Tariff and Equilibrium Indeterminacy–(I)

Yan Zhang

New York University

April 2008

Tariff and Equilibrium Indeterminacy–(I)

Yan Zhang*

Department of Economics

New York University

March 3, 2008

Abstract

Schmitt-Grohe and Uribe (1997, henceforth SGU) prove that in a standard neo-classical growth model the fiscal increasing returns induced by the endogenous factor income tax rate (assuming that the government expenditure is exogenous) has a close correspondence with the production increasing returns in Benhabib and Farmer (1994) model. Wen and Aguiar-Conraria (2005, 2006, henceforth WAC) extend the Benhabib-Farmer model to open economy by introducing imported foreign production factors. We prove that in a modified WAC model without increasing returns, using the tariff revenue from the imported production factor to finance the exogenous government expenditure, we can also have indeterminacy. From this perspective, factor income tax and tariff share similar channels to generate indeterminacy.

*Chapter one of my Ph.D dissertation. It should be the joint work of my advisor, Prof. Jess Benhabib, who initiates this project and corrects several mistakes as I write this paper. I also thank Wen Yi, Pierpaolo Benigno, Paul Dower, Martin Uribe and Viktor Tsyrennikov for their valuable comments. Correspondence: Zhang Yan, Economics Department, New York University, NY, 10003, USA. Tel.:1-212-992-9777; *E-mail address*: laurencezhang@yahoo.com (Y. Zhang). For a recent extensive survey of the literature, see Benhabib and Farmer (1999).

KeyWords : Indeterminacy, Endogenous Tariff Rate, Small Open Economy, Exogenous Government Expenditure

JEL Classification Number: Q43, F41

1. Motivation

Benhabib and Farmer (1999) provide the sources of the indeterminacy and sunspots in macroeconomics (pp 390):

> "Sunspots cannot occur in finite general equilibrium models with complete markets since their existence would violate the first welfare theorem; risk averse agents will generally prefer an allocation that doesn't fluctuate to one that does. Examples of departures from Arrow-Debreu structure that permit the existence of sunspots include (1) incomplete participation in insurance markets as in the OLG model, (2) incomplete markets due to transactions costs or asymmetric information, (3) increasing returns to scale in the technology, (4) market imperfections associated with fixed costs, entry costs or external effects, and (5) the use of money as a medium of exchange."

Tariff as a special kind of tax (or a kind of transaction cost in international trade) can also be a source of the indeterminacy.

The channel of the indeterminacy generated by the factor income taxes in a one sector neoclassical growth model was challanged by Schmitt-Grohe and Uribe (1997). In their model, they conclude that, "Under a balanced budget rule the rational expectations equilibrium may exist. In order to obtain this result, the presence of the endogenous distortionary taxes is crucial: it is straightforward to show that endogenous fluctuations are impossible when the balanced budget rule consists of fixed income tax rates and endogenous government expenditures."

Do tariff and factor income taxes (in SGU model) deliver indeterminacy in the same way? This paper gives a positive answer. Wen and Aguiar-Conraria (2005, 2006) extend the Benhabib-Farmer model into an open economy by introducing imported foreign factors of production. They show that reliance on foreign energy has a potentially important effect on economic activity, it destabilizes the economy by increasing the likelihood of indeterminacy, making fluctuations driven by self-fulfilling expectations more likely to occur. Leung (1999) presents an endogenous growth model in which the tariff revenue collected from the imported production factor finances the government expenditure in a small open economy.[1] Endogeneous tariff rates are also used by Loewy (2004) and Mourmouras (1991) in a two-country open economy endogenous growth model and a small open economy OLG model respectively. This approach originates from Ramsey (1927).

Following similar methodology as SGU, we present another reason why a balanced-budget rule can be destabilizing. We embed a balanced budget rule into the open economy version of Benhabib and Farmer model without increasing returns in the production side and assume that the fiscal authority finances a **pre-set** level of expenditure with the tariff revenue.[2] We show that under this type of policy, persisitent and recurring fluctuations in

[1] The revenue motive behind the imposition of trade taxes is well documented. See, Kindleberger and Lindert (1978, p. 143), and Riezman and Slemrod (1987). Recently, Manoj Atolia (2006) used the tariff and income taxes revenue to finance the government public investment.

[2] For simplicity, we assume that the government doesn't impose the consumption taxes or factor income taxes on the goods or production factors. Adding other taxes changes nothing as long as part of the exogenous spending must be still financed by the tariff on the imported factor. See Velasco(1996) for a similar explanation to fiscal increasing returns induced by taxes on domestic capital.

aggregate activity become possible in the absence of shocks to the fundamentals. Specifically, under a balanced budget rule, the rational expectation equilibria can be indeterminate and stationary sunspot equilibria may exist. Thus, an endogenous tariff rate could also be a source of fiscal increasing returns.

To show the main result of the paper analytically, in section 2, we consider a simple case in which government expenditures are constant and the only source of the government revenues is from the tariff.[3] We adopt the assumption that labor is indivisible, as in Hansen (1985) and SGU (1997). We show that the necessary and sufficient condition for a balanced budget rule to generate indeterminacy requires that the steady state tariff rate is greater than the share ratio of capital and imported factors in the production function and is less than a critical value τ^*.

From a policy standpoint, my results suggest that if the proposed balanced budget rule for the European countries is to avoid endogenous aggregate instability, it should be combined either with restrictions on the government ability to change the tariff rate in response to innovations in the state of the economy or with a reduction in the level of tariff rates currently in place. Relating the current high tariff rate prevailing in EU in 2002, some countries like Denmark and Netherlands, which are economies quite dependent on the imported exhaustible natural resources, can be easily pushed into destabilization.[4] I use the WAC's estimation of the imported goods share in the two countries and find that the high tariff rate on oil in the EU leads the two countries into destabilization.

[3] The tariff revenue in this model can also be interpreted as oil tax revenue. Miguel and Manzano (2006) consider a small open economy, in which the government finances an exogenous flow of public spending by using consumption and oil taxes and by issuing debt.

[4] Although throughout the paper, we did the numerical case for the developed countries, the results also hold for the less-developed countries which productions are dependent on the imported factors.

Similarly, the energy taxes which the EU countries have tried to impose recently also bring the potential dangers of destabilization into those countries which are economies largely dependent on imported non-reproducible resources. Those countries like Denmark and Netherlands whose production are heavily dependent on the imported factor of oil should pay close attention to the control of energy taxes in order to stabilize the economy. This is particularly true when we regard the energy taxes as the optimal tariff rate since David Newbery (2005) says the energy taxes seem to be very high in some EU countries.

In sections 3 and 4, we compare our model with Benhabib and Farmer, SGU and WAC models and find that (1) the indeterminacy condition obtained here also has a close correspondence with the one obtained in the increasing returns model of Benhabib and Farmer (1994); (2) if the imported factor is mainly a labor substitute, the indeterminacy may not easily arise; and (3) the larger the imported energy share in GDP, the easier it is for the economy to be subject to multiple equilibria. Section 5 concludes.

2. The Basic Model

This is a modified open economy version of Benhabib-Farmer model without increasing returns. There are two production sectors in the economy, the final goods and the intermediate goods sector. The final goods sector is competitive and uses a continuum of intermediate goods to produce final output according to the technology.

$$Y = (\int_{i=0}^{1} y_i^{\lambda} di)^{\frac{1}{\lambda}}$$

where $\lambda \in [0, 1]$ measures the degree of factor substitution among intermediate goods. Let p_i be the relative price of the ith intermediate good in terms of the final good, the profits of the final good producer are given by

$$\Pi = Y - \int_{i=0}^{1} p_i y_i di$$

First order conditions for profit maximization lead to the following inverse demand functions for intermediate goods:

$$p_i = Y^{1-\lambda} y_i^{\lambda - 1}$$

The technology for producing the intermediate goods is given by

$$y_i = k_i^{a_k} n_i^{a_n} o_i^{a_0}$$

where the third factor in production, non-reproducible natural resources, o_t, is imported, and $a_k + a_n + a_0 = 1$ (constant returns to scale without externality or increasing returns in BF or WAC models).[5] Assuming the firms are price takers in the factor markets, the profits of the ith intermediate goods producer are given by

$$\pi_i = p_i y_i - (r + \delta) k_i - w n_i - p^o (1 + \tau) o_i$$

where $(r + \delta)$ denotes the user cost of capital, w denotes the real wage, and p^o denotes the

[5] The model is based on the standard DSGE models that incorporate foreign energy as a third production factor. This class of models have been used widely to study the business-cycle effects of oil price shocks. This literature includes Finn (2000), Rotemberg and Woodford (1996), Wei (2003), Aguiar-Conraria and Wen (2006).

real price of the imported good.[6] τ is the tariff rate imposed on the imported good, such as oil, which is uniform to all firms. The intermediate goods producers are monopolistically competitive facing downward sloping demand curves for intermediate goods, hence the profit can be written as

$$\pi_i = Y^{1-\lambda} y_i^{\lambda} - (r+\delta)k_i - wn_i - p^o(1+\tau)o_i$$

This function will be concave as long as $\lambda(a_k + a_n + a_0) \leq 1$. Profit maximization of each intermediate goods producing firm leads to the following first order conditions

$$r + \delta = \lambda a_k \frac{p_i y_i}{k_i}$$

$$w = \lambda a_n \frac{p_i y_i}{n_i}$$

$$p^o(1+\tau) = \lambda a_0 \frac{p_i y_i}{o_i}$$

In a symmetric equilibrium, we have $n_i = n$, $k_i = k$, $o_i = o$, $y_i = y = Y$, $\pi_i = \pi$, $p_i = 1$ and

$$\Pi = Y - (\int_{i=0}^{1} y_i^{\lambda} di)^{\frac{1}{\lambda}} = 0$$

[6] $\delta \in (0,1)$ denotes the depreciation rate of capital, r_t is the rental rate of capital.

$$\pi = (1 - \lambda(a_k + a_n + a_0))Y = (1 - \lambda)Y$$

Perfect competition in final goods will make the firms earn zero profits and impefect competition in the intermediate goods sector leads to positive profits if $\lambda < 1$.

The government collects the tariff revenue to finance its expenditure as in SGU. The tariff rate is endogenous and we assume that the foreign input is perfectly elastically supplied, i.e , p^o is independent of the factor demand for o_i .

$$p^o \tau o = G$$

A representative consumer maximizes the utility function that SGU and WAC adopt:

$$\sum_{t=0}^{\infty} \beta^t (\log c_t - b n_t)$$

subject to

$$c_t + s_{t+1} = (1 + r_t) s_t + w_t n_t + \pi_t$$

where s_t is aggregate saving. Here the aggregate factor payment, $p^o(1 + \tau_t)o_i$ goes to the foreigners ($p^o o_i$) and the government ($p^o \tau_t o_i$). The first order conditions with respect to labor supply and savings are given by

$$\begin{aligned} b &= \frac{w_t}{c_t} \\ \frac{1}{c_t} &= \frac{\beta(1 + r_{t+1})}{c_{t+1}} \end{aligned}$$

In equilibrium, $s_t = k_t$, and factor prices equal marginal products, the first order conditions and the budget constraint then become

$$bn_t = \frac{1}{c_t}\lambda a_n y_t \tag{1}$$

$$\frac{1}{c_t} = \frac{\beta(1 - \delta + \lambda a_k \frac{y_{t+1}}{k_{t+1}})}{c_{t+1}} \tag{2}$$

$$c_t + k_{t+1} = (1-\delta)k_t + (1-\lambda a_0)y_t,\ y_t = k_t^{a_k} n_t^{a_n} o_t^{a_0} \tag{3}$$

$$p^o \tau_t o_t = G \tag{4}$$

We can substitute out o in the production function using

$$o_t = \lambda a_0 \frac{y_t}{p^o(1+\tau_t)}$$

to obtain the following reduced form production function:

$$y_t = A k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}} \tag{4a}$$

where $A = (\frac{\lambda a_0}{p^o(1+\tau_t)})^{\frac{a_0}{1-a_0}}$ as the "technology coefficient" in a neoclassical growth model, which is inversely related to the foreign factor price and endogenous τ. In this reduced-form production function, the "effective return to scale" with respect to the capital and labor is measured by

$$\frac{a_k + a_n}{1 - a_0} = 1$$

This A term generates fiscal increasing returns since the tariff rate now is regressive with respect to the output. We can see this because $p^o \tau_t o_t = G = \frac{\tau_t \lambda a_0 y_t}{(1+\tau_t)}$ implies $\frac{\partial \tau}{\partial y} < 0$.

Proposition 1. *If the tariff rate is exogenous, the model doesn't display increasing returns to scale since A term is a constant. (in this case, the government expenditure is not exogenous under the balanced budget rule)*

Proposition 2. *If the government expenditure is exogenous, the tariff rate is regressive with respect to the tax base ($p^o o_t$), or the output, under the balanced budget rule, i.e. $\frac{\partial \tau}{\partial y} < 0$.*[7]

From these propositions, we can see that the countercyclical tariff rate ($\frac{\partial \tau}{\partial y} < 0$) can induce increasing returns to scale with respect to capital and labor. Guo and Harrison (2004) illustrate that under perfect competition and constant returns-to-scale, Schmitt-Grohé and Uribe's indeterminacy result depends crucially on a balanced-budget requirement whereby the tax rate decreases with the household's taxable income. In our model, we get a similar result that requires the countercyclical rate to generate indeterminacy.[8] Once we fix the tariff rate (or oil tax rate) like Miguel and Manzano (2006), the model doesn't display increasing returns to scale, so indeterminacy cannot arise.

[7] *This relation doesn't violate the evidence of a negative relationship between tariffs and growth, especially among the world's rich countries like those in EU, which is documented by David N.Dejong and Marla Ripoll,2005)*

[8] In order to compare our model with BF, SGU models,under perfect competition in factor and product markets, we use the continuous time model in the next subsection to show this point. We also think that the progressive tariff rate may make the economy against the sunspots in WAC model.

To facilitate interpretation of this model, we map the current model with the intermediate goods into a one-sector Benhabib and Farmer (1994) competitive model without production externalities, in which the aggregate production function is replaced by

$$y_t = k_t^{a_k} n_t^{a_n} o_t^{a_0}$$

and the reduced form production function is replaced by

$$y_t = A k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}} \tag{4a}$$

where $A = (\frac{\lambda a_0}{p_0(1+\tau_t)})^{\frac{a_0}{1-a_0}}$, $\lambda = 1$. With this change in the framework, the first order conditions, budget constraint of the household and the government balanced budget requirement become:

$$bn_t = \frac{1}{c_t} a_n y_t \tag{1a}$$

$$\frac{1}{c_t} = \frac{\beta(1-\delta+a_k \frac{y_{t+1}}{k_{t+1}})}{c_{t+1}} \tag{2a}$$

$$c_t + k_{t+1} = (1-\delta)k_t + (1-a_0)y_t \tag{3a}$$

$$G = \frac{\tau_t a_0 y_t}{(1+\tau_t)} \tag{*}$$

Note that the international trade balance is always zero. Foreigners are paid in goods. This is clear in equation (3a), according to which domestic production is divided between consumption, investment, imports and government expenditure ($c_t + i_t + p_t^o o_t + G = y_t$, $i_t = k_{t+1} - (1-\delta)k_t$). So part of what is produced domestically is used to pay for the imports. This is the interpretation of Finn (2000), Wei (2003) and Aguiar-Conraria and Wen (2006) in similar models.

It can be easily shown that a steady state exists in this economy for reasonable level of government expenditure. To study indeterminacy, we substitute y_t by utilizing equation (4a) and (*) and log linearize equations (1a)-(3a) around the steady state. This gives (here the τ_{ss} denotes the steady state value of the endogenous tariff, see appendix B).

$$\hat{y_t} = \frac{a_k}{1 - a_0(1+\tau_{ss})}\hat{k_t} + \frac{a_n}{1 - a_0(1+\tau_{ss})}\hat{n_t} \tag{4b}$$

note that $\frac{a_k + a_n}{1 - a_0(1+\tau_{ss})} > 1$, increasing returns to scale comes from the endogenous tariff rate.

$$[1 - \frac{a_n}{1 - a_0(1+\tau_{ss})}]\hat{n_t} = \frac{a_k}{1 - a_0(1+\tau_{ss})}\hat{k_t} - \hat{c_t} \tag{1b}$$

$$\hat{c_{t+1}} - \hat{c_t} = [1 - \beta(1-\delta)][(\frac{a_k}{1 - a_0(1+\tau_{ss})} - 1)\hat{k_{t+1}} + \frac{a_n}{1 - a_0(1+\tau_{ss})}\hat{n_{t+1}}] \tag{2b}$$

$$(1-s)\hat{c_t} + \frac{s}{\delta}\hat{k_{t+1}} = [\frac{a_k}{1 - a_0(1+\tau_{ss})} + \frac{s(1-\delta)}{\delta}]\hat{k_t} + \frac{a_n}{1 - a_0(1+\tau_{ss})}\hat{n_t} \tag{3b}$$

where s is the adjusted steady-state saving rate (investment to national income ratio) given by

$$s = \frac{\delta k}{(1-a_0)y} = \frac{\delta \beta a_k}{(1-a_0)(1-\beta(1-\delta))}$$

The above system of linear equations can be reduced to

$$M_1 \begin{bmatrix} \hat{k}_{t+1} \\ \hat{c}_{t+1} \end{bmatrix} = M_2 \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix}$$

where

$$M_1 = \begin{bmatrix} M_{1,1} & -\{1+[1-\beta(1-\delta)]\frac{a_n}{(1-a_0(1+\tau_{ss})-a_n)}\} \\ \frac{s}{\delta} & 0 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 0 & -1 \\ \frac{s(1-\delta)}{\delta} + \frac{a_k}{1-a_0(1+\tau_{ss})}(1+\frac{a_n}{(1-a_0(1+\tau_{ss})-a_n)}) & s - \frac{(1-a_0(1+\tau_{ss}))}{(1-a_0(1+\tau_{ss})-a_n)} \end{bmatrix}$$

where $M_{1,1} = [1-\beta(1-\delta)][(\frac{a_k}{1-a_0(1+\tau_{ss})} - 1) + \frac{a_n a_k}{(1-a_0(1+\tau_{ss}))(1-a_0(1+\tau_{ss})-a_n)}]$

We propose a numerical case based on WAC 2005 model without increasing returns: $a_k = 0.09$, $a_n = 0.7$, $a_o = 0.21$, $\delta = 0.025$, $\beta = 0.99$.[9] Suppose the steady state tariff rate in the country for oil import is $\tau_{ss} = 0.6$, the two roots of the matrix $B = M_1^{-1}M_2$ are $0.5738 \pm 0.5496i$, with modulas 0.7945. We have multiple equilibria induced by the endogneous tariff rate. The high tariff rate ($\frac{\text{import tariff}}{\text{import price}} = \frac{15.6\$/bbl}{26\$/bbl} = 0.6$ optimal tariff rate of oil from David Newbery (2005)) is consistent with the EU in 2002.[10]

[9] In this numerical case, following WAC(2005), we set $a_0 = 0.21$ which is the cost share of foreign inputs in domestic production in Netherlands. Here we assume that the tariff rate in Netherlands is 0.6 (the one in EU, 2002, calculated by Newbery 2005.) Implicitly, we assume that the cost share a_0 keeps unchanged throughout the years.

[10] In an exercise paper of Chen and Zhang (2008), we introduce intrinsic uncertainty in the form of exogenous productivity and government purchases shocks into this model and investigate the propagation

The conclusions in the model also hold for the energy taxes. As we see, the energy taxes[11] as the optimal tariff argument[12] are relatively high in some European countries. For instance, oil is heavily taxed in Denmark, the effective tax rate on domestic fuels exceeds 0.8. It will push the Denmark's economy into destabilizing easily. ($a_k = 0.1$, $a_n = 0.7$, $a_o = 0.2$, $\delta = 0.025$, $\beta = 0.99$ based on WAC 2005 , $\tau_{ss} = 0.8$, two roots are $0.8591 \pm 0.3387i$).

2.1. Steady State and Local Indeterminacy

In order to derive analytical formulas of the indeterminacy conditions and facilitate the comparison with Benhabib and Farmer (1994) and SGU (1997) models, we transform our model into continous time. The present discounted value of the lifetime utility, ($\rho \in (0, 1)$ is the subjective discount rate in the continuous time model)

$$\int_0^\infty e^{-\rho t}(\log c_t - bn_t)dt$$

subject to

$$\dot{k}_t = r_t k_t + w_t n_t - c_t$$

mechanism of sunspot and fundamental shocks under a balanced-budget rule in the tariff model. Following SGU's method, we find that neither the first-order serial correlations, the contemporaneous correlations with output, nor the standard deviation relative to output of tariff rate, output, hours, and consumption is affected by the relative volatility of the sunspot shock or its correlation with the technology shock. Therefore it validates the equivalence between the factor income taxes (in SGU) in closed economy and the tariff in open economy, in the sense that they share similar propagation mechanism of sunspot and fundamental shocks under a balanced-budget rule.

[11] the energy tax revenue is overwhelmingly oil tax revenue in some EU countries, see David Newbery (2005).

[12] the need to correct externalities such as global warming.

where $r = a_k \frac{y}{k} - \delta$ is the rental rate of capital minus depreciation rate (the government doesn't transfer the tariff revenue to the agent, instead consumes this revenue by itself), the first order conditions become

$$\frac{1}{c_t} = \Lambda_t$$

$$b = \Lambda_t w$$

$$\dot{\Lambda}_t = (\rho - r)\Lambda_t$$

where Λ_t denotes the marginal utility of income. The single good is produced with a Cobb-Douglas production technology with three inputs : $y_t = k_t^{a_k} n_t^{a_n} o_t^{a_0}$ (or $y_t = A k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}}$ where $A = (\frac{a_0}{p^o(1+\tau)})^{\frac{a_0}{1-a_0}}$). Perfect competition in factor and product markets implies that factor demands are given by:

$$w_t = a_n \frac{y_t}{n_t}$$

$$r_t + \delta = a_k \frac{y_t}{k_t}$$

and

$$p^o(1 + \tau_t) = a_0 \frac{y_t}{o_t}$$

Market clearing requires that aggregate demand equal aggregate supply, that is,

$$c_t + G + \dot{k}_t + \delta k_t + o_t p^o = y_t$$

Government expenditure (a pre-set constant level) satisfies: $G = p^o \tau_t o_t$. When we replace the consumption with $\frac{1}{\Lambda_t}$, transform wage rate and rental rate into functions of capital and labor, the equilibrium conditions can be reduced to four equations:

$$b = \Lambda_t a_n A k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}-1} \tag{5}$$

$$\frac{\dot{\Lambda}_t}{\Lambda_t} = \rho + \delta - a_k A k_t^{\frac{a_k}{1-a_0}-1} n_t^{\frac{a_n}{1-a_0}} \tag{6}$$

$$\dot{k}_t = (1 - \frac{a_0}{1+\tau}) y_t - \delta k_t - \frac{1}{\Lambda_t} - G \tag{7}$$

and

$$G = \frac{\tau_t a_0 y_t}{(1+\tau_t)}, y_t = A k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}} \tag{8}$$

We first claim that for a given tariff rate, a steady state exists and is unique (same logic as SGU, government expenditure is endogenous in this case). Secondly, we claim that the number of tariff rates that generate enough revenue to finance a given level of government purchases can be 0, 1 or 2 (for the endogenous tariff rate case: see the appendix B).[13]

[13] SGU (1997) show that the revenue maximizing tax rate is the least upper bound of the set of taxes rate for which the rational expectations equilibrium is indeterminate. But in our endogenous tariff rate case, this

Consider the log linear approximation of the equilibrium conditions (5)-(8) around the steady state. Let λ_t, $\hat{k_t}$, $\hat{n_t}$, $\hat{\tau}$ denote the log deviations of Λ_t, k_t, n_t and τ from their respective steady states. The log linearized equilibrium conditions then are

$$0 = \lambda_t - \frac{\tau_{ss}\hat{\tau}}{\frac{1-a_0}{a_0}(1+\tau_{ss})} + \frac{a_k}{1-a_0}(\hat{k_t} - \hat{n_t}) \tag{9}$$

$$\dot{\lambda_t} = (\rho+\delta)[\frac{a_n}{1-a_0}(\hat{k_t} - \hat{n_t}) + \frac{\tau_{ss}\hat{\tau}}{\frac{1-a_0}{a_0}(1+\tau_{ss})}] \tag{10}$$

$$\dot{\hat{k_t}} = [(1-a_0)\frac{(\rho+\delta)}{1-a_0(1+\tau_{ss})} - \delta]\hat{k_t} + \frac{a_n(\rho+\delta)(1-a_0)}{a_k[1-a_0(1+\tau_{ss})]}\hat{n_t} + [-\delta + \frac{(1-a_0)}{a_k}(\rho+\delta)]\lambda_t \tag{11}$$

$$\hat{y_t} = -\frac{1}{1+\tau_{ss}}\hat{\tau} = \frac{a_k}{1-a_0(1+\tau_{ss})}\hat{k_t} + \frac{a_n}{1-a_0(1+\tau_{ss})}\hat{n_t} \tag{12}$$

Combining the (9) and (12), we can imply

$$\hat{n_t} = \frac{\lambda_t}{\frac{a_k}{1-a_0} - \frac{\tau_{ss}}{\frac{1-a_0}{a_0}}\frac{a_n}{1-a_0(1+\tau_{ss})}} + \frac{\frac{a_k}{1-a_0(1+\tau_{ss})}}{\frac{a_k}{1-a_0} - \frac{\tau_{ss}}{\frac{1-a_0}{a_0}}\frac{a_n}{1-a_0(1+\tau_{ss})}}\hat{k_t}$$

Using this expression to eliminate the $\hat{n_t}$in the (10) and (11) results in the following system:

---

property doesn't hold.

$$\begin{bmatrix} \dot{\lambda}_t \\ \dot{\hat{k}}_t \end{bmatrix} = \begin{bmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} \\ \mathbf{J}_{21} & \mathbf{J}_{22} \end{bmatrix} \begin{bmatrix} \lambda_t \\ \hat{k}_t \end{bmatrix}, J = \begin{bmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} \\ \mathbf{J}_{21} & \mathbf{J}_{22} \end{bmatrix}$$

where

$$\mathbf{J}_{11} = -(\rho+\delta)\frac{\frac{a_n}{1-a_0} + \frac{\tau_{ss}}{\frac{1-a_0}{a_0}}\frac{a_n}{1-a_0(1+\tau_{ss})}}{\frac{a_k}{1-a_0} - \frac{\tau_{ss}}{\frac{1-a_0}{a_0}}\frac{a_n}{1-a_0(1+\tau_{ss})}}$$

$$\mathbf{J}_{12} = (\rho+\delta)\{[\frac{a_n}{1-a_0} - \frac{\tau_{ss}}{\frac{1-a_0}{a_0}}\frac{a_k}{1-a_0(1+\tau_{ss})}] - \frac{[\frac{a_n}{1-a_0} + \frac{\tau_{ss}}{\frac{1-a_0}{a_0}}\frac{a_n}{1-a_0(1+\tau_{ss})}]\frac{a_k}{1-a_0(1+\tau_{ss})}}{\frac{a_k}{1-a_0} - \frac{\tau_{ss}}{\frac{1-a_0}{a_0}}\frac{a_n}{1-a_0(1+\tau_{ss})}}\}$$

$$\mathbf{J}_{21} = [-\delta + \frac{(1-a_0)}{a_k}(\rho+\delta)] + \frac{\frac{a_n(\rho+\delta)(1-a_0)}{a_k[1-a_0(1+\tau_{ss})]}}{\frac{a_k}{1-a_0} - \frac{\tau_{ss}}{\frac{1-a_0}{a_0}}\frac{a_n}{1-a_0(1+\tau_{ss})}}$$

$$\mathbf{J}_{22} = [(1-a_0)\frac{(\rho+\delta)}{1-a_0(1+\tau_{ss})} - \delta] + \frac{\frac{a_k}{1-a_0(1+\tau_{ss})}}{\frac{a_k}{1-a_0} - \frac{\tau_{ss}}{\frac{1-a_0}{a_0}}\frac{a_n}{1-a_0(1+\tau_{ss})}}\frac{a_n(\rho+\delta)(1-a_0)}{a_k[1-a_0(1+\tau_{ss})]}$$

Proposition 3. *The equilibrium is indeterminate iff* $trace(\mathbf{J}) = \mathbf{J}_{11} + \mathbf{J}_{22} < 0 < \mathbf{J}_{22}\mathbf{J}_{11} - \mathbf{J}_{12}\mathbf{J}_{21} = \det(\mathbf{J})$, *or,* $\frac{a_k}{a_0} < \tau_{ss} < \tau^* = \frac{[(\rho+\delta)a_n(1-a_0)-\delta a_n a_k]}{[(\rho+\delta)a_0(1-a_0)+\delta a_0 a_k]}$

The indeterminacy requires that $trace(J) = \frac{a_k}{a_k - a_0\tau_{ss}}(\rho+\delta) - \delta < 0$ if and only if $\tau_{ss} > \frac{a_k}{a_0}$

After some manipulations, the determinant of the Jacobian can be written as

$$\det(J) = \frac{(\rho+\delta)}{a_k - a_0\tau_{ss}}\{\delta(a_n + a_0\tau_{ss}) - \frac{(\rho+\delta)}{a_k - a_0\tau_{ss}}[a_n(1-a_0) - a_0\tau_{ss}\frac{1-a_0}{a_k}(1-a_0(1+\tau_{ss}))]\}$$

The positive $det(J)$ requires that (conditional on $a_k - a_0\tau_{ss} < 0$), $G(\tau_{ss}) = [\frac{(\rho+\delta)a_0^2(1-a_0)}{a_k} + \delta a_0^2]\tau_{ss}^2 - \tau_{ss}[\frac{(\rho+\delta)a_0(1-a_0)^2}{a_k} + \delta a_0(a_k - a_n)] + [(\rho+\delta)a_n(1-a_0) - \delta a_n a_k] < 0$, we find that $G(\frac{a_k}{a_0}) = 0$, $G(0) > 0$, the necessary and sufficient condition of $G < 0$ is

$$\frac{a_k}{a_0} < \tau_{ss} < \tau^* \tag{13}$$

where $\tau^* = \frac{[(\rho+\delta)a_n(1-a_0) - \delta a_n a_k]}{[(\rho+\delta)a_0(1-a_0) + \delta a_0 a_k]} > \frac{a_k}{a_0}$

A sufficient condition for the set of tariff rates satisfying (13) to be nonempty is that the labor share is larger than the capital share (i.e., $a_n > a_k$).

The economic intuition behind the existence of stationary sunspot equilibria in the presence of a balanced budget rule is quite straightforward. Suppose that agents expect future tariff rates to be above average. This implies that, for any given capital stock, future oil imports and the rate of return on capital will be lower (the latter is due to the fact that the marginal product of capital is increasing in the oil input). The decrease in the expected rate of return on capital, in turn, lowers the current oil demand, leading the current output decrease. Since the tariff rate is countercyclical $\frac{\partial\tau}{\partial y} < 0$, budget balance requires that the current tariff rate increase. Thus expectation of an above steady state tariff rate in the next period leads to higher current tariff rate. For certain choices of the parameter values, namely those satisfying $\frac{a_k}{a_0} < \tau_{ss} < \tau^*$, the expectation of an above steady state tariff rate in the next period leads to an increase in tariff rates today that is larger than the one expected for next period. Furthermore, for such parameter values, the tariff rate in period 0 is larger in absolute value than the tariff rate in period $t' > 0$, so that the sequence of tariff rates converges to the steady state and thus can be justified as an equilibrium outcome.

To help understand the intuition, consider the consumption Euler equation (in discrete time for ease of interpretation) as follows:

$$\frac{c_{t+1}}{c_t} = \beta(1 - \delta + a_k \frac{y_{t+1}}{k_{t+1}}) = \beta[1 - \delta + (1 + \tau_{t+1})^{-\frac{a_0}{1-a_0}} r_{t+1}^{bt}]$$

where $r_{t+1}^{bt} = a_k(\frac{a_0}{p_0})^{\frac{a_0}{1-a_0}} k_{t+1}^{\frac{a_k}{1-a_0}-1} n_{t+1}^{\frac{a_n}{1-a_0}}$ denotes the before-tariff return on capital, τ_{t+1} the tariff rate in period $(t+1)$. Households' optimistic expectations that lead to higher investment raise the left hand side of this equation, but result in a lower before-tariff return on capital r_{t+1}^{bt} due to the diminishing marginal products. The countercyclical tariff rate can increase the right hand side of the equation, thus validating the initial optimistic expectations.

3. Comparison with Benhabib-Farmer Model

The indeterminacy condition obtained above also has a close correspondence with the one obtained in the increasing returns model of Benhabib and Farmer (1994). In both models, a necessary condition for local indeterminacy is that the "equilibrium labor demand schedule" can be upward sloping and steeper than the labor supply schedule. In the Benhabib-Farmer model, the equilibrium labor demand is upward sloping due to increasing returns in the production function. In my model, on the other hand the equilibrium labor demand is upward sloping because increases in the aggregate employment are accompanied by decreases in the tariff rate. The labor demand function can be written as (in log deviations around the steady state):[14]

[14] Here we should emphasize that the $\hat{w_t}$ denotes the log deviation of the after-tariff wage rate from the steady state.

$$\hat{w_t} = \frac{a_k}{1 - a_0(1 + \tau_{ss})}\hat{k_t} + \frac{-(a_k - a_0\tau_{ss})}{1 - a_0(1 + \tau_{ss})}\hat{n_t}$$

Based on $\frac{a_k}{a_0} < \tau_{ss} < \tau^*$, $\frac{-(a_k - a_0\tau_{ss})}{1-a_0(1+\tau_{ss})} > 0$ the labor demand function now is upward sloping. Since the aggregate labor supply is infinitely elastic (for a given tariff rate and marginal utility of income), in our case $\hat{w_t} = \hat{c_t}$, the labor demand schedule will be steeper than the labor supply schedule whenever $\frac{a_k}{a_0} < \tau_{ss} < \tau^*$.

4. Comparison With SGU and WAC model

SGU proved that within a standard neoclassical growth model, a balanced budget rule can make expectations of higher tax rates self fulfilling if the fiscal authority relies on changes in labor income taxes to eliminate the short run fiscal imbalances. People will naturally think if the import factor is a labor substitute, the endogenous tariff rate imposed on imported oil will make the indeterminacy arise more easily. Although in the above sections, we follow WAC to assume that the imported factor is mainly a substitue for capital, we can not eliminate the possibility that imported factor is a substitute for labor.

We get the following proposition:

Proposition 1. *If we assume that the imported factor is mainly a labor substitute instead of a capital substitute, which means* $a_k = 0.3$*,* $a_n + a_0 = 0.7$ *(instead of* $a_n = 0.7$*,* $a_k + a_0 = 0.3$*), the indeterminacy may not easily arise in the labor substitute assumption.*

Example 2. *we give a simple example to prove this proposition. Let* $a_0 = 0.2$ *as we did in the numerical case,* $a_n = 0.5$*, the necessary and sufficient condition becomes* $\frac{a_k}{a_0} = 1.5 < \tau_{ss} < \tau^* = \frac{[(\rho+\delta)a_n(1-a_0)-\delta a_n a_k]}{[(\rho+\delta)a_0(1-a_0)+\delta a_0 a_k]} \approx 2.5$*. Compared with the case we did before: imported* *factor is capital substitute,* $a_k = 0.1$*,* $a_n = 0.7, a_0 = 0.2$*, the necessary and sufficient condition is* $\frac{a_k}{a_0} = 0.5 < \tau_{ss} < \tau^* = \frac{[(\rho+\delta)a_n(1-a_0)-\delta a_n a_k]}{[(\rho+\delta)a_0(1-a_0)+\delta a_0 a_k]} \approx 3.5$*. The former case will not make the indeterminacy more easily to arise since empirically speaking, tariff rate cannot be that high (exceeds 150%).*

From this proposition, we can see that although the channel of the tariff to deliver indeterminacy is similar as the one of the factor income taxes, they have different implications in generating indeterminacy. We can say that the "equivalence" relationship between them only holds through fiscal increasing returns by endogenizing rates and making the government spending exogenous. WAC find that if imported factor is a substitute for labor, then a larger a_0 has the same qualitative consequences (meaning the degree of the externality decreases), although less dramatic. Here we find that if imported factor is a substitute for labor, then a larger a_0 will need a larger tariff rate to generate indeterminacy.

WAC show that heavy reliance on imported energy can have a significant effect on economic instability in the presence of increasing returns to scale: the larger the imported energy share in GDP, the easier it is for the economy to be subject to multiple equilibria. We have the similar proposition below:

Example 3. *Given* $a_n = 0.7$*,* $a_k + a_0 = 0.3$ *(the imported goods is capital substitute), the larger the imported energy share in GDP, the easier it is for the economy to be subject to multiple equilibria. Since the lower bound of the indeterminacy region* $\frac{a_k}{a_0} < \tau_{ss} < \tau^* = \frac{[(\rho+\delta)a_n(1-a_0)-\delta a_n a_k]}{[(\rho+\delta)a_0(1-a_0)+\delta a_0 a_k]}$ *decreases as* a_0 *increases, it makes indeterminacy arise more easily under the scope of empirically reasonable tariff rates.*

5. Conclusion

We explore the "channel equivalence" between the factor income taxes and tariff to generate indeterminacy. The channel is through fiscal increasing returns by endogenizing rates and making the government spending exogenous. In the two sector model without "fiscal increasing returns" induced by the factor income taxes, Bond, Wang and Yip (1996) and Meng and Velasco (2003) prove that distortionary factor taxation nonetheless causes indeterminacy in a closed-economy, endogenous growth model and a small open RBC model respectively. Does the "channel equivalence" between factor income taxes and tariff still hold in a small open economy two sector model? This is one issue which deserves our further research.

Another future task is to see whether plausible parametrization can generate the kinds of economic fluctuations that we observe in real-life economies. This will show that this source of instability is not just a theoretical possibility but also occurs for empirically realistic parameter values. We plan to pursue this line of research in the future.

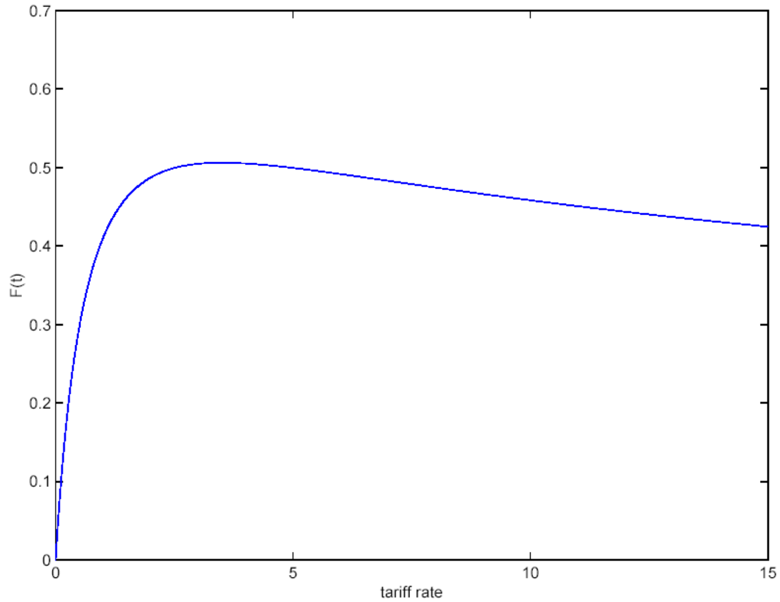
6. Appendix:

6.1. Appendix A: The discrete time model:

(i) $G = \frac{\tau_t a_0 y_t}{(1+\tau_t)}$ implies $(G - a_0 y_t)\hat{\tau} = a_0 y_t \hat{y}$, after some algebra, we can see that $\hat{y}_t = -\frac{1}{1+\tau_{ss}}\hat{\tau}$.

(ii) $A_t = (\frac{\lambda a_0}{p_0(1+\tau_t)})^{\frac{a_0}{1-a_0}}$ implies $\hat{A}_t = -\frac{\tau_{ss}\hat{\tau}}{\frac{1-a_0}{a_0}(1+\tau_{ss})}$

(iii) $\hat{y}_t = \hat{A}_t + \frac{a_k}{1-a_0}\hat{k}_t + \frac{a_n}{1-a_0}\hat{n}_t = \frac{a_k}{1-a_0(1+\tau_{ss})}\hat{k}_t + \frac{a_n}{1-a_0(1+\tau_{ss})}\hat{n}_t$



6.2. Appendix B: The continuous time case:

Claim 1. *The steady state in the continuous-time dynamic system (5)-(8) exists, given the proper level of government expenditure.*

We can derive steady state $\frac{k}{n} = (\frac{\rho+\delta}{a_k A})^{\frac{1-a_0}{-a_n}}$, $\Lambda = \frac{b}{a_n A}(\frac{\rho+\delta}{a_k A})^{\frac{a_k}{a_n}}$, $k = \frac{\frac{a_n A}{b}(\frac{\rho+\delta}{a_k A})^{-\frac{a_k}{a_n}}}{[\frac{1-a_0}{a_k}(\rho+\delta)-\delta]}$, $G = \frac{\tau}{(1+\tau)^{\frac{a_n+a_0}{a_n}}}$constant$=$ $F(\tau)$, constant$=\frac{(\frac{a_0}{p^o})^{\frac{a_0}{a_n}} a_0(\rho+\delta)a_n(\frac{\rho+\delta}{a_k})^{-\frac{a_k}{a_n}}}{a_k b[\frac{1-a_0}{a_k}(\rho+\delta)-\delta]}$. We can see $F(\tau)$ is non-monotone and the number of positive tariff rates that generate enough revenue to finance a given level of government purchases can be 0, 1 or 2.

(iv) $\mathbf{J}_{11} = -(\rho+\delta)\frac{a_n}{a_k - a_0\tau_{ss}}$, $\mathbf{J}_{22} = (\rho+\delta)\frac{1-a_0}{a_k - a_0\tau_{ss}} - \delta$, $\mathbf{J}_{12} = (\rho+\delta)\frac{-\tau_{ss}a_0}{a_k - a_0\tau_{ss}}$, $\mathbf{J}_{21} = (\rho+\delta)\frac{(1-a_0)}{a_k}\frac{1-(\tau_{ss}+1)a_0}{a_k - a_0\tau_{ss}} - \delta$

6.3. Appendix C: Close form correspondence

In this appendix, we show that there is a close correspondence between the equilibrium conditions of the model with a balanced-budget rule, distortionary tariff, and constant government purchases presented in this paper and that of the endogenous business cycles: the distortionary income taxes model in SGU (1997). Consider the case with tariff rate. The balanced-budget rule is then given by

$$G = \frac{\tau a_0 y_t}{(1+\tau)}$$

The following equilibrium conditions hold for both two models (in discrete time),

$$U_c(c_t, n_t) = \theta_t$$

$$U_n(c_t, n_t) = w_t \theta_t$$

$$\mathsf{Y}_t = c_t + k_{t+1} - (1-\delta)k_t$$

$$1 = \beta \frac{\theta_{t+1}}{\theta_t}(1 - \delta + r_{t+1})$$

where θ_t is the Lagrangian multiplier of the budget constaint of the household. In the balanced budget model, disposable income, Y_t, is given by

$$\mathsf{Y}_t = (1 - a_0)y_t = y_t - p^0 o_t - G$$

G represents a fixed cost that ensures that imperfectly competitive firms do not make pure profits in the long run (given that the foreign firms take away their payments). The after-tariff wage rate w_t, and the after-tariff rental rate r_t are given by

$$r_t^{bt} = a_k(\frac{a_0}{p^o})^{\frac{a_0}{1-a_0}} k_t^{\frac{a_k}{1-a_0}-1} n_t^{\frac{a_n}{1-a_0}} = \mu_t r_t$$

$$w_t^{bt} = a_n(\frac{a_0}{p^o})^{\frac{a_0}{1-a_0}} k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}-1} = \mu_t w_t$$

r_t^{bt}, w_t^{bt} denote the before-tariff return on capital and labor. In the balanced budget model, μ_t represents the wedge between marginal product and after tariff factor prices introduced by distortionary tariff. Specifically,

$$\mu_t = (1 + \tau_t)^{\frac{a_0}{1-a_0}} = (1 - \frac{1 - a_0}{a_0}\frac{G}{\mathsf{Y}_t})^{-\frac{a_0}{1-a_0}} = \mu(\frac{G}{\mathsf{Y}_t})$$

We can see that the markup μ_t is countercyclical with respect to y_t.

References

[1] Benhabib, J., Farmer, R.E., 1994. Indeterminacy and increasing returns. Journal of Economic Theory 63, 19–41.

[2] Benhabib, J., Farmer, R.E., 1999. Indeterminacy and sunspots in macroeconomics. In: Taylor, J.B., Woodford, M. (Eds.), Handbook of Macroeconomics, vol. 1A. North-

Holland, New York, pp. 387–448

[3] Bond, E., Wang P., Yip C. K., 1996. A general two-sector model of endogenous growth model with human and physical capital: balanced growth and transitional dynamics. Journal of Economic Theory 68, 149-173.

[4] Carlos de Miguel, Baltasar Manzano, 2006. Optimal oil taxation in a small open economy. Review of Economic Dynamics 9, 438–454

[5] Chen Yan, Zhang Yan, 2008. Tariff and Equilibrium Indeterminacy–An Exercise. Manuscript, New York University and Shandong University.

[6] David N.Dejong and Marla Ripoll, 2005. Tariffs and growth: An empirical Exploration of Contingent Relationship, Forthcoming in Review of Economics and Statistics.

[7] Finn, M., 2000, Perfect competition and the effects of energy price increases on economic activity, Journal of Money, Credit, and Banking, 32, 400-417.

[8] Guo Jang-Ting, Harrison Sharon G., 2004, Balanced-Budget Rules and Macroeconomic (In)stability, Journal of Economic Theory, Vol. 119, No. 2, December , 357-363

[9] Hansen, G., 1985, Indivisible labor and the business cycles, Journal of Monetary Economics 16:309- 327.

[10] Kindleberger, C. and P. Lindert, 1978, International economics (Irwin, Homewood).

[11] Leung, Charles Ka Yui, Income Tax, Property Tax, and Tariff in a Small Open Economy. Review of International Economics, 7(3), 541-554, 1999

[12] Manoj Atolia, 2006, Tax Evasion, Public Capital, and Welfare Effects of a Tariff Reform (Revised: March, 2006), Florida State University

[13] Meng, Qinglai, Velasco Andres, 2003. Market Imperfections and the Instability of Open Economies, Journal of International Economics, Vol. 64 (2), 2004.

[14] Michael B Loewy, 2004, Optimal tariffs, optimal taxes and economic development, Journal of International Trade & Economic Development, Volume 13, Number 4 / December 2004

[15] Mourmouras, A., 1991, Infant governments and the fiscal role of tariffs, inflation, and reserve requirements, Journal of International Economics 31, 271 – 90.

[16] Newbery, D., 2005, Why Tax Energy? Towards a More Rational Policy, The Energy Journal, Vol. 26, No. 3.

[17] Ramsey, F. P., 1927, A contribution to the theory of taxation, Economic Journal, 37, 47 – 61.

[18] Riezman, R. and J. Slemrod, 1987, Tariffs and collection costs, Weltwirtschaftliches Archiv. 123, no. 3, 545-550.

[19] Rotemberg, J. and Woodford, M., 1996, Imperfect competition and the effects of energy price increases on economic activity, Journal of Money, Credit, and Banking, 28, 549-577.

[20] Stephanie Schmitt-Grohe, Martin Uribe, Balanced-Budget Rules, Distortionary Taxes, and Aggregate Instability. Journal of Political Economy, October 1997, 105, 976-1000

[21] Velasco, A., 1996. Animal spirits, capital repatriation and investment. Journal of International Money and Finance 15, 221– 238.

[22] Wen, Yi, Luis Aguiar-Conraria, Understanding the Large Negative Impact of Oil Shocks, forthcoming: Journal of Money, Credit, and Banking

[23] Wen, Yi, Luis Aguiar-Conraria, Foreign trade and equilibrium indeterminacy, Federal Reserve bank of St. Louis, 2005-041a

[24] Wen, Yi, Luis Aguiar-Conraria, Oil dependence and economic Instability, Federal Reserve bank of St. Louis, 2006

[25] Wei, C., 2003, Energy, the stock market, and the putty-clay investment model, American Economic Review, 93, 311-323.