A Tautologies-Founded IS-LM Model

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A Tautologies-Founded IS-LM Model

By Martin Hiermeyer*

The paper combines five tautologies to come up with a model that is structurally similar to the IS-LM model but has five advantages vis-à-vis the IS-LM model. The model also has seven advantages vis-à-vis simple New Keynesian models such as Romer's IS-MP model or Carlin's and Soskice's IS-MR model.

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I. Introduction

Today, complex models dominate macroeconomic research. At the same time, simple models preserve niches such as undergraduate teaching or policy advice.

Among simple models, the "incumbent" is still the IS-LM[-AS] model of Hicks (1937) and Hansen (1953). It features prominently in such widely used undergraduate textbooks as Abel, Bernanke and Croushore (2017), Blanchard (2017), Dwivedi (2015), Froyen (2013), Gordon (2012), Heijdra (2017) or Mankiw (2016). The model’s influence in the corridors of power is unsurpassed at times. Cohen-Setton and Kessler (2011) cite Lawrence Summers as saying that the White House policy response to the financial crisis was all IS-LM augmented by a liquidity trap whereas other models played no role at all.

Yet, the IS-LM[-AS] model’s status is increasingly under attack from simple New Keynesian models that retain a variant of the IS curve but replace the LM curve with a monetary policy rule that describes how the central bank changes its policy rate in response to changes in inflation and/or output. Examples of such models include the IS-MP[-IA] model of Romer (2000), the IS-[PC-]MR model of Carlin and Soskice (2005) and

Those simple New Keynesian models reflect what is going on in complex New Keynesian models. For example, the model of Clarida et al. (1999) also uses a variant of the IS curve, a Phillips inflation adjustment curve and a monetary policy rule instead of an LM curve.

This paper goes back to the IS-LM[-AS] model. It combines five tautologies to come up with a model that is structurally similar to the IS-LM model but arguably superior both to the IS-LM model and to simple New Keynesian models.

II. Deriving the Tautologies-Founded IS-LM Model

The tautologies-founded IS-LM model is structurally similar to the IS-LM model.

A. IS-LM Model As Comparison

The IS-LM model is given by equations (IS) and (LM).

(IS) \[ Y = C(Y) + I(i) + G, \] where \( 0 \leq C'(Y) < 1 \), and \( I'(i) < 0 \)

(LM) \[ M = L(i, Y), \] where \( L'(i) < 0 \), and \( L'(Y) > 0 \)

The variables are:

\begin{align*}
Y: \text{Output} & \quad i: \text{Interest Rate} \\
C: \text{Consumption Spending} & \quad G: \text{Government Spending} \\
I: \text{Investment Spending} & \quad M: \text{Money}
\end{align*}

Equation (IS) is the IS equation. Equation (LM) is the LM equation.

B. Five Tautologies For The Tautologies-Founded IS-LM Model

Equations (1) through (5) give five tautologies that can be combined to form the tautologies-founded IS-LM model.

\[ Y \equiv C + I + G \]
(2) \[ M_1 V_1 \equiv PY \]
(3) \[ M_1 \equiv CHP + D \]
(4) \[ HPM \equiv CHP + ER + RR \]
(5) \[ r_r \equiv RR/D \text{ with } 0 < r_r \leq 1 \]

The variables, if not already defined, are:

- **M1**: M1 Money
- **HPM**: High-Powered Money
- **V1**: Velocity of M1 Money
- **ER**: Excess Reserves
- **P**: Price Level
- **RR**: Required Reserves
- **CHP**: Currency Held By the Public
- **rr**: Reserve Ratio
- **D**: Demand Deposits

Equation (1) is the national income identity for a closed economy; it also forms the basis for equation (IS) above.

Equation (2) is Fisher’s (1911) equation of exchange, also known as the quantity equation of money, given here with respect to M1 money.

Equation (3) defines the components of M1 money.

Equation (4) defines the components of high-powered money.

Equation (5) defines the reserve ratio.

**C. Seven Assumptions For The Tautologies-Founded IS-LM Model**

Equations (6) through (12) contain seven assumptions for the variables from the tautologies.

(6) \[ C = C(Y) \text{ with } 0 \leq C'(Y) < 1 \]

(7) \[ I = I(i) \text{ with } I'(i) < 0 \]

(8) \[ P = 1 \]

(9) \[ V_1 = 1 \]

(10) \[ HPM = HPM(pr) \text{ with } HPM'(pr) < 0 \]

(11) \[ CHP = CHP(i) \text{ with } CHP'(i) < 0 \]

(12) \[ ER = ER(i) \text{ with } ER'(i) < 0 \]
In equation (10), \( pr \) denotes the policy rate used by the central bank. All other variables have already been defined.

All assumptions contained in equations (6) through (12) are either not restrictive or plausible.

Equation (6) assumes that consumption spending may be positively related to output. This is plausible and a standard assumption of the IS-LM model. As such, the assumption is also contained in equation (IS) above.

Equation (7) assumes that investment spending is negatively related to the interest rate. This is plausible and a standard assumption of the IS-LM model. As such, the assumption is also contained in equation (IS) above.

Equation (8) assumes that the price level is exogenous and can be set equal to 1. This assumption, which is also a standard assumption of the IS-LM model, is not restrictive. The price level can be made endogenous anytime by adding an aggregate supply schedule that distributes changes in aggregate demand between changes in the price level and changes in output (“IS-LM-AS model”).

Equation (9) assumes that velocity of M1 money, \( V_1 \), is exogenous and can be set equal to 1. Like the assumption on the price level, the assumption is not restrictive as \( V_1 \) can be made endogenous anytime by adding a schedule. In this case, a schedule that distributes changes in aggregate demand between changes in M1 money and changes in \( V_1 \).

Equation (10) assumes that high-powered money is negatively related to the central bank’s policy rate. This is plausible as the policy rate is usually the interest rate that banks charge one another for overnight loans of high-powered money. As banks’ demand for high-powered money is presumably lower if the interest rate on it is higher, assuming a negative relationship between the two variables makes sense.

Equation (11) assumes that currency held by the public is negatively related to the interest rate. This is plausible. A higher interest rate increases the public’s opportunity cost of holding currency as paying the currency into an interest-bearing account becomes more attractive.
Equation (12) assumes that excess reserves are negatively related to the interest rate. This is plausible. A higher interest rate increases banks’ opportunity cost of holding excess reserves as making loans becomes more attractive.

**D. Tautologies-Founded IS-LM Model**

Combining equations (1), (6), (7), (8) and (9) yields a tautologies-founded IS curve.

\[ \text{(TIS)} \quad Y = C(Y) + I(i) + G, \text{ where } Y \equiv M1, \]
\[ \quad 0 \leq C'(Y) < 1, \text{ and } I'(i) < 0 \]

Combining equations (2), (3), (4), (5), (8), (9), (10), (11) and (12) yields a tautologies-founded LM curve.

\[ \text{(TLM)} \quad \text{HPM}(\text{pr}) = (1-\tau)r \text{CHP}(i) + \text{ER}(i) + \tau r Y, \text{ where } Y \equiv M1, \]
\[ \quad \text{HPM}'(\text{pr}) < 0, \quad \text{HPM}'(i) = (1-\tau)\text{CHP}'(i) + \text{ER}'(i) < 0, \]
\[ \quad \text{and } \text{HPM}'(Y) = \tau r \text{ with } 0 < \tau r \leq 1 \]

Figure 1 compares the IS-LM model of equations (IS) and (LM) to the tautologies-founded IS-LM model of equations (TIS) and (TLM). In the figure, a + symbol indicates that an exogenous increase in a variable acts expansionary while a – symbol indicates that an exogenous increase in a variable acts contractionary.
The slope of the IS curve is given by equation (13) and the slope of the tautologies-founded IS curve is given by equation (14).

\[
\frac{d\bar{Y}}{d\bar{Y}} = \frac{1-C'(\bar{Y})}{I'(\bar{i})} < 0
\]

(13)

\[
\frac{d\bar{Y}}{d\bar{Y}} = \frac{d\bar{M}_1}{d\bar{M}_1} = \frac{1-C'(\bar{Y})}{I'(\bar{i})} < 0
\]

(14)

The slope is the same for both curves. The only difference is that the slope of the tautologies-founded IS curve refers not only to an interest rate-output space but also to an interest rate-M1 money space.

The slope of the LM curve is given by equation (15) and the slope of the tautologies-founded LM curve is given by equation (16).

\[
\frac{d\bar{Y}}{d\bar{Y}} = \frac{-L'(\bar{Y})}{L'(\bar{i})} > 0
\]

(15)

\[
\frac{d\bar{Y}}{d\bar{Y}} = \frac{d\bar{M}_1}{d\bar{M}_1} = \frac{-rr}{[(1-rr)\text{CHP}'(\bar{i}) + \text{ER}'(\bar{i})]} > 0
\]

(16)

The slope of both curves is structurally similar. For \(L'(\bar{Y}) = rr\) and \(L'(i) = (1-rr)\text{CHP}'(i) + \text{ER}'(i)\) it is even identical. Like the tautologies-founded IS curve, the tautologies-founded LM curve refers not only to an interest rate-output space but also to an interest rate-M1 money space.

**III. Five Advantages Vis-à-vis The IS-LM Model**

Vis-à-vis the IS-LM model, the tautologies-founded IS-LM model has five advantages.

\textit{A. Central Bank Targets A Policy Rate Instead Of Money}

The tautologies-founded IS-LM model assumes that the central bank targets a policy rate which is negatively related to high-powered money. With that, it describes monetary policy realistically (i.e. central bank decides to increase or decrease the policy rate, and then decreases or increases high-powered money so that the policy rate is met). This is a step forward when compared to the IS-LM model which unrealistically assumes that the central bank targets money (and here presumably high-powered money, although this too is not entirely clear, see Romer 2000).
B. An In-Depth Look At Liquidity Traps Is Possible

The tautologies-founded IS-LM model integrates currency held by the public and excess reserves into the IS-LM model. This is a good thing as those are important economic variables: Currency held by the public played an important role during the Great Depression (Friedman and Schwartz 1963). Excess reserves ballooned over the last decade. In the US, for example, excess reserves were in 2016 about 1,600 times (sic!) above their 1987-2007 average (see Federal Reserve 2017).

The inclusion of currency held by the public and excess reserves allows for a more in-depth analysis of liquidity traps. In the IS-LM model, such a situation is described by a LM curve that is horizontal because of $L'(i) \to -\infty$. The equivalent condition in the tautologies-founded IS-LM model is $[(1-rr)CHP'(i) + ER'(i)] \to -\infty$. The condition clarifies that in a liquidity trap the central bank's high-powered money may be “trapped” in currency held by the public and/or in excess reserves.

C. Different Reserve Regimes Can Be Evaluated

Reserve requirements vary widely between countries. While the UK has a 0% reserve requirement, Switzerland is on the brink of a referendum regarding a possible introduction of a 100% reserve requirement. Most other countries have reserve requirements somewhere in the middle.

The tautologies-founded IS-LM model can be used to evaluate different reserve regimes. As equation (16) shows, for a reserve ratio of 0% the tautologies-founded LM curve is horizontal. While the central bank can increase high-powered money, its control over output and M1 money is zero as equilibrium in the tautologies-founded IS-LM model is in this case solely determined by the tautologies-founded IS curve. Goodhart’s (2017) suggestion that banking is a service industry, which sets the terms and conditions whereby the private and government sector can create additional money for itself, rings particularly true in this case.
For a reserve ratio greater than 0%, the tautologies-founded LM curve becomes steeper. With it, the central bank’s control over output and M1 money increases. For a 100% reserve ratio, the tautologies-founded LM curve is fairly steep, although still not vertical as long as banks can destroy or create M1 by holding more or less excess reserves.

D. A Fully-Fledged Banking System Is Included

In the tautologies-founded IS-LM model, banks borrow high-powered money from the central bank, comply with reserve requirements, take in demand deposits from the public, and make loans. Thus, the tautologies-founded IS-LM model includes a fully-fledged banking system. Its working is spelled out in the Appendix, which gives the tautologies-founded IS-LM story for a fiscal expansion and for a monetary expansion. The IS-LM model, in contrast, has no banking system (see also the Appendix).

E. Spending Is Explicitly Paid For

The tautologies-founded IS-LM model introduces the following constraint: \(M1 \equiv Y\). The constraint is similar to a cash-in-advance constraint (Clower 1967), although it refers to M1 money and not just to cash.

Requiring buyers to have M1 money in advance makes sense as payment with M1 money – currency, check, direct debit or bank wire transfer – is generally the only accepted method of payment. Of course, purchases can also take place on credit. However, even there M1 money is needed in advance – this time by the seller. We can see this from the fact that if a business sells a good on credit, its accounts receivable balance sheet item goes up. Accounts receivable is an asset which firms, if higher, have to match with an additional liability, that is, with a ceteris paribus higher borrowing of M1 money.

With \(M1 \equiv Y\) firmly in place, the tautologies-founded IS-LM model makes it clear that additional spending requires additional money.

This is an advantage vis-à-vis the IS-LM model. The IS-LM model can be confusing for students because government spending increases output
while money is assumed to be constant. Some students argue that, by closer inspection, the IS-LM model must hence be wrong. After all, they say, the money which the government additionally spends has to come from somewhere. Since money is assumed to be constant, it is not printed. Thus, it has to be either taxed or borrowed out of the economy. Given that those from whom the money is taken might have spent it as well, how can the IS-LM model be sure that output increases?

The argument, which is sometimes also called the “Treasury view” as staff of the UK Treasury advanced a similar logic in the 1930ies (Peden 2004) of course overlooks that velocity of money can increase – as it indeed does in the IS-LM model in such a case.

IV. Seven Advantages Vis-à-vis Simple New Keynesian Models

Vis-à-vis simple New Keynesian models, the tautologies-founded IS-LM model has seven advantages.

A. High-Powered Money Is Included

Central banks have to manipulate high-powered money in order to set the policy rate. Models that include the central bank policy rate therefore ideally also include high-powered money. The tautologies-founded IS-LM model does so. Simple New Keynesian models do not, despite their maybe even greater focus on the policy rate.

With quantitative easing, having high-powered money in the model has become even more valuable as central banks, in quantitative easing, increase high-powered money directly through asset purchases.

B. An In-Depth Look At Liquidity Traps Is Possible

As discussed in section III.B, the tautologies-founded IS-LM model allows for an in-depth look at liquidity traps. Of course, a liquidity trap can in principle also be added to simple New Keynesian models as is done in complex New Keynesian models. However, doing so is complex and predictions are often paradoxical (Cochrane 2017).
C. Different Reserve Regimes Can Be Evaluated

As discussed in section III.C, the tautologies-founded IS-LM model can analyze different reserve regimes. In contrast, simple New Keynesian models cannot as they do not include reserves or deposits in general.

D. A Fully-Fledged Banking System Is Included

As discussed in section III.D, the tautologies-founded IS-LM model includes a fully-fledged banking system. This is an advantage vis-à-vis simple New Keynesian models which include no banking system.

E. All Variables Can Be Directly Observed

The variables in the tautologies-founded IS-LM model can be directly observed. In contrast, the simple New Keynesian models often include variables such as potential output or a “stabilizing” rate of interest that are not directly observable.

F. Unconventional Monetary Policy Can Be Evaluated

In the tautologies-founded IS-LM model, if the central bank finances government spending, both the tautologies-founded IS curve and the tautologies-founded LM curve shift to the right. By this, nearly unlimited increases in aggregate demand can be caused. All past hyperinflations came about through that mechanism.

A tiny dose of such a potential hyperinflation is currently discussed as a possible way to reflate low-inflation economies. Catchwords are “helicopter money” and “people’s quantitative easing”. In helicopter money, central banks distribute newly created money to households, hoping to increase their consumption spending. In people’s quantitative easing, central banks finance public investment projects.

Unlike the tautologies-founded IS-LM model, simple New Keynesian models cannot inform those debates as they do not include any money.
G. The Model Is Derived Solely From Tautologies

The tautologies-founded IS-LM model is derived solely from tautologies. In that sense, the model is “safe”. This cannot be said for the simple New Keynesian models.

V. Conclusion

If “simplicity is the ultimate sophistication”, as Leonardo da Vinci is alleged to have said, then simple models have a place. The paper combines five tautologies to come up with a new simple model. The new model, called the “tautological-founded IS-LM model”, is structurally similar to the IS-LM model but has five advantages vis-à-vis the IS-LM model as explained in section III and summarized in Table 1 below.

| A. Central Bank Targets A Policy Rate Instead Of Money | ✓ | x |
| B. An In-Depth Look At Liquidity Traps Is Possible | ✓ | x |
| C. Different Reserve Regimes Can Be Evaluated | ✓ | x |
| D. A Fully-Fledged Banking System Is Included | ✓ | x |
| E. Spending Is Explicitly Paid For | ✓ | x |

Also, the tautological-founded IS-LM model has seven advantages vis-à-vis simple New Keynesian models as explained in section IV and summarized in Table 2 below.

| A. High-Powered Money Is Included | ✓ | x |
| B. An In-Depth Look At Liquidity Traps Is Possible | ✓ | x |
| C. Different Reserve Regimes Can Be Evaluated | ✓ | x |
| D. A Fully-Fledged Banking System Is Included | ✓ | x |
| E. All Variables Can Be Directly Observed | ✓ | x |
| F. Unconventional Monetary Policy Can Be Evaluated | ✓ | x |
| G. The Model Is Derived Solely From Tautologies | ✓ | x |
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Appendix

The Appendix compares the IS-LM story to the tautologies-founded IS-LM story. The following parameters are assumed:

<table>
<thead>
<tr>
<th>IS-LM Story</th>
<th>Tautologies-founded IS-LM Story</th>
</tr>
</thead>
<tbody>
<tr>
<td>C'(Y) = 0</td>
<td>C'(Y) = 0</td>
</tr>
<tr>
<td>Γ'(i) = -12</td>
<td>Γ'(i) = -12</td>
</tr>
<tr>
<td>L'(Y) = 0.25</td>
<td>r = 0.25</td>
</tr>
<tr>
<td>L'(i) = -3</td>
<td>CHP'(i) = -2 and ER'(i) = -1.5</td>
</tr>
<tr>
<td></td>
<td>HPM'(pr) = -6</td>
</tr>
</tbody>
</table>

While the IS-LM model consists only of an interest rate-output (i-Y) space, the tautologies-founded IS-LM model features both an interest rate-output (i-Y) space and an interest rate-M1 money (i-M1) space as Figure 1 showed.

The tautologies-founded IS-LM story interprets that latter space as the market for M1 money where the IS curve reflects demand for M1 money and the LM curve reflects supply of M1 money. The interest rate i equates both.

With respect to the tautologies-founded IS-LM story, it is useful to reformulate equation (TLM) as follows.

\[(A-1) \quad M1 = [HPM(pr) - (1-r)r)CHP(i) - ER(i)]/rr\]

This equation now shows how M1 money is created or destroyed. This is an important part of the tautologies-founded IS-LM model story as we will see next.

Table A-2 gives the IS-LM story and the tautologies-founded IS-LM story for a fiscal expansion. Table A-3 gives the IS-LM story and the tautologies-founded IS-LM story for a monetary expansion.
Increase in Government Spending (24 Units).—The economy is at point A when the government increases its spending G by 24 units from 40 units to 64 units. Given C’(Y)=0, the IS curve shifts to the right by 24 units, crossing through point B.

Increase in Output (12 Units).—Output increases, but not by the full 24 units. This is because higher output comes with higher money demand as higher output means additional purchases of goods and services (and thus additional transactions for which money is needed). Equilibrium is reached in point C. The higher money demand drives up the interest rate i by 1 percentage point from 4% to 5%. Given I’(i)=-12, this reduces investment spending by 12 units. At the goods market (IS curve), government spending is up by 24 units and investment spending is down by 12 units, for an overall increase in output of 12 units. At the other end, the higher money demand drives up the interest rate i by 1 percentage point from 4% to 5% as the economy moves to point C. There, equilibrium is restored, partly through lower demand for M1 money by firms (12 units) and partly through higher supply of M1 money by banks (12 units).

Lower Demand for M1 Money (12 Units).—Given I’(i)=-12, the higher interest rate makes firms borrow 12 units of M1 money less as they de-
money market (LM curve), the unchanged money supply is matched by an unchanged money demand (down by 3 units because of \( L'(i) = -3 \), but also up by 3 units because of \( L'(Y) = 0.25 \)).

Increase their investment spending by 12 units.

*Higher Supply of M1 Money (12 Units).*—Given \( \text{CHP}'(i) = -2 \), the higher interest rate makes the public pay 2 units of currency into interest-bearing demand deposit accounts. Demand deposits increase by 2 units, and banks experience an inflow of currency of 2 units. Of those 2 units, 0.5 units are needed to fulfil the higher reserve requirement that comes with the additional demand deposits. The other 1.5 units add to banks’ excess reserves. However, banks do not wish to increase excess reserves. Rather, they wish to decrease excess reserves – by 1.5 units, given \( \text{ER}'(i) = -1.5 \). Hence, banks lend out 3 units of excess reserves. Given \( r_r = 0.25 \), this translates into 12 additional units of loans. Banks make those loans by crediting the demand deposits account of the borrower with a demand deposit of the size of the loan. Hence, demand deposits increase by 12 units. Demand deposits were already up by 2 units because of the public’s deposit, so they are now up by 14 units. Since currency held by the public is down by 2 units, M1 money, the sum of currency held by the public and demand deposits, is up by 12 units – in line with equation (A-1).

Increase in Output (12 Units).*—Output
increases by the increase in government spending (24 units) minus the crowded out borrowing by firms (12 units). The intuition is that those 24 units of M1 money, which the government additionally spends, have to come from somewhere. Here, 12 of the 24 units come from higher supply of M1 money as banks create 12 additional units of M1 money by making 12 additional units of loans. This is good for output. The other 12 units come from lower demand for M1 money as firms borrow 12 units of M1 money less. This is not so good for output. After all, firms that borrow 12 units less also spend 12 units less as borrowing and spending go hand in hand.

In the easiest case, the government borrows the additional bank loans (12 units) and takes over the crowded out lending of firms (12 units) for its needed total of 24 units. In reality, things may be somewhat more complicated given that the government generally borrows at the bond market rather than from banks. This is, however, not a problem as the bond market and the bank lending market are interconnected. Here, for example, firms may borrow less from the bond market and more from banks, thus making room for government bond market borrowing.
Increase in Money (6 Units).—The economy is at point A when the central bank increases money M by 6 units from 90 units to 96 units. Given \( L'(Y) = 0.25 \), the LM curve shifts to the right by 24 units, crossing through point B. The increase in money drives down the interest rate \( i \).

Increase in Output (12 Units).—Investment spending and output increase, although not by the full 24 units. This is because higher output comes with higher money demand as higher output means additional purchases of goods and services (and thus additional transactions for which money is needed). This dampens the decrease in the interest rate \( i \).

Equilibrium is reached in point C. The interest rate \( i \) decreases by 1 percentage point from 5\% to 4\%. Given \( I'(i) = -12 \), this increases investment spending, and hence output, by 12 units. At the goods market (IS curve), investment spending and output is up by 12 units.

Cut in the Central Bank Policy Rate (1 Percentage Point).—The economy is at point A when the central bank cuts its policy rate \( pr \) by 1 percentage point from 2\% to 1\%.

Increase in High-Powered Money (6 Units).—Given \( HPM'(pr) = -6 \), the central bank’s traders must lend 6 additional units of high-powered money \( HPM \) to banks (96 units instead of 90 units) in order to meet the lower policy rate. Given \( r = 0.25 \), this translates into 24 additional units of loans. Banks make those loans by crediting the demand deposits account of the borrower with a demand deposit of the size of the loan. Hence, demand deposits increase by 24 units. With them, M1 money increases by 24 units – in line with equation (A-1). The TLM curve, which reflects supply of M1 money, shifts to the right by 24 units, crossing through point B.

Excess Supply of M1 Money at Point B (24 Units).—At point B, there is ex-
At the money market (LM curve), the 6-unit increase in money supply is matched by a 6-unit increase in money demand (up by 3 units because of \( L'(i)=-3 \), and up by another 3 units because of \( L'(Y)=0.25 \)).

In turn, the interest rate \( i \) decreases by 1 percentage point (from 5% to 4%) as the economy moves to point C. There, equilibrium is restored, partly through higher demand for M1 money by firms (12 units) and partly through lower supply of M1 money by banks (12 units).

**Higher Demand for M1 Money (12 Units).**—Given \( I'(i)=-12 \), the lower interest rate makes firms borrow 12 additional units of M1 money as they increase their investment spending by this amount.

**Lower Supply of M1 Money (12 Units).**—Given \( CHP'(i)=-2 \), the lower interest rate makes the public withdraws 2 units of currency from its demand deposit accounts to reduce shoe leather costs. Demand deposits decrease by 2 units, and banks experience an outflow of excess reserves of 2 units. On the other hand, 0.5 units of required reserves are now freed up given the fewer demand deposits. Thus, banks’ excess reserves decrease overall by 1.5 units. However, banks do not wish to decrease excess reserves. Rather, they wish to increase excess reserves – by 1.5 units, given \( ER'(i)=-1.5 \). Hence, banks lend out 3 units of excess reserves less, thus destroying 12 units of M1 money – in
line with equation (A-1).

*Increase in Output (12 Units).*—In sum, banks create 12 additional units of M1 money (plus 24 units because of the additional high-powered money, and minus 12 units because of the lower interest rate i). While monetary policy is thus partly self-defeating, there is still a 12-unit net increase in M1 money which is borrowed and spent by firms.