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Abstract

This study analyzes how financial shocks in one country transmit to another country through international trade. To this end, it develops a dynamic general equilibrium model of two-country Ricardian trade with a continuum of goods. Financial frictions exist in each country and the two countries can be asymmetric in terms of the degree of frictions, which can be a novel source of comparative advantage. In the case of a permanent credit crunch, we can analytically show that such a shock reduces the long-run investment, GDP, wage income, and aggregate income of heterogeneous entrepreneurs in both countries. We also numerically investigate the transitory responses to a temporal credit shock and show that such an internationally synchronized economic downturn is also observed during transition periods.

JEL classification: E22, E32, E44, F11, F44
Keywords: Financial Frictions, Dynamic Ricardian Trade with a Continuum of Goods, Heterogeneous Agents, Asymmetric Countries, Credit Crunch

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1 Introduction

The recent globally synchronized economic downturn during the financial crisis of 2007–2009 drew more attention toward the importance of interdependence among countries. How does a domestic business cycle shock in one country affect other countries? Recent theoretical studies emphasize the critical role of frictions in domestic financial markets for transmitting a shock from one country to another. Examples of such contributions using dynamic two-country models include Devereux and Yetman (2010), Devereux and Sutherland (2011), Kollmann et al. (2011), and Perri and Quadrini (2017). As explained below, their common finding is that under a higher level of financial integration, a country-specific shock leads to a more synchronized decline in economic activities.

All these studies assume a single consumption/investment good economy, thereby ignoring the possible transmission channel through the intra-temporal trade of multiple goods. While there is little doubt that financial globalization played an important role in the international co-movement seen during the recent financial crisis, the fact remains that not only international financial transactions, but also international trade in goods is the engine of globalization. On that basis, a financial shock in one country is also likely to spread through the latter channel of globalization. For instance, Lane and Milesi-Ferretti (2011) find that openness to trade had significant effects on the severity of affected countries' recessions. Moreover, by employing firm-level micro data for 42 countries, Claessens et al. (2012) find that the 2007–2009 crisis had a larger negative impact on firms in countries more open to trade. These results suggest that international trade may also have contributed to the global recession in a non-trivial way.

Against this background, this study theoretically explores how financial shocks in one country propagate to its partner country through international trade in goods alone. For this purpose, it incorporates financial frictions and international trade into a two-country dynamic general equilibrium framework. To simply embed financial frictions, this study borrows the heterogeneous agent framework of Buera and Moll (2015), who examine how a financial shock, modeled as a tightening of borrowing constraints, affects aggregate efficiency in a closed economy.[1] In our model, each country consists of homogeneous workers and heterogeneous entrepreneurs who engage in investment projects to produce capital. Heterogeneity arises in their investment technologies by receiving idiosyncratic shocks. In addition, they can borrow

[1]Since the seminal works by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), macroeconomic models with financial frictions have been major workhorses in business cycle studies. For instance, recent studies such as Jermann and Quadrini (2012) and Buera and Moll (2015) have focused on the shocks on the credit constraint itself as a key influence on business cycles.

from domestic lenders only up to a proportion of their own funds (i.e., they are credit constrained). These two assumptions jointly generate a cutoff that classifies the entrepreneurs into those actively investing and those inactively investing.

In addition, to describe the international production reallocation induced by financial shocks, this study extends the Ricardian trade model with a continuum of goods developed by Dornbusch et al. (1977) to the framework of endogenous capital accumulation.[2] In the present model, the two countries trade a continuum of intermediate goods used for the domestic production of a single final good. This final good is used for domestic consumption and investment. The advantage of employing such a continuum-good Ricardian framework is that it allows us to explore how each country experiences changes in its extensive margins of exports and imports. This, in turn, enables us to simply examine how financial shocks in one country affect the major macroeconomic variables in its partner country through international trade.

Within this framework, the present study analyzes the impacts of two kinds of financial shocks, namely a permanent and a temporal tightening of borrowing constraints (i.e., a credit crunch) in one country. Considering the former case is helpful to understand the qualitative characteristics of the model. In this case, we can analytically obtain the following two results. First, a credit crunch in one country changes the trade patterns in the long-run equilibrium such that this country experiences a decrease in its extensive margin of exports. Second, such a credit crunch reduces the investment, GDP, wage income, and aggregate income of the entrepreneurs in both countries. That is, international trade can work as the driver of a synchronized economic downturn.

The mechanism is explained as follows. Suppose that the borrowing constraint tightens in one country. This induces an inefficient reallocation of financial resources in that country from relatively high productive entrepreneurs to less productive ones who are otherwise inactive investors. From such a misallocation, investment efficiency declines on average, thereby making capital in this country endogenously scarce relative to labor. This means that the credit crunch affects the labor productivity of each sector in this country. Thus, in this model, the degree of financial frictions is a key determinant of the comparative advantage in the steady-state equilibrium.

At the same time, this naturally raises the domestic price of capital in the country that experiences the credit crunch, which is, in turn, reflected by an increase in the price of the intermediate goods produced in that country. Since they are exported, not only the country, but also its partner country faces upward pressure on the price of the domestic final good. Then,

[2]See Eaton and Kortum (2012) for reviews of recent developments in Ricardian trade theory.

to offset this upward pressure, the wage rate must fall in the partner country. Consequently, motivated by their cost-minimizing motive, the final good producers in the partner country reduce demand for capital. Consequently, investment in the partner country also decreases. These two analytical results are obtained without specifying the distribution function for the heterogeneity of entrepreneurs or assuming symmetry across the two countries.

Next, we turn to the more realistic case of a temporal credit shock. Under simply calibrated parameter values, our numerical experiment shows that these two results are also observed during transition periods. In addition, it is found that the degree of international co-movement increases as the intermediate goods become more complementary. This result is consistent with Heathcote and Perri's (2002) finding in a two-country international RBC model that the degree of international co-movement is decreasing in the elasticity of substitution between the tradable intermediate goods. Thus, this study obtains the following new theoretical finding that a credit shock in a country can be not only a source of business cycles in that country, but also combined with international trade to have a key influence on international co-movement.

The rest of this paper is organized as follows. After Section 1.1 discusses the related literature, Section 2 describes the setup of the model. Section 3 shows the uniqueness and local stability of the steady-state equilibrium. Section 4 pursues the qualitative characteristics of the model by considering a permanent credit crunch in one country. Section 5 calibrates the parameter values in this model and conducts a simple numerical analysis about a temporal credit shock to obtain the transitory responses of major macroeconomic variables in two countries. Section 6 concludes.

1.1 Related Literature

The results of this study complement the growing literature on the international transmission of domestic shocks under financial frictions. As introduced in the previous section, Devereux and Yetman (2010) construct a two-country model abstracting capital accumulation and consider the international transmission of a productivity shock in one country. They numerically examine how such a transmission is affected by the binding of borrowing constraints for investors who invest in domestic and foreign productive assets internationally. Devereux and Sutherland (2011) use Devereux and Yetman's framework to examine the effects of a credit crunch in one country. They investigate the transmission mechanisms in two financial integration settings, namely integration in a bond market and integration in both bond and equity markets, and show that integration in the equity market is crucial for generating international co-movement.[3]

[3]They also extend Devereux and Yetman's (2010) model to allow for capital accumulation.

Kollmann et al. (2011) assume a representative global bank that collects deposits from households in both countries and makes loans to entrepreneurs. They then quantitatively examine how the increase in the loan default rate in one country contributes to business cycle fluctuations in both countries. More recently, Perri and Quadrini (2017) develop a two-country model within which the firms in each country face a borrowing constraint: whether such a constraint is binding depends on agents' self-fulfilling expectations. Therefore, a credit crunch endogenously occurs in their model.

In contrast to the aforementioned work that investigates the transmission mechanisms in various financial integration settings, this study focuses on financial frictions as a determinant of the comparative advantage. Given that both international trade and international financial transactions serve as engines of globalization in reality, the results of this study can complement the theoretical findings in the literature. Recently, within the framework of a symmetric two-country DSGE model, Imura and Thomas (2016) conduct a quantitative analysis of how a temporal credit crunch in one country propagates to the partner country through trade in two kinds of intermediate goods. By doing so, they retain the assumption of international symmetry and assume that trade patterns are exogenous by employing the Armington assumption.

This study also contributes to the literature on international trade and financial frictions. Among previous empirical studies, Beck (2002, 2003) suggests that countries with better developed systems in their domestic financial markets have higher export shares in industries that use more external finance. He also reports evidence that such countries have a higher export share in GDP. On the theoretical side, the role of financial frictions in the equilibrium patterns of trade is examined in some studies. Examples of such studies include Matsuyama (2005), Antras and Caballero (2009), and Ju and Wei (2011). The role of financial frictions in this model is significantly different from their static trade models. In the dynamic model where the steady-state level of capital is endogenously determined, capital is no longer an exogenous endowment, but rather it changes over time and indirectly influences labor productivity in each sector. That is, in such a dynamic economy, trade patterns are determined in a Ricardian manner rather than under a Heckscher–Ohlin model irrespective of the number of production factors.[4] Our model thus highlights the role of financial frictions in capital accumulation.

[4]Baxter (1992) was the first to point out this Ricardian property by using a dynamic two-country, two-good, two-factor model. She shows that international heterogeneity in capital income tax rates leads at least one country to perfectly specialize in one good. By contrast, Chen (1992) retains the assumption that preferences, the production technology of each good, and public policies are identical between the countries, showing that if this is the case, the Heckscher–Ohlin theorem holds even in a dynamic framework.

2 Model

Time is discrete and indexed by $t = 0, 1, 2, \dots$ The world consists of two countries, namely home (denoted by H) and foreign (F), hereafter indexed by $j \in \{H, F\}$. In each country, a single non-tradable final good can be used for domestic consumption and investment. The final good is produced from a continuum of tradable intermediate goods with unit measure. Each variety of intermediate goods is produced from non-tradable capital and labor. Since we focus on exploring how credit shocks in one country are transmitted to the other country through international trade in goods alone, we do not consider international financial transactions.[5]

In each country j, there are two types of infinitely lived agents: one is a continuum of heterogeneous entrepreneurs with unit measure and the other is that of homogeneous workers with measure $L_j > 0$. The entrepreneurs accumulate capital and rent it to the domestic intermediate good firms. In each period, they face an idiosyncratic productivity shock on their investment technologies, and only highly productive entrepreneurs produce capital by their investment, as shown in Section 2.2. Although each entrepreneur faces such a stochastic environment, there is no aggregate uncertainty.

The two countries can be asymmetric in terms of residents' discount factor, the degree of financial frictions, and the distribution of entrepreneurs' heterogeneity.

2.1 Production of Goods

The representative final good firm in country j combines a continuum of intermediate goods to produce the final good according to the following CES production function:

$$Y_{j,t} = \left(\int_0^1 x_{j,t}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}},$$

where $Y_{j,t}$ is the output of the final good, $x_{j,t}(\omega)$ is demand for the intermediate good of variety $\omega \in [0, 1]$, and $\sigma > 1$ is the elasticity of substitution between any two varieties.

All varieties of the intermediate goods are freely traded and thus there is no international price gap. Let $p_t(\omega)$ stand for the price of variety ω and $P_{j,t}$ denote the price of the final good. Under perfect competition, the final good firm in country j chooses $(x_{j,t}(\omega))_{\omega \in [0,1]}$ to maximize

[5] Heathcote and Perri (2002) show that models without international financial transactions can generate international synchronization more closely fitted to the data than models with financial integration. Inspired by their finding, such a "financial autarky" assumption is often employed in studies that focus on international trade as a potential source of international co-movement. See, for instance, Kose and Yi (2006) and Arkolakis and Ramanarayanan (2009).

its profit $P_{j,t}Y_{j,t} - \int_0^1 p_t(\omega)x_{j,t}(\omega)d\omega$ subject to its production function. Profit maximization implies that

$$x_{j,t}(\omega) = \left(\frac{p_t(\omega)}{P_{j,t}}\right)^{-\sigma} Y_{j,t} \quad \forall \omega \in [0,1],$$

$$P_{j,t} = \left(\int_0^1 p_t(\omega)^{1-\sigma} d\omega\right)^{\frac{1}{1-\sigma}} \quad \forall j \in \{H, F\},$$

where the right-hand side of the second equation represents the price index of the intermediate goods. Throughout the paper, the final good in one country is chosen as the numeraire. As the above equation shows, the free trade in the intermediate goods means that the price in the other country also becomes unity: $P_{j,t} = 1$ for all j.

Turn to the intermediate good sector. Let $X_{j,t}(\omega)$ denote the output of variety ω in country j. Each intermediate good is produced according to the following Cobb–Douglas technology:

$$X_{j,t}(\omega) = \frac{1}{\psi_j(\omega)} \left(\frac{K_{j,t}(\omega)}{\alpha}\right)^{\alpha} \left(\frac{L_{j,t}(\omega)}{1-\alpha}\right)^{1-\alpha},$$

where $K_{j,t}(\omega)$ and $L_{j,t}(\omega)$ are demand for capital and labor, respectively. $\psi_j(\omega) > 0$ captures the exogenous and country-specific productivity parameter for variety ω. Thanks to the specification such that the share of capital $\alpha \in (0,1)$ does not vary across varieties, the specialization pattern in equilibrium is determined in the same way as Dornbusch et al. (1977), the detail of which is explained in Section 3.

Let $q_{j,t}$ and $w_{j,t}$ denote the rental price of capital and wage rate in country j, respectively. The unit cost function in country j is given by

$$\begin{aligned} mc_{j,t}(\omega) &= \min_{e_K, e_L} \left\{ q_{j,t}e_K + w_{j,t}e_L \;\middle|\; 1 = (\psi_j(\omega))^{-1} (e_K/\alpha)^{\alpha} (e_L/(1-\alpha))^{1-\alpha} \right\} \\ &= \psi_j(\omega) q_{j,t}^{\alpha} w_{j,t}^{1-\alpha}. \end{aligned}$$

Perfect competition results in $p_t(\omega) = \min_j\{mc_{j,t}(\omega)\}$.

Following Dornbusch et al. (1977), the varieties are indexed so that

$$\frac{d(\psi_F(\omega)/\psi_H(\omega))}{d\omega} < 0.$$

In other words, all other things being equal, the home (foreign) country has a comparative advantage in low-indexed (high-indexed) goods. Let ω_t^c denote the cutoff variety of the extensive margin of exports in each country. Under the assumed technology distribution, any variety no more (less) than ω_t^c is produced in the home (foreign) country. The cutoff is implicitly determined from $mc_{H,t}(\omega) = mc_{F,t}(\omega)$, which is rewritten as

$$\frac{\psi_F(\omega_t^c)}{\psi_H(\omega_t^c)} \equiv \left(\frac{q_{H,t}}{q_{F,t}}\right)^{\alpha} \left(\frac{w_{H,t}}{w_{F,t}}\right)^{1-\alpha}.$$

In Section 3, it is shown that ω_t^c is interior of $[0,1]$. Let $\Omega_{H,t} \equiv [0, \omega_t^c]$ and $\Omega_{F,t} \equiv [\omega_t^c, 1]$ stand for the sets of the varieties produced in the home and foreign countries, respectively. The price of each variety is given by

$$p_t(\omega) = \begin{cases} \psi_H(\omega) q_{H,t}^{\alpha} w_{H,t}^{1-\alpha} & \text{for } \omega \in \Omega_{H,t}, \\ \psi_F(\omega) q_{F,t}^{\alpha} w_{F,t}^{1-\alpha} & \text{for } \omega \in \Omega_{F,t}, \end{cases}. \tag{1}$$

The price index of the intermediate goods accordingly satisfies

$$1 = \left[\left(q_{H,t}^{\alpha} w_{H,t}^{1-\alpha} \right)^{1-\sigma} \int_{\Omega_{H,t}} \psi_H(\omega)^{1-\sigma} d\omega + \left(q_{F,t}^{\alpha} w_{F,t}^{1-\alpha} \right)^{1-\sigma} \int_{\Omega_{F,t}} \psi_F(\omega)^{1-\sigma} d\omega \right]^{1/(1-\sigma)}. \tag{2}$$

Throughout the paper, it is ensured that

Assumption 1. $\int_{\omega \in \Omega_{j,t}} \psi_j(\omega)^{1-\sigma} d\omega > 0$. *for all* $j \in \{H, F\}$.

For example, when $\psi_j(\omega)$ is specified as $\psi_j(\omega) = \omega^{-\phi_j}$ with the restriction $-(\phi_F - \phi_H) < 0$, the above assumption is satisfied as long as $1 - \phi_j(1-\sigma) > 0$ is satisfied.

2.2 Entrepreneurs

In country j, there exists a unit measure of entrepreneurs, indexed by $i_j \in [0,1]$. The preferences of agent i_j are given by the following utility function:

$$EU_{j,t}^i = E_t \left[\sum_{\tau=t}^{\infty} (\beta_j)^{\tau-t} \log c_{j,\tau}^i \right],$$

where c_j^i is consumption and $\beta_j \in (0,1)$ is the discount factor, which can vary across countries, but is the same among the entrepreneurs within a country.

For simplicity, capital fully depreciates in one period. The investment technology of agent i_j in period t is

$$k_{j,t+1}^i = \theta_{j,t}^i z_{j,t}^i,$$

where $z_{j,t}^i$ and $k_{j,t+1}^i$ are investment and capital, respectively.[6] Entrepreneurs differ in the efficiency of investment technologies, denoted by $\theta_{j,t}^i \in [\underline{\theta}, \overline{\theta}]$. Throughout the paper, $\theta_{j,t}^i$

[6] The results are qualitatively the same even when capital depreciates only partially as long as the remaining capital is liquidated before the new investment. If this is the case, the investment technology equation remains the same, whereas the budget constraint is replaced by $(q_{j,t} + 1 - \delta_j) k_{j,t}^i - (1 + r_{j,t}) d_{j,t-1}^i + d_{j,t}^i = c_{j,t}^i + z_{j,t}^i$, where $\delta_j \in (0,1)$ is the depreciation rate. By doing so, the autonomous dynamic system given in Section 3 becomes slightly more complex. However, the unique existence of the steady-state equilibrium is shown in the same way as the model of perfect depreciation. Furthermore, the comparative statics of the steady state are qualitatively the same and hence the main results presented in this paper, summarized in Propositions 1–3, are also obtained in this case. See the separate appendix of the paper (not intended for publication).

is continuous and its upper limit $\overline{\theta}$ is sufficiently large. At the end of each period, each entrepreneur draws a new productivity from the time-invariant distribution, which is captured by $G_j(\theta) \equiv \text{Prob}\left(\theta^i_{j,t} \leq \theta \mid j\right)$ and the corresponding density function $g_j(\theta) = dG_j(\theta)/d\theta$. Thus, $\theta^i_{j,t}$ is independent and identically distributed not only across agents but also over periods. The shape of the distribution can be country-specific.

The budget constraint is

$$q_{j,t}k^i_{j,t} - (1 + r_{j,t})d^i_{j,t-1} + d^i_{j,t} = c^i_{j,t} + z^i_{j,t},$$

where $r_{j,t}$ is the interest rate and $d^i_{j,t}$ is the end-of-period stock of the one-period bonds issued by entrepreneur i_j (i.e., his/her debt). In this model, each entrepreneur faces the following credit constraint:

$$d^i_{j,t} \leq \pi_{j,t}z^i_{j,t},$$

where $\pi_{j,t} \in [0, 1]$. Such a formulation is analytically convenient to capture credit market imperfections. It states that at most a proportion $\pi_{j,t}$ of investment can be externally financed. By varying $\pi_{j,t}$, we can trace out all degrees of financial frictions: $\pi_{j,t} = 1$ corresponds to a perfect credit market and $\pi_{j,t} = 0$ to the case where there is no financial market.

Let us introduce the following new variables:

$$\begin{aligned}
m^i_{j,t} &\equiv q_{j,t}k^i_{j,t} - (1 + r_{j,t})d^i_{j,t-1}, \\
a^i_{j,t} &\equiv z^i_{j,t} - d^i_{j,t}, \\
\lambda_{j,t} &\equiv \pi_{j,t}/(1 - \pi_{j,t}) \in [0, \infty).
\end{aligned}$$

In short, $m^i_{j,t}$ is an entrepreneur's net income flow and $a^i_{j,t}$ is his/her own funds (or cash-on-hand) for capital investment. By using these variables, the budget and credit constraints are respectively simplified to $m^i_{j,t} = c^i_{j,t} + a^i_{j,t}$ and $d^i_{j,t} \leq \lambda_{j,t}a^i_{j,t}$, the latter of which is equivalent to

$$z^i_{j,t} \leq (1 + \lambda_{j,t})a^i_{j,t}.$$

Namely, $\lambda_{j,t}$ captures the leverage ratio.

We are now ready to describe the optimization problem of an entrepreneur. Following Buera and Moll (2015), we assume that each entrepreneur can decide $z^i_{j,t}$ and $d^i_{j,t}$ after observing his/her investment efficiency $\theta^i_{j,t}$. Thus, both $m^i_{j,t}$ and $\theta^i_{j,t}$ are state variables in period t. This means that the optimization problem contains both static and dynamic maximization problems, solved stage by stage. In the first stage, each entrepreneur decides his/her consumption and cash-on-hand, motivated by the intertemporal consumption smoothing. This problem is recursively described as the following Bellman equation:

$$V_{j,t}(m,\theta) = \max_a \left\{ \log(m-a) + \beta_j \int_{\underline{\theta}}^{\overline{\theta}} V_{j,t+1}(m_{j,t+1}(a,\theta),\theta')dG_j(\theta') \right\}, \tag{3}$$

where $V_{j,t}(m,\theta)$ is the value function in period t and $m_{j,t+1}(a,\theta)$ is the maximized net income flow in the next period, defined as the solution to the following second-stage static optimization problem:

$$m_{j,t+1}(a,\theta) = \max_z \left\{ q_{j,t+1}\theta z - (1+r_{j,t+1})(z-a) \mid 0 \le z \le (1+\lambda_{j,t})a \right\}. \tag{4}$$

In other words, in the second stage, the entrepreneur decides his/her investment by taking his/her own funds in period t as given.

We can solve these problems in a backward manner from the second stage. By solving problem (4), the optimal investment and borrowing are given by

$$(z_{j,t}(a,\theta), d_{j,t}(a,\theta)) = \begin{cases} (0,\ -a) & \text{if } \theta < \theta^c_{j,t} \\ ((1+\lambda_{j,t})a,\ -\lambda_{j,t}a) & \text{if } \theta \ge \theta^c_{j,t}, \end{cases} \tag{5}$$

where $\theta^c_{j,t}$ is the cutoff efficiency of investment, defined as

$$\theta^c_{j,t} = \frac{1+r_{j,t+1}}{q_{j,t+1}}.$$

(5) suggests that the credit constraint is necessarily binding when an entrepreneur is actively investing. When he/she is not, he/she lends all financial funds to other active entrepreneurs. From (4) and (5), the maximized net income flow is given by

$$\begin{aligned} m_{j,t+1}(a,\theta) &= R_{j,t+1}(\theta)a, \\ R_{j,t+1}(\theta) &\equiv \begin{cases} 1+r_{j,t+1} & \text{if } \theta < \theta^c_{j,t} \\ (1+\lambda_{j,t})q_{j,t+1}\theta - \lambda_{j,t}(1+r_{j,t+1}) & \text{if } \theta \ge \theta^c_{j,t}. \end{cases} \end{aligned}$$

Now, we turn to the intertemporal optimization problem. The first-order condition of problem (3) is

$$\frac{1}{m-a} = \beta_j E_t \left[\frac{\partial m_{j,t+1}(a,\theta)}{\partial a} \frac{\partial V_{j,t+1}(m',\theta')}{\partial m'} \right].$$

Since there is no aggregate uncertainty, $\frac{\partial m_{j,t+1}(a,\theta)}{\partial a} = R_{j,t+1}(\theta)$ is not stochastic. Then,

$$\frac{1}{m-a} = \beta_j R_{j,t+1}(\theta) E_t \left[\frac{\partial V_{j,t+1}(m',\theta')}{\partial m'} \right]. \tag{6}$$

Appendix A shows that a is given by $a = \beta_j m$.

Let $A_{j,t} = \int_0^1 a^i_{j,t} di$. Hereafter, we refer to $A_{j,t}$ as the "aggregate wealth" in country j, since an entrepreneur's cash-on-hand $a^i_{j,t}$ is at the same time his/her net worth at the end of a period. Since net income $m^i_{j,t}$ has already been determined when the value of $\theta^i_{j,t}$ is realized, $a^i_{j,t}(=\beta_j m^i_{j,t})$ is independent of $\theta^i_{j,t}$. Therefore, $A_{j,t}$ is expressed as $\int a dF_{j,t}(a)$, where $F_{j,t}$ is the resulting distribution of $a^i_{j,t}$. Furthermore, since θ is iid across agents, no information on $F_{j,t}$ is required to obtain the aggregate values. (5) provides the aggregate investment $Z_{j,t}$ as

$$\begin{aligned} Z_{j,t} &\equiv \int \int_{\underline{\theta}}^{\overline{\theta}} z_{j,t}(a,\theta) dG_j(\theta) dF_{j,t}(a) \\ &= (1+\lambda_{j,t}) A_{j,t} (1 - G_j(\theta^c_{j,t})). \end{aligned}$$

Accordingly, the resulting amount of aggregate capital $K_{j,t+1}$ is given by $\mu_j(\theta^c_{j,t}) Z_{j,t}$, where $\mu_j(\cdot)$ is a tail-conditional average of θ:

$$\mu_j(\theta^c_{j,t}) \equiv (1 - G_j(\theta^c_{j,t}))^{-1} \int_{\theta^c_{j,t}}^{\overline{\theta}} \theta dG_j(\theta).$$

This captures the average productivity of the aggregate investment. It is easily shown that $d\mu_j(\theta^c_{j,t})/d\theta^c_{j,t} > 0$. Finally, the aggregate wealth in the next period $A_{j,t+1}$ is given by

$$\begin{aligned} A_{j,t+1} &\equiv \beta_j \int \int_{\underline{\theta}}^{\overline{\theta}} m_{j,t+1}(a,\theta) dG_j(\theta) dF_{j,t}(a) \\ &= \beta_j \Big[q_{j,t+1} K_{j,t+1} + (1 + r_{j,t+1}) B_{j,t} \Big], \end{aligned} \tag{7}$$

where

$$B_{j,t} \equiv A_{j,t} - Z_{j,t} = [1 - (1+\lambda_{j,t})(1 - G_j(\theta^c_{j,t}))] A_{j,t}.$$

That is, $B_{j,t}$ is the domestic excess supply of financial funds. In other words, $-B_{j,t}$ is the net supply of the one-period bonds issued by the entrepreneurs. The aggregate consumption of entrepreneurs, denoted by $C^E_{j,t}$, is obtained as $(1-\beta_j) A_{j,t}/\beta_j$.

2.3 Workers

Each worker is endowed with one unit of labor and he/she inelastically supplies it in each period to earn the wage rate, $w_{j,t}$. Following Buera and Moll (2015), the workers do not have investment opportunities or cannot borrow/save (i.e., they are hand-to-mouth consumers):

$$C^W_{j,t} = w_{j,t} L_j,$$

where $C^W_{j,t}$ is the aggregate consumption of workers in country j.

2.4 Market-clearing Conditions

The model is closed by the market-clearing conditions. These conditions for capital and labor in country j are respectively given by

$$K_{j,t} = \int_{\Omega_{j,t}} K_{j,t}(\omega) d\omega, \tag{8}$$

$$L_j = \int_{\Omega_{j,t}} L_{j,t}(\omega) d\omega. \tag{9}$$

The market-clearing condition of the intermediate goods is given by $X_{j,t}(\omega) = x_{H,t}(\omega) + x_{F,t}(\omega)$ for all $\omega \in \Omega_{j,t}$. In other words,

$$x_{H,t}(\omega) + x_{F,t}(\omega) = \begin{cases} X_{H,t}(\omega) & \text{for } \omega \in \Omega_{H,t}, \\ X_{F,t}(\omega) & \text{for } \omega \in \Omega_{F,t}. \end{cases} \tag{10}$$

Without international financial transactions, the bonds are in zero net supply:

$$B_{j,t} = 0. \tag{11}$$

Finally, the market-clearing condition for the final good in each country is

$$Y_{j,t} = C^E_{j,t} + C^W_{j,t} + Z_{j,t}. \tag{12}$$

From (8)–(12) with the firms' zero profit conditions and aggregate budget constraint,[7] the following trade balance automatically implies from Walras' law: $\int_{\omega^c_t}^1 p_t(\omega) x_{H,t}(\omega) d\omega = \int_0^{\omega^c_t} p_t(\omega) x_{F,t}(\omega) d\omega$.

3 Analytical Characterization of the Equilibrium

In this section, we analytically characterize the equilibrium. The market-clearing condition of the bonds (11) shows that $Z_{j,t} = A_{j,t}$. From the definition of $Z_{j,t}$, this is rewritten as

$$1 = (1 + \lambda_{j,t})(1 - G_j(\theta^c_{j,t})). \tag{13}$$

Given $\lambda_{j,t}$, (13) has the unique interior solution for $\theta^c_{j,t}$. This is hereafter denoted by $\Theta^c_j(\lambda_{j,t})$:

$$\frac{d\Theta^c_j(\lambda_{j,t})}{d\lambda_{j,t}} = \frac{1 - G_j(\cdot)}{(1 + \lambda_{j,t}) g_j(\cdot)} > 0.$$

Since $Z_{j,t} = A_{j,t}$, the level of capital in the next period is $K_{j,t+1} = \Gamma_j(\lambda_j) A_{j,t}$, where $\Gamma_j(\lambda_{j,t}) \equiv \mu_j(\Theta^c_j(\lambda_{j,t}))$ is the average productivity of the aggregate investment in equilibrium:

$$\frac{d\Gamma_j(\lambda_{j,t})}{d\lambda_{j,t}} = (1 - G_j(\cdot)) \left(\mu_j(\Theta^c_j) - \Theta^c_j \right) > 0.$$

[7] The aggregate budget constraint in country j is given by $A_{j,t} = q_{j,t} K_{j,t} + (1 + r_{j,t}) B_{j,t-1} + w_{j,t} L_j - C^E_{j,t} - C^W_{j,t}$.

The dynamic equation of the aggregate wealth is therefore given by $A_{j,t+1} = \beta_j q_{j+1}\Gamma_j(\lambda_{j,t})A_{j,t}$.

We next define the following new variables:

$$\widetilde{A}_t \equiv A_{H,t}/A_{F,t}, \quad \widetilde{q}_t \equiv q_{H,t}/q_{F,t}, \quad \widetilde{w}_t \equiv w_{H,t}/w_{F,t}.$$

We refer to $\widetilde{A}_t$ as relative wealth in the home country to that in the foreign country and $(\widetilde{q}_t, \widetilde{w}_t)$ as the factoral terms of trade in the home country. Appendix B shows that the autonomous system of equilibrium dynamics is given by

$$\widetilde{A}_{t+1} = \frac{\beta_H \Gamma_H(\lambda_{H,t})}{\beta_F \Gamma_F(\lambda_{F,t})} \widetilde{q}_{t+1} \widetilde{A}_t, \tag{14}$$

$$\widetilde{w}_{t+1} = \Pi_1(\omega^c_{t+1}, \widetilde{q}_{t+1}) \equiv \widetilde{q}_{t+1}^{\,-\alpha/(1-\alpha)} \left(\frac{\psi_F(\omega^c_{t+1})}{\psi_H(\omega^c_{t+1})} \right)^{1/(1-\alpha)}, \tag{15}$$

$$\widetilde{w}_{t+1} = \Pi_2(\omega^c_{t+1}, \widetilde{q}_{t+1}) \equiv \left(\widetilde{q}_{t+1}^{\,\alpha(1-\sigma)} \frac{L_F}{L_H} \frac{\int_0^{\omega^c_{t+1}} \psi_H(\omega)^{1-\sigma} d\omega}{\int_{\omega^c_{t+1}}^1 \psi_F(\omega)^{1-\sigma} d\omega} \right)^{\frac{1}{1+(1-\alpha)(\sigma-1)}}, \tag{16}$$

$$\widetilde{A}_{t+1} = \frac{\Gamma_F(\lambda_{F,t})}{\Gamma_H(\lambda_{H,t})} \frac{\widetilde{w}_{t+1}}{\widetilde{q}_{t+1}} \frac{L_H}{L_F}. \tag{17}$$

This system consists of the dynamic equation (14) and the static system of equations (15)–(17). (14) is the dynamic equation of relative wealth. This equation shows that given the endogenous variables $\widetilde{A}_t$ and $\widetilde{q}_{t+1}$, relative wealth in the next period depends on the following three kinds of international asymmetries: (i) asymmetry in the entrepreneurs' discount factor (β_j); (ii) asymmetry in the degree of financial frictions $(\lambda_{j,t})$; and (iii) asymmetry in the distribution of θ $(G_j(\cdot))$.

Given $\widetilde{A}_t$, the static system (15)–(17) determines the factoral terms of trade $(\widetilde{q}_{t+1}, \widetilde{w}_{t+1})$ and the cutoff variety ω^c_{t+1}. (15) is the equation that defines ω^c_{t+1}, and (16) is obtained from the labor market equilibrium in both countries. In this model, the capital market equilibrium in each country is given by $K_{j,t+1} = \alpha w_{j,t+1} L_j / [(1-\alpha) q_{j,t+1}]$. (17) is then obtained from this condition considering that $K_{j,t+1}$ is given by $K_{j,t+1} = \Gamma_j(\lambda_{j,t})A_{j,t}$. As in a standard continuum-good Ricardian model, (15) and (16) determine the relative wage and cutoff variety.[8]

Lemma 1. *Given $\widetilde{q}_{t+1} > 0$, there uniquely exists the pair $(\widetilde{w}_{t+1}, \omega^c_{t+1})$, which solves (15) and (16), where ω^c_{t+1} is in the interior of $[0,1]$.*

Proof. Note that Π_1 is a decreasing function of ω^c. By contrast, since $1 + (1-\alpha)(\sigma - 1) > 0$, Π_2 is an increasing function of ω^c with $\Pi_2(0, q) = 0$ and $\Pi_2(1, q) = \infty$. This implies that there uniquely exists $\omega^c_{t+1} \in (0,1)$ that solves $\Pi_1(\omega^c_{t+1}, \widetilde{q}_{t+1}) = \Pi_2(\omega^c_{t+1}, \widetilde{q}_{t+1})$. □

[8]Thus, these two equations play the same role as the corresponding equations in Dornbusch et al. (1977). To see why, consider the special case of $\alpha \to 0$ and $\sigma \to 1$. If this is the case, (15) and (16) respectively become $\widetilde{w}_{t+1} = \psi_F(\omega^c_{t+1})/\psi_H(\omega^c_{t+1})$ and $\widetilde{w}_{t+1} = \omega^c_{t+1} L_F / [(1-\omega^c_{t+1}) L_H]$, which appear in Dornbusch et al. (1977).

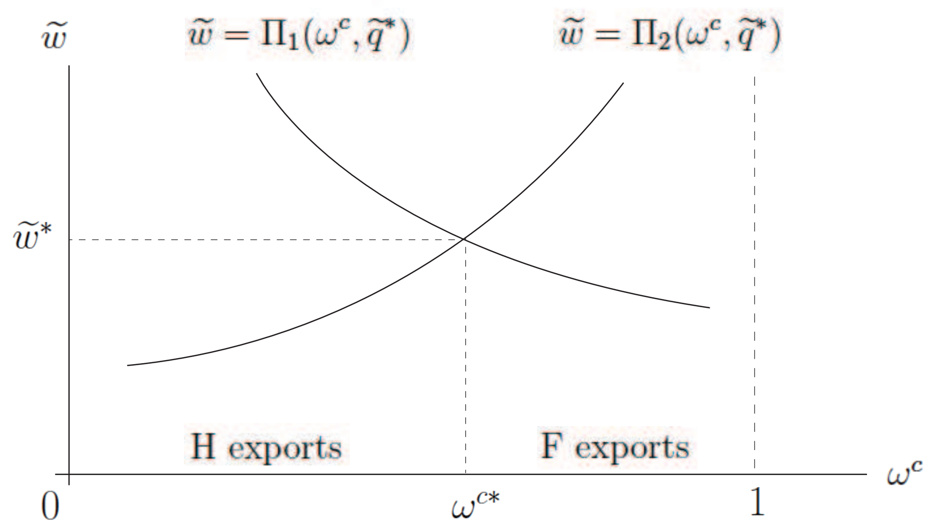

Figure 1: Patterns of Trade in the Steady-State Equilibrium

This lemma ensures that $\widetilde{w}_{t+1}$ and ω^c_{t+1} are given by the functions of $\widetilde{q}_{t+1}$. From this result and (17), $\widetilde{q}_{t+1}$ is given by the function of $\widetilde{A}_t$. Substituting this result into (14) yields the autonomous dynamic equation of $\widetilde{A}_{t+1}$. In the initial period, the net income flow of each agent, $m^i_{j,0}$ is historically given. Since $a^i_{j,0} = \beta_j m^i_{j,0}$, its aggregate value $A_{j,0}$ is also shown to be historically given. Thus, given the initial condition $\widetilde{A}_0 (\equiv A_{H,0}/A_{F,0})$ and the exogenous sequences of $\{\lambda_{j,t}\}$, (14)–(17) jointly constitute the autonomous dynamic system of the model.

Consider the steady-state equilibrium where $\lambda_{j,t}$ is given by an exogenous constant:

$$\lambda_{j,t} = \lambda_j > 0.$$

From (14) with $\widetilde{A}_{t+1} = \widetilde{A}_t$, the steady-state value of $\widetilde{q}_t$ is readily given by $\widetilde{q}^* = \frac{\beta_F \Gamma_F(\lambda_F)}{\beta_H \Gamma_H(\lambda_H)}$, where a superscript asterisk represents the steady-state equilibrium. Lemma 1 ensures that (15) and (16) provide $\widetilde{w}^*$ and ω^{c*}. The steady state of relative wealth, $\widetilde{A}^*$, is then uniquely determined from (17). Accordingly, all the other variables can be determined by substituting $(\widetilde{A}^*, \widetilde{q}^*, \widetilde{w}^*, \omega^{c*})$ back into the appropriate equations. For instance, the steady-state value of the wage rate in the foreign country w^*_F is determined from the following equation that comes from (2):

$$1 = {q^*_F}^{\alpha} {w^*_F}^{1-\alpha} \left[\left(\widetilde{q}^{*\alpha} \widetilde{w}^{*1-\alpha} \right)^{1-\sigma} \int_0^{\omega^{c*}} \psi_H(\omega)^{1-\sigma} d\omega + \int_{\omega^{c*}}^1 \psi_F(\omega)^{1-\sigma} d\omega \right]^{1/(1-\sigma)}, \qquad (18)$$

where $q^*_j = 1/(\beta_j \Gamma_j(\lambda_j))$.

Figure 1 depicts how $\widetilde{w}^*$ and ω^{c*} are determined in the steady-state equilibrium, which thus graphically shows the range of exporting varieties in each country. (15) is depicted as a downward sloping curve since a higher $\widetilde{w}_{t+1}$ raises the marginal cost of the intermediate good firms in the home country relative to those in the foreign country. This results in the decrease in the varieties exported in this country. By contrast, since $\sigma > 1$ is assumed, (16) is depicted as an upward sloping curve. It is upward sloping since a higher ω_{t+1}^c implies that more varieties are produced in the home country and this increases demand for labor in this country. Thus, the wage rate in the home country relative to that in the foreign country goes up. Note that a similar figure is used to characterize the equilibrium patterns of trade in static continuum-good Ricardian models such as Dornbusch et al. (1977).

However, there is an important difference between our model and static models in the following respect that in our model the location of each curve is affected by capital prices q_j^*. This means that the international difference in capital prices can affect the patterns of trade in intermediate goods. Moreover, such a difference is generated from the aforementioned international asymmetries. If there is no international asymmetry, in other words, if $\beta_H = \beta_F$, $\lambda_H = \lambda_F$, and $G_H(\cdot) = G_F(\cdot)$, then $q_H^* = q_F^*$ (i.e., $\widetilde{q}^* = 1$) holds and the patterns of trade eventually depend solely on the technological factors. By contrast, if there are international asymmetries in some respects, $\widetilde{q}^*$ can deviate from unity and hence such asymmetries can influence the patterns of trade in the steady-state equilibrium. Thus, each asymmetry can work as a source of comparative advantage. In the next section, we therefore pursue how the tightening of credit constraints in one country affects the equilibrium patterns of trade.

We now turn to the stability of the steady state. In Appendix C, a log-linear approximation of the system (14)–(17) is given, and it is shown that the steady state $\widetilde{A}^*$ is locally stable as long as the exogenous sequence $\{\lambda_{j,t}\}$ monotonically converges to λ_j.

Lemma 2. *The steady state of the economy is unique and locally stable.*

Proof. See Appendix C. □

4 Qualitative Nature of the Steady-State Equilibrium

In this section, to understand the qualitative nature of the steady-state equilibrium, we examine how a permanent credit crunch (i.e., the tightening of credit constraints) in one country (i.e., $d\lambda_j < 0$, $d\lambda_n = 0$) affects it and its partner countries in the long run.

As shown in the previous section, the cutoff productivity of investment in country j, $\Theta_j^c(\lambda_j)$, depends only on the degree of domestic financial frictions and shape of the distribution in this country. Since the capital price at the steady state is $q_j^* = 1/(\beta_j \Gamma_j(\lambda_j))$, it immediately follows that $dq_j^*/q_j^* = -\xi_j d\lambda_j/\lambda_j$, where $\xi_j \equiv \lambda_j \Gamma_j'(\lambda_j)/\Gamma_j(\lambda_j) > 0$. Thus, in the long run, a credit crunch in one country raises the capital price only in that country. The intuition is explained as follows. A credit crunch decreases the borrowing capacity of entrepreneurs, which induces the entry of less productive entrepreneurs who otherwise become inactive investors. Such an inefficient reallocation of financial resources lowers the average productivity of the aggregate investment. Consequently, the supply of capital decreases and its price rises.

From (15) and (16), we obtain

$$\frac{d\omega^{c*}}{\omega^{c*}} = -\frac{\alpha}{\zeta^*}\frac{d\widetilde{q}^*}{\widetilde{q}^*}, \tag{19}$$

$$\frac{d\widetilde{w}^*}{\widetilde{w}^*} = -\frac{\alpha[(\sigma-1)\phi^* + \varepsilon^*]}{\zeta^*}\frac{d\widetilde{q}^*}{\widetilde{q}^*}, \tag{20}$$

where $d\widetilde{q}^*/\widetilde{q}^* \equiv -\left(\xi_H d\lambda_H/\lambda_H - \xi_F d\lambda_F/\lambda_F\right)$. In these equations, ϕ^*, ε^* and ζ^* are defined in Appendix C as

$$\phi^* \equiv \frac{\omega^{c*}\psi_H'(\omega^{c*})}{\psi_H(\omega^{c*})} - \frac{\omega^{c*}\psi_F'(\omega^{c*})}{\psi_F(\omega^{c*})} > 0,$$

$$\varepsilon^* \equiv \varepsilon_H^* + \varepsilon_F^* > 0; \quad \varepsilon_j^* \equiv \frac{\omega^{c*}\psi_j(\omega^{c*})^{1-\sigma}}{\int_{\Omega_j^*}\psi_j(\omega)^{1-\sigma}d\omega} > 0,$$

$$\zeta^* \equiv [1 + (\sigma-1)(1-\alpha)]\phi^* + (1-\alpha)\varepsilon^* > 0.$$

Since $d\widetilde{q}^*/\widetilde{q}^*$ is positive (negative) when the credit crunch occurs in the home (foreign) country, (19) and (20) show the following proposition.

Proposition 1. *Consider the steady-state equilibrium. A permanent credit crunch in one country triggers*

1. *A decrease (increase) in the extensive margin of exports in its (the partner) country; and*
2. *A decrease in its wage relative to the partner country.*

This proposition, especially its first claim, has the following two clear-cut implications. First, a change in the degree of financial frictions alters the comparative advantage in the steady-state equilibrium. In this dynamic model, capital is no longer an exogenous endowment, but rather it changes over time. Thus, the steady-state level of capital is endogenously determined and this level influences labor productivity. As shown above, the capital price increases in the country that directly experiences a credit crunch. This fact implies that labor supply in this country becomes relatively abandoned, the marginal productivity of which then declines.

Second, this proposition suggests that following a credit crunch in one country, its effects can propagate to the partner country through international trade. First, the following proposition shows that wages fall in both countries from a unilateral credit crunch.

Proposition 2. *Consider the steady-state equilibrium. A permanent credit crunch in one country decreases the wages in both countries.*

Proof. See Appendix D. □

In the process of the proof, we can obtain the following formula, showing the decomposition of the effect on the wage in each country:[9]

$$\frac{dw_H^*}{w_H^*} = \frac{1}{1-\alpha}\left[\alpha\xi_H\frac{d\lambda_H}{\lambda_H} - \kappa_H^*\phi^*\frac{d\omega^{c*}}{\omega^{c*}}\right],$$
$$\frac{dw_F^*}{w_F^*} = \frac{1}{1-\alpha}\left[\alpha\xi_F\frac{d\lambda_F}{\lambda_F} + \kappa_F^*\phi^*\frac{d\omega^{c*}}{\omega^{c*}}\right],$$

where $\kappa_j^* \in (0,1)$ is the long-run import share of the final good firms in country j: $\kappa_j^* \equiv (q_n^{*\alpha}w_n^{*1-\alpha})^{1-\sigma}\int_{\Omega_n^*}\psi_n(\omega)^{1-\sigma}d\omega \in (0,1)$, where $j, n(\neq j) \in \{H, F\}$.[10] In each equation, the first term in the brackets represents the direct effect of a credit crunch and the second term captures the effect induced by the trade in intermediate goods. Note that while the former effect appears only in the country that directly experiences the credit crunch, the latter effect always exists in both countries, and from Proposition 1 it can be positive or negative depending on the location of the triggering event. For instance, suppose that a credit crunch occurs in the home country, i.e., $d\lambda_H < 0$ and $d\lambda_F = 0$. If this is the case, $dq_H^*/q_H^* > 0$, $dq_F^*/q_F^* = 0$, and $d\omega^{c*}/\omega^{c*} < 0$. From the above formula, we can easily verify $dw_F^*/w_F^* < 0$. By contrast, in the home country, the indirect effect is competing with the direct effect. Appendix D shows that the direct effect always dominates. Note that despite such a competing effect, the wage rate in the home country declines more sharply since in this case $d\widetilde{w}^*/\widetilde{w}^* < 0$ from the second claim of Proposition 1.

Such an internationally synchronized change in wages in turn plays an important role in the synchronization of the other macroeconomic variables. The amount of capital at the steady state is given by $K_j^* = \alpha w_j^* L_j/[(1-\alpha)q_j^*]$. From Proposition 2, w_j^*/q_j^* drops in both countries. This finding implies that the long-run level of capital decreases in both countries. Furthermore, Proposition 2 derives the following results.

Proposition 3. *Consider the steady-state equilibrium. A permanent credit crunch in one country decreases the long-run levels of aggregate investment, entrepreneurs' income, and GDP in both countries.*

[9] Indeed, these equations correspond to (33) and (34) in Appendix D.

[10] $\kappa_{j,t}$ is primarily defined as $\kappa_{j,t} \equiv \int_{\Omega_{n,t}} p_t(\omega)x_{j,t}(\omega)d\omega/Y_{j,t}$. From $x_{j,t}(\omega) = p_t(\omega)^{-\sigma}Y_{j,t}$ and the fact that $p_t(\omega) = \psi_n(\omega)q_{n,t}^{\alpha}w_{n,t}^{1-\alpha}$ for all $\omega \in \Omega_{n,t}$, we obtain $\kappa_{j,t} = \left(q_{n,t}^{\alpha}w_{n,t}^{1-\alpha}\right)^{1-\sigma}\int_{\Omega_{n,t}}\psi_n(\omega)^{1-\sigma}d\omega$. From (2), we can verify that $\sum_j \kappa_{j,t} = 1$.

Proof. Recall that $Z_j^* = A_j^*$. Since $K_{j,t+1}$ is given by $\Gamma_j(\lambda_j)A_{j,t}$, $A_j^* = K_j^*/\Gamma_j(\lambda_j)$ holds in the steady-state equilibrium. Then, the rate of change in aggregate investment is

$$\begin{aligned}\frac{dZ_j^*}{Z_j^*} &= \frac{dK_j^*}{K_j^*} - \xi_j \frac{d\lambda_j}{\lambda_j} \\ &= \frac{dw_j^*}{w_j^*},\end{aligned}$$

where we use $dq_j^*/q_j^* = -\xi_j d\lambda_j/\lambda_j$. From Proposition 2, the above equation implies that aggregate investment decreases in both countries. Since the aggregate income of entrepreneurs is given by $q_j^* K_j^* = \alpha w_j^* L_j/(1-\alpha)$, we can easily verify that the aggregate income of entrepreneurs decreases in both countries.[11]

Finally, long-run GDP is given by $\int_{\Omega_j^*} p_t(\omega)X_{j,t}(\omega)d\omega$. From the zero-profit conditions of the intermediate good firms and market-clearing conditions (8) and (9), this is reduced to $w_j^* L_j/(1-\alpha)$. □

All the results in this section are obtained without relying on the specification of the distribution function or assuming symmetry across the two countries.

Finally, to understand the qualitative nature of the equilibrium more in depth, it is helpful to examine what happens if the trade patterns are exogenously given. If this is the case, (15) disappears and the relative wage at the steady state is determined solely from (16) with $\widetilde{q}^*$ and ω^{c*} given. Then, the equation for its rate of change (20) is replaced by

$$\left.\frac{d\widetilde{w}^*}{\widetilde{w}^*}\right|_{d\omega^{c*}=0} = \frac{-\alpha(\sigma-1)}{1+(\sigma-1)(1-\alpha)}\frac{d\widetilde{q}^*}{\widetilde{q}^*},$$

where $d\widetilde{q}^*/\widetilde{q}^* \equiv -(\xi_H d\lambda_H/\lambda_H - \xi_F d\lambda_F/\lambda_F)$ is the same as before. The steady-state wage in the foreign country is determined from (18). The logarithmic differentiation of this equation under the hypothetical situation that $d\omega^{c*}=0$ yields

$$\begin{aligned}\left.\frac{dw_F^*}{w_F^*}\right|_{d\omega^{c*}=0} &= -\frac{1}{1-\alpha}\left[\alpha\frac{dq_F^*}{q_F^*} + \kappa_F^*\left(\alpha\frac{d\widetilde{q}^*}{\widetilde{q}^*} + (1-\alpha)\left.\frac{d\widetilde{w}^*}{\widetilde{w}^*}\right|_{d\omega^{c*}=0}\right)\right] \\ &= -\frac{1}{1-\alpha}\left[\alpha\frac{dq_F^*}{q_F^*} + \kappa_F^*\frac{\alpha}{1+(\sigma-1)(1-\alpha)}\frac{d\widetilde{q}^*}{\widetilde{q}^*}\right].\end{aligned}$$

Consider the situation that the credit crunch occurs in the home country: $d\lambda_H < 0$ and $d\lambda_F = 0$. Then, the above equation shows

$$\left.\frac{dw_F^*}{w_F^*}\right|_{d\omega^{c*}=0} = -\frac{\alpha\kappa_F^*}{1-\alpha}\frac{\xi_H}{1+(\sigma-1)(1-\alpha)}\frac{d\lambda_H}{\lambda_H}.$$

[11]From the aggregate income, $q_j^* K_j^* - (1+r_j^*)\lambda_j[1-G_j(\Theta_j^c(\lambda_j))]A_j^*$ is distributed to agents who invest, whereas $(1+r_j^*)G_j((\Theta_j^c(\lambda_j)))A_j^*$ is distributed to agents who lend, where $r_j^* \equiv q_j^*\Theta_j^c(\lambda_j) - 1$.

By using (19), the rate of actual change for w_F^* in this model is given by

$$\frac{dw_F^*}{w_F^*}\Big|_{d\omega^{c*}=0} = -\frac{\alpha\kappa_F^*}{1-\alpha}\frac{\phi^*\xi_H}{\zeta^*}\frac{d\lambda_H}{\lambda_H}.$$

Lemma 3. *The degree of the reduction in the wage rate induced by the international transmission is smaller when trade patterns are endogenously determined.*

Proof. From the definition of ζ^* in Appendix C, $\zeta^* \equiv [1 + (\sigma - 1)(1 - \alpha)]\phi^* + (1 - \alpha)\varepsilon^*$. This is strictly larger than $[1 + (\sigma - 1)(1 - \alpha)]\phi^*$ because $\alpha \in (0, 1)$ and $\varepsilon^* > 0$. Then,

$$\frac{\phi^*}{\zeta^*} < \frac{1}{1 + (\sigma - 1)(1 - \alpha)},$$

which implies that the absolute value of $dw_F^*/w_F^*|_{d\omega^{c*}=0}$ is strictly larger than that of dw_F^*/w_F^*. □

This lemma shows that the change in trade patterns itself acts as a buffer against the international transmission of shocks. To grasp its intuition, suppose that a credit crunch occurs in the home country. As already explained, this distorts the financial resources among domestic entrepreneurs and pushes up the capital price in that country. As (18) shows, this places upward pressure on the price index of the intermediate goods. Since the price of the final good is fixed here, such pressure must be offset. Then, the wage in the foreign country eventually declines. When trade patterns are endogenously determined, however, the rise in the capital price simultaneously changes the equilibrium trade patterns. The firms in the home country with a smaller comparative advantage are replaced by foreign ones. Consequently, the upward pressure on the price index is mitigated by such an international production reallocation.

4.1 Welfare Implications

At this point, we briefly discuss the implications for entrepreneurs' expected utilities in each country. For analytical convenience, suppose that the economy has arrived at its steady-state equilibrium in period t.

Pick an entrepreneur in country j whose net income is given by m_t. His/her expected utility before drawing θ in that period is given by

$$EV_j^{SS}(\lambda_j|m_t) = \int_{\underline{\theta}}^{\overline{\theta}} V_j^*(m_t, \theta) dG_j(\theta),$$

where $V_j^*(m, \theta)$ is the value function defined by the Bellman equation (3), with the market variables $q_{j,t+1}$ and $r_{j,t+1}$ now given by their steady-state values. Recall that $V_{j,t}$ is the value function evaluated after drawing θ in period t. Therefore, we have to calculate its expected value to obtain the ex-ante value function. As we show in Appendix E, EV_j^{SS} is given by

$$EV_j^{SS}(\lambda_j|m_t) = \frac{1}{1-\beta_j}\left\{\frac{\beta_j}{1-\beta_j}\int_{\underline{\theta}}^{\overline{\theta}} \log\left[R_j^*(\theta)\right] dG_j(\theta) + \log m_t + \frac{\beta_j}{1-\beta_j}\log\beta_j + \log(1-\beta_j)\right\},$$

where

$$R_j^*(\theta) \equiv \begin{cases} q_j^*\theta(1+\lambda_j) - (1+r_j^*)\lambda_j & \text{if } \theta \geq \Theta_j^c(\lambda_j), \\ 1+r_j^* & \text{if } \theta < \Theta_j^c(\lambda_j). \end{cases}$$

Recall that q_j^* or $r_j^*(= q_j^*\Theta_j^c - 1)$ never depends on λ_n $(n \neq j)$. Thus, at least in the long run, their expected utilities in a country are unaffected unless this country directly experiences a credit crunch. By contrast, for the country that experiences the credit crunch, the welfare effect is generally ambiguous.

Lemma 4. *Consider the steady-state equilibrium. Suppose that the entrepreneurs' welfare in each country is defined as an entrepreneur's ex-ante utility. Then, a permanent credit crunch in one country does not affect welfare in the other country, while it harms welfare in that country if*

$$\int \frac{dR_j^*(\theta)/d\lambda_j}{R_j^*(\theta)} dG_j(\theta) > 0,$$

that is, if the expectation of the rate of change in his/her income dR_j^*/R_j^* is negative.

The sign of $R_j^{*-1}(dR_j^*/d\lambda_j)$ depends on θ. For example, assume that θ follows a Pareto distribution: $\overline{\theta} \to \infty$ and G_j is specified as

$$G_j(\theta) = 1 - (\theta/\underline{\theta})^{-\eta_j},$$

where $\eta_j > 1$. From such a specification of G_j, the cutoff $\Theta_j^c(\lambda_{j,t})$ and average productivity of the aggregate investment $\Gamma_j(\lambda_{j,t})$ are respectively given by

$$\Theta_j^c(\lambda_{j,t}) = \underline{\theta}(1+\lambda_{j,t})^{1/\eta_j},$$
$$\Gamma_j(\lambda_{j,t}) = \frac{\eta_j}{\eta_j - 1}\underline{\theta}(1+\lambda_{j,t})^{1/\eta_j} > \Theta_j^c(\lambda_j).$$

Since $q_j^* = 1/(\beta_j\Gamma_j(\lambda_0)))$, the interest rate r_j^* is given by

$$1 + r_j^* \left(= q_j^*\Theta_j^c(\lambda_0)\right) = \frac{\eta_j - 1}{\beta_j\eta_j}. \tag{21}$$

Then, $dR_j^*/d\lambda_j = 0$ if $\theta < \Theta^c(\lambda_j)$, while

$$\begin{aligned}\frac{dR_j^*}{d\lambda_j} &= q_j^*(\theta - \Theta_j^c(\lambda_j)) + \theta(1+\lambda_j)\frac{dq_j^*}{d\lambda_j} \\ &= q_j^*\frac{\eta_j - 1}{\eta_j}\left(\theta - \Gamma_j(\lambda_j)\right),\end{aligned}$$

if $\theta \geq \Theta_j^c(\lambda_j)$. We easily verify $dR_j^*/d\lambda_j > 0$ if $\theta > \Gamma_j(\lambda_j)$, whereas $dR_j^*/d\lambda_j < 0$ if $\Theta_j^c(\lambda_j) < \theta \leq \Gamma_j(\lambda_j)$.

By contrast, from Proposition 2, the workers in both countries always suffer damage from such a unilateral credit crunch as long as their indirect utility is increasing in their wage income.

5 Transitory Responses to a Temporal Credit Crunch

Thus far, we have examined how a permanent shock to financial frictions affects the long-run equilibrium from a qualitative perspective in order to make the characteristics of the model easily understandable. At the same time, however, real-world financial shocks such as credit crunches are thought to be transitory phenomena. It is therefore important to check how macroeconomic variables respond to a temporal shock. In this section, we numerically derive their transitory responses by simply calibrating the parameter values in the model. Since the present model is not prepared for a full-scale quantitative analysis (e.g., full depreciation of capital and absence of trade costs), in some dimensions the model's predictions will depart from reality. The purpose in this section is thus to explore the way in which a credit crunch in one country internationally transmits through trade during the transitions.

5.1 Calibration

Suppose that in period 0, the economy has already arrived at the steady-state equilibrium, where the degree of financial frictions is given by

$$\lambda_{H,0} = \lambda_{F,0} = \lambda_0 > 0.$$

It is hereafter assumed that θ follows a Pareto distribution as at the end of the previous section, and the technology parameter $\psi_j(\omega)$ is specified as $\psi_j(\omega) = \omega^{-\phi_j}$, where $\phi_j \geq 0$, and $\phi \equiv \phi_F - \phi_H > 0$. Under this specification, ϕ^* is always given by the parameter ϕ.

Since the model is stripped down, the parameters are limited (see Table 1). As for β_j and α their values are exogenously chosen as conventional values in standard macroeconomic models. For the elasticity of substitution, we use the value in Arkolakis and Ramanarayanan (2009) as the baseline ($\sigma = 1.5$).[12] Since trade in the intermediate goods plays a key role in the international transmission, we also consider the case of when the intermediate goods are relatively substitutable ($\sigma = 3.8$). The population of workers is normalized to unity in both countries.

[12]Arkolakis and Ramanarayanan in turn borrow this value from the benchmark in Backus et al. (1994).

Table 1: Parameter values

β_j	0.91	Discount factor
α	0.33	Cost share of capital
σ	1.5	Elasticity of substitution among varieties (baseline)
	3.8	— (for comparison)
L_j	1	Population of workers
λ_0	2.226	The initial value of λ
η_j	12.36	Shape parameter of distribution
$\underline{\theta}$	0.836	Scale parameter (Minimum value of θ)
ϕ_H	0	Comparative advantage
ϕ_F	20.28	Comparative advantage

λ_0, η_j, $\underline{\theta}$, and ϕ_j are calibrated in the following way. First, we use the property that the interest rate r_j^* does not depend on λ_j in the steady state as long as G_j is specified as a Pareto distribution (see (21)). Given that, η_j is chosen so that $r_j^* = 0.01$ in both countries. From (21) and $\beta_H = \beta_F = 0.91$, we obtain $\eta_H = \eta_F \simeq 12.36$ for all j. Next, let $D_j^* \equiv \lambda_0 A_j^*/(1+\lambda_0)$ denote the aggregate debt of the investing entrepreneurs. The ratio of that to the aggregate capital is therefore given by

$$\frac{D_j^*}{K_j^*} = \frac{\lambda_0}{(1+\lambda_0)\Gamma(\lambda_0)},$$

where the subscript j of $\Gamma_j(\cdot)$ and η_j is hereafter omitted because $\eta_H = \eta_F$. To obtain the parameter values, it is assumed that

$$\Gamma(\lambda_0) = 1 \Leftrightarrow \frac{\eta}{\eta-1}\underline{\theta}(1+\lambda_0)^{1/\eta} = 1. \tag{22}$$

In other words, at the steady state, the final good is transformed into capital on a one-to-one basis on average as in standard macroeconomic models. The value of λ_0 is then chosen so that $D_j^*/K_j^* = \lambda_0/(1+\lambda_0) = 0.69$. We borrow this value from Buera and Nicolini (2017), who choose this value to match the average ratio of liabilities to non-financial assets for the U.S. non-financial business sector between 1997:Q3 and 2007:Q3. Then, by substituting the resulting values of λ_0 and η into (22), the value of the scale parameter $\underline{\theta}$ is numerically obtained. Finally, for the parameters of comparative advantage ϕ_j, we first assume $\sigma = 1.5$ as the baseline. Given that, $\phi(= \phi_F - \phi_H)$ is chosen so that the import share in the home country κ_H^* is equal to about

0.1 (i.e., Imports are about 10% of GDP in the home country).[13] This is roughly consistent with the actual imports-to-GDP ratio in the United States. The parameter ϕ_H is normalized to zero.

5.2 Transitory Responses

Consider the situation that at the end of period 1, a credit crunch unanticipatedly occurs in the home country: $\lambda_{H,1} < \lambda_0$. It then deterministically recovers over time according to

$$\log \lambda_{H,t+1} = \rho \log \lambda_{H,t} + (1-\rho) \log \lambda_0, \tag{23}$$

where $\rho \in (0,1)$ captures the degree of persistence of this shock. Since $a^i_{H,1}$ is given by $\beta_H m^i_{H,1}$, individual net worth is predetermined before this credit shock occurs. Accordingly, its aggregate value $A_{H,1}(= Z_{H,1})$ is predetermined. Furthermore, from the equilibrium system (14)–(17), all the market variables in period 1 depend only on $A_{j,0}$ and $\lambda_{j,0}$. Thus, in period 1, only the cutoff $\theta^c_{H,1}$ responds to the shock in $\lambda_{H,1}$, and this makes $K_{H,2}(= \Gamma(\lambda_{H,1})A_{H,1})$ deviate from its steady-state level. The world economy then follows the dynamic system now described by (14)–(17) and (23) together with $\lambda_{F,t} = \lambda_0$ for all t. Throughout the paper, the parameter ρ is chosen as 0.90. Given this value, $\lambda_{H,1}$ is assumed to be 1.22, about a 45% decrease from λ_0. This value is chosen so that GDP in the home country decreases by about 1% from peak to trough.

Figures 2 and 3 illustrate the responses of the major variables in the home and foreign countries, respectively. In each panel, the vertical axis is the percentage deviation of the variable from its initial steady state, except for the interest rate (percentage points). The responses are simulated by using the log-linear approximation of the equilibrium system around the initial steady state. Appendix F provides the derivation of the approximated system.

[13] $\kappa_{j,t}$ is primarily defined as the ratio of imports to the value-added of the final good in country j (see footnote 9). However, the following relationship holds:

$$\begin{aligned} Y_{j,t} &= \int_0^1 p_t(\omega) x_{j,t}(\omega) d\omega \\ &= \int_{\Omega_{j,t}} p_t(\omega) x_{j,t}(\omega) d\omega + \int_{\Omega_{n,t}} p_t(\omega) x_{j,t}(\omega) d\omega \\ &= \underbrace{\int_{\Omega_{j,t}} p_t(\omega) X_{j,t}(\omega) d\omega}_{\text{GDP}} + \underbrace{\int_{\Omega_{n,t}} p_t(\omega) x_{j,t}(\omega) d\omega - \int_{\Omega_{j,t}} p_t(\omega) x_{n,t}(\omega) d\omega}_{=0 \ \because \text{trade balance}} \\ &= \text{GDP}_{j,t}. \end{aligned}$$

In this model, $\kappa_{j,t}$ also represents the ratio of imports-to-GDP in country j.

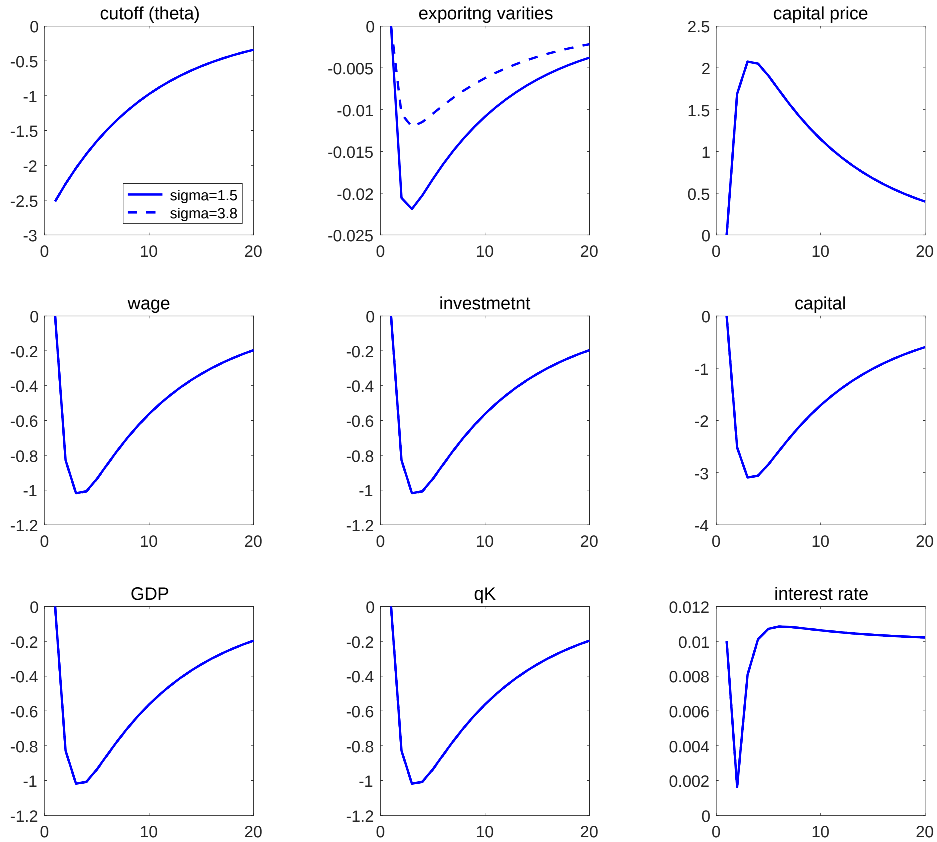

Figure 2: Responses in the home country (domestic responses)

From these figures, we can find several properties of the model. First, the international synchronization observed in the steady-state equilibrium is now also observed during transition periods. From the panels showing the responses of wages, investment, capital, GDP, and entrepreneurs' aggregate income in these figures, we can see that even without international financial transactions, a country-specific credit shock in one country triggers a synchronized international downturn. Recall that workers' aggregate consumption $C^W_{j,t}$ is $w_{j,t}L_j$ and entrepreneurs' aggregate consumption $C^E_{j,t}$ is given by $\beta_j Z_{j,t}/(1-\beta_j)$. Therefore, the fourth and fifth panels in these figures also represent the responses of workers' and entrepreneurs' consumption, respectively.

Second, such an international co-movement is closely related to the change in the equilibrium trade patterns of the intermediate goods. Further, this property has already been verified analytically in the steady-state equilibrium. To see how this property holds during the tran-

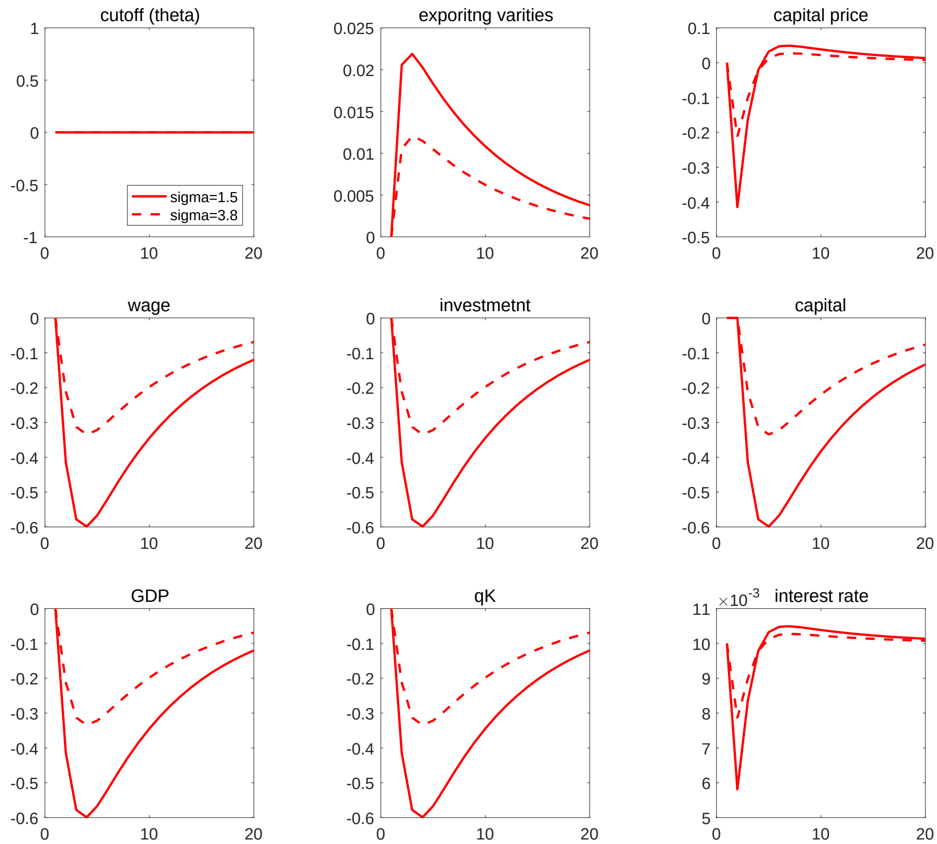

Figure 3: Responses in the foreign country (international transmissions)

sitions, the following equations are helpful, the derivations of which are given in Appendix F:

$$\widehat{A}_{F,t+1} = \widehat{w}_{F,t+1} = \alpha\widehat{A}_{F,t} + \kappa_F^*\phi^*\widehat{\omega}_{t+1}^c,$$
$$\widehat{q}_{F,t+1} = -(1-\alpha)\widehat{A}_{F,t} + \kappa_F^*\phi^*\widehat{\omega}_{t+1}^c.$$

In these equations, a hat over a variable represents the rate of deviation from the initial steady state (e.g., $\widehat{A}_{F,t} = (A_{F,t} - A_F^*)/A_F^*$). The responses of the major variables in the foreign country are derived by using these equations. On the right-hand side of each equation, $-\widehat{\omega}_{t+1}^c > 0$ represents the rate of the increase in the extensive margin of exports in the foreign country. Thus, from the standpoint of the foreign country, a credit shock that occurs in the partner country behaves as if it is a trade shock, meaning that the intermediate good firms in the home country suddenly lose their international competitiveness.

Third, as shown in Figure 3, the degree of international co-movement increases as the intermediate goods become more complementary. As shown in the above equations, the co-movement arises since the variables in the foreign country negatively respond to its increase in the extensive margin of exports. Indeed, as shown in the third and fourth panels in Figure 3, the factor prices in the foreign country fall after the shock. The second panel of Figure 3 shows that the change in ω^c increases if the intermediate goods become more complementary. Thus, if the intermediate goods become more complementary, such a decline in the factor prices becomes more serious and then the movement of the macroeconomic variables in both countries becomes more internationally synchronized. In the baseline case of $\sigma = 1.5$, GDP in the foreign country decreases by roughly 0.6% from peak to trough. This is 60% of the decrease in the home country.

6 Concluding Remarks

This study proposes a simple two-country dynamic general equilibrium framework for studying how financial shocks in one country propagate to the other country through international trade. Many studies that have analyzed the international transmission of financial shocks assume a one-good economy and focus on the transmission mechanisms in various types of financial integration (e.g., integration in bond markets, in equity markets, and in both). By contrast, this study focuses on financial frictions as a determinant of the comparative advantage. It shows that a credit crunch in one country misallocates domestic financial resources to less productive entrepreneurs. On the one hand, this changes the equilibrium trade patterns. On the other hand, this triggers a synchronized international downturn even without international financial transactions. In the case of a permanent credit crunch, these results are analytically obtained without relying on the specification of the distribution function or assuming symmetry across the two countries. In this sense, this study is the first to develop a tractable dynamic general equilibrium model simultaneously incorporating financial frictions, endogenous trade patterns, and asymmetric countries.

To obtain the qualitative results, this study developed a highly stylized model. Therefore, the following two points should be kept in mind. First, we abstract from any trade barriers such as iceberg trade costs and/or import tariffs. Second, we do not incorporate the international integration of equity markets, which plays an important role in the international transmission in existing studies. Thus, introducing these elements into the framework presented in this paper appears to be a promising extension.

Appendix A Derivation of the Policy Function

We can solve problem (3) by using the method of guess and verify. Guess that the value function takes the form $V_{j,t}(m,\theta) = v_{j,t}(\theta) + b\log m$, where $v_{j,t}(\cdot)$ and b are unknown. Since $E_t V_{j,t+1}(m',\theta') = E_t v_{j,t+1}(\theta') + b\log m'$, (6) implies $a = \beta_j bm/(1+\beta_j b)$. Moreover, as $m(a,\theta) = R_{j,t+1}(\theta)a$, the Bellman equation (3) therefore becomes

$$v_{j,t}(\theta) + b\log m = \log\left(\frac{1}{1+\beta_j b}m\right) + \beta_j\left\{E_t v_{j,t+1}(\theta') + b\log\left[R_{j,t+1}(\theta)\left(\frac{\beta_j b}{1+\beta_j b}m\right)\right]\right\}, \quad (24)$$

which gives $b = 1/(1-\beta_j)$. Then, a is given by $a = \beta_j m$, or equivalently $a^i_{j,t} = \beta_j m^i_{j,t}$.

Appendix B Derivations of (14)–(17)

Since the derivations of (14) and (15) are straightforward, here we derive (16) and (17). By applying Shepherd's lemma to the cost function, $mc_{j,t}(\omega)X_{j,t}(\omega)$, we have $K_{j,t}(\omega) = \alpha p_t(\omega)X_{j,t}(\omega)/q_{j,t}$ and $L_{j,t}(\omega) = (1-\alpha)p_t(\omega)X_{j,t}(\omega)/w_{j,t}$ for all $\omega \in \Omega_{j,t}$. By substituting these equations respectively into (8) and (9) and using (1), (10), and $x_{j,t}(\omega) = p_t(\omega)^{-\sigma}Y_{j,t}$, we obtain

$$q_{j,t}K_{j,t} = \alpha\left(q^{\alpha}_{j,t}w^{1-\alpha}_{j,t}\right)^{1-\sigma}(Y_{H,t}+Y_{F,t})\int_{\omega\in\Omega_{j,t}}\psi_j(\omega)^{1-\sigma}d\omega, \quad (25)$$

$$w_{j,t}L_j = (1-\alpha)\left(q^{\alpha}_{j,t}w^{1-\alpha}_{j,t}\right)^{1-\sigma}(Y_{H,t}+Y_{F,t})\int_{\omega\in\Omega_{j,t}}\psi_j(\omega)^{1-\sigma}d\omega. \quad (26)$$

Replace the subscript t with $t+1$. By dividing (26) for $j=H$ by that for $j=F$, we obtain (16). Furthermore, by using (26), (25) is expressed as

$$q_{j,t+1}K_{j,t+1} = \frac{\alpha}{1-\alpha}w_{j,t+1}L_j.$$

By substituting $K_{j,t+1} = \Gamma(\lambda_{j,t})A_{j,t}$ into this equation and following the same procedure, we obtain (17).

Appendix C Proof of Lemma 2

Let $\widehat{x}_t \equiv \log x_t - \log x^* \simeq (x_t - x^*)/x^*$. From (14)–(17), a log-linear approximated system around $(\widetilde{A}^*, \omega^{c*}, \widetilde{w}^*, \widetilde{q}^*)$ is given by

$$\widehat{A}_{t+1} = \widehat{q}_{t+1} + \widehat{A}_t + \xi_H\widehat{\lambda}_{H,t} - \xi_F\widehat{\lambda}_{F,t}, \quad (27)$$

$$-\phi^*\widehat{\omega}^c_{t+1} = \alpha\widehat{q}_{t+1} + (1-\alpha)\widehat{w}_{t+1}, \quad (28)$$

$$\widehat{w}_{t+1} = (1-\sigma)\left[\alpha\widehat{q}_{t+1} + (1-\alpha)\widehat{w}_{t+1}\right] + \varepsilon^*\widehat{\omega}^c_{t+1}, \quad (29)$$

$$\widehat{A}_t = \widehat{w}_{t+1} - \widehat{q}_{t+1} - \left(\xi_H\widehat{\lambda}_{H,t} - \xi_F\widehat{\lambda}_{F,t}\right), \quad (30)$$

where a tilde over the symbols is omitted. ξ_j, ϕ^*, and ε^* are respectively defined as

$$\xi_j \equiv \frac{\lambda_j \Gamma_j'(\lambda_j)}{\Gamma_j(\lambda_j)},$$
$$\phi^* \equiv \frac{\omega^{c*}\psi_H'(\omega^{c*})}{\psi_H(\omega^{c*})} - \frac{\omega^{c*}\psi_F'(\omega^{c*})}{\psi_F(\omega^{c*})} > 0,$$
$$\varepsilon^* \equiv \varepsilon_H^* + \varepsilon_F^* > 0; \quad \varepsilon_j^* \equiv \frac{\omega^{c*}\psi_j(\omega^{c*})^{1-\sigma}}{\int_{\Omega_j^*}\psi_j(\omega)^{1-\sigma}d\omega} > 0.$$

The sign of ϕ^* comes from that $\psi_F(\omega)/\psi_H(\omega)$ is decreasing with respect to ω, and the sign of ε^* comes from Assumption 1. From (28) and (29), we obtain

$$\widehat{\omega}_{t+1}^c = -\frac{\alpha}{\zeta^*}\widehat{q}_{t+1}, \tag{31}$$
$$\widehat{w}_{t+1} = -\frac{\alpha[(\sigma-1)\phi^* + \varepsilon^*]}{\zeta^*}\widehat{q}_{t+1}, \tag{32}$$

where

$$\zeta^* \equiv [1 + (\sigma-1)(1-\alpha)]\phi^* + (1-\alpha)\varepsilon^* > 0.$$

By substituting (32) into (30), we obtain $\widehat{q}_{t+1} = -\nu^*\left(\widehat{A}_t + \xi_H\widehat{\lambda}_{H,t} - \xi_F\widehat{\lambda}_{F,t}\right)$, where

$$\nu^* \equiv \left\{1 + \frac{\alpha[(\sigma-1)\phi^* + \varepsilon^*]}{\zeta^*}\right\}^{-1} \in (0,1).$$

Then, the dynamic equation of $\widetilde{A}_t$ is given by $\widehat{A}_{t+1} = (1-\nu^*)(\widehat{A}_t + \xi_H\widehat{\lambda}_{H,t} - \xi_F\widehat{\lambda}_{F,t})$. Since $(1-\nu^*) \in (0,1)$, this equation shows that given any $\widetilde{A}_0 > 0$, $\widetilde{A}_t$ converges to $\widetilde{A}^*$ as long as $\widehat{\lambda}_{j,t} = 0$ holds in the long run. That is, the steady state $\widetilde{A}^*$ is locally stable.

Appendix D Proof of Proposition 2

First, by logarithmically differentiating (18), we obtain $\alpha dq_F^*/q_F^* + (1-\alpha)dw_F^*/w_F^* + \Delta^* = 0$, where Δ^* is

$$\Delta^* = \frac{\left(q_F^{*\alpha}w_F^{*1-\alpha}\right)^{1-\sigma}}{1-\sigma}\left\{\begin{array}{l}\left[\left(\widetilde{q}^{*\alpha}\widetilde{w}^{*1-\alpha}\right)^{1-\sigma}\psi_H(\omega^{c*})^{1-\sigma} - \psi_F(\omega^{c*})^{1-\sigma}\right]d\omega^{c*} \\ \quad +(1-\sigma)\left(\widetilde{q}^{*\alpha}\widetilde{w}^{*(1-\alpha)}\right)^{1-\sigma}\int_0^{\omega^{c*}}\psi_H(\omega)^{1-\sigma}d\omega\left[\alpha\frac{d\widetilde{q}^*}{\widetilde{q}^*} + (1-\alpha)\frac{d\widetilde{w}^*}{\widetilde{w}^*}\right]\end{array}\right\}.$$

On the right-hand side of this equation, the first term in the brackets becomes zero because of (15). Moreover, by using the same equation, we can rewrite $\alpha d\widetilde{q}^*/\widetilde{q}^* + (1-\alpha)d\widetilde{w}^*/\widetilde{w}^*$ in the second term as $-\phi^* d\omega^{c*}/\omega^{c*}$. Then, we can obtain $\Delta^* = -\kappa_F^*\phi^* d\omega^{c*}/\omega^{c*}$, where $\kappa_j^* \in (0,1)$ is defined in the main body of the text. We obtain dw_F^*/w_F^* as

$$\frac{dw_F^*}{w_F^*} = -\frac{1}{1-\alpha}\left[\alpha\frac{dq_F^*}{q_F^*} - \kappa_F^*\phi^*\frac{d\omega^{c*}}{\omega^{c*}}\right]. \tag{33}$$

From (19), (20), and (33),

$$\frac{dw_H^*}{w_H^*} \equiv \frac{d\widetilde{w}^*}{\widetilde{w}^*} + \frac{dw_F^*}{w_F^*}$$
$$= -\frac{1}{1-\alpha}\left[\alpha \frac{dq_H^*}{q_H^*} + \kappa_H^* \phi^* \frac{d\omega^{c*}}{\omega^{c*}}\right]. \tag{34}$$

By using (19) and the fact that $dq_j^*/q_j^* = \xi_j d\lambda_j/\lambda_j$, we can more simply express (33) and (34) as

$$\frac{dw_j^*}{w_j^*} = -\frac{\alpha}{1-\alpha}\left[\left(1 - \frac{\kappa_j^*\phi^*}{\zeta^*}\right)\xi_j \frac{d\lambda_j}{\lambda_j} + \frac{\kappa_j^*\phi^*}{\zeta^*}\xi_n \frac{d\lambda_n}{\lambda_n}\right], \quad j, n(\neq j) \in \{H, F\}. \tag{35}$$

On the right-hand side, the first term in the brackets is the direct effect of a credit crunch and the second term is the indirect effect induced by international trade. The definition of ζ^* in Appendix C shows that $\zeta^* > \phi^*$. From this fact and $\kappa_j^* \in (0, 1)$, we can verify that $1 - \kappa_j^*\phi^*/\zeta^* > 0$ for all j. Then, (35) shows that the wage rates in both countries decline.

Appendix E Derivation of $EV_j^{SS}(\lambda_j|m)$

From Appendix A, we obtain $V_{j,t}(m,\theta) = v_{j,t}(\theta) + (1-\beta_j)^{-1}\log m$. From (24), $v_{j,t}(\theta)$ satisfies

$$v_{j,t}(\theta) = \beta_j \int_{\underline{\theta}}^{\overline{\theta}} v_{j,t+1}(\theta')dG_j(\theta') + \frac{\beta_j}{1-\beta_j}\log\left[R_{j,t+1}(\theta)\right] + \frac{\beta_j}{1-\beta_j}\log\beta_j + \log(1-\beta_j).$$

In general, the functional form of $v_{j,t}(\cdot)$ will vary, as it is affected by changes in the market variables, say, $q_{j,t+1}$ and $r_{j,t+1}$. In the steady-state equilibrium, however, these variables are given by their steady state. This implies that the functional from of v_j becomes stationary. $v_{j,t}(\theta) = v_{j,t+1}(\theta) = v_j^*(\theta)$ for all θ. By using this, we obtain $V_j^*(m,\theta) = v_j^*(\theta) + (1-\beta_j)^{-1}\log m$, and

$$\int v_j^*(\theta)dG_j(\theta) = \frac{1}{1-\beta_j}\left\{\frac{\beta_j}{1-\beta_j}\int_{\underline{\theta}}^{\overline{\theta}} \log\left[R_j^*(\theta)\right]dG_j(\theta) + \frac{\beta_j}{1-\beta_j}\log\beta_j + \log(1-\beta_j)\right\}.$$

Then, we can obtain $EV_j^{SS}(\lambda_j|m)$ in the main body of the text.

Appendix F Brief Derivations of the Transitory Responses

A log-linear approximation of the system for $(\widetilde{A}_{t+1}, \omega_{t+1}^c, \widetilde{w}_{t+1}, \widetilde{q}_{t+1})$ around the steady state is given by (27)–(30) in Appendix C, where $\widehat{\lambda}_{j,t}$ is now specified as

$$\widehat{\lambda}_{H,t+1} = \rho\widehat{\lambda}_{H,t}, \quad \widehat{\lambda}_{F,t} = 0. \tag{36}$$

By solving these equations, we can obtain

$$\widehat{A}_{t+1} = (1-\nu^*)\left(\widehat{A}_t + \xi_H \widehat{\lambda}_{H,t}\right), \tag{37}$$

$$\widehat{\omega}^c_{t+1} = \frac{\alpha\nu^*}{\zeta^*}\left(\widehat{A}_t + \xi_H \widehat{\lambda}_{H,t}\right), \tag{38}$$

$$\widehat{w}_{t+1} = \frac{\alpha[\phi^*(\sigma-1)+\varepsilon^*]\nu^*}{\zeta^*}\left(\widehat{A}_t + \xi_H \widehat{\lambda}_{H,t}\right), \tag{39}$$

$$\widehat{q}_{t+1} = -\nu^*\left(\widehat{A}_t + \xi_H \widehat{\lambda}_{H,t}\right), \tag{40}$$

where a tilde over the symbols is omitted. Given the initial conditions $\widehat{A}_1 = 0$ and $\widehat{\lambda}_1 \simeq (\lambda_{H,1} - \lambda_0)/\lambda_0 < 0$, (36) and (37) provide the response of $\widetilde{A}_t$. Once this is obtained, the responses of $(\omega^c_{t+1}, \widetilde{q}_{t+1}, \widetilde{w}_{t+1})$ are given by (38)–(40).

The responses of the other major variables in each country are accordingly determined. The system for $(A_{F,t+1}, q_{F,t+1}, w_{F,t+1})$ is given by

$$\begin{aligned} A_{F,t+1} &= \beta_F q_{F,t+1}\Gamma_F(\lambda_{F,t})A_{F,t}, \\ \Gamma_F(\lambda_{F,t})A_{F,t} &= \frac{\alpha}{1-\alpha}\frac{w_{F,t+1}}{q_{F,t+1}}L_F, \\ 1 &= q^{\alpha}_{F,t+1}w^{1-\alpha}_{F,t+1}\left[\left(\widetilde{q}^{\alpha}_{t+1}\widetilde{w}^{1-\alpha}_{t+1}\right)^{1-\sigma}\int_0^{\omega^c_{t+1}}\psi_H(\omega)^{1-\sigma}d\omega + \int_{\omega^c_{t+1}}^1 \psi_F(\omega)^{1-\sigma}d\omega\right]^{1/(1-\sigma)}. \end{aligned}$$

The first equation is the dynamic equation of $A_{F,t}$. The second equation corresponds to the market-clearing condition for capital in the foreign country. The third equation is the final good firms' zero-profit condition, which is the condition that the final good price equals the price index of the intermediate goods. Noting that $\lambda_{F,t}$ does not change from λ_0 (i.e., $\widehat{\lambda}_{F,t} = 0$), a log-linear approximation of the above system is

$$\begin{aligned} \widehat{A}_{F,t+1} &= \widehat{q}_{F,t+1} + \widehat{A}_{F,t}, \\ \widehat{A}_{F,t} &= \widehat{w}_{F,t+1} - \widehat{q}_{F,t+1}, \\ 0 &= \alpha\widehat{q}_{F,t+1} + (1-\alpha)\widehat{w}_{F,t+1} - \kappa^*_F\phi^*\widehat{\omega}^c_{t+1}. \end{aligned}$$

From these equations, we readily obtain

$$\widehat{A}_{F,t+1} = \widehat{w}_{F,t+1} = \alpha\widehat{A}_{F,t} + \kappa^*_F\phi^*\widehat{\omega}^c_{t+1}, \tag{41}$$

$$\widehat{q}_{F,t+1} = -(1-\alpha)\widehat{A}_{F,t} + \kappa^*_F\phi^*\widehat{\omega}^c_{t+1}. \tag{42}$$

Then, we obtain

$$\widehat{A}_{H,t+1} = \widehat{A}_{t+1} + \widehat{A}_{F,t+1}, \tag{43}$$

$$\widehat{q}_{H,t+1} = \widehat{q}_{t+1} + \widehat{q}_{F,t+1}, \tag{44}$$

$$\widehat{w}_{H,t+1} = \widehat{w}_{t+1} + \widehat{w}_{F,t+1}. \tag{45}$$

Since the investment $Z_{j,t}$ is given by $A_{j,t}$, capital $K_{j,t+1}$ is given by $\Gamma_j(\lambda_{j,t})A_{j,t}$, workers' and entrepreneurs' aggregate consumption, $C^W_{j,t}$ and $C^E_{j,t}$, are respectively given by $w_{j,t}L$ and $\beta_j A_{j,t}/(1-\beta_j)$, and GDP is given by $q_{j,t}K_{j,t} + w_{j,t}L_j$, the responses of all the major variables in this model are obtained by using (36)–(45).

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Separate Appendix (not for publication): Model with the Partial Depreciation of Capital

In this section, it is assumed that capital depreciates only partially, while the remaining capital is liquidated before the new investment. In this case, the investment technology equation remains the same, whereas the budget constraint is replaced by

$$(q_{j,t} + 1 - \delta_j)k^i_{j,t} - (1 + r_{j,t})d^i_{j,t-1} + d^i_{j,t} = c^i_{j,t} + z^i_{j,t},$$

where $\delta_j \in (0, 1)$ is the depreciation rate. Then, the second-stage optimization problem (4) is replaced by

$$m_{j,t+1}(a, \theta) = \max_z \Big\{(q_{j,t+1} + 1 - \delta_j)\theta z - (1 + r_{j,t+1})(z - a) \mid 0 \le z \le (1 + \lambda_{j,t})a\Big\}.$$

The cutoff of investment technology $\theta^c_{j,t}$ is now given by

$$\theta^c_{j,t} = \frac{1 + r_{j,t+1}}{q_{j,t+1} + 1 - \delta_j},$$

and (7) is replaced by $A_{j,t+1} = \beta_j\left[(q_{j,t+1} + 1 - \delta_j)K_{j,t+1} + (1 + r_{j,t+1})B_{j,t}\right]$. Since the market-clearing condition for the one-period bonds does not change, $B_{j,t} = 0$. Then, the dynamic equation of relative wealth (14) becomes

$$\widetilde{A}_{t+1} = \frac{\beta_H}{\beta_F}\frac{\Gamma_F(\lambda_{H,t})}{\Gamma_F(\lambda_{F,t})}\frac{\widetilde{q}_{t+1}q_{F,t+1} + 1 - \delta_H}{q_{F,t+1} + 1 - \delta_F}\widetilde{A}_t. \tag{46}$$

Thus, the system of equations (15)–(17) and (46) no longer constitute the autonomous dynamic system, since it lacks the equation to obtain $q_{F,t+1}$. The autonomous system in this case is given by (15)–(17), (46), and the following subsystem:

$$A_{F,t+1} = \beta_F(q_{F,t+1} + 1 - \delta_F)\Gamma_F(\lambda_{F,t})A_{F,t}, \tag{47}$$

$$\Gamma_F(\lambda_{F,t})A_{F,t} = \frac{\alpha}{1-\alpha}\frac{w_{F,t+1}}{q_{F,t+1}}L_F, \tag{48}$$

$$1 = q^{\alpha}_{F,t+1}w^{1-\alpha}_{F,t+1}\left[\left(\widetilde{q}^{\alpha}_{t+1}\widetilde{w}^{1-\alpha}_{t+1}\right)^{1-\sigma}\int_0^{\omega^c_{t+1}}\psi_H(\omega)^{1-\sigma}d\omega + \int_{\omega^c_{t+1}}^1 \psi_F(\omega)^{1-\sigma}d\omega\right]^{1/(1-\sigma)}. \tag{49}$$

These constitute the system for $(A_{F,t+1}, q_{F,t+1}, w_{F,t+1})$. The first equation is the dynamic equation of $A_{F,t}$. The second equation corresponds to the market-clearing condition for capital in the foreign country. The third equation is the final good firms' zero-profit condition, which is the condition that the final good price equals the price index of the intermediate goods.

We can easily show the unique existence of the steady-state equilibrium. To this end, assume that $\lambda_{j,t}$ becomes a constant. From (47) with $A_{F,t} = A_{F,t+1}$, we obtain $q_F^* = 1/(\beta_F \Gamma_F(\lambda_F)) - (1-\delta_F)$. By substituting this into (46) and imposing $\widetilde{A}_{t+1} = \widetilde{A}_t$, $\widetilde{q}^*$ is now given by

$$\begin{aligned}\widetilde{q}^* &= \frac{1}{q_F^*}\left[\frac{\beta_F \Gamma_F(\lambda_F)}{\beta_H \Gamma_H(\lambda_H)}(q_F^* + 1 - \delta_F) - (1-\delta_H)\right] \\ &= \frac{1/(\beta_H \Gamma_H(\lambda_H)) - (1-\delta_H)}{1/(\beta_F \Gamma_F(\lambda_F)) - (1-\delta_F)} \left(\equiv \frac{q_H^*}{q_F^*}\right).\end{aligned}$$

We assume that $\Gamma_j(\lambda_j)$ is sufficiently small that $\widetilde{q}^*$ is positive. Then, substituting q^* into (15)–(17) yields $(\omega^{c*}, \widetilde{w}^*, \widetilde{A}^*)$. Once these are found, (w_F^*, A_F^*) are accordingly determined from (48) and (49).

We can obtain the same results for the comparative statics in the main body. In other words, we can show Propositions 1–3. To see why, first note that (19) and (20) are also satisfied in this case. Moreover, as the above equation shows, $\widetilde{q}$ is increasing in λ_H, whereas it is decreasing in λ_F. Then, we can obtain the same result as Proposition 1. Next, note that (33) and (34) in Appendix D, which gives the proof of Proposition 2, also hold in this case, since these equations come from (18)–(20), which are satisfied irrespective of the existence of partial depreciation. This implies that Proposition 2 holds in this case. Finally, from its proof, we can find that Proposition 3 is established if Proposition 2 holds true. Then, we can obtain the same results as in the main body of the text.