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# Economics versus psychology. Risk, uncertainty and the expected utility theory

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## **Abstract.**

The present contribution examines the emergence of expected utility theory by John von Neumann and Oskar Morgenstern, the subjective the expected utility theory by Savage, and the problem of choice under risk and uncertainty, focusing in particular on the seminal work "The Utility Analysis of Choices involving Risk" (1948) by Milton Friedman and Leonard Savage to show how the evolution of the theory of choice has determined a separation of economics from psychology.

Keywords: Rational Choice, Risk, Uncertainty, Expected Utility Theory.

JEL Classification: C70, D80, D81.

## **1. Introduction**

During the first half of the 20th century, rational choice theory has become the core of neoclassical economics. In particular, the evolution of the theory of decision-making was marked by an "anti-psychologist" movement. A new theoretical framework has been established by crystallizing, through analytical terms and mathematical constraints, the process of rational choice, stripping it of any psychological and descriptive content. Rationality was construed as consistency of preference or of choice, so that an individual is rational if he has coherent behaviors and consistent preferences. In other words, rational is the individual that is able to think clearly and to make decisions and judgments that are based on reason. Consequently, there is a clear separation between economics, which has become the center of the normative theories, (i.e. how the individual should act), and psychology which has become the field of study of actual behavior. Although discrepancies between normative prescriptions and behavior were well known, they were considered a minor problem.

This paper examines the emergence of expected utility theory by John von Neumann and Oskar Morgenstern, the subjective the expected utility theory by Savage, and the problem of choice under risk and uncertainty, focusing in particular on the seminal work "The Utility Analysis of Choices involving Risk" (1948) by Milton Friedman and Leonard Savage to show how the evolution of the theory of choice has determined a separation of economics from psychology.

## **2. Choice under risk and uncertainty**

The rational choice under risk and uncertainty was based on the notion of probability. In fact, probability theory was developed as a rational approach to risk and uncertainty, and the great

mathematicians that gave fundamental contributions to this theory in XVIII and XIX centuries implicitly assumed that their models should have allowed persons exposed to risky decisions to behave rationally. The efforts of mathematicians were only partially successful. In fact, despite these efforts, the progressive edification of the theory of probability did not remove some "irrationalities" in gambler's behavior. Still today there are some typical lottery conditions in which gamblers exhibit systematic discrepancies from the normative prescriptions of the theory.

The best-known phenomenon in this respect is now called "gambler's fallacy": it happens when the sequence of numbers extracted in repetitive runs of a lottery appears to gamblers not to be random<sup>1</sup>. This line of thinking is incorrect because past events do not change the probability that certain events will occur in the future.

Let's analyze "gambler's fallacy" in more details.

In a random sequence of tosses of a fair coin, for example, gamblers expect sequences where the proportion of heads and tails in any short segment stays far closer to .50 .50 than probability theory would predict. In other words, gamblers expect that also a short sequence of heads on the toss of a coin is balanced by a tendency for the opposite outcome (e.g., tails) and bet accordingly.

But each toss of the coin is an independent event, which means that any previous toss of coin has no bearing on future toss. That some gamblers' beliefs about probability are systematically biased have become more and more evident in parallel with the progressive construction of the theory. Already in 1796, Pierre Simon de Laplace was concerned with errors of judgment that he gave the first published account of the gambler's fallacy.

### 3. The expected utility theory

The expected utility theory (EUT) is a special instance of the theory of choice under objective uncertainty, or risk. The theory's main concern is the representation of individual attitudes toward risk. Its basic premises are (Karni, 2014, p. 4):

- (a) because the outcomes,  $x_i$ , are mutually exclusive, the evaluation of risky prospects entails separate evaluations of the outcomes;
- (b) these evaluations are quantifiable by a cardinal utility,  $u$ ;
- (c) the utilities of the alternative outcomes are aggregated by taking their expectations with respect to the objective probabilities,  $p_1; \dots ; p_n$ .

EUT states that the decision maker chooses between risky and uncertain prospects by comparing their expected utility values, i.e. the weighted sum obtained by adding the utility values of outcomes multiplied by their respective probabilities. In EUT, the probabilities are a primitive concept representing the objective uncertainty.

The expected-utility theory of behavior under risk, or, more precisely, the principle of maximizing expected monetary values had been first advanced by Daniel Bernoulli in 1738 as a resolution of the St. Petersburg Paradox. Bernoulli resolved the paradox by assuming a logarithmic utility function of

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<sup>1</sup> The gambler's fallacy is also known as Monte Carlo fallacy.

wealth, whose essential property was “diminishing marginal utility.” So, Bernoulli (1738) had devised the notion of expected utility, wherein he decomposed the valuation of a risky venture as the sum of utilities from outcomes weighted by the probabilities of outcomes.

In the St. Petersburg game people were asked how much they would pay for the following prospect: if tails come out of the first toss of a fair coin, to receive nothing and stop the game, and in the complementary case, to receive two guilders and stay in the game; if tails come out of the second toss of the coin, to receive nothing and stop the game, and in the complementary case, to receive four guilders and stay in the game; and so on *ad infinitum*. The expected monetary value of this prospect is

$$\sum_{n=1}^{\infty} \frac{1}{2^n} (2^n) = + \infty$$

Since the people always set a definite, possibly quite small upper value on the St. Petersburg prospect, it follows that they do not price it in terms of its expected monetary value. Bernoulli argued in effect that they estimate it in terms of the utility of money outcomes, and defended the Log function as a plausible idealization, given its property of quickly decreasing marginal utilities. Because the resulting series,

$$\sum_{n=1}^{\infty} \frac{1}{2^n} (\text{Log} 2^n),$$

is convergent, Bernoulli's hypothesis is supposed to deliver a solution to the paradox. Thus, Bernoulli's hypothesis counts as the first systematic occurrence of expected utility theory (Mongin, 1997).

However, Bernoulli's assumption of diminishing marginal utility seemed to imply that, in a gamble, a gain would increase utility less than a decline would reduce it. Consequently, many scholars (economists, statisticians) concluded, the willingness to take on risk must be “irrational”, and thus the issue of choice under risk or uncertainty was viewed suspiciously, or at least considered to be outside the realm of an economic theory which assumed rational actors.

### 3.1 Von Neumann and Morgenstern's theory of expected utility under risk

The great task of John von Neumann and Oskar Morgenstern was to lay a rational foundation for decision-making under risk according to expected utility rules. Thus, the theory of expected utility under risk received its first axiomatic characterization in von Neumann and Morgenstern's *Theory of games and economic behavior* (1947, 2nd edition)<sup>2</sup>. In particular, von Neumann and Morgenstern state a series of axioms about the individual's preferences over indifference classes of lotteries, and offer a proof that an individual obeying these axioms will then follow expected utility theory. Consequently, the preference structure is depicted by a set of axioms. In the normative interpretation,

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<sup>2</sup> It is important to observe that von Neumann and Morgenstern developed a utility theory intended to serve as auxiliary to their main goal: a theory of games, which they planned to make the most adequate description of economic activity in general (Mengov, 2015).

these axioms are regarded as tenets of rational choice and should be judged by their normative appeal. In fact, if an individual does not maximize his expected utility he violates in his choice some precise axiomatic principles, which are rationally binding (Schilirò, 2013). Actually, von Neumann and Morgenstern's expected utility theory has been generally accepted as a normative model of rational choice.

In different terms, von Neumann and Morgenstern analyzed the strategic behavior of players in noncooperative zero-sum games in which no pure strategy equilibrium exists. In such games, the equilibrium may require the employment of mixed strategy. By adopting the axiomatic approach to depict the decision maker's preference relation on the set of objective risks, von Neumann and Morgenstern identified necessary and sufficient conditions for the existence of a utility function on a set of outcomes that captures the decision maker's risk attitudes, and represented his/her choice as expected utility maximizing behavior (Karni, 2014).

Let's analyze in more analytical details the EUT.

The expected utility theory under risk is concerned with the evaluation of risky prospects, depicted as lotteries over an arbitrary set of outcomes (or prizes).

Let

$$X = \{x_1; \dots; x_n\}$$

be the set of outcomes and denote a risky prospect by

$$(x_i; p_i; \dots, x_n; p_n);$$

where for each  $i$ ,  $p_i$  indicates the probability of the outcomes  $x_i$ . In the model of expected utility theory under risk, risky prospects are evaluated according to the following functional form:

$$\sum_{i=1}^n u(x_i)p_i;$$

where  $u$  is the real-valued function on  $X$  representing the decision maker's tastes.

In other words,  $i$  are the states of the world. In each state of the world,  $i$ , the individual receives  $x_i$  (euros/dollars). The probability of receiving  $x_i$  is  $p_i$ . An individual will prefer a risky lottery over another if their utility is higher in the first lottery compared to the second. It is important to note that in their theoretical framework, Von Neumann and Morgenstern provide a method to measure cardinally the marginal utility of money. But this method does not refer whether the marginal utility of money diminishes or increases.

This EUT states that the decisions of economic agents conform to an expected utility function of the outcomes<sup>3</sup>. In practice, individuals should always choose in risky situations the alternatives that offer them the highest utility, i.e. the alternatives that offer ever higher earnings or the lowest losses. The theory is built with the minimum set of reasonable assumptions (axioms). The main axioms of EUT

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<sup>3</sup> The expected utility model derives its name from the fact the preference function  $V(\cdot)$  consists of the mathematical expectation of the von Neumann-Morgenstern utility function  $U(\cdot)$ , with respect to the probability distribution  $P$ . Of course, there is a variety of attitudes toward risk depending upon the shape of the von Neumann-Morgenstern utility function  $U(\cdot)$ .

according to von Neumann and Morgenstern are: completeness; transitivity; continuity of preferences<sup>4</sup>. The set of axioms provides criteria for the rationality of choices. What do these axioms of expected utility mean? First, all individuals are assumed to make completely rational decisions (reasonable). Second, people are assumed to make these rational decisions among thousands of alternatives. Finally, uncertainty or risk does not possess utility or disutility on its own.

The great task of von Neumann and Morgenstern was to lay a rational foundation for decision-making under risk according to expected utility rules. An important positive aspect of von Neumann and Morgenstern's contribution is that, through the axioms of the expected utility theory, the underlying logic of the decision-making behavior is greatly simplified. Though the novelty of using the axiomatic method caused that most economists of the time would find their contribution inaccessible. However, restatements and re-axiomatizations by Jacob Marschak (1950) and Paul Samuelson (1952)<sup>5</sup> did much to make the theory more accessible. At the same time, a growing number of instances emerged that clearly showed how the theory was unable to explain real economic behavior. But, in any case, it remains a cornerstone in the development of theory of choice and, more generally, of science.

### **3.2 Savage Subjective Expected Utility theory**

Years after the contribution of von Neumann and Morgenstern, Leonard Savage in *The Foundations of Statistics* (1954) proposed the first complete axiomatic subjective expected utility theory, focusing on uncertainty rather than risk. Subjective expected utility theory is another relevant instance of the theory of choice under uncertainty, while the expected utility hypothesis was originally formulated to be used with specified or objective probabilities.

Savage introduced his new analytical framework, which was a synthesis of the ideas of de Finetti (1937)<sup>6</sup> and von Neumann and Morgenstern (1947). He provided necessary and sufficient conditions for the existence and joint uniqueness of utility and probability, as well as the characterization of individual choice in the face of uncertainty as expected utility maximizing behavior (Karni, 2014).

In Savage's approach the notion of probability does not appear as a primitive concept in his model. One important goal of Savage's theory, with regard to probability, was to furnish the prior probability, that constituted the missing ingredient necessary to complete Bayes' model<sup>7</sup>.

The model of subjective expected utility by Savage (1954) postulates a preference structure that permits (Karni, 2014, p.11):

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<sup>4</sup> Von Neumann and Morgenstern do not feature an assumption corresponding to what today we call the 'Independence Axiom' (Moscati, 2016).

<sup>5</sup> Samuelson not only gave a major contribution to re-axiomatizations of the von Neumann and Morgenstern's theoretical approach, but he also became a supporter of the expected utility hypothesis (Moscati, 2016).

<sup>6</sup> de Finetti devised the same notion of probability already suggested by Ramsey (1926). The basic idea behind the Ramsey-de Finetti derivation is that by observing the bets people make, one can presume this reflects their personal beliefs on the outcome of the race. Thus, Ramsey and de Finetti argued that subjective probabilities can be inferred from observation of people's actions. de Finetti's model, in particular, was based on the notion of expected value maximizing behavior, or linear utility.

<sup>7</sup> Bayes outlined a method, since known as Bayes' rule, for updating probabilities in light of new information. Bayes' method does not specify how the original, or prior, probabilities to be updated are determined (Karni, 2014).

- (i) the numerical expression of the decision maker's valuation of the consequences by a utility function;
- (ii) the numerical expression of the decision maker's degree of beliefs in the likelihoods of events by a finitely additive, probability measure;
- (iii) the evaluation of acts by the mathematical expectations of the utility of their consequences with respect to the subjective probabilities of the events in which these consequences materialize. In this model, the utility of the consequences is independent of the underlying events, and the probabilities of events are independent of the consequences assigned to these events by the acts.

In analytical terms, Savage proved that, if the decision-maker adheres to axioms of rationality, believing an uncertain event has possible outcomes  $x_i$ , each with a utility of  $u(x_i)$ , the choices of the individual can be explained by this utility function combined with the subjective belief that there is a probability of each outcome,  $P(x_i)$ . Therefore, the subjective expected utility is the resulting expected value of the utility.

$$\sum_i u x_i P(x_i).$$

However, the descriptive validity of Savage's model, in particular, the specific functional form of the representation and the separability and linearity in the probabilities, has been questioned.

The most severe criticism in this regard is due to Ellsberg (1961), who demonstrated, by using simple thought experiments, that individuals display choice patterns that are inconsistent with the existence of beliefs representable by a probability measure.

#### 4. The Contribution by Friedman and Savage

The properties of the expected utility hypothesis when payoffs are univariate (i.e. money) were further developed in the post-war period. In 1948, Milton Friedman and Leonard Savage published in the *Journal of Political Economy* a seminal article 'The Utility Analysis of Choices Involving Risks', wherein they developed their own utility function (known as the Friedman-Savage utility function). In this article, the authors attempt "to provide a crude empirical test by bringing together a few broad observations about the behavior of individual in choosing among alternatives involving risk and investigating whether these observations are consistent with the hypothesis revived by von Neumann and Morgenstern. It turns out that these empirical observations are entirely consistent with the hypothesis if a rather special shape is given to total utility curve of money" (Friedman and Savage, 1948, p. 282).

Friedman and Savage start from the question: why do people buy both lottery tickets and insurance against losses? It is well known that when a person gets an insurance policy, he pays to escape or avoid risk.

"An individual who buy fire insurance on a house he owns is accepting the certain loss of a small sum (the insurance premium) in preference to the combination of a small chance of a much larger loss (the value of the house) and a large chance of no loss. That is, he is choosing certainty in preference of uncertainty" (Friedman and Savage, 1948, p. 279).

But when it buys a lottery ticket, he gets a small chance of a large gain, whereas is subjecting himself to a large chance of losing a small amount. Thus, he assumes risk. So that he is choosing uncertainty in preference to certainty<sup>8</sup>. In addition, the empirical evidence for the willingness of persons of all income classes to buy insurance or lottery tickets is extensive.

Friedman and Savage remind us that this choice among different degree of risk so prominent in insurance and gambling is also very important and present in a much broader range of economic choices (e.g. in labor market, financial markets, business activities, etc.). But, according to Friedman and Savage, although economic theorists (e.g. Marshall, Edgeworth, Fisher, Pareto) explained the choices among riskless alternatives in terms of maximization of utility, on the other hand they rejected the utility maximization as an explanation of choices among different degrees of risk.

Friedman and Savage, instead, basing their analysis on von Neumann and Morgenstern's book *Theory of Games and Economic Behavior* (1947, 2nd edition), do not reject the utility maximization as an explanation of choices among different degrees of risk. In fact, von Neumann and Morgenstern challenged the rejection of maximization of expected utility and showed the conditions under which the expected value of utility is maximized in choosing among alternatives involving risk. However, the axiomatization of expected utility theory that Friedman and Savage (1948, pp. 287-288) have advanced consisted of three assumptions, which are not logically equivalent to von Neumann and Morgenstern's axioms, as questioned by Paul Samuelson in his "Japanese paper" of 1950 (Moscati, 2016).

But apart from the axiomatization of the expected utility hypothesis, Friedman and Savage theorized that individuals who both gamble and buy insurance may exhibit utility functions with both concave and convex segments. Then, Friedman and Savage consider and illustrate a utility function consistent with willingness of a low-income consumer unit both to purchase insurance and to gamble (*ibid.*, p. 285).

Following their arguments in more details, these authors maintain that a single individual could have different utility functions depending on their initial wealth. The implication of an individual being, at the same time, risk-loving and averse, determines that its utility function has different curvatures, based upon the amount of wealth the individual has, in fact part of the utility function is concave, and part is convex, as shown in the Figure 1. This figure is consistent with the five statements by Friedman and Savage describing the features of the observable behavior concerning individual. The five statements are the following (*ibid.*, p. 294):

1. Consumer units prefer larger to smaller certain incomes;
2. low-income consumer units buy, or are willing to buy, insurance;
3. low-income consumer units buy, or are willing to buy, lottery tickets;
4. many low-income consumer units buy, or are willing to buy, both insurance and lottery tickets;
5. lotteries typically have more than one prizes.

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<sup>8</sup> The extensive market for highly speculative stocks is a border-line case that could equally well be designated as investment or gambling (Friedman and Savage, 1948, p. 285).



The reason why these particular statements are selected, according to Friedman and Savage, is that they are convenient to handle and the restrictions imposed to rationalize them turn out to be sufficient to rationalize all the behavior described by them in their article (*ibid.*, pp. 283-287).

Let's focus on Figure 1, where  $W$  represents the wealth of the individual and  $U$  is the total utility function.

Across the lower range the individual wishes to play it safe, but above a certain margin he is willing to take gambles. In the first part the utility function ( $U_0 U_a$ ) is concave to the origin, so the individual exhibits risk aversion and is inclined to buy insurance against potential losses; in the second part ( $U_a, U_c$ ) the function is convex to the origin, so the individual becomes risk loving. In the third part (above  $U_c$ ), the curve would become concave again at a suitably high level of wealth<sup>9</sup>.

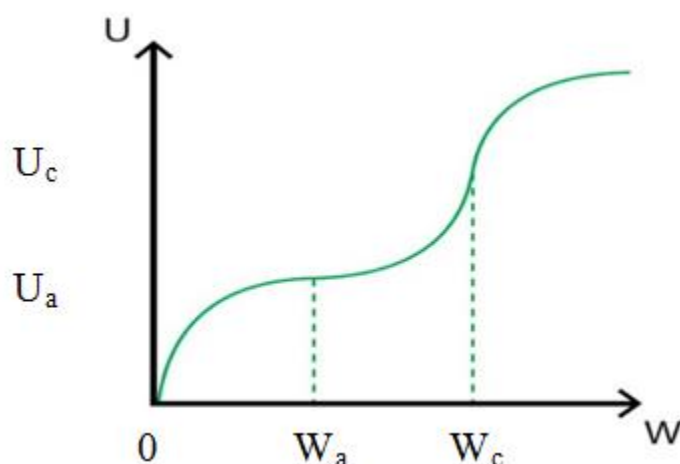


Figure 1

The conclusions by Friedman and Savage (1948, p. 303) regarding the behavior of consumer units in choosing among alternatives involving risk are that an individual behaves as if

1. it had a consistent set of preferences;
2. it chose among the alternatives involving risk that one for which the expected utility (as contrasted with the utility of the expected income) is largest;
3. the function describing the utility of money income had in general the following properties:
  - a) utility rises with income, i.e. marginal utility of money income everywhere positive;
  - b) it is convex from above below some income (i.e. diminishing marginal utility of money income for incomes below some income), concave between that income and some larger income (i.e. increasing marginal utility of money for incomes between that income and some larger income), and convex for all higher incomes (i.e. diminishing marginal utility of money income for all higher incomes);
4. most consumer units tend to have incomes that place them in the segments of the utility function for which marginal utility of money income diminishes.

Friedman and Savage underline that point 2 is an ancient idea revived<sup>10</sup> and given new content by von Neumann and Morgenstern, while points 3b and 4 are the consequence of the attempt of their

<sup>9</sup> Fig. 1 is equivalent to Fig.3 showed in Friedman and Savage (1948, p.297).

<sup>10</sup> Friedman and Savage (1948, p. 281) recall Bernoulli's analysis of the St. Petersburg paradox.

paper to use this idea (in point 2) to rationalize existing knowledge about the choices people make among alternatives involving risk.

An important feature of this version of expected utility theory provided by Friedman and Savage is that it has been considered by the authors as a descriptive model of economic behavior, not just a normative model. In fact, in the subsection of the article where Friedman and Savage (1948) (pp. 297-298) discuss the descriptive realism of the hypothesis, they maintain that the validity of their hypothesis depends solely on whether it yields sufficiently accurate predictions about the class of decisions with which the hypothesis deals. The reasoning relative to the approach “as if”, used in the Friedman and Savage’s article, will become the “as if” methodology in Friedman’s famous essay ‘The Methodology of Positive Economics’ (1953).

Later, Harry Markowitz (1952) criticized the Friedman-Savage utility function. In fact, he proposed a utility function that explains gambling and insurance which differs significantly from Friedman and Savage’s (1948) utility function. Markowitz argued that the final concavity of their function assumes that individuals with the highest incomes would never gamble. He raised a few important issues, later on confirmed by experimental studies. First, Markowitz proposed measuring utility based on a reference level instead of in absolute values. This implied that, to individuals, small gains would provide an increasing utility, while big gains would provide a decreasing utility, anticipating somehow Kahneman and Tversky’s prospect theory (1979)<sup>11</sup>. So, he claimed that not only total wealth but also change of wealth may be a factor in the decision-making process, and second, that “temporary” changes in the utility function might take place and, therefore, a distinction should be made between “customary” wealth and present wealth. Moreover, he also suggested that the inflection point temporarily “travels” along the utility function (Levy and Wiener (2013)). Thus, the Markowitz’s utility function is the following as represented in Figure 2.

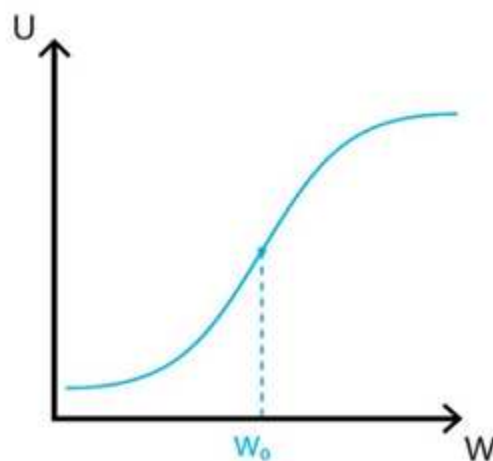


Figure 2

The contribution of Markowitz (1952), according to Rabin and Thaler (2001) (p.223) “provides simple, clear, and decisive illustrations of how the combined convex/concave functions lead to a host of patently false predictions”.

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<sup>11</sup> For a discussion on prospect theory, Schilirò (2016).

However, in 1952 Friedman and Savage published another article ‘The Expected-Utility Hypothesis and the Measurability of Utility’, wherein they constructed an expected utility curve which, they claimed, provided a reasonably accurate representation of human behavior at the aggregate level. In this article, Friedman and Savage consider the individual’s expression of preferences as irrelevant and, consequently, not to be submitted to empirical control. Deviations from rational decision making were supposed to be detectable only at the aggregate level, and many attempts were made to justify the persuasion that, on average, individuals behave rationally.

Friedman, in particular, suggested an evolutionary defense of full rationality by claiming that those who failed to conform to rational behavior would be gradually excluded by market selection. A fortiori, according to this view, the psychological aspects of decision making were not considered worthy of investigation, because non-rational behaviors were thought to be a minor aspect of market economies.

## **Conclusions**

This paper has examined the expected utility theory by von Neumann and Morgenstern (1947) (2nd edition), the subjective the expected utility theory by Savage (1954), and the problem of choice under risk and uncertainty, focusing in particular on the seminal work ‘The Utility Analysis of Choices involving Risk’ (1948) by Milton Friedman and Leonard Savage. The major goal of the paper has been to show that the evolution of the theory of choice was marked by an “anti-psychologist” approach, thus creating a situation of separation of economics from psychology.

For several decades, the expected utility theory has been the paradigmatic model of decision-making under risk and uncertainty. The expected utility model acquired its dominant position because it is founded on normatively compelling principles, and its representation has an appealing functional form (Karni (2014)). However, although the expected utility theory has had great success, since it makes the mathematical modeling of the decision-making process very simple, it neglects some important variables involved in the decision process such as the complexity of the task, the limits of the cognitive resources of the agents, etc. (Schilirò (2012)).

The contributions by Friedman and Savage (1948, 1952), even more so, have placed economics in an autonomous position with respect to psychology; the former being stripped of any psychological elements. However, Maurice Allais (1953), shortly after, will undermine this theoretical position with his epistemology of exact descriptions and the related falsifications of rational choice theory (Schilirò (2012, 2013)).

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