# Endogenous Private Leadership under Subsidy Policy on the Social Enterprises

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# Endogenous Private Leadership under Subsidy Policy on the Social Enterprises[†]

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*We investigate a mixed oligopoly model in which private enterprises compete with social enterprises under government subsidy policy, and examine the endogenous choice of private leadership. We show that private leadership is socially desirable, but the numbers of private and social enterprises affect endogenous choices and welfare consequences. We also show that the role of government in choosing the optimal subsidy will be significant when there are more than one private enterprises but its number is smaller than that of the social enterprises.*

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**JEL Classification** L13; D45; H23

## 1. Introduction

During the last decades, non-profit organizations have been developed and expanded since social sector organizations, such as co-operatives and social enterprises, have entered social economy with more entrepreneurial characters. For instance, G8 Social Impact Investment Taskforce reports that social sector organizations already account for more than 5% of GDP in several countries, including Canada, Germany, the UK and the US. In some EU countries such as Italy and France, they employ more than 10% of the workforce. According to the European Commission, one in four companies in the EU falls into the social economy category.[1]

Contrary to private enterprises (PEs), which pursue their own profits, the initiatives of the social enterprises (SEs) supply social goods[2] and generate social values such as supporting public welfare such as health care, creating jobs for the underprivileged, reducing poverty and undernourishment, and so on. Thus, understanding the competition in the context of mixed market configuration between SEs and PEs has now challenged economic performances and social impacts in the society.

However, SEs have different financial structures and sometimes they are financially supported by

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[1] Impact investment: the invisible heart of markets (2014, p.9) and Insight social entrepreneurship: on response to the crisis, Europolitics (2014).

[2] Social goods (or merit goods) are private goods that have excludability and rivalry because they have marketability by nature, but they provide social values to the society as an externality, which is caused mostly by public concern about income inequality or justice.

government subsidy. Thus, the performances of SEs in the social economy depend heavily upon market structure and government regulation. For example, Remploy Ltd. in the UK supports the employment of people with disabilities and promotes the creation of sustainable employment.[3] It has its own operating revenue but government grants account for 40% of its total revenue. The Goodwill in the US has also its own operating revenue but government grants account for 20%.[4]

Several recent studies have examined the competition between SEs and PEs in mixed oligopolies, and the performances of government subsidies. Two approaches are suggested in the formation of the objectives of SEs. On the one hand, SEs are confined to pursue both profit and social concerns such as consumer surplus and/or environmental damage. For example, Kopel and Brand (2012) Lambertini and Tampieri (2015), Brand and Grothe (2015), Liu, *et al.* (2015), Flores and Gracia (2016) and Bian, *et al.* (2016) analyzed the PEs with corporate social responsibilities. They show that firms' profits and social welfare can be improved when firms consider consumer welfare and/or environmental pollution to be social concerns. Regarding the endogenous competition structure of mixed oligopolies, for example, Matsumura and Ogawa (2012, 2014) and Scrimitore (2013, 2014) and Haraguchi and Matsumura (2016) showed that competition structure is changed if one firm is a welfare maximizer and the other firm is a profit maximizer. This suggests the possibility that non-profit maximizing objectives may change the competition structure

On the other hand, Crémer *et al.* (1989), Estrin and de Meza (1995), Bennett and La Manna (2012) and Cho and Lee (2017) incorporated the social concerns solely into the objective function of SEs. In their analyses, SEs produce their outputs and support social values under a break-even constraint, which encompasses a wide range of social activities such as expanding employment level and improving consumer surplus. In special, Cho and Lee (2017) considered social activity as an externality of production and investigated market competition between PE and SE under subsidy policy. They showed that private leadership is better for total social welfare and the number of PEs affects not only profitability but welfare.

In this study, adopting the second approach, we consider a mixed oligopoly where multiple SEs compete with PEs under government subsidy, and investigate the interactions between PEs and SEs. Further, we endogenize the choice of market role between private leadership and private followership, and find the relationship between the relative numbers of PEs and SEs and the equilibrium of endogenous choices. We also examine the welfare consequences in both cases between first-mover and second-mover of government choice on subsidization.

We summarize the main findings of this paper. First, the optimal subsidy for SEs under private

[3] Remploy is a government-led social enterprise for the disabled people. It was founded in 1945 as a full-fledged government fund under "Disability Employment Act" in the UK, enacted in 1944. In the employment law, some companies, such as Ford and Unilever, are employing disabled workers in connection with Remploy like Tesco.

[4] Kim, *et al.* (2015, Chapter 3) examined social enterprise employment for the disabled.

leadership is higher than that under private followership. This is because private leadership increases the output of the PEs but decreases the output of the SEs. Second, the output of SE under private followership is higher than that under private leadership, while both output of the PE and total market outputs under private followership are lower than those under private leadership. This implies that the SEs expand their outputs more under private followership to increase social value, while the PEs expand their outputs more under private leadership. Third, total social welfare under private followership is lower than that under private leadership. Therefore, private leadership is socially desirable than private followership because private leadership can induce PEs to behave aggressive, which increases total market outputs. Finally, we also show that the role of government in choosing the optimal subsidy policy will be significant when there are more than one PEs but its number is smaller than that of SEs. In particular, the regions for the private leadership for being an equilibrium of endogenous competition mode is reduced when the government chooses the optimal subsidy after the PEs choose the competition mode, in which the total social welfare can be reduced. Therefore, the role of government should be emphasized in deciding subsidy policy.

The remainder of this paper is organized as follows. In section 2, we provide the basic model with three-stage game. In section 3, we analyze two modes of market competition in the third stage, private leadership and private followership, respectively, and examined the endogenous choice of market role in the second stage. In section 4, we provide the optimal subsidy policy on the SEs in the first stage. In section 5, we discuss policy implications. Finally, in section 6, we conclude the study.

## 2. The Model

We consider an oligopoly market in which $m+n$ enterprises produce homogeneous products, where $m\ (\geq 1)$ social enterprises (SEs) and $n\ (\geq 1)$ private enterprises (PEs) compete with outputs in the market trade. We denote $q_j$ as the output of the SE $j\ (=1,\dots,m)$, and $q_i$ as the output of the PE $i\ (=1,\dots,n)$. Market price $P$ is given by a linear inverse demand function:

$$P = A - Q \tag{1}$$

where $A$ is the market size and $\mathrm{Q} = \sum_{j=1}^{m} q_j + \sum_{i=1}^{n} q_i$. Then, consumer surplus can be denoted by $\mathrm{CS} = \frac{1}{2}Q^2$.

We assume that the identical quadratic cost functions of both types of enterprises,[5] $c(q_k) = \frac{1}{2}q_k^2$

[5] We adopt traditional assumption in mixed oligopolies that there are diseconomies of scale and thus the optimal productions exist. See, for example, Kopel and Brand (2012), Lambertini and Tampieri (2015), Brand and Grothe (2015), Liu, *et al.* (2015), Flores and Gracia (2016) and Bian, *et al.* (2016). If we assume a constant marginal cost, however, corner solutions appear in the equilibrium, depending on the relative size of marginal social value, *b*, and the number of SE, *m*

where $k = i, j$. We also assume that government can provide an output subsidy with the rate of *s* to the SEs to support their social benefits in the social economy. Then, the profits of the SE and PE are respectively as follows:

$$\pi_j = (A - \sum_{j=1}^{m} q_j - \sum_{i=1}^{n} q_i)q_j - \frac{1}{2}q_j^2 + sq_j, \;\; j = 1, 2 \ldots m \tag{2}$$

$$\pi_i = \left(A - \sum_{j=1}^{m} q_j - \sum_{i=1}^{n} q_i\right)q_i - \frac{1}{2}q_i^2, \;\; i = 1, 2 \ldots n. \tag{3}$$

While the PE maximizes its profit, the SE aims to create social value such as the well-being of co-operatives and large employment of disadvantaged and/or aged workers despite incurring high costs. We also assume that the SE maximizes both economic market value, which is defined as consumer surplus in the market trade, and intrinsic social value, which is created from its social activities. For example, job creation or wage expenditure on the underprivileged can be counted as social concerns, as these can be used to combat poverty and undernourishment. We consider this social value as an externality of production activity and thus, assume that social value is proportional to the SE's output level. In specific, the objective of the SE is to maximize the following function under non-negative profit:

$$G_j = \mathrm{CS} + b\, q_j, \;\; s.t \;\; \pi_j \geq 0 \tag{4}$$

where $b\ (> 0)$ is interpreted as the marginal social value of the economic activities of the SE.
Note that there is an externality in output of social enterprises and the activities of the SE are constrained by the economic constraint of non-negative profit, which supports its survival under subsidy policy. This formulation with a break-even constraint is sufficiently general to cover a wide range of public concerns such as employment level and social activities.[6] To analyze the interior solutions in the equilibrium, we assume that the market size is sufficiently large, i.e., $0 < bm < A$.

Finally, the government can subsidize the SEs under the financial requirement that its economic profit should be non-negative in (4). We assume that the benevolent government maximizes total social welfare, which is defined as the sum of economic welfare and social value, where economic welfare contains consumer surplus ($CS$) and producer surplus ($\sum_{j=1}^{m} \pi_j + \sum_{i=1}^{n} \pi_i$) minus the subsidy expenditures of the government ($s \sum_{j=1}^{m} q_j$) as follows:[7]

[6] Because the break-even constraint prevents the imposition of the fully optimal, so called first-best, marginal cost prices, we refer to prices which maximize total surplus subject to breaking even as optimal second-best. See Cremer *et al.* (1989), Estrin and de Meza (1995), and Bennett and La Manna (2012). In particular, Cho and Lee (2017) explicitly considered the social value created by the social activities of the SEs to be a positive externality of market activities.

[7] From the normative perspective, we can show that SE produces positive outputs in the market equilibrium at the first-best optimum, which maximizes (5) without subsidization if $A > bm$, but it cannot earn non-negative

$$W = CS + b\sum_{j=1}^{m} q_j + \sum_{j=1}^{m} \pi_j + \sum_{i=1}^{n} \pi_i - s\sum_{j=1}^{m} q_j \quad (5)$$

The timing and structure of the game are as follows: In the first stage, government decides output subsidy to maximize total social welfare. In the second stage, PEs and SEs decide the mode of market competition. Following Hamilton and Slutsky (1990), we consider the observable delay game in choosing the competition mode. That is, PEs and SEs simultaneously and independently chooses whether to move early or late. Then, simultaneous choice becomes Cournot, while sequential choice becomes Stackelberg. In the last stage, PEs and SEs choose outputs simultaneously or sequentially under the committed competition mode in the second stage. We solve the subgame perfect Nash equilibrium by backward induction.

Before proceeding the analysis, it is noteworthy that the market role of the SEs does not affect the market equilibrium when the PEs follow because the optimal reactions of the PEs are the same under the zero-profit condition of SEs.[8] Hence, we focus on two different modes of market competition between the SEs and PEs under subsidization. The first case is Cournot competition in which both the SEs and the PEs are followers and thus, choose the outputs simultaneously. The other case is Stackelberg competition in which the PEs lead and the SEs follow sequentially. Note again that the reverse case of Stackelberg competition where the SE leads and the PEs follow sequentially is the same case of Cournot competition where the SEs and PEs choose their outputs simultaneously.

## 3. Endogenous Choice of Market Competition

In the last stage, PEs and SEs choose outputs simultaneously or sequentially under the competition mode between Cournot and Stackelberg. In the below, we analyze and compare the results, respectively.

### 3.1 Private Followership: Cournot Competition

In Cournot competition, SEs and PEs choose outputs simultaneously. The profit-maximization condition of the PEs in (3) provides the following reaction function:

$$q_i = \frac{A - \sum_{j=1}^{m} q_j}{2+n} \quad (6)$$

For the case of the SEs, we know that $G_j$ in (4) is increasing in $q_j$. This implies that the zero-profit condition of the SEs is binding at the optimum. That is, the SEs do not behave strategically by choosing the optimal decision, which is simply determined by the zero-profit condition. Thus, the zero-profit condition of the SEs in (2) provides the following reaction function :

profits without government subsidization.

[8] We can easily show that the simultaneous choice between the SEs and PEs provides the same results as the sequential choice where the SE leads and the PEs follow sequentially.

$$q_j = \frac{2(A+s-\sum_{i=1}^{n} q_i)}{1+2m} \tag{7}$$

Note that products are strategic substitutes and the reaction function of the SEs is less sensitive, i.e., $\frac{\partial q_j}{\partial q_i} = -\frac{2}{1+2m} < 0$. Thus, we have the following equilibrium outcomes from the symmetric output of the PEs:

$$q_j = \frac{4A+2(2+n)s}{2+4m+n}, \quad q_i = \frac{A-2ms}{2+4m+n}, \text{ and } \quad Q = \frac{4Am+An+4ms}{2+4m+n} \tag{8}$$

Note that $q_j > q_i$ irrespective of the number of firms and the subsidy rate. The marginal social value does not directly affect equilibrium output. As the subsidy increases, the output of the SE increases while that of the PE decreases. The equilibrium output of the SE is greater than that of the PE if the subsidy is non-negative.

The profits of the SEs and PEs and total social welfare are as follows:

$$\pi_j = 0 < \pi_i = \frac{3(A-2ms)^2}{2(2+4m+n)^2}, \tag{9}$$

$$W = \frac{8Abm(2+4m+n)+A^2(3n+(4m+n)^2)-4Am(4+3n)s+4ms(b(2+n)(2+4m+n)-4s-(4+n)(m+n)s)}{2(2+4m+n)^2} \tag{10}$$

### 3.2 Private Leadership: Stackelberg Competition

In Stackelberg private leadership competition, profit-maximizing PEs play market leaders and thus move first with the SEs following sequentially. The SEs does not choose its output strategically and thus the reaction function is the same as that in (7). Then, the profit function of the PEs becomes:

$$\pi_i = \left(A - \sum_{j=1}^{m}\left(\frac{2(A+s-nq_i)}{1+2m}\right) - \sum_{i=1}^{n} q_i\right) q_i - \frac{1}{2}q_i^2 = 0 \tag{11}$$

The first-order condition of the PEs is as follows:

$$\frac{\partial \pi_i}{\partial q_i} = \frac{A-2ms-(2+2m+n)q_i}{1+2m} = 0 \tag{12}$$

Solving these equations provides the following equilibrium outcomes:

$$q_j = \frac{4A(1+m)+2(2+n+2m(1+n))s}{(1+2m)(2+2m+n)}, \quad q_i = \frac{A-2ms}{2+2m+n}, \text{ and } \quad Q = \frac{A(n+2m(2+2m+n))+4m(1+m)s}{(1+2m)(2+2m+n)} \tag{13}$$

Note that $q_j \gtrless q_i$ if $s \gtrless -\frac{A(3+2m)}{2(2+3m+2m^2+n+2mn)}$. The marginal social value does not directly affect equilibrium output. As the subsidy increases, the output of the SEs increases while that of the PEs decreases. The equilibrium output of the SE is greater (smaller) than that of the PE if the subsidy is

large (small).

The profits of the SEs and PEs and total social welfare are as follows, respectively:

$$\pi_j = 0 < \quad \pi_i = \frac{(3+2m)(A-2ms)^2}{2(1+2m)(2+2m+n)^2} \tag{14}$$

$$W = \frac{\begin{matrix}(8Abm(1+m)(1+2m)(2+2m+n)+A^2(16(m+m^2)^2+(1+2m)^2(3+4m)n+(n+2mn)^2)-4Am(4+3n+4m(2+m)(1+n))s \\ +4ms(b(1+2m)(2+2m+n)(2+n+2m(1+n))-(4(1+m)^3+(1+2m)(4+m(5+2m))n+(n+2mn)^2)s))\end{matrix}}{2(1+2m)^2(2+2m+n)^2} \tag{15}$$

### 3.3 Endogenous Choice of Competition Mode

In the second stage, PEs decide the mode of market competition by comparing market equilibria between private followership and private leadership.

**Proposition 1***: Suppose that $A > 2ms$. The output of SE under private followership is higher than that under private leadership, while both output of the PE and total market outputs under private followership are lower than those under private leadership.*

*Proof*: Comparing the output levels yield the followings:

$q_j^F - q_j^L = \frac{4mn(A-2ms)}{(1+2m)(2+2m+n)(2+4m+n)} > 0$, $q_i^F - q_i^L = -\frac{2m(A-2ms)}{(2+2m+n)(2+4m+n)} < 0$ and

$Q^F - Q^L = \frac{2mn(-A+2ms)}{(1+2m)(2+2m+n)(2+4m+n)} < 0$. Q.E.D.

Proposition 1 implies that consumer surplus under private followership is lower than that under private leadership. But, the social value created by the SEs under private followership is higher than that under private leadership. Thus, from the viewpoint of SEs, there is a trade-off between private followership and private leadership.

**Proposition 2***: Suppose that $A > 2ms$. The profit of the PE under private followership is lower (higher) than that under private leadership when $n^2 \gtreqless \frac{1}{4}(1 + 2m + \sqrt{3}\sqrt{3 + 4m(2 + m)})^2$.*

*Proof*: Comparing the profits of the PEs yields the followings:

$$\pi_i^F - \pi_i^L = -\frac{1}{2}\frac{4m(2+5m+2m^2-n-2mn-n^2)(A-2ms)^2}{(1+2m)(2+2m+n)^2(2+4m+n)^2} \gtreqless 0 \quad \text{if} \quad 2 + 5m + 2m^2 - n - 2mn - n^2 \lesseqgtr 0 \quad .$$

Q.E.D.

Proposition 2 implies that the equilibrium of the endogenous choice on market competition in the second stage does not depend on the subsidy rate, but depends on the number of SEs and PEs. Thus, the optimal decision on the subsidy rate in the first stage does not affect the endogenous choice on

competition mode in the second stage.

Then, given the number of SEs, $m$, we have $n^*(m) = \frac{1}{2}(-1 - 2m + \sqrt{3}\sqrt{3 + 8m + 4m^2})$ from$2 + 5m + 2m^2 - n - 2mn - n^2 = 0$. Fig. 1 shows that the profits under private followership is higher if the number of PEs is large, i.e., $n > n^*(m)$. On the other hand, the profits under private leadership is higher if $n < n^*(m)$. Thus, the private followership (leadership) tends to be the equilibrium of the endogenous choice if the number of PEs is large (small)

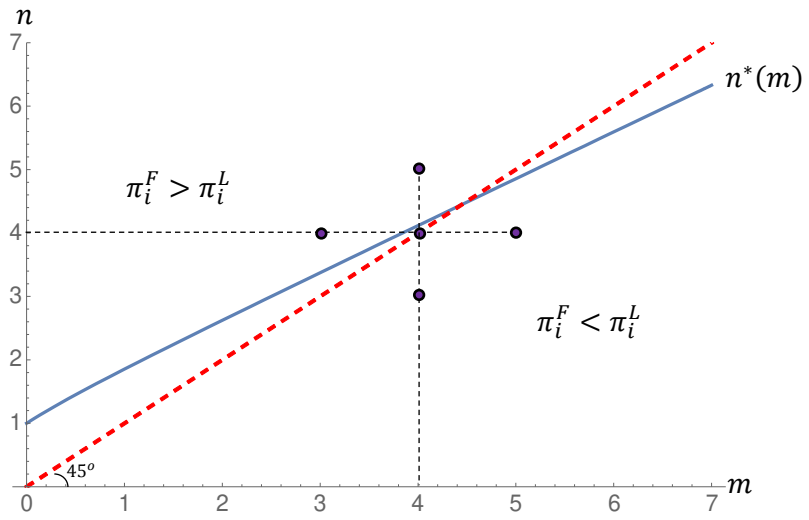


Fig. 1 The profits ranks of PEs under the same subsidy

Fig. 1. provides a few of interest findings. First, when the number of PEs is larger than that of SEs, i.e., $n > m$, private followership is an equilibrium for all $m \geq 1$. Second, when the number of PEs is equal to that of SEs, i.e., $n = m$, private followership is an equilibrium if $m \geq 5$, but private leadership is an equilibrium if $m < 5$. Finally, when the number of PEs is smaller than that of SEs, i.e., $n < m$, private leadership is an equilibrium for $n \leq 5$. Hence, we can conclude that a larger number of PEs yields private followership, while a smaller number of PEs yields private leadership only when the number of SEs is small. For example, consider a single SE, $m = 1$, which is examined by Cho and Lee (2017, Proposition 3). Then, private leadership is preferred when $n = 1$, while private followership is preferred when $n \geq 2$. It supports that the previous results that the first-mover under Stackelberg competition has a higher output effect compared with the price effect, which increases profits.[9] However, when the number of SEs is more than four, $m \geq 4$, the first-mover advantage depends on the number of PEs. Thus, the relative numbers between PEs and SEs affect the endogenous choice of private

[9] Ono (1978) examined a homogeneous product market with cost asymmetry, while Van Damme and Hurkens (2004) and Amir and Stepanova (2006) analyzed a differentiated product market.

leadership.[10] In particular, as the number of the PEs is relatively large, competition between private leaders intensifies and thus the first-mover advantage disappears.

## 4. Endogenous Choice of Subsidization Policy

In the first stage, government chooses the optimal subsidy on the SEs. Due to the independency between the subsidy and the endogenous choice on market competition, as shown in proposition 2, we have two cases. The government decides the optimal subsidy rate under the private followership when the number of PEs is large, $n > n^*(m)$, while it decides the optimal subsidy rate under the private leadership when the number of PEs is small, $n < n^*(m)$.

### 4.1 Private Followership under Optimal Subsidy

From the total social welfare under private followership in (9), the differentiation of $W$ with respect to $s$ yields the following optimal subsidy where the superscript F stands for private followership:

$$s^F = \frac{b(2+n)(2+4m+n)-A(4+3n)}{2((2+n)^2+m(4+n))} \gtreqless 0 \text{ if } b \gtreqless \frac{A(4+3n)}{(2+n)(2+4m+n)} \tag{16}$$

From the assumption that $bm < A$, we have $s^F < \frac{A}{2m}$. The comparative statics yield:

$$\frac{\partial s^F}{\partial b} = \frac{(2+n)(2+4m+n)}{2((2+n)^2+m(4+n))} > 0, \ \frac{\partial s^F}{\partial m} = \frac{(4+3n)(b(2+n)^2+A(4+n))}{2((2+n)^2+m(4+n))^2} > 0 \text{and}$$

$$\frac{\partial s^F}{\partial n} = -\frac{(A-bm)(8m-(2+n)(2+3n))}{2((2+n)^2+m(4+n))^2} \lessgtr 0 \text{ if } m \gtreqless \frac{1}{8}(2+n)(2+3n).$$

It represents that the optimal subsidy rate increases as the marginal social value increases or the number of SEs increases. As expected, an increased marginal social value or increased number of SEs raises the subsidy rate for encouraging the production of the SEs. However, the optimal subsidy depends on the number of PEs. The government decreases (increases) the subsidy rate when the number of SEs is larger (smaller) than that of the PEs.

Substituting $s^F$ provides the following equilibrium output:

$$q_j^F = \frac{b(2+n)^2+A(4+n)}{(2+n)^2+m(4+n)}, \ q_i^F = \frac{(A-bm)(2+n)}{(2+n)^2+m(4+n)}, \text{ and } Q^F = \frac{2bm(2+n)+A(n(2+n)+m(4+n))}{(2+n)^2+m(4+n)} \tag{17}$$

Then, $q_j^F \gtreqless q_i^F$ if $b \gtreqless -\frac{2A}{(2+n)(2+m+n)}$. Note that when the marginal social value of the SEs is large, its output is greater than that of the PEs under private followership.

[10] Daughety (1990) and Ino and Matsumura (2012) investigated a Stackelberg model in which $m$ leaders and $n-m$ followers compete in a homogeneous goods market with identical cost functions and found non-monotonicity between $m$ and economic performance.

The market price is as follows:

$$p^F = \frac{2(A-bm)(2+n)}{(2+n)^2+m(4+n)}. \tag{18}$$

The profit of the PEs and total social welfare are respectively as follows:

$$\pi_i^F = \frac{3(A-bm)^2(2+n)^2}{2((2+n)^2+m(4+n))^2} \tag{19}$$

$$W^F = \frac{b^2m(2+n)^2+2Abm(4+n)+A^2(n(3+n)+m(4+n))}{2((2+n)^2+m(4+n))} \tag{20}$$

### 4.2 Private Leadership under Optimal Subsidy

From the total social welfare under private followership in (15), the differentiation of $W$ with respect to $s$ yields the following optimal subsidy where the superscript L stands for private leadership:

$$s^L = \frac{-4A(1+m)^2-A(3+4m(2+m))n+b(1+2m)(2+2m+n)(2+n+2m(1+n))}{2(4(1+m)^3+(1+2m)(4+m(5+2m))n+(n+2mn)^2)} \gtrless 0\,,$$

$$\text{if } b \gtrless \frac{A(4+3n+4m(2+m)(1+n))}{(1+2m)(2+2m+n)(2+n+2m(1+n))} \tag{21}$$

Again, from the assumption that $bm < A$, we have $s^L < \frac{A}{2m}$. The comparative statics yield:

$$\frac{\partial s^L}{\partial b} = \frac{(1+2m)(2+2m+n)(2+n+2m(1+n))}{2(4(1+m)^3+(1+2m)(4+m(5+2m))n+(n+2mn)^2)} > 0,$$

$$\frac{\partial s^L}{\partial m} = \frac{\begin{matrix}16(A+b)(1+m)^4+4(1+m)\Big(A\Big(6+4m\big(5+2m(3+m)\big)\Big)+b\Big(7+m\big(25+4m(7+2m)\big)\Big)\Big)n\\ +(1+2m)(16b(1+m)^3+A(3+2m)(5+4m(2+m)))n^2+(1+2m)^2(4A+b(3+4m(1+m)))n^3\end{matrix}}{2(4(1+m)^3+(1+2m)(4+m(5+2m))n+(n+2mn)^2)^2} > 0 \text{ and}$$

$$\frac{\partial s^L}{\partial n} = \frac{(1+2m)(-A+bm)(-4(1+m)^2-8(1+m)^2(1+2m)n-(3+2m)(n+2mn)^2)}{2(4(1+m)^3+(1+2m)(4+m(5+2m))n+(n+2mn)^2)^2} > 0,$$

$$\text{iff } n^2 \gtrless \frac{4(1+m)^4}{(2+\sqrt{(1+3m+2m^2)^2(1+6m+4m^2)}+2m(4+m(5+2m)))^2}$$

Similar to private followership, the optimal subsidy rate increases as the marginal social value increases or as the number of SEs increases. However, contrary to private followership, the optimal subsidy increases as the number of PEs rises. That is, when tough competition occurs under private leadership, the government increases the subsidy rate to encourage the production of the SEs

Substituting $s^L$ provides the equilibrium output levels:

$$q_j^L = \frac{4A(1+m)^2+A(1+2m)^2n+b(2+n+2m(1+n))^2}{4(1+m)^3+(1+2m)(4+m(5+2m))n+(n+2mn)^2},\ q_i^L = \frac{(1+2m)(A-bm)(2+n+2m(1+n))}{4(1+m)^3+(1+2m)(4+m(5+2m))n+(n+2mn)^2}\ ,\ \text{and}$$

$$Q^L = \frac{4(A+b)m(1+m)^2+(1+2m)(2bm(1+m)+A(2+m(3+2m)))n+A(n+2mn)^2}{4(1+m)^3+(1+2m)(4+m(5+2m))n+(n+2mn)^2} \tag{22}$$

Thus, $q_j^L \gtrless q_i^L$ if $b \gtrless -\frac{2A(1+m)}{(2+2m+n+2mn)(2+3m+2m^2+n+2mn)}$. Note that when the marginal social value of the SEs is large, its output is greater than that of the PEs under private leadership.

The market price is as follows:

$$P^L = \frac{2(1+m)(A-bm)(2+n+2m(1+n))}{4(1+m)^3+(1+2m)(4+m(5+2m))n+(n+2mn)^2} \tag{23}$$

The profit of the PEs and total social welfare are respectively as follows:

$$\pi_i^L = \frac{(A-bm)^2(3+4m(2+m))(2+n+2m(1+n))^2}{2(4(1+m)^3+(1+2m)(4+m(5+2m))n+(n+2mn)^2)^2} \tag{24}$$

$$W^L = \frac{\begin{matrix} b^2m\left(2+n+2m(1+n)\right)^2+2Abm\left(4+n+4m(2+m+n+mn)\right) \\ +A^2(4m(1+m)^2+(3+m(9+4m(2+m)))n+(n+2mn)^2) \end{matrix}}{2(4(1+m)^3+(1+2m)(4+m(5+2m))n+(n+2mn)^2)} \tag{25}$$

### 4.3 Endogenous Choice of Optimal Subsidy

We compare the optimal subsidy policies between private followership and private leadership.

**Proposition 3:** *The optimal subsidy under private followership is lower than that under private leadership.*

*Proof:* Comparing the optimal subsidies in (16) and (21) yields the followings:

$$s^F - s^L = \frac{2m(-A+bm)n(2m^2(1+n)+(1+n)(2+n)+m(3+2n(3+n)))}{((2+n)^2+m(4+n))(4(1+m)^3+(1+2m)(4+m(5+2m))n+(n+2mn)^2)} < 0. \quad \text{Q.E.D.}$$

It represents that the optimal subsidy under private leadership (private followership) should be high-powered (low-powered) to increase the outputs of SEs. This is because private leadership increases the output of the PEs but decreases the output of the SEs, as shown in the following proposition.

**Proposition 4:** *Under the optimal subsidy, the output of SE under private followership is higher than that under private leadership, while both output of the PE and total market outputs under private followership are lower than those under private leadership.*

*Proof:* Comparing the output levels in (17) and (22) yields the followings:

$$q_j^F - q_j^L = \frac{4m(A-bm)n(2+m+n+mn)}{((2+n)^2+m(4+n))(4(1+m)^3+(1+2m)(4+m(5+2m))n+(n+2mn)^2)} > 0.$$

$$q_i^F - q_i^L = (A-bm)\left(-\frac{2m(4+8m+4m^2+4n+9mn+4m^2n+n^2+2mn^2)}{(4+4m+4n+mn+n^2)(4+12m+12m^2+4m^3+4n+13mn+12m^2n+4m^3n+n^2+4mn^2+4m^2n^2)}\right) < 0.$$

$$Q^F - Q^L = \frac{2m(-A+bm)n(2m^2(1+n)+(2+n)^2+m(4+n(7+2n)))}{((2+n)^2+m(4+n))(4(1+m)^3+(1+2m)(4+m(5+2m))n+(n+2mn)^2)} < 0. \quad \text{Q.E.D.}$$

Proposition 4 supports the previous results in a duopoly with homogeneous products, in which the first-mover advantage increases the output of the PEs and total output compared to Cournot competition.[11] However, in our model where SE has different objective function, the SEs expand their outputs more under private followership to increase social value, while the PEs care only for their profits and expand their output more under private leadership.

**Proposition 5:** *Total social welfare under private followership is lower than that under private leadership.*

*Proof:* Comparing the social welfare in (19) and (24) yields the following:

$$W^F - W^L = -\frac{2m(A-bm)^2 n(2+m+n+mn)}{((2+n)^2+m(4+n))(4(1+m)^3+(1+2m)(4+m(5+2m))n+(n+2mn)^2)} < 0. \quad \text{Q.E.D.}$$

Proposition 5 indicates that private leadership is socially desirable than private followership because private leadership can induce PEs to behave aggressive, which increases total market outputs. This result is consistent with Cho and Lee (2017), who examined competition with single SE, but we confirm their result with multiple SEs. This finding also supports the previous result in the context of mixed markets where a profit-maximizing PE competes against a welfare-maximizing SE in a duopoly setting. For example, Pal (1998), Lu (2006), Ino and Matsumura (2010) and Matsumura and Ogawa (2014) examined the role of public and private firms with asymmetric payoffs in a mixed market and showed that either private or public leadership can improve welfare.

## 5. Discussions

We have assumed that the government can choose the optimal subsidy rate in the first stage, as a first-mover. As proposition 2 states, the subsidy rate does not matter on the endogenous choice in the second stage. However, we wonder whether this result is invariant even if the government is the second-mover. If so, which condition is necessary? Thus, we will examine the reverse case that the government chooses the optimal subsidy rate in the second stage after the PEs choose the competition mode in the first stage. We assume that in the final stage, PEs and SEs choose outputs simultaneously or sequentially under the committed competition mode in the first stage.

The equilibrium results in final stage are the same and described in Section 3.1 and 3.2. Also, the government choice on the optimal subsidy is also the same and described in Section 4.1 and 4.2. Thus, we need to examine the first stage where the PEs decide the endogenous choice on market competition. Then, from the profits ranks of PEs between the private followership and private leadership, Fig. 2 shows the locus of $n^{**}(m)$ and $n^{*}(m)$.

[11] See, for example, Gal-Or (1985) and Dowrick (1986).

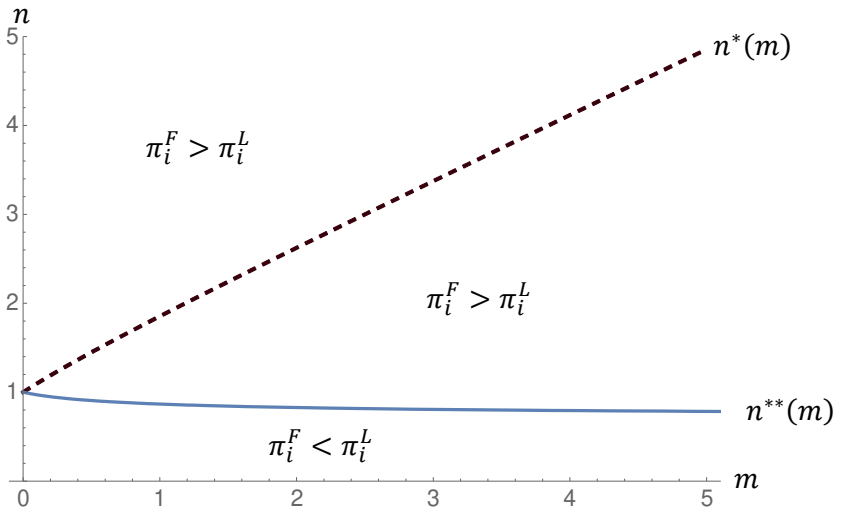


Fig. 2 The profits ranks of PEs under the optimal subsidy

**Proposition 6:** *Under the optimal subsidy, the profit of the PE under private leadership is higher than that under private followership when* $n = 1$ *or* $n < n^{**}(m)$.

*Proof:* Comparing the profits of PEs in (18) and (23) yields the following: $\pi_i^F - \pi_i^L = \frac{1}{2}\frac{(A-bm)^2 K(m,n)}{H(m,n)}$ where $H(m,n) = ((2+n)^2 + m(4+n))^2(4(1+m)^3 + (1+2m)(4+m(5+2m))n + (n+2mn)^2)^2$ and $K(m,n) = (4(1+m)^3 + (1+2m)(4+m(5+2m))n + (n+2mn)^2)^2 3(2+n)^2 - (3+4m(2+m))(2+n+2m(1+n))^2((2+n)^2 + m(4+n))^2$. Since $A > bm$ and $H(n,m) > 0$, the comparison depends on the sign of $K(m,n)$. Then, we have $K(m,1) < 0$. Further, when $n \geq 2$, there exists $\hat{m}$, which satisfies $K(\hat{m},n) = 0$, since $\lim_{n\to\infty} K(m,n) > 0$ and $\lim_{m\to\infty} K(m,n) < 0$ when $n \geq 2$. Hence, given the number of SEs, $m$, we can get $n^{**}(m)$, which satisfies $K(m,n) = 0$ when $n \geq 2$. Q.E.D

As shown in Fig. 2, the regions for the private leadership for being an equilibrium of endogenous competition mode is reduced when the government is the second-mover, in which it chooses the optimal subsidy after the PEs choose the competition mode. Then, proposition 5 and 6 support that total welfare is also reduced in that case. Hence, there is a first-mover advantage to support the private leadership equilibrium.

**Proposition 7:** *When the government decides the optimal subsidy after PEs choose the competition mode, the total social welfare is reduced when* $n^{**}(m) < n < n^*(m)$.

Proposition 7 also states that the equilibrium of endogenous competition mode does not changed either when $n < n^{**}(m)$ or $n > n^{*}(m)$. In particular, the first-mover advantage disappears either when $n = 1$ or when $n > m$. It implies that either (i) when the number of PE is one or (ii) when the number of PE is larger than that of SE, there is no welfare change between first-mover and second-mover of government choice on subsidization. Therefore, when $n^{**}(m) < n < n^{*}(m)$, the role of government should be emphasized in policy considerations when deciding subsidy policy.

## 6. Conclusion

The recent emergence of SEs and market competition with PEs has initiated the economic analysis on the performance of market structure and its welfare consequences. In this study, we have investigated the market role of PEs under government subsidy and examined the endogenous choice of private leadership. We showed that private leadership is better from the viewpoint of total social welfare, but the numbers of private and social enterprises affect the endogenous choices and welfare consequences. We also showed that the role of government in choosing the optimal subsidy policy will be significant when the number of PEs is larger than one but smaller than that of SEs. Otherwise, the first-mover advantage of the government does not exist.

However, better understanding on the market role of the SEs requires further examination on the different organizational structure between shareholders (investors) and stakeholders (managers, employees, business partners, and consumers). In particular, the internal relations on managerial delegation in SEs are more significant because it will affect the social activities and the profit-sharing scheme.[12] Thus, the practical and innovative analysis on the social activities of the SEs and the evaluation process in deciding government subsidy policy are promising topics for future research.

## References


Amir R. and Stepanova, A. (2006). Second-mover Advantage and Price Leadership in Bertrand Duopoly. *Games and Economic Behavior*, 55(1), 1-20.

Bennett, J. and la Manna, M. (2012), "Mixed oligopoly, public firm behavior, and free private entry," *Economics Letters*, 117(3), 767-769

Bian, J., Li, K.W., Guo, X. (2016). A Strategic Analysis of Incorporating CSR into Managerial Incentive Design. *Transportation Research Part E: Logistics and Transportation Review*, 86, 83-93.

Brand, B. and Grothe, M. (2015). Social Responsibility in a Bilateral Monopoly. *Journal of Economics*, 115(3), 275–289


[12] Nakamura Y., (2017) consider the endogenous choice problem of strategy contracts for public and private firms in a managerial mixed duopoly with differentiated goods. He clarifies that the equilibrium market structure depends on the bargaining power of the manager within the private firms.

Cho, S. M. and Lee, S. H. (2017). Subsidization Policy on the Social Enterprise for the Underprivileged. *Korean Economic Review*, 33(1), 153-178

Cremer, H., Marchand, M., and Thisse, J.-F. (1989). The Public Firm as an Instrument for Regulating an Oligopolistic Market," *Oxford Economic Papers*, 41(2), 283–301.

Daughety, A. F. (1990), "Beneficial Concentration," *American Economic Review*, 80(5), 1231-1237.

Dowrick, S. (1986). von Stackelberg and Cournot duopoly: Choosing roles," *Rand Journal of Economics*, 17(2), 251-260.

Estrin, S. and D. de Meza, (1995). Unnatural Monopoly. *Journal of public Economics*, 57(3), 471-488.

Flores, D. and Garcia, A. (2016). On the Output and Welfare Effects of a non-Profit Firm in a Mixed Duopoly: A Generalization. *Economic Systems,* 40, 631-637

Gal-Or, E. (1985). First Mover and Second Mover Advantages," *International Economic Review*, 26(3), 649-653.

Haraguchi, J. and Matsumura, T. (2016). Cournot–Bertrand Comparison in a Mixed Oligopoly. *Journal of Economics,* 117(2), 117–136.

Hamilton, J. H. and Slutsky, S. M. (1990). Endogenous Timing in Duopoly Games: Stackelberg or Cournot Equilibria. *Games and Economic Behavior*, 2(1), 29-46.

Ino, H. and Matsumura, T. (2010). What Role Should Public Enterprises Play in Free-Entry Markets?. *Journal of Economics*, 101(3), 213–30.

Ino, H. and Matsumura, T. (2012). How Many Firms Should Be Leaders? Beneficial Concentration Revisited. *International Economic Review*, 53(4), 1323-1340.

Kim, S. W., Kwag, S.H., Park, H.J., Lee, C.H., Seo, W, S., Lee, S.Y. and Kim, H.M. (2015), A Case Study on Social Enterprise for Disabled People, Chapter 3 Overseas Cases of Social Enterprise Employment for the Disabled, Policy Report on Korea Disabled People's Development Institute

Kopel, M. and Brand, B. (2012). Socially Responsible Firms and Endogenous Choice of Strategic Incentives. *Economic Modelling*, 29, 982-989

Lambertini, L. and Tampieri, A. (2015). Incentives, Performance and Desirability of Socially Responsible Firms in a Cournot Oligopoly. *Economic Modelling,* 50, 40-48

Liu, C.C., Wang, L.F.S. and Lee, S.H. (2015. Strategic Environmental Corporate Social Responsibility in a Differentiated Duopoly Market. *Economics Letters*, 129(4), 108-111

Lu, Y., (2006). Endogenous Timing in a Mixed Oligopoly with Foreign Competitors: the Linear Demand Case. *Journal of Economics,* 88(1), 49–68

Matsumura, T. and Ogawa, A. (2012). Price versus quantity in a mixed duopoly. *Economics Letters,* 116(2), 174–177.

Matsumura, T. and Ogawa, A. (2014). Corporate Social Responsibility or Payoff Asymmetry? A Study of an Endogenous Timing Game. *Southern Economic Journal*, 81(2), 457-473

Ono, Y. (1978). The Equilibrium of Duopoly in a Market for Homogeneous Goods. *Economica,* 45(179), 287–295

Pal, M. (1998). Endogenous Timing in a Mixed Oligopoly. *Economics Letters,* Vol. 61(2),181–185

Roeger, K., Blackwood, A. and Pettijohn, S. (2012). The Nonprofit Almanac 2012. Washington, DC: The Urban Institute.

Report on the Social Impact Investment Taskforce Established under the UK's Presidency of the G8, (2014). Impact Investment: The Invisible Heart of Markets: Harnessing the power of entrepreneurship, innovation and capital for public good.

Scrimitore, M. (2013). Price or Quantity?: The Strategic Choice of Subsidized Firms in a Mixed Duopoly. *Economics Letters,* 118(2), 337–341.

Scrimitore, M. (2014). Profitability under Commitment in Cournot and Bertrand Mixed Markets. *Journal of Institutional and Theoretical Economics,* 170(4), 684–703.

Van Damme, E. and Hurkens, S. (2004). Endogenous Price Leadership. *Games and Economic Behavior*, 47(2), 404–420

Nakamura, Y. (2017). Endogenous Market Structures in a Mixed Oligopoly with a Public Firm whose Managerial Contract is based on Welfare and Bargaining over the Managerial Contract of a Private Firm. *Economia Politica*, 34(2), 189–209.