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A Quantitative Theory of Time-Consistent Unemployment Insurance *

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ABSTRACT

During recessions, the U.S. government substantially increases the duration of unemployment insurance (UI) benefits through multiple extensions. This paper seeks to understand the incentives driving these extensions. Because of the trade-off between insurance and job search incentives, the classic time-inconsistency problem arises. We endogenize a time-consistent UI policy in a stochastic equilibrium search model, where a government without commitment to future policies chooses the UI benefit level and expected duration each period. A longer duration increases the unemployed workers' consumption but reduces their job search incentives, leading to higher future unemployment. We use the framework to evaluate the effects of the 2008-2013 benefit extensions on unemployment and welfare.

Keywords: Time-consistent policy, Unemployment insurance, Labor market, Business cycle
JEL classifications: E61, J64, J65, H21

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1 INTRODUCTION

The U.S. government has extended unemployment insurance (UI) benefits in response to higher unemployment since the 1950s. During the Great Recession, benefit durations were extended up to 99 weeks. A big debate in the literature is whether benefit extensions worsened unemployment during recessions due to adverse incentives (e.g. [Nakajima \(2012\)](#), [Hagedorn, Manovskii, and Mitman \(2015\)](#), [Johnston and Mas \(2016\)](#), and [Chodorow-Reich and Karabarbounis \(2017\)](#)). At the same time, the literature finds the optimal UI policy under the assumption of perfect government commitment differs from the extensions policy implemented (e.g. [Mitman and Rabinovich \(2015\)](#) and [Jung and Kuester \(2015\)](#)). We relax the perfect commitment assumption and look at a time-consistent policy over the business cycle. We find that the time-consistent policy is both qualitatively and quantitatively consistent with the UI extensions during 2008-2013. Furthermore, because the endogenously generated extensions create expectations of further extensions in equilibrium, they worsen unemployment in recessions.

Time-consistent policy has been used to study issues related to monetary policy and taxation. Our application of the concept to UI policies is a natural way to model the decision of a government during recessions. Because the optimal policy with commitment is not optimal *ex post*, there is always incentive for the government to renege on a commitment plan, and in the absence of effective commitment devices, the optimal UI policy under commitment is not credible.¹ This is especially true during recessions when political pressure is high for the government to act out of current interests.

We analyze the government’s choice of UI policy in an equilibrium business cycle model with search and matching. Risk-averse unemployed workers choose search intensity in order to be matched with job vacancies posted by risk-neutral firms. We use the concept of Markov-perfect equilibrium to characterize the decisions of a government without commitment. Because the equilibrium restricts the government’s policy rules to depend only on current payoff-relevant states, the policies are time-consistent. Specifically, a welfare-maximizing central government chooses the UI benefit level and the probability that the benefit expires (“benefit exhaustion probability”) each period depending on the current levels of unemployment and aggregate productivity.²

Modeling the benefit exhaustion probability rather than a fixed length of benefits keeps the government’s decision tractable. At the same time, the inverse of benefit exhaustion probability gives the expected duration, which allows for comparison with empirical evidence on benefit extensions. A key assumption here is that once benefits expire, the unemployed worker does not regain eligibility before he finds a job. In other words, the government commits to excluding these ineligible unemployed workers from receiving UI benefits. Under this assumption, the unemployed workers with benefits search less than those without benefits. As a result, benefit duration policy today,

¹ Section 6.4 discusses the time inconsistency issue in UI policy and the main differences between commitment and non-commitment policies.

² Because we want to use the model to study observed UI variations, we keep the policy parameters as close to reality as possible. For example, we abstract from the type of employment history-dependent UI policy analyzed in [Hopenhayn and Nicolini \(1997\)](#).

through changing the proportion of insured unemployed workers, directly impacts the states of the economy (unemployment and the measure of benefit-eligible unemployed workers) inherited by the future government and thus the future policies.

The private sector's decisions are modeled using a search-matching model with risk-averse workers, endogenous search intensity by the unemployed, and business cycles driven by shocks to aggregate labor productivity. Unemployed workers search for jobs, while firms post vacancies. Both parties make decisions given the government's policy choices. Because future government policies affect their expected future value, their decisions also depend on the expectations about future government policies. Generous future benefit policies reduce worker's incentives to search, which in turn lowers firm's incentives to create vacancies. Since the government's duration policy directly affects the future states of the economy and in turn affecting the private sector's expectations about future policies, the government's policy decision has to take into account the effect of expectations on private choices.

The main trade-off associated with the government's duration policy is between insurance and incentives. A longer duration increases the UI coverage today and thus raising the average insurance for the unemployed workers. It also reduces the average job search through an increase in the share of unemployed workers receiving benefits, which raises the future unemployment and alters the private sector's expectations about future policies. Over the business cycle, UI duration is strongly countercyclical. In response to a drop in productivity, the expected future productivity is low, which implies a low marginal return to production tomorrow and a low marginal gain from job creation today. As a result, the cost of a higher expected duration is low, and the government raises UI duration. As the unemployment rate rises, the marginal gain from increasing UI duration is higher as more unemployed workers receive benefits, and as a result, the expected duration increases further.³

Given these empirically consistent cyclical movements of UI policy, we then apply the model to the U.S. economy between 2008 and 2013. We feed in exogenous job separation rates taken from the data and calibrate exogenous labor productivity so that it matches the observed path of unemployment rates during the period. Overall, our model matches the variations in benefit durations very well, generating the correct timing of duration changes as well as 80% of the overall increase in UI duration.

An implication of our theory is that the Markov policy, by increasing UI duration in recessions, contributes to higher unemployment. Using the calibrated model, we find that at the peak of unemployment between 2008 and 2013, about 3 percentage point increase in the unemployment rate can be accounted for by rising UI benefit extensions. Of this unemployment gap, more than

³ The idea that the welfare gains and costs of UI vary over the business cycle is not new. For example, [Krueger and Meyer \(2002\)](#) argue that the efficiency loss from reduced search effort may be smaller during a recession than during a boom. More recently, [Kroft and Notowidiglo \(2015\)](#) empirically estimate the moral hazard cost and consumption smoothing benefit of UI benefits, and they find that the marginal welfare cost from generous benefits is procyclical and the marginal welfare gain is modest and varies positively with unemployment rate. While they focus on the changing moral hazard effect of UI benefits on individual workers, we investigate the optimal government's response to the changing efficiency loss.

two-thirds can be explained by unemployed workers' expectations of longer future benefit durations. Compared to other structural evaluations of the benefit extension, our approach allows workers and firms to form expectations about future extension policies which are chosen endogenously by the government.⁴

The rest of the paper proceeds as follows. Section 2 describes the model environment and defines the private-sector competitive equilibrium. Section 3 defines the Markov-perfect equilibrium. We characterize the solution to the government's problem and solve the equilibrium. Section 4 describes the parametrization strategy. We conduct equilibrium analysis in Section 5 by presenting the Markov government's policy rules and discussing their implications for the labor market. Section 6 provides quantitative analysis of UI benefit extensions during recessions. Section 7 concludes.

2 MODEL

In this section, we describe the model environment and characterize the competitive equilibrium. The model is based on a search-matching framework with aggregate productivity shocks.

2.1 Model environment

Time is discrete and infinite. The model is inhabited by a mass of infinitely lived workers and firms. The measure of workers is normalized to one. In any given period, a worker can be either employed or unemployed. Some unemployed workers receive UI benefits. Workers are risk averse and maximize expected lifetime utility given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [U(c_t) - v(s_t)]$$

where \mathbb{E}_0 is the period 0 expectation factor, and β is the time discount factor. Period utility comprises of utility from consumption of goods $U(c)$ and disutility from job search activity $v(s)$. Utility is increasing in c and decreasing in s . To study the insurance incentive of the government we assume that $U(\cdot)$ is a concave function. Only unemployed workers choose positive search intensity; that is, there is no on-the-job search. Each period, an employed worker gets paid wages from production. Wage determination technology is specified later in this section. An unemployed worker, if on unemployment benefits, receives b from the government. In addition, an unemployed worker also produces h , which we take as the combined value of leisure and home production. There are no private insurance markets and workers cannot save or borrow.

Firms are risk neutral and maximize the expected discounted sum of profits, with the same discount factor β . A firm can be either matched to a worker (and producing) or vacant. A vacant

⁴ [Mitman and Rabinovich \(2016\)](#) estimate a government's policy response function using historical extensions and unemployment data. Similar to our approach, they allow the private sector to form expectations about future policies. The difference is the expectations are based on the exogenous policy rules instead of endogenously chosen policy like in our case.

firm posting a vacancy incurs a flow cost κ .

Unemployed workers and vacancies form new matches. Let I and V denote the aggregate search by unemployed workers and the aggregate vacancy posting by firms, respectively. Then the number of new matches formed in a period is given by the matching function $M(I, V)$. The matching function exhibits constant returns to scale, is strictly increasing and strictly concave in both arguments, and is bounded above by the number of expected matches: $M(I, V) \leq \min\{I, V\}$. The job-finding probability per efficiency unit of search intensity, f , and the job-filling probability per vacancy, q , are functions of labor market tightness, $\theta = V/I$. More specifically,

$$\begin{aligned} f(\theta) &= \frac{M(I, V)}{I} = M(1, \theta) \\ q(\theta) &= \frac{M(I, V)}{V} = M\left(\frac{1}{\theta}, 1\right). \end{aligned}$$

Following the assumptions made on M , $f(\theta)$ is increasing in θ and $q(\theta)$ is decreasing in θ . The job-finding probability for an unemployed worker searching with intensity s is $sf(\theta)$. Existing matches are destroyed exogenously with job separation probability δ .

Each period, a matched pair of a worker and a firm produces z , where z is the aggregate labor productivity. z is equal to \bar{z} at the steady state.

2.2 Government policy

The government cannot borrow or save; instead, it balances the budget each period. The government finances unemployment benefits b through a lump sum tax, τ , on all workers, both employed and unemployed. The government budget constraint is

$$\tau = u_{benefit} b. \tag{1}$$

The government decides the generosity of the UI program by varying (1) benefit level, $b \geq 0$, and (2) the benefit exhaustion probability d ($1/d$ is the maximum expected benefit duration). Once the benefit level and exhaustion probability are determined, benefit-eligible unemployed workers receive benefits b with probability $1 - d$. Government policies are taken as exogenous in a decentralized competitive equilibrium but are chosen endogenously in the government's problem.

Two things are worth noting about the exhaustion probability. First, using a probability instead of introducing individual history dependency means an unemployed worker could receive UI benefits for more or less than the maximum expected benefit duration ($1/d$). But this setup allows for tractability by using the proportion of benefit-eligible workers to proxy parsimoniously for the duration that a worker stays on UI.⁵

⁵ Similar ways of modeling are used in the monetary policy ("Calvo fairy") and in the sovereign debt literature (modeling long duration debt).

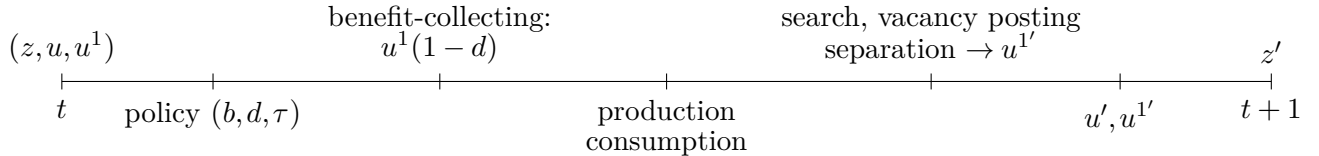


Figure 1: Timing of events.

Second, as noted in the introduction, a key assumption about exhaustion probability is that the government can commit to excluding benefit-ineligible unemployed workers from receiving benefits: once an unemployed worker loses benefit eligibility, he has to find work first before becoming eligible for benefits again. The assumption is consistent with how UI policy normally works.⁶

2.3 Timing

The timing of events within a period is illustrated in Figure 1 and is as follows. The economy enters period t with a measure of the total unemployed workers u and a measure of the benefit-eligible unemployed workers u^1 . The aggregate shock z then realizes. (z, u, u^1) are the aggregate states of the economy.

Once government policies (b, d, τ) for the period are announced, $u_{benefit} = u^1(1 - d)$ workers collect benefit. In other words, with probability d , benefit-eligible unemployed workers lose benefit status in this period.

Employed workers produce z and receive wages w . Unemployed workers produce h and, if collecting benefits, receive b . All workers pay a lump sum tax τ .

Given aggregate states and government policies for the period, unemployed workers with and without benefits choose search intensity s^1 and s^0 , respectively. At the same time, firms decide how many vacancies to post, at cost κ per vacancy. The aggregate search is then $I = u^1(1 - d)s^1 + (u - u^1(1 - d))s^0$, aggregate vacancy posting is V , and market tightness is equal to $\theta = V/I$. The fraction of unemployed workers with and without benefits who find jobs is $f(\theta)s^1$ and $f(\theta)s^0$, respectively. At the same time, a fraction δ of the existing $1 - u$ matches are exogenously destroyed. Newly unemployed workers and unemployed workers with benefits constitute next period's state u^1 .⁷

⁶ But during the most recent recession and especially during 2008-2009, many unemployed workers who had previously exhausted their benefits became eligible for new tiers of extensions. In other words, during this period the government does not have commitment to *not* bring ineligible unemployed workers back into the eligible pool. We interpret this non-commitment as coming from convenience instead of optimality concerns: the government did not optimally *choose* to let these unemployed workers' benefits expire and then give them more tiers of extension. As such, our original assumption about the exhaustion probability is an abstraction of what happened during this period.

⁷ Effectively, newly unemployed workers receive benefits with next period probability $1 - d$, the same probability an unemployed worker with benefits today keeps collecting tomorrow. In reality, newly unemployed workers qualify for benefits with at least two quarters of earnings and must pass an "earnings test" that depends on individual state policies. We model it as a probability for simplicity here.

The laws of motion of unemployed workers are

$$\begin{aligned}
\text{total unemployment: } u' &= \underbrace{\delta(1-u)}_{\text{newly unemployed}} + \underbrace{(1-f(\theta)s^0)(u-u^1(1-d)) + (1-f(\theta)s^1)u^1(1-d)}_{\text{previously unemployed who didn't find job}} \\
\text{benefit-eligible unemployed: } u^1 &= \underbrace{\delta(1-u)}_{\text{newly unemployed}} + \underbrace{(1-f(\theta)s^1)u^1(1-d)}_{\text{unemployed with benefits who didn't find job}}
\end{aligned} \tag{3}$$

2.4 Workers

Denote by g the government policy (b, d, τ) . In what follows we suppress the functional arguments in θ , which is an object determined in equilibrium. Wage w depends on the states of the economy, and may be an equilibrium object. The wage determination process is specified later. An unemployed worker with benefits consumes $h + b - \tau$ and chooses search intensity s^1 ; an unemployed worker without benefits consumes $h - \tau$ and chooses search intensity s^0 . With probability $f(\theta)s$, $s = \{s^0, s^1\}$, he finds a job and starts working the following period. Let $V^e(z, u, u^1; g)$, $V^1(z, u, u^1; g)$ and $V^0(z, u, u^1; g)$ be the value of an employed worker, an unemployed worker with and without benefits, respectively, given the aggregate states and government policy for the period.

The optimization problem of an unemployed worker *without* benefits (superscript 0 denotes no benefits) is

$$\begin{aligned}
V^0(z, u, u^1; g) &= \max_{s^0} U(h - \tau) - v(s^0) + f(\theta)s^0\beta\mathbb{E}V^e(z', u', u^1; g') \\
&\quad + (1 - f(\theta)s^0)\beta\mathbb{E}V^0(z', u', u^1; g'),
\end{aligned} \tag{4}$$

and the problem of an unemployed worker *with* benefits is

$$\begin{aligned}
V^1(z, u, u^1; g) &= \max_{s^1} U(h + b - \tau) - v(s^1) + f(\theta)s^1\beta\mathbb{E}V^e(z', u', u^1; g') \\
&\quad + (1 - f(\theta)s^1)\beta\mathbb{E} \left[d'V^0(z', u', u^1; g') + (1 - d')V^1(z', u', u^1; g') \right],
\end{aligned} \tag{5}$$

where if still unemployed next period, then with probability d' he loses benefits.

A worker entering a period employed produces and consumes his wage w minus tax τ . With probability δ , he loses his job and becomes unemployed the following period. There is no intra-temporal search, so a newly separated worker remains unemployed for at least one period. The Bellman equation of an employed worker is then

$$\begin{aligned}
V^e(z, u, u^1; g) &= U(w - \tau) + (1 - \delta)\beta\mathbb{E}V^e(z', u', u^1; g') \\
&\quad + \delta\beta\mathbb{E} \left[d'V^0(z', u', u^1; g') + (1 - d')V^1(z', u', u^1; g') \right],
\end{aligned} \tag{6}$$

where if separated from his current job, then with probability $1 - d'$ he has benefits the next period.

2.5 Firms

An unmatched firm posts a vacancy to be matched with a worker and start production.⁸ A firm that posts a vacancy incurs a flow cost κ . With probability $q(\theta)$, a vacancy is filled and ready for production the following period. Let $J^u(z, u, u^1; g)$ and $J^e(z, u, u^1; g)$ be the value of an unmatched and a matched firm, respectively. The Bellman equation of an unmatched firm is

$$J^u(z, u, u^1; g) = -\kappa + q(\theta)\beta\mathbb{E}J^e(z', u', u^{1'}; g') + (1 - q(\theta))\beta\mathbb{E}J^u(z', u', u^{1'}; g'), \quad (7)$$

In the equilibrium and under free-entry condition, the firm will post vacancies $v(z, u, u^1; g)$ until $J^u(z, u, u^1; g) = 0$.

A matched firm receives output net of wages $z - w$. With constant probability δ , a match is destroyed at the end of the period and the firm becomes vacant. The Bellman equation of a matched firm is

$$J^e(z, u, u^1; g) = z - w + (1 - \delta)\beta\mathbb{E}J^e(z', u', u^{1'}; g') + \delta\beta\mathbb{E}J^u(z', u', u^{1'}; g'). \quad (8)$$

2.6 Wage determination

When a match is formed, the economic rent is shared between the firm and the worker. It is well known that to generate realistic cyclical movements in search and matching models wage rigidity is needed. There are many ways to do this in the literature (e.g. [Hagedorn and Manovskii \(2008\)](#), [Hall and Milgrom \(2008\)](#)). To introduce wage rigidity, we set wages to be a function of productivity. In particular, wages increase in labor productivity z but less than one to one. This way, workers and firms share the risk of fluctuating aggregate labor productivity.

While Nash bargaining is widely used in the search and matching framework, other wage determination specifications have been used in the literature. Works in the UI literature have used alternate specifications to introduce wage rigidity: [Landais, Michaillat, and Saez \(2010\)](#) and [Nakajima \(2012\)](#) use specifications similar to the one we use. The main advantage of our specification is it allows us to calibrate the degree of wage rigidity directly from the data. The main drawback is because wages do not depend on workers' outside options, benefit policies have no effect on wages, which is the macro channel emphasized in [Hagedorn, Karahan, Manovskii, and Mitman \(2013\)](#).⁹ Nevertheless, we choose this exogenous wage specification as the benchmark because the government's non-commitment optimal problem is complicated enough without introducing endogenous wage settings. Using an exogenous wage setting, we can more easily illustrate the non-commitment mechanism which is the highlight of this paper.¹⁰

⁸ The firms can be viewed as a representative firm with a collection of jobs and several vacancies.

⁹ More recently, [Chodorow-Reich and Karabarbounis \(2017\)](#) find small macro effects of UI extensions.

¹⁰ In the online appendix we present some partial quantitative results using Nash bargaining, which illustrate interactions between wage and UI policy.

2.7 Competitive equilibrium

DEFINITION 1. (Competitive equilibrium) Given a policy $g = (b, d, \tau)$ and initial conditions (z^-, u^-, u^{1-}) , a competitive equilibrium consists of (z, u, u^1) -measurable functions for worker's search intensities $s^0(z, u, u^1; g)$ and $s^1(z, u, u^1; g)$, market tightness $\theta(z, u, u^1; g)$, total unemployment $u'(z, u, u^1; g)$ and the measure of benefit-eligible unemployed workers $u^1(z, u, u^1; g)$, and value functions $V^e(z, u, u^1; g)$, $V^0(z, u, u^1; g)$, $V^1(z, u, u^1; g)$, $J^e(z, u, u^1; g)$, and $J^u(z, u, u^1; g)$, such that for all $(z, u, u^1; g)$:

- the value functions satisfy the worker's and firm's Bellman equations (4)-(8);
- the search intensities s^0 and s^1 solve the unemployed worker's maximization problems of (4) and (5), respectively;
- the market tightness θ is consistent with the free-entry condition, $J^u(z, u, u^1; g) = 0$;
- the measures of unemployment satisfy the laws of motion (2)-(3).

2.8 Characterization of private-sector optimality

The competitive equilibrium can be characterized by three optimality conditions.¹¹ The online appendix contains a derivation of the optimality conditions. In what follows, primes denote variables of the following period, and subscripts denote derivatives.

Worker's Search Incentive The optimal choice of search intensity s^0 and s^1 for the unemployed worker is characterized by

$$\text{no benefit: } \frac{v_s(s^0)}{f(\theta)} = \beta \mathbb{E} \left[U(w' - \tau') - U(h - \tau') + v(s^{0'}) + (1 - f(\theta')s^{0'}) \frac{v_s(s^{0'})}{f(\theta')} - \delta \frac{v_s(s^{1'})}{f(\theta')} \right] \quad (9)$$

$$\begin{aligned} \text{with benefit: } \frac{v_s(s^1)}{f(\theta)} &= \beta \mathbb{E} d' \left[U(w' - \tau') - U(h - \tau') + v(s^{0'}) + (1 - f(\theta')s^{0'}) \frac{v_s(s^{0'})}{f(\theta')} - \delta \frac{v_s(s^{1'})}{f(\theta')} \right] \\ &\quad + \beta \mathbb{E} (1 - d') \left[U(w' - \tau') - U(h + b' - \tau') + v(s^{1'}) + (1 - f(\theta')s^{1'} - \delta) \frac{v_s(s^{1'})}{f(\theta')} \right]. \end{aligned} \quad (10)$$

The worker's optimality conditions state that the marginal cost (left-hand side) of increasing the job-finding probability equals the marginal gain (right-hand side). The marginal cost is the marginal disutility from search weighted by the aggregate job-finding rate per efficiency unit of search. The marginal gain is the sum of the utility gain from being employed the next period and the benefit of economizing on future search cost. We can make two useful observations.

PROPOSITION 1. Unemployed workers with benefits search less than those without benefits, $s^1 < s^0$, given $v_s(s) > 0$ and $v_{ss}(s) > 0$, and $0 < \mathbb{E} d' < 1$.

For the unemployed worker with benefits (equation 10), his marginal gain from search is a weighted sum of the gains if he loses benefit eligibility (first line) and if he stays benefit eligible

¹¹ To economize on notation, we suppress the dependence on $(z, u, u^1; g)$. It should be understood throughout that the equilibrium allocations are functions with arguments $(z, u, u^1; g)$.

next period (second line). The first part is identical to the marginal gain for the unemployed worker without benefits (equation 9) and is larger than the second part. So the marginal gain from search and thus the search incentive is smaller for the unemployed worker with benefits as long as the expected future exhaustion probability d' is bounded away from 1. Given an increasing marginal search cost function, it then implies that the unemployed worker with benefits search less. In other words, a non-zero expected probability of receiving benefits *tomorrow* ($\mathbb{E}(1 - d')$) creates a moral hazard problem today for the unemployed worker with benefits.

PROPOSITION 2. A lower expected future benefit exhaustion probability d' or a higher future benefit b' reduces the search incentive for the unemployed workers with benefits.

Because the first part of the marginal gain from search (equation 10) is larger than the second part, the total marginal gain and hence the search incentive is lower when the expected future benefit exhaustion probability is lower. At the same time, the second part is decreasing in the future benefit, so the search incentive is lower when the future benefit is expected to be higher.

Firm's Job Posting Incentive From firm's free-entry condition

$$\frac{\kappa}{q(\theta)} = \beta \mathbb{E} \left[z' - w' + (1 - \delta) \frac{\kappa}{q(\theta')} \right], \quad (11)$$

where the marginal cost (left-hand side) equals the marginal gain (right-hand side) of a filled vacancy. The marginal cost is the flow cost of posting a vacancy weighted by the probability of filling that vacancy. The marginal gain is the profits from a filled vacancy. Because a newly formed match does not become operational until the next period, the gain from production only has components from the next period. The current productivity level z therefore does not have a direct impact on the firm's current hiring decision. Instead, due to persistence in the productivity process it affects the firm's expectation of future productivity and hence its current hiring decision.

3 MARKOV EQUILIBRIUM

In this section, we define the Markov-perfect equilibrium in our economy. We assume the government is a utilitarian planner who maximizes the expected value of a worker's utility. The government policy instruments include benefit level b , expected duration $1/d$, and lump-sum tax τ . We do not pose assumptions on the government's ability to commit to future policies, and we consider government policies that are time consistent using the Markov-perfect equilibrium, à la [Klein, Krusell, and Ríos-Rull \(2008\)](#).

Intuitively, one can think of the economy as having a different government each period. Each successive government chooses only the current policy, taking all future governments' policies as given. In other words, today's government cannot directly choose future policies. Instead, both today's government and the private sector form expectations about future government policy rules.

Like [Klein, Krusell, and Ríos-Rull \(2008\)](#), we focus on equilibria where government policy depends *differentiably* on the aggregate states of the economy.¹²

The timing of events is illustrated in [Figure 1](#). Because each worker and firm is infinitely small, they take future government policies as given.¹³ The equilibrium described above can be equivalently stated as an equilibrium where the government chooses policy and private-sector allocations together given the states of the economy. To reduce the number of policy instruments in the government's problem, we use the following function derived from the government's budget constraint to express tax

$$\mathcal{T}(u^1, b, d) := u^1(1-d)b.$$

The *government period welfare function* is equal to the average welfare of all workers, and is given by

$$\begin{aligned} R(z, u, u^1, b, d, s^0, s^1) = & \quad (1-u)U(w(z) - \mathcal{T}(u^1, b, d)) && \text{worker} \\ & + (u - u^1(1-d)) [U(h - \mathcal{T}(u^1, b, d)) - v(s^0)] && \text{unemployed without benefit} \\ & + u^1(1-d) [U(h + b - \mathcal{T}(u^1, b, d)) - v(s^1)] && \text{unemployed with benefit} \end{aligned}$$

DEFINITION 2. (Markov-perfect equilibrium) A Markov-perfect equilibrium consists of a government's value function G , government policy rules Ψ^b and Ψ^d , and private decision rules $\{S^0, S^1, \Theta, \Gamma, \Gamma^1\}$ such that for all aggregate states (z, u, u^1) , $b = \Psi^b(z, u, u^1)$, $d = \Psi^d(z, u, u^1)$, $s^0 = S^0(z, u, u^1)$, $s^1 = S^1(z, u, u^1)$, $\theta = \Theta(z, u, u^1)$, $u' = \Gamma(z, u, u^1)$, and $u^{1'} = \Gamma^1(z, u, u^1)$ solve

$$\max_{b, d, s^0, s^1, \theta, u', u^{1'}} R(z, u, u^1, b, d, s^0, s^1) + \beta \mathbb{E}G(z', u', u^{1'})$$

subject to

- The worker's laws of motion

$$f_1(u, u^1, d, s^0, s^1, \theta, u') := u' - \delta(1-u) - f(\theta)(s^0 - s^1)u^1(1-d) - (1-f(\theta)s^0)u = 0 \quad (12)$$

$$f_2(u, u^1, d, s^1, \theta, u^{1'}) := u^{1'} - \delta(1-u) - (1-f(\theta)s^1)u^1(1-d) = 0; \quad (13)$$

- The private-sector optimality conditions below, writing $\mathcal{O} = (z, u, u^1)$ to economize on notation

¹² While there is no proof for the existence and uniqueness of Markov-perfect equilibrium, [Chatterjee and Eyigungor \(2014\)](#) provide argument for the existence of Markov-perfect equilibrium with continuous decision rules; and [Pei and Xie \(2015\)](#) use numerical method to provide evidence for a unique *differentiable* equilibrium in a simpler setup.

¹³ The current government policies are decided before the private sector moves. The future government policies depend on future states, which are affected by how the private sector moves today. In our setup, workers and firms do not take into account how their action will affect future policies through changing future states.

$$\begin{aligned}
& \eta_1(s^0, \theta, \mathcal{O}'; \Psi^b, \Psi^d, S^0, S^1, \Theta) \\
& := \frac{v_s(s^0)}{f(\theta)} - \beta \mathbb{E} \left[U(w(z') - \mathcal{T}(u^{1'}, \Psi^b(\mathcal{O}'), \Psi^d(\mathcal{O}'))) - U(h - \mathcal{T}(u^{1'}, \Psi^b(\mathcal{O}'), \Psi^d(\mathcal{O}'))) + v(S^0(\mathcal{O}')) \right] \\
& \quad - \beta \mathbb{E} \left[(1 - f(\Theta(\mathcal{O}'))) S^0(\mathcal{O}') \frac{v_s(S^0(\mathcal{O}'))}{f(\Theta(\mathcal{O}'))} - \delta \frac{v_s(S^1(\mathcal{O}'))}{f(\Theta(\mathcal{O}'))} \right] = 0
\end{aligned} \tag{14}$$

$$\begin{aligned}
& \eta_2(s^1, \theta, \mathcal{O}'; \Psi^b, \Psi^d, S^0, S^1, \Theta) \\
& := \frac{v_s(s^1)}{f(\theta)} - \beta \mathbb{E} \Psi^d(\mathcal{O}') \left[U(w(z') - \mathcal{T}(u^{1'}, \Psi^b(\mathcal{O}'), \Psi^d(\mathcal{O}'))) - U(h - \mathcal{T}(u^{1'}, \Psi^b(\mathcal{O}'), \Psi^d(\mathcal{O}'))) + v(S^0(\mathcal{O}')) \right] \\
& \quad - \beta \mathbb{E} \Psi^d(\mathcal{O}') \left[(1 - f(\Theta(\mathcal{O}'))) S^0(\mathcal{O}') \frac{v_s(S^0(\mathcal{O}'))}{f(\Theta(\mathcal{O}'))} - \delta \frac{v_s(S^1(\mathcal{O}'))}{f(\Theta(\mathcal{O}'))} \right] \\
& \quad - \beta \mathbb{E} (1 - \Psi^d(\mathcal{O}')) \left[U(w(z') - \mathcal{T}(u^{1'}, \Psi^b(\mathcal{O}'), \Psi^d(\mathcal{O}'))) - U(h + \Psi^b(\mathcal{O}') - \mathcal{T}(u^{1'}, \Psi^b(\mathcal{O}'), \Psi^d(\mathcal{O}'))) + v(S^1(\mathcal{O}')) \right] \\
& \quad - \beta \mathbb{E} (1 - \Psi^d(\mathcal{O}')) [1 - f(\Theta(\mathcal{O}'))) S^1(\mathcal{O}') - \delta] \frac{v_s(S^1(\mathcal{O}'))}{f(\Theta(\mathcal{O}'))} = 0
\end{aligned} \tag{15}$$

$$\eta_3(\theta, \mathcal{O}'; \Theta) := \frac{\kappa}{q(\theta)} - \beta \mathbb{E} \left[z' - w(z') + (1 - \delta) \frac{\kappa}{q(\Theta(\mathcal{O}'))} \right] = 0; \tag{16}$$

- The government value function satisfies the functional equation

$$G(\mathcal{O}) \equiv R(\mathcal{O}, \Psi^b(\mathcal{O}), \Psi^d(\mathcal{O}), S^0(\mathcal{O}), S^1(\mathcal{O})) + \beta \mathbb{E} G(z', \Gamma(\mathcal{O}), \Gamma^1(\mathcal{O})).$$

The government chooses current policy to maximize current and expected future welfare, knowing how the private sector will behave given the policy. More specifically, the current government weighs the trade-off between the current and future welfare. By choosing a longer expected duration $1/d$, the current government increases the share of unemployed workers receiving benefits today, thus raising the current welfare. At the same time, because of moral hazard problem, unemployed workers on benefits choose a lower search intensity, and as a result higher duration reduces the average search intensity, leading to higher future unemployment and lower future welfare.¹⁴

In the equilibrium, all successive governments follow the same set of policy rules. The current government, by choosing the current policy, affects the policies of future governments through changing the future states of the economy. This *disciplining effect*, through the private-sector's expectations of future policies, affects the job search of unemployed workers with benefits today, and through general equilibrium effects, affects the job search of unemployed worker without benefits. The current government correctly anticipates this effect when choosing today's policy. Proposition 3 provides the conditions that characterize the government's optimal decisions. The proof involves deriving the Generalized Euler Equation (GEE) and is included in the online appendix.

PROPOSITION 3. Given the aggregate states of the economy and the private-sector optimality con-

¹⁴ A secondary effect exists through taxation. With longer duration, more unemployed workers receive benefits and the lump-sum tax is higher. The size of this effect is small relative to the other two effects.

ditions, the unemployment benefit policy b in the Markov-perfect equilibrium is characterized by¹⁵

$$R_b = 0, \quad (17)$$

and policy d associated with the expected benefit duration can be characterized by the GEE

$$\begin{aligned} 0 &= R_d - f_{1d}\lambda \\ &+ \frac{f_{2d}}{f_{2u^{1'}}} \left\{ \frac{\eta_{1u^{1'}}}{\eta_{1s^0}} [R_{s^0} - \lambda f_{1s^0}] + \frac{\eta_{2u^{1'}}}{\eta_{2s^1}} \left[R_{s^1} - \lambda f_{1s^1} - \frac{f_{2s^1}}{f_{2d}} (R_d - \lambda f_{1d}) \right] + \dots \right. \\ &\dots + \frac{\eta_{3u^{1'}}}{\eta_{3\theta}} \left[-\lambda f_{1\theta} - \frac{f_{2\theta}}{f_{2d}} (R_d - \lambda f_{1d}) - \frac{\eta_{1\theta}}{\eta_{1s^0}} (R_{s^0} - \lambda f_{1s^0}) - \frac{\eta_{2\theta}}{\eta_{2s^1}} \left(R_{s^1} - \lambda f_{1s^1} - \frac{f_{2s^1}}{f_{2d}} (R_d - \lambda f_{1d}) \right) \right] \left. \right\} \\ &+ \beta \mathbb{E} \left(-\frac{f_{2d}}{f_{2u^{1'}}} \right) [R'_{u^1} - \lambda' f'_{1u^1}] \\ &+ \beta \mathbb{E} \left(-\frac{f_{2d}}{f_{2u^{1'}}} \right) \left(-\frac{f'_{2u^1}}{f'_{2d}} \right) [R'_d - \lambda' f'_{1d}], \end{aligned} \quad (18)$$

where λ is the shadow price of unemployment characterized by

$$\begin{aligned} 0 &= \lambda \\ &+ \left\{ \frac{\eta_{1u'}}{\eta_{1s^0}} [R_{s^0} - \lambda f_{1s^0}] + \frac{\eta_{2u'}}{\eta_{2s^1}} \left[R_{s^1} - \lambda f_{1s^1} - \frac{f_{2s^1}}{f_{2d}} (R_d - \lambda f_{1d}) \right] + \dots \right. \\ &\dots + \frac{\eta_{3u'}}{\eta_{3\theta}} \left[-\lambda f_{1\theta} - \frac{f_{2\theta}}{f_{2d}} (R_d - \lambda f_{1d}) - \frac{\eta_{1\theta}}{\eta_{1s^0}} (R_{s^0} - \lambda f_{1s^0}) - \frac{\eta_{2\theta}}{\eta_{2s^1}} \left(R_{s^1} - \lambda f_{1s^1} - \frac{f_{2s^1}}{f_{2d}} (R_d - \lambda f_{1d}) \right) \right] \left. \right\} \\ &- \beta \mathbb{E} [R'_e - \lambda' f'_{1e}] \\ &- \beta \mathbb{E} \left(-\frac{f'_{2u}}{f'_{2d}} \right) [R'_d - \lambda' f'_{1d}]. \end{aligned} \quad (19)$$

Benefit level b affects only the current welfare and does not have an effect on the future states of economy and so it does not affect future policies.¹⁶ As a result, b is set at a level that equates the current marginal gain (higher consumption for unemployed workers with benefits) and marginal cost (higher lump-sum tax). The equation $R_b = 0$ captures these incentives.

Interpretation of the GEE In contrast, because d affects future states (u', u^1) , its choice is more complex. Here we give a heuristic interpretation of the different effects based on the GEE (Equation 18). Section 5.2 provides a comparative static analysis of the government's different incentives. From (18), a change in d has four effects, holding policy b unchanged. First, it directly affects the trade-off between current consumption and future unemployment (first line). In particular, a lower d (higher expected duration) increases the current welfare by increasing the share of unemployed workers receiving benefits. This is the *insurance* effect. At the same time, a lower d also reduces average

¹⁵ Subscripts denote partial derivatives, for example, $f_{1d} = \partial f_1 / \partial d$.

¹⁶ While the current benefit level does not affect search behavior, higher expected future benefit levels reduce the current search incentive of an unemployed worker with benefits.

search, thus increasing the future unemployment.¹⁷ Second, through changing the expectation of future benefit duration it affects the current job search of the unemployed workers with benefits. This is the *moral hazard* effect. Changes in the search intensity of these unemployed workers in turn affect the average job search and vacancy posting through general equilibrium effects (second and third lines). Third, any change in d affects future consumption through changing the future unemployment (fourth line). This and the second effect together represent the “search/leisure” trade-off: a lower d reduces today’s job search but increases future unemployment. Lastly, through changing d' , any change in d changes the future trade-off between consumption and unemployment (last line). The weight on the last line can be thought of as dd'/dd holding the two flow equations at zero and unemployment after the next period unchanged. The government determines the current d by setting the sum of the four marginal effects of d to zero.

Note that the choice of d changes the *average* search because unemployed workers with and without benefits search differently (Proposition 1). It does not have a direct moral hazard effect on the individual’s search decision; instead, through its effects on the expected future policies (d', b') , it creates a moral hazard effect on the search of unemployed workers with benefits (Proposition 2). This absence of a direct moral hazard effect is inherent in the timing of the model, that is, policy takes effect at the beginning of each period and consumption and production take place before job creation.¹⁸

The Markov-perfect equilibrium is then characterized by a system of *functional* equations (12)–(16), and (17)–(19). An analytical characterization of the Markov-perfect equilibrium is not possible; instead, we solve for the equilibrium numerically by approximating the government policy rules and the private-sector decision rules using the Chebyshev collocation method.¹⁹

4 PARAMETRIZATION

We describe our calibration strategy in this section. The model period is one month. We calibrate the parameters by matching the steady state moments of the Markov equilibrium to the long-run empirical moments of the U.S. labor market between 2003.I and 2007.IV. We do this under the assumption that the government behaves as a benevolent utilitarian welfare-maximizer without commitment to future policies and the equilibrium economy with such a government mirrors the U.S. economy. We then evaluate untargeted model moments and model-generated policy paths during recession (in Section 6) to validate this model assumption.²⁰

¹⁷ $f_{1d} = f(\theta)(s^0 - s^1)u^1$ is the marginal change in unemployment when d changes.

¹⁸ Chetty (2008), Krusell, Mukoyama, and Sahin (2010), Nakajima (2012) and Mitman and Rabinovich (2016) among others adopt the timing of consumption before job creation in their analyses. Mitman and Rabinovich (2015) use a different timing whereby job creation takes place before consumption and production.

¹⁹ The GEE contains derivatives of policy rules, which make solving the Markov equilibrium different from solving a standard growth model or the optimal policy problem of a government with commitment.

²⁰ We calibrate to the Markov equilibrium here. An alternative is to calibrate to the competitive equilibrium of the model economy imposing an exogenous UI policy rule which mimics the UI policy in the U.S. over the last 50 years—similar to Mitman and Rabinovich (2016)—and then solve the Markov equilibrium using these calibrated

The utility function is

$$U(c, s) = \frac{c^{1-\sigma}}{1-\sigma} - v(s),$$

where $v(\cdot)$ is the search cost function. We assume $v(\cdot)$ is a non-negative, strictly increasing, and convex function, with the property that $v(0)$ is bounded and $v(0) \geq 0$. We specify the search cost function to be consistent with the literature:

$$v(s) = \alpha \frac{s^{1+\phi}}{1+\phi}.$$

For any $\alpha > 0$, v exhibits positive and increasing marginal cost, $v_s(s) > 0$ and $v_{ss}(s) > 0$, and $v(0) = v_s(0) = 0$.

We adopt the matching function from [Den Haan, Ramey, and Watson \(2000\)](#), which is also used in [Hagedorn and Manovskii \(2008\)](#) and [Krusell, Mukoyama, and Sahin \(2010\)](#) among others,

$$M(I, V) = \frac{V}{[1 + (V/I)^\chi]^{1/\chi}},$$

where I is the aggregate job search and V is the aggregate vacancy posting in the economy. This matching function guarantees that both the job-finding rate,

$$f(\theta) = \frac{\theta}{[1 + \theta^\chi]^{1/\chi}},$$

and the job-filling rate,

$$q(\theta) = \frac{1}{[1 + \theta^\chi]^{1/\chi}},$$

are always strictly less than 1.

We pick three parameters related to preferences. The discount factor β is set at $0.99^{1/3}$, giving a quarterly discount factor of 0.99. The coefficient of relative risk aversion σ is set to 1 (log utility). Finally, the search cost curvature parameter ϕ is set to 1 following the average estimate in the literature.²¹

The externally calibrated parameters are summarized in [Table 1](#). Following the methodology outlined in [Shimer \(2005\)](#), we calculate the average monthly job separation rate from the aggregate-level CPS data.²² This gives an average job-finding rate during 2003.I-2007.IV of 0.40, and an average separation rate $\delta = 0.02$.²³ We set the costs of vacancy creation κ to be 58% of monthly

parameters.

²¹ Imposing ϕ equal to 1 gives a quadratic search cost function. This restriction is consistent with estimates by [Yashiv \(2000\)](#), [Christensen et al. \(2005\)](#), and [Lise \(2013\)](#), and calibration work of [Nakajima \(2012\)](#).

²² To be consistent with our model, we do not adjust for time aggregation error when computing the job separation rate. Therefore, the job separation rate from the data is $\delta_t = u_{t+1}^s / e_t$, where u^s is the short-term (one to four weeks) unemployment, and e is the total employment.

²³ Although some may argue that the U.S. economy during 2003.I-2007.IV is *above* the long-run trend, we believe it is an appropriate period to target for the labor market, especially because of the secular downward trend in job

Table 1: Externally Calibrated Parameters

Parameter	Description	Value
δ	U.S. job separation rate	0.02
κ	Vacancy posting cost	0.58
ρ	Persistence of productivity	0.968
σ_ϵ	Standard deviation of innovation to productivity	0.0060
ϵ_w	Elasticity of wage with respect to productivity	0.446

Note: Calibration targets are monthly statistics of the U.S. economy.

labor productivity following [Hagedorn and Manovskii \(2008\)](#).

As in [Shimer \(2005\)](#), labor productivity z is taken to be the average real output per employed person in the non-farm business sector. This measure is taken from the seasonally adjusted quarterly data constructed by the Bureau of Labor Statistics. We normalize the mean productivity to be $\bar{z} = 1$, and assume an AR(1) process for the shock to z :

$$\log z' = \rho \log z + \sigma_\epsilon \epsilon,$$

where $\rho \in [0, 1)$, $\sigma_\epsilon > 0$, and ϵ are i.i.d. standard normal random variables. We target a quarterly autocorrelation of 0.762 and an unconditional standard deviation of 0.013 for the HP-filtered productivity process. At a monthly frequency this means setting $\rho = 0.9680$ and $\sigma_\epsilon = 0.006$.

Wages are determined by the following function of productivity,

$$w(z) = \exp(\log \bar{w} + \epsilon_w \log z),$$

where \bar{w} represents the steady-state share of output for the worker, and ϵ_w is the elasticity of the average wage with respect to aggregate productivity. We use the data on labor productivity and real wages (constructed using labor shares data) between 1951.I and 2014.IV to estimate $\epsilon_w = 0.446$. This means a 1 percentage point increase in labor productivity is associated with a 0.446 percentage point increase in real wages. Our estimate is close to the estimate of 0.449 for 1951.I-2004.IV obtained by [Hagedorn and Manovskii \(2008\)](#).

We jointly calibrate four parameters using steady-state moments. The four parameters are (1) the value of nonmarket activity h , (2) the matching function parameter χ , (3) the level parameter of search cost α , and (4) the steady-state wage level \bar{w} . We use four steady-state moments as targets: (1) the expected UI replacement ratio,²⁴ (2) the average job-finding rate, (3) the average job-filling

separation rate documented by, for example, [Fujita \(2012\)](#). The online appendix also documents a declining trend in job-finding rate since 1951. Given these trends, using the average job-finding and separation rates over a longer horizon would overestimate the recent steady-state numbers.

²⁴ Unlike [Shimer \(2005\)](#) and [Hagedorn and Manovskii \(2008\)](#), the benefit level in our model is endogenously chosen by the government and is a function of nonmarket activity in the steady state. This is why we can target replacement

Table 2: Internally Calibrated Parameters

Parameter	Description	Value
h	Value of nonmarket activity	0.595
α	Disutility of search	1.706
χ	Matching parameter	3.462
\bar{w}	Steady-state wage	0.979
Target	Data	Model
Average replacement ratio	40%	38.1%
Average job-finding rate	0.40	0.416
% unemployed with benefits	45	45.8
Average job-filling rate	0.66	0.661

Note: Calibration targets are monthly statistics of the U.S. economy 2005.I-2007.IV.

rate, and (4) the proportion of unemployed workers with benefits.²⁵ We follow [Shimer \(2005\)](#) and set the replacement ratio at 40%. The average job-finding rate is the monthly rate at which unemployed workers become employed, and it is 0.40 for 2003.I-2007.IV. Over the same period, the job-filling rate is 0.66.²⁶ Table 2 reports these internally calibrated parameter and the matching of calibration targets. The calibrated model delivers a benefit duration of 26.3 weeks (untargeted), very close to the benefit duration of 26 weeks in the U.S. during normal times, thus delivering the first model validation.

Table 3 compares key labor market statistics in the pre-2008 U.S. economy and the calibrated Markov economy.²⁷ The calibrated model does a good job of generating the relevant cyclical properties, which provides the second model validation. The model also produces a negative correlation between unemployment and vacancy, thus preserving the shape of the Beveridge-curve (inverse relation between unemployment and vacancy). Two parameters are critical to generating the cyclical properties. First, we calibrate the elasticity of wage with respect to productivity to match data counterparts. The relatively low wage elasticity means firm's profit and hence vacancy posting are volatile over the business cycle. Second, in the equilibrium, unemployed workers expect higher benefit durations when the productivity is low, which lowers search even more and leads to larger cyclical responses in search and hence unemployment.

ratio to pin down the value of nonmarket activity.

²⁵ We use a derivative-free algorithm for least-squares minimization to perform joint calibration. See [Zhang, Conn, and Scheinberg \(2010\)](#) for details.

²⁶ The job-filling rate is calculated as the job-finding rate divided by the vacancy-unemployment ratio, where the latter is computed using the national unemployment rate reported by the BLS and the nonfarm job openings from the Job Openings and Labor Turnover Survey. The estimate for the 2003.I-2007.IV period is close to [Den Haan, Ramey, and Watson \(2000\)](#) who use plant-level data during 1972.II-1988.IV and get a job-filling rate of 0.71.

²⁷ Here we use the long-run average job separation rate in the model to see if the model can generate the second-moments in the data. In Section 6, we focus on the period of 2008-2013 and use the realized path of job separation rates to generate period-to-period movements.

Table 3: Summary Statistics: Cyclicity

Statistic	Productivity		Unemployment		Vacancy		v-u ratio	
	z		u		v		v/u	
<i>Quarterly U.S. data 1951.I-2007.IV</i>								
Standard deviation		0.013		0.123		0.142		0.257
Correlation matrix	z	1		-0.271		0.392		0.339
	u	-		1		-0.889		-0.951
	v	-		-		1		0.980
	v/u	-		-		-		1
<i>Calibrated Markov economy</i>								
Standard deviation		0.013		0.147		0.167		0.273
Correlation matrix	z	1		-0.908		0.919		0.982
	u	-		1		-0.698		-0.909
	v	-		-		1		0.933
	v/u	-		-		-		1

Note: Seasonally adjusted unemployment series, u , is constructed by the BLS from the CPS. Vacancy-posting, v , is [Barnichon \(2010\)](#)'s spliced series of seasonally adjusted help-wanted advertising index constructed by the Conference Board and the job-posting data from the JOLTS. Both u and v are quarterly averages of monthly series. All variables are reported in logs as deviations from an HP trend with smoothing parameter 1,600.

5 EQUILIBRIUM ANALYSIS

In this section, we present the Markov government policy rules and discuss their effects on the equilibrium labor market.

5.1 Markov equilibrium policy rules

Figure 2 plots the Markov equilibrium UI policy rules holding productivity at the steady state level.²⁸ In each plot, the solid line represents the policy rule, and the dashed line marks the steady-state unemployment.

The expected UI duration $1/d$ increases in the total unemployment. The government's decision on the UI duration involves a trade-off between insurance (for higher current consumption) and job-creation (for higher future welfare). When the unemployment is high, both the insurance and the job-creation incentives are high—the former because as more people are unemployed a longer duration gives more unemployed workers benefits, and the latter because a shorter duration increases the job-search incentives for more people. In the equilibrium, the increase in the insurance incentive outweighs the higher job creation incentive, and the expected duration increases in the total

²⁸ We also hold the proportion of unemployed workers with benefits at the steady-state level.

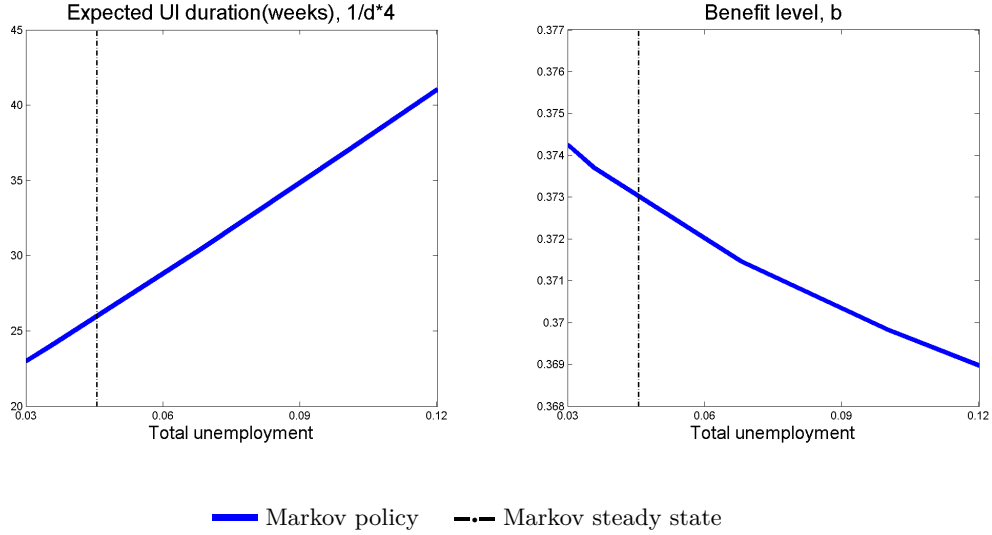


Figure 2: Markov equilibrium government policy rules holding productivity and proportion of benefit-eligible unemployed workers at steady state.

unemployment.

In contrast, the UI benefit level b is lower at higher unemployment, but the size of variations is minuscule, falling by less than 1% from the steady state level when the unemployment is at 10%. Intuitively, when the unemployment increases, the rise in the cost of taxation is almost entirely offset by the higher gain from insurance.

Figure 3 plots the Markov equilibrium UI policy rules, holding the unemployment (both total unemployment and benefit-eligible unemployment) at the steady-state levels. The expected benefit duration increases drastically with a lower labor productivity, especially when the productivity is below its steady state level. This is because when the productivity is low, the expected productivity next period is also low, assuming a persistent productivity process. As such, the marginal return from production tomorrow (for both workers and firm) is low, as is the cost of a low level of job creation today (or equivalently a high unemployment tomorrow). As a result, the marginal cost of a longer duration (lower job creation) is low, and the government chooses a long duration. The unemployment benefit level b , in contrast, increases with a higher labor productivity, but the slope is fairly small.

Overall, the plots show that the Markov equilibrium benefit duration is countercyclical, higher when the unemployment is higher or when the productivity is lower, whereas the benefit level is almost acyclical (slightly procyclical). These properties are broadly consistent with the U.S. policies during recessions. In Section 6 we quantitatively evaluate how close the policies are to reality. Before that, we use comparative static and impulse response analyses to provide some intuition for the equilibrium results.

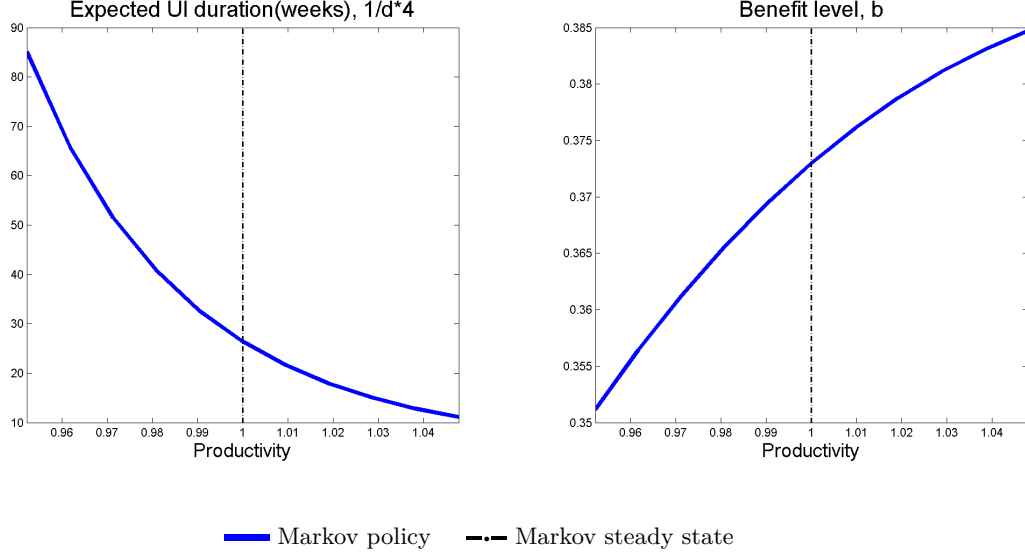


Figure 3: Markov equilibrium government policy rules holding unemployment at steady state.

5.2 Comparative static analysis of government incentives

Because the government's choice of d (which directly translates into expected duration) involves the trade-off between insurance and moral hazard, we conduct a comparative static analysis to understand the changes in these two incentives that drive the movements in the benefit duration. Figure 4 shows the responses of these two incentives to changes in unemployment (left panel) and productivity (right panel).²⁹ The two incentives are equalized at the steady state (dotted vertical line). Off steady state, when the marginal gain (solid blue line) is higher than the marginal cost (dashed red line) the government has an incentive to increase the benefit duration.

As the total unemployment rises, both the insurance gains and the moral hazard cost are higher. The marginal insurance gain is mainly captured by R_d in the GEE (Equation 18)

$$- \underbrace{u^1}_{\text{extensive margin}} \times \underbrace{\left[U(h + b - \tau) - v(s^1) - \left(U(h - \tau) - v(s^0) \right) \right]}_{\text{welfare gain from giving benefits to an additional worker}} \quad (20)$$

With a higher unemployment (u) (and fixing the proportion of benefit-eligible unemployed workers), u^1 in (20) is larger, which increases the marginal welfare gain from a smaller d (longer duration). At the same time, the larger moral hazard cost comes from the fact that a higher unemployment makes the future unemployment more sensitive to changes in search. The left panel of Figure 4 shows that a 1% increase in the total unemployment (from the dotted vertical line to the dashed vertical

²⁹ The responses over unemployment hold the proportion of benefit-eligible unemployed workers and productivity at steady state levels. The responses over productivity hold total and benefit-eligible unemployment at steady states.

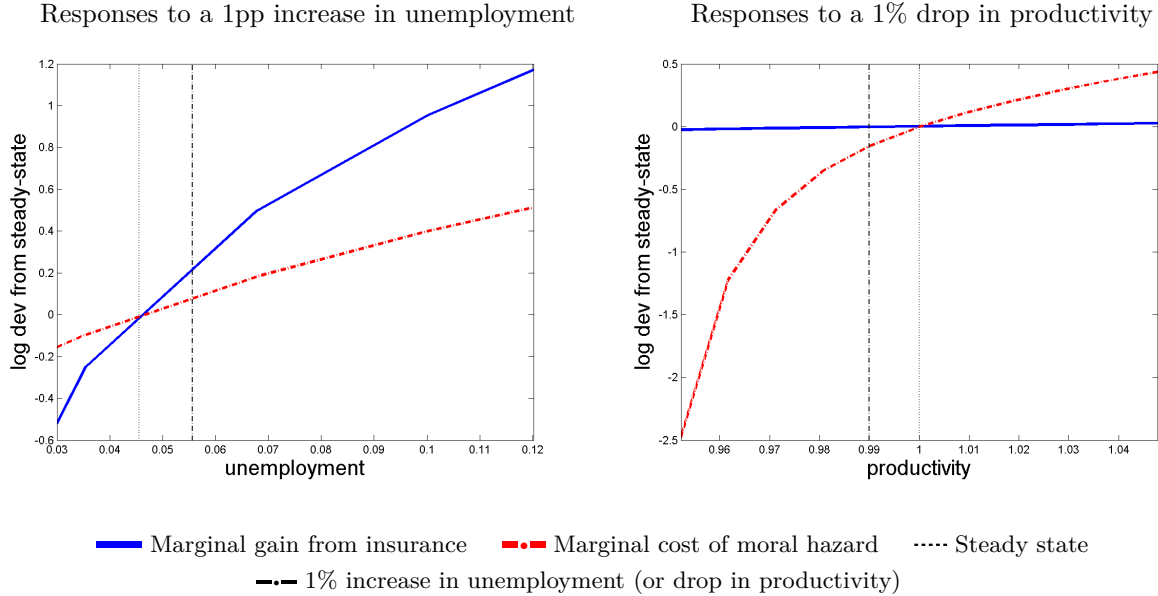


Figure 4: Responses of marginal gain from insurance and marginal moral hazard cost to a 1 percentage-point increase in unemployment or 1% drop in productivity, holding government policies at the steady state.

line) raises the insurance incentive by 16% and moral hazard cost by 9%. The higher increment in the insurance incentive means that at a higher unemployment level the government has a stronger incentive to increase the UI duration.

In response to a drop in productivity, both the insurance gains and the moral hazard cost are lower. In particular, a lower productivity leads to lower wages, which increases the employed workers’ marginal utility of consumption and reduces the marginal gain from insurance. This effect is small, and disappears if the worker is risk neutral. In contrast, the drop in moral hazard cost is large. At a lower productivity level, the expected future productivity and wages are also lower, which means that an increase in the future unemployment leads to a smaller reduction in the average consumption. In other words, there is a lower moral hazard cost associated with a longer benefit duration, and the government can “afford” to choose a longer duration. The drop in the moral hazard cost is amplified by a drop in job posting—result of the lower productivity and hence a lower expected future profit—which lowers the response of future unemployment to changes in the duration policy. This amplification accounts for the nonlinear shape of the duration policy with respect to productivity.

The variations of the marginal welfare gain and cost here are consistent with recent empirical findings by [Kroft and Notowidiglo \(2015\)](#). First, they find that the moral hazard cost is procyclical. The marginal cost of moral hazard here varies positively with both the unemployment and productivity and is overall procyclical.³⁰ Second, they find that the marginal welfare gain from consumption

³⁰ While we distinguish between a drop in productivity and an increases in unemployment, the empirical work of [Kroft and Notowidiglo \(2015\)](#) does not. So their finding that moral hazard cost is higher when the unemployment rate is lower should not be directly compared to the left panel of Figure 4.

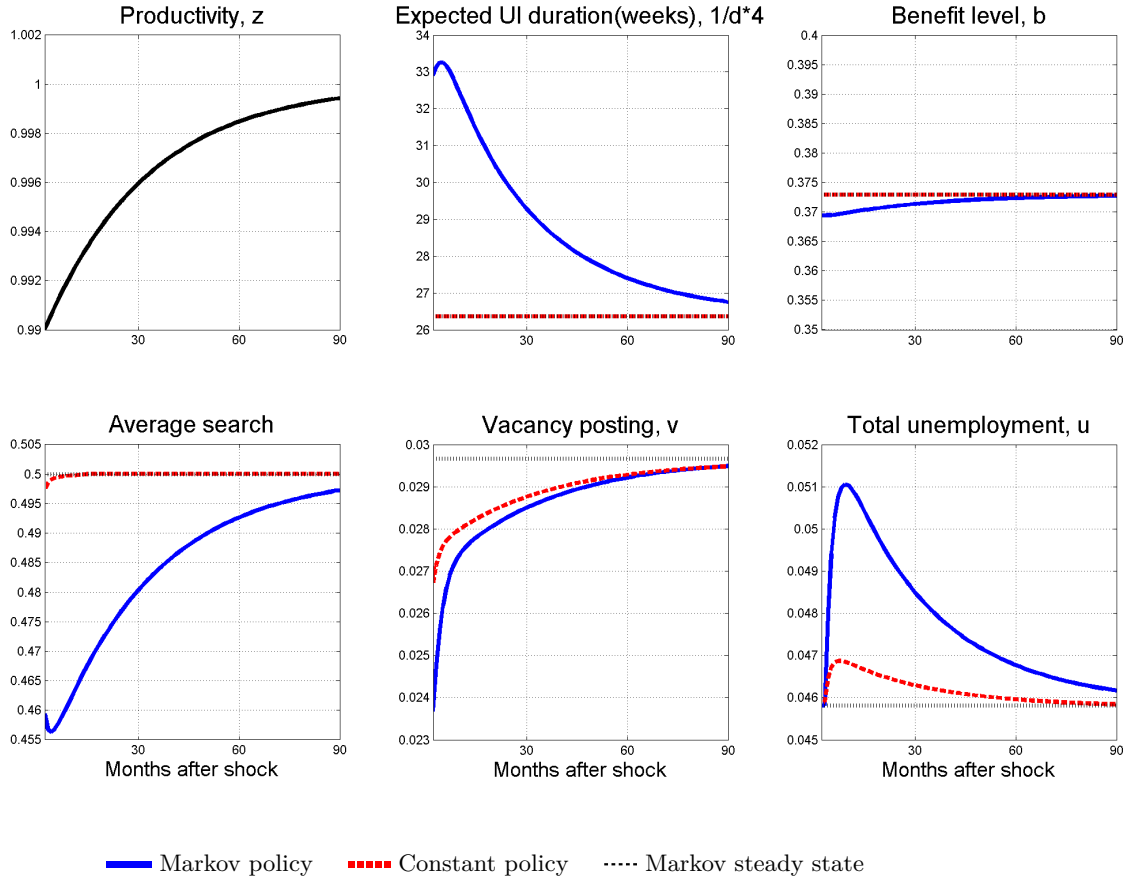


Figure 5: Impulse response to a 1% drop in productivity.

smoothing varies positively with the unemployment but the variations are small. The marginal gain from insurance in our mechanism also varies positively with unemployment, but the scale of variation is large. This is because the gain from consumption smoothing that they document correspond to the *intensive* margin in (20), which is only part of the gain from insurance in our mechanism. Most of the variations in the gain from insurance in the left panel of Figure 4 come from the *extensive* margin.

5.3 Impulse response in policy and labor market

To illustrate how government policy and the private sector interact in the Markov equilibrium we now consider the economy's response to a one-time, unanticipated drop in productivity. Figure 5 shows the response of the economy to a 1% drop in productivity z at time 0. We first focus on the responses in the Markov equilibrium (solid blue lines). We then compare the Markov equilibrium to an economy without the government policy changes (dotted red lines) to understand the driving force behind the movements in the labor market.³¹ Because the transitional dynamics are relatively

³¹ The online appendix provides impulse responses of additional labor market statistics.

slow, it takes a long time for the economy to return to a steady state. In Figure 5, the time horizon is 90 months or approximately seven and a half years.

Upon shock, the benefit duration rises immediately from 26.3 weeks to 33 weeks and then falls slowly as productivity recovers. By month 30, the benefit duration has fallen to 29 weeks. Since the unemployment is a slow-moving process, it peaks at around month 7, when productivity has already recovered one-fifth of the 1% drop. Because the benefit duration increases in unemployment, the drop in the benefit duration after the initial rise is slowed down by the rising unemployment. Benefit level, in contrast, falls initially to below steady state, with less than 1% total change, and slowly recovers to the pre-shock steady state as both productivity and unemployment recover.³²

Search by the unemployed workers both with and without benefits fall initially, which drives the average search down by about 10%. While the drop in search by those without benefits primarily comes from lower expected future wages, the drop for the unemployed workers with benefits is additionally driven by a longer expected benefit duration. Vacancy posting also falls initially but the recovery is much quicker than the overall job search recovery. By month 6, vacancy posting is more than half-way back to the pre-shock steady-state level. This is because vacancy posting depends on the expected future productivity and aggregate search. As search by the individual unemployed workers recovers, and with high unemployment during the first few months after shock, *aggregate* search is high. Because higher aggregate search increases the marginal return from vacancy posting, in equilibrium vacancy posting responds to the aggregate (and not average) search.³³ Total unemployment increases rapidly to peak in month 7, before gradually falling back to its steady-state level.

To understand to what extent the rise in unemployment is driven by changes in productivity versus policy, we shut down the changes in government policy. Compared to the unemployment increase with policy changes (solid blue lines), the increase without policy change (dotted red lines) is much more muted (1pp versus 5pp). Underlying this difference in unemployment change are smaller drops in both search and vacancy posting without policy change. In particular, the average search drops by less than 1%, compared to a 9% decrease with policy changes. The drop in vacancy posting without policy changes is about half of the drop with policy change. This shows that job search incentives are directly distorted by policy changes whereas vacancy posting incentives respond mostly to productivity.

³² The benefit level in our model is slightly procyclical. This is because during a recession, lower wages and higher total unemployment raise the marginal cost (in utility terms) of providing benefits. Even though the scale of changes is small, the procyclical benefits go against what happens in a typical recession. To be more realistic, it is reasonable to think that during a recession, the government has a more relaxed budget which allows it to keep the benefit level constant.

³³ Mechanically, in our setup because wages are exogenous, labor market tightness does not respond to changes in search or vacancy posting. So when the aggregate search increases, vacancy posting also increases to keep tightness constant.

6 UI DURATION EXTENSIONS IN RECESSION

Because the cyclical properties of the Markov equilibrium policy rules are consistent with those of the U.S. policy, in this section we use the theoretical framework to study recessions. We first validate the model by using the model to account for the benefit duration variations during and after the Great Recession (December 2007 to December 2013). We then compare the Markov policy to alternative UI policies to study the impacts on unemployment and welfare.

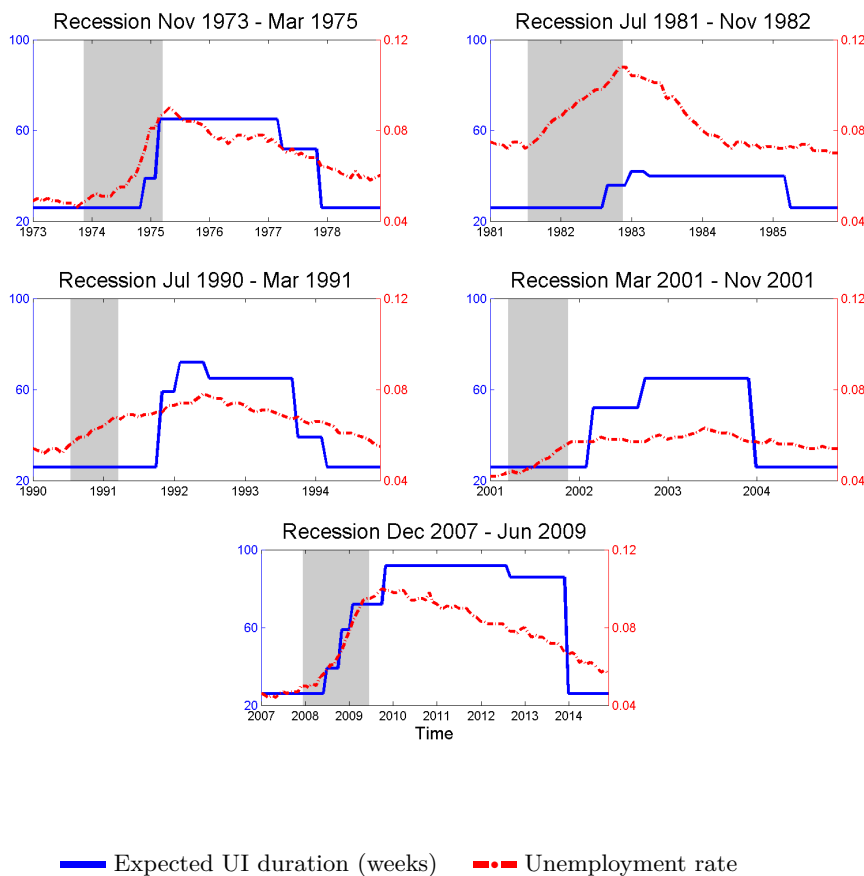


Figure 6: Empirical changes in unemployment (right axis) and UI duration (left axis) during recessions since the 1970s.

6.1 Empirical evidence of UI benefit extensions in recessions

We first document the variations in UI duration during each recession since the 1970s. Figure 6 plots the variations in unemployment and UI duration during all five recession episodes.³⁴ The shaded regions mark National Bureau of Economic Research (NBER) official recession dates. For each recession

³⁴ The recession from January to July 1980 was both shorter and milder than the other recessions. In addition, it was followed immediately by the much longer recession from July 1981 to November 1982. We therefore leave out the former recession period.

sion episode, the dotted red line (right axis) plots the unemployment rate, and the solid blue line (left axis) plots the maximum expected UI duration in weeks. The timing and the size of changes in the UI duration follow the specifics of the federal unemployment compensation laws, which are available from the U.S. Department of Labor Employment and Training Administration (DOLETA) website. Two things are worth noting. First, during all recession episodes, the UI duration reached its highest level around the time the unemployment peaked. Second, comparing across recessions, the recession with higher unemployment is in general associated with higher expected UI durations, except for the 1980s recession. Our Markov equilibrium benefit duration rises with the total unemployment, which is consistent with the above historical evidence.

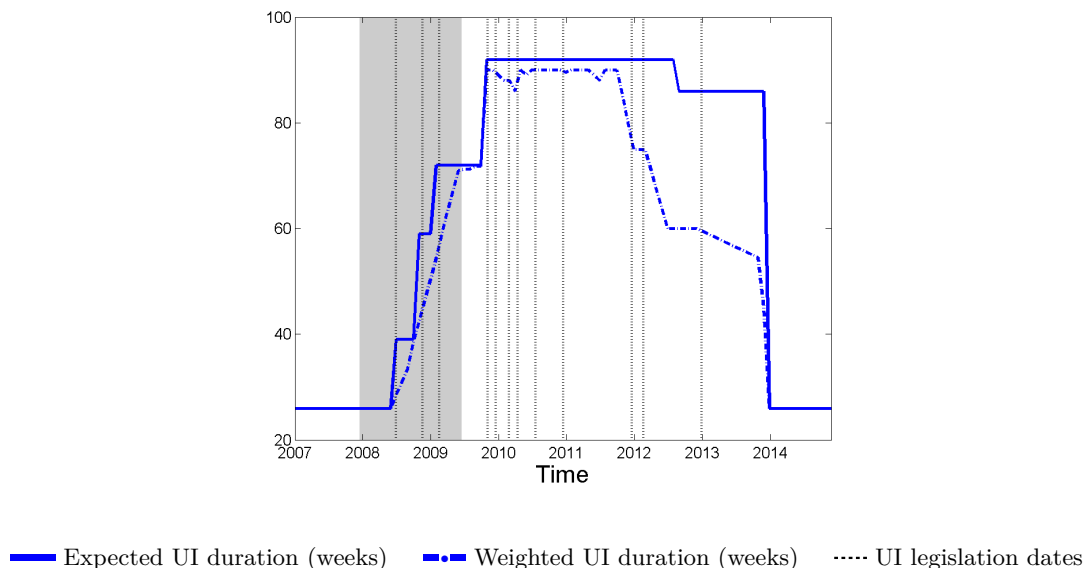


Figure 7: Empirical changes in UI duration and timing of UI-related legislation during the Great Recession.

Because more detailed data are available for the Great Recession, we document the frequency of legislation on UI policy during and following this recession in Figure 7. The vertical dotted lines indicate the timings of legislation. The frequency of legislation increased substantially from the mid-2008, especially from the late 2009, to 2011. This provides evidence that during the recessions the federal government does not follow a prescribed policy rule and instead makes policy choices depending on the contemporary states of the economy.³⁵ This observation motivates our choice to use the Markov equilibrium policy, which is time consistent, to describe the policy changes during the recession.

Because the state-level implementations of UI benefit extension are conditional on the state’s

³⁵ There is an automatic benefit extensions program called Extended Benefits (EB), whereby the benefit duration is automatically extended when a state’s unemployment rate exceeds 6.5% or 8%. The EB extensions are triggered in a state regardless of the national economic conditions or what the Congress decides. So in a sense this is a committed extensions program, in contrast to the discretionary extensions implemented in recessions. During the Great Recession, the EB extensions represent about one-third of the total overall maximum extensions. We thank an anonymous reviewer for pointing this out.

economic conditions, especially on the state’s insured unemployment rate (IUR) and total unemployment rate (TUR), we use the two statistics to compute whether the state was eligible for longer durations in the month a UI-related legislation was passed during the Great Recession. We then create a weighted measure of the maximum expected UI durations across states using the number of total insured unemployed workers in each state as weights. Appendix A provides more details on the dates of policy changes and the construction of this weighted measure of UI duration. Figure 7 plots the weighted expected UI duration (dashed blue line). For the quantitative analyses, we use this weighted average series as the empirical counterpart for a more accurate description of the UI duration policy implemented.

6.2 The Great Recession

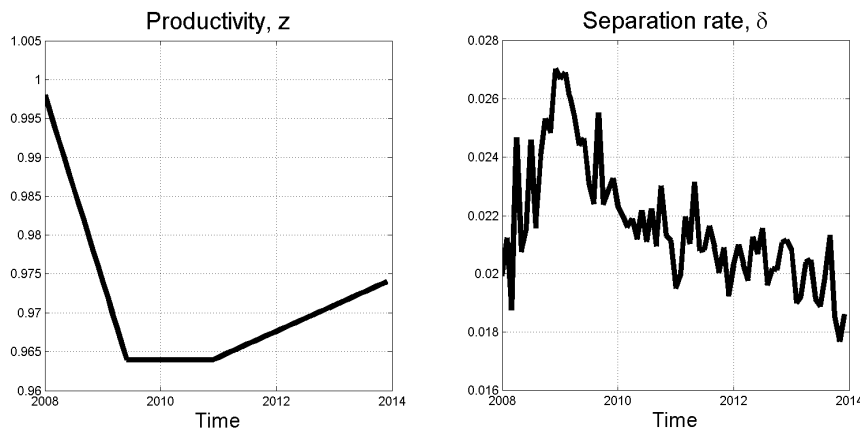


Figure 8: Exogenous shock processes during the Great Recession.

To further our theory, we put the model in an environment similar to the U.S. economy during and following the Great Recession from December 2007 to December 2013. Because our theory focuses on UI, we specify a path for productivity to match the observed unemployment path during the period.³⁶ We use a piecewise linear productivity path consisting of the decline, the trough and the recovery. It turns out that this simplified way of calibrating the productivity path generates a good fit for the path of unemployment. We use an exogenous series of job separation rates over this period calculated from the aggregate-level CPS data. This path is then fed into the model assuming they are unexpected shocks and the agents expect future separation rate to return to its steady state. In Section 6.2.3 we look at the case where the agents have an expectation about how future

³⁶ The online appendix includes two alternative calibrations where we use productivity z to either match the path of UI duration or get a best fit for both unemployment and duration. The benchmark calibration of the productivity path here is the preferred one because while unemployment is a smooth process (and reported on a weekly basis), UI duration is not, because Congress meets on a relatively fixed schedule that typically does not respond to changes in the economic conditions. Therefore, matching to the UI duration series is subject to the underlying assumptions about meeting schedules.

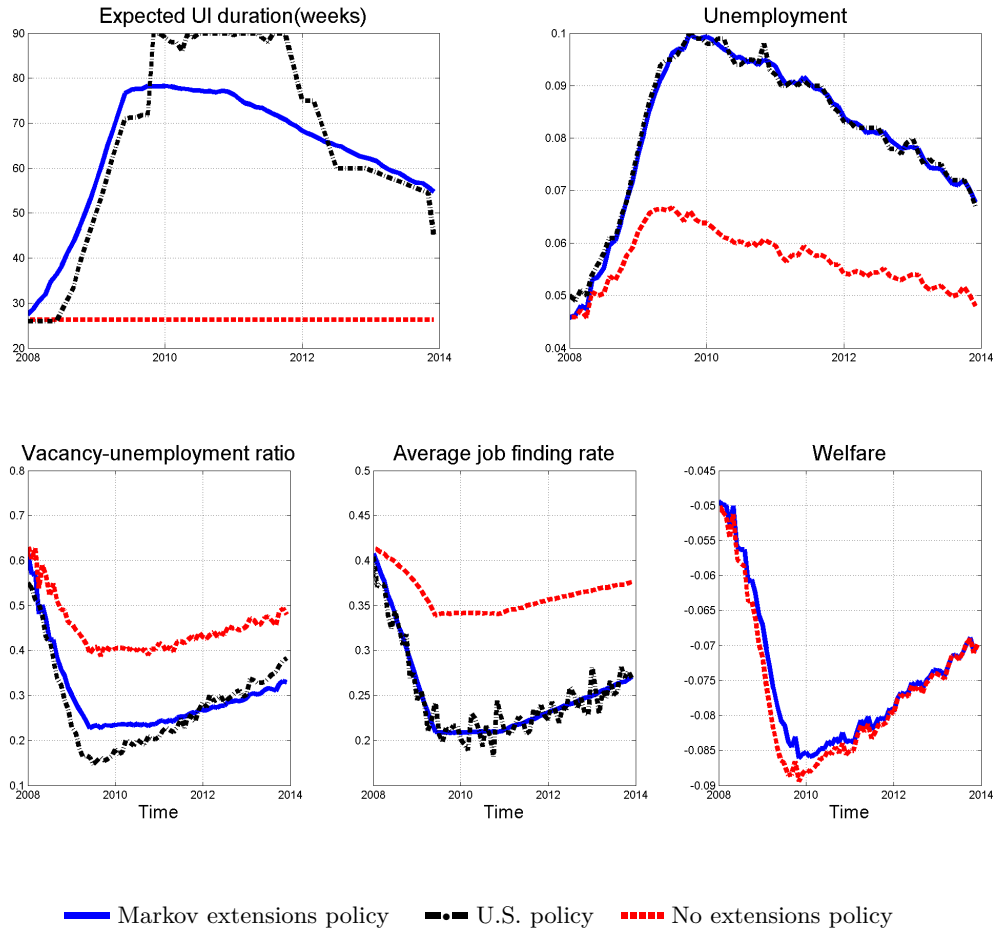


Figure 9: UI duration, unemployment, and welfare during the Great Recession: Model, data, and counterfactual policies.

job separation rate evolves. Figure 8 plots these two shock paths.³⁷

Model Fit Given the exogenous shock paths, Figure 9 plots the variations in the UI duration, labor market variables, and average welfare generated by the Markov equilibrium (solid blue lines). Compared to the U.S. economy (dashed black lines), the Markov equilibrium matches well the variations of UI duration and the vacancy-unemployment ratio. In particular, the Markov equilibrium benefit duration policy rises from 26 weeks to slightly below 80 weeks, compared to the maximum of 90 weeks in the U.S. economy. The Markov equilibrium also generates a decline in the benefit duration,

³⁷ The path we specify for labor productivity is in fact not far-fetched. Labor productivity as measured by the average production per person in the nonfarm business sector fell by 3% from the end of 2007 to the beginning of 2009, which is more than 4% in detrended terms. The difference between our calibrated path and the actual path is the recovery part. The U.S. labor productivity recovered swiftly to the pre-2008 level by the end of 2010, whereas our path follows a slower recovery process. For theories on slow and/or jobless recovery following the Great Recession, see, for example, [Stock and Watson \(2012\)](#), [Shimer \(2012\)](#), and [Heathcote and Perri \(2015\)](#). In addition, [McGrattan and Prescott \(2010, 2014\)](#) suggest that labor productivity calculated from the data—especially during the 1990s and the Great Recession—are mismeasured. This paper does not take a stand on what the true labor productivity is, and instead uses the observed unemployment path to discipline productivity.

but the decline starts sooner than in the U.S. economy. The Markov equilibrium captures the sudden drop and the slow rise in the vacancy-unemployment ratio as observed in the U.S. economy, but underestimates the scale of the drop. One reason for the smaller drop in the vacancy-unemployment ratio is that the model does not have job-to-job transition. During recessions, job-to-job transition, in addition to unemployment-to-job transition, declines, and as a result, vacancy posting should decline more when job-to-job transition is taken into account.

6.2.1 Policy evaluation

One interesting question we can answer using the model is whether and how much the benefit extensions contributed to the higher unemployment rates during the Great Recession. We analyze a counterfactual where the government does not do benefit extensions but instead keeps benefit duration at 26 weeks throughout the recession (and the private sector fully understands it). Figure 9 shows that, in contrast to the no-extension policy (dotted red line), the Markov equilibrium benefit extensions lead to higher unemployment rates. At the peak, the unemployment is lower by more than 3 percentage points in the economy without extensions.³⁸

A key prediction of our theory is that search effort is procyclical, that it falls during recessions. This feature is present in the standard search model with endogenous search effort. The empirical findings on the cyclicity of search effort are mixed.³⁹ More recently, [Gomme and Lkhagvasuren \(2015\)](#), after controlling for the heterogeneity in the unemployed worker’s past wages and hours, find evidence that search is procyclical, consistent with the prediction of the structural search literature.

Welfare Evaluations We compute the consumption equivalent variation under a utilitarian welfare function; that is, the welfare function in each period is

$$\begin{aligned}
 W(c^w, c^0, c^1, s^0, s^1, u, u^1, d) = & \quad (1 - u)U(c^w) && \text{worker} \\
 & + (u - u^1(1 - d)) [U(c^0) - v(s^0)] && \text{unemployed without benefit} \\
 & + u^1(1 - d) [U(c^1) - v(s^1)] && \text{unemployed with benefit.}
 \end{aligned}$$

This is also the period welfare function of the government in the Markov equilibrium. It captures the effects of higher income (both wages and benefits), larger proportions of unemployed workers on benefits, lower lump-sum taxes, and gains from saving on search effort.

We perform the welfare evaluation of extensions at two points in time. First, consistent with how the effect on unemployment is evaluated, we look at the *ex-post* welfare. This is computed at each

³⁸ Even though both productivity and job separation rate fluctuations contribute to the cyclical movements in the model, the productivity changes actually drive most of the movements. The online appendix provides an analysis where productivity is kept at its steady-state level to isolate the effect of the shock to job separation rate.

³⁹ [Shimer \(2004\)](#) and [Mukoyama, Patterson, and Şahin \(2006\)](#), for example, find countercyclical search effort, while [DeLoach and Kurt \(2013\)](#) find evidence of procyclical search effort. See [Gomme and Lkhagvasuren \(2015\)](#) for a brief review of the empirical literature on search effort.

point in time over the transition path after the realization of shocks each period:

$$\sum_{t=2008.1}^{2013.12} \beta^t W(c_t^w, c_t^0, c_t^1, s_t^0, s_t^1, u_t, u_t^1, d_t) = \sum_{t=2008.1}^{2013.12} \beta^t W(\tilde{c}_t^w(1 + \Delta), \tilde{c}_t^0(1 + \Delta), \tilde{c}_t^1(1 + \Delta), \tilde{s}_t^0, \tilde{s}_t^1, \tilde{u}_t, \tilde{u}_t^1, \tilde{d}_t)$$

where the left-hand side is the aggregate welfare of the Markov equilibrium economy, \tilde{x} on the right-hand side are the equilibrium allocations under the no-extension policy, and Δ is the consumption equivalent variation. We allow the distribution of workers to vary in the welfare calculation to capture the distributional effects of different policies on welfare. The last panel of Figure 9 plots the welfare during this period. Even though the Markov equilibrium extensions substantially raised the unemployment, the average welfare is higher with the Markov extensions. The difference in the average welfare over the transition path roughly translates into 0.16% of average monthly consumption.

The *ex-post* welfare gap between the Markov equilibrium and no-extensions economies in Figure 9 are driven by two opposing forces: the higher unemployment in the Markov equilibrium reduces the average welfare in the economy relative to the no-extensions policy, whereas the higher proportion of unemployed workers on benefits increases the average welfare of unemployed workers in the Markov economy. Figure 10 illustrates these two forces.

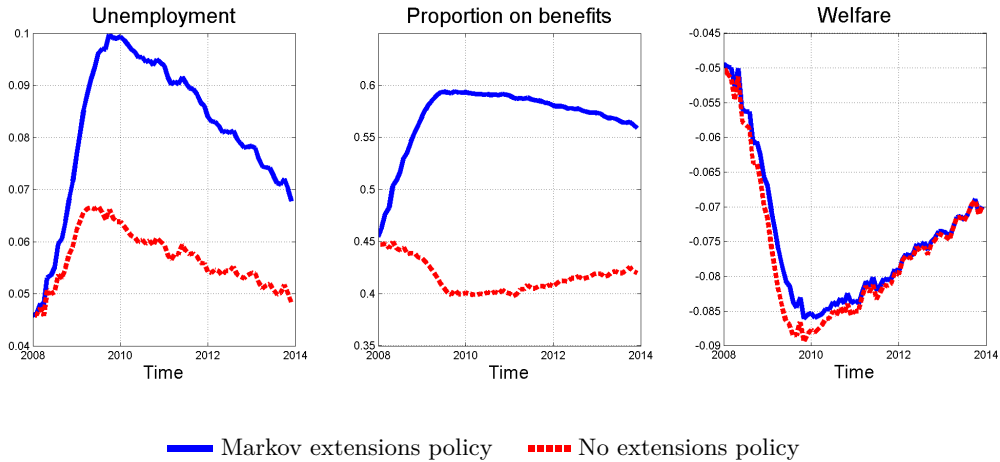


Figure 10: Drivers of welfare gap between Markov extensions and no extensions policies: Unemployment and proportion unemployed on benefits.

The proportion of unemployed workers on benefits (middle panel) is calculated as $u^1(1 - d)/u$. Under the no-extensions policy (dotted red lines), the proportion falls early in the recession. While both the measure of benefit-eligible unemployed worker, u^1 , and the total unemployment, u , increase in response to rising job separation rates and falling productivity, the rise in the total unemployment is larger because job-finding rates of the unemployed workers both with and without benefits fall. In contrast, in the Markov equilibrium, the initial rise in the proportion is mainly driven by the longer duration policy (lower d).

The gap in benefit coverage between the two policies increases over time. At the time when the

unemployment peaks, about 60% of unemployed workers are covered by UI benefits in the Markov equilibrium, whereas only 40% have benefits under the no-extensions policy. This higher benefit coverage leads to higher average welfare among the unemployed workers in the Markov equilibrium. The welfare gap (right panel) between the Markov and no-extensions policies is then the result of the welfare cost of higher unemployment being outweighed by the welfare gain from a higher benefit coverage ratio. An important reason for this result is that productivity is low during the recession, which lowers the marginal cost of unemployment, making welfare higher in the high-unemployment-long-duration economy.

Second, we compare the welfare at the start of the recession. This is an *ex-ante* comparison:

$$\begin{aligned} & \mathbb{E}_{2008.1} \sum_{t=2008.1}^{\infty} \beta^t W(c_t^w, c_t^0, c_t^1, s_t^0, s_t^1, u_t, u_t^1, d_t) \\ = & \mathbb{E}_{2008.1} \sum_{t=2008.1}^{\infty} \beta^t W(\tilde{c}_t^w(1 + \Delta), \tilde{c}_t^0(1 + \Delta), \tilde{c}_t^1(1 + \Delta), \tilde{s}_t^0, \tilde{s}_t^1, \tilde{u}_t, \tilde{u}_t^1, \tilde{d}_t) \end{aligned}$$

The difference here is the *ex-ante* evaluation is done without the knowledge of how the recession would pan out, and agents in the economy expect future shocks to follow an AR(1) process. We assume that an unexpected productivity shock occurs in January 2008, and perform the welfare comparison after the realization of this shock. The question addressed by this exercise is, “*should* the government follow the Markov rule or the no-extensions policy given this shock?” Interestingly, the no-extensions policy gives slightly higher (less than 0.1% in consumption equivalent) average welfare than the Markov policy rule *ex ante*. The reason for the different *ex-ante* and *ex-post* welfare results is that during this period the productivity slump turns out to be both long and severe, which gives more justification *ex-post* for the Markov equilibrium extensions.

6.2.2 The effect of expectation

Private sector’s expectations of future policy play an important role in the Markov equilibrium. To isolate the effect of expectation in our result, we shut down the channel of private sector’s expectations. In particular, in this exercise we assume that *ex ante* the private sector expects the future benefit duration (and benefit level) to stay at the steady-state level, but *ex post* the government implements the benefit extensions as in the Markov equilibrium. In other words, all benefit extensions above the regular 26 weeks are “unanticipated.” Figure 11 illustrates the experiment by comparing the economy under (1) the Markov equilibrium benefit extensions policy (solid blue lines), (2) the no-extensions policy (dotted red lines), and (3) the unexpected extensions policy (dashed green lines).

The unemployment gap between the Markov extensions and the unexpected extensions policies is large, accounting for about 70% of the unemployment gap between the Markov extensions and the no-extensions economies. This reflects the importance of expectations. When the unemployed workers with benefits rationally expect the government to follow the Markov equilibrium policy, they reduce their job search activities when the productivity is low or when the unemployment is high,

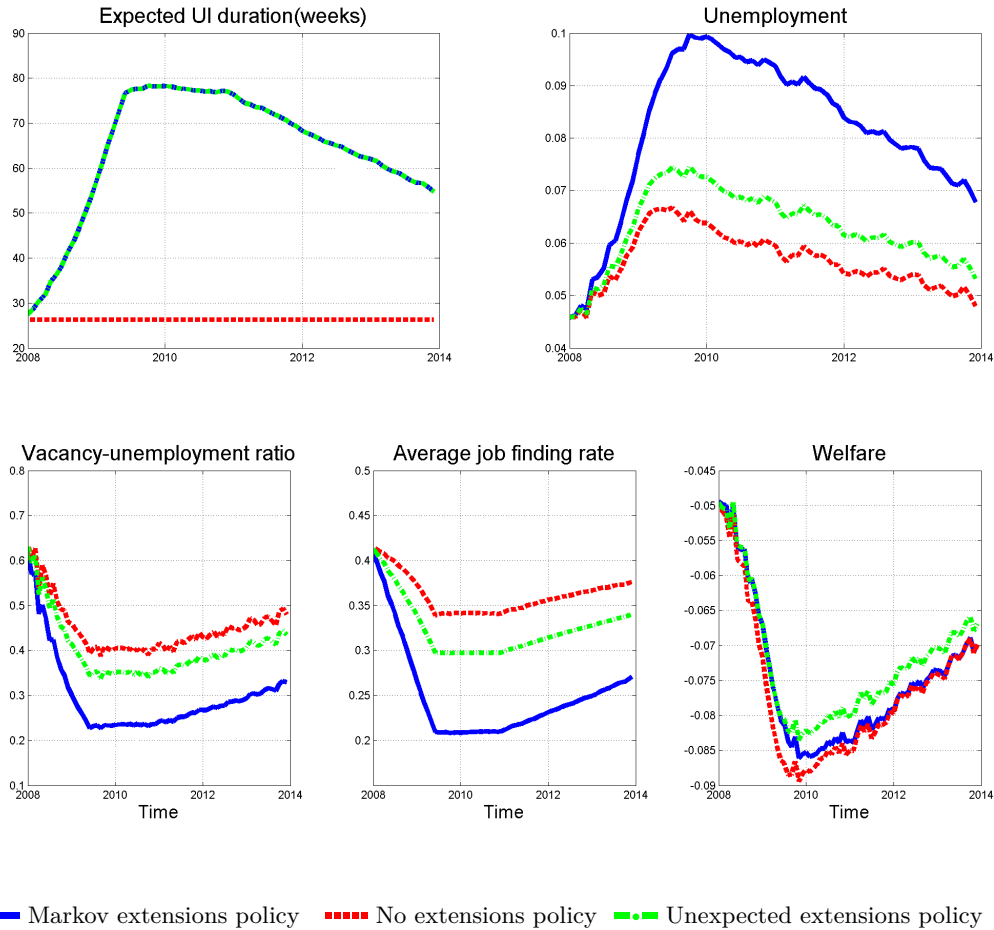


Figure 11: UI duration, unemployment, and welfare during the Great Recession: Effect of expectation.

because the expected long benefit duration next period distorts their search incentives. In contrast, when the unemployed workers expect the government to maintain a no-extensions policy, they do not reduce search as much.

Because the distortion on the search incentives works only through the unemployed worker’s expectation of future UI policies, this exercise is also a decomposition of the unemployment gap into the “search” wedge and the “composition” wedge. Specifically, the unemployment difference between the economies with the Markov equilibrium extensions (solid blue lines) and the unexpected extensions (dashed green lines) represents the effect of policy distortion on search behavior; the unemployment difference between the economies with the unexpected extensions and no extensions (dotted red lines) represents the compositional effect—a longer duration increases the proportion of unemployed workers with benefits, thus reducing average search.

Interestingly, the average welfare in the unexpected extensions economy is higher than in the Markov equilibrium. This is not surprising, because the former has both high current consumption—from the *ex-post* benefit extensions—and low future unemployment—from the expectation of a no-

extensions policy *ex ante*. But such an economy is in a sense “unsustainable” as it requires the government to be always able to “fool” the private sector.⁴⁰

6.2.3 Cyclical job separation risk

In the benchmark the cyclical job separation rate is both exogenous and unexpected. As a robustness check, we make the job separation shock contingent on productivity in the model, so that the private sector takes into consideration the cyclical job separation rate—and form expectations accordingly—when making decisions. We specify the job separation rate process as

$$\delta(z) = \bar{\delta} + I_{\delta}(z - \bar{z}),$$

where $\bar{\delta}$ is the steady-state job separation rate, and $I_{\delta} < 0$ is the rate of change of the separation rate with respect to the aggregate productivity. Note that the private sector forms expectations based on this process, but the realized separate rate is still the series taken from data as before. This formulation has the natural interpretation that the job separation rate increases when profits are low. When labor productivity is low, wages are low as wages are also a function of productivity. Because the elasticity of wages with respect to productivity is less than 1, lower productivity means lower profit, or $z - w$ in the model. To estimate this process we use the job separation and labor productivity data over 1951.I-2014.IV.

As before, the productivity shock z is exogenously specified to match the unemployment process. The resulting labor productivity path has a smaller drop than before—3.2% as opposed to 3.6% in the benchmark when the separation rate changes are unexpected. This is because a countercyclical separation rate shock reinforces the effect of the productivity shock. When the productivity is low, firms expect a large future separation rate, and as a result they reduce vacancy postings even more than in the benchmark. Figure 12 shows the transition paths for this alternate specification.

6.3 Other recessions

As noted in the empirical analysis, benefit extensions are not unique to the Great Recession. Comparing across recession episodes since the 1970s, those with higher unemployment were associated with, in general, higher benefit durations.⁴¹ In this section, we test whether our model delivers this characteristic. Because of declining secular trends in job-finding and separation rates, we need to recalibrate the model parameters to the pre-recession economy for each recession episode. Table 4 summarizes the labor market statistics for the pre-recession window for each recession.

⁴⁰ This is similar to the Lucas (1972, 1976) argument that only unexpected inflation has a positive impact on real output, and attempts by governments to exploit this unexpectedness will lead agents to revise their inflation expectations.

⁴¹ Even though benefit extension programs existed since the 1950s, it was not until the 1973-1975 recession that the federal government started actively adjusting extensions. Such active adjustment corresponds to our interpretation of a discretionary policy. We thus focus on recessions since the mid-1970s.

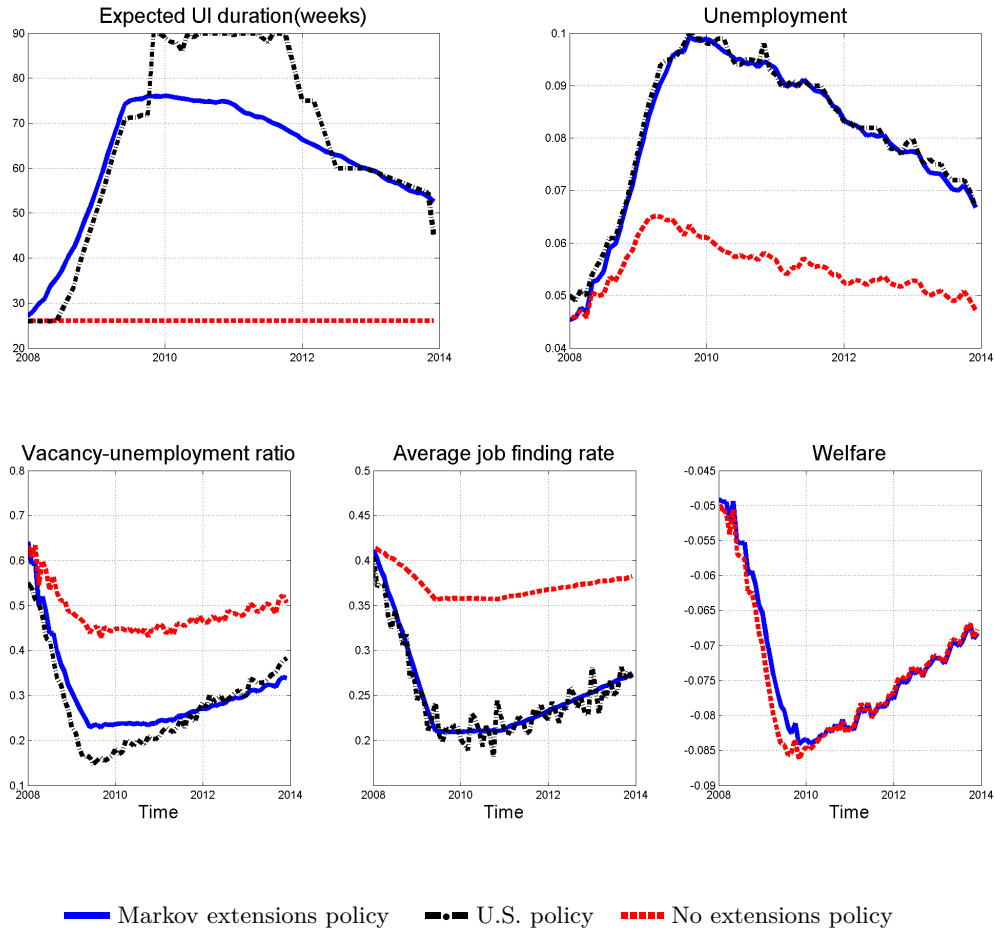


Figure 12: With cyclical separation risk: UI duration, unemployment, and welfare during the Great Recession.

As with the Great Recession, we use the path of job separation rate from data, and target the observed unemployment path to recover the path of productivity for each recession. Figure 13 displays the model-generated UI duration (solid blue lines), unweighted UI duration from the data (dashed blue lines), and the model-generated unemployment (broken red lines, right axis) for each recession documented in the empirical analysis. Three observations are worth noting. First, the model matches the increases in the UI duration reasonably well, producing more than 60% of the increases (solid blue line vs. dashed blue line) in each recession. Second, consistent with patterns documented from the data, during all four recessions, the model-generated UI duration reached its highest level around the time the unemployment peaked. Lastly, recessions where the unemployment was higher (broken red line) also had, in general, higher model-generated UI duration (solid blue line), except in the 1980s recession. This evidence shows that, as an additional model validation, our theory is able to generate not only quantitatively significant UI duration increases in recessions, but also cross-time patterns consistent with the data.

While our model can rationalize the benefit extensions during recessions, our theory is not suc-

Table 4: Calibration Targets for Other Recessions

Recession	Pre-recession period	Pre-recession labor market statistics		
		Separation	Job finding	Job filling
Nov 1973-Mar 1975	1973.I-1973.III	0.026	0.51	0.71
Jul 1981-Nov 1982	1980.II-1981.I	0.033	0.41	0.71
Jul 1990-Mar 1991	1988.I-1990.II	0.027	0.47	0.71
Mar 2001-Nov 2001	1999.I-2000.IV	0.020	0.49	0.66

Note: Job-filling rate pre-2000 are from [Den Haan, Ramey, and Watson \(2000\)](#).

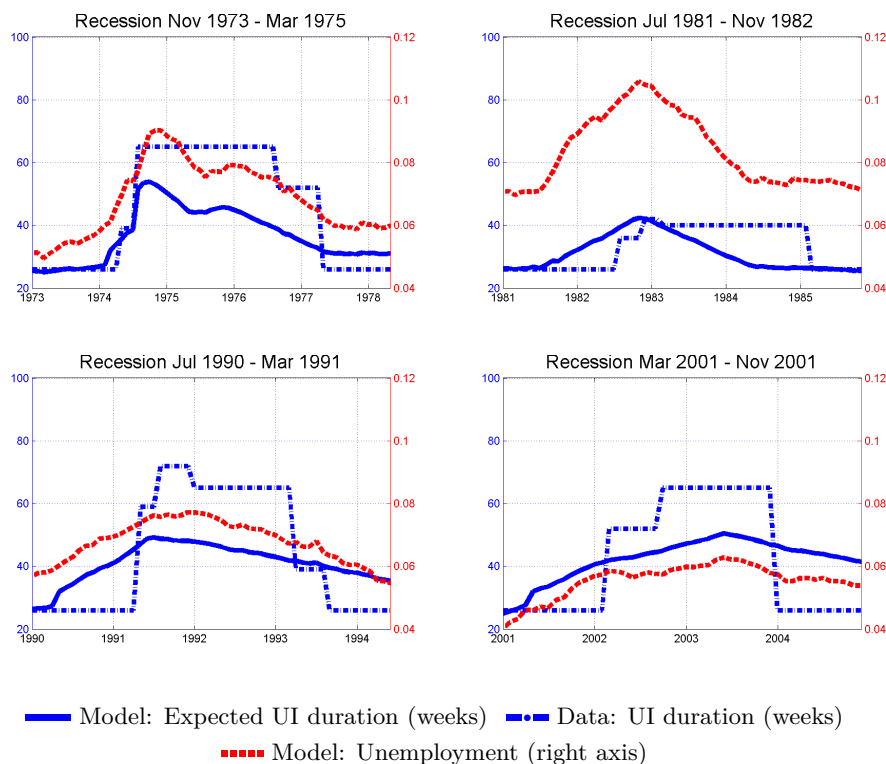


Figure 13: UI duration and unemployment in other recessions: Model versus data

successful in generating the lack of actions during non-recessionary times. In fact, our model predicts when productivity rises and unemployment falls, the government has incentives to reduce benefit duration to below 26 weeks. However, it is worth noting that the response of the Markov equilibrium policy to productivity is not entirely symmetric. Instead, the changes in the duration policy are very nonlinear; its increase over regions of low productivity is much faster than its decrease over regions of high productivity.⁴² As a result, even though the government has incentives to reduce UI duration during non-recessionary times, the reduction is much smaller than the increase during a recession.

⁴² Shown in Figure 3 and discussed in Section 5.2.

6.4 Discussion

6.4.1 Relation to commitment (Ramsey) policy

A recent strand of the UI literature looks at the optimal UI policy over the business cycle under the assumption of perfect commitment by the government (e.g. [Mitman and Rabinovich \(2015\)](#), [Jung and Kuester \(2015\)](#)). Our time-consistent no-commitment policy complements this literature by relaxing the commitment assumption on the government. A key difference between the two policies is that the government without commitment does not internalize the effect of future policy on today's search incentives, whereas the government with commitment does. As a result, the optimal commitment (Ramsey) policy is less generous, and the steady state unemployment is lower than that under the time-consistent no-commitment policy. Over the business cycle when the unemployment is high, the government with commitment (Ramsey) realizes that a short benefit duration creates search incentive for more workers, so it is ex ante optimal to promise a short benefit duration. Consistently, [Mitman and Rabinovich \(2015\)](#) find that the optimal benefit duration under commitment is procyclical⁴³. In contrast, a government without commitment cannot make a promise, which is why we find time-consistent non-commitment policy is countercyclical.

Note that commitment is assumed in the Ramsey case. If given the choice to break a promise, the government will find it optimal at any time to deviate from the promised policy schedule. More specifically, the government at time t has an incentive to promise a short future benefit duration to encourage search, because current search (mainly search of the benefit-collecting unemployed workers) is higher when the expected future benefit duration is shorter. However, after the employment outcome in period t is realized, the government has an incentive to provide insurance to more unemployed workers by choosing a longer duration. This incentive to deviate from the original plan is what constitutes the time inconsistency in the Ramsey problem. By allowing the government to break its promise, the time-consistent policy that we look at is arguably closer to reality.

6.4.2 Relation to [Hagedorn, Manovskii, and Mitman \(2015\)](#)

The comparison between the Markov economy and the no-extensions economy is in line with [Hagedorn, Manovskii, and Mitman \(2015\)](#), who exploit discontinuity at state borders to identify the effect of unemployment benefit extensions. In particular, one way to interpret their empirical result is as follows. In states where firms and workers expect good exogenous shocks to the economy, e.g. an oil boom, they also expect lower or no benefit extensions. This increases the expected value of

⁴³ [Jung and Kuester \(2015\)](#) find the response of the optimal UI benefits to business cycle fluctuations is short-lived because they consider multiple policy instruments; when the government is only allowed to change UI benefits, they find the policy responses are similar to [Mitman and Rabinovich \(2015\)](#). [Landais, Michailat, and Saez \(2010\)](#) find the optimal (Ramsey) UI benefits should be countercyclical, higher when the unemployment is higher; however, as [Mitman and Rabinovich \(2015\)](#) point out, the analysis in [Landais, Michailat, and Saez \(2010\)](#) rests on a steady state comparison and is thus not directly comparable to our results. We find the Ramsey policy in our setup behaves very similarly to that in [Mitman and Rabinovich \(2015\)](#).

employment to a worker and in turn increases job creation compared to states with bad economic outlooks. This interpretation is very similar to our theory. With the Markov equilibrium policy, firms and workers *expect* longer benefit durations in recessions—analogueous to states with bad economic outlooks—whereas in the no-extensions economy, the private sector *expects* no-extensions policy—an extreme case of states with good exogenous shocks.

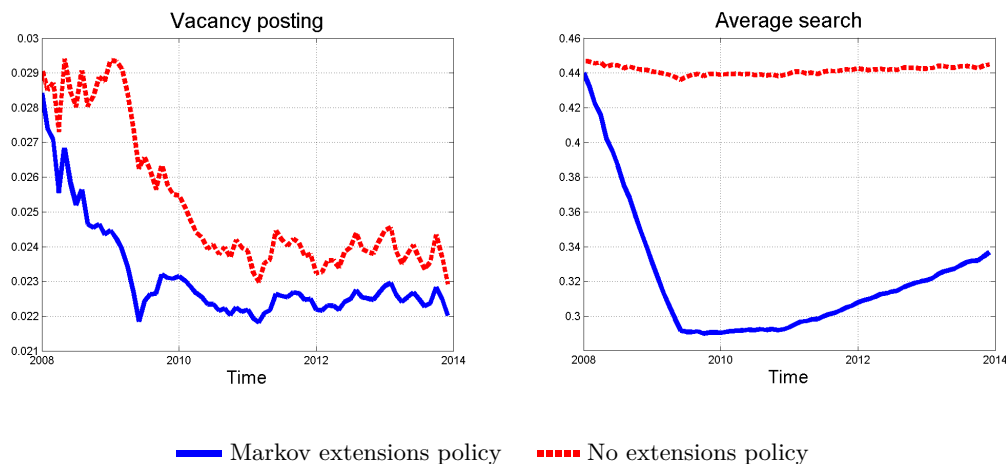


Figure 14: Markov extensions versus no extensions policy: Vacancy posting and average search.

One difference between our theory and [Hagedorn, Manovskii, and Mitman \(2015\)](#) lies in the mechanism underlying our results. In their model, a longer benefit duration increases the unemployed worker’s outside option, thus increasing their reservation wage. Higher reservation wages then reduce the firm’s vacancy posting by reducing profit margin. In our model, a longer expected duration makes unemployment less painful for workers and thus reduces job search activity. Reduced search activity then reduces the marginal return to vacancy posting, leading to a lower vacancy posting in equilibrium.⁴⁴ To see how much of the unemployment gap in [Figure 9](#) comes from differences in vacancy posting as opposed to job search, [Figure 14](#) compares the Markov equilibrium and the no-extensions economy along these two dimensions. Both vacancy posting and average search are higher in the no-extensions economy, but the gap is much larger for average search. This result indicates that in our model both vacancy posting and search contribute to the unemployment gap, but average search is a quantitatively more important channel.

6.4.3 Other policy instruments

In our analysis we consider UI as the only policy instrument. In reality, the government can use a number of other labor market and social insurance policies alongside UI. [Jung and Kuester \(2015\)](#) analyze the optimal mix of unemployment insurance, hiring subsidy, production tax and layoff tax

⁴⁴ With exogenous wages, labor market tightness does not respond to changes in search or vacancy posting. Mechanically, when aggregate search is lower, vacancy posting is also lower to keep tightness constant. As a result, in equilibrium the marginal return to search and vacancy posting stays constant for the same productivity level.

over the business cycle. They find that, compared to unemployment insurance, hiring subsidies and layoff taxes stimulate job creation without sacrificing the unemployed workers' consumption in recessions. As a result, during recessions the government does not rely on UI as much as when it was the only instrument available. In other words, policy instruments affecting the firm side crowd out UI.

In our setup, such firm-side policies, would similarly crowd out the use of UI extension policies during recessions. We choose to focus on UI primarily because this is the policy with the largest cyclical variations in the U.S., especially during the most recent recession. Underlying this observation is the possibility that the firm-side policies may be harder to implement and so the government relies heavily on UI during recessions.⁴⁵

7 CONCLUSION

This paper develops a quantitative theory of the government's choice of UI policy without the assumption of perfect commitment by the government. We study the Markov-perfect equilibrium of a search and matching model, which delivers a time-consistent UI policy where the government makes UI benefit level and duration choices each period contingent on the current payoff-relevant aggregate states of the economy. Our methodology is empirically relevant as supported by the frequent and haphazard benefit extensions during recessions, which we document. We find that the time-consistent benefit policies are consistent with the countercyclical UI extensions during recessions. Quantitatively, the endogenous benefit duration generated by the model matches the timing and the size of benefit extensions during the Great Recession.

Using a calibrated version of the model to evaluate the benefit extensions between 2008 and 2013, we find that the endogenous variations in the UI duration accounted for a 3 percentage-point increase in the unemployment rate at its peak, of which two-thirds was driven by the unemployed workers' expectation of longer future benefit durations. Our results thus highlight the importance of expectations in Markov equilibrium and in policy implementation. Another interesting finding is that even though the benefit extensions created higher unemployment, it actually improved welfare *ex post* during 2008-2013 compared to a no-extensions policy.

Several simplifying assumptions are made for tractability. First, neither workers nor government can save or borrow. Because savings provide self-insurance to workers, allowing workers to save will reduce the need for government-provided insurance policy. At the same time, credit access may reduce search by the unemployed (see, for example, [Herkenhoff \(2015\)](#)), thus exacerbating moral hazard problem. The reduced need for insurance and increased moral hazard problem will likely reduce the cyclical response of benefit duration. Allowing government access to the credit market will increase the cyclical fluctuations of benefit duration and likely make the benefit level less procyclical (or more acyclical). This is because the government can borrow to finance a generous

⁴⁵ More recently, hiring subsidies are introduced in the Hiring Incentives to Restore Employment Act of 2010.

benefit policy in bad times and pay back the debt with tax revenue in good times.

Another assumption is that government policy takes effect right away. In reality, there often is a time lag between legislation and implementation. Allowing the government to announce policy changes before implementation gives workers and firms time to react to the potential changes, which may mitigate the effect of policy changes (“announcement effect”). However, by looking at UI legislations during the Great Recession, we find that most extensions came into effect shortly after announcement. Furthermore, the announcement effect, if any, is likely small quantitatively. [Nakajima \(2012\)](#), for example, incorporates announcement effect in his evaluation of benefit extensions and finds small quantitative effect associated with policy announcement.

REFERENCES

- ALBERTINI, J. AND A. POIRIER (2015): “Unemployment Benefit Extension at the Zero Lower Bound,” *Review of Economic Dynamics*, 18, 733–751.
- BARNICHON, R. (2010): “Building a Composite Help-Wanted Index,” *Economics Letters*, 109, 175–178.
- CHATTERJEE, S. AND B. EYIGUNGOR (2014): “Continuous Markov Equilibria with Quasi-Geometric Discounting,” Federal Reserve Bank of Philadelphia Working Paper No. 14-6.
- CHETTY, R. (2008): “Moral Hazard versus Liquidity and Optimal Unemployment Insurance,” *Journal of Political Economy*, 116, 173–234.
- CHODOROW-REICH, G. AND L. KARABARBOUNIS (2017): “The Limited Macroeconomic Effects of Unemployment Benefit Extensions,” Working Paper.
- CHRISTENSEN, B. J., R. LENTZ, D. T. MORTENSEN, G. R. NEUMANN, AND A. WERWATZ (2005): “On-the-job Search and the Wage Distribution,” *Journal of Labor Economics*, 23, 31–58.
- DELOACH, S. B. AND M. KURT (2013): “Discouraging Workers: Estimating the Impacts of Macroeconomic Shocks on the Search Intensity of the Unemployed,” *Journal of Labor Research*, 34, 433–454.
- DEN HAAN, W. J., G. RAMEY, AND J. WATSON (2000): “Job Destruction and Propagation of Shocks,” *American Economic Review*, 90, 482–498.
- DI MAGGIO, M. AND A. KERMANI (2016): “The Importance of Unemployment Insurance as an Automatic Stabilizer,” Working Paper.
- FUJITA, S. (2012): “Declining Labor Turnover and Turbulence,” Federal Reserve Bank of Philadelphia Research Working Papers No. 11-44/R.
- GOMME, P. AND D. LKHAGVASUREN (2015): “Worker Search Effort as an Amplification Mechanism,” *Journal of Monetary Economics*, 75, 106–122.
- HAGEDORN, M., F. KARAHAN, I. MANOVSKII, AND K. MITMAN (2013): “Unemployment Benefits and Unemployment in the Great Recession: The Role of Macro Effects,” National Bureau of Economic Research Working Papers 19499.

- HAGEDORN, M. AND I. MANOVSKII (2008): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited,” *American Economic Review*, 98, 1692–1706.
- HAGEDORN, M., I. MANOVSKII, AND K. MITMAN (2015): “The Impact of Unemployment Benefit Extensions on Employment: The 2014 Employment Miracle?” National Bureau of Economic Research Working Papers 20884.
- HALL, R. E. AND P. R. MILGROM (2008): “The Limited Influence of Unemployment on the Wage Bargain,” *American Economic Review*, 98, 1653–1674.
- HEATHCOTE, J. AND F. PERRI (2015): “Wealth and Volatility,” National Bureau of Economic Research Working Papers 20994.
- HERKENHOFF, K. (2015): “The Impact of Consumer Credit Access on Unemployment,” Working Paper.
- HOPENHAYN, H. A. AND J. P. NICOLINI (1997): “Optimal Unemployment Insurance,” *Journal of Political Economy*, 105, 412–438.
- JOHNSTON, A. C. AND A. MAS (2016): “Potential Unemployment Insurance Duration and Labor Supply: The Individual And Market-Level Response to A Benefit Cut,” Working Paper.
- JUNG, P. AND K. KUESTER (2015): “Optimal Labor-Market Policy in Recessions,” *American Economic Journal: Macroeconomics*, 7, 124–156.
- KEKRE, R. (2015): “Unemployment Insurance in Macroeconomic Stabilization,” Working Paper.
- KLEIN, P., P. KRUSELL, AND J.-V. RÍOS-RULL (2008): “Time-Consistent Public Policy,” *Review of Economic Studies*, 75, 789–808.
- KROFT, K. AND M. J. NOTOWIDIGLO (2015): “Should Unemployment Insurance Vary with the Unemployment Rate? Theory and Evidence,” *Review of Economic Studies*, Forthcoming.
- KRUEGER, A. AND B. MEYER (2002): “Labor Supply Effects of Social Insurance,” in *Handbook of Public Economics*, ed. by A. Auerbach and M. Feldstein, Amsterdam: North-Holland.
- KRUSELL, P., T. MUKOYAMA, AND A. SAHIN (2010): “Labour-Market Matching with Precautionary Savings and Aggregate Fluctuations,” *Review of Economic Studies*, 77, 1477–1507.
- LANDAIS, C., P. MICHAILLAT, AND E. SAEZ (2010): “A Macroeconomic Theory of Optimal Unemployment Insurance,” National Bureau of Economic Research Working Papers 16526.
- LISE, J. (2013): “On-the-job Search and Precautionary Savings,” *Journal of Labor Economics*, 80, 1086–1113.
- LUCAS, R. E. (1972): “Expectations and the Neutrality of Money,” *Journal of Economic Theory*, 4, 103–124.
- (1976): “Econometric Policy Evaluation: A Critique,” *Carnegie-Rochester Conference Series on Public Policy*, 1, 19–46.
- MCGRATTAN, E. R. AND E. C. PRESCOTT (2010): “Unmeasured Investment and the Puzzling US Boom in the 1990s,” *American Economics Journal: Macroeconomics*, 2, 88–123.

- (2014): “A Reassessment of Real Business Cycle Theory,” *American Economic Review: Papers & Proceedings*, 104, 177–182.
- MITMAN, K. AND S. RABINOVICH (2015): “Optimal Unemployment Insurance in an Equilibrium Business Cycle Model,” *Journal of Monetary Economics*, 71, 99–118.
- (2016): “Do Unemployment Benefit Extensions Explain the Emergence of Jobless Recoveries?” Working Paper.
- MUKOYAMA, T., C. PATTERSON, AND A. ŞAHİN (2006): “Job Search Behavior over the Business Cycle,” Federal Reserve Bank of New York Staff Reports No. 689.
- NAKAJIMA, M. (2012): “A Quantitative Analysis of Unemployment Benefit Extensions,” *Journal of Monetary Economics*, 59, 686–702.
- PEI, Y. AND Z. XIE (2015): “Government Commitment and Unemployment Insurance over Business Cycle,” Manuscript, University of Minnesota.
- SHIMER, R. (2004): “Search Intensity,” Mimeo.
- (2005): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies,” *American Economic Review*, 95, 25–49.
- (2012): “Wage Rigidities and Jobless Recoveries,” *Journal of Monetary Economics*, 59, S65–S77.
- SHIMER, R. AND I. WERNING (2007): “Liquidity and Insurance for the Unemployed,” *American Economic Review*, 98, 1922–1942.
- STOCK, J. AND M. WATSON (2012): “Disentangling the Channels of the 2007-2009 Recession,” National Bureau of Economic Research Working Papers 18094.
- WHITTAKER, J. M. AND K. P. ISAACS (2013): “Unemployment Insurance: Programs and Benefits,” CRS Report for Congress, Congressional Research Service.
- YASHIV, E. (2000): “The Determinants of Equilibrium Unemployment,” *American Economic Review*, 90, 1297–1322.
- ZHANG, H., A. R. CONN, AND K. SCHEINBERG (2010): “A Derivative-Free Algorithm for Least-Squares Minimization,” *SIAM Journal on Optimization*, 20, 3555–3576.

A UNEMPLOYMENT INSURANCE BENEFIT EXTENSIONS IN THE GREAT RECESSION

The U.S. government has extended unemployment insurance (UI) benefit duration during recessions since the 1950s. During normal nonrecessionary times, an eligible unemployed worker may receive UI benefits for up to 26 weeks in most states under the regular UI program. During economic downturns, automatic benefits extensions are triggered under the Extended Benefits (EB) program, whereby additional 13 or 20 weeks are added on top of the usual 26 weeks of maximum benefits. The number of additional weeks depends on the state's insured unemployment rate (IUR) or the total unemployment rate (TUR). In addition, during each recession since the 1970s, the federal government has financed extra benefit extensions depending on the economy. The Emergency Unemployment Compensation (EUC08) were launched in 2008 in response to high unemployment. The program extended benefits in waves based on evaluations of unemployment and the underlying economic situation. Four waves called "Tiers" were implemented progressively between Nov 2008 and the end of 2013. Tiers I was effective without any conditions on the states' unemployment level. Tiers II, III and IV required certain condition on the states' IUR and/or TUR to take place.

We use the series of the IUR and TUR for 51 U.S. states to compute if the state was eligible for the EB and the EUC08 programs during any month between 2008 and 2013. This gives us the maximum duration of UI benefits for each state over time. We follow the methodology in [Albertini and Poirier \(2015\)](#) and weight these series to build an aggregate indicator. We use the the number of total insured unemployed workers in the state divided by the total insured unemployed workers in the U.S. as weights. Statistics on insured unemployment population come from the U.S. Department of Labor Employment and Training Administration (DOLETA). [Table 5](#) reports a timeline for policy changes and unweighted expected maximum duration under the EUC08 and EB programs.

Table 5: Federally-Funded Unemployment Insurance Extensions 2008-2013

Start date	Program extension of EUC08	End date	Additional Weeks
Jul 2008	13 weeks for all states	Nov 2008	13
Nov 2008	Tier I: 20 weeks for all states Tier II: 13 weeks for states with $TUR \geq 6\%$	Mar 2009	33
Mar 2009	keep existing structure	Nov 2009	33
Nov 2009	Tier I - 20 weeks for all states Tier II: 14 weeks for all states Tier III: 13 weeks if states $TUR \geq 6\%$ Tier IV: 6 weeks if states $TUR \geq 8.5\%$	Dec 2009	53
Dec 2009	keep existing structure	Aug 2010	53
Mar 2010	keep existing structure	Sep 2010	53
Apr 2010	keep existing structure	Nov 2010	53
Jul 2010	keep existing structure	May 2011	53
Dec 2010	keep existing structure	Jun 2012	53
Dec 2011	keep existing structure	Aug 2012	53
Feb 2012	Tier I: 20 weeks for all states Tier II: 14 weeks for all states Tier III: 13 weeks if states $TUR \geq 6\%$ Tier IV: 6 weeks if states $TUR \geq 8.5\%$ (16 weeks if no active EB and $TUR \geq 8.5\%$)	May 2012	53
Jun 2012	Tier I: 20 weeks for all states Tier II: 14 weeks if states $TUR \geq 6\%$ Tier III: 13 weeks if states $TUR \geq 7\%$ Tier IV: 6 weeks if states $TUR \geq 9\%$	Sep 2012	53
Sep 2012	Tier I: 14 weeks for all states Tier II: 14 weeks if states $TUR \geq 6\%$ Tier III: 9 weeks if states $TUR \geq 7\%$ Tier IV: 10 weeks if states $TUR \geq 9\%$	Dec 2012	47
Jan 2013	keep existing structure	Dec 2013	47
Start date	Program extension of EB	End date	Additional Weeks
Feb 2009	6.5% 13 week IUR and $IUR \geq 110\%$ of prior 3 years 8% 13 week IUR and $IUR \geq 110\%$ of prior 3 years	Dec 2013	13 26

Source: DOLETA, [Whittaker and Isaacs \(2013\)](#), [Albertini and Poirier \(2015\)](#)