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Abstract

This study explores the effects of patent protection and R&D subsidies on innovation and income inequality using a Schumpeterian growth model with heterogeneous households. We find that although strengthening patent protection and raising R&D subsidies have the same macroeconomic effects of stimulating innovation and economic growth, they have drastically different microeconomic implications on income inequality. Specifically, strengthening patent protection increases income inequality whereas raising R&D subsidies decreases (increases) it if the quality step size is sufficiently small (large). An empirically realistic quality step size is smaller than the threshold, implying a negative effect of R&D subsidies on income inequality. We also calibrate the model to provide a quantitative analysis and find that strengthening patent protection causes a moderate increase in income inequality and a negligible increase in consumption inequality whereas raising R&D subsidies causes a relatively large decrease in both income inequality and consumption inequality.

JEL classification: D30, O30, O40
Keywords: R&D subsidies, patents, income inequality, economic growth

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1 Introduction

The seminal study by Solow (1956) shows that economic growth in the long run must come from technological progress. The development of technologies in turn is driven by innovation and R&D. Therefore, patent protection and R&D subsidies are two important policy instruments that determine technological progress and economic growth. Since the development of the innovation-driven growth model by Romer (1990), many studies have used variants of the innovation-driven growth model to explore the macroeconomic effects of patent protection and R&D subsidies on innovation and economic growth. However, the microeconomic implications of these two policy instruments on the income distribution have received much less attention. Therefore, in this study, we explore the effects of patent protection and R&D subsidies on innovation as well as income inequality. We find that whether the relationship between innovation and inequality, which are both endogenous variables, is positive or negative depends on the underlying exogenous driving force (i.e., patent policy versus R&D subsidy).

The growth-theoretic framework that we consider is the Schumpeterian growth model. We extend it by allowing for heterogeneous households who have different levels of asset holdings. As Piketty (2014) argues, an unequal distribution of wealth is an important cause of income inequality. Within this growth-theoretic framework, we find that although strengthening patent protection and raising R&D subsidies have the same macroeconomic effects of stimulating innovation and economic growth, they have drastically different microeconomic implications on income inequality. Therefore, it is important to consider beyond aggregate effects and investigate distributional implications when evaluating the overall effects of a policy instrument.

When a strengthening of patent protection or a raise in R&D subsidies leads to a higher rate of economic growth, the real interest rate also rises leading to an increase in asset income, which is the cause of inequality in the model. As a result, strengthening patent protection and raising R&D subsidies both have a positive effect on income inequality via this *interest-rate* channel. Intuitively, the higher interest rate increases the income of asset-wealthy households relative to asset-poor households. Furthermore, the two policy instruments carry an *asset-value* effect that affects income inequality. By increasing monopolistic profits, strengthening patent protection increases asset value and causes an additional positive effect on income inequality. In contrast, raising R&D subsidies suppresses income inequality by reducing asset value through creative destruction[1] and consequently causing a decrease in asset income. As a result of the opposing interest-rate and asset-value effects, raising R&D subsidies has an overall ambiguous effect on income inequality. Specifically, if the quality step size is smaller (larger) than a threshold, then raising R&D subsidies leads to a lower (higher) degree of income inequality. An empirically realistic quality step size is smaller than the threshold implying a negative effect of R&D subsidies on income inequality. In contrast, a strengthening of patent protection causes a positive effect on income inequality. This theoretical result is consistent with the empirical finding in Adams (2008) who uses an index of patent rights constructed by Ginarte and Park (1997) and finds that strengthening patent

[1] This creative-destruction effect on asset value is also present in the case of patent protection but is offset by its monopolistic-profit effect.

protection has a positive and statistically significant effect on income inequality. Therefore, it may seem that pro-growth policies tend to worsen income inequality; however, our analysis shows that this may be true for patent policy but not so for R&D subsidies.

The above results are partly due to an implicit assumption that R&D subsidies affect only new inventions whereas patent breadth affects both new inventions and previously patented inventions, which are assets owned by households. This assumption is realistic because R&D subsidies only compensate firms for carrying out new innovation whereas increasing patent breadth enhances protection for future and current patents.[2] It is also important to emphasize that the above results are not due to a common misinterpretation of the Schumpeterian model that innovation comes from entrants but not incumbents. Cozzi (2007) shows that the correct interpretation of creative destruction in the Schumpeterian model is that incumbents' choice of R&D is simply indeterminate, so that the aggregate economy behaves *as if* innovation is targeted only by entrants. In other words, creative destruction in the Schumpeterian model can be consistent with the empirical observation that incumbents often target innovation at their own industries.

We also explore the effects of patent protection and R&D subsidies on consumption inequality. We find that strengthening patent protection increases consumption inequality whereas raising R&D subsidies continues to have an overall ambiguous effect on consumption inequality. Finally, we calibrate the model to investigate the quantitative effects of patent protection and R&D subsidies on growth and inequality. The policy experiments that we consider are to increase separately the rate of R&D subsidies and the level of patent protection such that the R&D share of GDP increases by one-tenth in each case, which in turn leads to the same proportional increase in the equilibrium growth rate. We find that the increase in patent protection causes a moderate increase in income inequality and a negligible increase in consumption inequality whereas the increase in R&D subsidies causes a relatively large decrease in both income inequality and consumption inequality. These results are robust to a number of robustness checks.

This study relates to the literature on R&D and economic growth. The seminal studies in this literature are Romer (1990), Segerstrom *et al.* (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992). Subsequent studies in this literature apply variants of the R&D-based growth model to explore the effects of patent protection and R&D subsidies on innovation and economic growth;[3] see for example Peretto (1998), Li (2001), Lin (2002, 2015), Furukawa (2007), Futagami and Iwaisako (2007), Horii and Iwaisako (2007), Iwaisako and Futagami (2013), Zeng and Zhang (2007), Zeng *et al.* (2014), Impullitti (2010), Acemoglu and Akcigit (2012), Minniti and Venturini (2014) and Kiedaisch (2015). These studies focus on a representative-household framework and do not consider the distributional implications

[2] It is useful to note that patent breadth refers to how broad claims in patents are (or are expected to be) interpreted by patent judges in court.

[3] Many empirical studies have investigated the effects of these two R&D policy instruments. Recent examples include Minniti and Venturini (2017), who find that R&D tax credits have positive effects on productivity growth, and Brown *et al.* (2017), who find that protection for intellectual property has positive effects on R&D. Other studies such as Jaffe and Lerner (2004), Bessen and Meurer (2008) and Boldrin and Levine (2008), find that patent protection may have blocking effects on future innovation. We do not consider blocking patents in this study; see for example Chu (2009), Chu *et al.* (2012), Cozzi and Galli (2014) and Yang (2017) for an analysis of blocking patents in the Schumpeterian model.

of patent protection and R&D subsidies. The current study fills this gap in the literature by exploring the effects of patent protection and R&D subsidies on income inequality in addition to growth within a Schumpeterian model with heterogeneous households.

Some studies in the literature consider heterogeneous workers and explore the relationship between innovation and wage inequality. For example, Acemoglu (1998, 2002) develops an R&D-based growth model with two R&D sectors and two types of workers to explore how the direction of innovation affects the skill premium. Li (1998) and Grossman and Helpman (2016) also consider heterogeneous workers and wage inequality in an R&D-based growth model with a uniform distribution of workers' productivity in the former and a general distribution of workers' productivity in the latter. Some studies such as Spinesi (2011), Pan *et al.* (2012) and Cozzi and Galli (2014) analyze the effects of patent protection on the skill premium. The present study differs from these studies by considering wealth heterogeneity instead of worker heterogeneity and by exploring income inequality instead of wage inequality.

A small number of studies in the literature consider income and/or wealth heterogeneity in the R&D-based growth model. Representative studies include Chou and Talmain (1996), Zweimuller (2000), Foellmi and Zweimuller (2006), Aghion *et al.* (2015) and Jones and Kim (2017). These studies focus on the relationship between income inequality and innovation. Our study complements these interesting studies by showing that if patent policy (R&D subsidy) changes, then the relationship between innovation and inequality would be positive (negative). Furthermore, we explore the effects of policy instruments not only on income inequality but also on consumption inequality. Chu (2010) and Kiedaisch (2016) also explore the effects of patent policy on inequality; however, they do not consider R&D subsidies. Therefore, this study generalizes the analysis in Chu (2010) and Kiedaisch (2016) by providing a comparative analysis of two popular policy instruments, which appear to have similar aggregate effects but drastically different distributional implications. Furthermore, unlike Chu (2010) and Kiedaisch (2016), we consider a lab-equipment innovation process under which R&D uses final goods (instead of labor) as input. Under the lab-equipment specification, strengthening patent protection causes the positive asset-value effect in addition to the positive interest-rate effect on income inequality.[4]

The rest of this study is organized as follows. Section 2 presents the model. Section 3 explores the effects of patent protection and R&D subsidies. Section 4 provides a quantitative analysis. Section 5 considers a number of extensions of the model. The final section concludes.

[4] If we instead considered the knowledge-driven specification under which R&D uses labor as the factor input, then the positive interest-rate effect of both patent protection and R&D subsidies would still be present. However, the positive asset-value effect of patent protection would be absent because the monopolistic-profit effect would be exactly offset by the creative-destruction effect in this case. As for the negative asset-value effect of R&D subsidies, it is robust to either R&D specification; therefore, our finding on the different effects of patent protection and R&D subsidies on inequality is also robust. See Section 5.1.

2 A Schumpeterian growth model with heterogeneous households

In this section, we extend the Schumpeterian quality-ladder model in Grossman and Helpman (1991), which is a workhorse model in the literature, to allow for heterogeneous households with different asset holdings. Furthermore, we consider two policy instruments, patent breadth and R&D subsidies, in order to perform a comparative policy analysis. We consider these two policy instruments in our analysis because they have the same implications at the macroeconomic level, which makes them easy to compare, by having the same effects on innovation and economic growth but drastically different implications at the microeconomic level by having different effects on inequality. Finally, we also modify the R&D specification by assuming a lab-equipment innovation process as in Rivera-Batiz and Romer (1991).[5]

2.1 Households

There is a unit continuum of households indexed by $h \in [0,1]$ with identical preferences over consumption $c_t(h)$ but different levels of asset holdings. Each household h has the following utility function:[6]

$$u(h) = \int_0^\infty e^{-\rho t} \ln c_t(h) dt. \tag{1}$$

The parameter $\rho > 0$ is the subjective discount rate. Each household h supplies one unit of labor[7] to earn wage income and makes consumption-saving decision to maximize utility subject to the following asset-accumulation equation:

$$\dot{a}_t(h) = r_t a_t(h) + (1 - \tau_t) w_t - c_t(h). \tag{2}$$

$a_t(h)$ is the real value of financial assets (i.e., the share of monopolistic firms) owned by household h. r_t is the real interest rate. w_t is the real wage rate. $\tau_t \in (0,1)$ is the rate of a wage income tax collected by the government.[8] From standard dynamic optimization, the

[5] In Section 5.1, we present an alternative version of our model with knowledge-driven innovation.

[6] Here we consider a log utility function for simplicity. In the case of an isoelastic utility function, the positive interest-rate effect of patent protection and R&D subsidies on income inequality would remain unchanged because the real interest rate would still be increasing in the equilibrium growth rate. As for the asset-value effect on income inequality, it would still be different for the two instruments: patent protection would have a positive asset-value effect whereas R&D subsidies would have a negative asset-value effect on income inequality.

[7] For simplicity, we assume inelastic labor supply in which case all households have the same labor income implying that labor income of an individual is independent of the individual's share of wealth in the economy. Under elastic labor supply, a negative relationship between wealth and labor income would emerge (Chu, 2010), consistently with evidence - see the empirical studies summarized in García-Peñalosa and Turnovsky (2006) who also consider the relationship between growth and inequality but in an AK model. The growth-inequality relationship in our model would continue to hold under elastic labor supply.

[8] Alternatively, one can consider an asset income tax. Here we consider a wage income tax for two reasons. First, it is non-distortionary and does not affect aggregate equilibrium allocations. Second, if we financed R&D subsidies by a tax on asset income - which is the source of inequality in the model - then raising R&D subsidies would cause an additional negative effect on inequality, which would reinforce our finding of a negative effect of R&D subsidies on inequality.

Euler equation is given by

$$\frac{\dot{c}_t(h)}{c_t(h)} = r_t - \rho, \tag{3}$$

which shows that the growth rate of consumption is the same across households such that $\dot{c}_t(h)/c_t(h) = \dot{c}_t/c_t$ for all $h \in [0,1]$, where $c_t \equiv \int_0^1 c_t(h)dh$ is aggregate consumption.

2.2 Final good

Competitive firms produce final good y_t using the following Cobb-Douglas aggregator over a unit continuum of differentiated intermediate goods:

$$y_t = \exp\left(\int_0^1 \ln x_t(i)di\right), \tag{4}$$

where $x_t(i)$ denotes intermediate good $i \in [0,1]$. The conditional demand function for $x_t(i)$ is given by

$$x_t(i) = \frac{y_t}{p_t(i)}, \tag{5}$$

where $p_t(i)$ is the price of $x_t(i)$.

2.3 Intermediate goods

There is a unit continuum of industries, which are also indexed by $i \in [0,1]$, producing differentiated intermediate goods. In each industry i, there is a monopolistic industry leader, who holds a patent on the latest technology and dominates the market until the arrival of the next innovation.[9] The production function of the leader in industry i is

$$x_t(i) = z^{n_t(i)} l_t(i), \tag{6}$$

where the parameter $z > 1$ is the step size of each quality improvement, $n_t(i)$ is the number of quality improvements that have occurred in industry i as of time t, and $l_t(i)$ is the amount of labor employed in industry i. Given the productivity level $z^{n_t(i)}$, the marginal cost function of the leader in industry i is $w_t/z^{n_t(i)}$. From Bertrand competition, the profit-maximizing price is a constant markup over the marginal cost such that

$$p_t(i) = \mu \frac{w_t}{z^{n_t(i)}}, \tag{7}$$

where the markup $\mu \leq z$ is a policy parameter determined by the level of patent protection in the economy.[10] Given (7), the amount of monopolistic profit in industry i is

$$\pi_t(i) = \frac{\mu - 1}{\mu} p_t(i) x_t(i) = \frac{\mu - 1}{\mu} y_t, \tag{8}$$

[9] See Cozzi (2007) for a discussion of this Arrow replacement effect.

[10] The presence of monopolistic profit attracts potential imitation; therefore, stronger patent protection allows monopolistic producers to charge a higher markup without losing their markets to potential imitators. This formulation of patent breadth captures Gilbert and Shapiro's (1990) seminal insight on "breadth as the ability of the patentee to raise price".

and the wage payment in industry i is

$$w_t l_t(i) = \frac{1}{\mu} p_t(i) x_t(i) = \frac{1}{\mu} y_t, \tag{9}$$

where the second equality of (8) and (9) follows from (5).

2.4 R&D

Equation (8) shows that $\pi_t(i) = \pi_t$ for all intermediate goods $i \in [0,1]$. Therefore, the value of inventions is also the same across industries such that $v_t(i) = v_t$ for all $i \in [0,1]$.[11] The no-arbitrage condition that determines v_t is

$$r_t = \frac{\pi_t + \dot{v}_t - \lambda_t v_t}{v_t}, \tag{10}$$

which states that the rate of return on v_t must equal the interest rate. The return on v_t is the sum of monopolistic profit π_t, capital gain $\dot{v}_t$ and expected capital loss $\lambda_t v_t$, where λ_t is the rate of creative destruction.

Competitive entrepreneurs devote R_t units of final goods to R&D. The free-entry condition of R&D is

$$\lambda_t v_t = (1-s) R_t, \tag{11}$$

where the policy parameter $s \in (0,1)$ is the rate of R&D subsidies and $\lambda_t v_t$ is the expected return on R&D. We assume that λ_t is an increasing function in R&D spending R_t given by

$$\lambda_t = \frac{\varphi R_t}{Z_t}, \tag{12}$$

where Z_t is the level of technology in the economy and captures in a simple way increasing R&D difficulty due to an increasing-complexity effect of technology.[12] Combining (11) and (12) yields

$$v_t = \frac{1-s}{\varphi} Z_t, \tag{13}$$

which shows that invention value v_t is proportional to technology level Z_t and that v_t is decreasing in R&D subsidy s for a given Z_t. Intuitively, for a given Z_t, the free-entry condition implies that an increase in subsidy s makes R&D cheaper and leads to a decrease in the price of inventions. In equilibrium, this decrease in the value of inventions is caused by a higher rate of creative destruction.

[11] We follow the standard approach in the literature to focus on the symmetric equilibrium. See Cozzi *et al.* (2007) for a theoretical justification for the symmetric equilibrium as the unique rational-expectation equilibrium in the Schumpeterian model.

[12] Venturini (2012) provides empirical evidence for the presence of increasing R&D difficulty.

2.5 Government

The government decides on the level μ of patent protection in the economy. Also, it collects tax revenue to finance R&D subsidies and non-productive government expenditure G_t subject to the following balanced-budget condition:

$$\tau_t w_t = sR_t + G_t, \tag{14}$$

where $G_t = \gamma y_t$ is assumed to be proportional to output. The parameter $\gamma \equiv G_t/y_t \geq 0$ is the ratio of government expenditure to output.

3 Solving the model

In this section, we proceed to solve the model as follows. Section 3.1 defines the equilibrium. Section 3.2 shows that the aggregate economy always jumps to a unique balanced growth path and explores the effects of patent protection and R&D subsidies on the aggregate growth rate of the economy. Section 3.3 shows that the wealth distribution is stationary, and hence, it is exogenously determined by its initial condition. Sections 3.4 and 3.5 show that income and consumption distributions are also stationary, but they are endogenously determined by patent protection and R&D subsidies.

3.1 Decentralized equilibrium

The equilibrium is a time path of allocations $\{c_t(h), a_t(h), y_t, x_t(i), l_t(i), R_t\}$ and a time path of prices $\{w_t, r_t, p_t(i), v_t\}$. Also, at each instance of time, the following conditions hold:

- households $h \in [0, 1]$ maximize utility taking $\{w_t, r_t\}$ as given;
- competitive firms produce final good y_t to maximize profit taking prices as given;
- each monopolistic firm i produces intermediate good $x_t(i)$ and chooses $\{l_t(i), p_t(i)\}$ to maximize profit taking w_t as given;
- competitive R&D entrepreneurs choose R_t to maximize expected profit taking $\{w_t, v_t\}$ as given;
- the market-clearing condition for labor holds such that $\int_0^1 l_t(i)di = 1$;
- the market-clearing condition for final goods holds such that $\int_0^1 c_t(h)dh + R_t + G_t = y_t$;
- the total value of household assets equals the value of all monopolistic firms such that $\int_0^1 a_t(h)dh = v_t$.

3.2 Aggregate economy

Aggregate technology Z_t is defined as

$$Z_t \equiv \exp\left(\int_0^1 n_t(i)di \ln z\right) = \exp\left(\int_0^t \lambda_\omega d\omega \ln z\right), \tag{15}$$

where the last equality uses the law of large numbers. From (9), we see that $l_t(i) = l_t$ for all $i \in [0,1]$. Therefore, substituting (6) into (4) yields

$$y_t = Z_t l_t = Z_t, \tag{16}$$

where the second equality uses $l_t = 1$. Differentiating the log of Z_t in (15) with respect to time yields the growth rate of technology given by

$$\frac{\dot{Z}_t}{Z_t} = \lambda_t \ln z, \tag{17}$$

where $\lambda_t = \varphi R_t / Z_t$ from (12). The following proposition shows that the aggregate economy jumps to a unique balanced growth path along which aggregate variables grow at the same rate as technology.

Proposition 1 *The aggregate economy jumps to a unique and saddle-point stable balanced growth path along which variables* $\{c_t, y_t, w_t, \pi_t, v_t\}$ *grow at the same rate as technology* Z_t*.*

Proof. See the Appendix. ■

Given Proposition 1, we impose balanced growth on (10) to derive

$$v_t = \frac{\pi_t}{r - g + \lambda} = \frac{\pi_t}{\rho + \lambda}, \tag{18}$$

where g denotes the steady-state growth rate of technology. Substituting (18) into (13) yields the steady-state arrival rate of innovation given by

$$\lambda = \frac{\varphi}{1-s}\frac{\pi_t}{Z_t} - \rho = \frac{\varphi}{1-s}\frac{\mu - 1}{\mu} - \rho, \tag{19}$$

where the second equality uses (8) and (16). Therefore, the steady-state growth rate of technology is

$$g = \lambda \ln z = \left(\frac{\varphi}{1-s}\frac{\mu-1}{\mu} - \rho\right)\ln z. \tag{20}$$

Differentiating (20) with respect to μ and s respectively yields

$$\frac{\partial g}{\partial \mu} = \frac{\varphi \ln z}{1-s}\frac{1}{\mu^2} > 0,$$

$$\frac{\partial g}{\partial s} = \frac{\varphi \ln z}{(1-s)^2} \frac{\mu - 1}{\mu} > 0,$$

which show that a strengthening of patent protection μ and a raise in R&D subsidy s both lead to an increase in the technology growth rate g. These aggregate effects of patent breadth and R&D subsidies are quite common in theoretical studies, such as Peretto (1998), Li (2001) and Chu (2011), and also consistent with empirical studies, such as Brown *et al.* (2017) and Minniti and Venturini (2017). The following proposition summarizes these results.

Proposition 2 *The steady-state equilibrium growth rate of technology is increasing in the level μ of patent protection and the rate s of R&D subsidies.*

Proof. Equation (20) shows that g is increasing in μ and s. ■

3.3 Wealth distribution

At time 0, the share of assets owned by household h is exogenously given by $\theta_{a,0}(h) \equiv a_0(h)/a_0$, which has a general distribution function f_a with a mean of one and a standard deviation of $\sigma_a > 0$. From (2), the aggregate value of financial assets evolves according to

$$\dot{a}_t = r_t a_t + (1 - \tau_t) w_t - c_t, \tag{21}$$

where $a_t \equiv \int_0^1 a_t(h) dh$ is the total value of financial assets owned by all households. Combining (2) and (21) yields the law of motion for $\theta_{a,t}(h) \equiv a_t(h)/a_t$ given by

$$\frac{\dot{\theta}_{a,t}(h)}{\theta_{a,t}(h)} = \frac{\dot{a}_t(h)}{a_t(h)} - \frac{\dot{a}_t}{a_t} = \frac{c_t - (1-\tau_t) w_t}{a_t} - \frac{c_t(h) - (1-\tau_t) w_t}{a_t(h)}, \tag{22}$$

which can be re-expressed as

$$\dot{\theta}_{a,t}(h) = \frac{c_t - (1-\tau_t) w_t}{a_t} \theta_{a,t}(h) - \frac{\theta_{c,t}(h) c_t - (1-\tau_t) w_t}{a_t}, \tag{23}$$

where consumption share $\theta_{c,t}(h) \equiv c_t(h)/c_t$ is a stationary variable. From (3), $\dot{c}_t(h)/c_t(h) = \dot{c}_t/c_t$, which in turn implies that $\dot{\theta}_{c,t}(h)/\theta_{c,t}(h) = 0$ and that $\theta_{c,t}(h) = \theta_{c,0}(h)$ for all $t > 0$. Then, recall that the aggregate economy is always on the balanced growth path along which c_t/a_t and w_t/a_t are stationary. We will also show that the steady-state equilibrium tax rate τ is stationary. Therefore, (23) is a one-dimensional differential equation, which describes the potential evolution of $\theta_{a,t}(h)$ given an initial $\theta_{a,0}(h)$. In the appendix, we show that the coefficient on $\theta_{a,t}(h)$ in (23) is positive. Together with the fact that $\theta_{a,t}(h)$ is a state variable, the only solution of (23) consistent with long-run stability is $\dot{\theta}_{a,t}(h) = 0$ for all t, which is achieved by consumption share $\theta_{c,0}(h)$ jumping to its steady-state value shown in the appendix.

Proposition 3 *For every household h, its asset share is constant over time and exogenously determined at time 0 such that $\theta_{a,t}(h) = \theta_{a,0}(h)$ for all t.*

Proof. See the Appendix. ■

Proposition 3 shows that as an equilibrium outcome, the initial wealth distribution remains unchanged over time. Therefore, the degree of wealth inequality is determined by the initial dispersion of asset holdings in the economy. However, as we will show in the next two sections, both the income distribution and the consumption distribution are endogenously determined, which in turn implies that the degrees of income and consumption inequality can be affected by policy instruments, such as patent protection and R&D subsidies.

3.4 Income distribution

Before-tax income earned by household h is

$$I_t(h) \equiv r_t a_t(h) + w_t. \tag{24}$$

Total before-tax income earned by all households is

$$I_t = r_t a_t + w_t. \tag{25}$$

Combining these two equations yields the share of income earned by household h given by

$$\theta_{I,t}(h) \equiv \frac{I_t(h)}{I_t} = \frac{r_t a_t \theta_{a,0}(h) + w_t}{r_t a_t + w_t}. \tag{26}$$

Let's begin by considering a simple distribution of $\theta_{a,0}(h)$. Suppose for now that there are just two types of household $h \in \{E, W\}$. Households indexed by W are workers, who receive wage income but do not own any financial asset. In this case, the share of income owned by any household W is $\theta_{I,t}(W) = w_t/(r_t a_t + w_t)$. Households indexed by E are entrepreneurs, who receive wage income and equally own all the financial assets. In this case, the share of income owned by any household E is $\theta_{I,t}(E) = (r_t a_t/e + w_t)/(r_t a_t + w_t)$, where $e \in (0, 1)$ denotes the mass of entrepreneurs among the unit continuum of all households. Income inequality measured by the relative income between an entrepreneur and a worker is $\theta_{I,t}(E)/\theta_{I,t}(W) = 1 + \frac{1}{e} r_t a_t/w_t$,[13] which indicates two effects of innovation on income inequality. First, $\theta_{I,t}(E)/\theta_{I,t}(W)$ is increasing in r_t capturing the effect of innovation via the interest rate on income inequality. Second, $\theta_{I,t}(E)/\theta_{I,t}(W)$ is increasing in a_t/w_t capturing the effect of innovation via the value of assets (relative to wage) on income inequality. Putting these two effects together, we have $r_t a_t/w_t$, which captures the effects of innovation on income inequality via asset income relative to wage income.[14]

[13] A small value of e captures the case in which the returns to innovation are appropriated by a small number of successful entrepreneurs, and implies a high degree of income inequality.

[14] Given that financial assets are usually owned by top-income earners, $r_t a_t/w_t$ also relates to the important concept of top-income inequality explored in Aghion *et al.* (2015) and Jones and Kim (2017).

Let's turn to a general distribution of $\theta_{a,0}(h)$. Equation (26) implies that the distribution of income share at time t has a mean of one and a standard deviation of

$$\sigma_{I,t} = \sqrt{\int_0^1 \left[\theta_{I,t}(h) - 1\right]^2 dh} = \frac{r_t a_t}{r_t a_t + w_t}\sigma_a = \frac{r_t a_t / w_t}{1 + r_t a_t / w_t}\sigma_a, \tag{27}$$

which is increasing in $r_t a_t / w_t$ capturing the above-mentioned interest-rate and asset-value effects of innovation on income inequality. From (3), we have $r_t = \rho + g$. From (13), we have $v_t = (1 - s)Z_t/\varphi$, and we also know that $a_t = v_t$. From (9) and (16), we have $w_t = Z_t/\mu$. Substituting these conditions into (27) yields

$$\sigma_{I,t} = \sigma_I = \frac{(\rho + g)(1 - s)\mu}{(\rho + g)(1 - s)\mu + \varphi}\sigma_a \tag{28}$$

for all t. Equation (28) implies that the distribution function f_I of income share has a mean of one and a standard deviation of σ_I. Here we measure income inequality by the standard deviation σ_I of income share, which is equivalent to the coefficient of variation of before-tax income.[15] Equation (28) shows that income inequality σ_I is lower than wealth inequality σ_a, and this finding is consistent with the evidence documented in Budria-Rodriguez *et al.* (2002). Substituting the steady-state equilibrium growth rate $g(\mu, s)$ from (20) into (28) yields

$$\sigma_I = \frac{\rho(1 - \ln z)(1 - s) + \frac{\mu - 1}{\mu}\varphi \ln z}{\rho(1 - \ln z)(1 - s) + \frac{\mu - 1}{\mu}\varphi \ln z + \varphi/\mu}\sigma_a = \frac{1}{1 + \frac{\varphi}{\rho(1 - \ln z)(1 - s)\mu + (\mu - 1)\varphi \ln z}}\sigma_a, \tag{29}$$

which is increasing in $\Omega \equiv \frac{\rho(1-\ln z)(1-s)\mu + (\mu-1)\varphi \ln z}{\varphi}$.[16] Differentiating Ω with respect to μ and s yields[17]

$$\frac{\partial \Omega}{\partial \mu} = \frac{\rho(1 - \ln z)(1 - s) + \varphi \ln z}{\varphi} > 0,$$

$$\frac{\partial \Omega}{\partial s} = -\frac{\rho(1 - \ln z)\mu}{\varphi},$$

which is negative (positive) if $1 - \ln z > 0$ $(1 - \ln z < 0)$.

[15] If we considered after-tax income, then the coefficient of variation of after-tax income would be $\sigma_I = \frac{r_t a_t}{r_t a_t + (1-\tau) w_t}\sigma_a = \frac{(\rho+g)(1-s)\mu}{(\rho+g)(1-s)\mu + (1-\tau)\varphi}\sigma_a$. For a given tax rate τ (which can be achieved by making γ endogenous to balance the government's budget constraint), the effects of s and μ on g and σ_I would be the same as the case of before-tax income. In the next section, we will explore how an endogenous tax rate τ responds to s and μ.

[16] If we captured the effects of education quality by φ, then income inequality would be decreasing in education quality because a larger φ increases wage income relative to asset income despite its positive effect on the interest rate.

[17] It is useful to note that $\sigma_I > 0$ requires $\rho(1 - \ln z)(1 - s) + \frac{\mu-1}{\mu}\varphi \ln z > 0$, which in turn implies that $\rho(1 - \ln z)(1 - s) + \varphi \ln z > 0$.

Proposition 4 *The degree of income inequality is increasing in the level μ of patent protection but decreasing (increasing) in the rate s of R&D subsidies if* $\ln z < 1$ $(\ln z > 1)$.

Proof. Equation (29) shows that σ_I is increasing in μ but decreasing (increasing) in s if $1 - \ln z > 0$ $(1 - \ln z < 0)$. ■

Recall that the steady-state equilibrium growth rate $g(\mu, s)$ is increasing in both μ and s. Equation (28) shows that an increase in g leads to an increase in income inequality σ_I by increasing the real interest rate and asset income, which is the cause of income inequality in the model. This is the *symmetric* interest-rate effect of patent protection μ and R&D subsidy s on income inequality σ_I. However, these two policy instruments have an additional asset-value effect on income inequality captured by the term $(1-s)\mu$ in (28), and this asset-value effect is *asymmetric* between μ and s. To understand this asymmetric asset-value effect, one can consider the ratio $a_t/w_t = v_t/w_t = (1-s)\mu/\varphi$ derived from (9), (13) and (16). Interestingly, a_t/w_t is decreasing in R&D subsidy s but increasing in patent protection μ. Intuitively, an increase in patent protection reduces the share of wage income as (9) shows and raises the share of profit income as (8) shows, thereby increasing asset income, which is heterogeneous across households and the source of income inequality in the model, relative to wage income. In contrast, an increase in R&D subsidies reduces asset income by decreasing the value of inventions as (13) shows. Therefore, while strengthening patent protection causes only positive interest-rate and asset-value effects on income inequality, raising R&D subsidies carries both a positive interest-rate effect and a negative asset-value effect on income inequality. The positive interest-rate effect is stronger when the quality step size is larger because $\partial g/\partial s$ is increasing in the quality step size z as (20) shows. Equation (29) shows that if $\ln z$ is larger (smaller) than one, then a raise in R&D subsidies would have a positive (negative) effect on income inequality. The empirical value of z is often considered to be less than 1.20;[18] see for example Laitner and Stolyarov (2013). Therefore, under an empirically realistic quality step size, raising R&D subsidies has an overall negative effect on income inequality.

The above results are driven by an implicit assumption that R&D subsidies affect only new inventions whereas patent breadth affects both new inventions and previously patented inventions, which are assets owned by households. The value of these assets is $a_t = v_t = \pi_t/(\rho + \lambda) = \frac{\mu-1}{\mu} y_t/(\rho + \lambda)$. Therefore, the value of assets relative to wage is $a_t/w_t = (\mu-1)/(\rho+\lambda)$, which is decreasing in the rate of creative destruction λ.[19] Suppose we assume that patent breadth affects only new inventions but not previously patented inventions. Then, before the arrival of new inventions, a larger patent breadth does not yet have a positive effect via the markup μ on the value of existing assets and only causes a negative effect on the value of assets by increasing the rate of creative destruction λ. In this case, the effect of patent breadth μ is similar to the effect of R&D subsidies until the arrival of new inventions at which point the positive effect of the markup μ on a_t/w_t appears.

[18] It is useful to note that $\ln(1.2) \approx 0.182 < 1$.

[19] For a given μ, a_t/w_t is decreasing in λ because creative destruction decreases the present value of future profits. However, the interest rate $r = \rho + g = \rho + \lambda \ln z$ is increasing in λ. Therefore, the overall effect of λ on income inequality, which is determined by ra_t/w_t, is ambiguous.

3.5 Consumption distribution

From (2), consumption by household h is

$$c_t(h) = \left[r_t - \frac{\dot{a}_t(h)}{a_t(h)}\right] a_t(h) + (1-\tau)w_t = \rho a_t(h) + (1-\tau)w_t, \tag{30}$$

where the second equality uses (3) and the balanced-growth condition $\dot{a}_t(h)/a_t(h) = \dot{c}_t(h)/c_t(h)$. Aggregate consumption is

$$c_t = \rho a_t + (1-\tau)w_t. \tag{31}$$

Combining (30) and (31) yields the share of consumption by household h given by

$$\theta_{c,t}(h) \equiv \frac{c_t(h)}{c_t} = \frac{\rho a_t(h) + (1-\tau)w_t}{\rho a_t + (1-\tau)w_t}. \tag{32}$$

Equation (32) implies that the distribution of consumption share at time t has a mean of one and a standard deviation of

$$\sigma_{c,t} = \sqrt{\int_0^1 \left[\theta_{c,t}(h) - 1\right]^2 dh} = \frac{\rho a_t}{\rho a_t + (1-\tau)w_t}\sigma_a, \tag{33}$$

Following the same derivations as in the previous section, we obtain

$$\sigma_{c,t} = \sigma_c = \frac{\rho(1-s)\mu}{\rho(1-s)\mu + (1-\tau)\varphi}\sigma_a, \tag{34}$$

where the steady-state equilibrium tax rate τ can be derived as follows by substituting (9), (12), (16) and (19) into (14):

$$\tau = \mu\frac{sR+G}{y} = \mu\left(\frac{s\lambda}{\varphi} + \gamma\right) = \mu s\left(\frac{1}{1-s}\frac{\mu-1}{\mu} - \frac{\rho}{\varphi}\right) + \mu\gamma, \tag{35}$$

which is increasing in s and μ. Substituting (35) into (34) yields

$$\sigma_c = \frac{\rho(1-s)\mu}{(\rho-\gamma\varphi)\mu + \varphi(1-s\mu)/(1-s)}\sigma_a = \frac{1}{\Theta}\sigma_a, \tag{36}$$

which is decreasing in $\Theta \equiv \frac{1}{\rho}\left[\frac{\rho-\gamma\varphi}{1-s} + \frac{\varphi}{(1-s)^2}\left(\frac{1}{\mu} - s\right)\right]$. Differentiating Θ with respect to μ and s respectively yields

$$\frac{\partial\Theta}{\partial\mu} = -\frac{\varphi}{\rho(1-s)^2}\frac{1}{\mu^2} < 0,$$

$$\frac{\partial\Theta}{\partial s} \equiv \frac{1}{\rho(1-s)^2}\left[\rho - (\gamma+1)\varphi + \frac{2\varphi}{1-s}\left(\frac{1}{\mu} - s\right)\right],$$

which can be positive or negative.

As before, we measure consumption inequality by the standard deviation of consumption share, which is equivalent to the coefficient of variation of consumption. Equation (33) shows that consumption inequality σ_c is independent of the interest rate because a higher interest rate leads to a higher saving rate such that consumption $c_t(h)$ is always a constant fraction ρ of asset $a_t(h)$ due to the log utility function in (1). Therefore, unlike income inequality, the interest-rate effect of patents and R&D subsidies is absent under consumption inequality.[20] Consequently, for a given tax rate τ, we are left with the asymmetric asset-value effect of patents and R&D subsidies captured by the term $(1-s)\mu$ in $v_t/w_t = (1-s)\mu/\varphi$ and in (34). As in the case of income inequality, a strengthening of patent protection has a positive asset-value effect on consumption inequality by raising asset income whereas an increase in R&D subsidies causes a negative asset-value effect. However, the tax rate τ is also a function of μ and s. When either μ or s increases, R&D spending increases, which in turn leads to a higher tax rate τ as (35) shows and worsens consumption inequality σ_c because the tax is levied on wage income w_t rather than asset income, which is the source of inequality in the model. Therefore, strengthening patent protection increases consumption inequality due to the positive asset-income and tax-rate effects whereas raising R&D subsidies has an overall ambiguous effect on consumption inequality due to the negative asset-income effect and the positive tax-rate effect.

Proposition 5 *The degree of consumption inequality is increasing in the level μ of patent protection but can be decreasing or increasing in the rate s of R&D subsidies.*

Proof. Equation (34) shows that σ_c is increasing in μ but can be decreasing or increasing in s. ■

4 Quantitative analysis

In this section, we calibrate the model to data in the US in order to provide a quantitative illustration on the effects of patent protection and R&D subsidies. The model features the following aggregate parameters: $\{\rho, s, \varphi, \mu, z, \gamma\}$. We set the discount rate ρ to a conventional value of 0.05. We follow Impullitti (2010) to set the R&D subsidy rate s in the US to 0.188. We calibrate the value of R&D productivity φ by setting the time between arrivals of innovation to $1/\lambda = 8$ years.[21] As for the patent protection level μ, we calibrate its value by setting the R&D share of GDP to $R/y = 0.028$. As for the quality step size z, we calibrate its value by setting the long-run growth rate g to 2%. Finally, the ratio γ of government spending to GDP is set to 0.2 as in Belo *et al.* (2013). These empirical moments are representative of the US economy. Table 1 summarizes the calibrated parameter values. Under these parameter values, consumption inequality σ_c is lower than income inequality

[20] In the case of an isoelastic utility function, this neutral interest-rate effect of the two policy instruments on consumption inequality would become ambiguous. Specifically, if the elasticity of intertemporal substitution were less (greater) than unity, then the interest-rate effect of the two instruments on consumption inequality would be positive (negative). As for the asset-value effect on consumption inequality, it would still be different for the two instruments.

[21] In the literature, studies have considered different values for the arrival rate of innovations. For example, Caballero and Jaffe (2002) use a structural model to estimate an innovation-arrival rate of 4% whereas Acemoglu and Akcigit (2012) use a growth model to calibrate an innovation-arrival rate of 33%. We consider an intermediate value of 12.5% within this range.

σ_I, and this finding is consistent with the evidence documented in Krueger and Perri (2006) and Blundell *et al.* (2008).

Table 1: Calibrated parameter values					
ρ	s	φ	μ	z	γ
0.050	0.188	4.464	1.033	1.174	0.200

The policy experiments that we consider are to increase separately the R&D subsidy rate s and the patent protection level μ such that the R&D share of GDP R/y increases by one-tenth from 0.0280 to 0.0308 in each case, which in turn leads to the same proportional increase in the equilibrium growth rate. Table 2 reports the resulting implications of each of these policy changes on economic growth g, income inequality σ_I and consumption inequality σ_c. Table 2a shows that in order to increase the R&D share of GDP R/y by one-tenth, patent protection level μ needs to increase from 1.033 to 1.035, in which case the growth rate g increases from 2.0% to 2.2%. This increase in the growth rate leads to a corresponding increase in the interest rate, which in turn drives up income inequality σ_I. In this case, the coefficient of variation of income increases by 3.06%.[22] As for consumption inequality σ_c, the coefficient of variation of consumption increases negligibly by 0.36% because the positive interest-rate effect is absent. Although the positive asset-value effect remains and the positive tax-rate effect appears, they are relatively minor in magnitude in the case of patent protection.

Table 2b shows that in order to increase the R&D share of GDP R/y by one-tenth, the R&D subsidy rate s needs to increase from 0.188 to 0.242. The increase in the growth rate g is the same as above and gives rise to a positive interest-rate effect on income inequality. However, the magnitude of the increase in s is large, which in turn gives rise to a strong negative asset-value effect on income inequality σ_I. In this case, the coefficient of variation of income decreases by 3.95%. As for consumption inequality σ_c, the coefficient of variation of consumption decreases even more by 6.33% because the positive interest-rate effect is now absent. Although the positive tax-rate effect appears, its magnitude is relatively minor because R&D spending is a small share of GDP, and hence, the negative asset-value effect remains the dominant force.

Table 2c considers the case in which both policy instruments change simultaneously. Specifically, the level of patent protection μ increases from 1.033 to 1.035 as in case a whereas the rate of R&D subsidies increases from 0.188 to 0.242 as in case b. In this case, we find that the overall effects are dominated by R&D subsidies such that the coefficient of variation of income decreases by 0.90% whereas the coefficient of variation of consumption decreases by 5.96%. This is due to the relatively large change in the R&D subsidy rate s. The smaller decrease in income inequality than before is due to the larger increase in the growth rate to 2.41%, which causes a larger interest-rate effect on income inequality than before.

[22] Here the change in income inequality σ_I is reported as proportional change (i.e., $\Delta\sigma_I \equiv \sigma_I^{new}/\sigma_I^{old} - 1$).

Table 2a: Effects of patent protection				
$\mu \to 1.035$	R/y	g	$\Delta\sigma_I$	$\Delta\sigma_c$
	0.0308	2.20%	3.06%	0.36%
Table 2b: Effects of R&D subsidies				
$s \to 0.242$	R/y	g	$\Delta\sigma_I$	$\Delta\sigma_c$
	0.0308	2.20%	-3.95%	-6.33%
Table 2c: Effects of both instruments				
$\mu \to 1.035$	R/y	g	$\Delta\sigma_I$	$\Delta\sigma_c$
$s \to 0.242$	0.0338	2.41%	-0.90%	-5.96%

In the rest of this section, we perform the following hypothetical experiments. In the US, the R&D share of GDP increases from 0.024 in 1995 to 0.028 in 2015. We use HP filter to extract the trend of the R&D share of GDP from 1995 to 2015. Then, we consider two hypothetical scenarios. First, suppose the increasing trend of the R&D share of GDP is due to a gradual increase in the level of patent breadth. Then, we plot the resulting effects on income and consumption inequality. Second, suppose the increasing trend of the R&D share of GDP is due to a gradual increase in the rate of R&D subsidies. Then, we plot the resulting effects on income and consumption inequality. These results are summarized in the following figure. Figure 1 simulates the percent changes in inequality and shows that if the increase in R&D were driven by a strengthening of patent protection, then income and consumption inequality would have increased by 4.10% and 0.47% respectively. If the increase in R&D were driven by an increase in R&D subsidies instead, then income and consumption inequality would have decreased by 5.68% and 8.81% respectively.

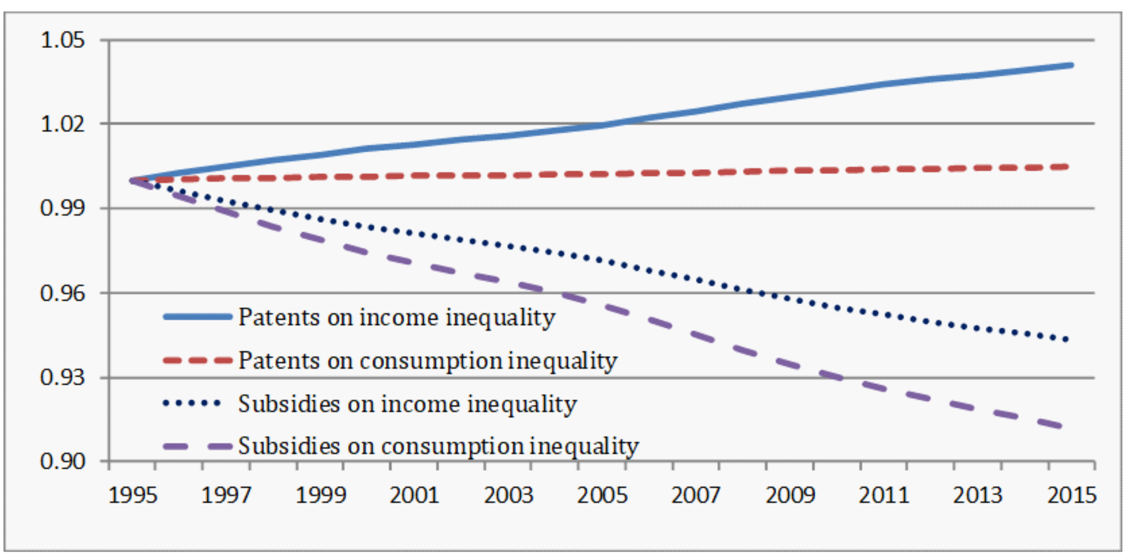

Figure 1: Effects of patents and subsidies on inequality

4.1 Robustness check: R&D share of GDP

Starting from this section, we consider a number of robustness checks on our simulation exercise to illustrate how the numerical results would change under different assumptions. In this section, we examine data on the R&D share of GDP. As Comin (2004) argues, data on R&D expenditures reported by firms may not capture all the resources devoted to innovation-related activities. Here we consider a rough exercise by doubling the R&D share of GDP from 0.028 to 0.056. Table 3 reports the re-calibrated parameter values and shows that because R/y is increasing in μ, the markup ratio μ increases from 1.033 to 1.068, which is a more realistic value.[23] The re-calibrated value of R&D productivity φ decreases because a lower R&D productivity is required (given a higher R&D spending R/y) in order to keep the innovation arrival rate λ at 0.125 as in the previous section. Table 4 shows that the increases in income and consumption inequality under patent protection become larger at 3.27% and 0.76% respectively whereas the decreases in income and consumption inequality under R&D subsidies become slightly smaller at 3.90% and 5.96% respectively. However, the qualitative pattern remains that income and consumption inequality increases under patent protection but decreases under R&D subsidies and that the magnitude of the changes under R&D subsidies is larger than under patent protection. Finally, when both instruments change simultaneously, the overall effects are still dominated by R&D subsidies such that income inequality decreases by 0.64% whereas consumption inequality decreases by 5.19%. Therefore, our results are robust to considering biases in R&D share R/y.

Table 3: Calibrated parameter values

ρ	s	φ	μ	z	γ
0.050	0.188	2.232	1.068	1.174	0.200

Table 4a: Effects of patent protection

$\mu \rightarrow 1.073$	R/y	g	$\Delta\sigma_I$	$\Delta\sigma_c$
	0.0616	2.20%	3.27%	0.76%

Table 4b: Effects of R&D subsidies

$s \rightarrow 0.242$	R/y	g	$\Delta\sigma_I$	$\Delta\sigma_c$
0.242	0.0616	2.20%	-3.90%	-5.96%

Table 4c: Effects of both instruments

$\mu \rightarrow 1.073$	R/y	g	$\Delta\sigma_I$	$\Delta\sigma_c$
$s \rightarrow 0.242$	0.0676	2.41%	-0.64%	-5.19%

4.2 Robustness check: technology growth rate

In the previous sections, we calibrate the value of the quality step size z by targeting the long-run growth rate of output per capita in the US. It is equally reasonable to calibrate the value of z by targeting the long-run growth rate of technology instead. In this case, we re-calibrate the parameter values by setting $g = 1\%$. Table 5 reports the re-calibrated

[23] For example, Jones and Williams (2000) report a range of estimates for the markup from 1.05 to 1.40. Laitner and Stolyarov (2004) use stock market data to estimate a markup of 1.10.

parameter values and shows that the calibrated value of z decreases from 1.174 to 1.083[24] (because a lower $g = \lambda \ln z$ implies a lower z) whereas other parameter values remain largely the same. Under the new parameter values, the long-run growth rate g increases by the same proportion of one-tenth from 1.0% to 1.1%. Table 6 shows that the pattern of changes in inequality is the same as before except that the increase in income inequality under patent protection becomes smaller at 2.11% whereas the decrease in income inequality under R&D subsidies becomes larger at 5.00%. Finally, when both instruments change simultaneously, the overall effects continue to be mostly driven by R&D subsidies such that income inequality decreases by 2.91% whereas consumption inequality decreases by 5.19%. The decrease in income inequality is larger than in the previous section because the increase in the growth rate is now smaller, which in turn implies that the positive interest-rate effect is also smaller. In any case, our results are robust to considering biases in the growth rate g.

Table 5: Calibrated parameter values					
ρ	s	φ	μ	z	γ
0.050	0.188	2.232	1.068	1.083	0.200

Table 6a: Effects of patent protection				
$\mu \to 1.073$	R/y	g	$\Delta\sigma_I$	$\Delta\sigma_c$
	0.0616	1.10%	2.11%	0.76%
Table 6b: Effects of R&D subsidies				
$s \to 0.242$	R/y	g	$\Delta\sigma_I$	$\Delta\sigma_c$
	0.0616	1.10%	-5.00%	-5.96%
Table 6c: Effects of both instruments				
$\mu \to 1.073$	R/y	g	$\Delta\sigma_I$	$\Delta\sigma_c$
$s \to 0.242$	0.0676	1.21%	-2.91%	-5.19%

5 Extensions of the model

In this section, we consider a number of extensions to the benchmark model in order to explore the robustness of our results. In Section 5.1, we change the lab-equipment innovation specification to the knowledge-driven innovation specification under which R&D uses labor as the factor input. In Section 5.2, we change the quality-ladder model to a variety-expanding model.

5.1 R&D labor

We now assume that R&D uses labor instead of final good as the factor input. Under this assumption, (12) is modified as follows:

$$\lambda_t = \varphi l_{r,t}. \tag{37}$$

[24]This value for the quality step size is closer to the one in Garcia-Macia *et al.* (2016). From their estimated parameter values, the average quality step size is 1.071 in the latest period.

In this case, the free-entry condition of R&D becomes

$$\lambda_t v_t = (1-s) w_t l_{r,t} \Leftrightarrow \varphi v_t = (1-s) w_t, \tag{38}$$

where the second equality uses (37). Substituting (8), (9) and (18) into (38) yields

$$\frac{\varphi(\mu - 1)}{\rho + \lambda} = \frac{1-s}{l_x}, \tag{39}$$

where l_x denotes production labor. Substituting (37) and the resource constraint $l_x + l_r = 1$ into (39) yields the equilibrium R&D labor given by

$$l_r(\underset{+}{\mu}, \underset{+}{s}) = \frac{1}{\mu - s}\left[\mu - 1 - (1-s)\frac{\rho}{\varphi}\right], \tag{40}$$

which is increasing patent breadth μ and R&D subsidy s. Therefore, the steady-state equilibrium growth rate $g = \lambda \ln z = (\varphi \ln z) l_r(\mu, s)$ is also increasing in μ and s.

From (27), the standard deviation of before-tax income share is

$$\sigma_{I,t} = \frac{r_t a_t}{r_t a_t + w_t}\sigma_a = \frac{(\rho + g)a_t/w_t}{1 + (\rho + g)a_t/w_t}\sigma_a, \tag{41}$$

where the relative value between assets and wage is given by $a_t/w_t = (1-s)/\varphi$ from (38). Therefore, an increase in the level of patent breadth leads to a higher degree of income inequality via only the interest-rate channel $r = \rho + g$. The asset-value effect $a_t/w_t = (1-s)/\varphi$ of patent breadth is now absent because the monopolistic-profit effect and the creative-destruction effect of μ exactly cancel each other in this case. In contrast, an increase in the rate of R&D subsidies has both positive and negative effects on income inequality. The positive effect arises via the interest-rate channel $r = \rho + g$ whereas the negative effect arises via the asset-value channel $a_t/w_t = (1-s)/\varphi$. In other words, although the positive asset-value effect of patent breadth becomes absent when R&D uses labor as the factor input, the overall effects of patent breadth and R&D subsidies on income inequality remain the same as before. We summarize these results in the following proposition.

Proposition 6 *When R&D uses labor as the factor input, the degree of income inequality is increasing in the level μ of patent protection but decreasing (increasing) in the rate s of R&D subsidies if z is below (above) a threshold.*[25]

Proof. Substituting $a_t/w_t = (1-s)/\varphi$ and $g = (\varphi \ln z) l_r$ into $(\rho + g)a_t/w_t$ yields

$$(\rho + g)\frac{a_t}{w_t} = \left[\rho + (\varphi \ln z) l_r(\underset{+}{\mu}, \underset{+}{s})\right]\frac{1-s}{\varphi}, \tag{42}$$

where $l_r(\mu, s)$ is given by (40). Equation (42) implies that $(\rho + g)a_t/w_t$ is increasing in μ but decreasing (increasing) in s if z is sufficiently small (large). Then, recall from (41) that σ_I is increasing in $(\rho + g)a_t/w_t$. ■

[25] It can be shown that $\ln z < 1$ is now a sufficient condition (but no longer necessary) for an increase in R&D subsidies to decrease income inequality. Derivations are available upon request.

5.2 Variety expansion

We now consider a variety-expanding growth model to examine the robustness of our results. To begin, we replace the Cobb-Douglas production function of final good in (4) by the following CES production function:

$$y_t = \left[\int_0^{N_t} x_t^{\varepsilon}(i) di \right]^{1/\varepsilon}, \tag{43}$$

where the parameter $\varepsilon \in (0, 1)$ determines the elasticity $\epsilon \equiv 1/(1-\varepsilon)$ of substitution between intermediate goods. Then, we replace the production function of intermediate goods in (6) by a simple one-to-one production function $x_t(i) = l_{x,t}(i)$. In this case, the familiar profit-maximizing price of $x_t(i)$ is given by $p_t(i) = w_t/\varepsilon$. To introduce patent breadth, we introduce a patent policy parameter μ such that $p_t(i) = \min\{\mu, 1/\varepsilon\} w_t$. In this case, the amount of profit earned by intermediate good $x_t(i)$ is given by

$$\pi_t(i) = p_t(i)x_t(i) - w_t x_t(i) = (\mu - 1)w_t l_{x,t}(i). \tag{44}$$

It can be shown that the equilibrium features symmetry such that $l_{x,t}(i) = l_{x,t}/N_t$ for all $i \in [0, N_t]$. In this case, the production function in (43) simplifies to

$$y_t = N_t^{1/\varepsilon} x_t(i) = N_t^{(1-\varepsilon)/\varepsilon} l_{x,t}, \tag{45}$$

which implies that the steady-state equilibrium growth rate of output is $g_y = g_N(1 - \varepsilon)/\varepsilon$.

As in Section 5.1, we consider the knowledge-driven innovation specification under which R&D uses labor such that

$$\dot{N}_t = \varphi N_t l_{r,t}. \tag{46}$$

In this case, the free-entry condition of R&D is given by

$$\dot{N}_t v_t = (1 - s)w_t l_{r,t} \Leftrightarrow \varphi N_t v_t = (1 - s)w_t, \tag{47}$$

where the second equality uses (46). The no-arbitrage value of an invention on the balanced growth path is given by

$$v_t = \frac{\pi_t}{r - g_\pi} = \frac{\pi_t}{\rho + g_y - g_\pi}, \tag{48}$$

where the second equality uses (3) and the steady-state equilibrium condition $g_c = g_y$. It can be shown that the steady-state equilibrium growth rate of π_t is given by $g_\pi = g_y - g_N$.[26] Therefore, we have $v_t = \pi_t/(\rho + g_N)$, which shows that for a given π_t, the value v_t of an invention is decreasing in the growth rate g_N of varieties. Intuitively, more varieties in the future will reduce the market share of each invention and lower its present value.

To solve for the steady-state equilibrium growth rate g_N of varieties, we substitute (44) and (48) into (47) to obtain

$$\frac{\varphi(\mu - 1)}{\rho + g_N} = \frac{1 - s}{N_t l_{x,t}(i)} = \frac{1 - s}{l_x}, \tag{49}$$

[26] It can be shown that w_t and y_t grow at the same rate on the balanced growth path.

where $g_N = \varphi l_r$. Substituting the resource constraint $l_x + l_r = 1$ into (49) yields

$$l_r(\underset{+}{\mu}, \underset{+}{s}) = \frac{1}{\mu - s}\left[\mu - 1 - (1-s)\frac{\rho}{\varphi}\right], \tag{50}$$

which is increasing patent breadth μ and R&D subsidy s. Therefore, the steady-state output growth rate $g_y = g_N(1-\varepsilon)/\varepsilon = \varphi l_r(\mu, s)(1-\varepsilon)/\varepsilon$ is also increasing in μ and s.

The standard deviation of before-tax income share is

$$\sigma_{I,t} = \frac{r_t a_t}{r_t a_t + w_t}\sigma_a = \frac{(\rho + g_y)a_t/w_t}{1 + (\rho + g_y)a_t/w_t}\sigma_a, \tag{51}$$

where the relative value between assets and wage is given by $a_t/w_t = N_t v_t/w_t = (1-s)/\varphi$ from (47). Therefore, an increase in the level of patent breadth leads to a higher degree of income inequality via only the interest-rate channel $r = \rho + g_y$. In contrast, an increase in the rate of R&D subsidies has both positive and negative effects on income inequality. The positive effect arises via the interest-rate channel $r = \rho + g_y$ whereas the negative effect arises via the asset-value channel $a_t/w_t = (1-s)/\varphi$. As in the knowledge-driven quality-ladder model in Section 5.1, although the positive asset-value effect of patent breadth is absent, the overall effects of patent breadth and R&D subsidies on income inequality remain the same as before. We summarize these results in the following proposition.

Proposition 7 *In the variety-expanding model, the degree of income inequality is increasing in the level μ of patent protection but decreasing (increasing) in the rate s of R&D subsidies if ε is above (below) a threshold.*

Proof. Substituting $a_t/w_t = (1-s)/\varphi$ and $g_y = \varphi l_r(\mu, s)(1-\varepsilon)/\varepsilon$ into $(\rho + g_y)a_t/w_t$ yields

$$(\rho + g_y)\frac{a_t}{w_t} = \left[\rho + \varphi\frac{1-\varepsilon}{\varepsilon}l_r(\underset{+}{\mu}, \underset{+}{s})\right]\frac{1-s}{\varphi}, \tag{52}$$

where $l_r(\mu, s)$ is given by (50). Equation (52) implies that $(\rho + g_y)a_t/w_t$ is increasing in μ but decreasing (increasing) in s if ε is sufficiently large (small). Then, recall from (51) that σ_I is increasing in $(\rho + g_y)a_t/w_t$. ■

6 Conclusion

In this study, we have explored the effects of innovation policies, such as patent protection and R&D subsidies, on innovation and economic growth as well as income inequality, which is often neglected by studies in the literature. We have shown that policy instruments may have similar aggregate effects on innovation and economic growth but very different distributional effects on inequality. Specifically, we find that strengthening patent protection causes a moderate increase in income inequality and consumption inequality whereas raising R&D subsidies causes a relatively large decrease in both income inequality and consumption inequality. These results suggest that if the objective of a government is to enhance economic growth and reduce inequality, then the government should raise R&D subsidies instead of (or at least in combination with) strengthening patent protection. In our analysis, we have focused on a non-distortionary tax instrument. Considering more realistic distortionary tax instruments may lead to additional insights and different implications on before-tax and after-tax income inequality.

In this study, we have considered a Schumpeterian model with heterogeneity in household asset holdings. Our model focuses on asset income inequality instead of wage income inequality for two reasons. First, wage inequality in the form of skill premium has received much attention in the literature, but only a relatively small number of studies have considered asset income inequality in the Schumpeterian growth model. Second, empirical studies, such as Atkinson (2000, 2003) and Piketty (2014), have shown that inequality in asset income is playing an increasingly important role.[27] Finally, although our model does not feature tangible assets such as physical capital, intangible assets are an important part of the modern economy.[28]

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Appendix

Proof of Proposition 1. From (8), (9), (13) and (16), we have

$$\frac{\dot{v}_t}{v_t} = \frac{\dot{Z}_t}{Z_t} = \frac{\dot{y}_t}{y_t} = \frac{\dot{w}_t}{w_t} = \frac{\dot{\pi}_t}{\pi_t}, \tag{A1}$$

which shows that $\{y_t, w_t, \pi_t, v_t\}$ grow at the same rate as technology Z_t. We also know that the value of inventions equals the value of assets such that $v_t = a_t$; therefore, a_t also grows at the same rate as technology Z_t. Recall from (17) that the growth rate of Z_t is $\dot{Z}_t/Z_t = \lambda_t \ln z$. In the rest of this proof, we will show that λ_t jumps to a unique and stable steady state, so that $\dot{Z}_t/Z_t$ also jumps to its steady-state value. From (12), we have

$$\lambda_t = \frac{\varphi R_t}{Z_t} = \varphi\left(1 - \gamma - \frac{c_t}{Z_t}\right), \tag{A2}$$

where the second equality uses $y_t = c_t + R_t + G_t$, $y_t = Z_t$ and $G_t = \gamma y_t$. If we define $\chi_t \equiv c_t/Z_t$, then $\lambda_t = \varphi(1 - \gamma - \chi_t)$, which shows that the dynamics of λ_t is solely determined by the dynamics of χ_t. Taking the log of χ_t and differentiating it with respect to time yield

$$\frac{\dot{\chi}_t}{\chi_t} = \frac{\dot{c}_t}{c_t} - \frac{\dot{Z}_t}{Z_t} = \frac{\dot{c}_t}{c_t} - \frac{\dot{v}_t}{v_t}, \tag{A3}$$

where the second equality uses (A1). Recall from (3) that $\dot{c}_t(h)/c_t(h) = \dot{c}_t/c_t$. Then, substituting (3) and (10) into (A3) yields

$$\frac{\dot{\chi}_t}{\chi_t} = \frac{\pi_t}{v_t} - \lambda_t - \rho = \frac{\pi_t}{v_t} - \varphi(1 - \gamma - \chi_t) - \rho, \tag{A4}$$

where the second equality uses (A2). Substituting (8), (13) and (16) into (A4) yields

$$\frac{\dot{\chi}_t}{\chi_t} = \frac{\varphi}{1-s}\frac{\mu - 1}{\mu} - \varphi(1 - \gamma - \chi_t) - \rho = \varphi\chi_t - \Phi, \tag{A5}$$

where we define a composite parameter $\Phi \equiv \varphi(1-\gamma) + \rho - \frac{\varphi}{1-s}\frac{\mu-1}{\mu}$, which is assumed to be positive by imposing parameter restrictions.[29] Equation (A5) shows that the dynamics of χ_t is characterized by instability, so that χ_t must jump to its unique and saddle-point stable steady state given by $\chi = \Phi/\varphi$. At the steady state, c_t and Z_t grow at the same rate given by $g = \lambda \ln z = \varphi(1 - \gamma - \chi)\ln z = [\varphi(1-\gamma) - \Phi]\ln z$ as in (20). ■

[29] Otherwise, χ_t and c_t would be negative.

Proof of Proposition 3. Substituting (9), (13), (16) and $a_t = v_t$ into (23) yields

$$\dot{\theta}_{a,t}(h) = \varphi \frac{\chi_t - (1-\tau)/\mu}{1-s} \theta_{a,t}(h) - \varphi \frac{\theta_{c,t}(h)\chi_t - (1-\tau)/\mu}{1-s}, \tag{A6}$$

which also uses $\chi_t \equiv c_t/Z_t$. Recall from (3) that $\dot{c}_t(h)/c_t(h) = \dot{c}_t/c_t$, which in turn implies $\dot{\theta}_{c,t}(h) = 0$ for all t. Substituting $\theta_{c,t}(h) = \theta_{c,0}(h)$ and $\chi_t = \Phi/\varphi$ for all t into (A6) yields

$$\dot{\theta}_{a,t}(h) = \frac{\Phi - \varphi(1-\tau)/\mu}{1-s} \theta_{a,t}(h) - \frac{\theta_{c,0}(h)\Phi - \varphi(1-\tau)/\mu}{1-s}. \tag{A7}$$

Therefore, the dynamic property of $\theta_{a,t}(h)$ depends on the sign of $[\Phi - \varphi(1-\tau)/\mu]/(1-s)$. To see that $\Phi > \varphi(1-\tau)/\mu$, one can use (35) to show that

$$\Phi > \frac{\varphi(1-\tau)}{\mu} \Leftrightarrow s < 1. \tag{A8}$$

Therefore, the coefficient on $\theta_{a,t}(h)$ in (A7) is positive, which in turn implies that $\dot{\theta}_{a,t}(h) = 0$ for all t is the only solution of (A7) consistent with long-run stability. Finally, imposing $\dot{\theta}_{a,t}(h) = 0$ on (A7) yields the steady-state value of $\theta_{c,t}(h)$ given by

$$\theta_{c,0}(h) = \frac{\varphi(1-\tau)}{\mu\Phi} + \left[1 - \frac{\varphi(1-\tau)}{\mu\Phi}\right] \theta_{a,0}(h). \tag{A9}$$

Equation (A9) shows that $\partial\theta_{c,0}(h)/\partial\theta_{a,0}(h) > 0$ given $\Phi > \varphi(1-\tau)/\mu$. ■