ is $L_t^a+L\int_{\underline{\beta}}^{\beta^*}\frac{\beta[Q_t-(K_{t+1}-K_t)](p_t^m-\beta w_t)}{p_t^m}gd\beta$. The supply of labor in period $t$ is $L$. Labor market equilibrium requires that

$$L_t^a+L\int_{\underline{\beta}}^{\beta^*}\frac{\beta[Q_t-(K_{t+1}-K_t)](p_t^m-\beta w_t)}{p_t^m}gd\beta=L\,. \tag{16}$$

Total output of the manufactured good $Q_t$ is the integration of output produced by firms with marginal costs in the interval $[\underline{\beta},\beta^*]$:

$$Q_t\equiv L\int_{\underline{\beta}}^{\beta^*}q_t gd\beta=L\int_{\underline{\beta}}^{\beta^*}\frac{[Q_t-(K_{t+1}-K_t)](p_t^m-\beta w_t)}{p_t^m}gd\beta\,. \tag{17}$$

For the goods market, with the homothetic utility function, each consumer spends $\theta$ percent of income on the agricultural good and $1-\theta$ percent of income on the manufactured good. Thus, the ratio of the value of all consumers' consumption of the agricultural good and that of the manufactured good is $\theta/(1-\theta)$. The value of agricultural good consumed in a period is $p_t^a L_t^a$. The value of manufactured good consumed in a period is the difference between the total value of manufactured output and the amount used for investment, or $p_t^m[Q_t-(K_{t+1}-K_t)]$. Goods market clearance requires that

$$\frac{\theta}{1-\theta}=\frac{p_t^a L_t^a}{p_t^m[Q_t-(K_{t+1}-K_t)]}\,. \tag{18}$$

A worker employed in the agricultural sector is paid the value marginal product of labor. Since a worker in the agricultural sector produces one unit of output, this agricultural worker's return is $p_t^a$. A worker employed in the manufacturing sector is paid $w_t$. Since an individual may become a worker either in the agricultural sector or the manufacturing sector, in equilibrium the returns to a worker in the two sectors should be equal:[10]

$$w_t = p_t^a \,. \tag{19}$$

For the market for capital, each manufacturing firm demands $z$ units of capital, and the total measure of manufacturing firms is $L\int_{\underline{\beta}}^{\beta^*} gd\beta$. Thus, total demand for capital in period $t$ is $zL\int_{\underline{\beta}}^{\beta^*} gd\beta$. Total supply of capital in period $t$ is $K_t$. The clearance of the market for capital requires that

$$zL\int_{\underline{\beta}}^{\beta^*} gd\beta = K_t \,. \tag{20}$$

For the market for assets, total demand for assets comes from the saving of the young generation.[11] From equation (8), a young individual saves $\frac{\rho}{1+\rho}$ percent of income, and total income of young individuals is the sum of labor income and profits $w_t L + L\int_{\underline{\beta}}^{\beta^*}\left\{\frac{(p_t^m - \beta w_t)^2[Q_t - (K_{t+1} - K_t)]}{p_t^m} - r_t z\right\}gd\beta$. Thus, total demand for assets is $\frac{\rho}{1+\rho}\left[w_t L + L\int_{\underline{\beta}}^{\beta^*}\left\{\frac{(p_t^m - \beta w_t)^2[Q_t - (K_{t+1} - K_t)]}{p_t^m} - r_t z\right\}gd\beta\right]$. Supply of assets takes the form of capital and is equal to $p_t^m K_{t+1}$. The clearance of the market for assets in a period requires that

$$\frac{\rho}{1+\rho}\left[w_t L + L\int_{\underline{\beta}}^{\beta^*}\left\{\frac{(p_t^m - \beta w_t)^2[Q_t - (K_{t+1} - K_t)]}{p_t^m} - r_t z\right\}gd\beta\right] = p_t^m K_{t+1} \,. \tag{21}$$

Together with the initial amount of capital stock $K_0$, equations (15)-(21) define the evolution of endogenous variables over time. For the rest of the paper, the manufactured good is used as the numeraire: $p_t^m \equiv 1$.

[10] In the real world, different sectors are likely to have different wage rates. Wage gap is an important topic in labor economics, and rural-urban wage difference is an important issue in economic development. In this paper, since wage gap is not our focus and we are mainly interested in the impact of firm heterogeneity, for simplicity we assume equalization of wage rates in different sectors.

[11] At the beginning of a period, capital is used for production. At the end of the period, owners of capital sell capital and use the revenue for consumption.

### 3. The steady state

In a steady state, endogenous variables such as the amount of capital do not change over time. We drop time subscript for variables in the steady state. From equations (15)-(21), the following equations are valid in the steady state:

$$Q(1-\beta^* w)^2 - z r = 0\,, \tag{15*}$$

$$L^a + \int_{\underline{\beta}}^{\beta^*} \beta Q(1-\beta w) g d\beta = L\,, \tag{16*}$$

$$L\int_{\underline{\beta}}^{\beta^*} (1-\beta w) g d\beta = 1\,, \tag{17*}$$

$$\frac{\theta}{1-\theta} = \frac{p^a L^a}{Q}\,, \tag{18*}$$

$$w = p^a\,, \tag{19*}$$

$$z L\int_{\underline{\beta}}^{\beta^*} g d\beta = K\,, \tag{20*}$$

$$\frac{\rho}{1+\rho}\left[wL + L\int_{\underline{\beta}}^{\beta^*} [(1-\beta w)^2 Q - rz] g d\beta\right] = K\,. \tag{21*}$$

Equations (15*)-(21*) form a system of seven equations defining a system of seven variables $r$, $p^a$, $w$, $Q$, $L^a$, $K$, and $\beta^*$ as functions of exogenous parameters. A steady state is a tuple ($r$, $p^a$, $w$, $Q$, $L^a$, $K$, $\beta^*$) satisfying equations (15*)-(21*).

To derive properties of the steady state, we reduce the system of seven equations characterizing the steady state to a smaller number of equations so that comparative statics can be conducted. From equations (15*)-(21*), we derive the following set of three equations defining three endogenous variables $w$, $Q$, and $\beta^*$ as functions of exogenous parameters:[12]

$$\Omega_1 \equiv 1 - L\int_{\underline{\beta}}^{\beta^*} (1-\beta w) g d\beta = 0\,, \tag{22a}$$

$$\Omega_2 \equiv \frac{\rho}{1+\rho}\left\{w + Q\int_{\underline{\beta}}^{\beta^*} [(1-\beta w)^2 - (1-\beta^* w)^2] g d\beta\right\} - z\int_{\underline{\beta}}^{\beta^*} g d\beta = 0\,, \tag{22b}$$

[12] The derivation of equations (22a)-(22c) is as follows. First, equation (22a) comes from equation (17*). Second, equation (22b) comes from plugging the value of $K$ from equation (20*) and the value of $rz$ from equation (15*) into equation (21*). Third, equation (22c) comes from plugging the value of $L^a$ from equation (18*) into equation (16*).

$$\Omega_3 \equiv Q\int_{\underline{\beta}}^{\beta^*} \beta(1-\beta w)g d\beta + \left(\frac{\theta}{1-\theta}\right)\frac{Q}{w} - L = 0\,. \tag{22c}$$

Partial differentiation of (22a)-(22c) with respect to $w$, $\beta^*$, $Q$, $z$, $\theta$, $\rho$, and $L$ yields

$$\begin{pmatrix} \frac{\partial\Omega_1}{\partial w} & \frac{\partial\Omega_1}{\partial\beta^*} & 0 \\ \frac{\partial\Omega_2}{\partial w} & \frac{\partial\Omega_2}{\partial\beta^*} & \frac{\partial\Omega_2}{\partial Q} \\ \frac{\partial\Omega_3}{\partial w} & \frac{\partial\Omega_3}{\partial\beta^*} & \frac{\partial\Omega_3}{\partial Q} \end{pmatrix} \begin{pmatrix} dw \\ d\beta^* \\ dQ \end{pmatrix} = -\begin{pmatrix} 0 \\ \frac{\partial\Omega_2}{\partial z} \\ 0 \end{pmatrix} dz - \begin{pmatrix} 0 \\ 0 \\ \frac{\partial\Omega_3}{\partial\theta} \end{pmatrix} d\theta - \begin{pmatrix} 0 \\ \frac{\partial\Omega_2}{\partial\rho} \\ 0 \end{pmatrix} d\rho - \begin{pmatrix} \frac{\partial\Omega_1}{\partial L} \\ 0 \\ \frac{\partial\Omega_3}{\partial L} \end{pmatrix} dL\,. \tag{23}$$

Let $\Delta$ denote the determinant of the coefficient matrix of endogenous variables of (23). Stability of the steady state requires that $\Delta < 0$ (Turnovsky, 1977, chap. 2).

Entry barriers into the manufacturing sector are common in developing countries. There are various motivations for the government to impose entry barriers. First, the government may believe that large firms are more efficient. By forcing small and less efficient firms exiting an industry, factors of production may be absorbed by large and more efficient firms. Thus, allocation of resources may be more efficient. Second, the government may also erect entry barriers for rent seeking purpose. De Soto (2002) has argued that entry barriers lead to a large informal sector in Latin American countries, such as Peru. An increase in the level of entry barrier is captured by an increase in the value of $z$. The following proposition studies the impact of an increase in the level of entry barriers.

Proposition 1: An increase in the level of entry barriers in the manufacturing sector decreases the steady state wage rate, and the cutoff level of marginal cost to become an entrepreneur is lower.

Proof: An application of Cramer's rule on the system (23) yields

$$\frac{dw}{dz} = \frac{\partial\Omega_1}{\partial\beta^*}\frac{\partial\Omega_2}{\partial z}\frac{\partial\Omega_3}{\partial Q}/\Delta < 0\,,$$

$$\frac{d\beta^*}{dz} = -\frac{\partial\Omega_1}{\partial w}\frac{\partial\Omega_2}{\partial z}\frac{\partial\Omega_3}{\partial Q}/\Delta < 0\,. \blacksquare$$

Since the equilibrium wage rate decreases, Proposition 1 shows that an increase in the level of entry barriers in a developing country such as Peru will not be beneficial to workers. To understand Proposition 1, when the level of entry barriers increases, the measure of manufacturing firms decreases because each firm needs a higher amount of capital to come into existence. This decrease in the number of firms decreases the demand for workers and thus the wage rate decreases. Since the measure of manufacturing firms is smaller, only more productive entrepreneurs can remain producing. That is, the cutoff level of marginal cost to become an entrepreneur is lower.

When there is an increase in the level of entry barriers into the manufacturing sector, total output in the manufacturing sector does not necessarily decrease. The reason is as follows. There are two effects on total output when the level of entry barriers increases. First, since the critical level of productivity to become an entrepreneur decreases, the measure of firms in the manufacturing sector decreases. Other things equal, this will tend to decrease total output in the manufacturing sector. Second, since the wage rate is lower, a firm will produce a higher level of output when marginal cost decreases. This will tend to increase total output in the manufacturing sector. Since the two effects work in opposite directions, the total impact on total output in the manufacturing sector is ambiguous. In addition, from equation (20), since $\beta^*$ decreases when $z$ increases, the overall impact of an increase in the level of entry barriers on the total capital stock is ambiguous.

Economic development is associated with an increase in the percentage of income spent on the manufactured good, which is captured by a decrease in the value of $\alpha$ here. The following proposition studies the impact of an increase in the percentage of income spent on the manufactured good.

Proposition 2: An increase in the percentage of income spent on the manufactured good increases the wage rate, and the cutoff level of marginal cost to become an entrepreneur increases. Total capital stock in the steady state increases.

Proof: An application of Cramer's rule on the system (23) yields

$$\frac{dw}{d\theta} = -\frac{\partial \Omega_1}{\partial \beta^*}\frac{\partial \Omega_2}{\partial Q}\frac{\partial \Omega_3}{\partial \theta} / \Delta < 0,$$

$$\frac{d\beta^*}{d\theta} = \frac{\partial \Omega_1}{\partial w}\frac{\partial \Omega_2}{\partial Q}\frac{\partial \Omega_3}{\partial \theta} / \Delta < 0.$$

From (20), the amount of capital in the steady state increases when $\beta^*$ increases. ■

When there is an increase in the percentage of income spent on the manufactured good, total output in the manufacturing sector does not necessarily increase. The reason is that there are two effects on total output working in opposite directions. While the measure of firms increases and this tends to increase output in the manufacturing sector, a higher wage rate increases marginal costs of production and decreases outputs of firms. Without imposing additional structure, it is not clear which effect dominates.

Developing countries differ in their saving rates and saving rates are affected by the degree of patience of an individual. An increase in the degree of patience of an individual is captured by an increase in the value of $\rho$ in this model. The following proposition studies the impact of an increase in the degree of the patience of an individual.

Proposition 3: An increase in the degree of patience of an individual increases the wage rate, and the cutoff level of marginal cost to become an entrepreneur is higher. Total capital stock in the steady state increases.

Proof: An application of Cramer's rule on the system (23) yields

$$\frac{dw}{d\rho} = \frac{\partial \Omega_1}{\partial \beta^*} \frac{\partial \Omega_2}{\partial \rho} \frac{\partial \Omega_3}{\partial Q} / \Delta > 0 ,$$

$$\frac{d\beta^*}{d\rho} = -\frac{\partial \Omega_1}{\partial w} \frac{\partial \Omega_2}{\partial \rho} \frac{\partial \Omega_3}{\partial Q} / \Delta > 0 .$$

From (20), the amount of capital in the steady state increases. ■

When there is an increase in the degree of patience of an individual, total output in the manufacturing sector does not necessarily increase. The reason is as follows. When the degree of patience increases, there are two effects affecting total output in the manufacturing sector. First, since more individuals choose to become entrepreneurs, the measure of firms increases. This tends to increase total output in the manufacturing sector. Second, with an increase in the wage rate, existing firms will produce a smaller amount of output and total output in the manufacturing sector tends to decrease. Because the two effects work in opposite directions and it is not clear which effect dominates, the total impact on the level of output in the manufacturing sector is ambiguous.

Similar to the proof of Propositions 1-3, it can be shown that the impact of an increase in population on the wage rate, the cutoff level of productivity, and total output in the manufacturing sector is ambiguous. The existence of fixed costs leads to increasing returns in the manufacturing sector. In a one-period model with constant returns in agriculture and increasing returns in manufacturing, an increase in population will increase the size of the market and thus the wage rate. However, in this overlapping-generations model, the amount of capital is endogenously determined by saving. Suppose an increase in population increased the wage rate. Other things equal, the profit of a firm would decrease. The amount of saving is a fraction of the sum of wage income and profits. An increase in the wage rate may not increase this sum because one component increases while the other component decreases. Without an increase in the amount of saving, the supply of capital in the next period would not increase. Without an increase in the amount of capital, no additional firms could enter and the wage rate could not increase. That is, the impact of population size on the wage rate is ambiguous. Similarly, the impact of an increase in population on the cutoff level of productivity and total output in the manufacturing sector is ambiguous.

## 4. Conclusion

In this paper, we have studied the impact of firm heterogeneity in economic development in a general equilibrium model in which manufacturing firms engage in oligopolistic competition and the amount of saving is endogenously determined. We show that an increase in the level of entry barriers does not necessarily decrease total output while an increase in population does not necessarily increase total output in the manufacturing sector.

There can be several interesting generalizations and extensions of the model. First, in this model, wage rates in the agricultural sector and manufacturing sector are equal. Due to barriers for individuals to move from rural areas to urban areas, the wage rate in the manufacturing sector can be higher than that in the agricultural sector. Inequality between wage rates in the agricultural sector and the manufacturing sector can be introduced into the model to study unemployment in the urban areas and incentives for migration. Second, this paper studies a closed economy. By studying an open economy, the impact of international trade on firm heterogeneity and economic development can be addressed. The opening of international trade can lead to the exit of less efficient firms in the manufacturing sector.

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**References**

Banerjee, Abhijit, and Esther Duflo. 2005. Growth theory through the lens of development economics. in: Philippe Aghion & Steven Durlauf (ed.) *Handbook of Economic Growth*, pp. 473-552.

Chandler, Alfred. 1990. *Scale and Scope: The Dynamics of Industrial Capitalism*. Cambridge, MA: Harvard University Press.

De Soto, Hernando. 2002. *The Mystery of Capital: Why Capitalism Triumphs in the West and Fails Everywhere Else*. New York: Basic Books.

Drazen, Allan, and Zvi Eckstein. 1988. On the organization of rural markets and the process of economic development. *American Economic Review* 78, 431-443.

Eaton, Jonathan. 1987. A dynamic specific-factors model of international trade. *Review of Economic Studies* 54, 325-338.

Fiori, Giuseppe, Giuseppe Nicoletti, Stefano Scarpetta, and Fabio Schiantarelli. 2008. Employment effect of product and labor market reforms: Are there synergies? *Economic Journal* 122, 79-104.

Griffith, Rachel, Rupert Harrison, and Gareth Macartney. 2007. Product market reforms, labor market institutions and unemployment. *Economic Journal*, 117, 142-166.

Hirschman, Albert. 1958. *The Strategy of Economic Development*. New Haven, CT: Yale University Press.

Holmes, Thomas, and James Schmitz. 1990. A theory of entrepreneurship and its application to the study of business transfers. *Journal of Political Economy* 98, 265-294.

Jean, Sebastien, and Giuseppe Nicoletti. 2004. Regulation and wage premium. CEPII Working Paper, No.12.

Lahiri, Sajal, and Yoshiyasu Ono. 2004. *Trade and Industrial Policy under International Oligopoly*. Cambridge, UK: Cambridge University Press.

Lucas, Robert, Jr. 1978. On the size distribution of business firms. *Bell Journal of Economics* 9, 508-523.

Melitz, Marc. 2003. The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica* 71, 1695-1725.

Montagna, Catia. 1995. Monopolistic competition with firm-specific costs. *Oxford Economic Papers* 47, 318-328.

Neary, Peter, J. 2003. The road less traveled: Oligopoly and competition policy in general equilibrium. In: R. Arnott, B. Greenwald, R. Kanbur and B. Nalebuff (eds.): *Economics for an Imperfect World: Essays in Honor of Joseph E. Stiglitz*, Cambridge, MA: MIT Press.

Nicoletti, Giuseppe, and Stefano Scarpetta 2005. Product market reforms and employment in OECD countries. OECD Working Paper, No. 59.

Pindyck, Robert, and Rubinfeld, Daniel. 2005. *Microeconomics*, sixth edition, Upper Saddle River, New Jersey: Pearson Education.

Qiu, Larry, D. and Wen Zhou. 2007. Merger waves: a model of endogenous mergers. *Rand Journal of Economics* 30, 214-226.

Rosen, Sherwin. 1981. The economics of superstars. *American Economic Review* 71, 845-858.

Rosenberg, Nathan, and Luther Earle Birdzell Jr. 1986. *How the West Grew Rich*, New York, NY: Basic Books.

Shiferaw, Admasu. 2007. Firm heterogeneity and market selection in Sub-Saharan Africa: Does it spur industrial progress? *Economic Development and Cultural Change* 55, 393-423.

Solow, Robert. 1956. A contribution to the theory of economic growth. *Quarterly Journal of Economics* 70, 65-94.

Turnovsky, Stephen. 1977. *Macroeconomic Analysis*. New York, NY: Cambridge University Press.

Zhang, Jie. 2002. Urbanization, population transition, and growth. *Oxford Economic Papers* 54, 94-117.

Zhou, Haiwen. 2010a. Oligopolistic competition, firm heterogeneity, and impact of international trade. *Eastern Economic Journal* 36, 107-119.

Zhou, Haiwen. 2010b. Globalisation and the size distribution of firms. *Economic Record* 86, 84-94.

Zhou, Haiwen, and Ruhai Zhou. 2016. A dynamic model of the choice of technology in economic development. *Frontiers of Economics in China* 11, 498-518.