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Firms’ Procurement Decisions: Is Input Specificity Always an Argument for Vertical Integration?*

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Abstract

The transaction cost theory predicts that firms are inclined to vertically integrate transactions in response to the specificity of their required inputs. Yet, reality proves that some firms engage in repeated transactions with external suppliers aimed at procuring highly specific inputs. To explain this phenomenon, this paper investigates a firm’s make-or-buy decision in a context with relational (i.e. non-enforceable) contracts, and exposes how this decision is affected by the required input specificity. This paper demonstrates that a high degree of input specificity can lead to repeated market transactions being favored over vertical integration because demanding more specific inputs is shown to (i) impose lower costs on firms to maintain repeated market transactions founded on relational contracts; and (ii), facilitate the self-enforcement of these relational contracts.

Keywords: Input specificity, vertical integration, market transactions, relational contracts, transaction cost theory.

JEL classification: D23, L22, L23

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1 Introduction

Since the seminal work of Coase [1937], a stream of economic literature has emerged to explain why firms exist and utilize non-market in place of market transactions.\(^1\) However, a fundamental question remains relatively unanswered: What are the conditions driving firms to integrate transactions? As Williamson [1971, 1979, 1985] and Klein, Crawford, and Alchian [1978] argued, high levels of quasi-rents due to relation-specific investments increase the likelihood of vertical integration.\(^2\) In the same vein, the procurement of highly specific inputs requires relation-specific investments such that input specificity can be considered as an important driver for vertical integration. Market transactions in competitive markets thus appear to be beneficial so long as buyers do not rely on relatively specific inputs. However, if firms were to require highly specific inputs, the value of market transactions appears to be limited because supplying firms may not make the necessary relation-specific investments in anticipation of hold-up.\(^3\) This suggests a strong relationship between the degree of input specificity and firms’ tendency to vertically integrate transactions. For instance, as reported by Monteverde and Teece [1982], car manufacturers like Ford and General Motors procure standardized inputs like mirrors, carpeting, safety belts, and wires from external suppliers. In contrast, engines and automatic transmissions—which are highly specific inputs that must be tailored to car manufacturers’ particular technical requirements—are internally produced.

Contrary to this prediction however, the dependency on highly specific inputs does not always lead to vertical integration. For example, Boeing and Airbus rely on highly specific turbofan engines for producing commercial airplanes. According to the transaction cost theory, we would expect both firms to vertically integrate the production of aircraft engines.


\(^2\)For more thoroughly discussions refer to Riordan and Williamson [1985], Joskow [1988], and Demsetz [1988].

\(^3\)The Fisher Body - General Motors case reported in Klein et al. [1978] and further analyzed by Klein [1988, 2000] is a widely used example for illustrating hold-up. For a summary of extreme examples of hold-up see Shavell [2005].
Instead, Boeing and Airbus acquire custom-tailored engines from external producers: Rolls Royce or CFM International (a joint venture between General Electric and the French company Snecma). The relationships between the suppliers of aircraft engines—Rolls Royce and CFM—and their customers—Boeing and Airbus—are characterized by (incomplete) long-term contracts imposing significant relation-specific investments on the supplying side. For example, Rolls Royce tailored the turbofan engine Trent XWB to the specific requirements of the Airbus A350 XWB family, and developed the Trent 900 series to power exclusively the new Airbus A380. Furthermore, CFM designed the turbofan engine CFM56-3 exclusively for Boeing aircrafts. Other engines produced by CFM, like the CFM56-5 series, are only used by Airbus for its commercial airplanes. Chiu [1998] makes a similar observation and argues that the correlation between relation-specific investments and vertical integration is not as strong as the theory predicts.\(^4\) Two questions therefore emerge: First, how does the degree of input specificity affect a firm’s input procurement? Second, can input specificity serve as a rationale for the aforementioned counterintuitive phenomenon? This paper endeavors to answer these questions by elaborating on a firm’s make-or-buy decision in light of the specificity of required inputs. By shedding light on how a firm’s input procurement is affected by the desired input specificity, this study closes a prevailing gap in the literature on the theory of the firm, and thus enhances our understanding of a firm’s choice for inter- vs. intra-firm transactions.

By utilizing a framework similar to the one developed by Baker, Gibbons, and Murphy [2002], this paper demonstrates that a firm relying on highly specific inputs can indeed favor repeated market transactions over vertical integration. The rationale for this observation is as follows. First, demanding more specific inputs impairs supplying firms’ bargaining positions in repeated trading relationships. This in turn imposes lower costs on demanding firms to maintain repeated market transactions founded on relational contracts. Second, a higher degree of input specificity facilitates the self-enforcement of relational contracts between firms, thereby inducing the efficient level of relation-specific investments. Thus, this paper offers a

\(^4\)Whinston [2003] points to the same observation by noting that some firms even increase their mutual dependency e.g. by agreeing upon exclusive contracts.
theoretical underpinning of the seemingly contradictory phenomenon that some firms utilize repeated market transactions for procuring highly specific inputs.

The framework analyzed in this paper is characterized by two distinctive features. First, this study considers a firm’s make-or-buy decision in a context with relational contracts. Generally, relational (or implicit) contracts refer to contracts, for which some elements are not enforceable by third parties [MacLeod and Malcomson, 1989], or are prohibitively costly to specify *ex ante* [Baker et al., 2002]. As documented in contemporary literature, relational contracts need to be self-enforcing in repeated games in order to eliminate opportunistic behavior. In particular, the framework in this paper comprises relational contracts within firms as analyzed by Bull [1987], Pearce and Stacchetti [1998], and Levin [2003]; and between firms as considered by Telser [1980], Klein and Leffler [1981], and Itoh and Morita [2006]. Second, to model varying degrees of input specificity, the productive party is assumed to be charged with implementing multidimensional effort. An underlying premise of this modeling technique resides in the fact that the implementation of differential effort allocations leads to intermediate products with distinguishable characteristics. An intuitive example concerns the implementation of enterprise-oriented technological solutions (e.g., SAP) in consultation with either an internal IT department or external IT consultants. The specific configuration of these technological solutions encompasses several components, and is thus multidimensional. By emphasizing different components, IT specialists have the choice of devising a generic technological solution which can be adopted by several firms versus one which matches the unique requirements of a particular firm (but may be unserviceable for others). Therefore, the implementation of different effort allocations (e.g., emphasizing varying combinations of technical components) can yield intermediate products (e.g., technological solutions) with distinctive characteristics. This approach allows the utilization of a simple and intuitive measure of input specificity, which keeps the analysis in this paper highly tractable. More precisely, the employed measure captures the extent to which the effort allocation—and thus the specific characteristics of the input—desired by a particular firm deviates from the one required by other market participants.
As emphasized above, the framework in this paper builds partially on the model devised by Baker et al. [2002], who analyzed the efficiency of vertical integration versus non-integration within the context of relational contracts. Particularly, they investigated how asset ownership facilitates the attainment of superior relational contracts within and between firms. Despite being rooted in the model developed by Baker et al. [2002], the present conceptualization differs in two fundamental aspects. First, contrary to Baker et al. [2002], the worker (in case of integration) lacks sufficient financial resources such that making him an autonomous input supplier (outsourcing) by selling him the asset is not feasible. For the approach pursued in this paper, non-integration refers to transactions performed with an existing market participant acting as an independent input supplier. Although this difference does not appear to be substantial at first glance, it is shown to exert a significant impact on a firm’s make-or-buy decision. A potential transfer price of the asset should naturally affect a firm’s outsourcing decision, which in turn will be highly sensitive to the applied valuation method of the asset. Because Baker et al. [2002] focused on the efficiency of alternative organizational forms (integration vs. non-integration) by comparing the total surplus generated under each alternative, this ‘asset valuation problem’ does not arise in their framework. By abstracting from incentive effects associated with asset ownership, it is well known that transferring the property rights of an asset at a certain price determines the allocation of economic rents, but leaves the entire surplus unaffected. The objective of this paper, however, is to shed light on how a firm’s make-or-buy decision is affected by the desired degree of input specificity. In this respect, the allocation of rents does matter. Further, the incorporation of a limited liability constraint on the side of the worker not only better reflects the trend that workers rarely possess sufficient resources to act as autonomous input suppliers, it is also a viable means to make the analysis in this paper and its findings independent of potential asset valuation methods. Finally, to identify a firm’s optimal make-or-buy decision, this paper compares the profitability of alternative methods of input procurement from a firm’s

More precisely, it is crucial whether the transfer price is assumed to reflect the market value of the asset, or the present value of the worker’s future profits which he expects to obtain as an independent input supplier.
perspective, which constitutes the second fundamental difference to Baker et al. [2002] as they contrast respective efficiencies from a social perspective.

Extant literature on the theory of the firm focused on two important drivers for vertical integration. Firstly, the property rights approach as devised by Grossman and Hart [1986], Hart and Moore [1990], and Bolton and Whinston [1993], paid attention to the efficient allocation of asset ownership as a means to induce sufficient relation-specific investments. The efficient ownership structure of assets in a context with relational contracts is further investigated by Garvey [1995], Baker, Gibbons, and Murphy [2001], Bragelien [2001], Baker et al. [2002], and Halonen [2002]. Second, asset specificity as a potential explanatory contribution to the theory of the firm—originated in the modern transaction cost theory a lá Williamson [1971, 1979, 1985]—is thoroughly analyzed by Riordan and Williamson [1985], Suzuki [2005], Kvaløy [2007], and Ruzzier [2007]. Asset specificity governs incentives for the contracting parties to behave opportunistically by taking advantage of the fact that investments in relation-specific assets are not entirely reversible. Accordingly, the need for significant investments in relation-specific assets is deemed as an argument for vertical integration.

This paper is organized as follows. Section 2 provides an overview of the analyzed economic environment. The optimal contracts under a firm’s alternative input procurements are derived in section 3. In section 4, a firm’s optimal make-or-buy decision is identified; and

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6 Input specificity differs from asset specificity in two principal aspects. First, using specific assets is not a necessity for custom-tailoring intermediate products. Even a generic asset can be utilized to produce highly specific intermediate products. The key is how this asset is being utilized in the production process. As a simple example consider the customization of marble plates for kitchen counters or window sills. The asset required to cut marble plates—a diamond saw—can be utilized to produce plates ranging from standard to highly specific sizes. In this sense, firms can produce intermediate products characterized by different degrees of specificity without necessarily investing in specific assets. Second, in contrast to the acquisition of specific assets, producing specific intermediate products is a repeated investment decision (which also involves the transfer of associated property rights), implying that a firm is not entirely tied to these (relation-specific) investments in the future.

7 See also the discussion by Masten [1984] and Whinston [2003].
investigated in light of how it is affected by different required degrees of input specificity. Section 5 summarizes the main results and concludes.

## 2 The Model

Consider a risk-neutral market participant (firm) henceforth referred to as the downstream party. In every period, the downstream party requires an intermediate product to sustain her production. The downstream party owns an asset which can be utilized to produce this input. However, the downstream party lacks either the ability or the time to produce the input by herself.

The downstream party can choose among two alternatives for procuring the required input: (i) she can purchase the input from another risk-neutral market participant (firm) henceforth referred to as the upstream party (market transaction); or (ii), she can vertically integrate its production (integration). The upstream party owns an identical asset as the downstream party such that their production technologies are comparable. If the downstream party decides to integrate the production of the required input, she depends on a worker as productive party. The worker is risk-neutral and financially constrained. For parsimony, his reservation utility is zero.

All parties interact for an infinite number of periods and share the same interest rate \( r \).\(^8\) In every period, depending on the chosen type of transaction, either the worker \((W)\) or the upstream party \((U)\) produces the input by implementing non-verifiable effort \( e_i = (e_{i1}, e_{i2})^T \in \mathbb{R}^{2+}, \ i = W, U \).\(^9\) Effort imposes strictly convex increasing costs \( C(e_i) = c_i e_i^T e_i / 2, \ i = W, U, \) where for parsimony \( c_W = 1 \). To reflect potential agency costs of production for the upstream party, let \( c_U \geq 1 \).

The characteristics of the input are determined by the implemented effort allocation. Let \( \mu = (\mu_1, \mu_2)^T \in \mathbb{R}^{2+} \) and \( \omega = (\omega_1, \omega_2)^T \in \mathbb{R}^{2+} \) represent the relative effort allocation—

\(^{8}\text{Infinitely living parties can be obtained by assuming overlapping generations of individuals who in turn live only a certain number of periods [Thomas and Worrall, 1988].}\)

\(^{9}\text{All vectors are column vectors, where ‘}T\text{’ denotes the transpose.}\)
and therefore the attributes of the input—desired by the downstream party and other market participants, respectively. Consequently, the degree of input specificity required by the downstream party can then be measured by the geometric relation of $\mu$ and $\omega$: the angle $\varphi$. The greater $\varphi$ is, the more specific is the intermediate product required by the downstream party. The degree of input specificity—measured by $\varphi$—is exogenously determined by the downstream’s processing technology which the input is required for.\footnote{I discuss in section 5 a firm’s choice with respect to the specificity of required inputs if the production technology allows to process intermediate products with different characteristics.} To reduce the notational burden, it is assumed that $\|\mu\| = \|\omega\| = 1$, i.e. the lengths of $\mu$ and $\omega$ are normalized to one.

As mentioned above, the implemented effort allocation—and hence the properties of the input—determines its value for the downstream party and for other market participants. In particular, the downstream’s (internal) value $I$ and the market (external) value $E$ can be either high (indexed by $H$) or low (indexed by $L$), where $\Delta I \equiv I_H - I_L$ and $\Delta E \equiv E_H - E_L$. The internal and external input values are observable by all involved entities, but non-verifiable by third parties. Let

$$\begin{align*}
\text{Prob}\{ I = I_H | e_i \} &= \min\{ \mu^T e_i, 1 \}, \\
\text{Prob}\{ E = E_H | e_i \} &= \min\{ \omega^T e_i, 1 \}
\end{align*}$$

be the conditionally independent probabilities that the high internal and high external input values will be realized. Observe that maximizing the expected internal value requires a different effort allocation than maximizing the expected external value if $\mu$ and $\omega$ are linearly independent. In this case, the downstream party requires a specific input with certain attributes diverging from those desired by other market participants. Independent of the realized input value however, the downstream party always prefers an internal use to sustain her production. Formally, this requires that $I_H > I_L \geq E_H > E_L$, where $E_L$ is normalized to zero. Finally, to ensure interior solutions, it is assumed that $\Delta I, \Delta E < 1$.\footnote{Alternatively, one can let $\Delta I, \Delta E > 1$ by assuming that $\mu^T \mu$ and $\omega^T \omega$ are sufficiently small. In this case, the lengths of $\mu$ and $\omega$ appear in the subsequent solutions. Since this does not provide additional insights, I opted for the first alternative for parsimony purposes.}
For both types of transactions being considered—vertical integration and market exchange—the downstream party can utilize either a spot contract (spot integration, spot market), or a relational contract contingent on the non-verifiable, but observable input values (relational integration, relational market). If the involved parties agree upon a relational contract, they play a grim trigger strategy: Once they detect a violation of implicit obligations, they will never rely on relational contracts with the violator again.

3 Alternative Input Procurements

In the subsequent sections, I elaborate on the downstream’s alternatives to procure the required input: (i) spot integration, (ii) relational integration, (iii) spot market; and (iv), relational market. This eventually allows me to identify the optimal input procurement from the downstream’s perspective, and to illustrate the effect of input specificity on her make-or-buy decision.

3.1 Spot Integration

Consider first the case where the downstream party recruits the worker as the productive party for one period. Since verifiable information about the realized input value is not available, the downstream party cannot provide the worker with a court-enforceable incentive contract. Nevertheless, the downstream party can promise the worker to pay a bonus in the event that the high internal input value $I_H$ is realized. Once this occurs however, the downstream party can take the input without paying the promised bonus since she owns the asset and possesses the associated property rights [Grossman and Hart, 1986, Hart and Moore, 1990]. Anticipating this opportunistic behavior, the worker maximizes his utility by implementing $e_W = (0, 0)^T$ such that the downstream party obtains $\Pi_{D|SI} = I_L$ under spot integration.
3.2 Relational Integration

If the downstream party repeatedly interacts with the worker, a relational incentive contract can be self-enforcing, and therefore credible from the worker’s perspective. In particular, to motivate effort, the downstream party can promise to pay the worker a bonus $\beta$ in addition to a fixed transfer $\alpha$ in the event that the high internal input value $I_H$ is realized. As a consequence of the implicit nature of this bonus contract, the downstream’s promise to pay $\beta$ needs to be reliable. Let $\tilde{\Pi}^{RI} \equiv \max\{\Pi^D|SI, \Pi^D|SM, \Pi^D|RM\}$ denote the downstream’s expected profit obtained under her best alternative after violating the relational employment contract with the worker. After behaving opportunistically by reneging on $\beta$, the downstream party can henceforth either choose spot integration ($SI$), or engage in market transactions with the upstream party based upon explicit ($SM$) or implicit ($RM$) contracts. Suppose for a moment that the high internal input value $I_H$ is eventually realized. Then, the downstream party honors her implicit obligation to pay $\beta$ if

$$-\beta + \frac{\Pi^{D|RI}}{r} \geq \frac{\tilde{\Pi}^{RI}}{r}. \tag{1}$$

The left side of (1) represents the downstream’s expected payoff when she delivers on her promise, i.e. paying the bonus $\beta$ but obtaining the expected profit under relational integration $\Pi^{D|RI}$ in the future. To be deterred from reneging, this payoff needs to be greater than the present value of her best fallback position $\tilde{\Pi}^{RI}$. If this self-enforcement condition is satisfied, the worker anticipates the downstream party to deliver on her promise to pay $\beta$ whenever $I = I_H$ such that he is motivated to implement effort.

The optimal relational employment contract maximizes the difference between the expected internal input value and the worker’s expected wage payment. Hence, the downstream’s problem can be formalized as follows:\textsuperscript{12}

\textsuperscript{12}Maximizing the downstream’s expected profit for a single period is equivalent to maximizing the present value of all future expected profits. This can be deduced because reneging does not occur in the reputational equilibrium, which in turn implies that the downstream’s expected profits for every single period are identical.
\[
\max_{\alpha, \beta, e_W} \Pi^D_{RI} = I_L + \Delta I e_W^T e_W - \alpha - \beta e_W^T e_W \tag{2}
\]

s.t.
\[
\alpha + \beta e_W^T e_W - \frac{1}{2} e_W^T e_W \geq 0 \tag{3}
\]
\[
e_W \in \arg\max_{\tilde{e}_W} \alpha + \beta \tilde{e}_W^T e_W - \frac{1}{2} \tilde{e}_W^T \tilde{e}_W \tag{4}
\]
\[
\alpha + \beta \geq 0 \tag{5}
\]
\[
\alpha \geq 0 \tag{6}
\]
\[
I_L + \Delta I e_W^T e_W - \alpha - \beta e_W^T e_W - \tilde{\Pi}^R \geq \beta r. \tag{7}
\]

Condition (3) is the worker’s participation constraint and ensures that it is in his interest to enter into this employment relationship. Further, (4) constitutes the worker’s incentive constraint, implying that he implements \( e_W = \beta \mu \) in order to maximize his expected utility. Constraints (5) and (6) guarantee that the relational employment contract is compatible with the worker’s liability limit. Finally, (7) is the self-enforcement condition (derived from (1)) ensuring that the downstream party is not tempted to renege on \( \beta \).

The subsequent proposition characterizes the downstream’s expected profit under the optimal employment contract \((\alpha^*, \beta^*)\) by utilizing two threshold interest rates, \(r^{RI}\) and \(\hat{r}^{RI}\). For parsimony, the optimal contracts as well as the threshold interest rates for this and subsequent propositions are characterized in the respective proof in the appendix.

**Proposition 1** By utilizing the optimal relational employment contract \((\alpha^*, \beta^*)\) under integration, the downstream’s expected profit is

\[
\Pi^D_{RI}(r) = \begin{cases} 
I_L + \frac{1}{4} (\Delta I)^2, & \text{if } r \leq r^{RI} \\
\frac{r}{2} [\Delta I - r + 2\phi] + \tilde{\Pi}^{RI}, & \text{if } r^{RI} < r \leq \hat{r}^{RI} \\
I_L, & \text{if } \hat{r}^{RI} < r,
\end{cases} \tag{8}
\]

where

\[
\phi \equiv \left[ \frac{1}{4} (\Delta I - r)^2 + I_L - \tilde{\Pi}^{RI} \right]^\frac{1}{2}.
\]
Proof All proofs are given in the appendix.

For a sufficiently low interest rate $r \leq r^{RI}$, the downstream party can credibly commit to pay the efficient bonus $\beta^* = \Delta I/2$. In this case, the value of a sustained employment relationship based upon a relational incentive contract eliminates the downstream’s temptation to renege on $\beta^*$. The worker anticipates that the downstream party would deliver on her promise to pay $\beta^*$ and is therefore motivated to implement the efficient (second-best) effort $e_W^* = \Delta I \mu/2$. For $r^{RI} < r \leq \hat{r}^{RI}$ however, the downstream’s promise to pay the efficient bonus $\beta^* = \Delta I/2$ is not credible from the worker’s perspective. This is because a higher interest rate $r$ imposes a less severe ‘penalty’ on the downstream party for violating the relational incentive contract with the worker. In such a situation, the worker anticipates that the downstream party would behave opportunistically by reneging on $\beta^*$, and would thus refuse to implement effort. To motivate the worker nonetheless to implement effort, the downstream party is compelled to adjust the bonus $\beta$ in order to ensure it satisfies the self-enforcement condition (7). However, the more the credible bonus $\beta^*(r)$ deviates from the efficient bonus $\beta^* = \Delta I/2$, the lower is the downstream’s expected profit $\Pi_D^{\text{RI}}(r)$. Finally, if $r > \hat{r}^{RI}$, the downstream party cannot find a strictly positive bonus which eliminates her reneging temptation. As a result, $\beta^*(r) = 0$, and the downstream party obtains the same expected profit as under spot integration.

3.3 Spot Market

Instead of utilizing an integrated production, the downstream party can alternatively procure the input from the upstream party through a spot market exchange. Under this arrangement, both parties negotiate in every period about a price $\Upsilon^{SM}$ in exchange for the input. As in Baker et al. [2002], this price is obtained by applying the Nash-Bargaining solution with equal bargaining powers. Accordingly, the downstream party pays the external input value $E$ plus half of the surplus $I - E$ for its internal use so that $\Upsilon^{SM} = [I + E]/2$. 
The upstream party chooses effort $e_U$ with the objective of maximizing his expected profit

$$\Pi_{U|SM} = E[\Upsilon_{SM}|e_U] - C(e_U),$$

which is equivalent to

$$\max_{e_U} \Pi_{U|SM} = \frac{1}{2} \left[ I_L + \Delta I \mu^T e_U + \Delta E \omega^T e_U \right] - \frac{1}{2} c_U e_U^T e_U. \quad (9)$$

As can be deduced from the first-order condition, the upstream party chooses

$$e_U^* = \frac{1}{2c_U} [\Delta I \mu + \Delta E \omega]. \quad (10)$$

Apparently, the upstream party intends to maximize the internal and external input value with the aim of improving his own expected bargaining position, and hence, the price he expects to obtain. Observe further that the upstream party does not perfectly tailor the input to the downstream’s requirements if $\mu \neq \lambda \omega$, $\lambda > 0$. In this case, adjusting the input to the downstream’s and to other market participants’ needs are two competing objectives. In other words, tailoring the input exclusively to the downstream’s requirements would constitute a relation-specific investment, which—due to the nature of one-time transactions with irreversible investments—would provoke opportunistic behavior on the side of the downstream party, commonly referred to as hold-up.

If the upstream party anticipates a spot market transaction with the downstream party, he implements $e_U^*$. Hence, his expected profit is

$$\Pi_{U|SM} = \frac{1}{2} I_L + \frac{1}{8c_U} \left[ (\Delta I)^2 + (\Delta E)^2 + 2\Delta I \Delta E \cos \varphi \right]. \quad (11)$$

Note that the upstream’s expected profit is decreasing in the degree of input specificity $\varphi$ which is desired by the downstream party. This can be observed because the upstream’s trade-off between tailoring the input to the downstream’s specific requirements and adjusting the input to the market’s (general) needs becomes more severe. This eventually deteriorates the upstream’s expected bargaining position, and as a consequence, leads to a smaller expected premium for selling the input to the downstream party in place of the other market participants.
If the downstream party decides in favor of spot market exchange with the upstream party, her expected profit $\Pi^{D|SM}$ is the difference between the expected internal input value and the price she expects to pay. Formally, $\Pi^{D|SM} = E[I - \Upsilon^{SM}|\epsilon^*_U]$, which is equivalent to

$$\Pi^{D|SM} = \frac{1}{2} I_L + \frac{1}{4c_U} [(\Delta I)^2 - (\Delta E)^2].$$

(12)

At first glance, the downstream’s expected profit under spot exchange is independent of the required input specificity measured by $\varphi$. However, from a closer inspection of the expected internal input value $E[I|\epsilon^*_U]$ and the expected price $E[\Upsilon^{SM}|\epsilon^*_U]$ it becomes clear that there are two countervailing effects of $\varphi$ on $\Pi^{D|SM}$. In particular, it can be shown that

$$\frac{\partial E[I|\epsilon^*_U]}{\partial \varphi} = \frac{\partial E[\Upsilon^{SM}|\epsilon^*_U]}{\partial \varphi} = -\frac{1}{2c_U} \Delta I \Delta E \sin \varphi. \quad \text{(13)}$$

Accordingly, an increase in $\varphi$ leads to a lower expected internal input value, but also to a lower price the downstream party expects to pay. Since the magnitudes of both effects are identical, they cancel each other out such that $\Pi^{D|SM}$ is eventually not affected by the desired degree of input specificity. Generally speaking, a lower expected price perfectly compensates the downstream party for the expected exchange of an insufficiently tailored input.

### 3.4 Relational Market

As demonstrated in the preceding section, utilizing spot market transactions with the upstream party leads to the procurement of insufficiently tailored inputs due to the upstream’s reluctance to make relation-specific investments. To ensure the exchange of perfectly tailored inputs however, the downstream party can promise the upstream party to pay a certain amount conditional on the realized internal input value $I$. This is aimed at motivating the upstream party to make relation-specific investments because tailoring inputs to the downstream’s requirements irretrievably impairs their expected market value.

Let $P_L$ denote the floor payment both parties consent to in an enforceable contract for exchanging the input, regardless of its final value. In addition, to motivate relation-specific investments in the sense of tailoring the input, the downstream party promises to pay the upstream party a higher price $P_H$ if $I = I_H$. Thus, this contract consists of an explicit
component $P_L$ and a non-enforceable premium $\Delta P \equiv P_H - P_L$. The upstream party however, is only motivated to tailor the input if the downstream’s promise to pay the premium $\Delta P$ is perceived as reliable.

To derive the self-enforcement condition for relational market transactions, let $\tilde{\Pi}^{RM} \equiv \max\{\Pi^{D|SI}, \Pi^{D|RI}, \Pi^{D|SM}\}$ denote the downstream’s expected profit which she obtains under her best fallback position. Now suppose for a moment that the high internal input value $I_H$ is eventually realized. The downstream party is not tempted to hold up the upstream party by reneging on $\Delta P$ if

$$-\Delta P + \frac{\Pi^{D|RM}}{r} \geq \frac{\tilde{\Pi}^{RM}}{r}.$$  \hspace{1cm} (14)

The downstream party adheres to her promise if paying the premium $\Delta P$ but perpetuating the long-term trading relationship with the upstream party provides her with a higher expected profit than her best fallback position.

In addition to satisfying the self-enforcement condition (14), the payments $P_L$ and $P_H$ need to guarantee that it is in the upstream’s interest to enter into a long-term trading relationship with the downstream party founded on relational contracts. This requires that the upstream party is at least weakly better off under relational market transactions with the downstream party than under his best alternative. It is crucial here to note that the upstream’s best alternative is directly linked to the downstream’s best fallback position. If the downstream’s best fallback is to utilize spot market exchange, the upstream’s reservation profit is $\Pi^{U|SM}$ as derived in section 3.3. By contrast, if vertical integration is the downstream’s best alternative, the upstream’s reservation profit is the one he obtains by selling the intermediate product on the market at the market price $\Upsilon^M = E$. In this case, it can be shown that the upstream party implements $e^*_U = \Delta E \omega / c_U$, which in turn provides him with the expected profit

$$\Pi^{U|M} = \frac{1}{2c_U}(\Delta E)^2.$$  \hspace{1cm} (15)
The objective of the optimal relational contract is to maximize the difference between the expected internal input value and the expected payment to the upstream party. The downstream’s problem can thus be formalized as follows:

\[
\max_{P_L, P_H, e_U} \Pi^{D|RM} = I_L + \Delta I_\mu^T e_U - P_L - \Delta P_\mu^T e_U
\]

s.t.

\[
P_L + \Delta P_\mu^T e_U - \frac{1}{2} c_U e_U^T e_U \geq \bar{\Pi}^U
\]

\[
e_U \in \arg \max_{\tilde{e}_U} P_L + \Delta P_\mu^T \tilde{e}_U - \frac{1}{2} c_U \tilde{e}_U^T \tilde{e}_U
\]

\[
I_L + \Delta I_\mu^T e_U - P_L - \Delta P_\mu^T e_U - \tilde{\Pi}^{RM} \geq \Delta Pr,
\]

where

\[
\bar{\Pi}^U = \begin{cases} 
\frac{1}{2} I_L + \frac{1}{8 c_U} [\Delta I_\mu + \Delta E \omega]^2, & \text{if } \tilde{\Pi}^{RM} = \Pi^{D|SM} \\
\frac{1}{2 c_U} (\Delta E)^2, & \text{if } \tilde{\Pi}^{RM} \neq \Pi^{D|SM},
\end{cases}
\]

represents the upstream’s reservation profit which is—as previously explained—conditional on the downstream’s best fallback position. Condition (17) is the upstream’s participation constraint guaranteeing that the proposed relational contract makes him at least weakly better off than his best alternative. Further, (18) is the upstream’s incentive constraint, implying that he implements \(e_U^* = \Delta P_\mu / c_U\) in order to maximize his expected profit. Finally, the self-enforcement condition (19) (derived from (14)) ensures that the downstream party is not tempted to renege on the non-enforceable premium \(\Delta P\).

As discussed above, the upstream’s reservation profit can take one of two values conditional on the downstream’s best fallback position. For the sake of lucidity, I subsequently consider both cases separately. The next proposition characterizes the downstream’s expected profit under relational market transactions if her best alternative is spot market exchange.
Proposition 2 Suppose $\Pi^{RM} = \Pi^{SM}$, i.e. the downstream’s best alternative is spot market exchange. Then, by utilizing the optimal relational contract $(P_L^*, P_H^*)$ for repeated transactions with the upstream party, the downstream’s expected profit is

$$\Pi^{D|RM}(r) = \begin{cases} 
\frac{1}{2} I_L + \frac{1}{2c_U}(\Delta I)^2 - \frac{1}{8c_U}[\Delta I \mu + \Delta E \omega]^2, & \text{if } r \leq r^{RM} \\
r \left[ \Delta I - r c_U + \frac{1}{2} \right] + \Pi^{D|SM}, & \text{if } r^{RM} < r \leq \hat{r}^{RM} \\
\Pi^{D|SM}, & \text{if } \hat{r}^{RM} < r,
\end{cases} \quad (21)$$

where

$$\phi \equiv (\Delta I - r c_U)^2 - \frac{1}{4} (\Delta I \mu + \Delta E \omega)^2 - c_U \left( 2 \Pi^{D|SM} - I_L \right).$$

As long as the mutually shared interest rate $r$ is sufficiently low (i.e., $r \leq r^{RM}$), the downstream party honors her non-enforceable obligation to pay the optimal premium $\Delta P^* = \Delta I$ if $I = I_H$.\(^\text{13}\) In this case, the upstream party anticipates that hold-up will not occur, and is therefore motivated to make the desired relation-specific investment in the sense of tailoring the input to the downstream’s requirements. In contrast, if $r^{RM} < r \leq \hat{r}^{RM}$, the downstream’s promise to pay the efficient premium $\Delta P^*(r)$ is not credible from the upstream’s perspective. In such a situation, motivating relation-specific investments necessitates the adjustment of the premium $\Delta P^*(r)$ targeted at ensuring it satisfy the self-enforcement condition (19). The provision of an inefficient (but credible) premium $\Delta P^*(r)$ however, inevitably leads to a lower expected profit for the downstream party. Finally, if $r > \hat{r}^{RM}$, the downstream party cannot find a strictly positive and reliable premium $\Delta P$ whose effectiveness would not be jeopardized by her temptation to hold up the upstream party. As a logical consequence, the upstream party refuses to make relation-specific investments such that the downstream party obtains the same expected profit as under spot market exchange.

To gain further insights, one can re-write $\Pi^{D|RM}$ for $r \leq r^{RM}$ as\(^\text{14}\)

$$\Pi^{D|RM} = \frac{1}{2} I_L + \frac{1}{2c_U}(\Delta I)^2 - \frac{1}{8c_U} \left[ (\Delta I)^2 + (\Delta E)^2 + 2\Delta I \Delta E \cos \phi \right]. \quad (22)$$

\(^{13}\)See proof of proposition 2 in the appendix for the characterization of the optimal relational contract.

\(^{14}\)To see this, note that $\mu^T \omega = \|\mu\| \|\omega\| \cos \phi$, where $\|\mu\| = \|\omega\| = 1$. 

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If spot market exchange constitutes her best alternative input procurement, the downstream’s expected profit \( \Pi^{D|RM} \) clearly depends on the degree of the desired input specificity measured by \( \varphi \). The rationale for this observation is as follows: Ensuring the upstream’s participation requires that he obtains at least the same expected profit in a long-term relationship with the downstream party founded on relational contracts as under mutual spot exchange. The floor payment \( P_L \) thus reflects the upstream’s reservation profit \( \Pi^{U|SM} \), which in turn is a function of the input specificity measure \( \varphi \), see section 3.3. As a natural consequence, the degree of input specificity also affects the downstream’s expected profit under relational market transactions if spot exchange is her best alternative. More precisely, as discussed in section 3.3, a higher degree of input specificity (greater \( \varphi \)) deteriorates the upstream’s expected profit \( \Pi^{U|SM} \) under spot market exchange. This in turn makes it less costly for the downstream party to ensure the upstream’s participation, which translates into a higher expected profit \( \Pi^{D|RM} \). The same inferences with respect to the downstream’s expected profit \( \Pi^{D|RM} \) can be made for the case where \( r^{RM} < r \leq \hat{r}^{RM} \).

To get a more complete picture of how the desired input specificity measured by \( \varphi \) affects the profitability of a long-term relationship with the upstream party, consider the threshold interest rate \( r^{RM} \), which is characterized in the proof of proposition 2 in the appendix:

\[
r^{RM} = \frac{1}{8\overline{I}_U} \left[ (\Delta I)^2 + (\Delta E)^2 - 2\Delta I\Delta E \cos \varphi \right].
\]  

The threshold interest rate \( r^{RM} \) is apparently increasing in the specificity measure \( \varphi \). This implies that the downstream’s promise to pay the efficient premium \( \Delta P^* = \Delta I \) becomes credible even for higher interest rates if the downstream party relies on more specific inputs. This inference is rooted in the previously made observation that a higher degree of input specificity augments the downstream’s expected profit under relational market transactions. This in turn imposes a more severe ‘penalty’ on the downstream party for reneging on the non-enforceable premium \( \Delta P \), which is reflected by a higher threshold interest rate \( r^{RM} \). It is straightforward to verify that the same conclusion can be drawn for \( \hat{r}^{RM} \). These observations are in line with Halonen [2002] who points out that less attractive outside options alleviate incentives to behave opportunistically.
To complete the analysis in this section, it remains to characterize the downstream’s expected profit under relational market transactions in case vertical integration constitutes her best alternative.

**Proposition 3** Suppose $\bar{\Pi}^{RM} \neq \Pi^{D|SM}$, i.e. the downstream’s best alternative is vertical integration. Then, by utilizing the optimal relational contract $(P_L^*, P_H^*)$ for repeated transactions with the upstream party, the downstream’s expected profit is

$$
\Pi^{D|RM}(r) = \begin{cases} 
I_L + \frac{1}{2c_U} [(\Delta I)^2 - (\Delta E)^2], & \text{if } r \leq r^{RM} \\
r \left[ \Delta I - r c_U + \phi \frac{1}{2} \right] + \bar{\Pi}^{RM}, & \text{if } r^{RM} < r \leq \hat{r}^{RM} \\
\Pi^{D|SM}, & \text{if } \hat{r}^{RM} < r,
\end{cases}
$$

(24)

where

$$
\phi \equiv (\Delta I - r c_U)^2 - (\Delta E)^2 - 2c_U \left( \bar{\Pi}^{RM} - I_L \right).
$$

In principle, the relationship between the mutually shared interest rate $r$ and the downstream’s expected profit as previously exposed for the case $\bar{\Pi}^{RM} = \Pi^{D|SM}$ also applies in the event that $\bar{\Pi}^{RM} \neq \Pi^{D|SM}$. Observe however, that the downstream’s expected profit $\Pi^{D|RM}(r)$ is now independent of the input specificity measure $\phi$. The reason is as follows: Since the downstream’s best fallback position is to vertically integrate the production of the input, selling the intermediate product on the market constitutes the upstream’s (only) alternative. As revealed in the previous analysis, the upstream’s reservation profit $\Pi^{U|SM}$ is then independent of the input specificity measure $\phi$, which in turn leaves the optimal relational contract $(P_L^*, P_H^*)$, and hence the downstream’s expected profit $\Pi^{D|RM}(r)$, also unaffected.

## 4 Input Procurement and Input Specificity

This section identifies the optimal method of input procurement from the perspective of the downstream party. This in turn facilitates a thorough analysis and discussion of how the downstream’s make-or-buy decision is affected by the required degree of input specificity.
To ease the subsequent characterization of the downstream’s optimal input procurement, let us first identify the optimal spot contract. To do so, suppose for a moment that the mutually shared interest rate \( r \) is sufficiently high such that both considered relational contracts (i.e. relational integration and relational market) are not feasible. Formally, this requires that \( r > \max\{\hat{r}^{RI}, \hat{r}^{RM}\} \). It is straightforward to verify that \( \Pi^{D|SM} \geq \Pi^{D|SI} \) for \( r > \max\{\hat{r}^{RI}, \hat{r}^{RM}\} \) is equivalent to

\[
(\Delta E)^2 \leq (\Delta I)^2 - 2c_U I_L \equiv \Theta
\]  

(25)

As long as relational contracts cannot be attained and \( (\Delta E)^2 \leq \Theta \), procuring the input through spot market exchange with the upstream party dominates an integrated production. Put differently, market transactions are superior to vertical integration if the market valuation towards the difference between the high and low external value of the input is sufficiently low. The rationale for this observation is rooted in the upstream’s bargaining position. If \( (\Delta E)^2 \leq \Theta \), selling the input to other market participants is relatively unprofitable for the upstream party, which in turn weakens his bargaining position, and translates into a sufficiently low exchange price \( \Upsilon^{SM} \) for the input. By contrast, if \( (\Delta E)^2 > \Theta \), the downstream party strictly prefers an integrated production of the input because spot market exchange would impose higher costs due to the upstream’s improved bargaining position.\(^{15}\)

Before identifying the downstream’s optimal make-or-buy decision, it is further necessary to compare both considered relational contracts—relational integration and relational market—with respect to their profitabilities. First, suppose that the interest rate \( r \) is sufficiently low such that both relational contracts comprising the efficient incentive schemes are self-enforcing. Formally, this requires that \( r \leq \min\{r^{RM}, r^{RI}\} \). I demonstrate in the

\(^{15}\)This particular requires that \( c_U < (\Delta I)^2/(2I_L) \), i.e. the upstream’s production of the input is not too inefficient relative to the downstream’s production. Otherwise, spot integration would be preferred for all values of \( (\Delta E)^2 \).
appendix that relational market dominates relational integration from the downstream’s perspective (i.e., $\Pi^{D|RM} \geq \Pi^{D|RI}$), if

$$\frac{(\Delta E)^2}{\Phi} \leq \frac{(\Delta E)^2}{\Phi}, \quad \text{if } \Pi^{D|SM} = \tilde{\Pi}^{RM}$$

$$\frac{(\Delta E)^2}{\Phi} \leq \frac{(\Delta E)^2}{\Phi}, \quad \text{if } \Pi^{D|SM} \neq \tilde{\Pi}^{RM}.$$  

(26)  

(27)

The downstream’s preference for a particular relational contract is apparently determined by two different thresholds: $\Phi$ and $\Phi$.\(^{16}\) This can be observed because the downstream’s expected profit for relational market is conditional on whether or not a spot market exchange is her best fallback position. From the perspective of the downstream party, a long-term trading relationship with the upstream party (relational market) is generally more profitable than vertical integration if $\Delta E$ is sufficiently low. As previously exposed, a lower $\Delta E$ diminishes the value of the upstream’s outside option and hence, his bargaining position. This in turn mitigates the downstream’s costs for ensuring the upstream’s participation in a long-term trading arrangement. Finally, it remains to identify the downstream’s preferred relational contract and the corresponding thresholds if $r^i < r \leq \hat{r}^i$, $i = RM, RI$. For this case, it has been shown in sections 3.2 and 3.4 that the downstream party is compelled to adjust the incentive schemes for both relational contracts aimed at ensuring their self-enforcement. Then, $\Pi^{D|RM}(r) \geq \Pi^{D|RI}(r)$, i.e., relational market dominates relational integration, if

$$\frac{(\Delta E)^2}{\Psi(r)} \leq \frac{(\Delta E)^2}{\Psi(r)}, \quad \text{if } \Pi^{RM} = \Pi^{D|SM}$$

$$\frac{(\Delta E)^2}{\Psi(r)} \leq \frac{(\Delta E)^2}{\Psi(r)}, \quad \text{if } \Pi^{RM} \neq \Pi^{D|SM},$$

(28)  

(29)

where $(\Delta E)^2 = \Psi(r)$ implies $\Pi^{D|RM}(r) = \Pi^{D|RI}(r)$ for $\Pi^{RM} = \Pi^{D|SM}$, and $(\Delta E)^2 = \Psi(r)$ for $\Pi^{RM} \neq \Pi^{D|SM}$ respectively.\(^{17}\)

After identifying all relevant thresholds, we are now equipped to investigate the downstream’s optimal input procurement and its sensitivity to the required degree of input speci-

\(^{16}\) For parsimony purposes, the characterization of these two thresholds is relegated to the appendix.

\(^{17}\) Due to the structure of $\Pi^{D|RM}(r)$ and $\Pi^{D|RI}(r)$ for $r^i < r \leq \hat{r}^i$, $i = RM, RI$, one cannot obtain a tractable closed form solution. Nevertheless, utilizing the implicit characterizations does not derogate the subsequent results.
ficity. As a first step towards a thorough discussion of how input specificity affects the downstream’s make-or-buy decision, the next proposition characterizes the optimal input procurement for different values of $r$ and $\Delta E$.

**Proposition 4** The downstream party chooses relational market in the intervals

$$
\begin{align*}
0 < (\Delta E)^2 &\leq \min\{\Phi, \Theta\} \quad \text{and} \quad \Theta < (\Delta E)^2 \leq \overline{\Phi}, \quad \text{if} \quad r \leq r^{RM}; \\
0 < (\Delta E)^2 &\leq \min\{\Psi(r), \Theta\} \quad \text{and} \quad \Theta < (\Delta E)^2 \leq \overline{\Psi}(r), \quad \text{if} \quad r^{RM} < r \leq \hat{r}^{RM}.
\end{align*}
$$

In contrast, relational integration is preferred for

$$
\begin{align*}
\Phi < (\Delta E)^2 &\leq \Theta \quad \text{and} \quad \overline{\Phi} < (\Delta E)^2, \quad \text{if} \quad r \leq r^{RI}; \\
\Psi(r) < (\Delta E)^2 &\leq \Theta \quad \text{and} \quad \overline{\Psi}(r) < (\Delta E)^2, \quad \text{if} \quad r^{RI} < r \leq \hat{r}^{RI}.
\end{align*}
$$

Finally, if $r > \hat{r}^{RM}$ or $r > \hat{r}^{RI}$ in the relevant intervals, the downstream party utilizes spot market if $(\Delta E)^2 \leq \Theta$, and spot integration otherwise.

The downstream’s optimal input procurement is illustrated in Figure 1, where the squared spread of the external input value $(\Delta E)^2$ as a measure of the upstream’s bargaining position is on the horizontal axis, and the interest rate $r$ on the vertical axis. Consider first the downstream’s preferred spot transaction, which eventually determines her best fallback position for relational contracts. Spot contracts are chosen whenever the mutually shared interest rate $r$ is sufficiently high such that superior relational contracts cannot be attained. As previously exposed, spot market exchange dominates spot integration if the market valuation towards the difference between the high and low external value of the input $\Delta E$—and thus the upstream’s bargaining position—is sufficiently low. By contrast, if the interest rate $r$ is adequately low, the downstream party can utilize a relational contract to provide either the upstream party (relational market) or the worker (relational integration) with effective incentives

18More precisely, Figure 1 represents the downstream’s make-or-buy decision under the premise that $\Phi, \Psi(r) > \Theta$ and $(\Delta I)^2 \cos^2 \varphi > \Theta$. The latter condition implies that $\hat{r}^{RM}$ is convex decreasing in $(\Delta E)^2$ for $0 < (\Delta E)^2 \leq \min\{\Psi(r), \Theta\}$. Furthermore, it can be shown that $\hat{r}^{RI}$ is convex increasing in $(\Delta E)^2$ if the application of spot market exchange constitutes the downstream’s best fallback position. In contrast, if spot integration is her best alternative, $\hat{r}^{RI}$ is constant in $(\Delta E)^2$.

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to tailor the intermediate product to the specific requirements. Particularly, due to the previously discussed reasons, the downstream party prefers a long-term trading relationship with the upstream party whenever $\Delta E$—as an indicator of the upstream’s bargaining position—is sufficiently low; and a repeated employment relationship with the worker otherwise.

To shed more light on the downstream’s make-or-buy decision, suppose for a moment that $r \leq \min\{\hat{r}^{RM}, \hat{r}^{RI}\}$, i.e. both relational contracts are self-enforcing. As can be deduced from proposition 4 or Figure 1, relational market is generally superior for low values of $\Delta E$ due to the relatively weak bargaining position of the upstream party. Nevertheless, it is important here to emphasize that relational integration can be temporarily preferred by the downstream party in the interval $\Psi(r) < (\Delta E)^2 \leq \Theta$, whereas for $\Theta < (\Delta E)^2 \leq \Psi(r)$, relational market is again more profitable, see proposition 4. The reason for obtaining spanned intervals is as follows. Different fallback positions for relational market—spot market exchange or integrated spot production—impose diverse costs on the downstream party for ensuring the upstream’s participation in a long-term trading arrangement. This in turn pro-
vides the downstream party with differential expected profits, and as a consequence, leads to diverse threshold interest rates reflecting the feasibility of the respective relational contract. Note however, that the previous argumentation applies only if $\Psi(r) < \Theta < \overline{\Psi}(r)$, which depends on the specific parameter values. Otherwise, there exists only one threshold level of $\Delta E$ where the downstream party is indifferent between relational market and relational integration.

To illustrate the effect of input specificity on the downstream’s make-or-buy decision, suppose that the downstream party requires a more specific intermediate product. Formally, $\varphi$ increases to $\varphi'$. As long as spot market exchange constitutes the downstream’s best fallback position for repeated transactions with the upstream party founded on a relational contract ($\bar{\Pi}^{RM} = \Pi^{D|SM}$), demanding a more specific input leads to the subsequent effects (see also the discussion in section 3.4). First, utilizing relational market transactions becomes more profitability for the downstream party due to the weakened bargaining position of the upstream party, which in turn causes the thresholds $\Psi(r)$ and $\Phi$ to increase. Second, as a direct consequence of the improved profitability of relational market transactions, the corresponding threshold interest rates $r^{RM}$ and $\hat{r}^{RM}$ increase.

The effect of varying degrees of input specificity on the downstream’s make-or-buy decision is also depicted in Figure 1, where the dashed lines characterize the new thresholds for $\varphi'$. Consider first area $A$. Here, the downstream party can engage in a superior long-term trading relationship with the upstream party (relational market), if she relies on a sufficiently specific input (high $\varphi$). Otherwise, she would be compelled to utilize less profitable spot market exchange. Recall that demanding a more specific input—characterized by a higher $\varphi$—deteriorates the upstream’s bargaining position for mutual spot exchange. This mitigates the downstream’s costs for ensuring the upstream’s participation in a long-term trading arrangement, and thus improves her expected profit under relational market. A higher expected profit under relational market further enhances the downstream’s potential 'penalty’ for violating the relational contract with the upstream party, which in turn is reflected by higher threshold interest rates $r^{RM}$ and $\hat{r}^{RM}$. Generally speaking, the downstream party can now
motivate the upstream party to make relation-specific investments in the sense of perfectly tailoring the intermediate product to the specific requirements even for higher interest rates.

Next, consider area $B$ in Figure 1. Here, the downstream party engages in a long-term trading relationship with the upstream party whenever she relies on sufficiently specific inputs (high $\varphi$). Otherwise, for more generic intermediate products, the downstream party would opt for an integrated production. Accordingly, in area $B$, demanding highly specific inputs leads to repeated transactions with an external input supplier (relational market) dominating vertical integration. Again, this observation is rooted in the impairment of the upstream’s bargaining position. As discussed, a weaker bargaining position of the upstream party eventually improves the profitability of repeated market transactions for the downstream’s party relative to vertical integration, which in turn is not only reflected by higher threshold interest rates $r_{RM}$ and $\hat{r}_{RM}$, but also by a higher threshold $\Phi'$, see Figure 1.

Summarizing the above observations, a firm depending on sufficiently specific intermediate products might favor repeated transactions with an external input supplier over vertical integration for their procurement. This conclusion is rooted in the impairment of the upstream’s bargaining position occurring whenever spot market exchange is the downstream’s best alternative to repeated market transactions. Thus, the analysis in this paper provides a potential explanation of the phenomenon whereby some firms utilize repeated transactions with external suppliers for procuring highly specific inputs.

5 Conclusion

Prior literature on the theory of the firm presents a number of reasons why firms might prefer integrated instead of market transactions. This paper elaborates on the specificity of required inputs and its effect on a firm’s make-or-buy decision. In doing so, it sheds light on the contradictory phenomenon that some firms engage in repeated market transactions despite a strong reliance on highly specific inputs.

The analysis in this paper highlights one import conclusion: A firm might favor repeated market transactions if the procured input is sufficiently specific. The rationale for this obser-
vation is as follows. First, demanding a more specific input impairs the supplier’s bargaining position for mutual market transactions since tailored intermediate products are less likely to be purchased by other market participants. This in turn imposes lower costs on demanding firms for sustaining repeated market transactions founded on relational contracts. Second, relying on more specific inputs facilitates the self-enforcement of relational contracts between firms, which induces the input supplier to make the required relation-specific investments. This can be observed because repeated transactions with an external input supplier become sufficiently beneficial for the buyer such that hold up is less likely to occur. This paper thus provides a theoretical underpinning of why some firms are inclined to favor repeated market transactions over vertical integration for procuring highly specific inputs.

Indeed, due to the aforementioned reasons, a firm might be better off by procuring highly specific instead of more standardized intermediate products. Throughout this paper, the desired specificity of an intermediate product is assumed to be exogenous. This is a reasonable assumption whenever the utilized technology for processing the input does not allow for variations in its characteristics. Alternatively, firms could invest in production technologies that give them some latitude to process inputs with differential properties. According to the analysis in this paper, firms might then prefer to procure the most specific inputs that can still be processed by their current technologies. In this case, a firm’s choice in terms of the specificity of the required inputs is made strategically with the aim of deteriorating the supplying firm’s bargaining position in repeated inter-firm trade.

From a pragmatic standpoint, a strong dependency on highly specific inputs may not necessarily imply that an internal production is optimal for a firm. Rather, as suggested in this paper, if inputs can be custom-tailored to the extent to which they cannot be processed by any other firms, a long-term trading arrangement founded on relational contracts with an external supplier could be the superior alternative for their procurement. In situations of relatively exclusive inputs, a firm can therefore exploit the low bargaining position of its supplier to induce a cost advantage which cannot be replicated within the organization.

It is crucial to bear in mind that the subsequent argumentation implicitly assumes that additional costs imposed by tailoring and processing intermediate products are sufficiently low.
Appendix

Proof of Proposition 1.

Note first that $e_{Wi} > 0$ for at least one $i \in \{1, 2\}$ requires $\beta > 0$. Thus, (5) is satisfied if (6) holds, and can therefore be omitted. Assume for a moment that (7) is satisfied for the optimal bonus contract. Let $\lambda$ and $\xi$ denote Lagrange multipliers. Since $\mu^T \mu = \|\mu\|^2 = 1$, the Lagrangian is

$$L(\alpha, \beta) = I_L + I \beta - \alpha - \beta^2 + \lambda \left[ \alpha + \frac{1}{2} \beta^2 \right] + \xi \alpha.$$  \hspace{1cm} (30)

The first-order conditions with respect to $\alpha$ and $\beta$ are

$$-1 + \lambda + \xi = 0,$$ \hspace{1cm} (31)

$$\Delta I + \beta (\lambda - 2) = 0.$$ \hspace{1cm} (32)

To find a solution of this problem, suppose for a moment that $\lambda > 0$. Then, $\alpha + \beta^2/2 = 0$ due to complementary slackness. Since $\alpha \geq 0$, this would imply that $\alpha^* = 0$ and $\beta^* = 0$, and hence, $e^* = (0, 0)^T$. Thus, $\lambda > 0$ cannot be a solution of this problem such that $\lambda = 0$, i.e. the worker’s participation constraint is not binding. We can then infer from (31) that $\xi = 1$. Hence, complementary slackness implies that $\alpha^* = 0$. Since $\lambda = 0$, it follows from (32) that $\beta^* = \Delta I/2$. Substituting $\beta^* = \Delta I/2$ in the downstream’s objective function leads to

$$\Pi_{D|RI} = I_L + (\Delta I)^2/4.$$ \hspace{1cm} (33)

By substituting $\Pi_{D|RI}$ and $\beta^* = \Delta I/2$ in (7), we obtain the threshold interest rate $r_{RI}$:

$$r \leq \frac{\Delta I}{2} - \frac{2}{\Delta I} \left[ \tilde{\Pi}_{RI} - I_L \right] \equiv r_{RI}.$$ \hspace{1cm} (34)

If $r > r_{RI}$, the efficient bonus $\beta^* = \Delta I/2$ (and any higher bonus $\beta > \beta^*$) would violate (7). In this case, the downstream party chooses the highest feasible $\beta$ such that (7) becomes binding. Let $\beta^*(r)$ denote the maximum value of $\beta$ which solves (7), or equivalently,

$$\beta^2 - (\Delta I - r)\beta - I_L + \tilde{\Pi}_{RI} = 0$$ \hspace{1cm} (35)

$$\Leftrightarrow \beta^*(r) = \frac{1}{2}(\Delta I - r) \pm \left[ \frac{1}{4}(\Delta I - r)^2 + I_L - \tilde{\Pi}_{RI} \right]^{1/2}.$$ \hspace{1cm} (36)
Since it is optimal to choose the highest feasible $\beta$, the upper bound is relevant. Note however, that there exits only a solution to (36) if
\[ \frac{1}{4}(\Delta I - r)^2 + I_L - \bar{\Pi}^{RI} \geq 0. \] (37)
Then, rearranging (37) yields the lower threshold $\hat{r}$:
\[ r \leq \Delta I \pm 2\left[\bar{\Pi}^{RI} - I_L\right]^{\frac{1}{2}} \equiv \hat{r}. \] (38)
Observe that the upper bound of $\hat{r}$ implies $\beta^*(r) < 0$, which cannot be a solution. Consequently, the lower bound of $\hat{r}$ is relevant. Substituting $\beta^*(r)$ for $r < r \leq \hat{r}$ in the downstream’s objective function yields
\[ \Pi^{D|RI}(r) = \frac{1}{2} \Delta I - r + 2\left[\frac{1}{4}(\Delta I - r)^2 + I_L - \bar{\Pi}^{RI}\right]^{\frac{1}{2}} + \bar{\Pi}^{RI}. \] (39)
Finally, if $r > \hat{r}$, every strictly positive $\beta$ would violate (7). Thus, the downstream party sets $\beta^*(r) = 0$. As a result, $e^*_U = (0, 0)^T$ such that $\Pi^{D|RI}(r) = I_L$. \hfill \Box

**Proof of Proposition 2.**

Note first that $\bar{\Pi}^{RM} = \Pi^{D|SM}$ implies $\bar{\Pi}^U = I_L/2 + [\Delta I \mu + \Delta E \omega]^2/(8c_U)$. To minimize costs, the downstream party sets $P_L$ such that the upstream’s participation constraint (17) becomes binding. By substituting $e^*_U = \Delta P \mu/c_U$ in (17) and solving for $P_L$, one get
\[ P_L = \frac{1}{2} I_L + \frac{1}{8c_U} (\Delta I \mu + \Delta E \omega)^2 - \frac{1}{2c_U} (\Delta P)^2. \] (40)
Now suppose for a moment that (19) is satisfied for the optimal premium $\Delta P^*$. Substituting $P_L$ and $e^*_U$ in the downstream’s objective function yields the simplified problem
\[ \max_{\Delta P} \Pi^{D|RM} = \frac{1}{2} I_L + \frac{1}{c_U} \Delta I \Delta P - \frac{1}{8c_U} (\Delta I \mu + \Delta E \omega)^2 - \frac{1}{2c_U} (\Delta P)^2. \] (41)
The first-order condition implies $\Delta P^* = \Delta I$. Hence, the downstream’s expected profit is
\[ \Pi^{D|RM} = \frac{1}{2} I_L + \frac{1}{2c_U} (\Delta I)^2 - \frac{1}{8c_U} [\Delta I \mu + \Delta E \omega]^2. \] (42)
By substituting $\Pi^{D\vert RM}$, $\Pi^{D\vert SM}$, and $\Delta P^* = \Delta I$ in (19), we obtain the threshold interest rate $r_{RM}$:

$$r \leq \frac{1}{8c_U \Delta I} \left[ \Delta I \mu - \Delta E \omega \right]^2 \equiv r_{RM}. \quad (43)$$

If $r > r_{RM}$, the efficient premium $\Delta P^* = \Delta I$ (and any higher premium $\Delta P > \Delta P^*$) would violate (19). In this case, the downstream party chooses the highest feasible $\Delta P$ such that (19) becomes binding. Let $\Delta P^*(r)$ denote the highest feasible premium which solves (19), or equivalently,

$$(\Delta P)^2 - 2(\Delta I - rc_U)\Delta P + \frac{1}{4} (\Delta I \mu + \Delta E \omega)^2 + c_U \left(2\Pi^{D\vert SM} - I_L\right) = 0. \quad (44)$$

Solving for $\Delta P$ yields

$$\Delta P^*(r) = \Delta I - rc_U \pm \left[ (\Delta I - rc_U)^2 - \frac{1}{4} (\Delta I \mu + \Delta E \omega)^2 - c_U \left(2\Pi^{D\vert SM} - I_L\right) \right]^{\frac{1}{2}}. \quad (45)$$

Since it is optimal to choose the highest feasible $\Delta P$, the upper bound is relevant. However, there exists only a solution to (45) if

$$(\Delta I - rc_U)^2 - \frac{1}{4} (\Delta I \mu + \Delta E \omega)^2 - c_U \left(2\Pi^{D\vert SM} - I_L\right) \geq 0. \quad (46)$$

Rearranging (46) yields the lower threshold $\hat{r}_{RM}$:

$$r \leq \frac{\Delta I}{c_U} \pm \frac{1}{c_U} \left[ \frac{1}{4} (\Delta I \mu + \Delta E \omega)^2 + c_U \left(2\Pi^{D\vert SM} - I_L\right) \right]^{\frac{1}{2}} \equiv \hat{r}_{RM}. \quad (47)$$

Notice that the upper bound of $\hat{r}_{RM}$ implies $\Delta P < 0$, which cannot be a solution. Thus, the lower bound of $\hat{r}_{RM}$ is relevant. Substituting $\Delta P^*(r)$ for $r_{RM} < r \leq \hat{r}_{RM}$ in the downstream’s objective function yields

$$\Pi^{D\vert RM}(r) = r \left[ \Delta I - rc_U + \left( (\Delta I - rc_U)^2 - \frac{1}{4} (\Delta I \mu + \Delta E \omega)^2 - c_U \left(2\Pi^{D\vert SM} - I_L\right) \right)^{\frac{1}{2}} \right] + \Pi^{D\vert SM}. \quad (48)$$

Finally, if $r > \hat{r}_{RM}$, every strictly positive premium $\Delta P$ would violate (19). Thus, the downstream party sets $\Delta P^* = 0$. As a result, relational market becomes synonymous to spot
market exchange such that \( \Pi^{D|RM}(r) = \Pi^{D|SM} \) for \( r > \hat{r}^{RM} \). \( \Box \)

**Proof of Proposition 3.**

First, note that \( \bar{\Pi}^{RM} \neq \Pi^{D|SM} \) implies \( \bar{\Pi}^{U} = (\Delta E)^2/(2c_U) \). Cost minimization requires the downstream party to set \( P_L \) such that the upstream’s participation constraint (17) becomes binding. By substituting \( e^*_U = \Delta P \mu/c_U \) in (17) and solving for \( P_L \), one get

\[
P_L = \frac{1}{2c_U} (\Delta E)^2 - \frac{1}{2c_U} (\Delta P)^2.
\]

(49)

Suppose for a moment that (19) is satisfied for the optimal premium \( \Delta P^* \). Substituting \( P_L \) and \( e^*_U \) in the downstream’s objective function leads to the simplified problem

\[
\max_{\Delta P} \Pi^{D|RM} = I_L + \frac{1}{c_U} \Delta I \Delta P - \frac{1}{2c_U} (\Delta E)^2 - \frac{1}{2c_U} (\Delta P)^2.
\]

(50)

The first-order condition yields \( \Delta P^* = \Delta I \). Thus, the downstream’s expected profit is

\[
\Pi^{D|RM} = I_L + \frac{1}{2c_U} [ (\Delta I)^2 - (\Delta E)^2 ].
\]

(51)

By substituting \( \Pi^{D|RM} \) and \( \Delta P^* = \Delta I \) in (19), we get the threshold interest rate \( r^{RM} \):

\[
r \leq \frac{1}{2c_U \Delta I} \left[ (\Delta I)^2 - (\Delta E)^2 - 2c_U \left( \bar{\Pi}^{RM} - I_L \right) \right] \equiv r^{RM}.
\]

(52)

If \( r > r^{RM} \), the efficient premium \( \Delta P^* = \Delta I \) (and any higher premium \( \Delta P > \Delta P^* \)) would violate (19). In this case, the downstream party chooses the highest feasible \( \Delta P \) such that (19) becomes binding. Let \( \Delta P^*(r) \) denote the highest feasible premium which solves (19), or equivalently,

\[
(\Delta P)^2 - 2(\Delta I - rc_U)\Delta P + (\Delta E)^2 + 2c_U \left( \bar{\Pi}^{RM} - I_L \right) = 0.
\]

(53)

Solving for \( \Delta P \) gives

\[
\Delta P^*(r) = \Delta I - rc_U \pm \left[ (\Delta I - rc_U)^2 - (\Delta E)^2 - 2c_U \left( \bar{\Pi}^{RM} - I_L \right) \right]^{1/2}.
\]

(54)

Again, it is optimal to choose the highest feasible \( \Delta P \), implying that the upper bound is relevant. Note however, that there exits only a solution to (54) if

\[
(\Delta I - rc_U)^2 - (\Delta E)^2 - 2c_U \left( \bar{\Pi}^{RM} - I_L \right) \geq 0.
\]

(55)
Rearranging (55) yields the lower threshold interest rate $\hat{r}^{RM}$:

$$r \leq \frac{\Delta I}{c_U} \pm \frac{1}{c_U} \left[ (\Delta E)^2 + 2c_U \left( \Pi^{RM} - I_L \right) \right]^{\frac{1}{2}} \equiv \hat{r}^{RM}. \quad (56)$$

Observe that the upper bound of $\hat{r}^{RM}$ implies $\Delta P < 0$, which cannot be a solution. Hence, the lower bound of $\hat{r}^{RM}$ is relevant. Substituting $\Delta P^*(r)$ for $r^{RM} < r \leq \hat{r}^{RM}$ in the downstream’s objective function yields

$$\Pi^{D|RM}(r) = r \left[ \Delta I - r c_U + \left[ (\Delta I - r c_U)^2 - (\Delta E)^2 - 2c_U \left( \Pi^{RM} - I_L \right) \right]^{\frac{1}{2}} \right] + \Pi^{RM}. \quad (57)$$

Finally, if $r > \hat{r}^{RM}$, every $\Delta P > 0$ would violate (19). Therefore, the downstream party is forced to set $\Delta P^* = 0$. As a result, relational market becomes equivalent to spot market exchange such that $\Pi^{D|RM}(r) = \Pi^{D|SM}$ for $r > \hat{r}^{RM}$. □

**Comparison of Relational Market and Relational Integration.**

Consider first the case $\Pi^{RM} = \Pi^{D|SM}$. Then, $\Pi^{D|RM} \geq \Pi^{D|RI}$ is equivalent to

$$(\Delta E)^2 + 2\Delta I \cos \varphi \Delta E + 4c_U I_L - (\Delta I)^2 (3 - 2c_U) \leq 0. \quad (58)$$

First, we can treat (58) as an equality. By applying the quadratic formula one get

$$\Delta E = -\Delta I \cos \varphi \pm \sqrt{(\Delta I)^2 \cos^2 \varphi - 4c_U I_L + (\Delta I)^2 (3 - 2c_U)}. \quad (59)$$

Since $\Delta E > 0$, the upper bound is relevant. Thus, $\Pi^{D|RM} \geq \Pi^{D|RI}$ requires

$$(\Delta E)^2 \leq \left[ (\Delta I)^2 \cos^2 \varphi + (\Delta I)^2 (3 - 2c_U) - 4c_U I_L \right]^{\frac{1}{2}} - \Delta I \cos \varphi \equiv \Phi. \quad (60)$$

In case $\Pi^{RM} \neq \Pi^{D|SM}$, it can be shown that $\Pi^{D|RM} \geq \Pi^{D|RI}$ is equivalent to

$$(\Delta E)^2 \leq \frac{1}{2} (\Delta I)^2 (2 - c_U) \equiv \overline{\Phi}. \quad (61)$$
Proof of Proposition 4.

Suppose first that \( r \leq r^i, i = RM, RI \). Then, it is necessary to identify whether the downstream party prefers different relational contracts for the same value of \((\Delta E)^2\), but different values of \( r \). To do so, consider first the intervals \( 0 < (\Delta E)^2 \leq \Phi \) and \( \Theta < (\Delta E)^2 \leq \Phi \), where \( \Pi^{D|RM} \geq \Pi^{D|RI} \). Suppose for a moment that \( r^{RM} \geq r^{RI} \), which is equivalent to

\[
\frac{\Pi^{D|RM} - \Pi^{RM}}{\Delta I} \geq \frac{2 \left[ \Pi^{D|RI} - \Pi^{RI} \right]}{\Delta I}.
\] (62)

Note that \( r^{RM} \geq r^{RI} \) further implies \( \tilde{\Pi}^{RM} = \max\{\Pi^{D|SM}, \Pi^{D|SI}\} \) and \( \tilde{\Pi}^{RI} = \Pi^{D|RM} \). Thus, (62) simplifies to

\[
3\Pi^{D|RM} \geq 2\Pi^{D|RI} + \max\{\Pi^{D|SM}, \Pi^{D|SI}\},
\] (63)

which is satisfied in the intervals \( 0 < (\Delta E)^2 \leq \Phi \) and \( \Theta < (\Delta E)^2 \leq \Phi \). Thus, \( r^{RM} \geq r^{RI} \) in these two intervals. Because \( \Pi^{D|RM} \geq \Pi^{D|RI} \) in these two intervals, the downstream party chooses relational market if \( r \leq r^{RM} \). Next, consider the intervals \( \Phi < (\Delta E)^2 \leq \Theta \) and \( \Phi < (\Delta E)^2 \), where \( \Pi^{D|RI} > \Pi^{D|RM} \). Suppose for a moment that \( r^{RI} \geq r^{RM} \), which implies \( \tilde{\Pi}^{RI} = \max\{\Pi^{D|SM}, \Pi^{D|SI}\} \) and \( \tilde{\Pi}^{RM} = \Pi^{D|RI} \). Consequently, \( r^{RI} \geq r^{RM} \) is equivalent to

\[
3\Pi^{D|RI} \geq 2\Pi^{D|RM} + \max\{\Pi^{D|SM}, \Pi^{D|SI}\},
\] (64)

which is satisfied in the intervals \( \Phi < (\Delta E)^2 \leq \Theta \) and \( \Phi < (\Delta E)^2 \) since \( \Pi^{D|RI} \geq \Pi^{D|RM} \). Consequently, \( r^{RI} \geq r^{RM} \) in these two intervals. Because \( \Pi^{D|RI} \geq \Pi^{D|RM} \) in these two intervals, the downstream party chooses relational integration if \( r \leq r^{RI} \). Finally, by substituting the respective fallback profits in \( r^{RM} \) and \( r^{RI} \), we obtain

\[
r^{RM} = \begin{cases} 
\frac{1}{8c_U \Delta I} [\Delta I \mu - \Delta E \omega]^2, & \text{if } \tilde{\Pi}^{RM} = \Pi^{D|SM} \\
\frac{1}{2c_U \Delta I} [(\Delta I)^2 - (\Delta E)^2], & \text{if } \tilde{\Pi}^{RM} = \Pi^{D|SI},
\end{cases}
\] (65)

\[
r^{RI} = \frac{\Delta I}{2} - \frac{2}{\Delta I} \left[ \max\{\Pi^{D|SM}, \Pi^{D|SI}\} - I_L \right].
\] (66)

Next, consider the case \( r^i < r \leq \hat{r}^i, i = RM, RI \). Recall that \( \Pi^{D|RM}(r) \geq \Pi^{D|RI}(r) \) in the intervals \( 0 < (\Delta E)^2 \leq \Psi(r) \) and \( \Theta < (\Delta E)^2 \leq \Psi(r) \), where the thresholds \( \Psi(r) \) and
$\Psi(r)$ are valid for all $r \in (r^i, \hat{r}^i]$, $i = RM, RI$. Since the credible incentive schemes $\beta^*(r)$ and $\Delta P^*(r)$ are decreasing in $r$ for $r^i < r \leq \hat{r}^i$, $i = RM, RI$, the downstream’s expected profits $\Pi^{D|RI}(r)$ and $\Pi^{D|RM}(r)$ are also decreasing in $r$. Hence, it is necessary to identify the threshold interest rate $\bar{r}$, where the downstream party is indifferent between the optimal relational contract (relational integration or relational market) and the best spot alternative (spot integration or spot market). Consider first relational integration. Suppose for a moment that $\bar{r} < \hat{r}^i$. This would imply that $\Pi^{D|RI}(\bar{r}) > \hat{\Pi}^R$, which is equivalent to

$$\frac{\hat{r}^R}{2} \left[ \Delta I - \bar{r}^R + 2 \left[ \frac{1}{4} (\Delta I - \bar{r}^R)^2 + I_L - \hat{\Pi}^R \right] \right] > 0,$$

$$\Leftrightarrow \frac{1}{4} (\Delta I - \bar{r}^R)^2 + I_L > \frac{1}{4} (\Delta I - \bar{r}^R)^2.$$

(67)

Since $\hat{\Pi}^R \geq I_L$, it follows that $\bar{r} < \hat{r}^R$ can never be satisfied. Thus, $\bar{r}^R \geq \hat{r}^R$, i.e. $\hat{r}^R$ is the relevant threshold interest rate for relational integration. Hence, the downstream party chooses relational integration for all $r \leq \hat{r}^R$ in the relevant intervals. By utilizing the same approach as for relational integration, one can verify that the downstream party prefers relational market for all $r \leq \hat{r}^{RM}$ in the relevant intervals. Finally, by substituting the respective fallback profits in $\hat{r}^{RM}$ and $\hat{r}^{RI}$, one get

$$\hat{r}^{RM} = \begin{cases} \frac{\Delta I}{c_U} - \frac{1}{c_U} \left[ \frac{1}{4} (\Delta I + \Delta E \omega)^2 + \frac{1}{2} [(\Delta I)^2 - (\Delta E)^2] \right]^{\frac{1}{2}}, & \text{if } \hat{\Pi}^{RM} = \Pi^{D|SM} \\ \frac{1}{c_U} [\Delta I - \Delta E], & \text{if } \hat{\Pi}^{RM} = \Pi^{D|SI} \end{cases},$$

$$\hat{r}^{RI} = \Delta I - 2 \left[ \max \{\Pi^{D|SM}, \Pi^{D|SI}\} - I_L \right]^{\frac{1}{2}}.$$
References


