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Es ist dabei selbst die historische Wahrheit eine Nebensache, ein erfundenes Beispiel könnte auch dienen; nur haben historische immer den Vorzug, praktischer zu sein und den Gedanken, welchen sie erläutern, dem praktischen Leben selbst näher zu führen.

(Carl von Clausewitz: *Vom Kriege*)

Abstract

We discuss the strategy-proofness of multistage tournaments. In a tournament with subsequent group stages, players are divided into groups in the preliminary and main rounds, where they play pairwise matches against each other. The higher ranked players qualify to the next stage such that matches are not repeated in the main round if two qualified players have already faced in the preliminary round. Players prefer to carry over better results to the main round, provided that they qualify. It is shown that these tournament systems, widely used in handball, are incentive incompatible. We also present some historical examples where a team was *ex ante* not interested in winning by a high margin.

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1 Introduction

Strategy-proofness is an extensively discussed concept in social choice theory since the famous Gibbard-Satterthwaite (Gibbard, 1973; Satterthwaite, 1975) and the more general Duggan-Schwartz (Duggan and Schwartz, 2000) impossibility theorems, which state that a fair voting rule is susceptible to tactical voting: there always exists a voter who can achieve a better outcome by being insincere. Nonetheless, there are several cases when an incentive compatible rule can be found, but a method used in practice is manipulable. For example, Tasnádi (2008) demonstrated that the Hungarian mixed-member electoral system, applied between 1990 and 2010, suffers from the 'population paradox' as the governing coalition may lose seats either by getting more votes or by the opposition obtaining fewer votes. Similarly, the invariant method (Pinski and Narin, 1976), characterised by Palacios-Huerta and Volij (2004), and used to quality-rank academic journals is subject to manipulation, too, because a journal can boost its performance by making additional citations to other journals (Kóczy and Strobel, 2009).

Analysis of sport ranking rules from this perspective has started recently. Kendall and Lenten (2017) is probably the first comprehensive review of sport regulations resulting in unexpected consequences. On the basis of these examples, we have identified three possible situations in which a team might prefer losing a game to winning it: (1) when a team might gain advantages in the next season; (2) when a lower ranked team still qualify and it might face a preferred competitor in the knockout stage; (3) when the team is strictly better off by losing due to ill-constructed rules.

The classical example for the first situation arises from the reverse order applied in the traditional set-up of player drafts, which supposedly increases competitive balance over time, but if a team is still certainly eliminated from the play-off, it creates a perverse incentive to tank the later games (Kendall and Lenten, 2017, Section 4.1). The second situation occurred in Badminton at the 2012 Summer Olympics – Women's doubles (Kendall and Lenten, 2017, Section 3.3.1), and has probably inspired some game theoretical works on the strategic manipulation problem (Pauly, 2014; Vong, 2017).

However, in the first case, the perverse incentives are not generated by tournament rules, and in the second case, the team gains only in expected terms. We will address here the remaining third situation, when tournament rules go awry such that a team is guaranteed to benefit from exerting a lower effort.

It is far from trivial to identify an ill-constructed rule in the real world since usually there is a low probability that it fails. Furthermore, a scandal has such an enormous cost that the rule is almost certain to be never used again. Perhaps the most famous case is a football match, Barbados vs. Grenada (1994 Caribbean Cup qualification), when a sudden-death goal scored in extra time counted as double, creating an incentive to concede a goal at the end of the match in order to gain additional time for a two-goal win (Kendall and Lenten, 2017, Section 3.9.4). The Barbadians exploited this perverse rule by scoring an own goal in the 87th minute (Dagaev and Sonin, 2017, Note 1). The match had not affected any third team, so one can accept the decision of FIFA not to penalize Barbados as the players were striving for the best outcome conditional upon the prevailing rules. Nevertheless, this rule was not applied since then.

A similar situation was prevented by a FIBA[1] rule saying that '*if a player deliberately*

[1] FIBA stands for *Fédération internationale de basket-ball*, French for International Basketball Federation. It is an association of national organizations, governing international competitions in basketball.

scores in the team's own basket, it is a violation and the basket does not count': in the men's tournament of the 2014 Asian Games Basketball Competition, a Philippine player shot at his own basket against Kazakhstan in order to force overtime and thus increase the margin of victory (Carpio, 2014).

In the absence of comparable clear-cut events, authors often outline hypothetical situations in order to motivate the importance of strategy-proofness. Dr. Andrei Brichkin described a possible scenario in the Russian Premier League in 2012 under which one team, Lokomotiv Moskva,[2] should lose the final game of the national championship as it would finish sixth independently of its last match, but losing would qualify the team for the Europa League (Dagaev and Sonin, 2017, Section titled 'A Real-World Example' and Note 5). Until the 2015-16 season, most UEFA[3] national federations applied an incentive incompatible allocation rule causing this problem. Dagaev and Sonin (2017) have shown that similar tournament systems, consisting of multiple round-robin and knock-out tournaments with noncumulative prizes, are strategy-proof only if all vacant slots are awarded to the teams from the round-robin tournament. Now UEFA Champions League and Europa League qualifications seems to be immune to manipulation.

Dagaev and Sonin (2013) have revealed and Dagaev and Sonin (2017) have mentioned in a sentence that the European qualification for the 2014 FIFA World Cup in Brazil also suffers from incentive incompatibility. Independently from these works, Csató (2017a) has presented that 2018 FIFA World Cup qualification (UEFA) violates strategy-proofness. Furthermore, Csató (2017b) has built a theoretical model containing this tournament, identified nine recent incentive incompatible qualifications, and suggested a correction mechanism. Csató (2017c) has presented that a scandal almost happened in 1995 due to the bad tournament design.

This paper will show that tournaments with subsequent group stages, where some matches from the preliminary round are carried over to the main round, suffer from incentive incompatibility. In this respect, it is one of the follow-up papers promised in Csató (2017b, Conclusions). Section 2 presents an example from handball, which may be more serious than the match Barbados vs. Grenada as a probably unfair behaviour of a team led to the elimination of a third team. Section 3 contains the theoretical model and proves that the aforementioned competition format violates strategy-proofness. Section 4 lists some recent tournaments designed in this way and discusses the implications of our formal results. Finally, Section 5 summarizes our findings.

2 A real-world example

The 11th Men's European Handball Championship (EHF[4] Euro 2014) was held in Denmark between 12 and 26 January, 2014.[5] 16 national teams participated in the tournament. In the preliminary round, they were divided into four groups (A-D), playing in a round-robin format. The top three teams in each group qualified to the main round. Teams from

[2] The English website of UEFA mentions the team, based in Moscow, under this name.

[3] UEFA stands for *Union of European Football Associations*, the administrative body for association football in Europe. However, several UEFA member states are primarily or entirely located in Asia. It is one of the six continental confederations of world football's governing body FIFA.

[4] EHF stands for *European Handball Federation*, the umbrella organization for European handball, which was founded in 1991.

[5] This section is mainly based on the Wikipedia page of 11th Men's European Handball Championship (EHF Euro 2014). We will cite only those official documents which concern the ranking of teams.

Table 1: 11th Men's European Handball Championship (EHF Euro 2014) – Group C

(a) Match results

Date	First team	Second team	Result
13 January 2014, 18:00	Serbia	Poland	20-19
13 January 2014, 20:15	France	Russia	35-28
15 January 2014, 18:00	Russia	Serbia	27-25
15 January 2014, 20:15	Poland	France	27-28
17 January 2014, 18:00	Poland	Russia	to be played
17 January 2014, 20:15	Serbia	France	to be played

(b) Standing after two matchdays

Pos = Position; W = Won; D = Drawn; L = Loss; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played 2 matches.

Pos	Team	W	D	L	GF	GA	GD	**Pts**
1	France	2	0	0	63	55	8	**4**
2	Serbia	1	0	1	45	46	-1	**2**
3	Russia	1	0	1	55	60	-5	**2**
4	Poland	0	0	2	46	48	-2	**0**

Groups A and B of the preliminary round composed the first main round group, while teams from Groups C and D of the preliminary round composed the second main round group. Main round groups were organized in a round-robin format, too, but all matches (consequently, results and points), played in the preliminary round between teams that were in the same main round group, were kept and remained valid for the ranking of the main round.

In the groups of the preliminary and main rounds, two points were awarded for a win, one point for a draw and zero points for a defeat. Teams were ranked by adding up their number of points. If two or more teams had an equal number of points, the following tie-breaking criteria were used after the completion of preliminary round matches, according to EHF (2014a, Article 9.12):

a) Higher number of points obtained in the group matches played amongst the teams in question;

b) Superior goal difference from the group matches played amongst the teams in question;

c) Higher number of goals scored in the group matches played amongst the teams in question;

d) Superior goal difference from all group matches (achieved by subtraction);

e) Higher number of goals scored in all group matches.

During the preliminary round, a strange situation emerged in Group C, which is worth further investigation. On 16 January, 2014, each team in the group had one more game to play. Table 1 shows the known results and the preliminary standing of the group.

Consider the situation from the perspective of Poland. The team is eliminated if it does not win against Russia. Poland carries over 0 points, 46 goals for and 48 goals against to the main round if it wins against Russia and Serbia plays at least a draw against France. If Poland wins by x goals against Russia and Serbia loses, there will be three teams with 2 points, which obtained 2 points in the group matches played among them. Consequently, the second tie-breaking criteria should be applied: Poland, Russia and Serbia will have head-to-head goal differences of $x-1$, $2-x$ and -1, respectively. Serbia is eliminated as the fourth team if $1 \leq x \leq 2$. Russia and Serbia has the same head-to-head goal difference if $x=3$, hence higher number of goals scored against the three teams with 2 points break the tie. It is 45 for Serbia and at least 27 for Russia, thus Russia qualifies if it scores at least 19 goals against Poland (if Poland vs. Russia is 21-18, then the third place will depend on the result of Serbia vs. France). If $x \geq 4$, then Serbia has a better head-to-head goal difference than Russia, so Serbia qualifies and Russia is eliminated.

Note that if Poland wins, it carries over its result against Russia (2 points) or Serbia (0 points) to the main round, thus Poland has a strong incentive to qualify together with Russia. Hence it is unfavourable for Poland to win by more than three goals against Russia as this scenario yields no gain in the main round, but may lead to a loss of 2 points if Serbia is defeated by France. Russia is clearly better off by a smaller defeat.

In fact, Poland vs. Russia was 24-22 and Serbia vs. France was 28-31, so France, Poland and Russia qualified to the main round with 4, 2 and 0 points, respectively. Naturally, it is not a proof that Poland manipulated, but the circumstances are at least suspicious: the result of Poland vs. Russia was 10-14 after 30 minutes (half-time), while the match stood at 21-16 in the 48th, 22-17 in the 50th, and 23-18 in the 52th minute (EHF, 2014b).

The probably unfair behaviour of Poland resulted in the elimination of a third, innocent team, Serbia, which makes the situation more worrying than any other case from the history of sport we know about. It seems to be a very persuading argument against the rules of 11th Men's European Handball Championship (EHF Euro 2014).

3 The model

In this section, we will build a model for a tournament consisting of round-robin preliminary and main rounds, where matches played in the preliminary round against teams qualified to the same main round group are carried over. It will be revealed that these systems are incentive incompatible, that is, they are vulnerable to a manipulation identical to the one presented in Section 2. Our notations follow Csató (2017b) in certain details since the qualification system discussed there is also based on groups organized in a round-robin format.

Definition 3.1. *Round-robin tournament*: Let X be a nonempty finite set of at least two teams, $x, y \in X$ be two teams and $v : X \times X \to \{(v_1; v_2) : v_1, v_2 \in \mathbb{N}\} \cup \{—\}$ be a function such that $v(x, y) = —$ if and only if $x = y$. The pair (X, v) is called a *round-robin tournament*.

Function v describes game results with the number of goals scored by the first and second team, respectively.

Definition 3.2. *Single round-robin tournament*: Round-robin tournament (X, v) is *single* if $v_1(x, y) = v_2(y, x)$ and $v_2(x, y) = v_1(y, x)$ for all $x, y \in X$.

In a single round-robin tournament, any two teams play each other only once (often at a neutral site), so the order of the teams has no significance.

Definition 3.3. *Double round-robin tournament*: Round-robin tournament (X, v) is *double* if $v_1(x, y) \neq v_2(y, x)$ and $v_2(x, y) \neq v_1(y, x)$ is allowed for all $x, y \in X$.

In a home-and-away round-robin tournament, any two teams play each other once at home and once at away. The first team is the one playing at home.

Definition 3.4. *Incomplete round-robin tournament*: Let X be a nonempty finite set of at least two teams, $x, y \in X$ be two teams and $v : X \times X \to \{(v_1; v_2) : v_1, v_2 \in \mathbb{N}\} \cup \{—\}$ be a function such that $v(x, y) = —$ if $x = y$. The pair (X, v) is called an *incomplete round-robin tournament*.

In an incomplete round-robin tournament, some matches between the teams remains to be played. Note that any round-robin tournament is an incomplete round-robin tournament, too.

Definition 3.5. *Ranking in incomplete round-robin tournaments*: Let $\mathcal{X}$ be the set of incomplete round-robin tournaments with a set of teams X. A *ranking method* R maps any characteristic function v of $\mathcal{X}$ into a strict order $R(v)$ on the set X.

Let (X, v) be an incomplete round-robin tournament, $R(v)$ be its ranking and $x, y \in X$, $x \neq y$ be two different teams. x is said to be ranked higher (lower) than y if and only if $x \succ_{R(v)} y$ ($x \prec_{R(v)} y$).

Let $x, y \in X$, $x \neq y$ be two different teams and $v(x, y) = (v_1(x, y); v_2(x, y))$. It is said that team x wins over team y if $v_1(x, y) > v_2(x, y)$ (home) or $v_1(y, x) < v_2(y, x)$ (away), team x loses to team y if $v_1(x, y) < v_2(x, y)$ (home) or $v_1(y, x) > v_2(y, x)$ (away) and teams x draws with team y if $v_1(x, y) = v_2(x, y)$.

Since in some team sports (basketball, ice hockey, volleyball, etc.) draws are not allowed, and we want to keep the model as general as possible, this possibility is excluded in the following.

Assumption 3.1. *No matches result in a draw: $v_1(x, y) \neq v_2(x, y)$ for any incomplete round-robin tournament (X, v) and teams $x, y \in X$.*

Ranking is usually based on points.

Definition 3.6. *Number of points*: Let (X, v) be an incomplete round-robin tournament and $x \in X$ be a team. Denote by $N_v^w(x)$ the number of wins and by $N_v^l(x)$ the number of losses of team x in (X, v), respectively. The *number of points* of team x is $s_v(x) = \alpha N_v^w(x) + \beta N_v^l(x)$ such that $\alpha > \beta$.

In other words, a win means α points and a loss means β points.

Number of points is not guaranteed to induce a strict order on the set of teams, hence some tie-breaking rules are needed.

Definition 3.7. *Goal difference*: Let (X, v) be an incomplete round-robin tournament and $x \in X$ be a team. The *goal difference* of team x is

$$gd_v(x) = \sum_{y \in X, y \neq x} (v_1(x, y) - v_2(x, y)) + \sum_{y \in X, y \neq x} (v_2(y, x) - v_1(y, x)) .$$

Goal difference is the difference of the number of goals scored for team x and the number of goals conceded by team x.

Definition 3.8. *Head-to-head results*: Let (X, v) be a round-robin tournament and $x \in X$ be a team. Denote by $L \subseteq X \setminus \{x\}$ a set of teams.

The *head-to-head number of points* of team x with respect to L in (X, v) is

$$\begin{aligned} s_v^L(x) &= \alpha \left(\left| \{ y \in L : v_1(x,y) > v_2(x,y) \} \right| + \left| \{ y \in L : v_1(y,x) < v_2(y,x) \} \right| \right) + \\ &\quad + \beta \left(\left| \{ y \in L : v_1(x,y) = v_2(x,y) \} \right| + \left| \{ y \in L : v_1(y,x) = v_2(y,x) \} \right| \right) \end{aligned}$$

The *head-to-head goal difference* of team x with respect to L in (X, v) is

$$gd_v^L(x) = \sum_{y \in L} \left(v_1(x,y) - v_2(x,y) \right) + \sum_{y \in L} \left(v_2(y,x) - v_1(y,x) \right).$$

In accordance with EHF (2014a, Articles 9.12 and 9.24), head-to-head results can be calculated only if all group matches were played.

Definition 3.9. *Monotonicity of group ranking*: Let $\mathcal{X}$ be the set of incomplete round-robin tournaments with a set of teams X, and R be a ranking method. S is *monotonic* if for any characteristic function v and for any different teams $x, y \in X$, $x \neq y$:

1. $s_v(x) > s_v(y) \Rightarrow x \succ_{R(v)} y$;
2. $s_v(x) = s_v(y)$ and $gd_v(x) > gd_v(y)$, furthermore, if (X, v) is a round-robin tournament, then $s_v^L(x) > s_v^L(y)$, or $s_v^L(x) = s_v^L(y)$ and $gd_v^L(x) > gd_v^L(y)$ where $z \in L$ if and only if $s_v(x) = s_v(y) = s_v(z) \Rightarrow x \succ_{R(v)} y$.

Monotonicity implies that (a) a team should be ranked higher if it has a greater number of points (criterion 1); (b) a team should be ranked higher compared to another with the same number of points, an inferior goal difference and worse head-to-head results against all teams with the same number of points (criterion 2). Monotonicity still does not imply that the ranking is unique. The complexity of Definition 3.8 is necessary in order to cover the two different tie-breaking rules, goal difference and head-to-head concepts. For example, in association football FIFA usually uses the former, while UEFA applies the latter.

Definition 3.10. *Preliminary round*: A *preliminary round* $\mathcal{G}$ consists of k groups of round-robin tournaments (X^1, v^1), (X^2, v^2), $\dots$, (X^k, v^k) such that $X^i \cap X^h = \emptyset$ for any $i \neq h$.

Definition 3.11. *Main round*: A *main round* $\mathcal{M}$ consists of ℓ groups of incomplete round-robin tournaments (Y^1, w^1), (Y^2, w^2), $\dots$, (Y^ℓ, w^ℓ) such that $Y^j \cap Y^h = \emptyset$ for any $j \neq h$.

Definition 3.12. *Qualification rule*: Let $\mathcal{G}$ be a preliminary round and $\mathcal{M}$ be a main round. A *qualification rule* is a mapping $\mathcal{Q} : \mathcal{X}^1 \times \mathcal{X}^2 \times \dots \times \mathcal{X}^k \to \mathcal{Y}^1 \times \mathcal{Y}^2 \times \dots \times \mathcal{Y}^\ell$.

Team $x \in X^i$ is said to be *qualified* to the main round if $x \in \cup_{j=1}^{\ell} Y^j$.

Definition 3.13. *Tournament with subsequent group stages*: A *tournament with subsequent group stages* is the triple $(\mathcal{G}, \mathcal{M}, \mathcal{Q})$ consisting of a preliminary round $\mathcal{G}$, a main round $\mathcal{M}$ and a qualification rule $\mathcal{Q}$.

Definition 3.14. *Regularity of a qualification rule*: Let $(\mathcal{G}, \mathcal{M}, \mathcal{Q})$ be a tournament with subsequent group stages. Qualification rule $\mathcal{Q}$ is *regular* if:

a) $\cup_{j=1}^{\ell} Y^j \subseteq \cup_{i=1}^{k} X^i$;

b) there exists a common monotonic ranking R in each group of the preliminary round $\mathcal{G}$ such that $x, y \in X^i$, $1 \leq i \leq k$ and $x \succ_{R(v^i)} y$, $y \in \cup_{j=1}^{\ell} Y^j$ imply $x \in \cup_{j=1}^{\ell} Y^j$;

c) $x, y \in X^i \cap Y^j$ implies $w(x, y) = v(x, y)$;

d) $x \in X^i$, $y \in X^h$, $h \neq i$ and $x, y \in Y^j$ imply $w(x, y) = —$;

e) there exists a common monotonic ranking S in each group of the main round $\mathcal{M}$.

The idea behind a regular qualification rule is straightforward. Some top teams of the preliminary round groups qualify to the main round (conditions *a)* and *b)*), where they are organized into new groups such that matches already played against other qualified teams are carried over to the main round (conditions *c)* and *d)*). Furthermore, rankings in the preliminary and main round groups should be monotonic (conditions *b)* and *e)*).

Perhaps these ideas have inspired the decision-makers of EHF.

Definition 3.15. *Manipulation*: Consider a tournament with subsequent group stages $(\mathcal{G}, \mathcal{M}, \mathcal{Q})$ and a set of preliminary round results $V = \left\{ v^1, v^2, \dots, v^i, \dots, v^k \right\}$. A team $x \in X^i$ can *manipulate* $(\mathcal{G}, \mathcal{M}, \mathcal{Q})$ if there exists a set of group results $\bar{V} = \left\{ v^1, v^2, \dots, \bar{v}^i, \dots, v^k \right\}$ such that $\bar{v}_2^i(x, y) \geq v_2^i(x, y)$ and $\bar{v}_1^i(y, x) \geq v_1^i(y, x)$ for all $y \in X^i$, furthermore, $x \in \cup_{j=1}^{\ell} Y^j$ according to both $\mathcal{Q}(V)$ and $\mathcal{Q}(\bar{V})$ such that $s_w(x) < s_{\bar{w}}(x)$, or $s_w(x) = s_{\bar{w}}(x)$ and $gd_w(x) < gd_{\bar{w}}(x)$.

Manipulation means that team x can increase its number of points, or improve its goal difference with the same number of points in the main round by conceding more goals in a match of the preliminary round.

Definition 3.16. *Strategy-proofness*: A tournament with subsequent group stages $(\mathcal{G}, \mathcal{M}, \mathcal{Q})$ is called *strategy-proof* if there exists no set of group results $V = \left\{ v^1, v^2, \dots, v^k \right\}$ under which a team can manipulate it.

Our main results concern the strategy-proofness of tournaments with subsequent group stages such that the qualification rule is regular. Note that manipulation certainly worsen a team's goal difference (and sometimes its number of points, too) in the given group as the ranking rule applied here is monotonic, but – provided that the team still qualifies – it may pay off in the main round, where some matches of the preliminary round are discarded.

Theorem 3.1. *Let $(\mathcal{G}, \mathcal{M}, \mathcal{Q})$ be a tournament with subsequent group stages such that $\mathcal{Q}$ is a regular qualification rule and the following conditions hold:*

- *there exists $x, y \in X^i \cup Y^j$ for some $1 \leq i \leq k$ and $1 \leq j \leq \ell$;*
- *for at least one $1 \leq i \leq k$, there exists $u, v \in X^i$ with $u \in Y^j$ implying $v \notin Y^j$ for all $1 \leq j \leq \ell$.*

Then the tournament with subsequent group stages $(\mathcal{G}, \mathcal{M}, \mathcal{Q})$ *does not satisfy strategy-proofness.*

Theorem 3.1 requires that the result of at least one match played in the preliminary round is carried over to main round, and the result of at least one such match is ignored.

Proof. An example is presented where a team can manipulate a tournament with subsequent group stages that satisfies all criteria of Theorem 3.1.

Table 2: Group 1 of Example 3.1

GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points.
Last but one row contains the group winner's benchmark results that are carried over to the main round.
Last row contains the group winner's alternative results that are carried over to the main round, obtained if it manipulates.

Position	Team	a	b	c	GF	GA	GD	**Pts**
1	a	—	0-1	4-0	4	1	3	$\alpha + \beta$
2	b	1-0	—	0-2	1	2	-1	$\alpha + \beta$
3	c	0-4	2-0	—	2	4	-2	$\alpha + \beta$
1	a	—	0-1	—	0	1	-1	β
1*	a*	—	—	2-0*	2*	0*	2*	α*

Example 3.1. Let $X^1 = \{a, b, c\}$ such that the group is a single round-robin tournament.

Consider the regular qualification rule $\mathcal{Q}$ with $\ell = 1$ and $x \in Y^1$ if and only if $\left\{z \in X^i : x \succ_{R(v^i)} z\right\} \neq \emptyset$. $\mathcal{Q}$ says that the group winner and the runner-up qualify to the main round.

A possible set of results in Group 1 is shown in Table 2. Team a is the group-winner since it has the best (head-to-head) goal difference (see criterion 2 of a monotonic group ranking method), and it is considered with $s_w(a) = \beta$ points in the main round, after discarding its match against team c, the last in Group 1 due to criterion 2 of a monotonic group ranking method (see the last but one row of Table 2).

However, examine what happens if $\bar{v}^1(a, c) = (2; 0)$ (thus $\bar{v}^1(c, a) = (0; 2)$). Then teams a, b and c remain with $\alpha + \beta$ points, but they have head-to-head goal differences of $+1$, -1 and 0, respectively, thus a is the first and c is the second according to criterion 2 of a monotonic group ranking method. Consequently, team a is considered with $s_{\bar{w}}(a) = \alpha > \beta = s_w(a)$ points in the main round (see the last row of Table 3).

To summarize, team a has an opportunity to manipulate this simple tournament with subsequent group stages under the set of group results V, so it is not strategy-proof.

Example 3.1 contains only three teams, which is minimal with respect to the conditions of Theorem 3.1. It is clear that the number of groups and the number of teams in them can be increased without changing the essence of the counterexample. Furthermore, groups can be double round-robin tournaments instead of single ones. □

Theorem 3.1 remains valid if draws are allowed in the tournament with subsequent group stages, too.

Remark 3.1. The 11th Men's European Handball Championship (EHF Euro 2014), discussed in Section 2, fits into the model presented above. The number of groups in the preliminary round is $k = 4$, the number of groups in the main round is $\ell = 2$, and the qualification rule is regular (EHF, 2014a):

a) $Y^1 \subset X^1 \cup X^2$ and $Y^2 \subset X^3 \cup X^4$;

b) Ranking in the preliminary round groups is monotonic as it is based on the number of points with tie-breaking through head-to-head results, and the first three teams qualify to the main round;

c) Matches played during the preliminary round against opponents which qualified to the main round are kept and remain valid for the ranking of the main round;

d) Matches of the main round are played in groups with each team facing three opponents which did not participated in its preliminary round group;

e) Ranking in the main round groups is monotonic as it is based on the number of points with tie-breaking through head-to-head results.

Proposition 3.1. *The 11th Men's European Handball Championship (EHF Euro 2014) is not strategy-proof.*

Proof. The scenario presented in Section 2 shows that team Poland $= x \in X^3$ can manipulate since there exist sets of group results $V = \{v^1, v^2, v^3, v^4\}$ and $\bar{V} = \{v^1, v^2, \bar{v}^3, v^4\}$ such that $\bar{v}^3 = v^3$, $\bar{v}^3_1(x,y) = v^3_1(x,y) = 26$ with the exception of $\bar{v}^3_2(x,y) = 24 > 22 = v^3_2(x,y)$, where team Russia $= y \in X^3$ and Poland qualifies according to $\mathcal{Q}(V)$ and $\mathcal{Q}(\bar{V})$, but $s_w(x) = 0 < 2 = s_{\bar{w}}(x)$.

Theorem 3.1 can also be applied due to Remark 3.1. □

Now we state a positive result, a 'pair' of Theorem 3.1.

Theorem 3.2. *Let $(\mathcal{G}, \mathcal{M}, \mathcal{Q})$ be a tournament with subsequent group stages such that $\mathcal{Q}$ is a regular qualification rule and at least one of the following conditions hold:*

- *there does not exist $x, y \in X^i \cup Y^j$ for any $1 \le i \le k$ and $1 \le j \le \ell$;*
- *$u, v \in X^i$ and $u \in Y^j$ implies $v \in Y^j$ for all $1 \le i \le k$.*

Then the tournament with subsequent group stages $(\mathcal{G}, \mathcal{M}, \mathcal{Q})$ is strategy-proof.

Proof. If all preliminary round results obtained against other qualified are carried over to the main round or ignored, then it makes no sense to exert a lower effort in the preliminary round. □

Theorem 3.2 practically says that teams qualifying from the same preliminary round group should be drawn into different main round groups (it is guaranteed if only one team qualifies from each preliminary round group), or all teams from a given preliminary round group should qualify to the same main round group, or some matches should be repeated in the tournament.

Consequently, our main result seems to be related to the finding of Vong (2017) that in general multistage tournaments, the necessary and sufficient condition of strategy-proofness is to allow only the top-ranked player to qualify from each group. However, in the model of Vong (2017), teams deliberately lose matches in order to meet preferred opponents in the next round, so they only gain in expected value. Contrarily, we have discussed the possibility that a team is strictly better off by exerting a lower effort.

4 Discussion

The Men's European Handball Championship is the official competition for senior men's national handball teams of Europe. It takes place every two years since 1994 and serves as a qualification for the Olympic Games and World Championship, too.

The tournaments between 1994 and 2000 consisted of a group stage followed by a knock-out stage, hence they were incentive compatible. Since 2002, its format is the same as outlined in Section 2: there is a preliminary round with four groups of four teams each such that the first three teams qualify to the main round with two groups of six teams each (three-three from two groups of the preliminary round), and they carry over the matches against the two teams in their preliminary round group. The winners and runners-up of the main round groups qualify to the semifinals.

During the 10th Men's European Handball Championship (EHF Euro 2012), a situation analogous to the one presented in Section 2 emerged: Slovenia plays its last match in Group D against Iceland such that Croatia had 4 points after it won against Iceland and Slovenia, Norway had 2 points because of its win against Slovenia by 28-27, and Iceland had also 2 points due to its win against Norway by 34-32. Consequently, Slovenia should win against against Iceland for qualification to the main round, but it would be better not to win by more than 3 goals in order to carry over the result against Iceland. The actual results were Iceland vs. Slovenia 32-34, and Croatia vs. Norway 26-22, so the manipulation of Slovenia turned out to be successful (with Iceland vs. Slovenia 31-34 or 32-35, Iceland still would have qualified, but 30-34, 31-35, or 32-36 would be unfavourable for Slovenia).

The Women's European Handball Championship is the official competition for senior women's national handball teams of Europe. The tournament also serves as a qualification for the Olympic Games and World Championship. It takes place in the same years as Men's European Handball Championships, and is organized in the same format, so it was also strategy-proof until 2000, but incentive incompatible from 2002.

The Women's EHF Champions League is an official competition for women's handball clubs of Europe, organized annually since the season of 1993-94. It is the most competitive and prestigious tournament for the top clubs of the continent's leading national leagues. It is organized with subsequent group stages since 2013-14. The preliminary round consists of four groups of four teams each, playing each other twice in home and away matches such that the best three teams qualify. In the main round, two groups of six teams are formed, and teams play those three teams they have not already faced twice, in home and away matches. The top four teams from each group advance to the quarter-finals.

Table 3 summarizes our findings on incentive incompatible tournaments; Men's (Women's) EHF Euro stands for Men's (Women's) European Handball Championship, and Women's EHF CL shortens Women's EHF Champions League. They all contain two subsequent group stages, and the number of qualified teams in the main round (see the last column) is the number of teams which can win the tournament, not taking into account the possible matches for fifth and seventh places.

It is clear from our theoretical results, presented in Section 3, that – contrary to tournament systems consisting of multiple round-robin and knockout tournaments (Dagaev and Sonin, 2017), or group-based qualification systems (Csató, 2017b) – there is no straightforward way to guarantee the incentive compatibility of tournaments with subsequent group stages. According to Theorem 3.2, strategy-proofness is met if either all matches from the preliminary round are considered in the main round, or all of them are discarded,

Table 3: Tournaments with subsequent group stages

S = single round-robin (in groups); D = double round-robin (in groups); Gr. = Number of groups in the preliminary and main round, respectively; Teams = Number of teams in each group of the preliminary and main round, respectively; Q = Number of teams qualified from each group of the preliminary and main round, respectively

Tournament	Sport	Type	Preliminary round Gr.	Preliminary round Teams	Preliminary round Q	Main round Gr.	Main round Teams	Main round Q
2002 Men's EHF Euro	Handball	S	4	4	3	2	6	2
2004 Men's EHF Euro	Handball	S	4	4	3	2	6	2
2006 Men's EHF Euro	Handball	S	4	4	3	2	6	2
2008 Men's EHF Euro	Handball	S	4	4	3	2	6	2
2010 Men's EHF Euro	Handball	S	4	4	3	2	6	2
2012 Men's EHF Euro	Handball	S	4	4	3	2	6	2
2014 Men's EHF Euro	Handball	S	4	4	3	2	6	2
2016 Men's EHF Euro	Handball	S	4	4	3	2	6	2
2018 Men's EHF Euro	Handball	S	4	4	3	2	6	2
2002 Women's EHF Euro	Handball	S	4	4	3	2	6	2
2004 Women's EHF Euro	Handball	S	4	4	3	2	6	2
2006 Women's EHF Euro	Handball	S	4	4	3	2	6	2
2008 Women's EHF Euro	Handball	S	4	4	3	2	6	2
2010 Women's EHF Euro	Handball	S	4	4	3	2	6	2
2012 Women's EHF Euro	Handball	S	4	4	3	2	6	2
2014 Women's EHF Euro	Handball	S	4	4	3	2	6	2
2016 Women's EHF Euro	Handball	S	4	4	3	2	6	2
2018 Women's EHF Euro	Handball	S	4	4	3	2	6	2
2013-14 Women's EHF CL	Handball	D	4	4	3	2	6	4
2014-15 Women's EHF CL	Handball	D	4	4	3	2	6	4
2015-16 Women's EHF CL	Handball	D	4	4	3	2	6	4
2016-17 Women's EHF CL	Handball	D	4	4	3	2	6	4
2017-18 Women's EHF CL	Handball	D	4	4	3	2	6	4

which is against the essence of these tournaments.[6]

Perhaps the only solution is to carry over *all* preliminary round results to the main round – not only the matches played against teams which qualify to the same main round group –, regardless that some matches were played against teams already eliminated from the tournament.

5 Conclusions

Design of sport ranking rules is an important topic of Operational Research. We have presented that practitioners, especially tournament organizers, should not miss analysing strategy-proofness since complex tournament formats may create perverse incentives for

[6] It is worth to note that 2001-02 UEFA Champions League also included two group stages: from the first group stage of eight groups with four teams each, eight winners and eight runners-up were drawn into four groups of four teams each, containing two group winners and two runners-up such that teams from the same country or from the same first round group could not be drawn together. The last condition would have guaranteed strategy-proofness even if matches would have allowed to carry over.

some competitors. While the actual probability of manipulation can be relatively small, and the audience does not always recognize the problem, it is not worth to risk a potential scandal which has enormous financial and reputational costs. The way of manipulation discussed here may also violate our sense of fairness as a third team is hurt by it.

It is somewhat surprising that we have not found any controversy about the probable manipulation presented in Section 2. We think it is because of the complex detection: compared to the football and basketball matches discussed in Section 1, it was enough to make some mistakes in defence or attack, scoring of own goals was unnecessary. One can understood that EHF remained silent on this issue, while the audience obviously did not studied the tie-breaking rules carefully. On the other hand, it is almost sure that coaches and players knew that they should not make great efforts to win by a higher margin. We hope the the paper contributes to placing this match in the category of the notorious 'Nichtangriffspakt (Schande) von Gijón'[7] (Kendall and Lenten, 2017, Section 3.9.1) in the history of sports.

There are at least two directions for future scientific research. First, other tournaments can be examined from the perspective of strategy-proofness and some suggestions can be made for the modification of their design. Second, by the quantification of team strengths and the modelling of match outcomes, the probability of manipulation can be estimated. We think it is not negligible in the case of European Handball Championships as two examples have been found from the $2 \times 8 \times 4 = 64$ observations represented by groups in the preliminary rounds of EHF Euros, which took place between 2002 and 2016.

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