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Abstract

This paper investigates economies of scale (ES) in financial intermediation as a source of equilibrium indeterminacy. Consumption in the model can be purchased with currency and deposits, and ES in intermediation implies that deposit costs are decreasing in aggregate deposits. The results suggest that indeterminacy does not depend on a large degree of ES nor a large intermediation sector, but on monetary policy and the determination of nominal interest rates. Monetary policies not targeting nominal rates allow for indeterminacy to arise for any degree of ES, while policies targeting nominal rates eliminates indeterminacy for all degrees of ES.

Keywords: Financial Intermediation; Economies of Scale; Equilibrium Indeterminacy; Monetary Policy

JEL: C62, E44, E52

1. Introduction

What are the conditions for economies of scale (ES) in financial intermediation to deliver equilibrium indeterminacy? This question is motivated by the extensive literature on indeterminacy due to increasing returns to scale in production (e.g. Benhabib and Farmer, 1994), as well as the literature on banking crises where a strategic complementarity in intermediation delivers multiple (steady-state) equilibria (e.g. Bryant, 1987). In contrast to the research on the production sector, this analysis builds on empirical evidence reported by Hughes and Mester (1998) that banks exhibit significant scale economies.[1] In contrast to the research on banking crises, ES in intermediation is explicitly captured in an infinitely lived, representative agent economy and its potential for delivering indeterminacy is analyzed. Combining elements of both literatures results in this paper being a first attempt at considering intermediation as an avenue through which agents' beliefs (or *animal spirits*) contribute to economic fluctuations.[2]

To illustrate how ES can deliver indeterminacy, suppose intermediaries possess decreasing marginal costs with managing household deposits, and pass these costs onto depositors. ES will distort household decisions regarding deposit and currency balances to be used to purchase consumption. If a household believes the deposit market will be thick (thin) and the costs to using deposits small (large), it will hold more (less) deposits resulting in the initial belief to become self-fulfilled. The sunspot shocks resulting from this strategic complementarity will influence nominal interest rates, distort real behavior, and be welfare reducing.

The intermediation technology described above is introduced into an otherwise standard monetary environment where households purchase consumption with both currency and deposits. The model follows Carlstrom and Fuerst (2001) (henceforth, CF) where cash-in-

[1] See Berger and Mester (1999) for references both in support and in contrast to this result. More recently, Wang (2003) and Allen and Liu (2007) have uncovered modest ES in intermediation.

[2] Cooper and Corbae (2002) use an OG model without capital to assess the importance of a banking crisis on the Great Depression. Their shock results in a lower steady state, where this analysis follows the production sector literature and considers fluctuations around a unique steady state.

advance timing allows the nominal interest rate to be interpreted as a tax (i.e. opportunity cost) on goods purchased with currency. Deposits provide an alternative medium of exchange capable of circumventing this tax, but their use incurs a transaction cost. A natural equilibrium condition in models with multiple mediums of exchange is that the marginal cost of using each medium must be equal. Therefore, if ES influences the cost of deposit use, then there exists a link between the source of indeterminacy and the opportunity cost of currency.

The results suggest that equilibrium indeterminacy does not heavily depend on the degree of ES in the intermediary or the size of the intermediation sector, but on monetary policy and the determination of nominal interest rates. When the monetary authority does not target interest rates, as it would when following an endogenous (or exogenous) money growth rule, the nominal rate is influenced by the cost of deposit use and indeterminacy arises *for any* degree of ES. When the monetary authority targets the nominal rate as it would when following a Taylor (1993)-type rule, it targets the opportunity cost of currency (and in equilibrium, deposits). The cost of deposit use is therefore realized and indeterminacy fails to arise for any degree of ES. In other words, when the monetary authority determines the cost of using deposits, it is impossible for household beliefs to become self-fulfilled.

These results have several implications. First, the model suggests that equilibrium indeterminacy arising from the financial intermediation sector is possible even though a small fraction of assets are intermediated and intermediaries possess ES very close to zero (i.e. constant marginal costs). This suggests that the impact of ES in intermediation might be a significant issue even though there has yet to be a consensus in the empirical banking literature on its existence. Second, while providing a situation where policies targeting nominal interest rates or money growth rates *are not* equivalent, this analysis adds an additional condition on subsequent research concerning the quantitative importance of this source of indeterminacy: not only must intermediaries possess ES, but monetary policy must allow (albeit, unknowingly) these intermediaries to deliver indeterminacy. This suggests that there could be many natural experiments in history which can be used to assess the economic significance of this indeterminacy. For example, how much of the decline in US economic volatility observed during the 1980s (termed the Great Moderation) was due to the monetary authority effectively eliminating sunspot shocks from the intermediation sector by switching from a money target to an interest rate target in 1984?

The remainder of the paper proceeds as follows. Section 2 outlines the economy and presents propositions concerning conditions for equilibrium indeterminacy as well as a sensitivity analysis. Section 3 concludes. Proofs are presented in an appendix.

2. The Model

2.1. Environment

The economy consists of a large number of identical and spatially distinct locations (i.e. islands) and a single monetary authority. Each location is populated by numerous identical and infinitely lived households, and a large number of perfectly competitive goods-producing firms and financial intermediaries. The only asset that can travel across locations are nominal bonds issued by the monetary authority, ensuring that each location faces an identical nominal interest rate. Since the analysis focuses on equilibrium determinacy, the environment is simplified to a deterministic economy without loss of generality. If the economy is subject to indeterminacy, it is possible to construct sunspot equilibria.

Households

A representative household has preferences over streams of consumption (c_t) and labor time (ℓ_t) given by

$$\sum_{t=0}^{\infty} \beta^t U\left(c_t, \ell_t\right), \tag{1}$$

where $\beta < 1$ is the discount rate. Instantaneous utility is assumed to take the form: $U(c, \ell) = (c^{1-\sigma} - 1) / (1 - \sigma) - \ell$ with $\sigma \geq 0$.

Period t begins with a household holding nominal financial wealth W_t and physical capital k_t. Capital evolves according to $k_{t+1} = (1 - \delta) k_t + i_t$, where δ denotes the depreciation rate and i_t denotes investment. The financial market opens first, where households receive a lump-sum currency transfer from the monetary authority (X_t) and buy and sell bonds (B_t) which are in zero net supply (across locations) and earn a gross nominal return $1 + R_t$. Any remaining financial wealth takes the form of currency (M_t) such that

$$M_t = W_t + X_t - B_t. \tag{2}$$

Before leaving the financial market, households divide their capital into a deposit with an intermediary (d_t) and a direct loan to the firm (a_t) at gross real returns r_t^d and r_t, respectively. This implies the constraint $k_t = a_t + d_t$.

Consumption can be purchased with both currency and deposits. Currency use is governed by a cash-in-advance constraint

$$P_t c_{1t} \leq M_t, \tag{3}$$

where P_t denotes the aggregate price level and c_{1t} denotes the portion of consumption purchased with currency. Deposit use implies

$$P_t c_{2t} \leq P_t d_t, \tag{4}$$

where $c_{2t}\,(= c_t - c_{1t})$ denotes the portion of consumption purchased with deposits.

It is assumed that checks clear, and interest payments and loans are simultaneously paid at the end of the period. The household's intertemporal constraint is given by

$$P_t\left(c_t+k_{t+1}\right)+W_{t+1}\leq P_t\left(w_t\ell_t+r_ta_t+r_t^dd_t\right)+M_t+\left(1+R_t\right)B_t, \tag{5}$$

where w_t is the real wage rate.

Productive Firms

Goods-producing firms combine rented capital and hired labor to produce according to a CRS technology: $f\left(k_t^T,\ell_t^T\right)=k_t^{T\alpha}\ell_t^{T(1-\alpha)}$, where a superscript T denotes aggregate quantities. Profit-maximization results in the marginal product of each input to equal its marginal cost.

$$f_{k^T}\left(k_t^T,\ell_t^T\right)=r_t-1+\delta \tag{6}$$

$$f_{\ell^T}\left(k_t^T,\ell_t^T\right)=w_t \tag{7}$$

Financial Intermediaries

An intermediary pools household deposits, creates interest-bearing checking accounts, and rents the deposited capital to firms. It is assumed that an intermediary provides no additional services such as loan screening or monitoring. From the perspective of a firm, loans from intermediaries and households are perfect substitutes and therefore must share the same rental rate $\left(r_t\right)$. This prevents the financial intermediary from having monopoly power over loan supply.

In the equilibria examined in this paper, fiat currency and deposits will both be held. Following Williamson (1986), for arbitrage opportunities to be absent and to guarantee intermediaries are solvent, the following must hold in equilibrium:

$$r_t-r_t^d=R_t. \tag{8}$$

This condition states that the opportunity cost of holding currency (R_t) is equal to the opportunity cost of holding capital in the form of deposits $\left(r_t - r_t^d\right)$. In other words, although intermediaries might be monopoly suppliers of deposits, they are not monopoly suppliers of liquidity because a household can choose to make all purchases with currency.

It is assumed that intermediaries have no minimum reserve requirements and lend all of their deposits.[3] The profit function of an intermediary is given by

$$r_t d_t^T - \left[r_t^d d_t^T + C\left(d_t^T\right)\right], \tag{9}$$

where d_t^T is total real deposits and $C\left(d_t^T\right)$ denotes operating costs. If these costs are marginally decreasing, the intermediary will exhibit ES. Assuming $C(D_t) = \Gamma D_t^{1+\theta}$, ES arises for any $\theta \in (-1, 0)$.

There is free entry into intermediation so if an intermediary receives strictly positive profits, another intermediary can enter, offer a higher deposit rate and capture the entire market. Given that competition with households for loans and the monetary authority for liquidity results in intermediaries taking r_t and r_t^d as given, zero profits in (9) generates the following average-cost pricing rule:

$$r_t^d = r_t - \tau_t, \tag{10}$$

where $\tau_t = C(D_t)/D_t = \Gamma D_t^{\theta}$ denotes average operating costs. Zero profits ensures that there will be no equilibrium entry or exit into the intermediation sector. Furthermore, while suggesting that there will be only one financial intermediary per location, competition with other agents prevents the intermediary from exploiting pricing powers usually associated with natural monopolies.[4]

[3] The results presented below are qualitatively unchanged if the intermediary is required to keep a minimum fraction of deposits in reserves.

[4] Of course, one could assume informational asymmetries that would make intermediation essential and allow deposits or loans to be priced in a monopolistically competitive fashion. However, monopolistic competition would still result in intermediaries passing on some of their reduced costs (as in Williamson, 1986), which is sufficient for the main points of this analysis to go through.

The Monetary Authority

The monetary authority's budget constraint is $X_t = W^T_{t+1} - W^T_t$, where W^T_{t+1} denotes the currency base available at each location at the end of period t. The base evolves according to $W^T_{t+1} = \mu_t W^T_t$ where μ_t is the gross growth rate. The analysis considers two specifications of how the monetary authority chooses μ_t: money growth rules and interest rate rules. Under a money growth rule, μ_t is pinned down and the nominal interest rate is determined by market conditions. Under an interest rate rule, μ_t is chosen in order for the nominal interest rate to achieve a specified target.

Equilibrium

An equilibrium is defined as a list of prices $\{P_t,\ R_t,\ r_t,\ w_t\}$ and allocations $\{c_{1t},\ c_{2t},\ \ell_t,\ B_t,\ d_t,\ W_{t+1},\ k_{t+1}\}$ such that: (i) households maximize (1) subject to (3), (4), and (5); (ii) firms maximize profits; (iii) intermediaries follow (10); (iv) all individual and aggregate quantities are equal (e.g. $k_t = k^T_t$); (v) the markets for goods ($f(k_t, \ell_t) = c_t + i_t + C(d_t)$), bonds ($B_t = 0, \forall t$), and money clear; and (vi) $\tau_t = \Gamma d^\theta_t$.

Household optimization is characterized by the binding conditions (3), (4) and (5), and the Euler equations

$$\frac{U_{\ell t}}{U_{ct}} = \frac{w_t}{1+R_t}, \tag{11}$$

$$\frac{U_{ct}}{1+R_t} = \beta r_{t+1} \frac{U_{ct+1}}{1+R_{t+1}}, \tag{12}$$

$$\frac{U_{ct}}{1+R_t} = \beta \frac{1+R_{t+1}}{\pi_{t+1}} \frac{U_{ct+1}}{1+R_{t+1}}, \tag{13}$$

$$R_t = r_t - r^d_t, \tag{14}$$

where $\pi_t = P_t/P_{t-1}$ is the inflation rate. Equations (11) through (13) equate the costs and benefits of marginal increases in ℓ_t, k_{t+1}, and W_{t+1}, respectively. Since labor income is not available for period t consumption purchases, (11) illustrates the cash versus non-cash distortion and allows the nominal rate to be interpreted as a consumption tax.

Equation (14) states that the optimal composition of currency versus deposit-goods is chosen such that the opportunity costs for using deposits $\left(r_t - r_t^d\right)$ and currency (R_t) are equal. This condition is identical to (8) and is necessary for an interior solution to the household's problem. Substitution of (14) into the intermediary's pricing function (10) results in

$$R_t = \tau_t \tag{15}$$

and states that the cost per deposit is equal to the opportunity cost per unit of currency. Removing τ_t with the intermediary's cost function results in

$$R_t = \Gamma d_t^{\theta}. \tag{16}$$

Equation (16) suggests that any change in marginal deposit costs must also influence the nominal rate (and vice-versa). Indeterminacy will therefore crucially depend on the ability for changes in deposit balances to influence nominal rates. Since nominal rates may be dictated by monetary policy, indeterminacy may depend on the type of monetary policy in use.

2.2. Equilibrium Dynamics

The baseline dynamics are assessed under two extreme versions of monetary policy. First, monetary policy is assumed to be exogenous $(\mu_t = \mu, \forall t)$.[5] Second, the monetary authority follows a Taylor-type rule where the nominal rate responds to past inflation,

$$1 + R_t = (1 + R)\left(\frac{\pi_{t-1}}{\pi}\right)^{\omega}, \tag{17}$$

[5] A following section considers endogenous money growth rules and confirms that with the exception of policy parameters, the results are identical to those under an exogenous rule.

where π denotes the long-run (steady state) inflation rate and $(1 + R) = \pi/\beta$. This 'backward-looking' rule with $\omega > 1$ was shown by CF to deliver equilibrium determinacy in a model without intermediation.[6]

The results under these two policies are formalized in the following propositions.

Proposition 1 *Under exogenous monetary policy, equilibrium indeterminacy arises for any θ satisfying*

$$\max\left\{-1, -\sigma\frac{1+R}{R}\frac{d}{c}\right\} < \theta < 0.$$

Proof: See Appendix.

The proof uses (16) to substitute R_t out of the dynamic system resulting in the eigenvalue associated with inflation to be dependent on θ. ES in the intermediary then delivers indeterminacy as long as θ satisfies the above condition. While leaving the discussion for later, this condition is only restrictive when there is no interior solution and no assets are being intermediated (i.e. $d = 0$).

For intuition on how indeterminacy arises in the model, consider the depositing decision of a household when the financial intermediary exhibits ES. Given that (10) is anticipated, a household decides how much consumption to purchase with their present currency holdings and how much to purchase with deposits. A strategic complementarity emerges from this decision: the more households choose to deposit, the higher the returns to using deposits (all else equal). Therefore, the size of the deposit market is determined in a non-cooperative fashion by the simultaneous choices of the (identical) households. If a household believes the size of the deposit market will be large (small) and the net returns on deposits high (low), it will hold more (less) deposits resulting in smaller (larger) deposit costs and higher (lower) effective deposit returns. The belief is supported by intermediaries possessing ES and becomes self-fulfilled.

[6] In addition to real indeterminacy, CF define nominal indeterminacy as an inability to pin down equilibrium prices. This analysis restricts attention to policies delivering both real and nominal determinacy.

Proposition 2 *If the monetary authority follows an interest rate rule*

$$1 + R_t = (1 + R) \left(\frac{\pi_{t-1}}{\pi} \right)^{\omega}$$

with $\omega > 1$, *equilibrium indeterminacy fails to arise for any* $\theta \in [-1, 0)$.

Proof: See Appendix.

With R_t predetermined due to a predetermined inflation rate, equation (16) removes θ from the dynamic system and the proof exactly follows that of CF's proposition pertaining to determinacy under the same policy rule. Although households hold deposits in equilibrium, determinacy is independent of θ and depends solely on the monetary authority's response to changes in past inflation (i.e. $\omega > 1$). This suggests that even though the intermediary may possess a large degree of ES, indeterminacy is eliminated when the monetary authority determines the cost to using currency, and in equilibrium, deposits. With the cost of deposits pinned down, it is not possible for belief shocks from the households to become self-fulfilled.

These results suggest that indeterminacy arising from the intermediary sector crucially depends on how nominal interest rates are determined. While this is the main point of the analysis, there are some finer details worth noting. First, when monetary policy is exogenous, Proposition 1 states that indeterminacy is possible for rather lax conditions: (i) that some assets be intermediated ($d > 0$), and (ii) $\theta < 0$. To illustrate the first condition, under log utility ($\sigma = 1$) and R equal to 6 percent ann., a deposit-currency ratio (d/c) greater than 0.0145 makes $-\sigma(1 + R)d/Rc < -1$ and θ can then take any value between 0 and -1. This deposit-consumption ratio is far below the post-war US value, and suggests indeterminacy is possible for *any* degree of ES without further conditions on the intermediation sector (e.g. relative sector size, direct-intermediated loan ratios, etc.).[7] Considering the debate in the empirical literature concerning ES in the intermediation sector referenced in the introduction,

[7] The post-war average of 'Demand Deposits at Commercial Banks' (series name: DEMDEPSL) divided by 'Personal Consumption Expenditures' (series name: PCE) is over 0.15, with a minimum of 0.029. Sources: Federal Reserve Board of Governors and the BEA.

small degrees of ES (i.e. θ close to zero) may be difficult to significantly estimate but could still deliver indeterminacy. Second, when monetary policy follows an endogenous interest rate rule, the determinacy of the economy exactly mimic those of CF. In other words, an interest rate rule with $\omega < 1$ would deliver indeterminacy even in the absence of ES (i.e. $\theta > 0$) because the monetary authority is accommodating changes in inflation. Finally, having θ enter into the inflation equation of the dynamic system indicates that although the deposit cost τ is real, indeterminacy enters through nominal channels by allowing inside money and prices to be susceptible to belief shocks. This stands in sharp contrast to the literature on indeterminacy from the production sector which focuses on real channels.

2.3. Sensitivity Analysis

Alternative monetary policies can be considered in order to assess the robustness of the results. First, μ_t can be endogenous and allowed to respond to changes in inflation, $\mu_t = \mu\left(\pi_t/\pi\right)^{\omega}$, or in nominal rates, $\mu_t = \mu\left(\frac{1+R_t}{1+R}\right)^{\omega}$,with elasticity ω. The exogenous policy considered previously is a special case of either rule with $\omega = 0$. Given $\omega < 1$, which is the condition insuring that the policy itself is not a source on indeterminacy, the results as well as the proofs for both rules are exactly the same as Proposition 1.[8] If the nominal rate responds to changes in deposits, indeterminacy arises for any degree of ES. Note that although the nominal rate appears in the second rule, it is not being targeted so the intuition under exogenous monetary policy goes through.[9]

To assess the results under interest rate rules, consider a rule using more than past inflation to establish the target.

$$1 + R_t = (1 + R)\left(\frac{\pi_t}{\pi}\right)^{\omega_1}\left(\frac{\pi_{t-1}}{\pi}\right)^{\omega_2} \qquad (18)$$

[8] Although the sensitivity results are stated in this section, they are formally stated in proposition form and proved in the appendix.

[9] 'Backward-looking' money growth rules result in bifurcations and are not considered (see Azariadis, 1993, ch 8).

Although this rule implies that R_t is not entirely predetermined, the result is identical to Proposition 2. In particular, $\omega_2 > 1$ is necessary for the policy rule to not be a source of indeterminacy, and this condition is independent of θ. Although the nominal rate is allowed to respond to current inflation, the result is identical to the result under a 'backward-looking' interest rate rule.[10]

3. Conclusion

The goal of this paper was to establish conditions for ES in the financial intermediation sector to deliver equilibrium indeterminacy in a monetary environment. While it is reasonable to believe that more sophisticated models (featuring non-linear leisure preferences, nominal rigidities, etc.) may yield additional conditions for indeterminacy to arise, if these environments possess multiple mediums of exchange then their marginal costs of use must be identical in equilibrium. Since this feature is central to the results presented here, it suggests that indeterminacy will be eliminated when nominal rates are targeted in these environments as well.

This result warrants some discussion. First, the model provides an example where interest rate rules and money growth rules are not equivalent policies, and the choice of the policy target may be important. Second, since the analysis identifies the class of policies which allow intermediation to deliver indeterminacy, research on the quantitative importance of this indeterminacy must be restricted to be in this class. In other words, when is it reasonable to believe that such (non-interest rate targeting) monetary policies were in use? For the US, policies other than explicit interest rate rules were in use before 1979 and during the monetary aggregate targeting experiment from 1979 to 1982 (see Meulendyke, 1989, ch 2). For other countries, particularly small-open economies, policies occasionally focus on global variables such as exchange rates rather than domestic interest rates. It is encouraging that

[10] Purely 'current-looking' or 'forward-looking' interest rate rules deliver (real and / or nominal) indeterminacy for all parameterizations and are not considered (see CF).

these candidates are associated with relatively large degrees of observed economic volatility. For example, the US economy experienced a large decline in economic volatility (referred to as the Great Moderation) around the same time the Federal Reserve began targeting nominal interest rates (see Blanchard and Simon, 2001). Since the results presented here suggest that the adoption of a nominal interest rate target may have removed intermediation as a source of equilibrium indeterminacy, then the reduction in economic volatility might be partly attributable to the removal of these sunspot shocks. It would therefore be of interest to assess the quantitative impact of indeterminacy arising from the intermediation sector on more sophisticated environments, as well as assessing the quantitative importance using these candidate episodes. These issues are presently being explored.

Appendix

Proposition 1 *Under exogenous monetary policy, equilibrium indeterminacy arises for any θ satisfying*

$$\max\left\{-1, -\sigma\frac{1+R}{R}\frac{d}{c}\right\} < \theta < 1.$$

Proof. Let $x_t = \ell_t / k_t$. The first order conditions, normalized by the currency base, are given by

$$\begin{aligned}
\frac{x_t^{\alpha}}{1-\alpha} &= \beta\frac{P_t}{\mu P_{t+1}}\left(1+R_{t+1}\right)\frac{x_{t+1}^{\alpha}}{1-\alpha}, \\
\frac{x_t^{\alpha}}{1-\alpha} &= \beta\left[\alpha x_{t+1}^{1-\alpha} + 1 - \delta\right]\frac{x_{t+1}^{\alpha}}{1-\alpha}, \\
k_{t+1} &= k_t x_t^{1-\alpha} + \left(1-\delta\right)k_t - c_{1t} - c_{2t} - \tau_t d_t, \\
\frac{U_{ct}}{1+R_t} &= \frac{x_t^{\alpha}}{1-\alpha} \\
P_t^{-1} &= c_{1t},\ d_t = c_{2t},\ R_t = \tau_t = \Gamma d_t^{\theta}.
\end{aligned}$$

Substituting c_{1t}, c_{2t}, d_t, and R_t out of the system results in the first three equations comprising the log-linearized system:

$$\begin{bmatrix} 1-\beta\left(1-\alpha\right)\left(1-\delta\right) & 0 \\ \alpha - \frac{\theta R}{1+R}A_1 & \frac{\theta R}{1+R}A_2 - 1 \end{bmatrix}\begin{bmatrix} \hat{x}_{t+1} \\ \hat{P}_{t+1} \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ \alpha & -1 \end{bmatrix}\begin{bmatrix} \hat{x}_t \\ \hat{P}_t \end{bmatrix}$$

where

$$A_1 = \frac{\alpha}{\sigma}\left[\frac{c_2}{c} + \frac{\theta R}{\sigma\left(1+R\right)}\right]^{-1} \text{ and } A_2 = \frac{c_1}{c}\left[\frac{c_2}{c} + \frac{\theta R}{\sigma\left(1+R\right)}\right]^{-1},$$

and the resource constraint $\hat{k}_{t+1} = H\left(\hat{x}_t, \hat{P}_t, \hat{k}_t\right)$. The resource constraint can be uncoupled from the remaining system and delivers one eigenvalue $e_1 = \left[1-\beta\left(1-\alpha\right)\left(1-\delta\right)\right]/\alpha\beta > 1$. The roots of the remaining system are given by

$$e_2 = \alpha / \left[1-\beta\left(1-\alpha\right)\left(1-\delta\right)\right] < 1, \text{ and } e_3 = \left[1 - \frac{\theta R}{1+R}A_1\right]^{-1}.$$

Since there is only one predetermined variable to the system, indeterminacy of the system depends on e_3 lying inside the unit circle or if $\theta A_1 < 0$. Economies of scale implies $\theta < 0$. Therefore, $A_1 > 0$ with $d = c_2$ implies $-\sigma \frac{(1+R)}{R} \frac{d}{c} < \theta$. ■

Proposition 2 *If the monetary authority follows an interest rate rule*

$$1 + R_t = (1 + R) \left(\frac{\pi_{t-1}}{\pi} \right)^{\omega}$$

with $\omega > 1$, *equilibrium indeterminacy fails to arise for any* $\theta \in (-1, 0)$.

Proof. It is easier for the first order conditions to be cast in real terms and given by

$$\frac{x_t^{\alpha}}{1-\alpha} = \beta \frac{(1 + R_{t+1})}{\pi_{t+1}} \frac{x_{t+1}^{\alpha}}{1-\alpha},$$
$$\frac{x_t^{\alpha}}{1-\alpha} = \beta \left[\alpha x_{t+1}^{1-\alpha} + 1 - \delta \right] \frac{x_{t+1}^{\alpha}}{1-\alpha},$$
$$k_{t+1} = k_t x_t^{1-\alpha} + (1-\delta) k_t - c_{1t} - c_{2t} - \tau_t d_t,$$
$$\frac{U_{ct}}{1 + R_t} = \frac{x_t^{\alpha}}{1-\alpha}$$
$$d_t = c_{2t},\ R_t = \tau_t = \Gamma d_t^{\theta},\ 1 + R_t = (1 + R) \left(\frac{\pi_{t-1}}{\pi} \right)^{\omega}.$$

Following the steps as in Proposition 1, the log-linearized system is given by

$$\begin{bmatrix} 1 - \beta(1-\alpha)(1-\delta) & 0 & 0 \\ \alpha & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_{t+1} \\ \hat{\pi}_{t+1} \\ \hat{\pi}_t \end{bmatrix} = \begin{bmatrix} \alpha & 0 & 0 \\ \alpha & -\omega & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_t \\ \hat{\pi}_t \\ \hat{\pi}_{t-1} \end{bmatrix}$$

and the resource constraint $\hat{k}_{t+1} = H\left(\hat{x}_t, \hat{P}_t, \hat{k}_t\right)$. The resource constraint can again be uncoupled from the remaining system and delivers one eigenvalue $e_1 = [1 - \beta(1-\alpha)(1-\delta)] / \alpha\beta > 1$. The roots of the remaining system are given by

$$e_2 = \alpha / [1 - \beta(1-\alpha)(1-\delta)] < 1,\ e_3 = 0 < 1, \text{ and } e_4 = \omega.$$

Since there are two predetermined variable to the system, determinacy of the system depends on e_4 lying outside the unit circle or $\omega > 1$. This condition is independent of θ. ■

Proposition 3 *If the monetary authority follows an endogenous money growth rule*

$$\mu_t = \mu \left(\frac{1 + R_t}{1 + R} \right)^{\omega}$$

with $\omega < 1$, equilibrium indeterminacy arises for any θ satisfying

$$\max \left\{ -1, -\sigma \frac{1 + R}{R} \frac{d}{c} \right\} < \theta < 0.$$

Proof. Following the proof to Proposition 1, the log-linearized system is given by

$$\begin{bmatrix} 1 - \beta (1 - \alpha) (1 - \delta) & 0 \\ \alpha - \frac{(1-\omega)\theta R}{1+R} A_1 & \frac{(1-\omega)\theta R}{1+R} A_2 - 1 \end{bmatrix} \begin{bmatrix} \hat{x}_{t+1} \\ \hat{P}_{t+1} \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ \alpha & -1 \end{bmatrix} \begin{bmatrix} \hat{x}_t \\ \hat{P}_t \end{bmatrix}$$

where

$$A_1 = \frac{\alpha}{\sigma} \left[\frac{c_2}{c} + \frac{\theta R}{\sigma (1 + R)} \right]^{-1} \text{ and } A_2 = \frac{c_1}{c} \left[\frac{c_2}{c} + \frac{\theta R}{\sigma (1 + R)} \right]^{-1},$$

and the resource constraint $\hat{k}_{t+1} = H \left(\hat{x}_t, \hat{P}_t, \hat{k}_t \right)$. The resource constraint again delivers one eigenvalue $e_1 = [1 - \beta (1 - \alpha) (1 - \delta)] / \alpha\beta > 1$. The roots of the remaining system are given by

$$e_2 = \alpha / [1 - \beta (1 - \alpha) (1 - \delta)] < 1, \text{ and } e_3 = \left[1 - \frac{(1 - \omega) \theta R}{1 + R} A_1 \right]^{-1}.$$

Indeterminacy of the system depends on e_3 lying inside the unit circle or if $(1 - \omega) \theta A_1 < 0$. Economies of scale implies $\theta < 0$. Therefore, $A_1 > 0$ and $\omega < 1$ with $d = c_2$ implies $-\sigma \frac{(1+R)}{R} \frac{d}{c} < \theta$. ■

Proposition 4 *If the monetary authority follows an interest rate rule*

$$1 + R_t = (1 + R) \left(\frac{\pi_t}{\pi} \right)^{\omega_1} \left(\frac{\pi_{t-1}}{\pi} \right)^{\omega_2}$$

with $\omega_2 > 1$, equilibrium indeterminacy fails to arise for any $\theta \in (-1, 0)$.

Proof. Following the proof to Proposition 2, the log-linearized system is given by

$$\begin{bmatrix} 1-\beta(1-\alpha)(1-\delta) & 0 & 0 \\ \alpha & \omega_1 - 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_{t+1} \\ \hat{\pi}_{t+1} \\ \hat{\pi}_t \end{bmatrix} = \begin{bmatrix} \alpha & 0 & 0 \\ \alpha & -\omega_2 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_t \\ \hat{\pi}_t \\ \hat{\pi}_{t-1} \end{bmatrix}$$

and the resource constraint $\hat{k}_{t+1} = H\left(\hat{x}_t, \hat{P}_t, \hat{k}_t\right)$. The resource constraint again delivers one eigenvalue $e_1 = [1-\beta(1-\alpha)(1-\delta)]/\alpha\beta > 1$. The roots of the remaining system are given by

$$e_2 = \alpha/[1-\beta(1-\alpha)(1-\delta)] < 1, \; e_3 = 0 < 1, \text{ and } e_4 = \omega_2.$$

Since there are two predetermined variable to the system, determinacy of the system depends on e_4 lying outside the unit circle or $\omega_2 > 1$. As in Proposition 2, this condition is independent of θ. ■

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