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Ein Fehler in der ursprünglichen Versammlung der Heere ist im ganzen Verlauf des Feldzuges kaum wieder gut zu machen.

(Helmuth Karl Bernhard von Moltke: *Taktisch-strategische Aufsätze aus den Jahren 1857 bis 1871*)

Abstract

Tournament organizers supposedly design rules such that a team cannot be better off by exerting a lower effort. It is shown that the European qualifiers to the 2018 FIFA World Cup are not strategy-proof in this sense: a team might be eliminated if it wins its last match in the group stage, while it advances to play-offs by playing a draw, provided that all other results remain the same. The scenario could have happened in October 2017, after four-fifth of all matches have already been played. We present a theoretical model and identify nine incentive incompatible qualifiers to recent UEFA European Championships and FIFA World Cups. A mechanism is suggested in order to seal the way of manipulation in similar group-based qualification systems.

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Keywords: OR in sport; 2018 FIFA World Cup; tournament ranking; mechanism design; strategy-proofness

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1 Introduction

One important role of scientific research is to inform decision-makers about the possible properties, especially failures of different rules and formulas. It is an essential issue on the field of sport, since a bad regulation can easily lead to public outrage, as illustrated through several historical examples (Kendall and Lenten, 2017). These negative events may have contributed to the increasing popularity of operations research analysis of sport ranking rules (Gerchak, 1994; Wright, 2009, 2014), and to the recent application of an axiomatic approach towards sport rankings (Berker, 2014; Csató, 2017a,b; Vaziri et al., 2017). We aim to continue this research direction by analysing qualification tournaments with respect to the axiom called manipulability, strategy-proofness, or incentive compatibility. If this condition does not hold, a team[1] might gain by performing worse in certain matches.

Specifically, the qualifiers to two prominent football competitions, the UEFA European Championships and FIFA World Cups (in the European Zone) will be discussed from this perspective. They are organized recently such that the top $k-1$ teams from each group of size ℓ or $\ell+1$ qualify, while the kth placed teams from each group advance to play-offs, with a slight adjustment that either the best kth placed team qualifies or the worst kth placed team is eliminated. Consequently, the kth placed teams – which have not played against each other – should be compared in a subtournament where they are considered with the same number of matches, implying the ignorance of some group matches if group sizes vary. We get a negative result as the monotonicity of rankings for each group and the separate subtournament for the kth placed teams is not enough to guarantee the strategy-proofness of the whole qualification system.

The problem has still been revealed by a column in the case of the European qualification for the 2014 FIFA World Cup in Brazil (Dagaev and Sonin, 2013), and has been described by Dagaev and Sonin (2017) in a sentence: '*Two months before the end of the tournament, with 80% of games completed, there still was a scenario under which a team might need to achieve a draw instead of winning to go to Brazil.*'[2]

This paper outlines a similar scenario for 2018 FIFA World Cup qualification (UEFA) by showing that a team is eliminated if it wins in the last matchday of group stage, but it advances to play-offs by playing a draw, provided that all other results remain unchanged. The example takes the outcome of matches played before October 2017 as given. After that, we formalize a model of group-based qualification tournaments. A pair of theorems lists the conditions of incentive incompatibility and strategy-proofness, respectively. They are applied to identify nine recent qualification systems that can be manipulated. Finally, we suggest a mechanism for tournament organizers in order to solve the problem.

The rest of the article is organized as follows. In Section 2, we overview the related literature. Section 3 describes the real-world example, the European section of the 2018 FIFA World Cup qualification. The incentive incompatibility of the tournament is proved in Section 4. Section 5 presents and analyses the theoretical model, which is applied to examine the strategy-proofness of some qualification systems in Section 6. Section 7 summarizes policy implications for tournament organizers, and Section 8 concludes.

The paper is written both for the sport and the scientific community. Decision-makers not interested in or not familiar with the theoretical background of manipulation can skip

[1] The word team is used because of the example, but it can also be a player in other settings.

[2] We have written the first version of the paper without knowing about Dagaev and Sonin (2013) or Dagaev and Sonin (2017). While it is not an excuse for the originality of the current research, this fact indicates that the failure of qualification rules is almost obvious. It is worth to note that Dagaev and Sonin (2017) also build on a 'borrowed' real-world example.

Sections 2, 5 and maybe 3 as well as 6, in order to focus on the example presented in Section 4 and the possible remedies presented in Section 7.

2 Related work

Ranking in sport tournaments is closely related to the problem of preference aggregation. The issue of strategy-proofness is an extensively discussed concept in social choice theory since the well-known Gibbard-Satterthwaite impossibility theorem (Gibbard, 1973; Satterthwaite, 1975), which state that fairness may lead to manipulation, that is, if a voting rule is fair, there always exists a voter who can achieve a better outcome by tactical voting.

Nonetheless, there are several cases when an incentive compatible rule can be found, but a method used in practice is manipulable. For example, Tasnádi (2008) proved that the Hungarian mixed-member electoral system, applied between 1990 and 2010, suffers from a 'population paradox': the governing coalition may lose seats either by getting more votes or by the opposition obtaining fewer votes. Similarly, (Kóczy and Strobel, 2009) found that the invariant method (Pinski and Narin, 1976), characterised by Palacios-Huerta and Volij (2004), and used to rank academic journals by quality violates strategy-proofness as a journal can boost its performance by making additional citations to other journals.

Some recent works address the incentive (in)compatibility of sport ranking rules. Stanton and Williams (2013) investigate double-elimination tournaments (a competition where no participant is eliminated until it lost two matches) and show that they are vulnerable to manipulation by a coalition of players who can improve their chance of winning by throwing matches. Russell and Walsh (2009) and Schneider et al. (2016) also discuss manipulation by coalitions through a collusion between several teams. Pauly (2014) develops a mathematical model of strategic manipulation in round-robin subtournaments and derives an impossibility theorem. Vong (2017) considers the strategic manipulation problem in multistage tournaments and shows that it is necessary to allow only the top-ranked player to qualify from each group in order to guarantee that all players exert full effort. Russell (2010) studies the complexity of manipulation strategies in knock-out and round-robin tournaments as well as presents some algorithms which are able to identify with high accuracy whether a coalition manipulates the tournament. Lasek et al. (2016) suggest some strategies for improving a team's position in the official ranking of international football teams compiled by FIFA.

Strategy-proofness is usually violated because being ranked lower in the group stage might lead to facing a more preferred competitor in the following knock-out stage, but this means an advantage only in expected terms. Dagaev and Sonin (2017) prove that tournament systems, consisting of multiple round-robin and knock-out tournaments with noncumulative prizes, are characteristically incentive incompatible in a stronger sense since a team may be *strictly better off* by exerting a lower effort. Tournaments with subsequent group stages, widely used in handball, also violate strategy-proofness if some results are carried over (Csató, 2017c). Furthermore, there were two historical matches where a team was *ex ante* disinterested in winning by a high margin.

In the following, we will discuss this strong version of manipulation when a team is guaranteed to gain from a worse result.

3 2018 FIFA World Cup qualification (UEFA)

2018 FIFA World Cup qualification (UEFA) is the European section of the 2018 FIFA[3] World Cup qualification, the qualifier of national association football teams which are members of UEFA[4] for the 2018 FIFA World Cup, to be held in Russia.[5] Russia automatically qualified as a host, so – after Gibraltar and Kosovo became a FIFA member in May 2016 – 54 teams competed in the qualification for 13 slots in the final tournament.

The qualifying format was confirmed by the UEFA Executive Committee meeting on 22-23 March 2015 in Vienna. The qualification structure is as follows:

- Group stage (first round): Nine groups of six teams each, playing home-and-away round-robin matches. The winners of each group qualify to the 2018 FIFA World Cup, and the eight best runners-up advance to play-offs (second round).
- Play-offs (second round): The eight best second-placed teams from the group stage play home-and-away matches over two legs. The four winners qualify to the 2018 FIFA World Cup.

We focus on the first round. FIFA (2016, Article 20.4a) specifies this stage.

The matches shall be played in accordance with one of the following three formats:

a) in groups composed of several teams on a home-and-away basis, with three points for a win, one point for a draw and no points for a defeat (league format).

Tie-breaking rules in the groups are detailed in FIFA (2016, Article 20.6).

In the league format, the ranking in each group is determined as follows:

a) greatest number of points obtained in all group matches;

b) goal difference in all group matches;

c) greatest number of goals scored in all group matches.

If two or more teams are equal on the basis of the above three criteria, their rankings shall be determined as follows:

d) greatest number of points obtained in the group matches between the teams concerned;

e) goal difference resulting from the group matches between the teams concerned;

f) greater number of goals scored in all group matches between the teams concerned;

[3] FIFA stands for *Fédération Internationale de Football Association*, French for International Federation of Association Football, which is the international governing body of association football, futsal, and beach soccer.

[4] UEFA stands for *Union of European Football Associations*, the administrative body for association football in Europe. However, several UEFA member states are primarily or entirely located in Asia. It is one of the six continental confederations of world football's governing body FIFA.

[5] This section is mainly based on the Wikipedia page of 2018 FIFA World Cup qualification (UEFA). We will cite only those official documents which concern the ranking of teams.

g) the goals scored away from home count double between the teams concerned (if the tie is only between two teams).

h) fair play points system in which the number of yellow and red cards in all group matches is considered according to the following deductions:

– *first yellow card:*	*minus 1 point*
– *second yellow card/indirect red card:*	*minus 3 points*
– *direct red card:*	*minus 4 points*
– *yellow card and direct red card:*	*minus 5 points;*

i) drawing of lots by the FIFA Organising Committee.

Strangely, FIFA (2016, Article 20.6) does not state explicitly that greater goal differences and fair play points are preferred.[6] Choice of the eight best second-placed teams is not addressed in this document. FIFA (2016, Article 20.8) only describes that

Should the best second- or third-placed team within a group stage qualify for the next stage or for the final competition, the criteria to decide such best second- or third-placed team shall depend on the competition format and shall require the approval of FIFA following proposals from the confederations.

We were not able to find the relevant regulation of UEFA. But, according to a FIFA (FIFA, 2017) Media Release – reinforced by an earlier UEFA news (UEFA, 2016) –,

the eight best runners-up will be decided by ranking criteria as stated in the 2018 FIFA World Cup Regulations, namely points, goal difference, goals scored, goals scored away from home and disciplinary ranking, with the results against teams ranked 6th not being taken into account.

Since head-to-head results are nonexistent in the comparison of runners-up, the ranking of second-placed teams strictly follow tie-breaking in groups, with the crucial difference of discarding two matches played against the last team of the group.[7]

Another confederation of FIFA, the AFC (Asian Football Confederation) has published a Media Release (AFC, 2015) on the ranking of runners-up, which provides an illustration

[6] The purpose of mixing words *greater* and *greatest* is not clear for us.

[7] However, there was some controversy around the ranking of second-placed teams. According to our knowledge, FIFA and UEFA did not publish the ranking of second-placed teams *before* the end of group stage. The Spanish Wikipedia page of 2018 FIFA World Cup qualification (UEFA) ranked the runners-up on the basis of all matches played even on 12 September 2017. Most Wikipedia pages of the qualification, like the English, French, or Hungarian have placed Sweden on the 6th and Montenegro on the 7th position on 12 September 2017, after eight matchdays were played. On the other hand, Montenegro was the 6th and Sweden the 7th in German and Italian. As one can check in Tables 2 and A.4, Sweden and Montenegro had the same goal difference (+3) and number of goals scored (10) at that time. Furthermore, both teams scored 4 goals away from home. There is a difference in the goals against them at home, other teams of the group (without the last) had scored 2 goals in Sweden and 3 in Montenegro. It may be a weak argument to rank Sweden higher, nevertheless, in the lack of exact rules, we are not sure. It is also possible that disciplinary points count, but then with or without the matches against the last team? Anyway, it is rather an academic issue as it obviously does not influence which teams advance to play-offs.

on how to calculate the ranking of second-placed teams when some group matches are discarded.[8]

It will turn out that this, seemingly minor, modification in the comparison of runners-up has some unintended consequences regarding manipulation.

4 The possible manipulation

In this section we will present a possible manipulation of the European qualifiers to the 2018 FIFA World Cup.[9] Matches of the first eight matchdays – to be played between 4 September 2016 and 5 September 2017 – are assumed to be given.

Proposition 4.1. *It might still happen after four-fifth of all matches are over that 2018 FIFA World Cup qualification (UEFA) can be manipulated by Bulgaria playing a draw instead of a win against Luxembourg in the last matchday, on 10 October 2017.*

Proof. We provide an example by generating hypothetical results for the last two matchdays, to be played between 5 October 2017 and 10 October 2017.[10] Eight groups are detailed in the Appendix: Table A.1 presents Group B; Table A.2 presents Group C; Table A.3 presents Group D; Table A.4 presents Group E; Table A.5 presents Group F; Table A.6 presents Group G; Table A.7 presents Group H; and Table A.8 presents Group I.

Table 1 shows a possible scenario in Group A. Note that some results of Table 1.b may be unreasonable, like Belarus defeating Netherlands by 7-0. They are necessary to create the appropriate conditions for manipulation. Nevertheless, this set of match results still had a positive probability after eight matchdays were over.

On the basis of standings in Group A-I, runners-up are ranked in Table 2. Only the eight best second-places team advance to play-offs, hence Bulgaria is eliminated.

However, consider what happens if Bulgaria plays a draw of 1-1 against Luxembourg in the last matchday on 10 October 2017. It is clear that this change worsen Bulgaria's standing in the group. Nevertheless, it remains the runner-up with 16 points as both Bulgaria and Sweden would have the same goal difference (+4) with Bulgaria scoring more goals in all group matches (22 vs 18). On the other hand, Luxembourg overtakes Belarus thanks to its newly obtained draw: it has the same goal difference (-6) with more goals scored (12 vs 11). In the ranking of second-placed teams, matches against the last team are discarded. Consequently, Bulgaria would have 13 points, placing it seventh among the runners-up according to Table 2 (it has the same goal difference as Greece with more goals scored). Thus Bulgaria would advance to play-offs instead of Montenegro if it would concede a goal against Luxembourg. □

[8] Runners-up should be ranked in the second round of the Asian section of the 2018 FIFA World Cup qualification, organized for national teams which are members of AFC. AFC (2015) lists the following criteria as tie-breaking rules in this case: *greatest number of points obtained from group matches; goal difference in group matches; greatest number of goals scored in group matches; fewer number of points calculated according to the number of yellow and red cards received by the team; drawing of lots.* Number of goals scored away from home does not appear among the criteria and the preferred direction of goal difference is not specified, although it is provided for fair play points in contrast to FIFA (2017, Article 20.6).

[9] Perhaps the best summary of 2018 FIFA World Cup qualification (UEFA) is its Wikipedia page, too. However, a national team in Group G was referred to as Macedonia (at least on 12 September 2017), while its official name used by FIFA and UEFA is FYR Macedonia, as the country was admitted by United Nations under the provisional description *the former Yugoslav Republic of Macedonia*.

[10] It is worth to note that all teams play one match home and one away in the last two matchdays, which is not true for two subsequent matchdays chosen arbitrarily.

Table 1: 2018 FIFA World Cup qualification – UEFA Group A

(a) Match results of the first eight matchdays

Position is given according to the eight matches already played
Home team is in the row, away team (represented by its position) is in the column
Dates are given for the matches to be played on the last two matchdays in 2017

Position	Team	1	2	3	4	5	6
1	France	—	2-1	4-0	4-1	0-0	10 Oct
2	Sweden	2-1	—	1-1	3-0	7 Oct	4-0
3	Netherlands	0-1	10 Oct	—	3-1	5-0	4-1
4	Bulgaria	7 Oct	3-2	2-0	—	4-3	1-0
5	Luxembourg	1-3	0-1	1-3	10 Oct	—	1-0
6	Belarus	0-0	0-4	7 Oct	2-1	1-1	—

(b) Hypothetical match results of the last two matchdays

Last row shows an alternative result, obtained if Bulgaria manipulates

Date	Home team	Away team	Result
7 October 2017	Sweden	Luxemburg	0-4
7 October 2017	Belarus	Netherlands	7-0
7 October 2017	Bulgaria	France	8-0
10 October 2017	France	Belarus	1-0
10 October 2017	Luxemburg	Bulgaria	0-1
10 October 2017	Netherlands	Sweden	3-0
10 October 2017*	Luxemburg*	Bulgaria*	1-1*

(c) Final standing with the runner-up results

Pos = Position; W = Won; D = Drawn; L = Loss; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played 10 matches.

Last but one row contains the second-placed team's benchmark results, adjusted for the ranking of the runners-up (matches played against the 6th team are discarded) according to FIFA (2017).

Last row contains the second-placed team's alternative results, adjusted for the ranking of the runners-up (matches played against the 6th team are discarded) according to FIFA (2017), obtained if Bulgaria manipulates.

Pos	Team	W	D	L	GF	GA	GD	**Pts**
1	France	6	2	2	16	13	3	**20**
2	Bulgaria	6	0	4	22	17	5	**18**
3	Sweden	5	1	4	18	14	4	**16**
4	Netherlands	5	1	4	19	18	1	**16**
5	Belarus	2	2	6	11	17	-6	**8**
6	Luxembourg	2	2	6	11	18	-7	**8**
2	Bulgaria	4	0	4	17	14	3	**12**
2*	Bulgaria*	4*	1*	3*	20*	16*	4*	**13***

The example used in the proof of Proposition 4.1 is robust with respect to Groups B-I. If one considers the actual match results for these groups instead of the hypothetical ones, Slovakia is the worst second-placed teams with 12 points and a goal difference of +5 among

Table 2: 2018 FIFA World Cup qualification (UEFA) – Ranking of second-placed teams
Pos = Position; W = Won; D = Drawn; L = Loss; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points.
Since matches played against the 6th team in each group are discarded (FIFA, 2017), all teams have played 8 matches taken into account.
Last row contains Bulgaria's alternative results, obtained if it manipulates.

Pos	Team	Group	W	D	L	GF	GA	GD	**Pts**
1	Portugal	B	6	1	1	23	5	18	**19**
2	Italy	G	6	1	1	14	8	6	**19**
3	Northern Ireland	C	4	2	2	9	3	6	**14**
4	Wales	D	3	5	0	8	5	3	**14**
5	Turkey	I	4	2	2	8	8	0	**14**
6	Slovakia	F	4	1	3	11	5	6	**13**
7	Greece	H	3	4	1	8	4	4	**13**
8	Montenegro	E	3	3	2	12	6	6	**12**
9	Bulgaria	A	4	0	4	17	14	3	**12**
7*	Bulgaria*	A	4*	1*	3*	20*	16*	4*	**13***

the runners-up. Bulgaria is still eliminated by winning against Luxembourg according to Table 1.c, but is advanced to play-offs if it plays a draw of 1-1. Hence manipulation mainly depends on the events in Group A.

5 Theoretical background

In the following, we build a model for the home-and-away round-robin group stage of a qualification.

Definition 5.1. *Home-and-away round-robin tournament*: Let X be a nonempty finite set of at least two teams, $x, y \in X$ be two teams and $v : X \times X \to \{(v_1; v_2) : v_1, v_2 \in \mathbb{N}\} \cup \{—\}$ be a function such that $v(x, y) = —$ if and only if $x = y$. The pair (X, v) is called a *home-and-away round-robin tournament*.

In a home-and-away round-robin tournament, any two teams play each other once at home and once at away. Function v describes game results with the number of goals scored by the home and away teams, respectively.

Definition 5.2. *Ranking in home-and-away round-robin tournaments*: Let $\mathcal{X}$ be the set of home-and-away round-robin tournaments with a set of teams X. A *ranking method* S is a function that maps any characteristic function v of $\mathcal{X}$ into a strict order $S(v)$ on the set X.

Let (X, v) be a home-and-away round-robin tournament, $S(v)$ be its ranking and $x, y \in X$, $x \neq y$ be two different teams. x is ranked higher (lower) than y if and only if $x \succ_{S(v)} y$ ($x \prec_{S(v)} y$).

Let $x, y \in X$, $x \neq y$ be two different teams and $v(x, y) = (v_1(x, y); v_2(x, y))$. It is said that team x wins over team y if $v_1(x, y) > v_2(x, y)$ (home) or $v_1(y, x) < v_2(y, x)$ (away), team x loses to team y if $v_1(x, y) < v_2(x, y)$ (home) or $v_1(y, x) > v_2(y, x)$ (away) and teams x draws with team y if $v_1(x, y) = v_2(x, y)$.

Definition 5.3. *Number of points*: Let (X, v) be a home-and-away round-robin tournament and $x \in X$ be a team. Denote by $N_v^w(x)$ the number of wins and by $N_v^d(x)$ the number of draws of team x in (X, v), respectively. The *number of points* of team x is $s_v(x) = \alpha N_v^w(x) + N_v^d(x)$ such that $\alpha \geq 2$.

In other words, a win means α points, a draw means 1 point and a loss means 0 points.

Number of points does not necessarily give a strict order on the set of teams, therefore some tie-breaking rules should be introduced.

Definition 5.4. *Goal difference*: Let (X, v) be a home-and-away round-robin tournament and $x \in X$ be a team. The *goal difference* of team x is

$$gd_v(x) = \sum_{y \in X, y \neq x} (v_1(x,y) - v_2(x,y)) + \sum_{y \in X, y \neq x} (v_2(y,x) - v_1(y,x)).$$

Goal difference equals to the number of goals scored for team x minus the number of goals scored against team x.

Definition 5.5. *Head-to-head results*: Let (X, v) be a home-and-away round-robin tournament and $x \in X$ be a team. Denote by $L \subseteq X \setminus \{x\}$ a set of teams.

The *head-to-head number of points* of team x with respect to L in (X, v) is

$$\begin{aligned} s_v^L(x) \quad &= \quad \alpha \left(\left| \{ y \in L : v_1(x,y) > v_2(x,y) \} \right| + \left| \{ y \in L : v_1(y,x) < v_2(y,x) \} \right| \right) + \\ &\quad + \left| \{ y \in L : v_1(x,y) = v_2(x,y) \} \right| + \left| \{ y \in L : v_1(y,x) = v_2(y,x) \} \right| \end{aligned}$$

The *head-to-head goal difference* of team x with respect to L in (X, v) is

$$gd_v^L(x) = \sum_{y \in L} (v_1(x,y) - v_2(x,y)) + \sum_{y \in L} (v_2(y,x) - v_1(y,x)).$$

Definition 5.6. *Monotonicity of group ranking*: Let $\mathcal{X}$ be the set of home-and-away round-robin tournaments with a set of teams X, and S be a ranking method. S is *monotonic* if for any characteristic function v and for any different teams $x, y \in X$, $x \neq y$:

1. $s_v(x) > s_v(y) \Rightarrow x \succ_{S(v)} y$;
2. $s_v(x) = s_v(y)$ and $gd_v(x) > gd_v(y)$, furthermore, $s_v^L(x) > s_v^L(y)$, or $s_v^L(x) = s_v^L(y)$ and $gd_v^L(x) > gd_v^L(y)$ where $z \in L$ if and only if $s_v(x) = s_v(y) = s_v(z) \Rightarrow x \succ_{S(v)} y$.

Monotonicity implies that (a) a team should be ranked higher if it has a greater number of points (criterion 1); (b) a team should be ranked higher compared to another with the same number of points, an inferior goal difference and worse head-to-head results against all teams with the same number of points (criterion 2). Monotonicity still does not always lead to a unique ranking. The complexity of Definition 5.6 is necessary in order to cover the different tie-breaking rules recently applied by FIFA (goal difference) and UEFA (head-to-head results). See Berker (2014, Table 6) for a short overview of them.

Definition 5.7. *Group-based qualifier*: A *group-based qualifier* $\mathcal{T}$ consists of k groups of home-and-away round-robin tournaments with the set of teams X^1, X^2, ..., X^k such that $X^i \cap X^h = \emptyset$ for any $i \neq h$.

Definition 5.8. *Allocation rule*: An *allocation rule* of a group-based qualifier $\mathcal{T}$ is a function $\mathcal{R} : \mathcal{X}^1 \times \mathcal{X}^2 \times \cdots \times \mathcal{X}^k \to \{0; 1; 2\}$.

Consider a group-based qualifier $\mathcal{T}$, its allocation rule $\mathcal{R}$, a set of group results $V = \left\{ v^1, v^2, \dots, v^k \right\}$ and a team $x \in \cup_{i=1}^k X^i$. Team x is said to be (a) directly qualified if $\mathcal{R}(V,x) = 2$; (b) advanced to the next round with a chance to qualify if $\mathcal{R}(V,x) = 1$; (c) eliminated if $\mathcal{R}(V,x) = 0$.

Definition 5.9. *Qualification system*: The pair $(\mathcal{T},\mathcal{R})$ of a group-based qualifier $\mathcal{T}$ and its allocation rule $\mathcal{R}$ is a *qualification system*.

In order to fit 2018 FIFA World Cup qualification (UEFA) into this model, the allocation rule is allowed to compare teams from different groups through the introduction of an *extra group*.

Definition 5.10. *Extra group function*: Let $(\mathcal{T},\mathcal{R})$ be a qualification system. *Extra group function* $\mathcal{G}$ associates to any set of group results $V = \left\{ v^1, v^2, \dots, v^k \right\}$ a set of teams $X^{k+1} \subseteq \cup_{i=1}^k X^i$ and a set $X_x^i \subseteq X^i \setminus \{x\}$ for each $x \in X^{k+1}$.

Definition 5.11. *Extra group ranking*: Let $(\mathcal{T},\mathcal{R})$ be a qualification system and $\mathcal{G}$ be an extra group function. An *extra group ranking method* Q is a function that maps any set of group results $V = \left\{ v^1, v^2, \dots, v^k \right\}$ into a strict order on the set X^{k+1}.

Definition 5.12. *Ranking in extra group*: Let $(\mathcal{T},\mathcal{R})$ be a qualification system and $\mathcal{G}$ be an extra group function. The *number of points in the extra group* of team $x \in X^{k+1}$ is

$$\begin{aligned} s_{\mathcal{G},V}^{k+1}(x) \quad &= \quad \alpha \left(\left| \left\{ y \in X_x^i : v_1^i(x,y) > v_2^i(x,y) \right\} \right| + \left| \left\{ y \in X_x^i : v_1^i(y,x) < v_2^i(y,x) \right\} \right| \right) + \\ &\quad + \left| \left\{ y \in X_x^i : v_1^i(x,y) = v_2^i(x,y) \right\} \right| + \left| \left\{ y \in X_x^i : v_1^i(y,x) = v_2^i(y,x) \right\} \right| \end{aligned}$$

The *goal difference in the extra group* of team $x \in X_{k+1}$ is

$$gd_{\mathcal{G},V}^{k+1}(x) = \sum_{y \in X_x^i} \left(v_1^i(x,y) - v_2^i(x,y) \right) + \sum_{y \in X_x^i} \left(v_2^i(y,x) - v_1^i(y,x) \right).$$

Note that teams of X^{k+1} have not necessarily played against each other – so head-to-head results may be missing – since there are no further matches in the extra group, but their number of points and goal difference can be defined on the basis of certain group matches.

Definition 5.13. *Monotonicity of extra group ranking*: Let $(\mathcal{T},\mathcal{R})$ be a qualification system and $\mathcal{G}$ be an extra group function. Extra group ranking Q is said to be *monotonic* if for any set of group results $V = \{v_1, v_2, \dots, v_k\}$ and for any different teams $x, y \in X^{k+1}$, $x \neq y$:

1. $s_{\mathcal{G},V}^{k+1}(x) > s_{\mathcal{G},V}^{k+1}(y) \Rightarrow x \succ_{Q(V)} y$;

2. $s_{\mathcal{G},V}^{k+1}(x) = s_{\mathcal{G},V}^{k+1}(y)$ and $gd_{\mathcal{G},V}^{k+1}(x) > gd_{\mathcal{G},V}^{k+1}(y) \Rightarrow x \succ_{Q(V)} y$.

Definition 5.13 is simpler than Definition 5.6 due to the lack of head-to-head results in the extra group.

Definition 5.14. *Fairness of an allocation rule*: Let $(\mathcal{T},\mathcal{R})$ be a qualification system. Allocation rule $\mathcal{R}$ is *fair* if:

- there exists a common monotonic ranking S in each group such that $x, y \in X^i$, $1 \leq i \leq k$ and $x \succ_{S(v^i)} y$ implies $\mathcal{R}(V,x) \geq \mathcal{R}(V,y)$;

- there exists an extra group function $\mathcal{G}$ such that $x, y \in X^{k+1}$ implies $|X_x^i| = |X_y^i|$;
- there exists a monotonic extra group ranking Q such that $x, y \in X^{k+1}$ and $x \succ_{Q(V)} y$ implies $\mathcal{R}(V, x) \geq \mathcal{R}(V, y)$.

The idea behind a fair allocation rule is straightforward. Application of a monotonic ranking in groups ensures that teams have no incentive to exert a lower effort in any match since they cannot achieve a higher position in the group by deliberately playing worse. As it is also required in the extra group, extra group ranking Q should be monotonic. The second condition is responsible for fairness in the comparison of teams from different groups: if their number of matches considered in the extra group is different, number of points is not a good measure of performance as it cannot decrease if more matches are played.

Definition 5.15. *Manipulation*: Consider a qualification system $(\mathcal{T}, \mathcal{R})$ and a set of group results $V = \left\{v^1, v^2, \dots, v^i, \dots, v^k\right\}$. A team $x \in X^i$ can *manipulate* the qualification system $(\mathcal{T}, \mathcal{R})$ if there exists a set of group results $\bar{V} = \left\{v^1, v^2, \dots, \bar{v}^i, \dots, v^k\right\}$ such that $\bar{v}_2^i(x, y) \geq v_2^i(x, y)$ and $\bar{v}_1^i(y, x) \geq v_1^i(y, x)$ for all $y \in X^i$ and $\mathcal{R}(V, x) < \mathcal{R}(\bar{V}, x)$.

Manipulation means that team x can improve its position with respect to qualification by letting its opponents to score more goals.

Definition 5.16. *Strategy-proofness*: A qualification system $(\mathcal{T}, \mathcal{R})$ is called *strategy-proof* if there exists no set of group results $V = \left\{v^1, v^2, \dots, v^k\right\}$ under which a team can manipulate it.

Our main results concern the strategy-proofness of qualification systems which have a fair allocation rule.

Theorem 5.1. *Let $(\mathcal{T}, \mathcal{R})$ be a qualification system such that $\mathcal{R}$ is a fair allocation rule and the following conditions hold:*

- *the number of groups is at least $k \geq 2$;*
- *there is a difference in the allocation of teams in the extra group, that is, if $x \in X^{k+1}$, then there exists $z \in X^{k+1}$ such that $\mathcal{R}(V, x) \neq \mathcal{R}(V, z)$;*
- *there exists a team $x \in X^i \cap X^{k+1}$ such that $\left|\left\{y \in X^i : x \succ_{S(v^i)} y\right\}\right| \geq \ell + 1$ and $z \in X^i \setminus \{x\}$ if and only if $\ell = \left|\left\{y \in X^i : z \succ_{S(v^i)} y\right\}\right| \geq 1$.*

Then qualification system $(\mathcal{T}, \mathcal{R})$ does not satisfy strategy-proofness.

According to the second requirement of Theorem 5.1, teams of the extra group are treated differently by the allocation rule. The third condition means that if a team is considered in the extra group where matches played against the lowest ranked ℓ teams of its group are discarded, then at least $\ell + 1$ teams ranked lower than it can be found in the group.

Proof. An example is presented where a team can manipulate a qualification system satisfying all criteria of Theorem 5.1.

Table 3: Group 1 of Example 5.1

GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points.

Last but one row contains the group winner's benchmark results, adjusted for ranking in the extra group (matches played against the last team are discarded) according to the allocation rule $\mathcal{R}$.

Last row contains the group winner's alternative results, adjusted for ranking in the extra group (matches played against the last team are discarded) according to the allocation rule $\mathcal{R}$, obtained if team a manipulates.

Position	Team	a	b	c	GF	GA	GD	**Pts**
1	a	—	3-0	4-0	7	2	5	$2\alpha + 1$
2	b	2-0	—	1-0	3	6	-3	2α
3	c	0-0	3-0	—	3	5	-2	$\alpha + 1$
1	a	—	—	—	3	2	1	α
1*	a*	—	—	—	4*	1*	3*	α*

Example 5.1. Let $k = 2$, $X^1 = \{a, b, c\}$ and $X^2 = \{d, e\}$.

Consider the fair allocation rule $\mathcal{R}$ such that $x \in X^3$ if and only if $x \succ_{S(v^i)} y$ for all $x, y \in X^i$, $x \neq y$, $X^i_x = \left\{ y \in X^i \setminus \{x\} : \left| \left\{ z \in X^i : z \succ_{S(v^i)} y \right\} \right| = 1 \right\}$ and

$$\mathcal{R}(V, x) = \begin{cases} 0 & \text{if } x \in X^i \text{ and there exists a team } y \in X^i : x \prec_{S(v^i)} y \\ 0 & \text{if } x \in X^3 \text{ and there exists a team } y \in X^{k+1} : x \prec_{Q(V)} y \\ 2 & \text{otherwise} \end{cases}$$

$\mathcal{R}$ says that teams not winning their group are eliminated, so the extra group consists of the two remaining teams and the one ranked higher by Q – after the two matches of the first team against the third in Group 1 are discarded – qualifies.

A possible set of results in Group 1 is shown in Table 3. Team a should be the first since it has the most points (see criterion 1 of a monotonic group ranking method), and it is considered in the extra group with α points and a goal difference of +1 after discarding its two matches against team c, which is the last in Group 1 due to criterion 1 of a monotonic group ranking method.

There are only two matches to be played in Group 2. Let v^2 be given such that $v^2(d, e) = (3; 0)$ and $v^2(e, d) = (1; 0)$. Then team d should be the first (see criterion 2 of a monotonic group ranking method) and would be considered in the extra group with α points and a goal difference of +2. Consequently, $\mathcal{R}(V, a) = 0$ and $\mathcal{R}(V, d) = 1$ due to criterion 2 of a monotonic extra group ranking method Q.

Now examine what happens if $\bar{v}^1(a, c) = (1; 0)$ instead of $v^1(a, c) = (4; 0)$. Then teams a, b and c have 2α points and head-to-head goal differences of +4, -3 and -1, respectively, thus a is the first and c is the second according to criterion 2 of a monotonic group ranking method. Therefore, team a is considered with α points and a goal difference of +3 in the extra group. Then $\mathcal{R}(\bar{V}, a) = 1$ and $\mathcal{R}(\bar{V}, d) = 0$ due to criterion 2 of a monotonic extra group ranking method Q.

To summarize, team a can manipulate this qualification structure under a set of group results V, so it is not strategy-proof.

Example 5.1 has the least possible number of teams, three in the first and two in the second group. It is clear that the number of groups and the number of teams in them as well as parameter ℓ can be increased without changing the essence of the counterexample. □

Remark 5.1. 2018 FIFA World Cup qualification (UEFA), discussed in Section 3, fits into the model presented above. The number of groups is $k = 9$ and the allocation rule $\mathcal{R}$ is as follows:

- S is monotonic because number of points is the first and goal difference is the second tie-breaker in groups (Definition 5.6);
- Q is monotonic because number of points is the first and goal difference is the second tie-breaker in the extra group (Definition 5.13);
- the first-placed team in each group qualifies: $\mathcal{R}(V,x) = 2$ for all $x \in X^i$ if and only if $\nexists y \in X^i : y \succ_{S(v^i)} x$;
- the third-, fourth-, fifth- and sixth-placed teams in each group are eliminated: $\mathcal{R}(V,x) = 0$ for all $x \in X^i$ if $\left| \left\{ y \in X^i : y \succ_{S(v^i)} x \right\} \right| \geq 2$;
- the extra group consists of the second-placed teams: $x \in X^{k+1}$ if and only if $\left| \left\{ y \in X^i : y \succ_{S(v^i)} x \right\} \right| = 1$;
- matches against the last team are discarded in the extra group: if $x \in X^{k+1}$, then $z \in X^i \setminus (X^i_x \cup \{x\})$ if and only if $y \succ_{S(v^i)} z$ for all $y \in X^i \setminus \{z\}$;
- the worst second-placed team is eliminated: $\mathcal{R}(V,x) = 0$ if $x \in X^{k+1}$ and $y \succ_{Q(V)} x$ for all $y \in X^{k+1} \setminus \{x\}$;
- the eight best second-placed teams advance to the next round: $\mathcal{R}(V,x) = 1$ if $x \in X^{k+1}$ and $\exists y \in X^{k+1} : x \succ_{Q(V)} y$.

According to Definition 5.14, allocation rule $\mathcal{R}$ is fair because of the monotonicity of ranking methods S and Q together with $|X^i_v| = 5$ for all $x \in X^{k+1}$.

Proposition 5.1. *2018 FIFA World Cup qualification (UEFA) is not strategy-proof.*

Proof. The scenario presented in the proof of Proposition 4.1 shows that team Bulgaria $= x \in X^1$ can manipulate since there exist sets of group results $V = \{v^1, v^2, \dots, v^9\}$ and $\bar{V} = \{\bar{v}^1, v^2, \dots, v^9\}$ such that $\bar{v}^1 = v^1$ with the exception of $\bar{v}^1_1(y,x) = 1 > 0 = v^1_1(y,x)$, where team Luxembourg $= y \in X^1$ and $\mathcal{R}(V,x) = 0 < 1 = \mathcal{R}(\bar{V},x)$.

Theorem 5.1 can also be applied because of Remark 5.1: the allocation rule is fair, the number of groups is $9 > 2$, $\mathcal{R}(V,x)$ can be 0 or 1 if team $x \in X^{k+1}$ is in the extra group, and, finally, $x \in X^i \cap X^{k+1}$ implies that $\left| \left\{ y \in X^i : x \succ_{S(v^i)} y \right\} \right| = 4$, furthermore, $z \in X^i \setminus \{x\}$ if and only if $\left| \left\{ y \in X^i : z \succ_{S(v^i)} y \right\} \right| = 1$. □

Theorem 5.1 also proves the incentive incompatibility of 2014 FIFA World Cup qualification (UEFA), which has already been verified by Dagaev and Sonin (2013).

Now we state a positive result, a 'pair' of Theorem 5.1.

Theorem 5.2. *Let $(\mathcal{T}, \mathcal{R})$ be a qualification system such that $\mathcal{R}$ is a fair allocation rule and at least one of the following conditions hold:*

a) there is no need for an extra group;

b) there is no difference in the allocation of teams in the extra group, that is, $\mathcal{R}(V,x) = \mathcal{R}(V,y)$ for all $x, y \in X^{k+1}$;[11]

c) there exists no team $x \in X^i \cap X^{k+1}$ such that $\left| \left\{ y \in X^i : x \succ_{S(v^i)} y \right\} \right| \geq \ell + 1$ and $z \in X^i \setminus \{x\}$ if and only if $\ell = \left| \left\{ y \in X^i : z \succ_{S(v^i)} y \right\} \right| \geq 1$;

d) X^i_x is independent of v^i for all $x \in X^{k+1}$.

Then qualification system $(\mathcal{T}, \mathcal{R})$ satisfies strategy-proofness.

Proof. If there is no need for an extra group, monotonicity of S provides strategy-proofness.

If there is no difference in the allocation of teams in the extra group, then a team may improve its position in the extra group, but it has no incentives to cheat.

If all group matches are taken into account in the extra group or there exists no team ranked lower in the original group X^i than any team $x \in X^i \cap X^{k+1}$ of the extra group such that matches against it are considered in the extra group, than team x cannot manipulate by changing the set of its matches to be ignored because of the monotonicity of S and Q.

If X^i_x is independent of v^i, then $\mathcal{R}(V,x) = \mathcal{R}(\bar{V},x)$ under any sets of group results $V = \left\{ v^1, v^2, \dots, v^k \right\}$ and $\bar{V} = \left\{ v^1, v^2, \dots, \bar{v}^i, \dots, v^k \right\}$, so team x has no way to manipulate. □

The message of Theorem 5.2 for tournament organizers will be discussed later.

6 Discussion

It is known from Dagaev and Sonin (2013) and Proposition 5.1 that 2014 and 2018 FIFA World Cup qualifications (UEFA) were not strategy-proof. Qualifications to recent UEFA European Championships (UEFA Euro) and FIFA World Cups in the European Zone (World Cup (UEFA)) are analysed with respect to this property in Table 4. We have devised Theorem 5.1 and Theorem 5.2 such that they will be enough to answer the question.

UEFA European Championship is hold every four years since 1960. The qualifications for the tournaments between 1960 and 1992 were organized without an extra group, so they were strategy-proof due to condition a) of Theorem 5.2. Incentive compatibility of the 2004 and 2008 qualifying is provided by condition b) of Theorem 5.2. UEFA Euro 2020 qualifying is linked with the 2018-19 edition of the UEFA Nations League, which gives countries a secondary route to qualify for the final tournament. This format is not covered by our theoretical model, but it also violates strategy-proofness because of the findings of Dagaev and Sonin (2017).

The first incentive incompatible FIFA World Cup qualifications in the European zone was the 1998 FIFA World Cup qualification (UEFA). Nevertheless, the fairness of the 1994 FIFA World Cup qualification (UEFA) seems to be questionable as it is less difficult to achieve the two top positions in a group containing five or six teams than in a group with seven teams. The 2002 FIFA World Cup qualification (UEFA) again satisfied strategy-proofness due to the lack of extra group.

[11] Note that allocation rule $\mathcal{R}$ does not take seeding in play-offs into account. It is not a problem if play-offs are drawn randomly (like in the UEFA Euro 2000 qualifying) or based on an exogenous ranking of the teams (like in the 2018 FIFA World Cup qualification (UEFA)). However, if, for example, the best half of all teams advanced to play-offs from the extra group are placed in Pot 1, then there is a difference in the allocation of teams in the extra group, although it is not reflected by the allocation rule $\mathcal{R}$.

Table 4: Qualifications to the UEFA European Championships and FIFA World Cups in the European Zone since 1990

Groups = Number of groups; Teams = Number of teams; Gr / T = Number of groups / Number of teams in each group; Slots = Number of teams qualified; DQ = Number of teams directly qualified; PLO = Number of teams advanced to play-offs; Discarded = Group matches that are discarded when teams from different groups are ranked; SP = Strategy-proofness (✔: yes; ✗: no)

Qualification	Groups	Teams	Gr / T	Slots	DQ	PLO	Discarded matches	SP
1992 UEFA Euro	7	33	5/5; 2/4	7	1st	—	—	✔
1996 UEFA Euro	8	47	7/6; 1/5	15	1st; six best 2nd	two worst 2nd	against 5th and 6th	✗
2000 UEFA Euro	9	49	4/6; 5/5	14	1st; best 2nd	eight worst 2nd	against 5th and 6th	✗
2004 UEFA Euro	10	50	10/5	15	1st	all 2nd	—	✔
2008 UEFA Euro	7	50	1/8; 6/7	14	1st; 2nd	—	—	✔
2012 UEFA Euro	9	51	6/6; 3/5	14	1st; best 2nd	eight worst 2nd	against 6th	✗
2016 UEFA Euro	9	53	8/6; 1/5	23	1st; 2nd; best 3rd	eight worst 3rd	against 6th	✗
1990 World Cup (UEFA)	7	32	4/5; 3/4	13	1st; six 2nd[1]	—	—	✔
1994 World Cup (UEFA)	6	37	1/7; 4/6; 1/5[2]	12	1st; 2nd	—	—	✔
1998 World Cup (UEFA)	9	49	4/6; 5/5	14	1st; best 2nd	eight worst 2nd	against 5th and 6th	✗
2002 World Cup (UEFA)	9	50	5/6; 4/5	13.5[3]	1st	all 2nd[4]	—	✔
2006 World Cup (UEFA)	8	51	3/7; 5/6	13	1st; two best 2nd	six worst 2nd	against 7th	✗
2010 World Cup (UEFA)	9	53	8/6; 1/5	13	1st	eight best 2nd	against 6th	✗
2014 World Cup (UEFA)	9	53	8/6; 1/5	13	1st	eight best 2nd	against 6th	✗
2018 World Cup (UEFA)	9	54	9/6	13	1st	eight best 2nd	against 6th	✗

[1] The second-placed teams in the four groups containing five teams directly qualified together with the two best second-placed teams in the three groups containing only four teams
[2] Group 5, originally containing six teams, ended up with five after Yugoslavia were suspended
[3] One team was advanced to an intercontinental play-off
[4] The runner-up of Group 2 was drawn randomly for an intercontinental play-off

Tu summarize, at least nine recent incentive incompatible qualifications (to the 1996, 2000, 2012 and 2016 UEFA European Championships as well as to the 1998, 2006, 2010, 2014 and 2018 FIFA World Cups in the European Zone) can be identified on the basis of the theoretical results presented in Section 5. This finding carries a really frightening message for FIFA and UEFA: it has had a positive probability that a serious scandal occurs during a recent qualification, for example, in October 2017 as shown in a detailed example. It would be especially disturbing because Luxembourg would have practically no incentive to interfere with the manipulation of Bulgaria in order to prevent the elimination of Montenegro, and it may even interested in scoring a goal to be the fifth in the group. Fortunately, the situation has not materialized, and currently we do not know about any attempt to manipulate these qualifications in the way presented above. Probably the closest case was France against Israel in the 1996 UEFA Euro qualifying, where France would have better measures among runners-up if it would have scored two own goals (Csató, 2017d).

7 A strategy-proof mechanism for qualifications

We think the lack of observations of dishonest behaviour does not reduce the value of strategy-proofness, especially if it can be satisfied without significant rule changes. For instance, in the 2018 FIFA World Cup qualification (UEFA), the root of the problem resides in the difference of group and second-places teams ranking by discarding the matches against the sixth-placed teams in the latter case. The greatest pity about this situation is that it could have been straightforward to avoid by UEFA ditching the strange policy of ignoring some group matches, since all groups would have six teams following the admission of Gibraltar and Kosovo. Yet they chose not to modify the rules. According to an UEFA News (UEFA, 2017), released on 10 October 2017, *after* the end of group stage: '*the exclusion of results against sixth-placed teams was retained to alleviate any possible imbalance between the qualifying groups caused by the late introductions of Gibraltar and Kosovo*'. While it is respectable that organizers wanted to prevent some *mathematically unprovable* imbalances between the groups, they have sacrificed the much more clear and important theroretical issue of incentive compatibility.

However, it seems to be necessary to present a incentive compatible design in order to argue against the rules of recent qualifications. Let us examine Theorem 5.2 and search for strategy-proof qualification systems:

a) *There is no need for an extra group*: It holds if all groups have the same number of teams, which may conflict with divisibility. However, we suggest to choose this solution when it remains possible.

b) *There is no difference in the allocation of teams in the extra group*: It practically means that all second- or third-placed teams either qualify or advance to play-offs regardless that some groups may have more teams. For example, UEFA Euro 2004 qualifying was strategy-proof since the top team in each group automatically qualified, and all runners-up were paired for play-offs.

c) *Teams of the extra group have played no matches considered in the extra group against teams ranked lower in their groups*: If there are around 50 competing teams, it requires small group sizes (with few group matches and an increased randomness), standard group sizes with more rounds (which increases the number of matches to be played by a given team), or the ignorance of a large number of matches in the extra group.

Nonetheless, this condition provides the strategy-proofness of 2018 European Men's Handball Championship qualification phase 2 where the 32 teams were divided into seven groups of four teams each such that the top two teams of each group and the best third-ranked team (determined by considering only the matches played against the first and second team) qualified. Since there was only one team ranked lower in the groups than the teams of the extra group, they cannot manipulate by changing the set of matches ignored in the extra group.

d) *Matches to be discarded in the extra group are independent of group results*: We think it could be the ultimate solution. Social choice theory usually opposes the violation of anonymity at all costs, but let us study how groups are seeded: if n teams should be drawn into k groups, there would be k teams in Pot 1, k teams in Pot 2 and so on, until Pot m with $k < \ell < 2k$ (as in UEFA Euro 2008 qualifying) or with $\ell \leq k$ (as in 2018 FIFA World Cup qualification (UEFA)) is reached. As the difference in group sizes is caused by Pot m, it is fair and straightforward to discard the matches played against the team from Pot m in the ranking of the extra group, which immediately provides strategy-proofness according to Theorem 5.2.

In the case of 2018 FIFA World Cup qualification (UEFA) it means to fix in advance that matches played against teams in Pot 6 (Luxembourg, Andorra, San Marino, Georgia, Kazakhstan, Malta, Liechtenstein as well as the lately introduced Gibraltar and Kosovo) are discarded in the comparison of second-placed teams. Since only Luxembourg and Georgia obtained a better (the fifth) position in the qualification, this policy does not make much difference in practice.

Nevertheless, a problem may arise when a team from Pot m should be considered in the extra group due its unexpectedly good performance.[12] The unlikely scenario[13] can be immediately solved by discarding the match against the team in Pot $m-1$ in the case of this particular team, which does not ruin strategy-proofness.

To summarize, we suggest to follow the subsequent mechanism in order to guarantee the strategy-proofness of a qualification system: (1) policy a) if the number of teams n can be divided by the number of groups k; (2) policy b) if n is not divisible by k but teams in the extra group can be treated uniformly (which is certainly impossible if the extra group contains an odd number of teams); (3) policy d) if the first two policies cannot be implemented.

Another solution might be an artificial reduction of the number of teams in order to achieve equal group sizes. For instance, the weakest teams (e.g. Gibraltar, Liechtenstein, San Marino etc.) can be relegated to a special group, where they play against each other without the possibility of direct qualification. The winner of this group may advance to a play-off with the best runner-up or third-placed team. Besides excluding manipulation, this solution has the further benefit of giving a chance for lower-ranked national teams, mainly composed of amateur players, to compete in their own league and enjoy more success than scoring some lucky goals against professional sportsmen.[14]

[12] We are grateful to Dénes Pálvölgyi for spotting this issue.

[13] One of the greatest surprise occurred in 2016 UEFA Euro qualifying when Greece finished as the last team in Group F despite it was drawn from Pot 1.

[14] This idea may partially inspired UEFA Nations League, which starts in September 2018 and is

It is also possible to organize a preliminary round for lower-ranked teams such as in the CEV qualification for the 2018 FIVB Volleyball Men's World Championship. One can also follow the solution applied in the European zone of qualification for the 1990 FIFA World Cup: the runner-up (or third-placed team) to be directly qualified / eliminated is chosen from the groups containing more / less teams without discarding any matches.

8 Conclusions

Design of appropriate sport ranking rules is an important theoretical problem of economics and operations research. Tournament organizers may face unpleasant situations when they miss analysing strategy-proofness. While manipulation is often a low-probability event, the potential costs can be enormous. We have demonstrated that decision makers have chosen a risky strategy in the case of qualification tournaments to some recent UEFA European Championships and FIFA World Cups.

We hope the paper has reinforced that the scientific community and the sports industry should work more closely together in studying the effects of potential rules and rule changes even before they are implemented. For example, the governing bodies of major sports may invite academics to identify possible loopholes in proposed regulations in order to prevent serious scandals.

There are at least two possible directions for future research. First, a number of sport ranking rules can be investigated from the perspective of incentive (in)compatibility. We plan to write some follow-up papers on this topic. Second, similarly to Berker (2014) and Lasek et al. (2016), the current theory-oriented investigation can be supplemented by estimating the probability of manipulation with the use of historical and Monte-Carlo simulated data.

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Appendix

Table A.1: 2018 FIFA World Cup qualification – UEFA Group B

(a) Match results of the first eight matchdays
Position is given according to the eight matches already played
Home team is in the row, away team (represented by its position) is in the column
Dates are given for the matches to be played on the last two matchdays in 2017

Position	Team	1	2	3	4	5	6
1	Switzerland	—	2-0	7 Oct	2-0	3-0	1-0
2	Portugal	10 Oct	—	3-0	5-1	6-0	4-1
3	Hungary	2-3	0-1	—	10 Oct	4-0	3-1
4	Faroe Islands	0-2	0-6	0-0	—	1-0	7 Oct
5	Andorra	1-2	7 Oct	1-0	0-0	—	0-1
6	Latvia	0-3	0-3	0-2	0-2	10 Oct	—

(b) Hypothetical but reasonable match results of the last two matchdays

Date	Home team	Away team	Result
7 October 2017	Faroe Islands	Latvia	0-0
7 October 2017	Andorra	Portugal	0-3
7 October 2017	Switzerland	Hungary	2-0
10 October 2017	Hungary	Faroe Islands	2-0
10 October 2017	Latvia	Andorra	1-1
10 October 2017	Portugal	Switzerland	1-1

(c) Final standing with the runner-up results
Pos = Position; W = Won; D = Drawn; L = Loss; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played 10 matches.
Last row contains the second-placed team's results, adjusted for the ranking of the runners-up (matches played against the 6th team are discarded) according to FIFA (2017).

Pos	Team	W	D	L	GF	GA	GD	**Pts**
1	Switzerland	9	1	0	21	4	17	**28**
2	Portugal	8	1	1	32	5	27	**25**
3	Hungary	4	1	5	13	11	2	**13**
4	Faroe Islands	2	3	5	4	17	-13	**9**
5	Latvia	1	2	7	4	19	-15	**5**
6	Andorra	1	2	7	3	21	-18	**5**
2	Portugal	6	1	1	23	5	18	**19**

Table A.2: 2018 FIFA World Cup qualification – UEFA Group C

(a) Match results of the first eight matchdays
Position is given according to the eight matches already played
Home team is in the row, away team (represented by its position) is in the column
Dates are given for the matches to be played on the last two matchdays in 2017

Position	Team	1	2	3	4	5	6
1	Germany	—	2-0	8 Oct	3-0	6-0	7-0
2	Northern Ireland	5 Oct	—	4-0	2-0	2-0	4-0
3	Azerbaijan	1-4	0-1	—	5 Oct	1-0	5-1
4	Czech Republic	1-2	0-0	0-0	—	2-1	8 Oct
5	Norway	0-3	8 Oct	2-0	1-1	—	4-1
6	San Marino	0-8	0-3	0-1	0-6	5 Oct	—

(b) Hypothetical but reasonable match results of the last two matchdays

Date	Home team	Away team	Result
5 October 2017	Azerbaijan	Czech Republic	1-1
5 October 2017	Northern Ireland	Germany	0-1
5 October 2017	San Marino	Norway	0-2
8 October 2017	Czech Republic	San Marino	3-0
8 October 2017	Germany	Azerbaijan	2-0
8 October 2017	Norway	Northern Ireland	0-0

(c) Final standing with the runner-up results
Pos = Position; W = Won; D = Drawn; L = Loss; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played 10 matches.
Last row contains the second-placed team's results, adjusted for the ranking of the runners-up (matches played against the 6th team are discarded) according to FIFA (2017).

Pos	Team	W	D	L	GF	GA	GD	**Pts**
1	Germany	10	0	0	38	2	36	**30**
2	Northern Ireland	6	2	2	16	3	13	**20**
3	Czech Republic	3	4	3	14	10	4	**13**
4	Norway	3	2	5	10	16	-6	**11**
5	Azerbaijan	3	2	5	9	15	-6	**11**
6	San Marino	0	0	10	2	43	-41	**0**
2	Northern Ireland	4	2	2	9	3	6	**14**

Table A.3: 2018 FIFA World Cup qualification – UEFA Group D

(a) Match results of the first eight matchdays

Position is given according to the eight matches already played
Home team is in the row, away team (represented by its position) is in the column
Dates are given for the matches to be played on the last two matchdays in 2017

Position	Team	1	2	3	4	5	6
1	Serbia	—	1-1	2-2	3-2	9 Oct	3-0
2	Wales	1-1	—	9 Oct	1-0	1-1	4-0
3	Republic of Ireland	0-1	0-0	—	1-1	1-0	6 Oct
4	Austria	6 Oct	2-2	0-1	—	1-1	2-0
5	Georgia	1-3	6 Oct	1-1	1-2	—	1-1
6	Moldova	0-3	0-2	1-3	9 Oct	2-2	—

(b) Hypothetical but reasonable match results of the last two matchdays

Date	Home team	Away team	Result
6 October 2017	Georgia	Wales	0-1
6 October 2017	Austria	Serbia	0-0
6 October 2017	Republic of Ireland	Moldova	2-0
9 October 2017	Moldova	Austria	1-2
9 October 2017	Serbia	Georgia	1-1
9 October 2017	Wales	Republic of Ireland	1-0

(c) Final standing with the runner-up results

Pos = Position; W = Won; D = Drawn; L = Loss; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played 10 matches.
Last row contains the second-placed team's results, adjusted for the ranking of the runners-up (matches played against the 6th team are discarded) according to FIFA (2017).

Pos	Team	W	D	L	GF	GA	GD	**Pts**
1	Serbia	5	5	0	18	8	10	**20**
2	Wales	5	5	0	14	5	9	**20**
3	Republic of Ireland	4	4	2	11	7	4	**16**
4	Austria	3	4	3	12	11	1	**13**
5	Georgia	0	6	4	9	14	-5	**6**
6	Moldova	0	2	8	5	24	-19	**2**
2	Wales	3	5	0	8	5	3	**14**

Table A.4: 2018 FIFA World Cup qualification – UEFA Group E

(a) Match results of the first eight matchdays

Position is given according to the eight matches already played
Home team is in the row, away team (represented by its position) is in the column
Dates are given for the matches to be played on the last two matchdays in 2017

Position	Team	1	2	3	4	5	6
1	Poland	—	8 Oct	3-2	3-1	2-1	3-0
2	Montenegro	1-2	—	5 Oct	1-0	4-1	5-0
3	Denmark	4-0	0-1	—	8 Oct	1-0	4-1
4	Romania	0-3	1-1	0-0	—	1-0	5 Oct
5	Armenia	5 Oct	3-2	1-4	0-5	—	2-0
6	Kazakhstan	2-2	0-3	1-3	0-0	8 Oct	—

(b) Hypothetical but reasonable match results of the last two matchdays

Date	Home team	Away team	Result
5 October 2017	Armenia	Poland	1-5
5 October 2017	Montenegro	Denmark	0-0
5 October 2017	Romania	Kazakhstan	2-0
8 October 2017	Denmark	Romania	1-1
8 October 2017	Kazakhstan	Armenia	1-0
8 October 2017	Poland	Montenegro	1-1

(c) Final standing with the runner-up results

Pos = Position; W = Won; D = Drawn; L = Loss; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played 10 matches.
Last row contains the second-placed team's results, adjusted for the ranking of the runners-up (matches played against the 6th team are discarded) according to FIFA (2017).
Note: Montenegro is ranked above Denmark due to FIFA (2016, Article 20.6d) because it has obtained 4 points against Denmark, while Denmark has obtained only 1 point against Montenegro.

Pos	Team	W	D	L	GF	GA	GD	**Pts**
1	Poland	7	2	1	24	13	11	**23**
2	Montenegro	5	3	2	19	8	11	**18**
3	Denmark	5	3	2	19	8	11	**18**
4	Romania	3	4	3	11	9	2	**13**
5	Armenia	2	0	8	9	25	-16	**6**
6	Kazakhstan	1	2	7	5	24	-19	**5**
2	Montenegro	3	3	2	12	6	6	**12**

Table A.5: 2018 FIFA World Cup qualification – UEFA Group F

(a) Match results of the first eight matchdays

Position is given according to the eight matches already played
Home team is in the row, away team (represented by its position) is in the column
Dates are given for the matches to be played on the last two matchdays in 2017

Position	Team	1	2	3	4	5	6
1	England	—	2-1	5 Oct	3-0	2-0	2-0
2	Slovakia	0-1	—	1-0	3-0	4-0	8 Oct
3	Slovenia	0-0	1-0	—	8 Oct	4-0	2-0
4	Scotland	2-2	5 Oct	1-0	—	1-1	2-0
5	Lithuania	8 Oct	1-2	2-2	0-3	—	2-0
6	Malta	0-4	1-3	0-1	1-5	5 Oct	—

(b) Hypothetical but reasonable match results of the last two matchdays

Date	Home team	Away team	Result
5 October 2017	England	Slovenia	2-1
5 October 2017	Malta	Lithuania	0-1
5 October 2017	Scotland	Slovakia	0-0
8 October 2017	Lithuania	England	1-3
8 October 2017	Slovakia	Malta	3-0
8 October 2017	Slovenia	Scotland	1-0

(c) Final standing with the runner-up results

Pos = Position; W = Won; D = Drawn; L = Loss; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played 10 matches.
Last row contains the second-placed team's results, adjusted for the ranking of the runners-up (matches played against the 6th team are discarded) according to FIFA (2017).

Pos	Team	W	D	L	GF	GA	GD	**Pts**
1	England	8	2	0	21	5	16	**26**
2	Slovakia	6	1	3	17	6	11	**19**
3	Slovenia	5	2	3	12	6	6	**17**
4	Scotland	4	3	3	14	11	3	**15**
5	Lithuania	2	2	6	8	21	-13	**8**
6	Malta	0	0	10	2	25	-23	**0**
2	Slovakia	4	1	3	11	5	6	**13**

Table A.6: 2018 FIFA World Cup qualification – UEFA Group G

(a) Match results of the first eight matchdays

Position is given according to the eight matches already played
Home team is in the row, away team (represented by its position) is in the column
Dates are given for the matches to be played on the last two matchdays in 2017

Position	Team	1	2	3	4	5	6
1	Spain	—	3-0	6 Oct	4-1	4-0	8-0
2	Italy	1-1	—	2-0	1-0	6 Oct	5-0
3	Albania	0-2	9 Oct	—	0-3	2-1	2-0
4	Israel	9 Oct	1-3	0-3	—	0-1	2-1
5	FYR Macedonia	1-2	2-3	1-1	1-2	—	9 Oct
6	Liechtenstein	0-8	0-4	0-2	6 Oct	0-3	—

(b) Hypothetical but reasonable match results of the last two matchdays

Date	Home team	Away team	Result
6 October 2017	Italy	FYR Macedonia	2-0
6 October 2017	Liechtenstein	Israel	0-1
6 October 2017	Spain	Albania	3-1
9 October 2017	Albania	Italy	1-2
9 October 2017	Israel	Spain	0-3
9 October 2017	FYR Macedonia	Liechtenstein	2-1

(c) Final standing with the runner-up results

Pos = Position; W = Won; D = Drawn; L = Loss; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played 10 matches.
Last row contains the second-placed team's results, adjusted for the ranking of the runners-up (matches played against the 6th team are discarded) according to FIFA (2017).

Pos	Team	W	D	L	GF	GA	GD	**Pts**
1	Spain	9	1	0	36	3	33	**28**
2	Italy	8	1	1	23	8	15	**25**
3	Albania	4	1	5	12	14	-2	**13**
4	Israel	4	0	6	10	17	-7	**12**
5	FYR Macedonia	3	1	6	12	17	-5	**10**
6	Liechtenstein	0	0	10	2	36	-34	**0**
2	Italy	6	1	1	14	8	6	**19**

Table A.7: 2018 FIFA World Cup qualification – UEFA Group H

(a) Match results of the first eight matchdays

Position is given according to the eight matches already played
Home team is in the row, away team (represented by its position) is in the column
Dates are given for the matches to be played on the last two matchdays in 2017
Bosnia and Herz. stands for Bosnia and Herzegovina

Position	Team	1	2	3	4	5	6
1	Belgium	—	4-0	1-1	10 Oct	8-1	9-0
2	Bosnia and Herz.	7 Oct	—	0-0	2-0	5-0	5-0
3	Greece	1-2	1-1	—	2-0	0-0	10 Oct
4	Cyprus	0-3	3-2	7 Oct	—	0-0	3-1
5	Estonia	0-2	10 Oct	0-2	1-0	—	4-0
6	Gibraltar	0-6	0-4	1-4	1-2	7 Oct	—

(b) Hypothetical but reasonable match results of the last two matchdays

Date	Home team	Away team	Result
7 October 2017	Gibraltar	Estonia	0-1
7 October 2017	Bosnia and Herzegovina	Belgium	0-3
7 October 2017	Cyprus	Greece	0-1
10 October 2017	Belgium	Cyprus	3-1
10 October 2017	Estonia	Bosnia and Herzegovina	1-2
10 October 2017	Greece	Gibraltar	3-0

(c) Final standing with the runner-up results

Pos = Position; W = Won; D = Drawn; L = Loss; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played 10 matches.
Last row contains the second-placed team's results, adjusted for the ranking of the runners-up (matches played against the 6th team are discarded) according to FIFA (2017).

Pos	Team	W	D	L	GF	GA	GD	**Pts**
1	Belgium	9	1	0	40	4	36	**28**
2	Greece	5	4	1	15	5	10	**19**
3	Bosnia and Herzegovina	5	2	3	21	11	10	**17**
4	Cyprus	3	1	5	9	16	-7	**11**
5	Estonia	3	2	5	8	19	-11	**11**
6	Gibraltar	0	0	10	3	41	-38	**0**
2	Greece	3	4	1	8	4	4	**13**

Table A.8: 2018 FIFA World Cup qualification – UEFA Group I

(a) Match results of the first eight matchdays

Position is given according to the eight matches already played
Home team is in the row, away team (represented by its position) is in the column
Dates are given for the matches to be played on the last two matchdays in 2017

Position	Team	1	2	3	4	5	6
1	Croatia	—	2-0	1-1	1-0	6 Oct	1-0
2	Iceland	1-0	—	2-0	2-0	3-2	9 Oct
3	Turkey	1-0	6 Oct	—	2-2	2-0	2-0
4	Ukraine	9 Oct	1-1	2-0	—	1-0	3-0
5	Finland	0-1	1-0	9 Oct	1-2	—	1-1
6	Kosovo	0-6	1-2	1-4	6 Oct	0-1	—

(b) Hypothetical but reasonable match results of the last two matchdays

Date	Home team	Away team	Result
6 October 2017	Croatia	Finland	2-0
6 October 2017	Kosovo	Ukraine	0-2
6 October 2017	Turkey	Iceland	2-1
9 October 2017	Finland	Turkey	0-1
9 October 2017	Iceland	Kosovo	1-0
9 October 2017	Ukraine	Croatia	0-0

(c) Final standing with the runner-up results

Pos = Position; W = Won; D = Drawn; L = Loss; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played 10 matches.
Last row contains the second-placed team's results, adjusted for the ranking of the runners-up (matches played against the 6th team are discarded) according to FIFA (2017).

Pos	Team	W	D	L	GF	GA	GD	**Pts**
1	Croatia	6	2	2	14	3	11	**20**
2	Turkey	6	2	2	14	9	5	**20**
3	Ukraine	5	3	2	13	7	6	**19**
4	Iceland	6	1	3	13	9	4	**19**
5	Finland	2	1	7	6	12	-6	**7**
6	Kosovo	0	1	9	3	23	-20	**1**
2	Turkey	4	2	2	8	8	0	**14**