Bubbles and Bluffs: Risk Lovers Can Survive Economically

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Risk Lovers Can Survive Economically

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Abstract

In economics, risk lovers have been generally ignored, most likely because it has generally been thought that they cannot survive economically. In this paper, I examine the possibility that risk lovers can exist continuously in the framework of an economic growth model. A bubble-like phenomenon (a so-called "bubble economy") can be generated if risk lovers undertake a very risky financial "bluff"—for example, if they purposely raise some important asset prices. I conclude that because risk-loving and risk-averse households can coexist at a state of sustainable heterogeneity, risk lovers can exist continuously in an economy. Therefore, it is likely that a bluff will be undertaken by risk lovers and a bubble-like phenomenon can be generated.

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Keywords: Risk lover; Bluff; Bubble; Sustainable heterogeneity; Risk averse

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1 INTRODUCTION

Theoretically, a rational bubble cannot exist if agents have infinite horizons (Blanchard and Watson, 1982; Santos and Woodford, 1997). An assumption of some kind of irrationality may therefore be necessary to explain the existence of bubble-like phenomena (so-called "bubble economies") that have actually been observed in many countries. On the other hand, Harashima (2015) showed that a bubble-like phenomenon can be generated rationally if a financial "bluff" is undertaken—that is, if private information about the rate of time preference of the representative household (RTP RH) is utilized. A bluff provides a tremendous payoff to the bluffer if it succeeds, but it is very risky. Hence, ordinary risk-averse people will not bluff. Risk lovers are needed for a bluff to be undertaken.

In conventional economic growth models, however, if the relative degree of risk aversion of a household is negative (i.e., the household is a risk lover), its consumption steadily declines to zero, and the household ceases to exist. Hence, the common belief has been that even if risk lovers exist at some point, they will not exist continuously. As a result, there have been few economic studies of risk lovers, and the economic influences of risk lovers have generally been ignored. Exceptionally, Crainich et al. (2013), Jindapon (2013), and Jindapon and Whaley (2015) explicitly studied risk lovers. Jindapon and Whaley (2015) showed that an equilibrium exists in a rent-seeking game when players are risk loving and imprudent, but this result is generated only in some rent-seeking games. Crainich et al. (2013) showed that mixed risk lovers are prudent in the framework of a binary lottery, but Ebert (2013) criticized the study and said that a more careful treatment of the issue was needed. The common problem of these studies is that risk lovers are studied in very narrow environments, such as rent-seeking games or a binary lottery. In this paper, the possibility that risk lovers can exist continuously in an economy is examined in the framework of an economic growth model.

Here, the existence of risk lovers is examined on the basis of the concept of sustainable heterogeneity shown by Harashima (2010, 2017) and is defined as the state at which all the optimality conditions of all heterogeneous households are indefinitely satisfied. Sustainable heterogeneity therefore means an economically harmonious coexistence of heterogeneous households. If risk-loving and risk-averse households can coexist at this state, risk lovers can exist continuously and therefore can bluff. I examine this possibility and conclude that risk lovers can exist indefinitely at the state of sustainable heterogeneity, and therefore, it is likely that a bluff will be undertaken by risk lovers and a bubble-like phenomenon can be generated.

2 BUBBLES, BLUFFS, AND RISK LOVERS

2.1 The bluff mechanism

2.1.1 Bubbles and bluffs

Harashima (2014a) showed that each household must expect RTP RH *ex ante* for it to behave optimally. Nevertheless, a household cannot directly observe the intrinsic RTP RH although it knows its own intrinsic rate of time preference, and therefore the expected RTP RH has to be generated. As a result, the economy is subject not to the intrinsic RTP RH but to the expected RTP RH. However, there is a problem with this approach. Even if the expected RTP RH is actually very different from the intrinsic RTP RH, the economy will appear to proceed quite "normally" for an indefinite period of time without any inconsistencies among observed economic indicators. Hence, the observed economic indicators alone cannot tell us whether the expected RTP RH is truly identical to the intrinsic RTP RH or whether or not the current economy is in a bubble-like state. Because of this nature, the expected RTP RH is intrinsically fragile (see Harashima, 2014b).

The fragility of the expected RTP RH indicates that there is room for an agent (probably a malicious one) to manipulate the expected RTP RH, for example, by intentionally disseminating misleading information. By manipulating the expected RTP RH, the economy can also be influenced. For example, the expected RTP RH may be manipulated by injecting huge amounts of money into financial markets and purposely raising some important asset prices, and if the manipulation succeeds, a bubble-like phenomenon may be generated. I call this manipulation of the expected RTP RH a "bluff." More correctly, a bluff herein is defined as the behavior of an agent who pretends to possess information (which is actually false) that the intrinsic RTP RH has changed. If the expected RTP RH is manipulated well (i.e., the bluff succeeds), the bluffer can obtain a huge payout.

2.1.2 The bluff model

This section presents a brief explanation of Harashima's (2015) bluff model, which is used as a basis for the examination in this paper. Suppose for simplicity that all bluffers are identical and, therefore, bluffers' actions (bluffs) are represented by a representative bluffer (hereafter, "the bluffer"). Let $\overline{\pi}$ be the subjective payoffs of a bluff to the bluffer if the expected RTP RH is successfully manipulated. Let p ($0 \leq p \leq 1$) be the probability that, after observing the information the bluffer disseminated, households decide that the expected RTP RH has changed. Because bluffs are very risky, p is very low. If households do not believe the information and do not change the expected RTP RH, the bluff fails and the bluffer suffers the loss $-\underline{\pi}$ where $0 < \underline{\pi} < \overline{\pi}$. It is assumed for simplicity that $\overline{\pi}$ is identical for any bluff and $\underline{\pi}$ is also identical for any bluff.

The expected payoff to the bluffer (Π) for a bluff is therefore

$$\begin{aligned} \Pi &= p\overline{\pi} + (1-p)(-\underline{\pi}) \\ &= p(\overline{\pi} + \underline{\pi}) - \underline{\pi} \end{aligned} \tag{1}$$

Equation (1) indicates that, even if p is very low, $\Pi > 0$ if $\overline{\pi}$ is sufficiently large. The variance of Π, σ^2, is

$$\sigma^2 = (p - p^2)(\overline{\pi} + \underline{\pi})^2 \quad . \tag{2}$$

Because $0 \leq p \leq 1$, then $p - p^2 \geq 0$, $\sigma^2 \geq 0$,

$$\frac{d\sigma^2}{d\overline{\pi}} = (p - p^2)2(\overline{\pi} + \underline{\pi}) > 0 \quad ,$$

and

$$\frac{d^2\sigma^2}{d\overline{\pi}^2} = 2(p - p^2) > 0 \quad .$$

In addition, because $0 \leq p \leq 1$,

$$\frac{d\sigma^2}{d\Pi} = 2(1-p)(\overline{\pi} + \underline{\pi}) > 0$$

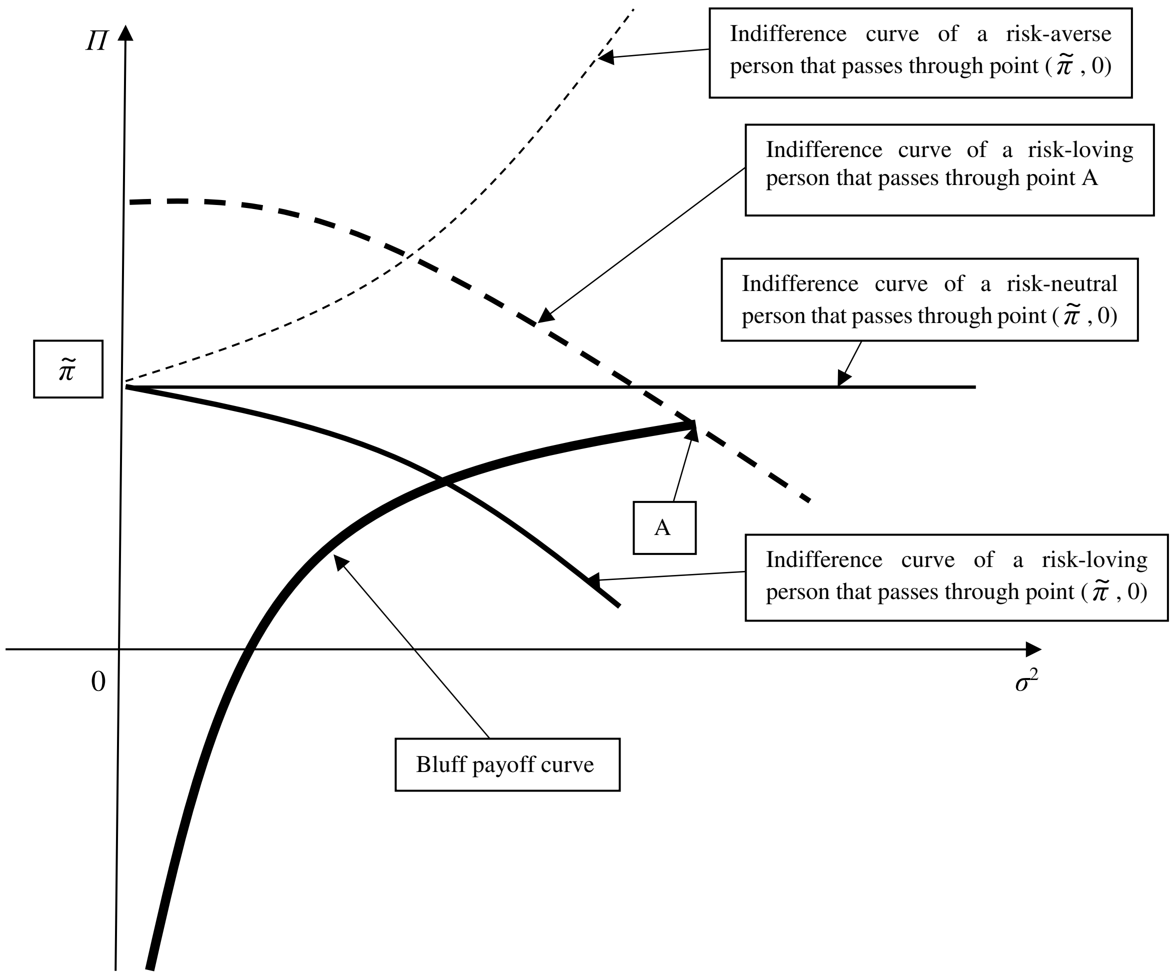
and

$$\frac{d^2\sigma^2}{d\Pi^2} = 2(p^{-1} - 1) > 0 \quad .$$

2.2 *Need for risk lovers*

Because p is very low, bluffs are very risky. By equations (1) and (2), a bluff payoff curve can be drawn as the bold line on the Π–σ^2 plane in Figure 1. Here, it is assumed for simplicity that, in every period, the chances of a bluff being undertaken at any point on the payoff curve occur randomly. The payoff curve breaks off at point A because p is very low and a higher σ^2 indicates a higher p if p is very low, as indicated by equation (2). This means that all bluffs have a very low p. Suppose for simplicity that investment opportunities are limited to either a bluff or a risk-free asset. Let $\tilde{\pi}$ be the payoffs of a risk-free asset. Suppose also that the value of Π at any point on the payoff curve is smaller than $\tilde{\pi}$ because a bluff is very risky and p is very low, even though $\tilde{\pi}$ is large.

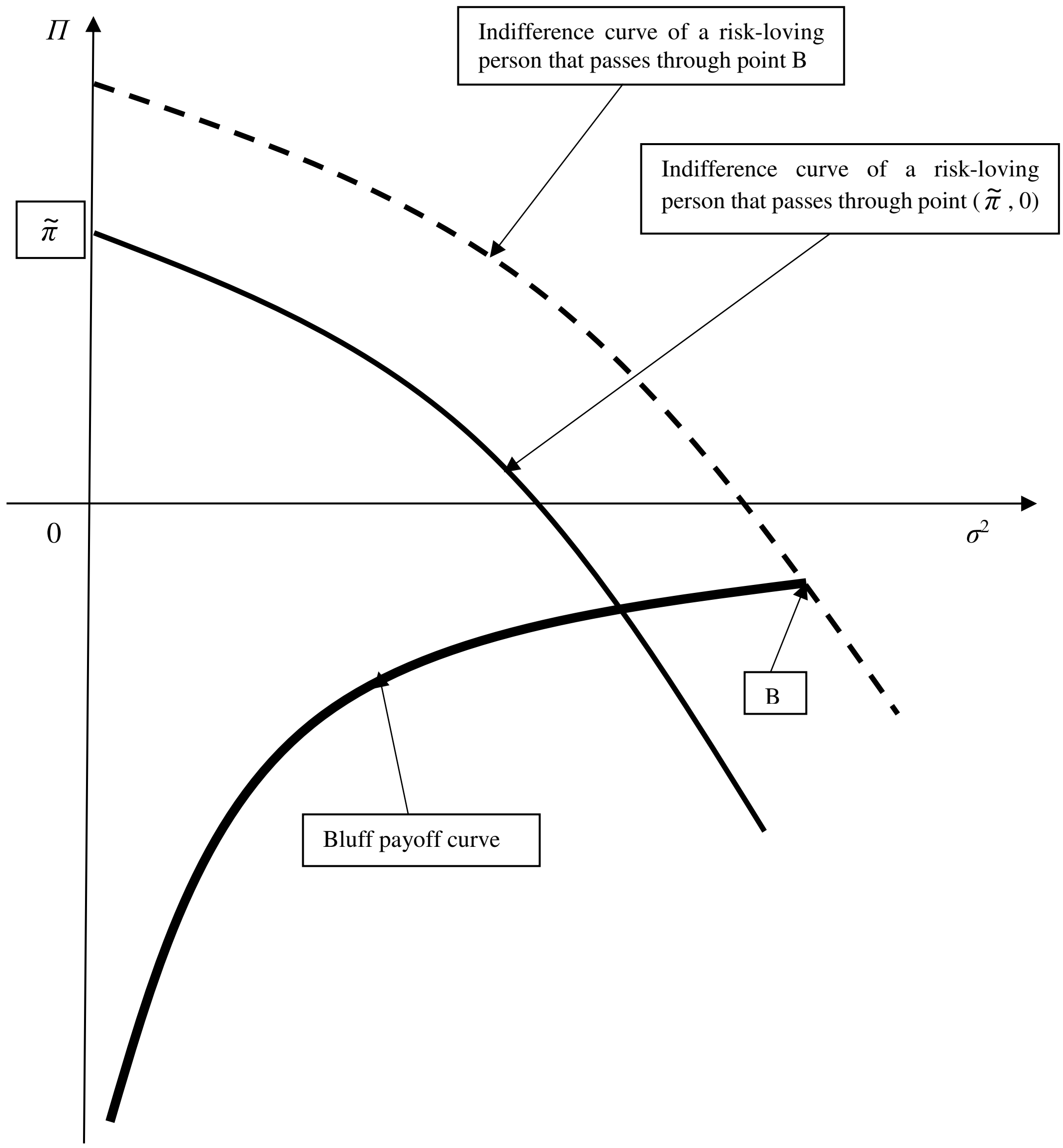
Figure 1: Payoff curve for a bluff undertaken by a risk lover



The other curves in Figure 1 indicate the indifference curves of risk-averse, risk-neutral, and risk-loving persons. Three of the curves pass through point ($\tilde{\pi}$, 0) where point (x, y) indicates the point for $\Pi = x$ and $\sigma^2 = y$. The indifference curves of risk-averse and risk-neutral persons that pass through point ($\tilde{\pi}$, 0) do not intersect the payoff curve, and therefore those persons will never undertake a bluff and instead invest in the risk-free asset. On the other hand, the indifference curve of a risk-loving person that passes through point ($\tilde{\pi}$, 0) intersects the payoff curve, so the risk-loving person will undertake a bluff. Furthermore, the indifference curve that passes through point A indicates a higher level of utility than the indifference curve that passes through point ($\tilde{\pi}$, 0) for the risk-loving person, and thereby the bluff corresponding to point A will be undertaken by the risk-loving person.

An important point is that the payoff (Π) at point A is lower than $\tilde{\pi}$. This means that even if the bluff payoff is lower than that of a risk-free asset, a risk-loving person still undertakes the bluff. Furthermore, even if the expected payoff of the bluff is negative, some risk lovers will undertake bluffs because the bluffs still give them a higher level of utility than investing in a risk-free asset. This situation is shown in Figure 2, in which point B gives a higher utility to the risk-loving person than point ($\tilde{\pi}$, 0). The reason for this behavior is that risk lovers enjoy risk and obtain utility from risk itself. For some risk lovers, the thrill and enthusiasm they feel from taking a risk overwhelms any worries about likely negative consequences. Therefore, if risk lovers exist, they will bluff even though it is very risky, and furthermore, even if its expected payoffs are negative.

Figure 2: A bluff with negative payoffs undertaken by a risk lover



2.3 Can risk lovers survive economically?

As shown in Section 2.2, for a bluff to be undertaken, risk lovers must exist in an economy; moreover, a sufficiently large number of risk lovers is necessary. A bluff requires a large amount of funds, and a single risk lover's funds will usually be limited. The funds of many risk lovers may have to be pooled consciously or unconsciously to make a bluff succeed. This means that a sufficiently large number of risk lovers needs to exist for a bluff to be undertaken. However, as noted in the Introduction, conventional economic theory teaches us that consumption by risk lovers will steadily decline to zero and that they will disappear from the economy. How, then, can so many risk lovers exist continuously in an economy?

3 SUSTAINABLE HETEROGENEITY

In this section, before examining the survivability of risk lovers, I briefly explain Harashima's (2010, 2017) concept of sustainable heterogeneity. Sustainable heterogeneity is defined as the state at which all the optimality conditions of all heterogeneous households are satisfied indefinitely. If sustainable heterogeneity is achieved, risk-averse and risk-loving households can coexist indefinitely, with both satisfying all of their optimality conditions.

3.1 The model

Three heterogeneities—time preference, risk aversion, and productivity—are considered. First, suppose that there are two economies—economy 1 and economy 2—that are identical except for time preference, risk aversion, or productivity. The economies are fully open to each other, and goods, services, and capital are freely transacted between them, but labor is immobilized in each economy. Each economy can be interpreted as representing either a country (the international interpretation) or a group of identical households in a country (the national interpretation). Usually, the concept of the balance of payments is used only for international transactions. However, because both national and international interpretations are possible, this concept and the related terminology are also used for the national models presented in this paper.

The degree of risk aversion (DRA) of economy 1 is $\varepsilon_1 = -\frac{c_{1,t}\, u_1''}{u_1'}$ and that of economy 2 is $\varepsilon_2 = -\frac{c_{2,t}\, u_2''}{u_2'}$, both of which are constant, where u_i and $c_{i,t}$, respectively, are the utility function and per capita consumption of economy i (i = 1, 2) in period t. The rate of time preference (RTP) of economy 1 is θ_1 and that in economy 2 is θ_2. The production function of economy 1 is $y_{1,t} = \omega_1^{\alpha} A_t^{\alpha} f(k_{1,t})$ and that of economy 2 is $y_{2,t} = \omega_2^{\alpha} A_t^{\alpha} f(k_{2,t})$, where $y_{i,t}$ and $k_{i,t}$, respectively, are per capita output and capital of economy i in period t. A_t is technology in period t, α is a constant ($0 < \alpha < 1$), and $\omega_1\ (0 < \omega_1 \le 1)$ and $\omega_2\ (0 < \omega_2 \le 1)$ are constants. The production functions are further specified as $y_{i,t} = \omega_i^{\alpha} A_t^{\alpha} k_{i,t}^{1-\alpha}$. The current account balance in economy 1 is τ_t and that in economy 2 is $-\tau_t$. The accumulated current account balance $\int_0^t \tau_s ds$ mirrors capital flows between the two economies. The economy with current account surpluses invests them in the other economy. Because the current account balance mirrors capital flows between the economies, the balance is a function of capital in both economies, such that $\tau_t = g(k_{1,t}, k_{2,t})$.

The representative household in economy 1 maximizes its expected utility

$$E\int_0^{\infty} u_1(c_{1,t})\exp(-\theta_1 t)dt \quad ,$$

subject to

$$\dot{k}_{1,t} = y_{1,t} + \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s \, ds - \tau_t - c_{1,t} - v\dot{A}_{1,t} \left(\frac{L_t}{2} \right)^{-1} \quad , \tag{3}$$

and that in economy 2 maximizes its expected utility

$$E \int_0^\infty u_2(c_{2,t}) \exp(-\theta_2 t) dt \quad ,$$

subject to

$$\dot{k}_{2,t} = y_{2,t} - \frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds + \tau_t - c_{2,t} - v\dot{A}_{2,t} \left(\frac{L_t}{2} \right)^{-1} \quad , \tag{4}$$

where E is the expectation operator, v (> 0) is a constant, and $\dot{A}_{i,t}$ is the increase in A_t by research and development activities in economy i in period t and $\dot{A}_t = \dot{A}_{1,t} + \dot{A}_{2,t}$.

3.2 Sustainable heterogeneity in the heterogeneous risk aversion model

Suppose that $\theta_1 = \theta_2 = \theta$ and $\omega_1 = \omega_2 = \omega$; that is, only risk aversion is heterogeneous between economies 1 and 2, and $\varepsilon_1 < \varepsilon_2$. In this model, if, and only if, $\lim_{t\to\infty} \frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty} \frac{\dot{c}_{2,t}}{c_{2,t}} =$ constant, all the optimality conditions of both economies are satisfied at steady state; that is, sustainable heterogeneity is achieved (see Harashima, 2010, 2017), and

$$\lim_{t\to\infty} \frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty} \frac{\frac{d\int_0^t \tau_s ds}{dt}}{\int_0^t \tau_s ds} = \lim_{t\to\infty} \frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty} \frac{\dot{c}_{2,t}}{c_{2,t}} = \lim_{t\to\infty} \frac{\dot{k}_{1,t}}{k_{1,t}} = \lim_{t\to\infty} \frac{\dot{k}_{2,t}}{k_{2,t}}$$

$$= \lim_{t\to\infty} \frac{\dot{y}_{1,t}}{y_{1,t}} = \lim_{t\to\infty} \frac{\dot{y}_{2,t}}{y_{2,t}} = \lim_{t\to\infty} \frac{\dot{A}_t}{A_t} = \text{constant.}$$

Sustainable heterogeneity is achieved on this path, which is called the multilateral path. The limit of the growth rate on the multilateral path is

$$\lim_{t\to\infty} \frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty} \frac{\dot{c}_{2,t}}{c_{2,t}} = \left(\frac{\varepsilon_1 + \varepsilon_2}{2} \right)^{-1} \left[\left(\frac{\varpi \alpha}{mv} \right)^{\alpha} (1-\alpha)^{-\alpha} - \theta \right] \tag{5}$$

where m (> 0) and $\varpi (> 1)$ are constants. In addition, on the multilateral path,

$$\lim_{t\to\infty} \frac{\tau_t}{k_{1,t}} = \lim_{t\to\infty} \frac{\tau_t}{k_{2,t}} = \Xi \quad ,$$

where Ξ is a constant, and

$$\Xi = \frac{(\varepsilon_1 - \varepsilon_2)\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta\right]}{(\varepsilon_1 + \varepsilon_2)\left\{\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\left(\frac{\varepsilon_1 + \varepsilon_2}{2}\right)\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta\right]^{-1} - 1\right\}} .$$

Furthermore, if $1 - \theta\left(\frac{\varpi\alpha}{mv}\right)^{-\alpha}(1-\alpha)^{-1+\alpha} < \frac{\varepsilon_1 + \varepsilon_2}{2}$,

$$\Xi < 0$$

(see Harashima, 2010, 2017). The condition $1 - \theta\left(\frac{\varpi\alpha}{mv}\right)^{-\alpha}(1-\alpha)^{-1+\alpha} < \frac{\varepsilon_1 + \varepsilon_2}{2}$ is generally satisfied for reasonable parameter values.

This two-economy model can be extended to a multi-economy model. Suppose that H economies are identical except for risk aversion, and the DRA of economy i is ε_i ($i = 1, 2, \dots, H$). In this case, if and only if

$$\lim_{t\to\infty}\frac{\dot{c}_{i,t}}{c_{i,t}} = \left(\frac{\sum_{q=1}^{H}\varepsilon_q}{H}\right)^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta\right] \tag{6}$$

for any i, all the optimality conditions of all heterogeneous economies are satisfied at steady state, and

$$\lim_{t\to\infty}\frac{\dot{c}_{i,t}}{c_{i,t}} = \lim_{t\to\infty}\frac{\dot{k}_{i,t}}{k_{i,t}} = \lim_{t\to\infty}\frac{\dot{y}_{i,t}}{y_{i,t}} = \lim_{t\to\infty}\frac{\dot{A}_t}{A_t} = \lim_{t\to\infty}\frac{\dot{\tau}_{i,j,t}}{\tau_{i,j,t}} = \lim_{t\to\infty}\frac{\frac{d\int_0^t \tau_{i,j,s}ds}{dt}}{\int_0^t \tau_{i,j,s}ds}$$

for any i and j ($i \neq j$). In this case, H economies can be seen as a combined economy with a DRA of $H^{-1}\sum_{q=1}^{H}\varepsilon_q$, and the combined economy proceeds on the multilateral path.

3.3 Unilateral path

If each economy behaves unilaterally in the sense that it sets τ_t without regard for the other economy's optimality conditions, then all the optimality conditions of all economies cannot be satisfied (see Harashima, 2010, 2017). This path is called the unilateral path, and heterogeneity is not sustainable on this path. Becker (1980) showed that, if the time preference is heterogeneous, the most patient household will eventually own all capital and substantial inequality will emerge. The unilateral path corresponds to the state depicted by Becker (1980).

4 UNILATERAL BEHAVIORS

4.1 Unilateral behavior of a risk lover

In this section, I examine the consequence if a risk lover chooses the unilateral path. Suppose that there are two economies that are identical except for DRA and that $\varepsilon_1 < 0 < \varepsilon_2$; that is, economy 1 is risk loving and economy 2 is risk averse.

First, suppose that the two economies are isolated, and there are no transactions between them. In this case, their growth rates are

$$\frac{\dot{c}_{1,t}}{c_{1,t}} = \varepsilon_1^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta\right] \tag{7}$$

and

$$\frac{\dot{c}_{2,t}}{c_{2,t}} = \varepsilon_2^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta\right] .$$

Because $\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} = \frac{\partial y}{\partial k}$ and $\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta > 0$ in an endogenously growing economy,

$$\frac{\dot{c}_{1,t}}{c_{1,t}} < 0$$

and

$$\frac{\dot{c}_{2,t}}{c_{2,t}} > 0 \ .$$

Hence, whereas risk-averse economy 2 can grow on a balanced growth path, risk-loving economy 1 will eventually cease to exist in the sense that $\lim_{t\to\infty} c_{1,t} = 0$. This result is the consequence that conventional economic theory proposes about risk lovers.

Next, suppose that the two economies are open to each other, and goods and services and capital can move smoothly between them, but labor is immobilized. Because

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \varepsilon_1^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} + \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\lim_{t\to\infty}\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}} - \lim_{t\to\infty}\frac{\partial \tau_t}{\partial k_{1,t}} - \theta\right] \tag{8}$$

(see Harashima, 2010, 2017), the consumption growth rate of economy 1 ($\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}$) can be constant only if either

$$\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}=\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} \tag{9}$$

or

$$\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}=\lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds}=\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} \quad . \tag{10}$$

Conversely, economy 1 has two paths on which $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}$ is constant. Equation (9) indicates the path on which

$$\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}}=\text{constant},$$

and equation (10) indicates the path on which

$$\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\left(\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t}\right)^{-1}=1$$

for any $\lim_{t\to\infty}\frac{\tau_t}{k_{1,t}}$.

Equation (9) corresponds to the multilateral path that will be examined in detail in Section 5. Equation (10) corresponds to the unilateral path, because if economy 1 behaves unilaterally and sets τ_t independently of economy 2's optimality conditions, then τ_t is a control variable in addition to c_t for economy 1. Therefore, the optimality condition

$$\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial\tau_t}=1 \tag{11}$$

is added to the optimality conditions of economy 1. If equation (11) (i.e., the unilateral path condition) is held, equation (10) is satisfied. Therefore, equation (10) corresponds to the unilateral path.

By equation (8), if economy 1 chooses the unilateral path that equations (10) and (11) indicate, the growth rate is

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\varepsilon_1^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\theta\right] \quad . \tag{12}$$

Because $\varepsilon_1 < 0$ and because $\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta > 0$ in an endogenously growing economy, by equation (12),

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} < 0 \quad .$$

Therefore, if risk-loving economy 1 behaves unilaterally and chooses the unilateral path, it will eventually perish in the sense that $\lim_{t\to\infty} c_{1,t} = 0$.

4.2 Unilateral behavior of a risk-averse household

As with risk-loving economy 1, risk-averse economy 2 can satisfy all of its optimality conditions on the multilateral path, but what will happen when economy 2 chooses the unilateral path? Because

$$\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \varepsilon_2^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\lim_{t\to\infty}\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{2,t}} + \lim_{t\to\infty}\frac{\partial \tau_t}{\partial k_{2,t}} - \theta\right]$$

(see Harashima, 2010, 2017), the consumption growth rate of economy 2 can be constant only if either

$$\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} \tag{13}$$

or

$$\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds} = \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} \quad . \tag{14}$$

Equation (13) indicates the multilateral path (sustainable heterogeneity). Similar to the unilateral behavior of economy 1, if economy 2 behaves unilaterally, equation (11) has to be added as an optimality condition of economy 2. In this case, economy 2 grows on the path such that

$$\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \varepsilon_2^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \theta\right] \quad , \tag{15}$$

thereby satisfying all optimality conditions of economy 2. The path indicated by equation (15) is identical to the path that satisfies equation (14). That is, equation (14) corresponds to the unilateral path. Note that equation (14) is identical to equation (10) for economy 1.

On the unilateral path of economy 2 indicated by equations (14) and (15),

$$\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} \neq \lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds} \quad ,$$

and

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} < 0 < \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} \quad .$$

Here,

$$c_{1,t} - c_{2,t} = 2\left(\frac{\partial y_{1,t}}{\partial k_{1,t}}\int_0^t \tau_s ds - \tau_t\right) = 2\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t \tau_s ds - \tau_t\right] \quad ,$$

(see Harashima, 2010, 2017), and

$$\lim_{t\to\infty}\left(c_{1,t} - c_{2,t}\right) = 0$$

is required because $\lim_{t\to\infty}\frac{\tau_t}{\int_0^t \tau_s ds} = \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}$ on the unilateral path by equation (14). However, because $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} < 0 < \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$ on the unilateral path, economy 1 must initially set consumption such that $c_{1,0} = \infty$, which violates its optimality condition. Hence, on the unilateral path of economy 2, economy 2 can satisfy its all optimality conditions but economy 1 cannot.

5 ECONOMICALLY SURVIVABLE RISK LOVING

5.1 Sustainable heterogeneity with risk lovers

As shown in Section 4, risk-loving economy 1 can satisfy all its optimality conditions only on the path indicated by equation (9). In this section, this path (i.e., the multilateral path) is examined in detail. By equation (8), for economy 1 to grow at a positive rate (i.e., $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} > 0$),

$$\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} + \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}\lim_{t\to\infty}\frac{\partial\left(\int_0^t \tau_s ds\right)}{\partial k_{1,t}} - \lim_{t\to\infty}\frac{\partial \tau_t}{\partial k_{1,t}} - \theta < 0 \tag{16}$$

needs to be satisfied. As shown in Section 4, if economy 1 chooses the unilateral path, equation (16) cannot be satisfied. However, if economy 1 chooses the multilateral path indicated by equation (9), another possibility emerges. On the multilateral path, $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}$ = constant and

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \varepsilon_1^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha} - \left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha}T + \lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}T - \theta\right] \quad (17)$$

is satisfied for an appropriate constant value T. Because $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}$ = constant, if

$$T > 1 + \frac{\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} - \theta}{\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{1-\alpha} - \lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}},$$

then

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} > 0$$

by equation (17). Therefore, if an appropriate value of T is realized, economy 1 can grow at a positive constant rate; that is, it is sustainable.

Here, if economy 2 satisfies equation (13) as economy 1 simultaneously satisfies equation (9),

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$$

because by equations (9) and (13),

$$\lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds} = \lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} .$$

Both economies grow at a common growth rate, indicated by equation (5), with an appropriate value of T.

Equation (5) indicates that, if $0 < \varepsilon_1 + \varepsilon_2$, $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} > 0$ and both economies can grow at an identical positive rate. That is, if $0 < \varepsilon_1 + \varepsilon_2$, sustainable heterogeneity can be achieved. If risk-loving economy 1 and risk-averse economy 2 jointly proceed on a multilateral path, economy 1 can exist indefinitely even though it is risk loving. Furthermore, in the multi-economy model, if equation (6) is satisfied and if $0 < \sum_{q=1}^{H}\varepsilon_q$, sustainable heterogeneity is also achieved. That is, a sufficiently large number of risk-loving people can exist indefinitely. Note, however, if $\varepsilon_1 + \varepsilon_2 < 0$ or $\sum_{q=1}^{H}\varepsilon_q < 0$, all economies perish on the multilateral path.

5.2 Characteristics of sustainable heterogeneity with risk lovers

As shown in Section 3.2, on the multilateral path with $0 < \varepsilon_1 + \varepsilon_2$, $\Xi < 0$ if $1 - \theta\left(\frac{\varpi\alpha}{mv}\right)^{-\alpha}(1-\alpha)^{-1+\alpha}$ $< \frac{\varepsilon_1 + \varepsilon_2}{2}$. The condition $1 - \theta\left(\frac{\varpi\alpha}{mv}\right)^{-\alpha}(1-\alpha)^{-1+\alpha} < \frac{\varepsilon_1 + \varepsilon_2}{2}$ is generally satisfied for reasonable parameter values unless $\frac{\varepsilon_1 + \varepsilon_2}{2}$ is close to zero. Inequality $\Xi < 0$ on the multilateral path with $0 < \varepsilon_1 + \varepsilon_2$ means that risk-loving economy 1 continues to borrow money from risk-averse economy 2 indefinitely. Because the current account of economy 1 is negative, economy 1 owes debts to economy 2 indefinitely. Note that Ξ is constant on this multilateral path, so the debt level does not explode.

If sustainable heterogeneity is achieved,

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}} = \lim_{t\to\infty}\frac{\dot{k}_{2,t}}{k_{2,t}} = \lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}} = \lim_{t\to\infty}\frac{\dot{y}_{2,t}}{y_{2,t}} = \lim_{t\to\infty}\frac{\dot{A}_t}{A_t} = \text{constant.}$$

Hence, a higher $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$ indicates a higher $\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}$. To achieve a higher $\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}$, a larger amount of investment in A_t is necessary. As (positive) DRA (ε) approaches zero, an economy invests more in A_t, and the growth rate of consumption increases, but as DRA (ε) becomes negative, investment in A_t, increases such that consumption has to continuously decline to zero. However, if both risk-averse and risk-loving economies exist and the risk-loving economy can borrow money from the risk-averse economy appropriately and indefinitely, the consumption of the risk-loving economy does not decline to zero and can be sustained. This is the mechanism behind inequality $\Xi < 0$. Sustainable heterogeneity therefore means risk-loving economy 1 owes appropriate and indefinite debts to risk-averse economy 2.

5.3 The choice of risk-averse economy

5.3.1 The best choice

Risk-loving economy 1 can achieve sustainable heterogeneity on the multilateral path, but does risk-averse economy 2 choose the multilateral path rather than the unilateral path? Unlike economy 1, economy 2 can satisfy its all optimality conditions on either path, but which path is best?

Suppose $0 < \varepsilon_1 + \varepsilon_2$. The growth rate of economy 2 on the multilateral path is indicated by equation (5), and the rate on the unilateral path is indicated by equation (15). The growth rate on the multilateral path is clearly higher than that on the unilateral path because $0 < \frac{\varepsilon_1 + \varepsilon_2}{2} < \varepsilon_2$. Considering this nature, it is highly likely that economy 2 prefers the multilateral path to the unilateral path as long as $0 < \varepsilon_1 + \varepsilon_2$ is satisfied. Economy 2 can enjoy a higher growth rate because of the existence of risk lovers. In other words, risk-averse economy 2 can exploit the opportunities that the existence of risk-loving economy 1 provides because economy 2 is better off appropriately lending money to economy 1. The multilateral path will therefore be favorable to both economies as long as $0 < \varepsilon_1 + \varepsilon_2$ is held.

5.3.2 Behavior of a household with a DRA is close to zero

More generally, in the multi-economy model, the multilateral path is favorable for most economies by equation (6) as long as $0 < \sum_{q=1}^{H} \varepsilon_q$ is held. However, in the multi-economy model, the multilateral path may not be favorable for economies with DRAs that are positive but close to zero, because the growth rates of these economies are higher on the unilateral path than they are on the multilateral path. By equation (7), as (positive) DRA approaches zero, the growth rate becomes extremely high and increases to infinity.

However, many of these economies will also choose the multilateral path because it seems likely that the DRA has the property of a Markov process. In this paper and in many economic studies, an infinitely long-lived household is assumed, which means there is a "dynasty" that consists of all its future generations. Some future generations in a dynasty may be relatively more risk averse and others may be less risk averse or even rather risk loving. This means that the DRA of a dynasty will possess the property of a Markov chain. If a state in this chain has a negative DRA, the dynasty may cease to exist during that period unless sustainable heterogeneity is formed with risk-averse dynasties. The states of a Markov chain of a dynasty with a positive near-zero expected DRA will generally include states with a negative DRA—that is, periods of risk loving. Therefore, dynasties with positive near-zero expected DRAs will generally behave in the same manner as risk lovers and choose the multilateral path.

Furthermore, even if some of these dynasties still behave unilaterally in periods of positive DRAs, the entire economy will still eventually proceed on the multilateral path, because when these dynasties' states change to a negative DRA they have to choose the multilateral path and reset their levels of consumption to be consistent with it. Transitions to states of negative DRAs will occur repeatedly in future generations, and on all such occasions the levels of consumption have to be reset to be consistent with the multilateral path. This means that these dynasties follow the multilateral path in the long run. Moreover, if these dynasties behave unilaterally, many other risk-averse dynasties will politically resist their unilateral behavior and governing authorities will intervene so as to achieve sustainable heterogeneity by constraining their unilateral behaviors (see Harashima, 2010, 2012, 2017). As a result, the entire economy will eventually proceed on the multilateral path even if some dynasties with positive but near-zero expected DRAs exist.

5.4 Degree of risk loving and number of risk lovers

If not only the DRA of an individual dynasty but the DRA of the combined economy (i.e., economies that commonly proceed on the multilateral path) is positive but close to zero (i.e., $H^{-1}\sum_{q=1}^{H} \varepsilon_q \cong 0$ for $H^{-1}\sum_{q=1}^{H} \varepsilon_q$), is the multilateral path still favorable for all economies? By equation (6), as $H^{-1}\sum_{q=1}^{H} \varepsilon_q$ approaches zero, the growth rate of the combined economy becomes extremely high and increases to infinity. However, an almost infinitely high growth rate seems quite unnatural. Therefore, in actuality, $H^{-1}\sum_{q=1}^{H} \varepsilon_q$ will be sufficiently distant from zero. This means that there exists an upper bound of the degree of risk loving among households and, in addition, an upper bound of the number of risk-loving households in a society. Hence, it seems likely that highly risk-loving people are a small minority in a society.

The reason for the existence of the upper bounds is that $H^{-1}\sum_{q=1}^{H} \varepsilon_q$ will have the same property as the DRA of an individual household in that it will also have the property of a Markov chain. If $H^{-1}\sum_{q=1}^{H} \varepsilon_q$ consists of states that include negative values, the combined economy will cease to exist in periods of negative $H^{-1}\sum_{q=1}^{H}\varepsilon_q$. As a result, $H^{-1}\sum_{q=1}^{H}\varepsilon_q$ will not consist of any state that includes negative values, and therefore the expected $H^{-1}\sum_{q=1}^{H}\varepsilon_q$ (i.e., the mean of $H^{-1}\sum_{q=1}^{H}\varepsilon_q$) will be sufficiently distant from zero.

5.5 Usefulness of risk lovers

Because of sustainable heterogeneity, risk lovers are useful for, and welcome in, a society unless their number and their degree of risk loving are too great. Suppose that the DRAs of H households in a country are heterogeneous and distributed uniformly between $\bar{\varepsilon}-\psi$ and $\bar{\varepsilon}+\psi$, where ψ is a positive constant. The average DRA of the country is therefore $H^{-1}\sum_{q=1}^{H}\varepsilon_q = \bar{\varepsilon}$. If there is no risk-loving household in the country, $0 < \bar{\varepsilon}-\psi$ and $\psi < H^{-1}\sum_{q=1}^{H}\varepsilon_q = \bar{\varepsilon}$. On the other hand, if risk-loving households with $-\psi < \bar{\varepsilon}-\psi < 0$ exist continuously under sustainable heterogeneity, the average DRA ($H^{-1}\sum_{q=1}^{H}\varepsilon_q = \bar{\varepsilon}$) can be lower than when no risk-loving households exist; that is, $0 < H^{-1}\sum_{q=1}^{H}\varepsilon_q = \bar{\varepsilon} < \psi$. Therefore, by equation (6), thanks to the existence of risk-loving households, the growth rate of the country is higher than when no risk-loving households exist, because of a lower $H^{-1}\sum_{q=1}^{H}\varepsilon_q = \bar{\varepsilon}$. In this sense, an adequate number of temperately risk-loving persons are useful for a society.

5 CONCLUDING REMARKS

A bubble-like phenomenon (the so-called "bubble economy") can be generated rationally by a bluff undertaken by risk lovers, but can risk lovers exist continuously in an economy? In conventional economic growth models, if a household's DRA is negative, its consumption steadily declines to zero and it ceases to exist. Because of this nature, there have been few economic studies on risk lovers. In this paper, the possibility that risk lovers can exist continuously is examined in the framework of an economic growth model on the basis of the concept of sustainable heterogeneity, which is the state at which all optimality conditions of all heterogeneous households are satisfied indefinitely. If risk-loving and risk-averse persons coexist at this state, risk lovers can exist continuously in an economy and a bluff can be undertaken.

I conclude that risk lovers can exist continuously at the state of sustainable heterogeneity, although their number and degree of risk loving have an upper bound. Therefore, it is likely that a bluff will be undertaken by risk lovers and that a bubble-like phenomenon can be generated.

Although risk lovers bluff, it is the bluff that deserves blame from a social perspective, not their risk-loving nature, which is not necessarily bad for society. Of course, a gambling addiction is socially unacceptable, but moderately risk-loving people may be socially desirable as long as their number is not too great (i.e., as long as $H^{-1}\sum_{q=1}^{H}\varepsilon_q$ has a sufficiently large positive value), because they contribute to a relatively high growth rate of an economy that consists of heterogeneous households.

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