A Flexible Fourier Form Nonlinear Unit Root Test Based on ESTAR Model

Güriş, Burak

Istanbul University

December 2017

A Flexible Fourier Form Nonlinear Unit Root Test Based on ESTAR Model

Assoc. Prof. Dr. Burak Güriş
Istanbul University, Faculty of Economics, Istanbul Turkey
E-mail: bguris@istanbul.edu.tr

Abstract

This study suggests a new nonlinear unit root test procedure with Fourier function. In this test procedure, structural breaks are modeled by means of a Fourier function and nonlinear adjustment is modeled by means of an Exponential Smooth Threshold Autoregressive (ESTAR) model. The Monte Carlo simulation results indicate that the proposed test has good size and power properties. This test eliminates the problems of over-acceptance of the null of nonstationarity to allow multiple smooth temporary breaks and nonlinearity together into the test procedure.

JEL classification: C12, C22

Keywords: Flexible Fourier Form, Unit Root Test, Nonlinearity

1. Introduction

The use of nonlinear unit root tests has become widespread in recent years. The main reason for this is that traditional unit root tests have low power in nonlinear processes. Structural break is also an important factor that reduces the power of unit root tests. If nonlinearity and structural break are not taken into account in the unit root tests, they produce results that have the tendency to accept the unit root null hypothesis.

The inclusion of structural breaks into the unit root test process was first carried out by Perron (1989). The studies carried out later have focused on modeling structural breaks, determining the history of structural break endogenously and determining the number of structural breaks. The approach used in the modeling of structural breaks in recent years is the use of the Fourier series. Becker *et al.* (2004, 2006) propose to use a Fourier series expansion. The main advantage of this method is that there is no need to know the number of structural breaks and form a priori.

Following these developments, Enders and Lee (2012) suggest a unit root test with a Fourier function in the deterministic term in a Dickey Fuller type regression framework.

The nonlinear unit root tests have been developed based on the Logistic Smooth Threshold Autoregressive (LSTAR) and the Exponential Smooth Threshold Autoregressive (ESTAR) model differentiation. Kapetanios *et al.* (2003) developed the unit root test based on the ESTAR model. Leybourne *et al.* (1998) combined the LSTAR model structure and Dickey Fuller type regression and developed a new test procedure.

Christopoulos and Leon-Ledesma (2010) made a significant contribution to literature by proposing new test procedures that combine Fourier transformation and nonlinearity. This procedure is based on using the Fourier form in the first stage and the KSS test in the second stage. This allows for modeling both nonlinearity and structural break.

In this paper, we extend the work of Christopoulos and Leon-Ledesma (2010) and suggest a new nonlinear unit root test procedure with Fourier function. In this test procedure, structural breaks are modeled by means of a Fourier function and nonlinear adjustment is modeled by means of an ESTAR model as proposed by Hu and Chen (2016). The two basic problems encountered by unit root tests have been eliminated thanks to this test procedure.

This paper is organized as follows. The next section describes the new test procedure, Section 3 presents the Monte Carlo results and measure the critical values, empirical size and the power of the test. Empirical applications are presented in Section 4 and Section 5 is the conclusion.

2. The Fourier Form Nonlinear Unit Root Test

Kapetanios *et al*. (2003) provide an alternative framework for a test of the null of a unit root process against an alternative of nonlinear exponential smooth transition autoregressive (ESTAR) process, which is globally stationary.

The ESTAR model,

$$\Delta y_t = \alpha y_{t-1} + \gamma y_{t-1}(1 - exp\{-\theta(y_{t-1} - c)^2\}) \qquad (1)$$

where c is location parameter and the authors make the restriction c=0. Contrary to the Kapetanios *et al.* (2003) study, Kruse (2011) has shown that a lot of empirical studies report significant estimates of c. In the Kruse (2011) study, the test equation proposed using a first order Taylor approximation to $G(y_{t-1}; \theta, c) = 1 - exp\{-\theta(y_{t-1} - c)^2\}$ around $\theta = 0$ is as follows:

$$\Delta y_t = \delta_1 y_{t-1}^3 + \delta_2 y_{t-1}^2 + \delta_3 y_{t-1} + \sum_{j=1}^{p} \varphi_j \Delta y_{t-j} + \varepsilon_t \qquad (2)$$

They impose $\delta_3 = 0$ to improve the power of the test and proceed with

$$\Delta y_t = \delta_1 y_{t-1}^3 + \delta_2 y_{t-1}^2 + \sum_{j=1}^{p} \varphi_j \Delta y_{t-j} + \varepsilon_t \qquad (3)$$

Kruse (2011) proposes a τ test here in order to test the null hypothesis of unit root) ($H_0: \delta_1 = \delta_2 = 0$) against the globally stationary ESTAR process ($H_1: \delta_1 < 0,\ \delta_2 \neq 0$). This test statistics formulated as

$$\tau = t^2_{\delta_2^{\perp}=0} + 1(\hat{\delta}_1 < 0)t^2_{\delta_1=0}$$

The critical values for this test statistics are tabulated in the Kruse (2011) study.

Hu and Chen (2016) are interested in the unit root test against the local explosive or local unit root but globally stationary ESTAR process. Kapetanios *et al.* (2003) demonstrate that if $|1 + \alpha + \gamma| < 1$ the process is globally stationary. The process is local explosive if $\alpha > 0$ or local unit root if $\alpha = 0$. Hu and Chen (2016) show that $\gamma < 0$ provided that the process is globally stationary. Therefore δ_3 in equation 2 is less than zero while δ_1 and δ_2 may take negative or positive values by Taylor aproximation which are dependent on the location parameter c.

According to Hu and Chen (2016), the null hypothesis is $H_0: \delta_1 = \delta_2 = \delta_3 = 0$ against $H_1: \delta_1 \neq 0, \delta_2 \neq 0, \delta_3 < 0$. Because two parameters are two sided under H_1, while the other is one sided this testing problem is nonstandard. To solve this, similar to Kruse(2011), Hu and Chen (2016) apply the Abadir and Distraso(2007) method and they suggest the modified Wald test. This test statistics is

$$\tau = \tau_I^2 + 1_{\tau_3 < 0}\tau_3^2$$

Where τ_3^2 is a squared t statistics for the hypothesis $\delta_3 = 0$ and τ_I^2 is a squared t statistics for the hypothesis $(\delta_1\ \delta_2) = (0\ 0)$ being orthogonal to δ_3.

Hu and Chen (2016) show that τ statistic has the following asymptotic distribution which is free of nuisance parameters

$$\tau \Rightarrow A\big(W(r)\big) + B(W(r))$$

where A(.) and B(.) are the function of standard Brownian motion W(r) (for details, see Hu and Chen (2016)).

Christopoulos and Leon-Ledesma (2010) suggest the new unit root test procedure which combines Fourier transformation and nonlinearity. This procedure is based on using the Fourier function in the first stage and the Kapetanios *et al.* (2003) test in the second stage. This allows for modeling both nonlinearity and structural break.

In this study, the method proposed by Christopoulos and Leon-Ledesma (2010) was expanded. The proposed new test procedure uses the Fourier function to model structural breaks and allows for the ESTAR type nonlinearity using the nonlinear unit root test proposed by Hu and Chen (2016). Thus, the proposed test not only allows for nonlinearity but also take into account the presence of multiple smooth temporary breaks.

The test procedure proposed in the study can be shown as follows similar to the study by Christopoulos and Leon-Ledesma (2010).

Step 1: The nonlinear deterministic component is specified in the first stage.

$$y_t = \alpha_0 + \alpha_1 sin\left(\frac{2\pi k^* t}{T}\right) + \alpha_2 cos\left(\frac{2\pi k^* t}{T}\right) + v_t$$

t is a trend term, T is the sample size, and $\pi = 3.1416$. k* is the optimal frequency and it will be obtained by assigning values to k changing between 1 to 5, then predicting the equation by using OLS and minimizing the total of the squares of error terms. The error terms of the equation predicted will be obtained.

$$v_t = y_t - \alpha_0 - \alpha_1 sin\left(\frac{2\pi k^* t}{T}\right) - \alpha_2 cos\left(\frac{2\pi k^* t}{T}\right)$$

Step 2: The test statistics is calculated predicting the equation below using the error terms obtained in the first stage:

$$\Delta v_t = \delta_1 v_{t-1}^3 + \delta_2 v_{t-1}^2 + \delta_3 v_{t-1} + \sum_{j=1}^{p} \varphi_j \Delta v_{t-j} + \varepsilon_t$$

By using this equation, the test statistics proposed by Hu and Chen(2016) $(\tau = \tau_I^2 + 1_{\tau_3<0}\tau_3^2)$ is calculated.

Step 3: If the null hypothesis of unit root is rejected, then the null hypothesis of linearity, $H_0: \alpha_1 = \alpha_2 = 0$, against the alternative of nonlinearity, $H_1: \alpha_1\ and/or\ \alpha_2 \neq 0$, is tested in this step using the F test $F(\tilde{k})$. If the null hypothesis is rejected, we can conclude that the variable is stationary around a breaking deterministic function. The critical values of this test are tabulated in Becker *et al*. (2006).

2. The Monte Carlo Results

The empirical size and power comparison of the critical values for the proposed flexible Fourier form nonlinear unit root test are presented in this section.

Asymptotic critical values of Fourier Hu and Chen test for nonzero mean and time trend are reported at significance levels of 1, 5 and 10%, respectively. As in the other version of test, the critical values for the null hypothesis of a unit root will depend only on the frequency k and the sample size T. The Asymptotic critical values are based on 50,000 replications for T= 50, 100, 250, 500 and k=1, 2, 3, 4, 5.

Table 1: Critical Values of Fourier Hu and Chen Test

		Level			Trend		
	k	1%	5%	10%	1%	5%	10%
T=50	1	24.64277	18.18961	15.40706	30.43297	23.1989	19.80694
	2	19.77758	14.1973	11.69259	27.44551	19.89235	16.73207
	3	18.24944	12.96865	10.67956	24.14909	17.58596	14.78757
	4	17.56503	12.2753	10.22886	22.72292	16.53164	13.86532
	5	16.94993	11.99176	9.89744	21.96334	15.88412	13.25555
T=100	1	22.5091	17.41234	14.95531	27.90102	21.70169	18.82442
	2	19.04813	14.1296	11.9371	24.96421	19.07019	16.28313
	3	17.45357	13.00571	10.89837	22.43881	16.74922	14.2288
	4	16.99545	12.60048	10.53413	21.68574	16.1272	13.77053
	5	16.51006	12.23702	10.39845	20.90523	15.60769	13.20255
T=250	1	21.79449	17.00648	14.73077	26.19611	20.9074	18.39804
	2	18.78079	14.28716	12.0166	24.35606	18.52812	15.97256
	3	17.85366	13.24554	11.1574	21.83227	16.67681	14.25411
	4	16.70747	12.78043	10.75943	20.7879	15.87466	13.65601
	5	17.00057	12.54512	10.62295	20.32961	15.60774	13.38647
T=500	1	21.79665	17.04212	14.66363	25.88905	20.71348	18.2934
	2	18.7254	14.18391	12.03263	23.31763	18.39493	15.94055
	3	17.59947	13.17847	11.14583	21.59063	16.79098	14.47213
	4	16.94453	12.72694	10.88562	21.00765	15.90085	13.68456
	5	16.78499	12.68118	10.75403	20.28085	15.67558	13.46488

The following data generating process (DGP) is used to investigate the size performance of Fourier Hu and Chen test. To evaluate the size of the test statistics,

$$y_t = \alpha_0 + \alpha_1 sin\left(\frac{2\pi k^* t}{T}\right) + \alpha_2 cos\left(\frac{2\pi k^* t}{T}\right) + v_t$$

$$v_t = v_{t-1} + \varepsilon_t$$

where $\varepsilon_t \sim N(0,1)$ and k^* stands for optimal frequency. The empirical size is considered for sample sizes $T = 50, 100, 250, 500$, values of k=1,2,3,4,5. Table 2 presents the simulation results in 2500 replications.

Table 2: Empirical Sizes of the Test

Level	k=1	k=2	k=3	k=4	k=5
50	0.0452	0.0528	0.0492	0.0532	0.0532
100	0.0508	0.0532	0.054	0.0444	0.0512
250	0.0512	0.0532	0.0428	0.046	0.052
500	0.0472	0.0552	0.0512	0.056	0.0488
Trend					
50	0.0452	0.0528	0.046	0.046	0.0476
100	0.044	0.0436	0.0588	0.0476	0.0576
250	0.0504	0.046	0.0534	0.05	0.05
500	0.0576	0.0544	0.0496	0.0506	0.0488

The results in Table 2 show that the size of proposed test is close to 5% in all cases with different values of k.

We next investigate the power properties of the unit root tests against using the following Fourier ESTAR model as a DGP:

$$y_t = \alpha_0 + \alpha_1 sin\left(\frac{2\pi k^* t}{T}\right) + \alpha_2 cos\left(\frac{2\pi k^* t}{T}\right) + v_t$$

$$\Delta v_t = \alpha v_{t-1} + \gamma v_{t-1}(1 - exp\{-\theta(v_{t-1} - c)^2\}) + \varepsilon_t$$

The parameter are set as follows $\alpha = 0, 0.1, 0.2$, $\theta = 0.001, 0.01, 0.1$, $\gamma = -0.5, -1, -1.5$, c is extracted from a uniform distribution with lower(upper) bound of -1.

Table 3: Power Analysis of Fourier Hu and Chen and Hu and Chen Tests

		Fourier Hu and Chen			Hu and Chen		
		k=1	k=2	k=3	k=1	k=2	k=3
γ	θ	$\alpha = 0$					
-0.5	0.001	**0.0692**	**0.0632**	**0.0776**	0.0496	0.0416	0.0372
	0.01	**0.126**	**0.1664**	**0.1776**	0.1132	0.0988	0.0968
	0.1	**0.6736**	**0.8636**	**0.91**	0.5728	0.5644	0.5508
-1	0.001	**0.0748**	**0.094**	**0.0748**	0.0628	0.058	0.0344
	0.01	**0.2176**	**0.314**	**0.3476**	0.216	0.1884	0.1832
	0.1	**0.9864**	**0.9992**	**0.996**	0.9176	0.918	0.9344
-1.5	0.001	**0.0784**	**0.098**	**0.0904**	0.0672	0.054	0.0432
	0.01	**0.3116**	**0.474**	**0.5244**	0.33	0.29	0.2652
	0.1	**1**	**1**	**1**	0.9864	0.988	0.9888
		$\alpha = 0.1$					
-0.5	0.001	0.04	0.0948	0.1344	**0.206**	**0.17**	**0.1512**
	0.01	0.1844	**0.2468**	**0.2936**	**0.1888**	0.178	0.1712
	0.1	**0.432**	**0.636**	**0.6832**	0.43	0.3972	0.3624
-1	0.001	0.1008	0.1744	**0.2248**	**0.2248**	**0.2024**	0.1872
	0.01	0.2072	**0.276**	**0.3388**	**0.2416**	0.2008	0.1828
	0.1	**0.9592**	**0.9952**	**0.9964**	0.862	0.8624	0.8692
-1.5	0.001	0.1404	**0.2192**	**0.2792**	**0.2488**	0.2152	0.214
	0.01	0.2928	**0.364**	**0.4432**	**0.3356**	0.2604	0.2516
	0.1	**0.9996**	**0.9992**	**1**	0.9696	0.9728	0.984
		$\alpha = 0.2$					
-0.5	0.001	0.1484	0.3456	0.4876	**0.6964**	**0.6692**	**0.62**
	0.01	0.4296	**0.61**	**0.6416**	**0.5284**	0.532	0.4907
	0.1	0.324	**0.4296**	**0.5136**	0.3728	0.304	0.2728
-1	0.001	0.2012	0.3632	0.4712	**0.6396**	**0.6004**	**0.5752**
	0.01	0.4896	**0.5628**	**0.6668**	**0.5188**	0.4688	0.4916
	0.1	**0.9004**	**0.9728**	**0.988**	0.8332	0.8224	0.7908
-1.5	0.001	0.302	0.4688	0.5612	**0.6544**	**0.6276**	**0.6092**
	0.01	0.4492	**0.5332**	**0.642**	**0.526**	0.4744	0.4656
	0.1	**0.998**	**1**	**1**	0.968	0.9668	0.9696

The results of power experiments are presented in Table 3. When $\alpha = 0$, Fourier Hu and Chen test is more powerful than the Hu and Chen(2016) test for all combinations of parameter values and frequencies. If $\alpha = 1$ or $\alpha = 2$, the Fourier Hu and Chen test is more powerful than the Hu and Chen(2016) test when k=2 or 3. The general outcome obtained from Table 3 is that the Fourier Hu and Chen test is more powerful than the Hu and Chen(2016) test especially k=2 or 3.

4. Empirical Application

For the empirical application of the proposed test, the validity of the PPP was tested in the G7 countries (Canada, France, Germany, Italy, Japan, the United Kingdom and the United States of America). A commonly preferred method for investigating the validity of PPP is unit root tests. If the real exchange rate is stable, this indicates that PPP is valid. If the real exchange rate includes a unit root, this means that the purchasing power parity is not valid.

Monthly data are employed and the time span is from January 1991 to December 2016 period. All of the Consumer Price Index and nominal exchange rates relative to the USD data are taken from the International Monetary Fund's International Financial Statistics. All the data was converted into natural logarithmic form before the empirical analysis. The real exchange rate series were obtained from the following equation.

$$y_{i,t} = s_{i,t} + p_{us,t} - p_{i,t}$$

Here $s_{i,t}$ indicates the logarithmic nominal exchange rate, $p_{us,t}$ the logarithmic price index of the USA and $p_{i,t}$ indicates the logarithmic price index of i country.

The validity of the purchasing power parity of G7 countries was tested using the Fourier Hu and Chen test and the test results for level case are presented in Table 4.

Table 4: Fourier Hu and Chen Test Results

	k	F Stat	Lag	Test Stats
Canada	1	829.0124	1	2.751107
France	1	284.3915	4	7.649759
Germany	1	467.2311	3	3.654647
Italy	1	120.8698	4	8.835064
Japan	1	160.8388	2	9.887423
UK	1	195.4541	3	7.266453

According to the results shown in Table 4, the PPP does not hold for all countries as the test statistics values in Table 1 for level case are lower than the critical values.

5. Conclusion

This study proposes a new unit root test allow nonlinearity and presence of multiple smooth temporary breaks. Due to this, compared with previous studies, the proposed test is more applicable in reality. The finite sample properties of the suggested test via Monte Carlo simulations were examined. It was found that the proposed test has greater power than the Hu and Chen(2016) test. This test eliminates the problems of over-acceptance of the null of nonstationarity to allow for multiple smooth temporary breaks and nonlinearity together into the test procedure. As an application, the PPP hypothesis was tested in G7 countries. According to the results obtained, the PPP does not hold for G7 countries.

References

Abadir, K. M., & Distaso, W. (2007). Testing joint hypotheses when one of the alternatives is one-sided. *Journal of Econometrics*, *140*(2), 695-718.DOI: 10.1016/j.jeconom.2006.07.022

Becker, R., Enders, W.,& Hurn., S., (2004). A general test for time dependence in parameters. *Journal of Applied Econometrics* 19, 899-906. DOI:10.1002/jae.751

Becker, R., Enders, W.,& Lee, J., (2006). A stationarity test in the presence of an unknown number of breaks. *Journal of time Series Analysis* 27, 381-409. DOI:10.1111/j.1467-9892.2006.00478.x

Christopoulos, D. K., & León-Ledesma, M. A. (2010). Smooth breaks and non-linear mean reversion: Post-Bretton Woods real exchange rates. *Journal of International Money and Finance*, *29*(6), 1076-1093. DOI: 10.1016/j.jimonfin.2010.02.003

Hu, J., & Chen, Z. (2016). A unit root test against globally stationary ESTAR models when local condition is non-stationary. *Economics Letters*, *146*, 89-94. DOI: 10.1016/j.econlet.2016.07.002

Enders, W., & Lee, J., (2012). The flexible form and Dickey-Fuller type unit root tests, *Economics Letters*, 117, 196-199. DOI: 10.1016/j.econlet.2012.04.081

Kapetanios, G., Shin, Y., & Snell, A. (2003). Testing for a unit root in the nonlinear STAR framework. *Journal of econometrics*, *112*(2), 359-379. DOI: 10.1016/S0304-4076(02)00202-6

Kruse, R. (2011). A new unit root test against ESTAR based on a class of modified statistics. *Statistical Papers*, 52(1), 71-85. DOI 10.1007/s00362-009-0204-1

Leybourne, S., Newbold, P., & Vougas, D. (1998). Unit roots and smooth transitions. *Journal of time series analysis*, *19*(1), 83-97. DOI: 10.1111/1467-9892.00078

Perron, P. (1989). The great crash, the oil price shock, and the unit root hypothesis. *Econometrica: Journal of the Econometric Society*, 1361-1401. DOI: 10.2307/1913712