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1985

Online at <https://mpra.ub.uni-muenchen.de/8356/>
MPRA Paper No. 8356, posted 21 Apr 2008 03:03 UTC

A reprint from

**PROCEEDINGS OF
SYMPOSIA IN
APPLIED MATHEMATICS**

**American Mathematical Society
Providence, Rhode Island**

**INTERNATIONAL TRADE IN RESOURCES:
A GENERAL EQUILIBRIUM ANALYSIS**

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1. Introduction

Physical considerations alone cannot explain the volatile behavior of resource prices, or the effects these have on different regions of the world. An optimization analysis may not suffice either, since typically there are several distinct objectives: conservation, cost-minimization, and the maximization of revenues by resource exporters. These issues require an economic analysis of markets.

Markets for resources interact rather strongly with other markets: for goods such as food or industrial products and for inputs to production such as labor and capital. Such interactions are best studied with general equilibrium tools. These tools explain trade and the determination of prices across different markets. In a general equilibrium model, different economic agents have typically different objectives, a useful feature for the study of resource markets. Trade in resources takes place largely across different regions, so one is dealing with international trade.

A general equilibrium model of a market economy consists of a set of simultaneous, generally nonlinear, equations. A solution determines market prices and the quantities employed, produced, consumed and traded by the various agents in different markets. One aims to understand the qualitative changes of a solution consequent upon changes in the exogenous parameters of the system. This is generally accomplished by studying relationships between the variables within the

manifold of solutions described as the exogenous parameter changes. This method is called "comparative statics": it seeks to explain what determines the outcomes, and how policies can be designed so as to obtain desirable solutions. The name "comparative statics" is somewhat misleading, since the same method is used to study the asymptotic behavior of dynamical systems in economics.

A system of simultaneous nonlinear equations can easily become unmanageable, and require computer analysis. Computer solutions cannot, however, disclose laws of economic behavior, nor can they explain why and how certain policies work. The challenge is therefore to represent the economy by a set of equations which is sufficiently simple to admit analytic or simple implicit solutions and the study of their qualitative behavior, while at the same time retaining the complexity needed to explore the issues involved. This paper will show how to perform this task and apply the results to study policy issues in the area of natural resources. We shall analyze:

1. The connections between resource exports, their prices, and the distribution of income within the exporting economy;
2. When a country should export more, and when less;
3. How the prices of resources affect employment, output and industrial prices within a resource importing region;
4. The effects of monopolistic resource prices on the prices of industrial goods which are traded in exchange for the resource;
5. Whether or not higher oil prices lead to lower GDP in an importing region, and whether there are cases where both the buyer and the seller of a resource may benefit from a given price change;
6. The effects of loans from a resource importer to a resource exporter when these are used to expand resource-extraction in the exporting region;
7. The effects of (unpaid) loans on the price of the resources exported;

8. The welfare effects of loans on the lending region: can an (unpaid) loan make the lender better off and the borrower worse off and if so, under what circumstances?

The following sections will present general equilibrium models of increasing complexity designed to analyze these questions, and theorems obtained on these issues.

$$E^B = \min(L^B/a_1^B, K^B/c_1^B)$$

$$E^I = \min(L^I/a_2^I, K^I/c_2^I)$$

where the superscripts B and I denote the sector in which inputs are used, and the superscript B denotes supply. Efficiency requires that firms always use factors in fixed proportions:

$$L^B/K^B = a_1^B/c_1^B \text{ and } L^I/K^I = a_2^I/c_2^I.$$

We can now write the equations that specify the model. Competitive behavior on the part of the firms ensures zero profits, so that

$$p_B = a_1^B w + c_1^B r \quad (2.1)$$

$$p_I = a_2^I w + c_2^I r \quad (2.2)$$

where p_B and p_I are the prices of B and I; w and r are the wages and the rate of return on capital. Equations (2.1) and (2.2) embody the information given by the production functions for B and I.

¹ Equations (2.1) and (2.2) are equivalent to the production functions when firms

2. The North-South Model

A. Specification of the Model

This section summarizes the general equilibrium model introduced in Chichilnisky (1981, 1983). There are two regions, North and South. The North represents the industrial countries, the South the developing countries. Each region produces and consumes two goods: basics (B) and industrial goods (I). There are two inputs to production: capital (K) and labor (L). Basics can also include a natural resource or a raw material. The two regions trade with each other.

Consider first the economy of the South. It produces basics and industrial goods using labor and capital, as described by the Leontief production functions

$$B^S = \min(L^B/a_1, K^B/c_1)$$

$$I^S = \min(L^I/a_2, K^I/c_2),$$

where the superscripts B and I denote the sector in which inputs are used, and the superscript S denotes supply. Efficiency requires that firms always use factors in fixed proportions:

$$L^B/K^B = a_1/c_1 \quad \text{and} \quad L^I/K^I = a_2/c_2.$$

We can now write the equations that specify the model. Competitive behavior on the part of the firms ensures zero profits, so that

$$p_B = a_1 w + c_1 r \tag{2.1}$$

$$p_I = a_2 w + c_2 r \tag{2.2}$$

where p_B and p_I are the prices of B and I; w and r are the wages and the rate of return on capital. Equations (2.1) and (2.2) embody the information given by the production functions for B and I.¹

¹ Equations (2.1) and (2.2) are equivalent to the production functions when firms

Labor and capital supplied are increasing functions of their rewards:

$$L^S = \alpha(w/p_B) + \bar{L} \quad (\alpha > 0), \quad (2.3)$$

$$K^S = \beta r + \bar{K} \quad (\beta > 0). \quad (2.4)$$

We now give market clearing conditions (superscript S denotes supply and D denotes demand):

$$L^S = L^D \quad (2.5)$$

$$K^S = K^D \quad (2.6)$$

$$L^D = L^B + L^I = B^S a_1 + I^S a_2 \quad (2.7)$$

$$K^D = K^B + K^I = B^S c_1 + I^S c_2 \quad (2.8)$$

$$B^S = B^D + X_B^S, \text{ where } X_B^S \text{ denotes exports of } B \quad (2.9)$$

$$I^D = X_I^D + I^S, \text{ where } X_I^D \text{ denotes imports of } I, \quad (2.10)$$

and $p_B X_B^S = p_I X_I^D, \quad (2.11)$

i.e., the value of exports equals the value of imports.²

are competitive. Profits are zero in this case, which means revenues equal costs, i.e., $p_B B^S = wL^B + rK^B$. Now from the production functions $B^S = L^B/a_1 = K^B/c_1$, so that $p_B B^S = a_1 w B^S + c_1 r B^S$, or $p_B = a_1 w + c_1 r$, equation (2.1). Similarly, one derives equation (2.2).

² It is worth noting that in this model when all markets clear, the value of domestic demand $p_B B^D + p_I I^D$ equals the value of domestic income $wL + rK$. This is called Walras' Law or the national income identity. From (2.1) - (2.11) one obtains:
 $p_B B^D + p_I I^D = p_B (B^S - X_B^S) + p_I (I^S + X_I^D) = p_B B^S + p_I I^S = (a_1 w + c_1 r) B^S + (a_2 w + c_2 r) I^S$
 $= wL + rK$. In view of this, and its homogeneity properties (solutions only depend on relative prices) the model is consistent with a standard Arrow-Debreu general equilibrium model, for some set of preferences.

The North is specified by a similar set of equations (2.1) - (2.11), with possibly different technology and factor supply parameters. In a world equilibrium, the prices of traded goods are equal across regions (factors K and L are not traded) and exports match imports:

$$p_I(S) = p_I(N) \quad (2.12)$$

$$p_B(S) = p_B(N) \quad (2.13)$$

$$X_B^S(S) = X_B^D(N) \quad (2.14)$$

$$X_I^S(N) = X_I^D(S) , \quad (2.15)$$

where letters S and N in brackets denote South and North respectively.

In each region there are eight exogenous parameters: $a_1, a_2, c_1, c_2, \alpha, \bar{L}, \beta$ and \bar{K} , making a total of sixteen exogenous parameters for the North-South model. When we add the price normalizing condition,³

$$p_I = 1 \quad (2.16)$$

we have a total of 26 independent equations: (2.1) - (2.11) for North; (2.1) - (2.11) for South, (2.12) through (2.14) and (2.16).⁴ There are in total 28 endogenous variables, 14 for each region: $w, r, L^S, L^D, K^S, K^D, B^S, B^D, X_B^S, I^S, I^D, X_I^D, p_B, p_I$. Therefore the system is underdetermined up to two variables.⁵ Thus, we now specify two more variables exogenously, industrial demand in the South, $I^D(S)$ and in the North, $I^D(N)$. Obviously we could have solved the model by specifying other variables, or else by postulating demand equations; this will be done in the following sections. The demand specifications of the model are chosen to meet two criteria: analytical tractability and empirical plausibility.

³ All relations in this model are homogeneous of degree zero in prices, i.e., only relative prices (rather than the price level) matter. Therefore, one normalizes the model by fixing the price of one good (the numeraire) equal to 1.

⁴ It is easy to see that (2.15) is always satisfied when (2.11) is satisfied in each region and (2.12) through (2.14) hold.

⁵ This is not surprising, since demand behavior, or preferences, have not been specified so far.

The North-South model is, therefore, a system of 26 equations in 26 variables, depending on 18 exogenous parameters: $a_1, a_2, c_1, c_2, \alpha, \bar{L}, \beta, \bar{K}$ for each region, plus $I^D(S)$ and $I^D(N)$.

B. Comparative Statics Results

The economies of the North and of the South are identical except possibly for the values of their exogenous parameters. Differences in the structural characteristics of the two regions are described by differences in their exogenous parameters. For instance, in the North the two sectors (B and I) use approximately the same technology, i.e., the economy is technologically homogeneous. This means that $a_1/c_1 \sim a_2/c_2$ so that the determinant $D(N)$ of the matrix of technical coefficients

$$\begin{pmatrix} a_1 & c_1 \\ a_2 & c_2 \end{pmatrix}$$

is close to zero in the North. In the South, instead, technologies are dualistic. The two sectors use factors very differently, and $D(S)$ is therefore large. In both regions $D(N)$ and $D(S)$ are positive, which indicates that the B-sector uses labor more intensively than the I-sector. Another difference arises in factor markets. In the North labor is relatively more scarce, i.e., less responsive to increases in the real wage w/p_B . This means $\alpha(N)$ is small. In the South the opposite is true, $\alpha(S)$ is large. The reciprocal relations hold in capital markets: $\beta(N)$ is large, and $\beta(S)$ is small. These parameter specifications can be presented so as to be independent of the units of measurements.

It is worth noting that while most equations are linear in the variables, some are not (e.g., (2.3) is nonlinear). The solutions also display nonlinearities, as we shall see in the following.

Recall that a North-South economy is defined by: equations (2.1) through (2.11) for each region, and (2.12) through (2.15). Exogenous parameters are $a_1, c_1, a_2, c_2, \alpha, \beta, \bar{L}, \bar{K}$ for each region, and $I^D(S), I^D(N)$. Endogenous parameters are $p_B, p_I, w, r, L^S, L^D, K^S, K^D, B^S, B^D, X_B^S, I^S, I^D, X_I^D$ for each region.

Therefore there are twenty-six independent equations in twenty-six variables, and eighteen exogenous parameters.

Proposition 1. A North-South economy has at most one solution. This solution can be computed explicitly by solving one equation which depends on all exogenous parameters of the model.

Proof. From

$$X_I^D(S) = X_I^S(N)$$

we have

$$I^D(S) - I^S(S) = I^S(N) - I^D(N) \quad (2.17)$$

Inverting (2.7) and (2.8) we obtain

$$B^S = \frac{c_2 L - a_2 K}{D} \quad (2.18)$$

$$I^S = \frac{a_1 K - c_1 L}{D} \quad (2.19)$$

and inverting (2.1) and (2.2),

$$w = \frac{p_B c_2 - c_1}{D} \quad (2.20)$$

$$r = \frac{a_1 - p_B c_2}{D} \quad (2.21)$$

We now rewrite (2.17) as a function of one variable only, p_B (which is the "terms of trade" of the South, since $p_I = 1$) and obtain:

$$p_B^2(A + A(N)) + p_B[C + C(N) + I^D(S) + I^D(N)] - (V + V(N)) = 0 \quad (2.22)$$

for

$$A = \beta a_1 a_2 / D^2$$

$$V = \alpha c_1^2 / D^2$$

and

$$C = \frac{1}{D} \left(c_1 \bar{L} - a_1 \bar{K} + \frac{(\alpha c_1 c_2 - \beta a_1 a_2)}{D} \right),$$

and where A , V and C have parameters for the South, and $A(N)$, $V(N)$ and $C(N)$ for the North. Solving equation (2.22) yields p_B^* as a function of all exogenous parameters of the system.

It is easy to check that (2.22) has at most one positive root p_B^* . From this, (2.20) and (2.21), one obtains the solutions w^* and r^* for each region; from (2.3) and (2.4) L^* and K^* for each region; from (2.18) and (2.19), $(B^S)^*$ and $(I^S)^*$ for each region. From (2.9) we then obtain $(B^D)^*$ for each region, and $(I^D(N))^*$ is computed from (2.11). All endogenous variables have been computed, and the solution is complete. ■

The next theorem looks at the qualitative behavior of the solutions as an exogenous parameter changes. We study changes in the levels of the exogenous parameter $I^D(N)$. Since for each $I^D(N)$ there is a unique equilibrium, we are looking at a one-dimensional family of equilibria. Along this family of equilibria the level of exports of the South changes. We are interested in the relationship across equilibria between the level of exports $X_B^S(S)$ and the terms of trade, real wages and consumption of the South.

Theorem 2. Consider a North-South economy as above where labor is very abundant in the South ($\alpha(S)$ large). An increase in the volume of exports $X_B^S(S)$ can lead to two different outcomes:

- (1) If at the initial equilibrium $c_2/D < 2w/p_B$ (e.g., technologies are dual in the South, i.e., D is large), then an increase in exports leads to lower terms of trade for the South (p_B), lower export revenues ($p_B X_B^S$), lower real wages (w/p_B), and lower domestic consumption (B^D) in the South.
- (2) When $c_2/D > 2w/p_B$ (e.g., technologies are more homogeneous, or real wages smaller) an increase in exports leads instead to higher export revenues ($p_B X_B^S$), higher terms of trade (p_B), higher real wages (w/p_B) and higher consumption in the South.

Proof: Across equilibria, the following equation is satisfied:

$$X_B^S(S) = B^S(S) - B^D(S) = \frac{(c_2 L - a_2 K)}{D} - \left(\frac{wL + rK - \bar{I}^D(S)}{p_B} \right), \quad (2.23)$$

where the second equality comes from (2.18) and the Walras Law of footnote 2.

Substituting L and K as functions of w and r , and w and r as functions of p_B (from (2.3), (2.4), (2.20) and (2.21)) we obtain

$$X_B^S(S) = \frac{\alpha c_1}{D^2 p_B} \left(c_2 - \frac{c_1}{p_B} \right) + \frac{\beta a_1}{D^2} \left(a_2 - \frac{a_1}{p_B} \right) + \frac{c_1 \bar{L} - a_1 \bar{K}}{D p_B} + \frac{\bar{I}^D(S)}{p_B} \quad (2.24)$$

We have therefore expressed the level of exports X_B^S as a function of the terms of trade p_B only. We can now consider the derivative across the manifold of equilibria of X_B^S with respect to p_B :

$$\frac{\partial X_B^S}{\partial p_B} = \frac{\alpha c_1}{D^2 p_B^2} \left(\frac{\alpha c_1}{p_B} - c_2 \right) + \frac{\beta a_1^2}{D^2 p_B^2} + \frac{a_1 \bar{K} - c_1 \bar{L}}{p_B^2} - \frac{\bar{I}^D(S)}{p_B^2}.$$

When α is large (and β small) $\partial X_B^S / \partial p_B$ has the sign of

$$\frac{1}{D} \left(\frac{2c_1}{p_B} - c_2 \right) = \frac{c_2}{D} - \frac{2w}{p_B}$$

by (2.20). Therefore, when $c_2/D < 2w/p_B$ (case (1)), an increase in the volume of exports leads to a lower solution value p_B^* ; when $c_2/D > 2w/p_B$ the opposite is true: as X_B^S rises, so does p_B^* . Our last task is to examine the impact of X_B^S on real wages, consumption and export revenues of the South.

It is easy to check that in case (1) ($c_2/D < 2w/p_B$) export revenues $p_B X_B^S$ fall as exports increase. This is because a higher level of X_B^S in this case leads to a lower p_B^* and from (2.20) to a lower $(w/p_B)^*$ and a higher r^* . By (2.3), (2.4) and (2.19), this means that $(I^S)^*$ increases. Since $\bar{I}^D(S)$ is a constant, X_I^D has decreased. Therefore by (2.11), $(p_B X_B^S)^*$ drops. The opposite effects happen in case (2): p_B increases, and so do $p_B X_B^S$ and w/p_B .

The effect of exports on domestic consumption is also easy to check.

In case (1) ($c_2/D < 2w/p_B$), p_B^* drops, $(w/p_B)^*$ drops and r^* increases. From (2.3) and (2.4) L^* drops and K^* increases. This means that $(B^S)^*$ drops, from (2.18). Since

$$B^D = B^S - X_B^S,$$

$(B^S)^*$ dropped and $(X_B^S)^*$ increases, $(B^D)^*$ must now be lower. The opposite happens when $c_2/D > 2w/p_B$. This completes the proof. ■

It is of interest to examine what drives this result. For exports to increase, the difference between domestic supply and demand for basics must be wider. If an increase in prices p_B leads to proportionately higher increase in demand than in supply, then exports can only rise when prices drop. This is case (1): the expression $c_2/D - 2w/p_B$ measures the relative strength of the supply and demand responses as prices p_B change, c_2/D coming from the supply and $2w/p_B$ from demand. Notice that we do not refer here to standard supply and demand responses to prices when all other variables are kept constant. We refer instead to general equilibrium effects: as prices change everything else does too: employment, real wages and thus also income. Wage income wL increases with higher prices p_B , so that the general equilibrium effect of higher prices on demand is positive. However, the standard partial equilibrium effect of prices on demand, when all other variables are constant, is in general negative. From the proof of Theorem 2 we obtain:

Corollary 3. The terms of trade of the South, p_B , are always positively associated with wage income wL and negatively associated with capital income rK in both regions.

The above results show how a system of 26 equations in 26 variables can be solved analytically to yield qualitative results. In particular we showed the connection across equilibria between the volume of exports of the South X_B^S and main aggregate variables of the South: real wages, consumption levels, export revenues.

Theorem 2 shows how to answer questions (1) and (2) in the Introduction. The connection between exports X_B^S and their price is given by equation (2.24) in Theorem 2. The distribution of income, measured here as the relative level of wages w (or real wages w/p_B) and the return on capital r , is in turn changing with exports through the relationships (2.20) and (2.21), as pointed out in Corollary 3. Finally, Theorem 2 shows when a country should export more (α large, $c_2/D > 2w/p_B$) or less (α large, $c_2/D < 2w/p_B$).

The next section will focus on the responses of the importing economy to exogenous changes in resource prices.

3. A Monopolistic Resource Exporter

A. Specification of the Model

The previous model dealt with trade in which prices adjust so as to clear the markets. No single agent influences prices; the economic agents are, therefore, "competitive." This section will deal instead with a case where one region fixes the price of the resource it exports. This agent is therefore called a monopolist.

The model of the last section will now be modified to include one more input of production, oil. The price of oil is now an exogenously set parameter. We shall focus on the effects of changes in oil prices on the economy of the importer. For this purpose we concentrate on modelling the economy of the North. This model was introduced in Chichilnisky (1981b).

The North produces basics (B) and industrial goods (I) using capital (K), labor (L) and oil (θ):

$$B^S = \min(L^B/a_1, \theta^B/b_1, K^B/c_1)$$

$$I^S = \min(L^I/a_2, \theta^I/b_2, K^I/c_2) .$$

The "zero profit" price equations, as explained in the previous sections, are

$$p_B = a_1 w + b_1 p_o + c_1 r p_I \quad (3.1)$$

$$p_I = a_2 w + b_2 p_o + c_2 r p_I \quad (3.2)$$

where $p_I r$ denotes here the "user's cost" of capital. Supplies of labor and capital depend on their rewards:

$$L^S = \alpha w / p_B , \quad \alpha > 0 \quad (3.3)$$

$$K^S = \beta r , \quad \beta > 0 . \quad (3.4)$$

Next we formulate a demand equation, which substitutes for the exogenously set industrial demand I^D of the previous section:

$$p_B B^D = wL, \quad (3.5)$$

i.e., wage income is spent on basics. This relation is not necessary to prove the results, but it simplifies considerably the computations.

The market clearing conditions are

$$L^D = L^B + L^I = a_1 B^S + a_2 I^S \quad (3.6)$$

$$K^D = K^B + K^I = c_1 B^S + c_2 I^S \quad (3.7)$$

$$\theta^D = \theta^B + \theta^I = b_1 B^S + b_2 I^S \quad (3.8)$$

$$K^S = K^D \quad (3.9)$$

$$L^D = L^S \quad (3.10)$$

$$\theta^D = \theta^S \quad (3.11)$$

$$B^D = B^S \quad (B \text{ is not traded internationally}) \quad (3.12)$$

$$I^D + X_I^S = I^S \quad (X_I^S \text{ are exports of } I) \quad (3.13)$$

$$p_I X_I^S = p_\theta X_\theta^D \quad (\text{value of exports of } I \text{ equals value of imports of oil}) \quad (3.14)$$

$$\theta^D = X_\theta^D \quad (\text{the North imports the oil it consumes}) \quad (3.15)$$

and basics are the numeraire

$$p_B = 1. \quad (3.16)$$

As before, the Walras Law or national income identity is identically satisfied when (3.1) through (3.15) hold: the value of domestic demand equals the value of domestic income

$$p_B B^D + p_I I^D = wL + rK. \quad (3.17)$$

The monopolistic oil-exporter model is specified as follows. Its exogenous parameters are the technical coefficients $a_1, a_2, b_1, b_2, c_1, c_2$; the supply parameters α and β ; and the price of oil, p_o .

There are sixteen endogenous variables: $p_B, p_I, r, w, L^D, L^S, K^D, K^S, B^D, B^S, I^D, I^S, X_I^S, \theta^D, X_\theta^D, \theta^S$; and sixteen independent equations (3.1) through (3.16). Therefore there are, as before, as many variables as independent equations, and the solutions will generally be (locally) unique. The following results explore the effects of changes in the exogenous price of oil p_o on the fifteen endogenous variables of the North. To simplify computations we now assume the following stylized conditions:

- I. $a_1 b_2 - a_2 b_1 = M > 0$, i.e., B is relatively more labor-intensive and I more oil-intensive;
- II. $c_1 = 0$, i.e., B requires no capital income (this condition is not necessary, but simplifies greatly the computations);
- III. b_1 is small, i.e., B requires relatively few oil inputs.

Proposition 4. The monopolistic oil-exporter model has at most one solution for each price of oil p_o . This solution can be explicitly computed by solving one equation.

Proof. Equations (3.6) and (3.7) imply

$$B^S = (c_2 L - a_2 K) / D \quad (3.18)$$

and

$$I^S = (a_1 K - c_1 L) / D \quad (3.19)$$

where $D = a_1 c_2 - a_2 c_1$.

The price equations (3.1) and (3.2) yield

$$w = \frac{(p_B - b_1 p_o) c_2 - (p_I - b_2 p_o) c_1}{D} \quad (3.20)$$

and

$$r = \frac{a_1 (p_I - b_2 p_o) - a_2 (p_B - b_1 p_o)}{D} \quad (3.21)$$

Substituting in (3.18) K and L from (3.3) and (3.4) and then w and r from (3.20) and (3.21) one obtains

$$B^S = \frac{(c_2 \alpha w - a_2 \beta r)}{D} = \frac{\alpha c_2}{D^2} (c_2 + p_o N - c_1 p_I) + \frac{\beta a_2}{D^2} \left(\frac{p_o}{p_I} M + \frac{a_2}{p_I} - a_1 \right) \quad (3.22)$$

where $M = a_1 b_2 - a_2 b_1$ and $N = c_1 b_2 - b_1 c_2$.

From $B^D = wL$ and $B^S = B^D$, one obtains, when $p_B = 1$,

$$\begin{aligned} \alpha c_2 (c_2 + p_o N - c_1 p_I) + \beta a_2 \left(\frac{p_o}{p_I} M + \frac{a_2}{p_I} - a_1 \right) \\ = \alpha [(1 - b_1 p_o) c_2 - (p_I - b_2 p_o) c_1]^2 \end{aligned} \quad (3.23)$$

When $c_1 = 0$, this yields an explicit relation between the price of oil and the price of industrial goods

$$p_I = \frac{a_2 + p_o M}{\gamma b_1 p_o (b_1 p_o - 1) + a_1} \quad (3.24)$$

where

$$\gamma = \frac{\alpha c_2^2}{\beta a_2}.$$

From equation (3.24), there exists at most one solution p_I^* for each p_o . Given p_I^* , we obtain all other endogenous variables as follows: w^* obtains from (3.20), r^* from (3.21), L^* and K^* from (3.3) and (3.4), $(B^S)^*$ and $(I^S)^*$ from (3.18) and (3.19) and $(B^D)^*$ from (3.5). $(\theta^D)^* = (X_\theta^D)^*$ obtains from (3.8), and

therefore from (3.14) one obtains $(X_I^S)^*$. All endogenous variables are uniquely determined. This completes the proof. ■

The next results explore the comparative statics properties of changes in the price of oil, p_o .

B. Comparative Statics Results

Theorem 5. Consider a monopolistic exporter model as in Proposition 4. An increase in the (monopolistically set) price of oil has the following effects: If the initial price of oil is low, increases in this price benefit the exporter; the volume of industrial goods traded in exchange for oil X_I^S increases. However, after a price \bar{p}_o has been reached, the volume of industrial exports X_I^S decreases with further increases in the price of oil; the monopolist loses from increasing the price p_o . There are two cases:

(A) $\alpha c_2^2 \leq 2\beta a_1 a_2$. In this case $\bar{p}_o = 1/2b_1$, and the rate of profit r rises and falls with the level of exports X_I^S , see Figure 1.

(B) $\beta c_2^2 > 2\beta a_1 a_2$. In this case

$$\bar{p}_o = \frac{1}{2b_1} \left(1 - \frac{(\gamma - 2a_1)^{1/2}}{\gamma} \right).$$

Between \bar{p}_o and $1/2b_1$, r increases and X_I^S decreases. Between $1/2b_1$ and $(1/b_1) - \bar{p}_o$ the opposite happens: X_I^S increases and r drops, see Figure 2.

Therefore in case (A) the oil exporter and the oil importer can both benefit from an increase in p_o when $p_o < 1/2b_1$ and from a decrease in p_o when $p_o > 1/2b_1$. This is provided that their aim is to maximize returns on capital (r) and the volume of industrial exports traded for oil (X_I^S) respectively.

In case (B), there are areas where both traders benefit from a price increase ($p_o < \bar{p}_o$) or from a price drop ($p_o > (1/b_1) - \bar{p}_o$). Outside those areas, the traders have conflicting interests.

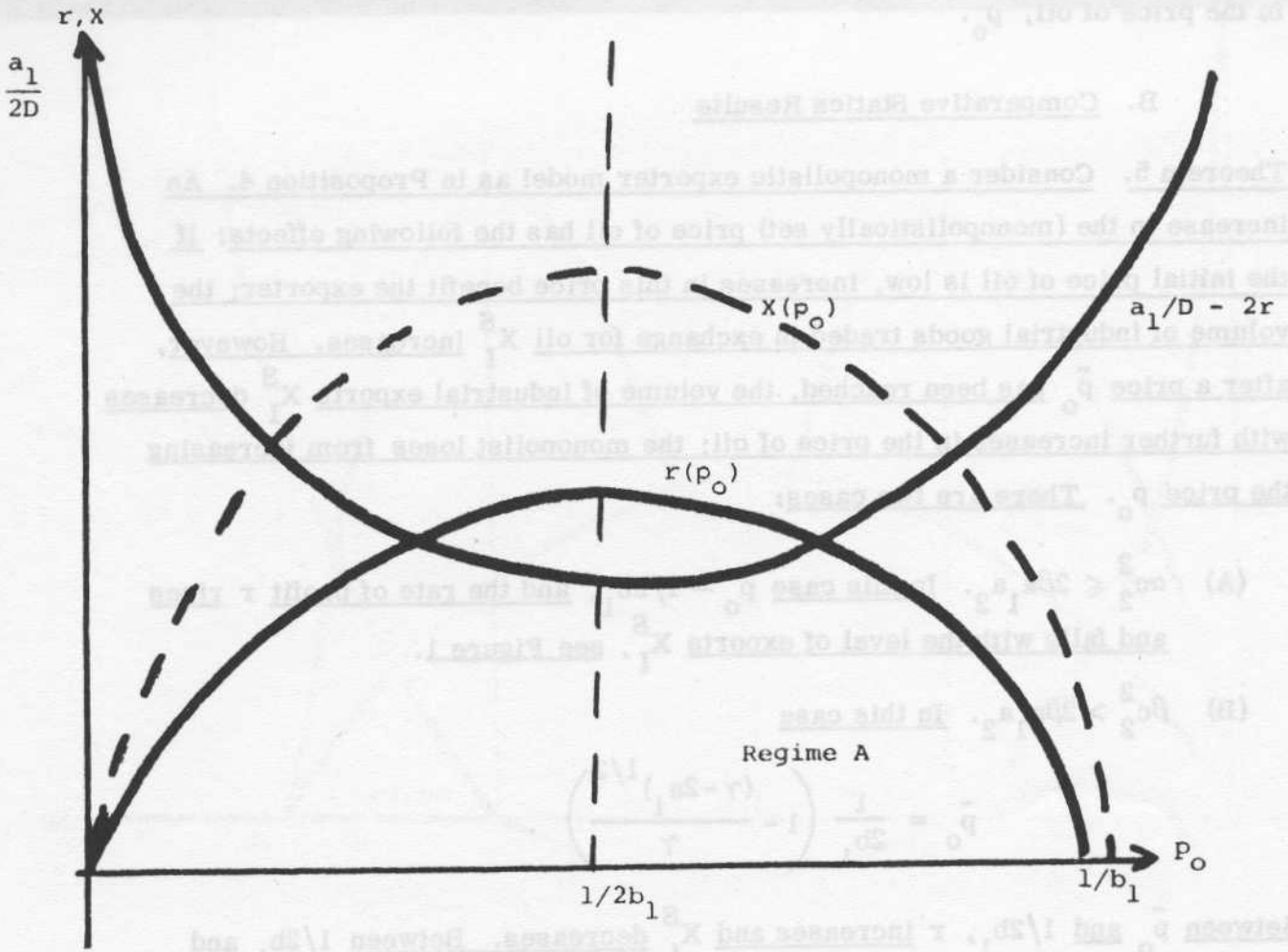


Figure 1. Case 1: $\alpha c_2^2 > 2\beta a_1 a_2$, i.e., r is always bounded below $a_1/2D$. In this case, as the price of oil p_o increases, the volume of industrial exports $X(p_o)$, and the rate of return on capital $r(p_o)$ increase and decrease together.

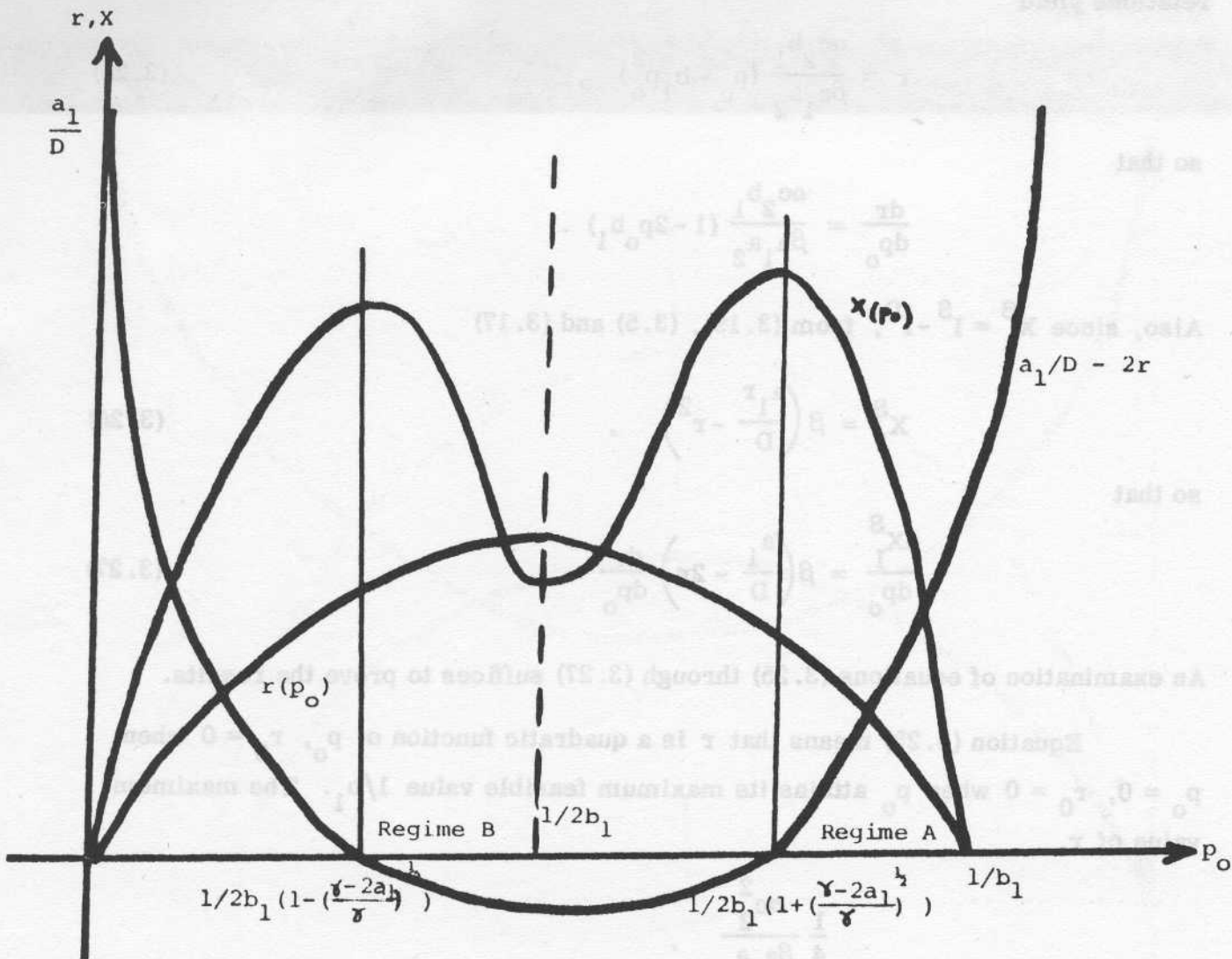


Figure 2. Case 2: In this case, the rate of return on capital r and the level of industrial exports traded for oil X , do not move in the same direction in regime B. There are therefore regions with community of interests for the traders, and others where there are conflicting interests.

Proof. By (3.21) r is a function of p_I and p_O . By (3.24) $p_I = p_I(p_O)$. These two relations yield

$$r = \frac{\alpha c_2 b_1}{\beta a_1 a_2} (p_O - b_1 p_O^2) \quad , \quad (3.25)$$

so that

$$\frac{dr}{dp_O} = \frac{\alpha c_2 b_1}{\beta a_1 a_2} (1 - 2p_O b_1) \quad .$$

Also, since $X_I^S = I^S - I^D$, from (3.19), (3.5) and (3.17)

$$X_I^S = \beta \left(\frac{a_1 r}{D} - r^2 \right) \quad , \quad (3.26)$$

so that

$$\frac{dX_I^S}{dp_O} = \beta \left(\frac{a_1}{D} - 2r \right) \frac{dr}{dp_O} \quad . \quad (3.27)$$

An examination of equations (3.25) through (3.27) suffices to prove the results.

Equation (3.25) means that r is a quadratic function of p_O , $r_0 = 0$ when $p_O = 0$, $r_0 = 0$ when p_O attains its maximum feasible value $1/b_1$. The maximum value of r ,

$$\frac{1}{4} \frac{\alpha c_2^2}{\beta a_1 a_2} \quad ,$$

is attained when $p_O = 1/2b_1$. From equation (3.27) when $a_1/D > 2 \max r$, then dX_I^S/dp_O has the same sign as dr/dp_O . Now, $a_1/D > 2 \max r$ if and only if

$$\frac{a_1}{D} > \frac{1}{2} \frac{\alpha c_2^2}{\beta a_1 a_2} \quad ,$$

i.e., $\alpha c_2^2 < 2\beta a_1 a_2$. This is case (A), see Figure 1. Case (B) is when $\alpha c_2^2 > 2\beta a_1 a_2$. In this case dX_I^S/dp_O has the opposite sign to dr/dp_O when $p_O^1 \leq p_O \leq p_O^2$, for

$$p_O^1 = \frac{1}{2b_1} \left(1 - \left(\frac{\gamma - 2a}{\gamma} \right) \right)^{1/2} \quad , \quad p_O^2 = \frac{1}{2b_1} \left(1 + \left(\frac{\gamma - 2a}{\gamma} \right)^{1/2} \right)$$

as in Figure 2. This completes the proof. ■

The previous theorem does not explain the impact of oil prices on the GDP of the importing region. This is measured as the total value of domestic output minus the value of imported inputs:

$$\text{GDP} = Y = p_B B + p_I I - p_O \theta.$$

Theorem 6. For very low prices of oil, an increase in p_O lowers the GDP of the importer. However, after a price p_O has been reached, further increases increase the GDP, up to p_O . From this latter point on, the effect of oil prices on GDP is again negative.

Proof: From (3.3), (3.4), (3.5) and (3.17),

$$Y = \alpha w^2 + p_I \beta r^2, \quad (3.28)$$

so that

$$\frac{dY}{dp_O} = \alpha 2w \frac{dw}{dp_O} + \beta \left(r^2 + \frac{dp_I}{dp_O} + p_I 2r \frac{dr}{dp_O} \right). \quad (3.29)$$

Note that from (3.20)

$$\frac{dw}{dp_O} = -b_1 c_2 / D < 0 \quad (3.30)$$

Also, since $p_B B^D = wL = \alpha w^2$,

$$\frac{dB^D}{dp_O} = \alpha 2w \frac{dw}{dp_O} < 0.$$

Furthermore, $r = 0$ when $p_O = 0$. Therefore (3.29) implies that for small values of p_O , Y is a decreasing function of p_O , $dY/dp_O < 0$. However, since $dr/dp_O > 0$ for $p_O \leq 1/2b_1$ and $b_1 \sim 0$, when p_O exceeds a small value (denoted p_O^3), Y is an increasing function of the price of oil. This is due to the fact that as p_O increases, the value of demand for I , $p_I I^D$ increases, since dr/dp_O and $dp_I/dp_O \geq 0$ for

$$p_O \leq \frac{1}{2b_1} \left(1 + \left(\frac{\gamma-2}{\gamma} \right) \right)^{1/2}.$$

Since b_1 is rather small, the increase in p_I^D within Y exceeds the decrease in p_B^D , so that Y increases with the price of oil. Figure 3 illustrates this result from a computer simulation of the model.

Finally, when p_o exceeds a value p_o^4 , dY/dp_o turns negative again, because $r = 0$ when p_o assumes its maximum value $1/b_1$, $dr/dp_o < 0$ for $p_o > 1/2b_1$, $dB^D/dp_o < 0$ and rdp_I/dp_o is bounded above by

$$-\frac{Mac_2b_1}{\beta a_1 a_2} p_o$$

since

$$\frac{dp_I}{dp_o} = \frac{M(a_1 + b_1 \gamma p_o (b_1 p_o - 1)) - (a_2 + p_o M) \gamma b_1 (2p_o b_1 - 1)}{(a_1 + \gamma b_1 p_o (p_o b_1 - 1))^2},$$

for $\gamma = \frac{\alpha}{\beta} c_2^2 / a_2$. This completes the proof. ■

The last two theorems answer questions 3, 4 and 5 of the Introduction. They show how the price of an imported resource affects prices, output and exports of the importing region. They show how a monopolistic resource exporter should take into account the effect of its prices on the importing economy: when to raise and when to decrease resource prices. Finally, they demonstrate cases where the importer and the exporter have a conflict of interest, and others where both could benefit from a rise, or a fall, in resource prices.

C. Computer Simulation

The following is a computer simulation of this model. The following values of the exogenous parameters are given:

Exogenous Data

$\alpha = 1.00$	$b_1 = 0.10$
$\beta = 2.00$	$b_2 = 0.20$
$a_1 = 0.30$	$c_1 = 0.001$
$a_2 = 0.20$	$c_2 = 0.60$

As explained in Proposition 4, a solution of the model is computed by resolving one equation in p_o , equation (3.23), and computing all other endogenous variables from p_o . The following table reproduces the numerical results for different values of the price of oil, p_o . Figures 3 and 4 reproduce graphically these results. Figure 3 illustrates the response of $GDP = Y$, the rate of return on capital r , the volume of industrial exports X , wages w and the price of industrial exports, as the price of oil varies. Figure 4 illustrates the connection between oil prices p_o and oil exports θ^S , across equilibria of the North-South economy.

p_o	Y	r	w	X	θ^S
10.0	3'00000	0'382833-01	0'	0'321284-19	0'282148
8.2	3'32041	0'332131	0'164303	1'21491	0'334803
6.0	3'24394	0'442210	0'230221	3'13049	1'11827
4.2	3'88221	0'211308	0'483308	2'18227	3'22101
2.0	3'30018	0'183328	0'621848	4'21086	4'24182
1.2	3'18328	0'633045	0'831888	2'14030	3'18893
1.0	4'30111	1'04123	0'382318	2'22181	10'3080
0.2	4'18041	1'13286	1'14881	2'10144	13'4201
0.0	2'76383	1'18820	1'37316	2'80880	16'4229
0.2	2'38881	1'33204	1'61163	2'13262	18'0334
0.0	2'32000	1'32112	1'84444	2'21801	18'2806
4.2	4'82248	1'34303	1'81383	2'30000	18'2124
4.0	4'33864	1'30101	1'88324	4'18216	18'2145
3.2	3'21824	1'14232	3'72384	4'33084	14'1328
3.0	3'61813	1'02883	3'25303	2'13151	17'6418
3.2	3'30842	0'848042	3'48313	3'13481	10'2418
3.0	1'80823	1'801018	3'28113	3'48223	8'44143
1.2	1'41338	0'844898	3'88030	1'04331	8'12423
1.0	1'10133	0'428232	3'88823	1'30823	8'44881
0.2	0'328213	0'343423	2'18761	0'280848	10'1321
0.0	0'88801	0'22143-03	3'3333	0'381204-18	11'1111

TABLE 1

Exogenous p_o	p_I	r	w	Endogenous p_I^*X	Y	θ^S
0.0	0.668901	0.5574d-02	3.3332	0.337150d-16	11.1111	1.01
0.5	0.858613	0.243452	3.16597	0.580648	10.1251	1.16
1.0	1.10132	0.456505	2.99832	1.20033	9.44897	1.20
1.5	1.41239	0.644688	2.83030	1.84227	9.18463	1.228
2.0	1.80953	1.807918	2.66179	2.48953	9.44742	1.244
2.5	2.30945	0.946045	2.49272	3.12497	10.3476	1.249
3.0	2.91912	1.05883	2.32303	3.73121	11.9419	1.243
3.5	3.61864	1.14595	2.15284	4.29064	14.1388	1.225
4.0	4.33894	1.20707	1.98254	4.78579	16.5742	1.196
4.5	4.95548	1.24203	1.81282	5.20000	18.5754	1.155
5.0	5.33000	1.25113	1.64444	5.51801	19.3906	1.103
5.5	5.38881	1.23504	1.47782	5.72565	18.6234	1.041
6.0	5.16262	1.19450	1.31278	5.80860	16.4559	0.968
6.5	4.75047	1.12996	1.14887	5.75144	13.4507	0.884
7.0	4.25777	1.04152	0.985218	5.53761	10.2080	0.791
7.5	3.76058	0.929042	0.821688	5.14990	7.16683	0.686
8.0	3.30079	0.792336	0.657949	4.57096	4.57735	0.571
8.5	2.89531	0.631209	0.493908	3.78351	2.55107	0.445
9.0	2.54664	0.445510	0.329551	2.77048	1.11951	0.307
9.5	2.25047	0.235131	0.164903	1.51491	0.276035	0.159
10.0	2.00000	0.385923d-01	0.	0.257568d-15	0.595746	0.000017

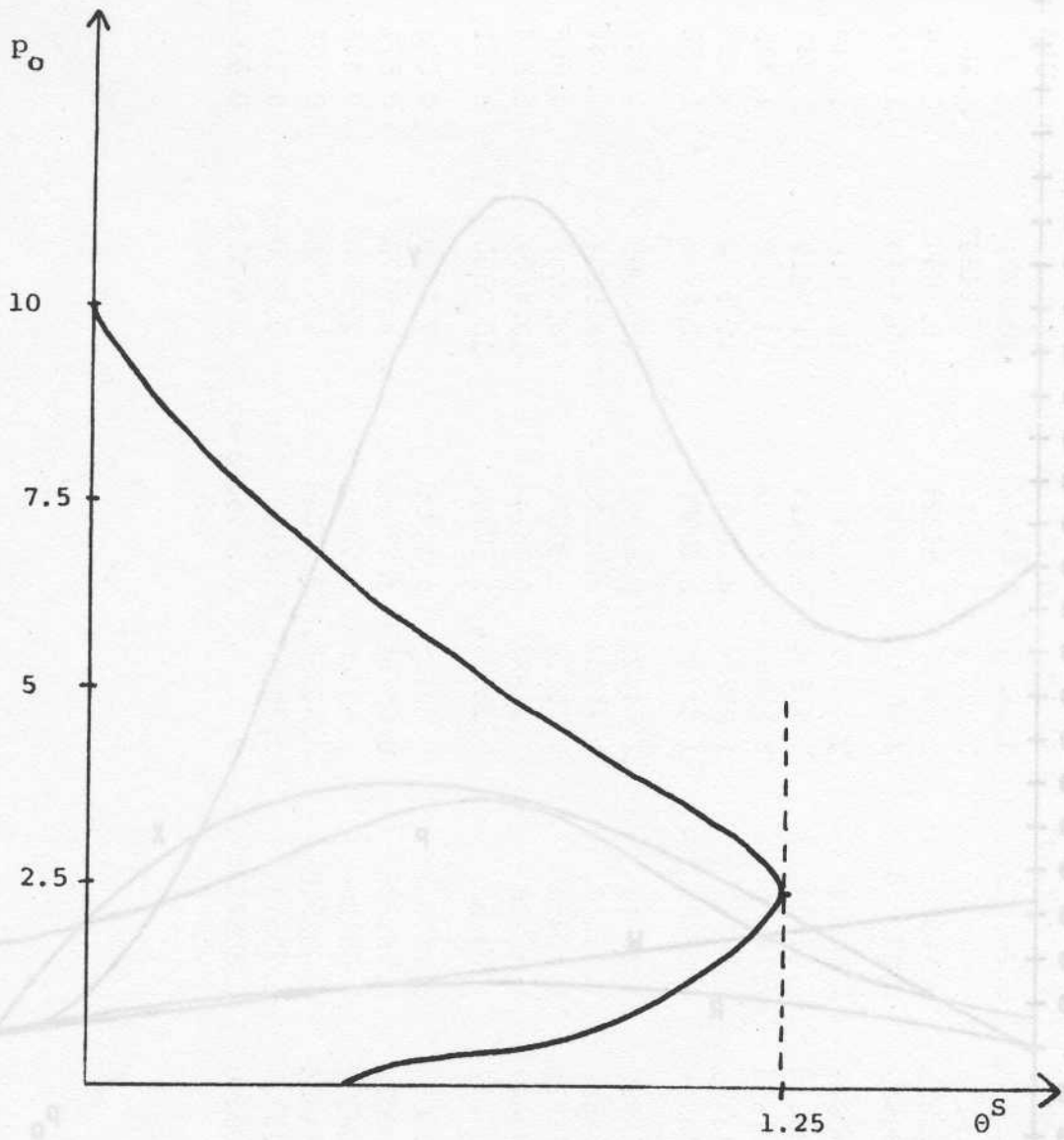


Figure 3. The volume of oil exports and the price of oil across equilibria.

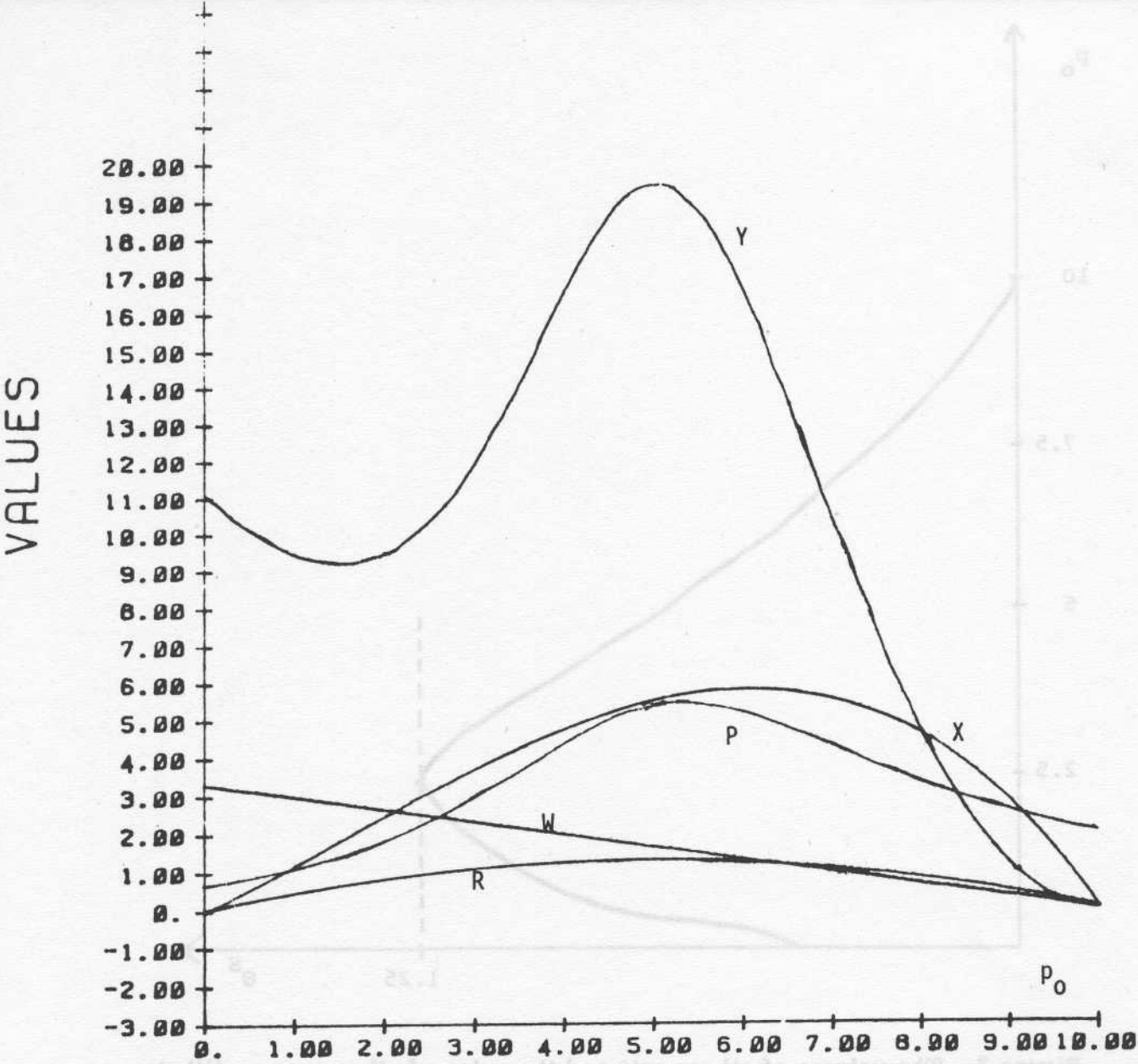


Figure 4. Results from a simulation: note that the GDP, Y , of the oil importer rises and then falls with increases in the price of oil, p_o .

4. Debt and Resource Exports

In this section we extend the North-South model to study the impact of debt on the resource-importing and resource exporting economies. We allow here for an imbalance in the trade account, which is matched by an inflow of overseas investment or a financial transfer. This imbalance represents the debt owed to foreigners, and is directed towards the expansion of oil supplied. Except for the wedge between export revenues and import costs, which represents the debt, the model is consistent with a standard competitive general equilibrium specification. The model was introduced in Chichilnisky, Heal and McLeod (1984).

The introduction of the debt wedge changes the main relations in the model: the operation of Walras' Law or the national income identity in both countries is altered. Overseas transfers lead to changes in oil supplies and consequently most variables adjust. As the debt increases, a new equilibrium emerges with different prices and levels of imports and exports. There are also changes in all domestic variables in both South and North: real wages, profits, domestic use of industrial and consumption goods, and employment of the factors labor, capital and oil. This allows us to trace the impact of the debt on the major macro variables of the two countries. The model could also be used to examine the impact of rescheduling, i.e., repaying the debt over a different time period, or of repaying it at a different rate of interest.

Following the macroeconomic impact analysis, two main questions emerge. The first is, who benefits and who loses from the accumulation of debt; and the second is, whether there exist debt-management policies that could make both countries better off, after taking fully into account the recycling effect of borrowing funds on imports from the lender.

The interest of the results lies in part in their simplicity and in part in the fact that they account for the impact of the debt on all markets simultaneously. Fairly simple analytical solutions are obtained to the rather complex questions posed. These are obtained, of course, at the cost of somewhat stylized assumptions.

We describe conditions under which increasing the debt leads the country to export more oil. In certain cases, this leads to lower prices of oil, lower volumes of industrial imports, lower real wages, and higher profits in the oil-exporting country. In other cases, the results are reversed, and real wages, consumption, and terms of trade all improve in the exporting country. The outcome depends on the technologies of the South and on the initial prices.

We also examine conditions under which the economy of the North actually benefits in macroeconomic terms from its loan to the South: because of lower oil prices, the consumption of both goods increases in the North when the transfer or loan increases. This occurs because the transfer leads to better terms of trade in the North, and because its production system is relatively homogeneous. This result is reminiscent of the argument that British investment overseas in the nineteenth century benefited the country by developing overseas supplies of food and raw material, thus making these supplies more elastic, keeping down prices, and improving the UK's terms of trade. Essentially we are specifying here conditions for overseas investment in material supplies to benefit the investing country even before any financial returns are paid, or in the case of a loan, before the loan is repaid.

A. Specification of the Model

There are two regions, the North and the South. Each produces two goods, denoted B and I, with three factors of production, capital, K; labor, L; and oil, θ . The South exports an input, oil, in exchange for a good, the "industrial" good I. The "basic" good B is produced domestically and not traded internationally.

We first specify the model for one region, namely the South. In what follows, the subscripts S and D will be used to denote supply and demand, and the superscripts N and S to denote variables or parameters referring to the North and South, respectively. All variables or parameters without a superscript refer to the South. The superscripts B and I after a factor (e.g., L^B , K^I) denote the amount of that factor used in sector B or I, respectively.

The basic good is produced according to the relation

$$B_S = \min[L^B/a_1, \theta^B/b_1, K^B/c_1] \quad (4.1)$$

and the industrial good according to

$$I_S = \min[L^I/a_2, \theta^I/b_2, K^I/c_2] \quad (4.2)$$

Labor and capital supplies are responsive to their rewards:

$$L_S = \alpha w/p_B, \quad \alpha > 0 \quad (4.3)$$

where w is the wage and p_B the price of B , and

$$K_S = \beta r, \quad \beta > 0 \quad (4.4)$$

where r is the rate of profit. p_I and p_θ will stand for the prices of industrial goods and of oil, respectively. The demand for B derives from wage income

$$p_B B_D = wL \quad (4.5)$$

The South produces oil (within given bounds) without using either domestic capital or labor. We shall assume that it uses the overseas borrowing or financial transfer FT to increase its oil supplies

$$\theta_S = \theta_S(FT), \quad \partial \theta_S / \partial FT > 0 \quad (4.6)$$

This completes the behavioral specification for the South.

The equilibrium conditions for the South are:

$$B_S = B_D \quad (4.7)$$

where B is not traded internationally.

$$I_D = I_S + M_I^S \quad (4.8)$$

where M_I^S denotes the South's imports of I .

$$\theta_S^S = \theta_D + X_\theta^S \quad (4.9)$$

where X_θ^S denotes oil exports by the South,

$$K_S = K_D \quad (4.10)$$

$$L_S = L_D \quad (4.11)$$

$$L_D = B_{S1}^a + I_{S2}^a \quad (4.12)$$

$$K_D = B_{S1}^c + I_{S2}^c \quad (4.13)$$

$$\theta_D = B_{S1}^b + I_{S2}^b \quad (4.14)$$

and the payments condition

$$p_\theta X_\theta^S = p_I M_I^S - FT, \quad (4.15)$$

where FT denotes financial transfers.

Note that FT could be either positive or negative, depending on the relative magnitudes of the debt service and the financial credit. However, the effect of a transfer (FT positive) may not be symmetric with that of a repayment (FT negative), because of the irreversibility of the investment in the oil sector: we assume that the financial transfer FT is used to purchase industrial goods to augment the supply of oil. This means that the new industrial investment in the oil sector is paid for by foreign loans. Hence, oil supplies θ_S change as the debt level changes; the debt is assumed to increase with increases in the level of the transfer (FT positive). The balance-of-payments condition (4.15) is that imports of industrial goods exceed export revenues by FT . As the demand for the basic good B comes entirely from wage income (4.5), the national income identity ((4.16) below) implies that the demand for industrial goods comes from the profit income rK , oil revenues $p_\theta X_\theta^S$, and the borrowing FT , with the last of these going to the oil sector. In the North we make a corresponding assumption, namely that the financial transfer to the South is taken from income that would otherwise have purchased industrial goods, so that the North's demand for industrial goods is $rK - FT$.

In an equilibrium situation, Walras' Law or the national income identity of the South is always satisfied (see Chichilnisky, 1981a), i.e.,

$$p_B B_D - p_I I_D = wL + rK + p_\theta \theta + FT \quad (4.16)$$

where $\theta = \theta_S$ is, as in (4.6), a function of FT. Equation (4.16) can also be rewritten as

$$p_B B_S + p_I (I_S + M_I^S) = wL + rK + p_\theta (\theta_D + X_\theta^S) + NF. \quad (4.16)'$$

This completes the specification of the South.

The model of the North consists of the same 15 equations, but with possibly different parameters $\alpha, \beta, a_1, a_2, b_1, b_2, c_1, c_2$. The following equation now substitutes for equation (4.6) in the South:

$$\theta_S = 0, \quad (4.6)'$$

Of course, the equations corresponding to (4.8) and (4.9) reflect the fact that the North imports oil and exports industrial goods. In a world trade equilibrium the prices of the traded goods must be equal:

$$p_\theta^S = p_\theta^N \quad (4.17)$$

$$p_I^S = p_I^N; \quad (4.18)$$

and traded quantities must also match:

$$X_\theta^S = M_\theta^N \quad (4.19)$$

$$X_I^N = M_I^S \quad (4.20)$$

where X_I^N and M_θ^N represent, respectively, the North's exports of I and imports of oil. There are therefore two sets of eight exogenous parameters each, one set for the North and the other for the South. Each set contains $\alpha, \beta, a_1, a_2, b_1, b_2, c_1$ and c_2 . These parameters are generally different in the two regions.

We make certain stylized assumptions to simplify computations: α is large in the South and relatively smaller in the North, indicating that labor is more "abundant" in the South. The corresponding parameters for capital exhibits the opposite behavior: β is large in the North than in the South. We shall also assume that c_1 is small in the South, i.e., the production of basic goods uses little capital, and a_2 is small in the North, i.e., Northern industry uses little labor.

There are a total of 33 independent equations for the complete North-South system: thirty correspond to two sets of (4.1) through (4.15), one set for each region, and three equations arise from the international trade conditions (4.17) through (4.20), since of these four, as usual, only three are linearly independent. There are 17 endogenously-determined variables each in the North and in the South: $p_I, p_\theta, p_B, w, r, L_S, L_D, K_S, K_D, B_S, B_D, I_S, I_D, M_I^S, \theta_S, \theta_D$ and X_θ^S . Finally, we have the transfer FT, making a total of 35 endogenous variables for the complete North-South system. We therefore have 33 equations in 35 unknowns. When we choose the numeraire ($p_\theta = 1$) an equilibrium is determined up to one variable. If we fix exogenously one variable, the equilibrium is (locally) unique. We choose this variable to be the value of the transfer FT. The transfer or loan thus becomes a policy variable. In the following we show how to compute explicitly a solution to the model, i.e., a value for each of the endogenous variables for each value of FT. In particular, we show that by successive substitutions the more important properties of the model can be obtained from the study of a single equation, giving an implicit relationship between the financial transfer FT and the relative prices of industrial goods and oil.

There are a number of determinants whose signs are important in the following, and which determine factor intensities in the different sectors. We have the following technical input-output coefficients:

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix}$$

in each region. The determinants to be used are:

$$D = a_1 c_2 - a_2 c_1, \quad M = c_1 b_2 - b_1 c_2, \quad Q = a_2 b_1 - a_1 b_2$$

The assumptions are:

$$D^N > 0, \quad D^S > 0, \quad M^S < 0, \quad Q^N < 0.$$

The positivity of the determinant D implies that the basic goods sector is relatively more labor intensive and the industrial goods sector relatively more capital intensive. The assumption (made above) that the basic goods sector uses very little capital in the South implies that c_1^S is small and therefore that $M^S < 0$. The industrial goods sector in the North was assumed to use little labor: hence a_2^N is small and $Q^N < 0$.

In order to solve the model we consider first the equation equating oil exported with oil imported:

$$X_\theta^S = M_\theta^N. \quad (4.21)$$

In view of (4.6), (4.9), and (4.6)', this equals

$$\theta_{S(FT)} - \theta_D = \theta_D^N, \quad (4.22)$$

where the left-hand-side variables are from the South. From (4.14), (4.12) and (4.13)

$$\begin{aligned} \theta_D &= \frac{b_1}{D} (c_2 L - a_2 K) + \frac{b_2}{D} (a_1 K - c_1 L) \\ &= -\frac{\alpha}{D} \frac{w}{p_B} M - \frac{\beta r}{D} Q, \end{aligned} \quad (4.23)$$

where

$$M = c_1 b_2 - b_1 c_2, \quad Q = a_2 b_1 - a_1 b_2.$$

Therefore, we may rewrite (4.22) as

$$\theta_{S(FT)} - \frac{\alpha}{D} \frac{w}{p_B} M - \frac{\beta r}{D} Q = \frac{a^N}{D^N} (w/p_B)^N M^N - \frac{\beta^N r^N Q^N}{D^N}. \quad (4.24)$$

Equation (4.24) is therefore an implicit equation in five variables, which we denote

$$\phi(\text{FT}, r, w/p_B, r^N, (w/p_B)^N) = 0 \quad (4.25)$$

Our next step is to write the rate of profit r and the wage w/p_B in the two regions as functions of the prices of basic and industrial goods, p_B and p_I . Recall that oil is the numeraire ($p_\theta = 1$). From the production functions (4.1) and (4.2) we obtain the associated competitive price equations

$$p_B = a_1 w + b_1 p_\theta + c_1 r, \quad p_I = a_2 w + b_2 p_\theta + c_2 r, \quad (4.26)$$

i.e.,

$$\begin{bmatrix} p_B - b_1 \\ p_I - b_2 \end{bmatrix} = \begin{bmatrix} a_1 & c_1 \\ a_2 & c_2 \end{bmatrix} \begin{bmatrix} w \\ r \end{bmatrix}$$

since $p_\theta = 1$. We therefore obtain the factor-commodity price relations

$$\begin{aligned} w &= \frac{c_2 p_B - c_1 p_I + M}{D} \\ w/p_B &= \frac{(p_B - b_1)c_2 + (b_2 - p_I)c_1}{D p_B} \\ r &= \frac{(b_1 - p_B)a_2 + a_1(p_I - b_2)}{D} = \frac{a_1 p_I - a_2 p_B + Q}{D} \end{aligned} \quad (4.27)$$

Substituting w/p_B and r from (4.27) into (4.25), we obtain a new implicit function, in four, rather than five, variables:

$$\psi(\text{FT}, p_I, p_B^N, p_B^S) = 0 \quad (4.28)$$

Recall that p_B^N may be different from p_B^S because B is not traded internationally. The last step is to substitute p_B^N and p_B^S as functions of p_I into (4.28). This will lead to an implicit function in two variables

$$\chi(\text{FT}, p_I) = 0 \quad (4.29)$$

Since FT is an exogenously given parameter, (4.29) is an analytic solution to the model: from (4.29) we may compute the equilibrium level of industrial prices $p_I^*(FT)$. It is easy to check that once p_I^* is known, we may solve for the equilibrium values of all other endogenous variables. This will be explained below.

Now, in order to obtain $p_B = p_B(p_I)$, we use another market-clearing condition, this time in the B-market:

$$B_S = B_D \quad (4.30)$$

From (4.12) and (4.13) this can be written as

$$\frac{c_2 L - a_2 K}{D} = \frac{w}{p_B} L \quad (4.31)$$

or

$$a \frac{c_2}{D} \frac{w}{p_B} - \frac{\beta a_2 r}{D} - a(w/p_B)^2 = 0$$

from which we obtain

$$\frac{w}{p_B} = \frac{c_2}{D} \pm \left(\frac{c_2^2}{4D^2} - \frac{\beta a_2 r}{Da} \right)^{1/2}, \quad (4.32)$$

a two-branched function relating w/p_B and r . The different parameter values will determine which is the appropriate branch in (4.32).

Using again the factor-commodity price relations, (4.32) yields an implicit relation between p_B and p_I as desired:

$$\frac{c_2}{aDp_B} \pm \frac{1}{p_B} \left(\frac{c_2^2}{4D^2} - \frac{\beta a_2}{D^2 a} (Q - a_2 p_B + a_1 p_I) \right)^{1/2} - \frac{M}{Dp_B} \frac{c_2}{D} + \frac{c_1 p_I}{Dp_B} = 0. \quad (4.33)$$

Substituting (4.33) into (4.28), we obtain the desired relation (4.29) between FT and p_I .

$$\chi(FT, p_I) = 0.$$

From (4.29) we may then compute $p_I^* = p_I^*(FT)$. From (4.33) we obtain $p_B^*(N)$ and $p_B^*(S)$, and from these three equilibrium prices we obtain the equilibrium rates of profit $r^*(N)$ and $r^*(S)$, and of real wages, $(w/p_B)^*(N)$ and $(w/p_B)^*(S)$. From these we obtain supply of labor and capital in the North and the South, and using the inversion of (4.32) and (4.33) we obtain the output of B and I in both regions. From the national income identity we may compute demand for I in the South, which determines imports from the North, and from exports of oil from the South. From (4.14) we obtain oil demanded in the South, thus completing the computation of the equilibrium.

B. Main Results: Trade and Debt

This section studies the impact of a change in the net transfer FT on the economies of the North and the South. Before going on to the algebra, it seems useful to explain the economics of this impact.

An increase in the transfer FT increases oil supplies θ_S , since the South invests borrowed funds in expanding the oil sector. At the new equilibrium, corresponding to higher FT , the total amount of oil utilized in the North and in the South therefore increases. This in turn alters the supplies of both goods in each region, possibly in different proportions. The composition of the product changes in both regions.

The changes in supplies lead to new equilibrium prices for the two goods. The prices of the factors labor and capital also change as relatively more or less labor and capital are employed. This implies that total income in the North and in the South are different at the new equilibrium. The results in this section give simple sufficient conditions for determining the signs of each of these effects.

The first theorem gives conditions under which an increase in oil supplies decreases the price of oil with respect to that of the industrial good. While it is intuitively plausible that the price of oil should drop as supplies increase, this is not always true. The second theorem gives conditions under which the relative price of oil increases as the transfer increases oil supplies. Whether one or the other result obtains depends on the relative strength of supply and demand effects,

and the general equilibrium solutions trace this in detail. The results are obtained from various assumptions on technologies and initial prices.

The next step is to explore the general equilibrium impacts of an increase in the relative price of industrial goods. The rate of profit rises both in the North and in the South. In the North, the rate of profit and the real wage move together, because the North's economy is rather homogeneous. Therefore, both wage and profit income increase in the North, and we show that there is also an increase in the consumption of both goods, even allowing for the loss of national income due to the transfer. All this occurs because the transfer has improved significantly the North's terms of trade.

In the South, because of the rather different technologies in the two sectors, the real wage moves in the opposite direction to the rate of profit. The transfer increases oil supplies and oil exports, but oil revenues in terms of industrial goods imported are reduced. Wage income and domestic consumption of basics decrease as well. If one sought to improve wage income without negatively affecting industrial consumption in the South, the economy of the South would have to be made more homogeneous.

The second theorem explores a different set of assumptions, and arrives at rather different conclusions. Now the transfer increases oil supplies, but it also increases the relative price of oil with respect to industrial goods. As the terms of trade of the South improve, its macro variables react differently, and so do the variables in the North. The conditions under which one or the other result obtains are therefore quite relevant for policy, and should be determined empirically. The simulations for the case of Mexico in Chichilnisky, Heal and McLeod (1984) are a first step in this direction.

A factor that plays an important role in determining the results of an increase in the transfer FT is the sign of the expression

$$\Delta = \left[c_2/D - 2w/p_B \right]$$

where D is the determinant of the matrix

$$\begin{bmatrix} a_1 & c_1 \\ a_2 & c_2 \end{bmatrix}.$$

The role and interpretation of this term have been discussed elsewhere (Chichilnisky, 1981a,b). Basically, the sign of this expression determines whether income effects will dominate price effects, so that increases in supplies will be proportionately larger or smaller than increases in demand as prices change. We refer to an economy as dual if $c_2/D < 2w/p_B$, since a large D would have this interpretation. Conversely, the economy is homogeneous if $c_2/D > 2w/p_B$. It should be noted that this condition can be written so as to be independent of the particular units of measurement used.

Theorem 7. Consider a North-South economy as defined above. Assume the economy of the North to be homogeneous ($c_2/D > 2w/p_B$) and that of the South to be dual ($c_2/D < 2w/p_B$). Suppose that at the initial equilibrium the price of industrial goods and the rate of profit are relatively high in the North ($p_I < b_2$ and $2r > a_1/D$). Labor is relatively abundant in the South (α large) and capital relatively abundant in the North (β large). In this case an increase in the transfer FT to the South has the following consequences:

- (i) Oil supplies and oil exports increase in the South.
- (ii) The North exports, and the South imports, fewer industrial goods. However, the terms of trade move in favor of the North (p_I increases) so much that its export revenues rise. There is a corresponding fall in oil export revenues of the South denominated in terms of its import I .
- (iii) Profits and real wages rise in the North, so much that its consumption of both goods increases.
- (iv) In the South, profits rise, but employment, real wages, and consumption of basics all fall.

Proof. We consider first the market-clearing condition in the oil market,

$$X_{\theta}^S = M_{\theta}^N . \quad (4.34)$$

From (4.6), (4.9), and (4.6)', this equals

$$\theta_{S(FT)}^S - \theta_D^S = \theta_D^N . \quad (4.35)$$

From (4.14),

$$\theta_D^S = b_1 B_S + b_2 I_S \quad (4.36)$$

and from inverting (4.12) and (4.13) we obtain

$$\theta_D^S = \frac{b_1}{D} (c_2 L - a_2 K) + \frac{b_2}{D} (a_1 K - c_1 L) . \quad (4.37)$$

In view of (4.3) and (4.4), we may rewrite (4.35):

$$\theta_{S(FT)}^S + \frac{\alpha}{D} \frac{w}{p_B} M + \frac{\beta r}{D} Q = - \frac{\alpha^N}{D^N} (w/p_B) M^N - \frac{\beta^N r^N}{D^N} Q \quad (4.38)$$

where M and Q are the determinants defined above. Equation (4.38) gives an implicit relation between real wages and the rates of profits in both regions, and the transfer FT , which we denote as

$$\phi[r^N, r^S, (w/p_B)^N, (w/p_B)^S, FT] = 0 . \quad (4.39)$$

Since factor prices are functions of commodity prices (see (4.27)), we obtain from the substitution of (4.27) into (4.38) a function linking the transfer FT to the prices of B and I :

$$\begin{aligned} \theta_{S(FT)}^S + \frac{\alpha^M}{D^2 p_B} (c_2 p_B^S - c_1 p_I + M) + \frac{\beta Q}{D^2} (p_I a_1 - p_B^S a_2 + Q) \\ + \frac{\alpha^N M^N}{(D^N)^2 p_B} (c_2 p_B^N - c_1 p_I + M^N) + \frac{\beta^N Q}{(D^N)^2} Q (p_I a_1 - p_B^S a_2 + Q) = 0 . \end{aligned} \quad (4.40)$$

Equation (4.40) is an implicit function of the form

$$\Gamma(\text{FT}, p_I, p_B^N, p_B^S) = 0 \quad .$$

However, the prices of basics p_B^S and p_B^N (which may be different since basics are not traded) are themselves functions of the price of industrial goods p_I in equilibrium.

From equation (4.33) we obtain:

$$p_B^N = p_B^N(p_I) \quad \text{and} \quad p_B^S = p_B^S(p_I) \quad .$$

Therefore, (4.40) is actually an implicit function of p_I and FT only

$$\Gamma(\text{FT}, p_I) = \Gamma(\text{FT}, p_I, p_B^S(p_I), p_B^N(p_I)) = 0 \quad . \quad (4.41)$$

It is then possible to differentiate implicitly across equilibria and obtain $\partial p_I / \partial \text{FT}$, or equivalently its reciprocal

$$\frac{d\text{FT}}{dp_I} = - \left[\frac{\partial \Gamma}{\partial p_I} \right] / \left[\frac{\partial \Gamma}{\partial \text{FT}} \right] \quad . \quad (4.42)$$

This equation represents the change in the price of industrial goods that follows an increase in the transfer FT. By (4.40) and (4.6)

$$\frac{d\Gamma}{d\text{FT}} = \frac{d\theta^S}{d\text{FT}} > 0 \quad .$$

Therefore the sign of (4.42) is always that of $-\partial \Gamma / \partial p_I$.

We may now compute the derivative $-\partial \Gamma / \partial p_I$. From (4.40) and (4.41) we obtain

$$\begin{aligned} -\frac{d\Gamma}{dp_I} = & -\frac{\partial p_B^S}{\partial p_I} \left(-\frac{\alpha M}{D^2 p_B^2} (M - c_1 p_I) - \frac{a_2 \beta Q}{D^2} \right) + \frac{\alpha c_1 M}{D^2 p_B} - \frac{\beta Q a_1}{D^2} \\ & -\frac{\partial p_B^N}{\partial p_I} \left(-\frac{\alpha^N M^N}{(D^N)^2 (p_B^N)^2} (M^N - c_1^N p_I) - \frac{a_2^N \beta^N Q^N}{(D^N)^2} \right) + \frac{\alpha^N c_1^N M^N}{(D^N)^2 p_B^N} - \frac{\beta^N Q^N a_1^N}{(D^N)^2} \end{aligned} \quad (4.43)$$

From expression (4.43) we may compute the changes in p_I as FT changes, provided we know the signs of the derivatives dp_B^S/dp_I and dp_B^N/dp_I across equilibria.

The next step is therefore to compute the signs of the derivatives of the price of basic goods p_B with respect to the price of industrial goods p_I across equilibria in each region. For this we utilize the expression relating the real wage and the rate of profit in each region, derived from the market-clearing condition $B_S - B_D = 0$:

$$\frac{\alpha c_2}{D} \frac{w}{p_B} - \frac{\beta a_2 r}{D} - \alpha (w/p_B)^2 = 0 \quad (4.44)$$

(see equation (4.31)), and also the equations relating factor prices to commodity prices:

$$r = \frac{p_I a_1 - p_B a_2 + Q}{D}, \quad w/p_B = \frac{p_B c_2 - p_I c_1 + M}{D p_B} \quad (4.45)$$

(see equation (4.27)). Equation (4.44) is an implicit expression between real wages and profits in each region, denoted $\Lambda(w/p_B, r) = 0$. Since equations (4.44) and (4.45) give real wages and profits as functions of commodity prices, (4.44) actually gives an implicit relation between commodity prices in each region, denoted

$$\psi(p_B, p_I) = \Lambda \left[\frac{w}{p_B}(p_B, p_I), r(p_B, p_I) \right] = 0 \quad (4.46)$$

From (4.46), by the implicit function theorem, in each region:

$$\begin{aligned} \frac{dp_B}{dp_I} &= - \left[\frac{\partial \psi}{\partial p_I} \right] / \left[\frac{\partial \psi}{\partial p_B} \right] \\ &= \left[- \frac{\partial \psi}{\partial (w/p_B)} \frac{\partial (w/p_B)}{\partial p_I} + \frac{\partial \psi}{\partial r} \frac{\partial r}{\partial p_I} \right] / \left[\frac{\partial \psi}{\partial (w/p_B)} \frac{\partial (w/p_B)}{\partial p_B} + \frac{\partial \psi}{\partial r} \frac{\partial r}{\partial p_B} \right] \end{aligned} \quad (4.47)$$

Furthermore, from (4.45) we find that the partial derivatives

$$\frac{\partial (w/p_B)}{\partial p_I} = - \frac{c_1}{D p_B} < 0 \quad (4.48)$$

$$\frac{\partial r}{\partial p_I} = \frac{a_1}{D} > 0 \quad (4.49)$$

$$\frac{\partial(w/p_B)}{\partial p_B} = \frac{p_I c_1 - M}{D p_B^2} > 0 \quad \text{when } p_I > b_2 \quad (4.50)$$

and

$$\frac{\partial r}{\partial p_B} = -\frac{a_2}{D} < 0 \quad (4.51)$$

We therefore obtain, from (4.47) and (4.51),

$$\frac{dp_B}{dp_I} = \left[\frac{c_1}{D p_B} \Delta + \frac{a_1 \beta a_2}{D^2} \right] / \left[\Delta \frac{(p_I c_1 - M)}{D p_B^2} + \frac{a_2^2 \beta}{D^2} \right] \quad (4.52)$$

where

$$\Delta = \alpha(c_2/d - 2w/p_B) \quad .$$

From relation (4.52) we may now determine the sign of dp_B/dp_I in both the North and the South. First note that dp_B/dp_I is always positive in the North since $p_I > b_2$, so that $p_I c_1 - M > 0$, and $\Delta > 0$ by assumption. In the South $\Delta < 0$, but β is rather small. Therefore, (4.52) is also positive in the South. With this information we may now return to equation (4.40) and compute $\partial \Gamma / \partial p_I$. As α is large in the South and β is large in the North, we have from (4.43) that the expression for $-\partial \Gamma / \partial p_I$ is dominated by the following terms:

$$\frac{\alpha M}{D^2 p_B^2} (M - c_1 p_I) \frac{dp_B^S}{dp_I} + \frac{\alpha c_1 M}{D^2 p_B^2} + \frac{dp_B^N}{dp_I} \frac{a_2^N \beta^N Q^N}{(D^N)^2} - \frac{\beta^N Q^N a_1^N}{(D^N)^2} \quad (4.53)$$

Here $M - c_1 p_I = c_1 b_2 - b_1 c_2 - c_1 p_I$ is negative as c_1 is small in the South. Hence the first term is positive (because $M^S < 0$) and dominates the second, which is multiplied by c_1 . As $Q^N < 0$, the third term is negative and the fourth positive. But a_2 is small in the North, so that the fourth term dominates. Hence we have that

$$-\frac{\partial \Gamma}{\partial p_I} > 0 .$$

Since $\partial \Gamma / \partial FT = d\theta^S / dFT > 0$, by (4.42) the price of industrial goods p_I rises as the transfer to the South increases, i.e.,

$$dp_I / dFT > 0 . \quad (4.54)$$

We next study the movements of the rate of return in the North r^N as p_I changes. From the national income identity

$$p_I I_D^N = rK - FT .$$

$$\text{As } I_D^N = I_S^N - X_I^N \text{ and } p_I X_I^N = X_\theta^S = \theta_D^N ,$$

$$p_I I_S^N = rK + \theta_D^N - FT .$$

In the North, β is large. We can therefore neglect terms other than those in β , giving

$$p_I = [(-Q/D) + r] / (a_1/D)$$

with

$$\frac{dp_I}{dr} = D/a_1 > 0 . \quad (4.55)$$

Hence as FT rises, p_I rises and the profit rate in the North r^N rises. Knowing how r^N moves enables us to find the sign of the change in the real wage in the North. By (4.31),

$$\frac{ac_2 w}{Dp_B} - \frac{\beta a_2 r}{D} - \alpha(w/p_B)^2 = 0$$

Implicit differentiation gives:

$$\frac{d(w/p_B)}{dr} = \frac{-a_2 \beta}{D\Delta} \quad (4.56)$$

where $\Delta = \alpha(c_2/D - 2w/p_B)$. As $\Delta < 0$ in the North by assumption, we have that

$$\frac{d(w/p_B)}{dr} > 0 \quad (4.57)$$

in the North. Hence an increase in FT raises the real wage in the North, as well as the profit rate. The next step is to show that the consumption levels of B and I rise in the North.

$$I_D^N = rK - FT = \beta r^2 - FT, \quad \frac{\partial I_D^N}{\partial FT} = 2\beta r \frac{\partial r}{\partial FT} - 1 \quad (4.58)$$

which is positive for large β . Also,

$$B_D^N = wL/p_B = \alpha(w/p_B)^2 \quad (4.59)$$

so that B_D^N also rises with FT by (4.57), (4.54), and (4.55). We have now proven point (iii) of Theorem 7.

Next we study the response of trade patterns to FT. By inverting (4.12) and (4.13),

$$X_I^N = I_S^N - I_D^N = \frac{a_1 K}{D} - \frac{c_1 L}{D} - rK + FT.$$

From (4.3) and (4.4)

$$X_I^N = \frac{a_1}{D} \beta r - \frac{c_1 \alpha w}{D p_B} - \beta r^2 + FT.$$

Hence

$$\frac{dX_I^N}{dr} = \beta \left(\frac{a_1}{D} - 2r \right) - \alpha \frac{c_1}{D} \frac{d(w/p_B)}{dr} + \frac{dFT}{dp_I} \frac{dp_I}{dr}.$$

By the conditions of the theorem, the first term is negative. By (4.57) the second term is negative, and by (4.56) it contains β . As β is large, these terms dominate, and therefore

$$dX_I^N/dr < 0, \quad (4.60)$$

i.e., the North's exports of the industrial good fall as FT and hence r^N rise. This

implies, of course, that the South's imports of industrial goods fall,

$$dM_I^S/dr^N < 0. \quad (4.61)$$

We next check what happens to the volume of oil traded. This equals oil demanded in the North, θ_D^N , which from (4.23) is

$$-\frac{\alpha w M}{D p_B} - \frac{\beta r Q}{D}.$$

Here β is large and Q is negative, by assumption. r rises, by (4.55). Hence

$$\frac{d\theta_D^N}{dFT} = \frac{dX_\theta^S}{dFT} > 0. \quad (4.62)$$

This proves points (i) and (ii) of Theorem 7.

What remains is to study the behavior of the Southern economy. We first show that r^S rises with FT. This is done by showing that $dM_I^S/dr^S < 0$. As

$$\frac{dM_I^S}{dr^S} = \frac{dM_I^S}{dr^N} \frac{dr^N}{dr^S},$$

this will imply from (4.61) that $dr^N/dr^S > 0$, which in conjunction with (4.54) and (4.55) gives $dr^S/dFT > 0$. From this last inequality the results follow. We therefore compute now the sign of dM_I^S/dr^S .

$$M_I^S = I_D^S - I_S^S = rK + \theta_S + FT - I_S^S = \beta r^2 - \frac{\beta r a_1}{D} + \frac{c_1 \alpha w}{D p_B} + \theta_S + FT$$

$$\frac{dM_I^S}{dr^S} = \beta(2r - a_1/D) + (c_1 \alpha/D) \left(\frac{\partial(w/p_B)}{\partial r} \right) + \left(\frac{d\theta_S}{dM_I^S} + \frac{dFT}{dM_I^S} \right) \frac{dM_I^S}{dr^S},$$

$$\text{i.e.,} \quad \frac{dM_I^S}{dr^S} \left(1 - \frac{d\theta_S}{dM_I^S} - \frac{dFT}{dM_I^S} \right) = \beta(2r - a_1/D) + c_1 \alpha/D \left(\frac{\partial(w/p_B)}{\partial r} \right).$$

Now

$$\frac{d\theta_S}{dM_I^S} = \frac{d\theta_S}{dr^N} \frac{dr^N}{dM_I^S} < 0$$

by (4.61), (4.54), (4.55) and (4.6). Similarly, $dFT/dM_I^S < 0$. By (4.56), $d(w/p_B)/dr < 0$ in the South. As by assumption α^S is large, this establishes that

$$dM_I^S/dr^S < 0 \quad \text{so that} \quad dr^S/dFT > 0. \quad (4.63)$$

It now follows from (4.56) and the fact that $\Delta^S < 0$ by assumption, that real wages in the South fall with FT. It follows immediately from (4.3) and (4.5) that employment and the consumption of basics also fall. This completes the proof of Theorem 7. ■

Theorem 8. Assume that $M^S > 0$, i.e., $c_1 b_2 - b_1 c_2 > 0$ in the South. Let p_B be small and $p_I > b_2^S$ at the initial equilibrium, with all other conditions as in Theorem 7. Then an increase in the financial transfer FT to the South has the opposite effects to those established in Theorem 7: it leads to a fall in the price of the industrial good p_I and a relative increase in the price of oil p_O , even though oil supplies have increased. The oil exporter's terms of trade therefore improve. In addition, oil exports and the rate of profit in the South decrease. The North exports more industrial goods. Real wages, employment, and consumption of basics increase in the South. In the North, the rate of profit and the real wage decrease.

Proof. As in the proof of Theorem 7, the sign of dFT/dp_I equals that of $-d\Gamma/dp_I$. This is given in equation (4.43), or approximately in (4.53). The latter may also be written as

$$\frac{\alpha M}{D^2 p_B} \left(\frac{dp_B}{dp_I} (M - c_1 p_I) \frac{1}{p_B} + c_1 \right) + \frac{\beta^N Q^N}{(D^N)^2} \left(\frac{dp_B^N}{dp_I} a_2^N - a_1^N \right), \quad (4.64)$$

where parameters and variables are of the South unless otherwise indicated. Now note from (4.52) that for large β^N ,

$$dp_B/dp_I \approx a_1/a_2.$$

Hence the second term in (4.64) is close to zero and (4.64) can be expressed as

$$\frac{\alpha M}{D^2 p_B} \left[\frac{dp_B}{dp_I} \frac{c_1}{p_B} (b_2 - p_I) - \frac{dp_B}{dp_I} \frac{c_2 b_1}{p_B} + c_1 \right]. \quad (4.65)$$

Under the conditions of Theorem 8 and since $dp_B/dp_I > 0$ by (4.52), (4.65) is negative, proving that the oil exporter's terms of trade improve, i.e., p_I falls with FT.

The rest of the theorem follows immediately. Inequality (4.55) implies that the profit rate in the North falls, and (4.56) implies that real wages in the North fall. Inequality (4.60) tells us that the North's exports (and the South's imports) of industrial goods will increase, and from (4.62) we then know that oil exports of the South fall. Equation (4.63) establishes that the rate of profit in the South falls, and using (4.56) again proves that real wages, employment, and consumption of basic goods all rise in the South. This completes the proof. ■

Theorems 7 and 8 give conditions for opposite effects of a financial transfer FT. The main difference in the conditions of Theorems 7 and 8 are: the sign of the determinant M^S , and the impact that the transfer has on the relative price of industrial goods p_I . The sign of M^S is positive in Theorem 8, and negative in Theorem 7. It seems more plausible that M^S should be negative, since this happens when the basic goods sector in the South uses few capital inputs. Theorem 8 assumes, instead, that the basic goods sector is more capital intensive. The impact of the transfer on prices seems also more plausible in Theorem 7: transfers which are used to develop oil supplies are likely to lead to lower, rather than higher, oil prices. We therefore should expect the relative price of industrial goods to increase. In Theorem 8, the transfer also increases oil supplies, but this leads to higher oil prices, an outcome which appears less plausible.

Clearly, an empirical analysis of the actual conditions is needed to ascertain which set of conditions is applicable. A priori, however, the conditions in Theorem 7 appear more intuitively natural than those in Theorem 8.

C. Conclusions

An inflow of foreign capital, whether used for consumption or investment, inevitably affects the internal equilibrium of the receiving country. Consumption

patterns, production patterns, and prices all change. The same is true of the lending country: it changes its consumption pattern by making a loan, and for this reason, and because the equilibrium of its trading partner changes, its own domestic equilibrium alters. A crucial factor in determining these macro effects of a loan is the change in relative prices (oil prices, industrial prices, and prices of basic goods that are not traded). A loan must be one of significant size before having a measurable impact on prices, and for the case of Mexico discussed in Chichilnisky, Heal and McLeod (1984), where the transfer is of the order of 100 billion US dollars, it certainly fits this description.

It is clear, then, that it is a complex matter to trace the full impacts of a loan from one trading country to another. The model has enabled us to identify these impacts in a rather simple fashion, because of our somewhat stylized assumptions, and to assess the gains and the losses arising from such a loan for different groups within the lending and borrowing countries.

One important feature to emerge is that the loan may have a beneficial effect on the equilibrium of the lending country. This happens when the borrowed funds are used to increase oil supplies, leading to more abundant oil, increased oil exports, and lower oil prices. The terms of trade of the lending country improve, and this leads to higher levels of consumption of both goods in the lending country. Theorem 7 establishes the conditions under which the welfare level in the lending country will rise as a result. In making a social cost-benefit analysis of such a loan, this is a point that should clearly be considered; there is a social return to the loan over and above the rate of interest paid on it. It is possible that even if a major rescheduling that delayed repayment were to happen, the lending country as a whole could nevertheless benefit.

Similar issues apply to the receiving country. The borrowing sector may benefit in commercial terms from the loan, but a social cost-benefit analysis of the loan should also take into account its effects on the overall economic equilibrium.

As Theorem 7 shows, these could be substantially negative. If there has been overspecialization in one sector, thus leading to lower terms of trade for the country, with correspondingly negative welfare effects. In summary, the fact that a loan, if large, may affect the equilibrium pattern of prices and quantities in both countries means that it will have macroeconomic consequences going far beyond its impacts on the profits of the borrowing and lending institutions.

Theorems 7 and 8 have indicated two very different possible outcomes. In one case, the effects are beneficial to the lending and harmful to the borrowing country, while in the other case the opposite is true. The distinguishing feature is the effect of the loan on the oil exporter's terms of trade. In the first case, they worsen, and in the second, they improve. Which of these two outcomes occurs depends on the patterns of factor intensities in the receiving country and the initial price levels. Once these are known, thus establishing whether the loan improves or worsens the receiver's terms of trade, everything else can be traced. Experience indicates that over the last three years, the terms of trade of oil exporters have worsened. While many factors have contributed to this price movement, this suggests that a policy of borrowing to invest in the oil sector might not have been the most favorable to the oil exporter. However, such a policy could be favorable to the lender; it yields more oil at lower prices. Such macro outcomes should be computed when discussing the present situation. The calculus of the debt must go beyond the financial aspects, and must include the macroeconomic effects on prices, imports, and exports of both countries.

It is important to emphasize that we have studied the consequences of granting a loan before this was repaid. The repayments will not have effects that are simply equal and opposite to those of the granting of the loan. The asymmetry arises because, when the loan is made, it is invested or consumed in sectors different than those that will pay the debt. For instance, in this paper the debt was used to build up the production capacity of the oil sector. However, when the loan is repaid, this will not, of course, coincide with running down this capacity. Investment is irreversible, and capital stock and machines depreciate. The loan will be

repaid by running a balance-of-trade surplus. The effects of running a trade surplus at a constant capacity level in the oil sector are not the opposite of those of running a trade deficit and using the capital inflow to expand oil-producing capacity. As a matter of fact, both could affect the major macro variables in the same direction.

Finally, we point out a connection between the problem that we have studied here and the extensive literature on the transfer problem in international economics. This literature is concerned with the possibility that a transfer of resources from one agent or country to another may benefit the donor and harm the recipient. Chichilnisky (1980) showed for the first time that such an outcome can occur in perfectly competitive markets with a unique and globally stable market equilibrium. This issue has so far been studied only in the context of a barter economy without production in the case of perfectly competitive general equilibrium models. For surveys of these results, see Chichilnisky (1980), Jones (1983), and Geanakoplos and Heal (1983). Our present Theorem 7 provides an example of the transfer paradox in a production economy: resources are transferred from lender to borrower, and the lender gains as a result, even though the receiver expands its production capacity.

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