How to deflate rigorously Economic Variables?

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Technical Note

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This brief technical note presents a judicious way to deflate rigorously economic variables by dissociating variations related to market fluctuations (or supply and demand) from monetary phenomenon driven by monetary policies. Below are two graphical examples that show how a standard way to deflate (using price indexes) can lead to biased estimations when measuring policies impacts. The text following next explains how to proceed technically for more accurate measures of price variations.

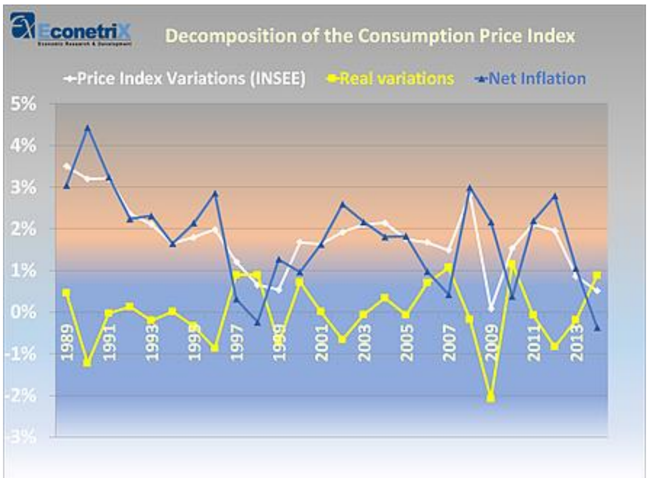

Figure 1. Consumption Price Variations

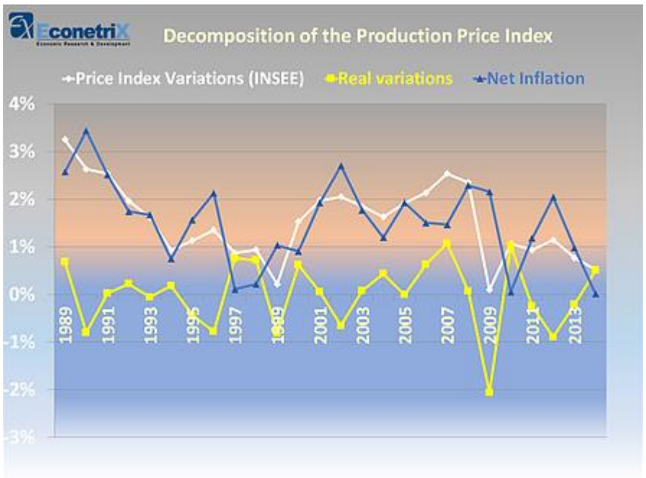

Figure 2. Production Price Variations

Using standard price indexes (for example from the INSEE) to deflate economic variables implies to remove variations represented by the white lines on both graphics. Those price variations result actually from two effects: supply and demand fluctuations (or market effects), represented by yellow lines, and pure monetary phenomenon related to monetary policies, represented by the blue lines. Clearly, variations of the white and blue lines can differ significantly sometimes, noticing that on average, the standard white line appears smoother than the blue one. A way to dissociate the two separate effects is presented below.

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Technical Explanations:

The following method is derived from our modelling tool.

Suppose for example the production function of a representative sector *j* is characterized by:

$$\begin{cases} Y_j = \dfrac{1}{\gamma_j} F(K_j, L_j) & (1) \\ K_j = \displaystyle\sum_{i=1}^{J} \gamma_i X_{ij} & (2) \\ X_{ij} = \mu_{ij} X_{1j} \qquad \forall i,j \in J & (3) \end{cases}$$

where γ_j is a sector-specific productivity parameter, K_j is an aggregate measure of capital calculated as a linear combination of the sectoral goods produced (X_{ij}) with for coefficients the sector-specific parameters γ_j, L_j is the labor quantity employed by sector *j*, and μ_{ij} are positive parameters that can be equal to zero. The function *F(.)* is HOD 1 and common to each sector. In other words, the technology assumed differs only on the scale parameter $1/\gamma_j$ and on a particular technical combination of factor goods that are perfect complement in this example (μ_{ij} are indeed defined as simple parameters).

Suppose business owners of sector *j* borrow a monetary capital M_j (to a non-profit bank) against a gross interest rate r in order to buy the capital goods required X_{ij} at their market prices p_i, that is $M_j = \sum_i p_i X_{ij}$. The amount received by sellers is saved in turn as long term deposits against a net interest rate $r' = r - \delta$ (where δ denotes a same depreciation rate for all goods and sectors). Suppose each sector remunerates labor a nominal wage w. The profit maximizing program of a sector is:

$$\underset{X_j L_j}{\text{Max}}\, \Pi_j = p_j \frac{1}{\gamma_j} F(K_j, L_j) - r \sum\nolimits_i p_i X_{ij} - w L_j \qquad (4)$$

By identification, we easily notice in that context that a possible set of equilibrium prices under perfect competition is:

$$\mathrm{p}_j^* = \gamma_j \qquad (5)$$

We get indeed that $\mathrm{M_j} = \mathrm{K_j}$, and that first order conditions simplify to standard ones; $F_K(K_j, L_j) = r$ and $F_L(K_j, L_j) = w$. By Euler's homogenous function theorem, we have:

$$r^* K_j + w^* L_j = F(K_j, L_j) = \gamma_j Y_j \qquad (6)$$

meaning that unit (and marginal) production costs are simply given by γ_j. We have also that $\mathrm{F_K}$ and $\mathrm{F_L}$ are homogenous of degree zero, which implies that each sector can produce its own level of output while satisfying equilibrium conditions. It implies also a simple aggregation rule over all sectors:

$$\sum_{j=1}^{J} r^* K_j + w^* L_j = \sum_{j=1}^{J} \lambda_j (r^* K + w^* L) = F(K, L) = \sum_{j=1}^{J} \gamma_j Y_j = P^Y Y \tag{7}$$

where $\lambda_j \in (0,1)$ are scale parameters that verify $\sum_{j=1}^{J} \lambda_j = 1$, K is the aggregate measure of all capital in the economy, that is, $K = \sum_{j=1}^{J} K_j = \sum_j \sum_i \gamma_i X_{ij} = \sum_j \sum_i p_i X_{ij} = \sum_{j=1}^{J} M_j$, and L is the aggregate level of labor defined as $L = \sum_{j=1}^{J} L_j$. We also let $P^Y Y$ be the aggregate monetary amount of output in the economy (or the theoretical counterpart of GDP) and M be the aggregate monetary capital borrowed by all sectors to the bank.

From a practical viewpoint, consider now that firms apply a specific margin rate to the marginal cost (as it is shown to be the case for most of European firms by Fabiani et al. (2006)):

$$p_j = (1 + \pi_j)\gamma_j \tag{8}$$

This assumption has slight implications on the model. The profit maximizing equation (4) becomes:

$$\underset{X_j L_j}{Max}\, \Pi_j = p_j \frac{1}{\gamma_j} F(K_j, L_j) - r^*(1 + \pi_{M_j}) \sum_i \gamma_i X_{ij} - w^* L_j \tag{9}$$

where π_{M_j} corresponds to an average profit given by $\pi_{M_j} = M_j / K_j$. Conditions become : $\partial F / \partial K_j = r^*. (1 + \pi_{M_j})/(1 + \pi_j)$ and $\partial F / \partial L_j = w^*/(1 + \pi_j), \forall j = 1,2,3$. Total factor cost is still given by $\gamma_j Y_j$ but total amount of output (or GDP) is now given by $P^Y Y = \sum_{j=1}^{J} (1 + \pi_j)\gamma_j Y_j$.

The empirical indicator "EBITDA" corresponds to the sum of capital and profit remunerations which is given here by:

$$r. \frac{1 + \pi_{M_j}}{1 + \pi_j}. K_j + \pi_j \gamma_j Y_j = (\alpha + \pi_j). \gamma_j Y_j \tag{10}$$

and the empirical ratio "EBITDA/Value Added" corresponds theoretically to $\frac{\alpha + \pi_j}{1 + \pi_j}$. Letting α be 0.3 for example, and normalizing γ_j to 1 (because knowing real quantities of goods across sectors is not necessary), we can easily deduce π_j and p_j. We obtain therefore measures of prices and markups reflecting real market fluctuations (i.e.; variations of those indicators are driven by supply and demand only).

References

Fabiani et al. (2006): “The Pricing Behaviour of Firms in the Euro Area: New Survey Evidence”, **Working Paper** · November 2005