New unit root tests with two smooth breaks and nonlinear adjustment

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Abstract

This paper proposes new three unit root testing procedures which consider jointly for two structural breaks and nonlinear adjustment. The structural breaks are modelled by means of two logistic smooth transition functions and nonlinear adjustment is modelled by means of ESTAR models. The Monte Carlo experiments display that the empirical sizes of tests are quite close to the nominal ones and in terms of power; the three new unit root tests are superior to the alternative tests. An empirical application involving crude oil underlines the usefulness of the new unit root tests.

Keywords: Smooth breaks, nonlinearity, unit root, ESTAR

JEL Classification: C12, C22

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1. Introduction

Beginning with the seminal work of Dickey and Fuller (1979), the debate as to whether economic time series are characterised as a trend stationary process or a difference stationary process has received much interest in time series econometrics. However as shown in Perron (1989), the Dickey-Fuller test fails to reject the null hypothesis of unit root when structural breaks are present in data generating process (DGP). Perron (1989) proposes a unit root test which takes into account structural breaks exogenously in the deterministic components and displays that the traditional unit roots tests detect incorrectly that the series have a unit root when in fact they are stationary with structural breaks. Therefore, Dickey-Fuller-type tests would be powerless to separate the behaviour of a unit process from the behaviour of a stationary process with structural breaks.

Following Perron (1989), Zivot and Andrews (1992) assume the knowledge of timing of the structural change as being endogenous and also propose a procedure testing the null of unit root against the alternative of stationarity with one structural break. Apart from Perron (1989) and Zivot and Andrews (1992), many authors like Lumsdaine and Papell (1997); Lee and Strazicich (2003); Perron and Rodriguez (2003) have developed unit root tests in order to take into account structural breaks. The main feature of these unit root tests is that the break time is known and so structural changes in level and trend are assumed to occur instantaneously, only in certain points of time.

However, the effect of structural changes on the level and trend could be gradual. As mentioned by Leybourne et. al. (1998), individual agents can react simultaneously to a given economic stimulus, while some may be able to react instantaneously and so will adjust with different time lags. Thus, when considering aggregate behavior, the time path of structural changes in economic series is likely to be better captured by a model whose deterministic component permits gradual rather than instantaneous adjustment between different values. From this point of view, some authors propose different unit root tests that consider smooth rather than sudden change. The main idea behind these tests is that nonlinearities can be present in time series as an asymmetric speed of mean reversion and autoregressive parameter varies depending upon the values of a variable. This nonlinear behavior implies that there is a central regime where the series behave as a unit root whereas for values outside the central regime, the variable tends to revert to the equilibrium (Cuestas and Ordóñez, 2014).

The nonlinear dynamics for unit root testing procedures and the joint analysis of nonlinearity and nonstationarity have been popularised in the last twenty years. Kapetanios et. al. (henceforth KSS) (2003) propose a unit root test within an exponential smooth transition autoregressive (ESTAR) model. Apart from KSS (2003), Rothe and Sibbertsen (2006), Sollis (2009), Kruse (2011) present invaluable contributions to the testing of unit roots considering nonlinearity. Although these studies consider asymmetric speed of mean reversion, they do not take into account nonlinearities in the deterministic components. They state that the modelling of intercepts and time trends is not straightforward in nonlinear models and suggest using de-meaned and/or de-trended data instead of modelling intercepts and trends.

On the other hand, Leybourne et. al. (1998) develop a set of unit root tests where the process under the alternative hypothesis is stationary around a smooth transition in the linear trend, which is intuitively appealing as it permits one structural break to occur gradually over time. Christopoulos and León-Ledesma (2010) propose tests for unit roots that account jointly for structural breaks and nonlinear adjustment. The prominent contribution of unit root test of Christopoulos and León-Ledesma (2010) is that this test takes into account asymmetric speed of mean reversion, as well as structural changes in the intercept, approximated by means of a Fourier function. Cuestas and Ordóñez (2014) also propose a unit root test which extends the unit root test of Leybourne et. al. (1998) and takes into account both sources of nonlinearities, i.e. in the deterministic components, approximated by a logistic smooth transition function not only in the intercept, but also in the trend, an asymmetric adjustment of mean reversion.

In this paper, we develop three new unit root testing procedures which consider jointly for two structural breaks and nonlinear adjustment. It would be possible that more than one structural change occurs in a time series so in our proposed test, the structural breaks are modeled by means of two logistic smooth transition functions that allows in the intercept, in the intercept under a fixed trend and in the intercept and trend following Harvey and Mills (henceforth HM) (2002). Nonlinear adjustment is modeled by means of ESTAR models separately as suggested by KSS (2003), Sollis (2009) and Kruse (2011).

The rest of the paper is organized as follows: Section 2 describes the proposed three test statistics and provides asymptotic critical values. Section 3 presents the results of size and power of our proposed tests via Monte Carlo simulation experiments. Section 4 contains the empirical application and the last section concludes the paper.

2. The New Unit Root Tests

The basic idea behind the unit root tests proposed in this section is to use two logistic smooth transition functions suggested by HM (2002). We combine the procedures of HM (2002) and KSS (2003), procedures of HM (2002) and Sollis (2009) and procedures of HM (2002) and Kruse (2011) and develop three new unit root tests for the case where the unit root null is tested against an alternative of nonlinear and stationary with two smooth breaks.

We consider the following three logistic smooth transition models in light of HM (2002):

Model A: $y_t = \alpha_1 + \alpha_2 S_{1t}\left(\lambda_1, \tau_1\right) + \alpha_3 S_{2t}\left(\lambda_2, \tau_2\right) + v_t$ (1)

Model B: $y_t = \alpha_1 + \beta_1 t + \alpha_2 S_{1t}\left(\lambda_1, \tau_1\right) + \alpha_3 S_{2t}\left(\lambda_2, \tau_2\right) + v_t$ (2)

Model C: $y_t = \alpha_1 + \beta_1 t + \alpha_2 S_{1t}\left(\lambda_1, \tau_1\right) + \beta_2 t S_{1t}\left(\lambda_1, \tau_1\right) + \alpha_3 S_{2t}\left(\lambda_2, \tau_2\right) + \beta_3 t S_{2t}\left(\lambda_2, \tau_2\right) + v_t$ (3)

where v_t in each model is assumed as a stationary process with zero mean and $S_{1i}\left(\lambda_i, \tau_i\right)$ represent logistic smooth functions based on a sample of size T and are defined by:

$$S_{1i}\left(\lambda_i, \tau_i\right) = \left[1 + \exp\left\{-\lambda_i\left(t - \tau_i T\right)\right\}\right]^{-1} \quad \lambda_i > 0 \quad i = 1, 2 \tag{4}$$

where τ_1 and τ_2 are the midpoints of two transitions and λ_1 and λ_2 determine the transition speeds which differ regime by regime. If we assume v_t is a zero-mean $I\left(0\right)$ process, then in model A y_t is stationary around a mean which changes from the initial value α_1 to the final value $\alpha_1 + \alpha_2 + \alpha_3$. Model B is similar to Model A, with the intercept changing from α_1 to $\alpha_1 + \alpha_2 + \alpha_3$, but it allows for a fixed slope term. Finally, in Model C, in addition to the change in intercept from α_1 to $\alpha_1 + \alpha_2 + \alpha_3$, the slope also changes contemporaneously, and with the same speed of transition β_1 to $\beta_1 + \beta_2 + \beta_3$.

Our new unit root tests take account of the possibility of two smooth breaks and asymmetric speed of adjustment toward equilibrium simultaneously. We assume that the adjustment speed is nonlinear and follows an exponential smooth transition autoregressive (ESTAR) process in each unit root test.

2.1. HM-KSS Unit Root Test

The null of unit root hypothesis which is our focus of interest may be stated as follows:

$$H_0 : y_t = \mu_t,\ \mu_t = \mu_{t-1} + \varepsilon_t \tag{5}$$

where ε_t is assumed to be an $I(0)$ process with zero mean. The test statistics are calculated via a two step procedure. In the first step, we use a nonlinear least squares (NLS) algorithm for estimating only deterministic components in model A, B and C, then we compute the NLS residuals,

$$\text{Model A: } \hat{v}_t = y_t - \hat{\alpha}_1 - \hat{\alpha}_2 S_{1t}\left(\hat{\lambda}_1, \hat{\tau}_1\right) - \hat{\alpha}_3 S_{2t}\left(\hat{\lambda}_2, \hat{\tau}_2\right) \tag{6}$$

$$\text{Model B: } \hat{v}_t = y_t - \hat{\alpha}_1 - \hat{\beta}_1 t - \hat{\alpha}_2 S_t\left(\hat{\lambda}_1, \hat{\tau}_1\right) - \hat{\alpha}_3 S_{2t}\left(\hat{\lambda}_2, \hat{\tau}_2\right) \tag{7}$$

$$\text{Model C: } \hat{v}_t = y_t - \hat{\alpha}_1 - \hat{\beta}_1 t - \hat{\alpha}_2 S_t\left(\hat{\lambda}, \hat{\tau}\right) - \hat{\beta}_2 t S_t\left(\hat{\lambda}, \hat{\tau}\right) - \hat{\alpha}_3 S_{2t}\left(\hat{\lambda}_2, \hat{\tau}_2\right) - \hat{\beta}_3 t S_{2t}\left(\hat{\lambda}_2, \hat{\tau}_2\right) \tag{8}$$

After computing the nonlinear residuals from the Models A, B and C, in the second step, for the HM-KSS test; we apply the unit root test of KSS (2003) to the residuals obtained in the first step. We consider an ESTAR model which is modified to our strategy as the following form:

$$\Delta \hat{v}_t = \gamma \hat{v}_{t-1}\left(1-\exp\left\{-\theta \hat{v}_{t-1}{}^2\right\}\right)+\varepsilon_t \tag{9}$$

where $\hat{v}_t$ is the estimated NLS residuals in the first step. KSS (2003) propose a first-order Taylor approximation for equation (9) and obtain the auxiliary regression shown at equation (10).

$$\Delta \hat{v}_t = \delta_1 \hat{v}_{t-1}^3 + \sum_{i=1}^{p} \psi_i \Delta \hat{v}_{t-i} + \varepsilon_t \tag{10}$$

In the auxiliary regression (10), the null hypothesis could be constituted $H_0 : \delta_1 = 0$ against $H_1 : \delta_1 < 0$. The test statistics of our new procedure are computed as t-type test statistics by following KSS (2003):

$$t_{2SNL\alpha},\ t_{2SNL\alpha(\beta)},\ t_{2SNL\alpha\beta} = \frac{\hat{\delta}_1}{SE\left(\hat{\delta}_1\right)} \tag{11}$$

where $\hat{\delta}_1$ is the OLS estimate of δ_1 and $SE\left(\hat{\delta}_1\right)$ is the standard error of $\hat{\delta}_1$. As mentioned by Leybourne et. al. (1998), we assume the residuals v_t are zero-mean $I(0)$ processes, and then y_t are also stationary processes in models A, B and C. Therefore, the asymptotic distributions of $t_{2SNL\alpha}$, $t_{2SNL\alpha(\beta)}$ and $t_{2SNL\alpha\beta}$ statistics have the same properties with the t_{NL} statistic of KSS (2003). (For proofs, see Appendix of KSS (2003)).

We call our new proposed test as HM-KSS and denote the value of test statistics as $t_{2SNL\alpha}$ if Model A is used to construct the $\hat{v}_t$, $t_{2SNL\alpha(\beta)}$ if Model B is used and $t_{2SNL\alpha\beta}$ if Model C is used. Thus, the critical values of $t_{2SNL\alpha}$, $t_{2SNL\alpha(\beta)}$ and $t_{2SNL\alpha\beta}$ test statistics are obtained via stochastic simulations at 1%, 5% and 10% significance levels based on 50,000 replications for $T = 50, 100, 250, 500$. The critical values are reported in Table 1.

Table 1: Critical Values for HM-KSS Unit Root Test

	$t_{2SNL\alpha}$			$t_{2SNL\alpha(\beta)}$			$t_{2SNL\alpha\beta}$		
T	**1%**	**5%**	**10%**	**1%**	**5%**	**10%**	**1%**	**5%**	**10%**
50	-3.491	-2.888	-2.605	-4.023	-3.385	-3.087	-4.023	-3.376	-3.072
100	-3.472	-2.907	-2.625	-2.618	-2.024	-1.736	-2.693	-2.124	-1.833
250	-3.472	-2.907	-2.625	-2.767	-2.220	-1.928	-2.569	-1.994	-1.704
500	-3.488	-2.932	-2.647	-3.957	-3.401	-3.119	-2.490	-1.941	-1.685

Notes: The initial values for parameters $\hat{\lambda}_1$, $\hat{\tau}_1$, $\hat{\lambda}_2$ and $\hat{\tau}_2$ in the nonlinear least squares estimation are set at 0.50, 0.25, 1.0, 0.50, respectively.

2.2. HM-Sollis Unit Root Test

In our new proposed test labeled the HM-Sollis test, the null hypothesis of unit root is as follows:

$$H_0 : y_t = \mu_t,\ \mu_t = \mu_{t-1} + \varepsilon_t \tag{12}$$

where ε_t is assumed to be an $I(0)$ process with zero mean. Once computing the NLS residuals from the Models A, B and C, then, for the HM-Sollis test; we apply the unit root test of Sollis (2009) to the residuals obtained in the first step. We consider an ESTAR model which employs an exponential function and a logistic function and is modified to our strategy as the following form:

$$\Delta\hat{v}_t = \left[1-\exp\left(-\theta_1 \hat{v}_{t-1}^2\right)\right]\left\{\left[1+\exp\left(-\theta_2 \hat{v}_{t-1}\right)\right]^{-1}\gamma_1 + \left(1-\left[1+\exp\left(-\theta_2 \hat{v}_{t-1}\right)\right]^{-1}\right)\gamma_2\right\}\hat{v}_{t-1} + \varepsilon_t \qquad (13)$$

where $\hat{v}_t$ is the estimated NLS residuals in the first step. Sollis (2009) proposes a first-order Taylor approximation for equation (13) and obtains the auxiliary regression shown at equation (14).

$$\Delta\hat{v}_t = \delta_1 \hat{v}_{t-1}^3 + \delta_2 \hat{v}_{t-1}^4 + \sum_{i=1}^{p} \psi_i \Delta\hat{v}_{t-i} + \varepsilon_t \qquad (14)$$

In the auxiliary regression (14), the null hypothesis could be constituted $H_0 : \delta_1 = \delta_2 = 0$ against $H_1 : \delta_1 \neq \delta_2 \neq 0$. The test statistics of our new procedure are computed as F-type test statistics by following Sollis (2009):

$$F_{2SNL\alpha},\ F_{2SNL\alpha(\beta)},\ F_{2SNL\alpha\beta} = \left(R\hat{\delta}-r\right)'\left[\hat{\sigma}^2 R\left\{\sum_t \hat{v}_t \hat{v}_t'\right\}^{-1} R'\right]^{-1}\left(R\hat{\delta}-r\right)\Big/ m \qquad (15)$$

where $\hat{v}_t = \left[\hat{v}_{t-1}^3, \hat{v}_{t-1}^4\right]'$, $m = 2$, R is a 2×2 identity matrix, $\hat{\delta} = \left[\hat{\delta}_1, \hat{\delta}_2\right]'$ where $\hat{\delta}_1$ and $\hat{\delta}_2$ are the OLS estimates of δ_1 and δ_2, $r = [0,0]'$ and $\hat{\sigma}^2$ is the OLS estimate of σ^2. As mentioned by Leybourne et. al. (1998), we assume the residuals v_t are zero-mean $I(0)$ processes, and then y_t are also stationary processes in models A, B and C. Therefore, the asymptotic distributions of $F_{2SNL\alpha}$, $F_{2SNL\alpha(\beta)}$ and $F_{2SNL\alpha\beta}$ statistics have the same properties with the F_{AE}, $F_{AE,\mu}$ and $F_{AE,t}$ statistics of Sollis (2009). (For proofs, see Appendix of Sollis (2009)).

We denote the value of test statistics as $F_{2SNL\alpha}, F_{2SNL\alpha(\beta)}$ and $F_{2SNL\alpha\beta}$ corresponding to Model A, Model B and Model C, respectively. Thus, the critical values of $F_{2SNL\alpha}$, $F_{2SNL\alpha(\beta)}$ and $F_{2SNL\alpha\beta}$ test statistics are obtained via stochastic simulations at 1%, 5% and 10% significance levels based on 50,000 replications for $T = 50, 100, 250, 500$. The critical values are reported in Table 2.

Table 2: Critical Values for HM-Sollis Unit Root Test

	$F_{2SNL\alpha}$			$F_{2SNL\alpha(\beta)}$			$F_{2SNL\alpha\beta}$		
T	**1%**	**5%**	**10%**	**1%**	**5%**	**10%**	**1%**	**5%**	**10%**
50	7.023	4.939	4.051	8.959	6.477	5.408	8.910	6.435	5.363
100	7.015	4.984	4.123	6.327	4.304	3.445	6.155	4.168	3.332
250	6.856	5.007	4.168	6.012	4.121	3.283	6.468	4.470	3.595
500	6.898	5.039	4.199	8.781	6.607	5.644	5.835	4.068	3.243

Notes: The initial values for parameters $\hat{\lambda}_1$, $\hat{\tau}_1$, $\hat{\lambda}_2$ and $\hat{\tau}_2$ in the nonlinear least squares estimation are set at 0.50, 0.25, 1.0, 0.50, respectively.

2.3. HM-Kruse Unit Root Test

The null hypothesis of the unit root of the HM-Kruse test is also defined as the same in the HM-KSS and HM-Sollis tests.

$$H_0 : y_t = \mu_t,\ \ \mu_t = \mu_{t-1} + \varepsilon_t \tag{16}$$

where ε_t is assumed to be an $I(0)$ process with zero mean. Also for the HM-Kruse test, after computing the nonlinear residuals from the Models A, B and C, in the second step, we apply the unit root test of Kruse (2011) to the residuals obtained in the first step. We allow for a nonzero location parameter c by following Kruse (2011) in the ESTAR model which is modified to our strategy as the following form:

$$\Delta\hat{v}_t = \gamma\hat{v}_{t-1}\left(1-\exp\left\{-\theta\left(\hat{v}_{t-1}-c\right)^2\right\}\right)+\varepsilon_t \quad (17)$$

where $\hat{v}_t$ is the estimated NLS residuals in the first step. Kruse (2011) proposes a first-order Taylor approximation for equation (17) and obtains the auxiliary regression shown at equation (18).

$$\Delta\hat{v}_t = \delta_1\hat{v}_{t-1}^3 + \delta_2\hat{v}_{t-1}^2 + \sum_{i=1}^{p}\psi_i\Delta\hat{v}_{t-i} + \varepsilon_t \quad (18)$$

In the auxiliary regression (18), the null hypothesis could be constituted $H_0 : \delta_1 = \delta_2 = 0$ against $H_1 : \delta_1 < 0,\ \delta_2 \neq 0$. It can be remarked that one parameter is one-sided and the other one is two-sided under the alternative hypothesis so a standard Wald type would be inconvenient to derive a test statistic. By following Kruse (2011) and applying the method of Abadir and Distaso (2007), the one-sided parameter is orthogonalized with respect to the two-sided one. The test statistics of our new procedure are computed as a modified Wald type test which builds upon the one-sided parameter and the transformed two-sided parameter:

$$\tau_{SNL\alpha},\ \tau_{SNL\alpha(\beta)},\ \tau_{SNL\alpha\beta} = \left(\hat{\psi}_{22}-\frac{\hat{\psi}_{21}^2}{\hat{\psi}_{11}}\right)\left(\hat{\delta}_2-\hat{\delta}_1\frac{\hat{\psi}_{21}}{\hat{\psi}_{11}}\right)^2 + 1\left(\hat{\delta}_1<0\right)\frac{\hat{\delta}_1^2}{\hat{\psi}_{11}} \quad (19)$$

which are the new statistics for a unit root hypothesis against nonlinear and stationary with one smooth break. $\hat{\psi}_{22}$, $\hat{\psi}_{11}$ and $\hat{\psi}_{21}$ are the elements of Variance-Covariance matrix. We denote the value of test statistics as $\tau_{2SNL\alpha}$ if Model A is used to construct the $\hat{v}_t$, $\tau_{2SNL\alpha(\beta)}$ if Model B is used and $\tau_{2SNL\alpha\beta}$ if Model C is used.

As mentioned by Leybourne et. al. (1998), we assume the residuals v_t are zero-mean $I(0)$ processes, and then y_t are also stationary processes in models A, B and C. Therefore, the asymptotic distributions of $\tau_{2SNL\alpha}$, $\tau_{2SNL\alpha(\beta)}$ and $\tau_{2SNL\alpha\beta}$ statistics have the same properties with the τ statistic of Kruse (2011). (For proofs, see Appendix of Kruse (2011)).

Thus, the critical values of $\tau_{2SNL\alpha}$, $\tau_{2SNL\alpha(\beta)}$ and $\tau_{2SNL\alpha\beta}$ test statistics are obtained via stochastic simulations at 1%, 5% and 10% significance levels based on 50,000 replications for $T = 50, 100, 250, 500$. The critical values are reported in Table 3.

Table 3: Critical Values for HM-Kruse Unit Root Test

	$\tau_{2SNL\alpha}$			$\tau_{2SNL\alpha(\beta)}$			$\tau_{2SNL\alpha\beta}$		
T	**1%**	**5%**	**10%**	**1%**	**5%**	**10%**	**1%**	**5%**	**10%**
50	13.387	9.567	8.021	17.356	12.467	10.451	17.349	12.408	10.407
100	13.550	9.838	8.294	14.454	10.102	8.257	16.887	12.615	10.740
250	13.510	10.029	8.474	20.563	14.981	12.369	16.585	12.730	10.931
500	13.534	9.833	8.232	17.108	12.897	11.053	16.892	12.671	10.796

Notes: The initial values for parameters $\hat{\lambda}_1$, $\hat{\tau}_1$, $\hat{\lambda}_2$ and $\hat{\tau}_2$ in the nonlinear least squares estimation are set at 0.50, 0.25, 1.0, 0.50, respectively.

3. Monte Carlo Study

In this section, we carry out a Monte Carlo experiment in order to investigate the small sample size and power properties of the tests suggested in the previous Section. First, we study the empirical size of test for different sample sizes i.e. $T = 50,100$ with a nominal size of 0.05. We consider the following data-generating process (DGP):

$$y_t = \mu_t, \quad \mu_t = \mu_{t-1} + \varepsilon_t, \quad \mu_0 = 0 \quad \varepsilon_t \sim NIID(0,1) \tag{20}$$

The results of empirical sizes of HM-KSS, HM-Sollis and HM-Kruse tests, based on 5000 replications, are presented in Tables 4-6.

Table 4: Size Properties of HM-KSS Unit Root Test

T	$t_{2SNL\alpha}$	$t_{2SNL\alpha(\beta)}$	$t_{2SNL\alpha\beta}$
50	0.053	0.053	0.048
100	0.048	0.053	0.049

Table 5: Size Properties of HM-Sollis Unit Root Test

T	$F_{2SNL\alpha}$	$F_{2SNL\alpha(\beta)}$	$F_{2SNL\alpha\beta}$
50	0.049	0.046	0.056
100	0.049	0.062	0.069

Table 6: Size Properties of HM-Kruse Unit Root Test

T	$\tau_{2SNL\alpha}$	$\tau_{2SNL\alpha(\beta)}$	$\tau_{2SNL\alpha\beta}$
50	0.051	0.044	0.047
100	0.048	0.045	0.060

Simulation results in Tables 4-6 display that empirical sizes of new unit root tests are quite close to the nominal one, 5%. Overall, the HM-KSS, HM-Sollis and HM-Kruse tests present good size properties and do not lead to over-rejections of the null hypothesis of the unit root.

In order to investigate the power of the HM-KSS unit root test, we generated series for $t_{2SNL\alpha}$, $t_{2SNL\alpha(\beta)}$ and $t_{2SNL\alpha\beta}$ test statistics from the following models, respectively:

$$y_t = 1 + \sqrt{T}\left[1 + \exp\left\{-\lambda_1\left(t - \tau_1 T\right)\right\}\right]^{-1} + \sqrt{T}\left[1 + \exp\left\{-\lambda_2\left(t - \tau_2 T\right)\right\}\right]^{-1} + v_t \quad (21)$$

$$y_t = 1 + \sqrt{T}t + \sqrt{T}\left[1 + \exp\left\{-\lambda_1\left(t - \tau_1 T\right)\right\}\right]^{-1} + \sqrt{T}\left[1 + \exp\left\{-\lambda_2\left(t - \tau_2 T\right)\right\}\right]^{-1} + v_t \quad (22)$$

$$y_t = 1 + \sqrt{T}t + \sqrt{T}\left[1+\exp\left\{-\lambda_1\left(t-\tau_1 T\right)\right\}\right]^{-1} + \sqrt{T}t\left[1+\exp\left\{-\lambda_1\left(t-\tau_1 T\right)\right\}\right]^{-1}$$
$$+\sqrt{T}\left[1+\exp\left\{-\lambda_2\left(t-\tau_2 T\right)\right\}\right]^{-1} + \sqrt{T}t\left[1+\exp\left\{-\lambda_2\left(t-\tau_2 T\right)\right\}\right]^{-1} + v_t \quad (23)$$

$$\Delta v_t = \gamma v_{t-1}\left(1-\exp\left\{-\theta v_{t-1}{}^2\right\}\right) + \varepsilon_t \quad (24)$$

where $\sqrt{T}$ is the square root of the number of observations. We set a broad range of parameter values for $\lambda_1 = 0.5$, $\lambda_2 = 1.0$ $\tau_1 = 0.25$, $\tau_1 = 0.50$, $\gamma = -1.5$ and $\theta = (0.01, 0.1, 1.0)$ for a general power comparison. For each Monte Carlo study, we compute the rejection probabilities of the null hypotheses. The nominal size of the tests are determined at 0.05, the number of replications at 5,000 and the sample size is considered for $T = 50,100$. The results of power experiments and power comparison with the HM and KSS tests are displayed in Table 7.

When interpreting the power performances of the HM, KSS and HM-KSS tests which are summarized at Table 7, we could observe that the HM-KSS unit root test is more powerful than the alternative tests which consider only the two smooth breaks (HM) and only the nonlinear adjustment (KSS) for Models A, B and C.

Table 7: Power Experiments and Comparison of HM, KSS and HM-KSS Unit Root Tests

Model A

	$\theta = 0.01$			$\theta = 0.1$			$\theta = 1.0$		
	HM	KSS	HM-KSS	HM	KSS	HM-KSS	HM	KSS	HM-KSS
$\gamma = -1.5$									
T=50	0.000	0.145	**0.162**	0.003	0.974	**0.979**	0.991	1.000	1.000
T=100	0.000	0.721	**0.737**	0.616	1.000	1.000	1.000	1.000	1.000

Model B

	$\theta = 0.01$			$\theta = 0.1$			$\theta = 1.0$		
	HM	KSS	HM-KSS	HM	KSS	HM-KSS	HM	KSS	HM-KSS
$\gamma = -1.5$									
T=50	0.000	0.043	**0.044**	0.000	0.910	**0.912**	0.957	1.000	1.000
T=100	0.000	0.390	**0.991**	0.228	1.000	1.000	1.000	1.000	1.000

Model C

	$\theta = 0.01$			$\theta = 0.1$			$\theta = 1.0$		
	HM	KSS	HM-KSS	HM	KSS	HM-KSS	HM	KSS	HM-KSS
$\gamma = -1.5$									
T=50	0.000	0.043	**0.045**	0.000	0.910	**0.913**	0.891	1.000	1.000
T=100	0.000	0.390	**0.986**	0.043	1.000	1.000	1.000	1.000	1.000

Notes: The values are rejection rates of HM, KSS and HM-KSS tests and the bold values display the cases where each test performs better. In power comparisons we consider $t_{NL,demeaned}$ statistic for model A and $t_{NL,\det rended}$ statistic for models B and C at KSS test and $s_{2\alpha}$, $s_{2\alpha(\beta)}$ and $s_{2\alpha\beta}$ statistics for models A, B and C, respectively at HM test.

In order to investigate the power of the HM-Sollis unit root test, we generate series for the $F_{2SNL\alpha}$, $F_{2SNL\alpha(\beta)}$ and $F_{2SNL\alpha\beta}$ test statistics from the models 21-23, respectively and also the following ESTAR model:

$$\Delta\hat{v}_t = \left[1-\exp\left(-\theta_1\hat{v}_{t-1}^2\right)\right]\left\{\left[1+\exp\left(-\theta_2\hat{v}_{t-1}\right)\right]^{-1}\gamma_1 + \left(1-\left[1+\exp\left(-\theta_2\hat{v}_{t-1}\right)\right]^{-1}\right)\gamma_2\right\}\hat{v}_{t-1} + \varepsilon_t \qquad (25)$$

We set a broad range of parameter values for $\lambda_1 = 0.5$, $\lambda_2 = 1.0$, $\tau_1 = 0.25$, $\tau_1 = 0.50$, $\gamma_1 = \gamma_2 = \left(-0.1, -0.3, -1.0\right)$, $\theta_1 = \left(0.1, 1.0\right)$, $\theta_2 = 1.0$ for a general power comparison. For each

Monte Carlo study, we compute the rejection probabilities of the null hypotheses. The nominal size of the tests is determined at 0.05, the number of replications at 5,000 and the sample size is considered for $T = 50,100$. The results of power experiments and power comparison with HM and Sollis tests are presented in Table 8.

Table 8: Power Experiments and Comparison of HM, Sollis and HM-Sollis Unit Root Tests

				Model A					
	$\gamma_1 = -0.1$ $\gamma_2 = -0.1$			$\gamma_1 = -0.3$ $\gamma_2 = -0.3$			$\gamma_1 = -1.0$ $\gamma_2 = -1.0$		
	HM	Sollis	HM-Sollis	HM	Sollis	HM-Sollis	HM	Sollis	HM-Sollis
$\theta_1 = 0.10$									
T=50	0.000	**0.003**	0.029	0.000	**0.156**	0.150	0.000	**0.869**	0.864
T=100	0.000	**0.099**	0.097	0.000	**0.688**	0.683	0.058	1.000	1.000
$\theta_1 = 1.0$									
T=50	0.000	**0.055**	0.052	0.000	**0.416**	0.405	0.521	0.999	0.999
T=100	0.000	**0.172**	0.168	0.012	**0.901**	0.899	1.000	1.000	1.000
				Model B					
	$\gamma_1 = -0.1$ $\gamma_2 = -0.1$			$\gamma_1 = -0.3$ $\gamma_2 = -0.3$			$\gamma_1 = -1.0$ $\gamma_2 = -1.0$		
	HM	Sollis	HM-Sollis	HM	Sollis	HM-Sollis	HM	Sollis	HM-Sollis
$\theta_1 = 0.10$									
T=50	0.000	0.007	**0.008**	0.000	0.044	**0.045**	0.000	0.657	**0.667**
T=100	0.000	0.026	**0.167**	0.000	0.365	**0.819**	0.004	0.995	**1.000**
$\theta_1 = 1.0$									
T=50	0.000	0.018	0.018	0.000	0.179	**0.185**	0.201	0.989	**0.990**
T=100	0.000	0.062	**0.276**	0.001	0.694	**0.949**	0.999	1.000	1.000
				Model C					
	$\gamma_1 = -0.1$ $\gamma_2 = -0.1$			$\gamma_1 = -0.3$ $\gamma_2 = -0.3$			$\gamma_1 = -1.0$ $\gamma_2 = -1.0$		
	HM	Sollis	HM-Sollis	HM	Sollis	HM-Sollis	HM	Sollis	HM-Sollis
$\theta_1 = 0.10$									
T=50	0.000	0.007	**0.008**	0.000	0.044	**0.048**	0.000	0.657	**0.676**
T=100	0.000	0.026	**0.190**	0.000	0.365	**0.841**	0.000	0.995	**1.000**
$\theta_1 = 1.0$									
T=50	0.000	0.018	**0.019**	0.000	0.179	**0.191**	0.059	0.989	**0.991**
T=100	0.000	0.062	**0.301**	0.000	0.694	**0.957**	0.989	1.000	1.000

Notes: The values are rejection rates of of HM, Sollis and HM-Sollis tests and the bold values display the cases where each test performs better. In power comparisons we consider $F_{AE,\mu}$ statistic for model A and $F_{AE,t}$ statistic for models B and C at Sollis test and $s_{2\alpha}$, $s_{2\alpha(\beta)}$ and $s_{2\alpha\beta}$ statistics for models A, B and C, respectively at HM test.

The results of power experiments and comparison of alternatives tests display that the new HM-Sollis unit root test is superior to the HM and Sollis tests in terms of power. In cases where the unit root tests are employed in the presence of the two smooth breaks in the mean only (Model A), the Sollis test performs better than the HM-Sollis test.

In order to investigate the power of the HM-Kruse unit root test, we generate series for the $\tau_{2SNL\alpha}$, $\tau_{2SNL\alpha(\beta)}$ and $\tau_{2SNL\alpha\beta}$ test statistics based on the models 21-23 respectively, and also the following ESTAR model:

$$\Delta\hat{v}_t = \gamma\hat{v}_{t-1}\left(1-\exp\left\{-\theta\left(\hat{v}_{t-1}-c\right)^2\right\}\right)+\varepsilon_t \tag{26}$$

We set a broad range of parameter values for $\lambda_1 = 0.5$, $\lambda_2 = 1.0$ $\tau_1 = 0.25$, $\tau_1 = 0.50$. The location parameter c is assigned by drawing from a uniform distribution with lower and upper bounds of $-5(-10)$ and $5(10)$, respectively. Analogously, the parameter θ is assigned by drawing from a uniform distribution with lower and upper bounds of $(0.001, 0.01)$ with slow transition between regimes (θl) and $(0.01, 0.1)$ with fast transition between regimes (θh), respectively. The nominal size of the tests are determined at 0.05, the number of replications is 5000 and the sample size is considered for $T = 50,100$. The results of power experiments and power comparison with the HM and Kruse tests are displayed in Table 9.

When interpreting the power performances of the HM, Kruse and HM-Kruse tests which are summarized at Table 9, we observe that the new HM-Kruse unit root test is more powerful than the alternative tests which consider only the two smooth breaks (HM) and only the nonlinear adjustment (Kruse) for Model A, B and C.

Table 9: Power Experiments and Comparison of HM, Kruse and HM-Kruse Unit Root Tests

	$c_{\pm 5}, \theta l$			$c_{\pm 5}, \theta h$			$c_{\pm 10}, \theta l$			$c_{\pm 10}, \theta h$		
Model A												
	HM	Kruse	HM-Kruse	HM	Kruse	HM-Kruse	HM	Kruse	HM-Kruse	HM	Kruse	HM-Kruse
T=50	0.000	0.190	**0.222**	0.468	0.802	**0.820**	0.068	0.721	**0.753**	1.000	1.000	1.000
T=100	0.002	0.337	**0.358**	0.835	0.885	**0.890**	0.760	0.928	**0.934**	1.000	1.000	1.000
Model B												
	HM	Kruse	HM-Kruse	HM	Kruse	HM-Kruse	HM	Kruse	HM-Kruse	HM	Kruse	HM-Kruse
T=50	0.000	0.094	**0.103**	0.282	0.735	**0.744**	0.015	0.566	**0.585**	0.999	0.999	**1.000**
T=100	0.000	0.207	**0.342**	0.776	0.845	**0.886**	0.525	0.878	**0.929**	1.000	1.000	1.000
Model C												
	HM	Kruse	HM-Kruse	HM	Kruse	HM-Kruse	HM	Kruse	HM-Kruse	HM	Kruse	HM-Kruse
T=50	0.000	0.094	**0.105**	0.000	0.735	**0.745**	0.000	0.566	**0.587**	0.000	0.999	**1.000**
T=100	0.000	0.207	**0.216**	0.001	0.845	**0.849**	0.000	0.878	**0.881**	0.001	1.000	1.000

Notes: The values are rejection rates of HM, Kruse and HM-Kruse tests and the bold values display the cases where each test performs better. In power comparisons we consider $\tau_{d_t=1}$ statistic for model A and $\tau_{[d_t=1\,t]'}$ statistic for models B and C at Kruse test and $s_{2\alpha}$, $s_{2\alpha(\beta)}$ and $s_{2\alpha\beta}$ statistics for models A, B and C, respectively at HM test.

4. Empirical Application

In this section, as an empirical example of our new test procedures which are labelled HM-KSS, HM-Sollis and HM-Kruse, we apply them to the monthly data set of crude oil prices (US$ per barrel). The data set covers the period from January 1980 to December 2016 with 444 observations. Figure 1 plots the crude oil prices in natural logarithm form. We also compute the test statistics for the alternative tests which are the HM (2002), KSS (2003), Sollis (2009) and Kruse (2011) tests for the comparison of the empirical results. We consider Model C for the HM (2002), HM-KSS, HM-Sollis and HM-Kruse tests and demeanad and detrended data for the KSS (2003), Sollis (2009) and Kruse (2011) tests. The empirical results are presented at Table 10.[1]

Figure 1: The Plot of Logarithm of Crude Oil Prices

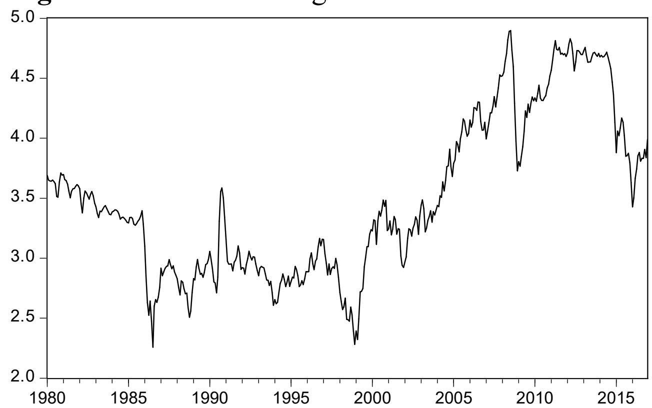

Table 10: Empirical Applications of Unit Root Tests

Unit Root Tests		Lag Length	Test Statistics
HM	$s_{2\alpha\beta}$	11	-5.898
KSS	$t_{NL,t}$	10	-2.841
Sollis	$F_{AE,t}$	10	4.040
Kruse	τ_t	11	8.265
HM-KSS	$t_{2SNL\alpha\beta}$	11	-5.466*
HM-Sollis	$F_{2SNL\alpha\beta}$	11	16.948*
HM-Kruse	$\tau_{2SNL\alpha\beta}$	11	34.097*

Notes: * denotes the rejection of unit root hypothesis at the 5% significance level. For all of the tests, lag lengths are determined through AIC.

As shown in Table 10, based on the $s_{2\alpha\beta}$, $t_{NL,t}$, $F_{AE,t}$ and τ_t test statistics, the null hypothesis of the unit root cannot be rejected at the 5% significance level. In other words, for the crude oil prices, the HM, KSS, Sollis and Kruse tests are unable to reject a unit root. However, according to the $t_{2SNL\alpha\beta}$, $F_{2SNL\alpha\beta}$ and $\tau_{2SNL\alpha\beta}$ test statistics, the null hypothesis of the unit root is rejected at the 5% significance level for the crude oil prices series. The HM-KSS, HM-

[1] The WinRATS codes to employ empirical application are available upon request.

Sollis and HM-Kruse tests yield evidence on the stationary of crude oil prices in comparison with their alternatives, the HM, KSS, Sollis and Kruse tests.

5. Conclusions

This paper proposes new three unit root testing procedures which consider jointly for two structural breaks and nonlinear adjustment. The structural breaks are modelled by means of two logistic smooth transition functions and nonlinear adjustment is modelled by means of ESTAR models. In the new unit root testing procedures labelled HM-KSS, HM-Sollis and HM-Kruse, the null hypothesis of the unit root is tested against the alternative of nonlinear and stationary with two smooth breaks. We study the finite sample properties and the power of proposed tests with Monte Carlo simulations and find that the empirical sizes of three unit root tests are quite close to the nominal one. We also find that our HM-KSS unit root test is more powerful than the alternative tests which are considering only the two smooth breaks (HM) and only the nonlinear adjustment (KSS). Similarly, HM-Sollis has greater power than the tests of HM (2002), but the Sollis (2009) the Sollis (2009) unit root test performs better than the HM-Sollis test for model A and our HM-Kruse unit root test is superior over the HM (2002) and Kruse (2011) tests. In all cases the new HM-KSS, HM-Sollis and HM-Kruse unit root tests are more powerful than the alternative tests which consider only the two smooth breaks (HM) and only the nonlinear adjustment (KSS, Sollis and Kruse).

As an empirical application to the crude oil prices, the HM (2002), the KSS (2003), the Sollis (2009) and the Kruse (2011) tests fail to reject the null hypothesis of the unit root. However the HM-KSS, HM-Sollis and HM-Kruse tests proposed here reject the unit root hypothesis. It could be concluded that our new proposed HM-KSS, HM-Sollis and HM-Kruse tests do not lead to over-rejections of the null hypothesis of the unit root.

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