The Bullionist Controversy: Theory and New Evidence

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April 2015

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Abstract

The Bullionist Controversy in the United Kingdom is one of the first debates about the determination of the price level and the exchange rate under a paper money standard. Despite the importance of the debate in the development of monetary theory, there remains little empirical evidence that uses modern, multivariate time series techniques. The evidence that does exist provides support for the Anti-Bullionist position. The purpose of this paper is to review the debate and develop a dynamic general equilibrium model that is capable of capturing key features of the 19th-century British financial system. The model is estimated using Bayesian procedures to test the competing hypotheses. The paper provides support for the Bullionist position.

Keywords Bullionist Controversy, Quantity Theory of Money, Bayesian estimation
JEL Classification B22, E42, E58, N13

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1 Introduction

From 1792 - 1815, the United Kingdom was involved in perpetual military conflict with France during the French Revolutionary Wars and the Napoleonic Wars. A few years after the start of the conflict, in 1795 and 1796, significant increases in military and political expenditures reduced the specie held in reserve at the Bank of England. Around the same time, poor harvests resulted in additional specie outflow due to the effect on the balance of trade. Finally, at the beginning of 1797, reports of a French invasion of Wales led to a run on banks. As a result of the drain of reserves at the Bank of England, in February 1797, the British government issued an order that suspended convertibility at the Bank. The suspension was meant to be temporary, but it lasted until 1821. This period from 1797 to 1821 is known as the Bank Restriction Period.

During the Bank Restriction Period, a paper money standard replaced the British gold standard. The suspension of convertibility also created a floating exchange rate regime. From the time of suspension until 1801, the price level increased by 50 percent. After a significant decline in the price level from 1801 to 1803, the price level increased uninterrupted until 1813. When the British government restored convertibility in 1821, the price level had already returned to the pre-Restriction level. The significant fluctuations in the price level resulted in considerable debate about the determination of the price level and the exchange rate under a paper money standard. These debates are known as the Bullionist Controversy.[1]

Participants in the debate did not have data on the price level. This not only complicated the debate, but is also the source of the debate's name.[2] The two parties to the debate, the Bullionists and Anti-Bullionists, were given their names as a result of their views on whether the paper price of the gold bullion accurately reflected a depreciation of the value of the currency. The Bullionists argued that rising prices were the result of an excessive issuance of bank notes on the part of the Bank of England. The

[1] One could think of the Bullionist Controversy as describing multiple debates occurring at once. For example, the debate about the deviation of the market price of gold from the mint price can, at times, be seen as distinct from issues involving the determination of the price level and the exchange rate. Other times, the discussion of the premium on the market price of gold serves as a proxy for the price level. This paper is primarily concerned with discussing the determination of the price level and the exchange rate under a paper money standard. In a recent paper, Antipa (2014) analyzes the premium of the market price of gold over the mint price, the "agio". In particular, he tests for structural breaks in the agio series and finds that they line up with significant events. Antipa argues that this is consistent with the fiscal theory of the price level. It is the view of this author that this work is complementary to the present analysis, especially given the fact that the present analysis finds evidence that the Bank of England effectively monetized some portion of government debt.

[2] Viner (1937: 126) notes: "The notion of an index number was still in its infancy. Evelyn had published his crude index number of English prices for the preceding two centuries in 1798 ... But no current index number yet existed for England, and there was but little information as to the prices prevailing in other countries."

early Anti-Bullionists countered that the rising price level was the result of shocks to the balance of trade due to large foreign remittances by the British government and poor harvests.

Despite the significance of these debates, there is very little empirical evidence for this time period. Much of the existing empirical evidence available was done by the participants in the debate and by economists in the early to mid-20th century.[3] More recently Nachane and Hatekar (1995) and Officer (2000) have revisited the Bullionist controversy using available data and modern multivariate time series analysis. These authors argue that the evidence favors the position of the Anti-Bullionists. Empirical evidence in support of the Bullionist position appears mixed, at best.

The Bank Restriction Period appears to follow a quintessentially monetarist story. The British government suspended the convertibility of bank notes into gold at the Bank of England in the midst of war and the price level rose substantially over the next decade and a half. Meanwhile, the French maintained their bimetallic standard without any evidence of a depreciation in the franc.[4] A natural hypothesis is that the British government used its political power in conjunction with the suspension of convertibility to obtain credit from the Bank of England, thereby effectively monetizing some portion of its debt. The failure of previous research to identify evidence in support of the standard monetarist position put forth by the Bullionists is therefore of some significance.

The purpose of this paper is to re-examine the conclusions of existing empirical work using modern macroeconomic theory as a guide. In particular, this paper develops an open economy monetary search model that captures some important characteristics of the 19th-century British financial system.[5] The model is then estimated using Bayesian procedures. The posterior estimates of the model parameters are then used to perform impulse response analysis. The importance of estimating impulse responses in this way is that the structural assumptions are consistent with the model and are therefore explicit. The estimates show that shocks to the supply of bank notes have a positive effect on the price level. In addition, the estimates provide evidence that some portion of the debt incurred by the British government was financed through the creation of bank notes by the Bank of England.

An important question is why the results differ from those of previous researchers. First, this paper

[3] For the empirical analysis of contemporaries, see Ricardo (1810-11) and Galton (1813). The analysis by economists of a later period, see Silbering (1923, 1924a, 1924b), Angell (1926), Viner (1937), and Gayer, Rostow, and Schwartz (1953).

[4] As Bordo and White (1991) point out, France's decision to remain on a bimetallic standard was the result of the French government's poor reputation in debt finance in the aftermath of the French Revolution.

[5] The model is a modification and extension of the monetary search model of Lagos and Wright (2005). Unlike conventional search models, there is spatial separation that creates the need for settlement as in Freeman (1996a, 1996b, 1999). For an overview of monetary search models and an application to settlement, see Nosal and Rocheteau (2011).

shows that the failure of previous researchers to identify cointegration between the logs of the supply of bank notes and the price level is because these variables do not have a unit root, but rather experience a change in the time trend following the Bank of England's voluntary resumption of bank note convertibility.

Second, the model is used to simulate data. This enables one to conduct Granger causality tests when the true data generating process is known. The inability to find evidence that the supply of bank notes "Granger cause" the price level is evident in both the simulated data and the real world data. Thus, it seems that the absence of Granger causation should not be interpreted as support for the Anti-Bullionist position, but rather as an inadequate test.

The paper therefore provides the first evidence in support of the Bullionist position using modern empirical techniques.

2 The Bullionist Controversy

2.1 An Overview of the Financial System in the 19th Century United Kingdom

The nineteenth-century United Kingdom had a rather sophisticated financial system. The banking system consisted of the Bank of England and various country banks. Each bank's liabilities consisted of privately issued bank notes that circulated alongside large denomination gold coins and small denomination silver coins.[6] Deposits were also used, but did not make up a large fraction of the money supply.

Credit was often bilateral, with bills of exchange a primary form of credit between firms. The following example illustrates the way in which bills of exchange functioned. Suppose that a merchant received goods from a producer or wholesaler. Instead of a cash payment, the producer would draw a bill of exchange on the merchant for the amount of the goods due at a short period in the future.[7] A merchant who accepted the bill agreed to settle the debt at the specified date in the future. Thus, upon acceptance, the bill of exchange represented an IOU from the merchant to the producer.

Bills of exchange differed from traditional bank loans in many ways. First, bills matured at the end of a short time interval. Second, bills represented a multilateral commitment by the acceptor. For example, in the case of a bank loan, the borrower engages in a bilateral commitment. The borrower agrees to pay back the loan to the bank. While bills of exchange were also forms of bilateral credit, bills represented

[6] Only underweight silver coins circulated (Viner, 1937).

[7] Sight bills could also be drawn, which were due at the time of issuance.

multilateral commitments. This is because a bill was redeemable at maturity by the bill-holder. This was true regardless of whether the bill-holder was the creditor who had originally drawn the bill.

The multilateral commitment implies that bills of exchange were negotiable. As a result, an initial bill-holder could reuse the bill of exchange. As Thornton (1802 [1807]: 31 - 2) details, a current bill-holder could endorse the bill and use it as a form of payment. In addition, in the event that the acceptor of the bill defaulted, the bill-holder could seek recourse with those who had previously endorsed the bill (Capie and Webber, 1985: 311-2).

The negotiability of bills of exchange also had important implications for banking. The discounting of bills was the primary form of bank lending. Specifically, a bill-holder could take the bill of exchange to the bank for sale. Banks would issue a liability in the form of new bank notes to purchase bills of exchange at a discounted price. The banks then presented the bills for collection at maturity, receiving the spread between the face value of the bill and the discounted price as profit.

Within this context, the Bank of England was the largest and most important bank. In particular, the Bank of England, located in London, had no competition from other banks in the surrounding area. Before 1797 and after 1821, the Bank of England's notes were convertible to specie. The bank notes issued by country banks were redeemable in specie, but also in notes issued by the Bank of England. In this way, country banks had a dual reserve system, holding both specie and Bank of England notes in reserve. During the Bank Restriction Period, Bank of England notes became the dominant form of reserves in the banking system. This was true despite the fact that convertibility was not suspended for the country banks.

2.2 The Debate

In 1793, the United Kingdom went to war with France. In the years that followed, the British economy was subject to considerable financial strain due to the increased spending on military and other foreign expenditures. In 1795 and 1796, bad harvests caused a sharp rise in imports in the U.K. The combination of these factors caused a considerable drain of gold reserves from the Bank of England. As a result, in February of 1797, the British government issued an order that suspended convertibility of bank notes at the Bank of England. The suspension lasted until 1821.

Figure 1 plots the price level for the Bank Restriction Period. As shown in Figure 1, the U.K. experienced a sharp increase in the price level that continued until 1801. After a brief period of price level

stability, the price level began to rise again in 1803. The increase continued until the second quarter of 1813 when military conflict with the French ended. After a decline in the price level, a resumption of inflation began after Napoleon's return in 1815. This increase lasted until the fourth quarter of 1818. In January of 1817, the Bank of England "began partial resumption at the old par, giving gold upon demand for certain categories of its notes, under the authority of a provision in the Restriction Act of 1797 permitting the banks to pay notes under £5 in cash" (Viner, 1937: 172). The British government restored convertibility in 1821, at which point the price level had already returned to its pre-restriction level.

The source of the persistent increases in prices over this period was subject to considerable debate.[8] Data on the price level did not exist at the time. As a result, the participants in the debate often refer to the differences between prices listed in terms of gold coins and prices listed in terms of paper bank notes as a measure of the depreciation of the currency. Others used foreign exchange rates, the price of bullion in terms of the paper pound, as well as other indicators of general increases in prices.

The debate consisted of two groups, the Bullionists and the Anti-Bullionists. The Bullionists argued that the increase in prices and the depreciation of the exchange rate was due to excess issuance of bank notes on the part of the Bank of England. Anti-Bullionists argued that fluctuations in prices and the exchange rate were due to remittances and subsidies paid by the British government. They also attributed changes to bad luck as a result of poor British harvests. In other words, Bullionists argued that fluctuations in the price level were due to shocks to the money supply whereas the Anti-Bullionists argued that fluctuations in the price level were the result of shocks to the balance of trade and supply shocks.

The evolving nature of the debate can be understood quite well by considering the views of Henry Thornton. In 1802, Thornton published *An Inquiry into the Nature and Effects of Paper Credit in Great Britain*. The book outlined a much more nuanced and comprehensive view of the Bullionist position than those given by Thornton's contemporaries. In addition, while his book outlined what would be considered a Bullionist argument, he also suggested that the Bank of England did not bear significant blame for the rise in the price level in the early stages of the Bank Restriction Period.

Later, as a member of the Bullion Committee, Thornton wrote, together with Francis Horner, the

[8]This debate has been summarized in great detail elsewhere. As such, what follows is a brief overview. For comprehensive overviews of the controversy, see Angell (1926) and Viner (1937).

Report of the Bullion Committee (1810). The report was very critical of the Bank of England and, in large part, relied on the principles set forth in Thornton (1802) to make these arguments. As a result, a thorough understanding of Thornton's views and the reason for his change in assessment of the blame help one to better understand the subtleties of the Bullionist Controversy.

2.3 Thornton and the Subtleties of the Debate

Thornton's *Paper Credit* outlines the Bullionist position in detail and presents a rather sophisticated view of the quantity theory. However, it also highlights some subtle points that help one to better understand the Bullionist position.

Paper Credit is notable, in part, for points that Thornton, unlike some other Bullionists, was willing to concede to the Anti-Bullionists. For example, Thornton (1802 [1807]: 168 - 70) readily acknowledged the validity of the arguments made by Anti-Bullionists regarding the effects of poor harvests and government remittances on the exchange rate. Nonetheless, in Thornton's view, shocks to the balance of trade could influence the price level or the exchange rate, but the dominant factor were shocks to the quantity of bank notes. Thornton (1802 [1807]: 167) explained the Bullionist position for explaining the rising price level in terms that would be familiar with a modern understanding of the quantity theory of money. Thornton (1802 [1807]: 186 - 7) also anticipated modern critiques of the quantity theory. Specifically, Thornton clarifies that if the demand for money is constant, the only factor that can effect the value of the Bank's notes is the supply. In other words, Thornton's discussion makes clear that while changes in the demand for money can have an effect on the supply of bank notes in circulation and/or the purchasing power of those notes, an excess issuance of bank notes will always cause an increase in prices. This is perhaps the clearest statement of the Bullionist position.

To conclude that this is all Thornton had to offer in terms of economic analysis would do a disservice to his work.[9] Thornton's views represent a rather sophisticated view of the determination of the supply of bank notes issued by the Bank of England. For example, throughout much of the debate, Anti-Bullionists

[9]Thornton's contributions to monetary economics are perhaps now well-known. It seems that for some time, Thornton was neglected in this debate. Niehans (1990: 110) notes that Thornton was "an authority during his lifetime, [but] he thus fell into near oblivion after his death." As Laidler (2000) notes, this is likely due to a confusion about whether Thornton's ideas were his own or instead represented ideas held by or developed by his brother, who was a governor of the Bank of England. Early scholars apparently viewed Thornton's work as something of an insiders account and it was discounted (unfairly), accordingly. His contribution was revived to some degree by Viner (1937) and later by Hayek (1939). Subsequent economists have emphasized the importance of Thornton's contributions to economics. See, for example, Schwartz (1981), Humphrey (1986), Skaggs (1995), and Laidler (2000).

largely took for granted the view that the Bank of England's supply of bank notes responded positively to supply shocks and shocks to the balance of trade to accomodate an increase in the demand for bank notes. The Anti-Bullionist view was that shocks to military expenditures, foreign subsidies, and poor harvests caused the exchange rate to depreciate and the price level to rise and the supply of bank notes to rise. Thornton argued that whether this would indeed occur depended on the Bank's credit policy. He also argued that the Bank could prevent such increases in the price level by restricting the supply of bank notes in response to these shocks.

Thornton's view is important because it represents an understanding of what modern economists would call the Bank of England's reaction function. Thornton (1802 [1807]: 248-9) outlined his view of the operations of the Bank of England as follows:

> To limit the total amount of paper issued, and to resort for this purpose, whenever the temptation to borrow is strong, to some effectual principle of restriction; in no case, however, materially to diminish the sum in circulation, but to let it vibrate only within certain limits; to afford a slow and cautious extension of it, as the general trade of the kingdom enlarges itself; to allow of some special, though temporary, encrease in the event of any extraordinary alarm or difficulty, as the best means of preventing a great demand at home for guineas; and to lean to the side of diminution, in the case of gold going abroad, and of the general exchanges continuing long unfavourable; this seems to be the true policy of the directors of an institution circumstanced like that of the Bank of England.

This quote illustrates Thornton's view of the role played by the Bank of England. Nonetheless, previous scholars seem to have interpreted this as a normative statement about what the Bank of England should do (Laidler, 2000). While it is true given the context of Thornton's entire argument that he advocated this type of policy, the quote above is meant to be a positive description of how the Bank actually operated. To illustrate this point, consider that just a page earlier, Thornton (1802 [1807]: 247-8) writes:

> The preceding observations explain the reason of a determination, adopted some time since by the bank directors, to limit the total weekly amount of loans furnished by them to the merchants. The adoption of a regulation for this purpose seems to have been rendered necessary by that impossibility of otherwise sufficiently limiting, at all times, the Bank of England paper, which it has been the design of this Chapter to point out.

The issue of whether to interpret Thornton's statement as a positive or normative argument is important. For example, within the literature it is claimed that Thornton's book "is hard to classify, because though its analytic content places it firmly in the Bullionist camp, it nevertheless defends the policies pursued by the Bank of England after 1797" (Laidler, 2000: 8). Laidler's view that Thornton's book is hard to

classify seems to imply a contradiction. The Bullionist position was that excess issuance on the part of the Bank of England caused the increase in the price level. A defense of the Bank would seem to support the Anti-Bullionist view. But there is no contradiction. Thornton's defense of the Bank of England in the early part of the Bank Restriction Period is a positive, not normative, analysis. For example, Thornton believed that changes in the price level could result from either a shock to the money supply or other shocks emphasized by the Anti-Bullionists. However, Thornton's view was that the importance of the shock depended on what economists would now call the Bank of England's reaction function. He didn't believe that remittances or adverse harvests would necessarily result in a change in prices, arguing that the effects of such shocks on the price level would depend on the central bank's response (Thornton, 1802 [1807]: 254).

Thornton's defense of the Bank of England in the early part of the debate amounts to little more than an implicit acknowledgement that the Bank responded differently to supply shocks than demand shocks. There is some indication that Thornton found this desirable. Elsewhere in the text, when discussing the role of the Bank of England under a gold standard, Thornton (1802 [1807]: 66-8) suggests that proportional reductions in the supply of bank notes in response to shocks to the balance of trade and the gold reserve would have adverse consequences on real economic activity if the shocks were temporary.[10] Nonetheless, when discussing the causes of the recent increase in prices, Thornton refrains from normative statements.

Perhaps further evidence that this defense of the Bank of England was strictly positive rather than normative is the fact that the defense was short-lived. As noted above, in 1809, a speculative boom and rising prices caused more consternation about the extent to which these were caused by the Bank of England. In 1810, the House of Commons formed a committee to studying rising price of bullion. The outcome of this report is the *Bullion Report*, which was largely written by Henry Thornton and Francis Horner. The *Report* places the blame on the Bank of England for the rising prices over the previous decade using arguments set forth in Thornton (1802) and described above. This latter characteristic is important because the *Report* is considered the defining document of the Bullionist position.

In assessing the Bullionist debate, it is the position of this paper that Thornton represents the most clear and coherent participant on either side of the debate.[11] The discussion of existing empirical work

[10] This is a point that Thornton's contemporaries, such as David Ricardo, either didn't accept or didn't understand.

[11] It is traditional in the literature on the Bullionist Controversy to heap praise on Thornton while similarly decrying the lack of clarity in his book (c.f. Laider, 2010). This author does not share the latter part of that judgment.

below and the theory and empirical work in this paper will be considered predominantly in light of this position.

2.4 Existing Empirical Evidence

Much of the existing empirical literature on the Bullionist Controversy predates modern multivariate time series analysis. This literature is not summarized here both because much of this literature consists of subjective interpretations of charts and tabular data and because it is summarized adequately elsewhere (Officer, 2008). The focus of this section is on the two papers that use modern empirical techniques.

The earliest attempt to use modern time series techniques to address the Bullionist Controversy was Nachane and Hatekar (1995). They use annual data from 1802 - 1838 on the price of gold, the exchange rate of sterling in Paris, a wholesale price index, the trade balance, and a measure of the money supply to examine the positions of the Bullionists and Anti-Bullionists. They measure the money supply as the ratio of the supply of bank notes and deposits to real output. The authors perform unit root tests for each of the variables. They fail to reject the null hypothesis of a unit root for each of the variables, except for the trade balance. They then use bivariate cointegration tests for each combination of variables. They only find evidence of one cointegrating relationship, which is between the logs of wholesale prices and the ratio of money to real output.[12]

In addition, they use Granger causality tests to determine whether lags of a given variable have an effect on a particular variable of interest. They perform these tests using each of the variables listed above as dependent variables. They find Granger causality in only two cases. Their results show that the trade balance Granger-causes the wholesale price index and that the exchange rate Granger-causes the wholesale price index. Each of these results is consistent with the Anti-Bullionist position. None of the results are consistent with the Bullionist position. Thus, Nachane and Hatekar (2005) conclude that their evidence strongly favors the Anti-Bullionist position.

There is reason for caution about these results. To the extent that the conclusions of each side of the debate were conditional on a monetary regime in which bank notes were inconvertible, it seems necessary to limit the scope of the empirical evidence to the Bank Restriction period. The use of annual data limits the authors in this respect and so they add data at the end of the sample. Also, the authors

[12] Curiously, they do not interpret this as evidence in favor of the Bullionist position, despite being consistent with Thornton's views above.

fail to find evidence of cointegration between all but two combinations of variables. This is problematic because one of the variables in this cointegrated relationship, the wholesale price index produced by Silberling (1924), is flawed (Gayer, Rostow, Schwartz, 1953: 463-7). In addition, the lack of cointegration might reflect a lack of a long-run equilibrium relationship between the variables or, alternatively the authors might have incorrectly concluded that the variables follow unit root processes. As there is so little evidence of cointegration, there is at least some reason to suspect that the latter might be true. The empirical work below considers this possibility.

Subsequent work by Officer (2000) is an improvement over Nachane and Hatekar (1995) for two reasons. First, Officer uses quarterly rather than annual data. This not only allows for greater degrees of freedom, but also allows him to limit his analysis to the Bank Restriction Period. Second, Officer uses the quarterly average of the price index developed by Gayer, Rostow, and Schwartz (1953). This index corrects for the shortcomings of Silberling's index.

Officer examines the positions of the Bullionists and Anti-Bullionists using Granger causality tests and a structural vector autoregresssion (VAR) model. His model includes the supply of bank notes, the price of wheat, the price level, military expenditures, and the exchange rate. Officer's Granger causality tests show that the supply of bank notes Granger causes both the price level and the exchange rate. There is also evidence that the price level and the price of wheat predict the supply of bank notes. Officer interprets this as mixed evidence since there seems to be causation running both ways between the price level and the supply of bank notes.[13]

Officer then estimates a structural VAR model using a Choleski decomposition to identify the economic shocks. Officer determines the orderings of the variables using the results of the Granger causality tests. After recovering the structural shocks, he then estimates the impulse response functions of each of the variables in response to each of the shocks. The impulse response functions show that the response of the price level to a shock to the supply of bank notes is not statistically significant for any point after the shock. In fact, the only shock that has a statistically significant effect on the price level is the shock to the price of wheat, which he views as a supply shock. Variance decompositions similarly cast doubt on the Bullionist position. Thus, Officer concludes that there is more support for the Anti-Bullionist position than for the Bullionist position.

[13]One should note here, as Officer himself does, that Granger causality is not necessarily economic causality. A point which is revisited below.

Nonetheless, there is reason for skepticism about Officer's results. Specifically, Granger causality tests depend critically on the nature and the role of policy. This is a case in which the Bullionist position described by Thornton is particularly important. If the Bank of England responded differently to demand-side shocks than supply-side shocks to the economy, then one would expect to find instances when the money supply lagged the price level as well as periods in which the money supply led the price level. Nonetheless, since the Granger causality tests are conducted for the entire sample, the test is averaging across these effects. In addition, if the Bank of England's credit policy was a function of the advances that they gave to the government, this would create a correlation between monetary and fiscal policy that would be hard to disentangle using Granger causality tests. These two criticisms suggest that Granger causality tests are potentially uninformative about the role of excess bank note issuance in explaining the price level.

In addition, Officer uses a Choleski decomposition to identify the model's structural shocks. This is potentially problematic because a Choleski decomposition requires assumptions about the contemporaneous relationships between the variables that is dependent on ordering. Given that the ordering can be arbitrary, Officer uses the results of his Granger causality tests to determine the ordering of the variables. Granger causality tests might not be useful for this purpose. They test the effects of a lagged variable on a variable of interest. These tests do not provide information about the contemporaneous relationships between the variables. Thus, it is possible that the structural shocks have not been properly identified.

With these criticisms in mind, the remainder of the paper develops an explicit theoretical model capable of replicating key characteristics of the British financial system described above. One can estimate the model directly to determine the effects of shocks to the supply of bank notes and to real government spending. The benefit of this approach is that the structural assumptions are explicitly known. Also, one can use the model to simulate data and construct Granger causality tests. One can then assess whether the existing evidence on Granger causality is informative toward understanding the source of fluctuations in the price level.

3 The Model

There are two countries, a home country and a foreign country. The countries are identical. The description below describes the home country, but is equally applicable to the foreign country. Time is

discrete and continues forever. Each time period is divided into three subperiods. For simplicity and ease of exposition, these time periods will be referred to as morning, afternoon, and night.[14] Agents are spatially separated across I different islands, where I is an even number. The islands are arranged in a circle around a central location. Each island has a neighboring island, but is separate from all other islands. Let $i \in \{1, 3, 5, \ldots, I-1\}$ be an index of odd numbered islands and $j \in \{2, 4, 6, \ldots, I\}$ be an index of even-numbered islands. The islands are numbered sequentially such that Island 1 is next to Island 2, Island 3 is next to Island 4, and so on. There are N agents on each odd-numbered island and each even-numbered island. The population on each island is assumed to be constant. A visualization of the environment is shown in Figure 2.

On all odd-numbered islands, agents produce a good that is unique to their island. Let $y(i)$ denote the quantity of the odd-island good produced by an agent on island i. Agents on odd-numbered islands use labor, h, to produce y. It is assumed that the production function is given as $y(i) = h(i)$ and that agents on odd-numbered islands can only produce at night. The odd-island good is assumed to be non-storable, even across subperiods. Henceforth, all agents on odd-numbered islands will be referred to as debtors, which foreshadows their role in the model.

On even-numbered islands, agents also produce a good that is unique to their island. Let $q(j)$ denote the quantity produced by agents on even-numbered island j. Let $f[q(j)]$ denote the cost of producing q for those on even-numbered islands, measured in terms of disutility. Agents on even-numbered islands only produce in the morning. The even-island good is also non-storable, even across subperiods. Henceforth, all agents on even-numbered islands are referred to as creditors, which again foreshadows their role in the model.

In addition to producing different goods with different production technologies, agents on odd- and even-numbered islands also differ in terms of consumption preferences. Debtors want to consume the good on their neighboring even-numbered island and creditors want to consume the good produced on odd-numbered islands. The expected lifetime utility of a debtor on island i is therefore given as

$$E_0 \sum_{t=0}^{\infty} \beta^t \{u[\varrho_t(i)] - h_t(i)\}$$

[14]The use of morning, afternoon, and night should not be taken literally. In fact, given the nature of bills of exchange, a more reasonable interpretation given the setup of the model would be that each period t is a quarter of one year, such that each subperiod represents a month's time.

where ϱ denotes the quantity of the creditor good that is consumed by the debtor, $-h_t(i)$ is the disutility of labor, u is the utility from consuming ϱ, and $u', -u'' > 0, u'(0) = \infty, u'(\infty) = 0$. Given the linear production technology, this can be re-written as

$$E_0 \sum_{t=0}^{\infty} \beta^t \{u[\varrho_t(i)] - y_t(i)\} \tag{1}$$

There are two types of creditors of equal size on each even-numbered island. The first group of creditors has deterministic preferences. Specifically, it is assumed that creditors on island j with deterministic preferences visit island $j \oplus 1 = j + 1 \mod I$ to consume good y. The expected lifetime utility of a creditor on island j with deterministic preferences is given as

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ c_t^{CD}(j) - f[q_t^{CD}(j)] \right\} \tag{2}$$

where q_t^{CD} is the quantity produced by creditors with deterministic preferences and $c_t^{CD}(j)$ is the consumption of the good produced on odd-numbered islands by creditors with deterministic preferences on island j.

The second group of creditors has random preferences. It is assumed that creditors on island j with random preferences want to consume a good produced by debtors on a random island $i \neq j-1$. In other words, the second group of creditors wants to consume a good produced by debtors, but the location of the debtor is random other than the fact that it is never the good produced by debtors on the neighboring island. The expected lifetime utility of a creditor on island j with random preferences is given as

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ c_t^{CR}(j) - f[q_t^{CR}(j)] \right\} \tag{3}$$

where $q_t^{CR}(j)$ denotes the quantity produced by creditors with random consumption preferences on island j, and $c_t^{CR}(j)$ is the consumption of the good produced on odd-numbered islands by creditors with random preferences on island j.

All agents can travel to the central location. There is a bank in the central location. In the morning that bank consumes good q^* from creditors in the foreign country. In the afternoon, the bank purchases debt from creditors at a discount and finances the debt by issuing bank notes. The bank also provides

settlement for debt at night. The bank finances consumption through seigniorage and/or the profits earned by discounting bills.[15]

All agents on a given island have complete information about the transaction histories of agents on the neighboring island. In addition, it is assumed that creditors with deterministic preferences always trade with the same debtor agent at night. Creditors with random preferences always trade with a random debtor. Each agent in the model can only travel to one other island each subperiod.

There are two assets in the model, bank notes and bills of exchange. Bank notes are perfectly recognizable, storable, and impossible to counterfeit. It is assumed that bills of exchange can be costlessly counterfeited, but that both a counterfeit bill and the counterfeiter can be identified when the bill is presented to the bank.

The timing of events is as follows.

1. *Morning.* The home bank consumes goods on the foreign creditor islands and the foreign bank consumes goods on the home creditor islands. Debtors then travel to their neighboring island to purchase the good produced by creditors. Debtors are matched with a creditor on the neighboring island with probability $\alpha \in (0, 1)$. Creditors and debtors have information about the past trading histories of all individuals on the neighboring island. Debtors can use bank notes held from the previous period to purchase the creditor's consumption good and/or the creditor can draw a bill of exchange on the debtor. After consuming, debtors travel back to their respective islands.

2. *Afternoon.* Creditors can visit the central island to sell bills of exchange to the bank in exchange for bank notes.

3. *Night.* Creditors travel to debtor islands to consume. Creditors with deterministic preferences travel to the same island and meet with the same debtor every period. Creditors with random preferences travel to different islands and trade with debtor agents in a centralized market. Debtors travel to the central location to repay their debt. The bank settles payments. The period ends.

3.1 The Role of Money and Debt

In the morning debtors travel to their neighboring island to purchase good q using bank notes and/or debt. Debt comes in the form of a bill of exchange. The bill of exchange is drawn by the creditor and

[15] The assumption here is akin to a consolidation of fiscal and monetary policy.

accepted by the debtor with a signature. The bill of exchange grants the creditor the right to a real quantity of goods produced by the debtor at night. As noted above, agents on neighboring islands have complete information about the transaction history of all the agents on the pair of islands. In addition, assume that if a debtor defaults on their debt, creditors on neighboring islands will refuse to trade with that debtor in perpetuity and can credibly commit to this action. The debtor will therefore never have an incentive to default.

Given that debtors never have an incentive to default, creditors will always lend to debtors in the morning. A creditor holding a bill of exchange will then have two options. First, the creditor can take the bill of exchange to the bank on the central location in the afternoon and sell the bill to the bank at a discount. Second, the creditor can take the bill of exchange to a debtor island and use the bill of exchange as a medium of exchange to purchase good y at night from a debtor. Thus, it is important to consider the conditions under which a creditor would choose between these alternatives.

Recall that creditors with deterministic preferences are assumed to be costlessly matched with the same debtor in every night of every period. Thus, they have an established trading relationship with debtors. Assume that debtors will refuse to trade with a creditor that passes a counterfeit bill in perpetuity and can credibly commit to these actions. This implies the following. If a debtor is in an established relationship with a creditor, they will accept bills of exchange drawn on another debtor in exchange for goods at night. If the meeting is at random, they will not accept these bills.

This implication can be understood as follows. Since it is costless to counterfeit, every creditor has an incentive to pass a counterfeit bill of exchange because this allows the creditor to generate consumption without any additional cost of producing. It was assumed above that once a counterfeit bill is given to the bank, the counterfeit is discovered and the counterfeiter is made public knowledge. Thus, for those in established relationships, the debtor can punish the creditor by refusing to trade in all future meetings if they pass a counterfeit bill. Thus, creditors in established relationships will never have an incentive to pass a counterfeit bill.

In contrast, creditors with random consumption preferences trade with debtors in a centralized market. These meetings are assumed to be anonymous. Thus, debtors do not have a punishment mechanism since they would not be able to identify the counterfeiter in future meetings and therefore will not accept bills of exchange from any creditor with random preferences.

Thus, creditors with random preferences will never bring bills of exchange for trading at night. They

will have to trade using bank notes. Creditors in established relationships will bring bills of exchange to trade at night if the consumption that they can purchase with bills of exchange exceeds the consumption that they can purchase with bank notes.

Proposition 1. *Define R_t as the (gross) real rate of interest. If $R_t \geq 1$, creditors with deterministic preferences will always bring bills of exchange to trade at night.*

Proof. See the appendix. □

3.2 Creditors

In the morning, creditors on island i produce $q(i)$. In the beginning of the morning, the foreign bank purchases an amount $(2/NI)g^*$ from each creditor. Subsequently, debtors arrive on the creditor islands to consume. When debtors arrive, they do not have any goods to trade. Debtors can offer bank notes to creditors in exchange for the creditor's production. If the debtor wants to purchase more than would be possible with bank notes, then the creditor can draw a bill of exchange on the debtor in exchange for the additional production. Let b_t^D denote the real value of a bill drawn by creditors with deterministic preferences and b_t^R denote the real value of a bill drawn by creditors with random preferences.

In the afternoon, the creditor can bring the bill of exchange received in the morning to the bill broker on the central island. Denote the (gross) real rate of interest as R_t.[16] The bill broker will offer an amount equal to $b_t^D(j-1)/R_t$ to creditors with deterministic preferences and $b_t^R(j-1)/R_t$ to creditors with random preferences. Alternatively, the creditor could skip the visit to the central island and simply bring the bill of exchange to a debtor at night and try to use it as a medium of exchange. Whether or not the creditor will visit the central island is determined as follows.

Creditors with deterministic preferences draw bills of exchange such that

$$b_t^D(j) = q_t^{CD}(j) - (2/NI)g_t^* - \phi_t m_t \tag{4}$$

where ϕ_t is the price of money, m_t is the quantity of bank note balances held by a debtor, $q_t^{CD}(i)$ is the quantity produced by creditors with deterministic preferences on island j and g^* denotes the consumption by the foreign bank.

[16]The term gross real interest rate has the following definition. Suppose that the real interest rate is 5%. The gross real interest rate is $R = 1.05$.

Creditors with random preferences draw bills of exchange such that

$$b_t^R(j) = q_t^{CR}(j) - (2/NI)g_t^* - \phi_t m_t \tag{5}$$

where $q_t^{CR}(j)$ is the quantity produced by creditors with random preferences on island j.

Creditors who have random consumption preferences bring banknotes into the night subperiod. They can consume an amount, c_t^{CR}, subject to the constraint

$$\phi_t m_t^{CR} \geq c_t^{CR}(j) \tag{6}$$

where m_t^{CR} denotes bank note balances held by creditors.

Proposition 2. *In any equilibrium in which bank notes are held, constraint (6) will always bind.*

Proof. See the Appendix. □

In any equilibrium where bank notes are held, it will be true that creditors will never hold more bank notes than are necessary to carry out transactions at night. It follows that the creditors' optimization problem is static.

Proposition 3. *The production decision of creditors with deterministic preferences is independent of the rate of return on money.*

Proof. Consumption for creditors with deterministic preferences satisfies

$$b_t^D(j) + \phi_t m_t + (2/NI)g_t^* = c_t^{CD}(j) \tag{7}$$

Their optimization problem is given as

$$\max_{c_t^{CD}(j), q_t^{CD}(j), b_t^D(j)} c_t^{CD}(j) - f[q_t^{CD}(j)]$$

subject to (4) and (7).

This implies that the optimization problem can be re-written as

$$\max_{q_t} q_t^{CD}(j) - f[q_t^{CD}(j)]$$

The first-order condition is given as

$$1 = f'[q_t^{CD}(j)] \tag{8}$$

The quantity of production done by creditors with deterministic preferences is independent of the rate of return on money. □

This result can be understood as follows. Since creditors with deterministic preferences can use the bills of exchange that they draw on debtors as a medium of exchange at night, they will choose to draw a quantity such that the marginal utility of consumption at night is equal to the marginal cost of production in the morning. This decision is independent of the price of money because creditors can substitute bills of exchange one-for-one with bank note balances. Thus, when the real value of bank notes is low, they can issue a larger quantity of bills of exchange and when the real value of bank notes is high, they can draw a smaller quantity of bills.

The same is not true for creditors with random preferences. Since creditors with random preferences meet anonymously with debtors at night, they must use bank notes to trade. Since debtors will not accept bills of exchange, creditors with random preferences must first visit the central location to discount the bills at the bank. If the (gross) real interest rate charged for discounting bills, R_t, is greater than one, these creditors cannot substitute bills of exchange one-for-one with bank notes. As such, the decision of creditors will be a function of the real interest rate.

The quantity of bank notes held by creditors with random preferences is equal to the sum of the discounted value of the bill of exchange they sold to the bank in the afternoon and the quantity of bank notes they received from the debtor in the morning. Thus, it follows that

$$\phi_t m_t^{CR} = \frac{b_t^R(j)}{R_t} + \phi_t m_t$$

Using equation (5) and the fact that (6) is binding, it follows that this can be re-written as

$$c_t^{CR} = \frac{q_t^{CR}(j) - (2/NI)g_t^* - \phi_t m_t}{R_t} + \phi_t m_t$$

Creditors therefore have the following maximization problem:

$$\max_{q_t^{CR}} \frac{q_t^{CR}(j) - (2/NI)g_t^*}{R_t} + \left(\frac{R_t - 1}{R_t}\right)\phi_t m_t - f[q_t^{CR}(j)]$$

The first-order condition is given as

$$\frac{1}{R_t} = f'[q_t^{CR}(j)] \tag{9}$$

3.3 Debtors

Debtors are matched with a creditor with probability $\alpha \in (0, 1)$. Debtors will either interact with a creditor with deterministic preferences or a creditor with random preferences. As noted above, creditors with deterministic preferences produce a quantity that is independent of the rate of return on money. Debtors that are matched with this creditor therefore consume this quantity. The analysis that follows is confined to debtors who are matched with creditors with random preferences.

Debtors come into the period with real bank note balances, $\phi_t m_t$. Debtors can use these bank note balances to purchase the good produced by creditors. Creditors can also draw bills of exchange on debtors to finance additional consumption. A debtor that uses a real quantity of bank notes, $\phi_t m_t$, to purchase good ϱ_t^{CR} receives a surplus from trade of $u(\varrho_t^{CR}) - \phi_t m_t$. The total surplus from trade between the debtor and creditor is given as $u(\varrho_t^{CR}) - f(\varrho_t^{CR})$. It is assumed that debtors and creditors split the surplus each period such that debtors and creditors get a fixed proportion of the surplus. Let θ denote the fraction of the surplus that goes to debtors. For a debtor that offers $d \leq m$ bank notes in exchange for the creditor's good, it follows that

$$u(\varrho_t^{CD}) - \phi_t d_t = \theta\Big[u(\varrho_t^{CR}) - f(\varrho_t^{CR})\Big] \tag{10}$$

where from (5) it must be true that $\phi_t d_t = q_t^{CR} - b_t^R$. It follows from Proposition 2 that in any monetary equilibrium, $d_t = m_t$.

At night, the debtor produces an amount, $y_t^{CR}(i)$. For a debtor that enters the period with real bank note balances, $\phi_t m_t$, the evolution of money balances of a debtor on island i over the course of the period is given as

$$\phi_t m_{t+1} = \phi_t m_t + y_t^l(i) + T_t \tag{11}$$

where y^l denotes the production of the odd-island good for a creditor of type $l \in \{CR, CD\}$, T_t is a lump-sum transfer/tax administered by the bank.

Let $W_t(m_t)$ denote the value function of debtors holding m_t banknotes in the morning and $V_t(m_t)$ denotes the value function of debtors carrying bank note balances, m_t, at night. It follows that the value function for debtors in the morning can be written as

$$W_t(m_t) = \alpha[u(\varrho_t^{CR}) + V_t(m_t - d_t)] + (1-\alpha)V_t(m_t) \tag{12}$$

For a debtor on island i, V_t satisfies

$$V_t(m_t) = \max_{y_t, m_{t+1}} -y_t^l(i) + \beta E_t W_{t+1}(m_{t+1})$$

Assuming an interior solution for y_t^l and using (11), this can be re-written as

$$V_t(m_t) = \max_{m_{t+1}} \left(T_t + \phi_t m_t - \phi_t m_{t+1} \right) + \beta E_t W_{t+1}(m_{t+1}) \tag{13}$$

The first-order condition is given as

$$\phi_t = \beta E_t W'_{t+1}(m_{t+1}) \tag{14}$$

The linearity of V_t implies that (12) can be re-written as

$$W_t(m_t) = \alpha[u(\varrho_t^{CR}) - \phi_t d_t] + V_t(m_t)$$

From (10) and (5) it follows that

$$W_t(m_t) = \alpha\theta[u(\phi_t m_t) - f(\phi_t m_t)] + V_t(m_t)$$

Differentiating this expression with respect to m_t yields

$$W'_t(m_t) = \alpha\theta\phi_t[u'(\varrho_t^{CR}) - f'(\varrho_t^{CR})] + \phi_t$$

Iterating forward and substituting this expression into (14) yields

$$E_t \frac{\phi_t}{\beta \phi_{t+1}} = \alpha \theta E_t \left[u'(\varrho_{t+1}^{CR}) - f'(\varrho_{t+1}^{CR}) \right] + 1 \tag{15}$$

3.4 The Bank

In the morning the bank consumes an amount, g_t, of the good q^*. In the afternoon the bank discounts bills of exchange. The bank's spending is financed using the profits from discounting bills and money creation. It follows that in equilibrium,

$$g_t = \phi_t (M_t - M_{t-1}) + (R_t - 1)(NI/4) b_t^R \tag{16}$$

where M_t is the aggregate supply of money.

3.5 Market-Clearing Conditions

The concavity of u ensures that $\varrho_t^{CD}(j) = \varrho_t^{CD}$ and $\varrho_t^{CR}(j) = \varrho_t^{CR}$, for all j. In addition, symmetry implies that $q_t^{CD}(j) = q_t^{CD}$ and $q_t^{CR}(j) = q_t^{CR}$, for all j, and $y_t^D(i) = y_t^D$ and $y_t^R(i) = y_t^R$, for all i.

Production by creditors satisfies

$$f'(q_t^{CD}) = 1$$

and

$$f'(q_t^{CR}) = 1/R_t$$

Equilibrium in the market for the good produced by creditors requires that the consumption purchased by the foreign government and the debtors must equal total production, such that

$$\varrho_t^{CD} + (2/NI) e_t g_t^* = 1 \tag{17}$$

$$\varrho_t^{CR} + (2/NI) e_t g_t^* = q_t^{CR} \tag{18}$$

where e_t is the exchange rate and is defined as

$$e_t = \frac{\phi_t}{\phi_t^*} \tag{19}$$

where variables with an asterisk denote the values for the foreign country.

The supply of debt issued by creditors with random preferences is given as

$$b_t^R = q_t^{CR} - [(1-\theta)u(\varrho_t^{CR}) + \theta f(\varrho_t^{CR})] - (4/NI)g_t \tag{20}$$

The aggregate real supply of money is given as

$$\phi_t M_t = (NI/4)[(1-\theta)u(\varrho_t^{CR}) + \theta f(\varrho_t^{CR})] \tag{21}$$

Since both countries are assumed to be identical, there is a symmetric set of equilibrium conditions for the foreign country:

$$g_t^* = \phi_t^*(M_t^* - M_{t-1}^*) + (R_t^* - 1)(NI/4)b_t^{R*} \tag{22}$$

$$\frac{\phi_t^*}{\beta\phi_{t+1}^*} = \alpha\theta\left[u'(\varrho_{t+1}^{CR*}) - f'(\varrho_{t+1}^{CR*})\right] + 1 \tag{23}$$

$$\varrho_t^{CD*} + (2/NI)(1/e_t)g_t = 1 \tag{24}$$

$$\varrho_t^{CR*} + (2/NI)(1/e_t)g_t = q_t^{CR*} \tag{25}$$

$$b_t^R* = q_t^{CR*} - [(1-\theta)u(\varrho_t^{CR*}) + \theta f(\varrho_t^{CR*})] - (4/NI)g_t^* \tag{26}$$

$$\phi_t^* M_t^* = (NI/4)[(1-\theta)u(\varrho_t^{CR*}) + \theta f(\varrho_t^{CR*})] \tag{27}$$

Equilibrium. *Suppose that:*

1. *The home money supply follows the process*

$$\ln(M_t) = \ln(M) + \rho_m \ln(M_{t-1}) + \gamma(1-\rho_m)\ln(g_t) + \epsilon_t^m$$

$$\epsilon_t^m = \rho_{em}\epsilon_{t-1}^m + \xi_t^m$$

where ϵ_t^m is an unexpected shock to the money supply, M is the steady state value of the home money supply, and $\rho_m \in (0,1)$ and γ are parameters.

2. *Home real bank spending follows the process*

$$\ln(g_t) = (1 - \rho_g)\ln(g) + \rho_g \ln(g_{t-1}) + \varepsilon_t^g$$

$$\epsilon_t^g = \rho_{eg}\epsilon_{t-1}^g + \xi_t^g$$

where ε_t^g is a shock to spending, g is the steady state level of consumption by the home bank, and $\rho_g \in (0,1)$ is a parameter.

3. *The foreign money supply follows the process*

$$\ln(M_t^*) = \ln(M^*) + \rho_m^* \ln(M_{t-1}^*) + \epsilon_t^{m*}$$

where ϵ_t^{m*} is an unexpected shock to the money supply, M^* is the steady state value of the foreign money supply, and $\rho_m^* \in (0,1)$ is a parameter

4. *Foreign real bank spending follows the process*

$$\ln(g_t^*) = (1 - \rho_g^*)\ln(g^*) + \rho_g^* \ln(g_{t-1}^*) + \varepsilon_t^{g*}$$

where ε_t^{g*} is a shock to spending, g^* is the steady state level of consumption by the foreign bank, and $\rho_g^* \in (0,1)$ is a parameter.

Given 1, 2, 3, and 4, equations (15) - (27) are sufficient to solve for $q_t^{CR}, R_t, \phi_t, b_t^R, \varrho^{CR}, \varrho_t^{CD}, q_t^{CR*}$, $R_t^*, \phi_t^*, b_t^R*, \varrho^{CR*}, \varrho_t^{CD*}, e_t$.

4 Discussion

4.1 Consistency with the 19th Century United Kingdom

The model is capable of capturing four key features of the financial system in the 19th century U.K.:

- Bills of exchange were the primary form of credit.
- The central bank controlled the money supply through the discounting of bills of exchange.

- Bills of exchange were often subject to multiple endorsements.[17]

  As noted by Thornton (1802), bills of exchange were often signed by the bill holder and exchanged in future transactions. Perhaps more remarkable, these bills of exchange traded at par value (Thornton, 1802 [1807]: 31 - 33). Whether this process was desirable was subject to debate. Some critics of this process referred to these types of bills of exchange as "fictitious" bills. Critics used the term "fictitious" since the bill was not backed by goods in process. Nonetheless, Thornton argues that this distinction is meaningless. This distinction confuses the backing of a bill with the redemption of a bill. For example, in the event that a bill was not repaid, there was recourse to go after the borrower. However, the individual or firm who had drawn the bill did not have the right to the goods that were to be produced or sold. Thus, reusing the bill did not alter the relationship between the bill-holder and the acceptor of the bill. Thornton's point was that bills of exchange were issued on the creditworthiness of the borrower and not the goods in process. Whereas the latter might enhance the capacity to repay the debt, the decision to draw a bill was based on the former.

  This characteristic is evident in the model. For example, bills circulate at par in the model because of established relationships between agents. Since agents have established and recurring relationships, they are willing to accept bills drawn on other agents as payment for production. The decision to do so is the result of the recurring relationship and the creditworthiness of the agent reusing the bill. This is true because the lack of a recurring relationship for creditors with random consumption preferences in the model prevents these agents from reusing bills. Their inability to reuse the bills is because those selling the goods do not have information about these creditors own creditworthiness. This characteristic is clearly at the center of Thornton's (1802 [1807]: 1 -2) view of bills.

- Bills of exchange were used as repayment for outstanding bills.

  In the 19th century United Kingdom, a debtor holding a bill drawn on another debtor could use this bill as repayment for their own debt. Again, this characteristic is evident in the model. The fact that bills of exchange are exchanged by some agents at par value implies that some debtors in

[17] A previous version of the paper referred to bills of exchange as serving a medium-of-exchange role. The author is grateful to George Selgin for pointing out the difference.

the model will sell their production in exchange for a bill drawn on another agent. A debtor who is matched with a creditor with deterministic preferences at night will receive bills of exchange and/or bank notes in exchange for production. Since the bank settles debt, these debtors can bring bills to the bank to repay their debt. To understand this, note the process of settlement in the model. The bank enters the night holding all of the bills drawn by creditors with random preferences. The debtors who are matched with creditors with deterministic preferences receive payment in the form of bills at night. Debtors, however, need to repay their own debt. Since the bank settles payments, they take the bill to the bank and present it for payment. Thus, every night the bank has the total quantity of bills issued in the morning. The bank nets out payments to debtors and collects a profit from the spread between the face value and the discounted value of the bills purchased in the afternoon.

4.2 Price Level and Exchange Rate Determination

The model can be used to characterize the positions of the Bullionists and the Anti-Bullionists. First, consider that the Bullionists argued that the depreciation of the exchange rate and the increase in prices more generally were caused by an excess issuance of currency. Within the model, the price level can be understood as the inverse of the price of money, $p_t = 1/\phi_t$. The intuition for the Bullionist position can illustrated by the equilibrium condition for the real supply of money. From equation (21), the price level can be written as

$$p_t = \frac{M_t}{(NI/4)[(1-\theta)u(\varrho_t^{CR}) + \theta f(\varrho_t^{CR})]}$$

As shown, the price level is equal to the ratio of the money supply to money demand. From this condition, fluctuations in the price level are the result of changes in the supply of money relative to the demand for money.

In addition, given this definition, one can write the equilibrium condition for the exchange rate as

$$e_t = \frac{\phi_t}{\phi_t^*} = \frac{[(1-\theta)u(\varrho_t^{CR}) + \theta f(\varrho_t^{CR})]}{[(1-\theta)u[(\varrho_t^{CR})^*] + \theta f[(\varrho_t^{CR})^*]]} \frac{M_t^*}{M_t}$$

It follows that, within the model above, an increase in the money supply can have the effects on the exchange rate and the price level predicted by the Bullionists. However, it is important to note that in

the model an increase in the money supply will also increase consumption done by debtors who interact with creditors with random preferences. Thus, the effect of a change in the money supply on the price level and the exchange rate is at least partially offset by the corresponding increase in money demand.

The Anti-Bullionists argued that government remittances to soldiers fighting abroad, government payments and subsidies to foreign governments, and poor harvests explained the depreciation in the exchange rate and the increases in prices. The model treats the consumption by the bank as being akin to government spending and this spending is only done abroad. In the model, an increase in this spending can cause depreciation in the exchange rate if it increases consumption in the foreign country.

With regard to this spending, it is important to consider the budget constraint explicitly:

$$g_t = (1/p_t)(M_t - M_{t-1}) + (R_t - 1)(NI/4)b_t^R$$

This constraint is of some importance to the debate. For example, while it might be true in a partial equilibrium context that an increase in government spending and remittances cause the exchange rate the depreciate, the spending has to be financed.

The bank constraint in this model resembles a consolidated government budget constraint. The choice of this constraint and the relation to a consolidated government budget constraint is a desirable characteristic given the nature of Bank of England's credit policies. For example, as Viner (1937: 153) explains:

> The powers of the Bank of England to expand its note issues, moreover, were not confined to its commercial discount activities. The Bank could also, and did, get its notes into circulation by advances to the government, by purchases of exchequer bills and public stocks in the open market, and by advances to investors in new issues of government stocks.

The way in which the Bank of England adjusted note issuances depends on how their policy toward discounting commercial bills adjusted in response to an increasing demand for credit from the British government. Modeling the bank's budget constraint as done in this paper therefore seems desirable. Silberling (1924a) argues that the Bank of England did an admirable job in its credit policy. He finds a negative correlation between commercial discounting and advances to the government. Silberling concludes that the Bank of England extended credit to the government only when commercial discounting was low. He then argues that the Bank's credit policy is not consistent with excess issuance. Viner (1937)

counters that one could just as easily conclude that the Bank reduces commercial credit when advances to the government were high. In addition, while this negative correlation is perhaps true looking at the early years of the Restriction Period, there is a clear positive upward trend in both commercial credit and advances to the government over the entire Restriction Period.[18] Viner (1937: 168) points out that "the Bank increased both its advances to the government and its commercial discounts substantially" and "the increase in its commercial discounts was proportionately much greater than the increase in its advances to the government." Even if Silberling is correct that government spending was not the driving force behind an excess issuance, one cannot ignore the substantial increase in bank notes implied by the extension of commercial credit.

Another point that Silberling ignores is that if the Bank of England exercised restraint in granting commercial credit, this would have resulted in an increase in the real rate of interest as a result of crowding out. In the present model, the change in the real interest rate causes a change in the demand for goods and a corresponding change in the demand for money. These general equilibrium concerns are absent from Silberling's assessment and from the writings of the Anti-Bullionists.

The present model considers alternative assumptions about the financing of bank spending (which, given the assumptions of the model, is analogous to government spending during the Restriction Period). Above, it was assumed that the money supply follows the process:

$$\ln(M_t) = ln(M) + \rho_m \ln(M_{t-1}) + \gamma(1-\rho_m)\ln(g_t) + \epsilon_t^m$$

Of particular importance is the magnitude of γ. If, for example, $\gamma \leq 0$, then this would correspond to the case in which the bank exercises restraint in the financing of spending. However, if $\gamma > 0$, then the bank monetizes some of the spending. Nonetheless, even in the case in which the money supply is constant, one must carefully consider the general equilibrium effects of the change in spending. For example, under the scenario in which the money supply is constant, the increase in spending would cause the real interest rate to rise as a result of the crowding out of private debt. In this case it is possible that the private response to higher bank spending has an offsetting effect. In the section below γ is estimated to determine the bank's credit policy.

[18] See Chart 2 and Table 3 in Chapter 3 of Viner (1937).

5 Empirical Analysis

The existing empirical evidence on the Bullionist Controversy provides little empirical support for the Bullionist position. In this section, it is argued that the theoretical framework presented above can be used to evaluate the competing hypotheses. The model is estimated with Bayesian procedures using four macroeconomic variables.[19] The estimated impulse response functions show that shocks to the supply of bank notes cause a statistically significant rise in the price level and a statistically significant depreciation in the exchange rate. In addition, there is evidence that the Bank of England monetized some of the spending of the British government.

The model is then used to simulate data and conduct Granger causality tests. The evidence suggests that Granger causality tests would cause a researcher to incorrectly determine that the supply of bank notes is unimportant. This casts doubt on previous empirical results. Taken together with the results of the estimated model, these results not only cast doubts on the methods and results of previous research, but also find new evidence in favor of the Bullionist position.

5.1 Data

For the estimation of the model, the vector of observables includes the supply of bank notes, real government spending, the price level, and the exchange rate. The sample is quarterly from 1797:Q1 - 1821:Q2. The data on bank notes are taken from Silberling (1923b), who obtained them from the *Parliamentary Papers*. The price level data comes from Gayer, Rostow, and Schwartz (1953). This index is a monthly index and it has been converted into quarterly averages. The data on the exchange rate are taken from Officer (2000). The exchange rate is measured as Flemish schillings per pound.[20] Real government spending is measured using the same series as Officer (2000). This series is an improvement by Officer (2000) over previous measures. The original source of this data is Silberling (1924), who calculated government spending as the sum of government expenditure on British armies in Europe and government remittances, which include subsidies and payments made by the British government to foreign governments. This series was subsequently updated by Sherwig (1969). Officer (2000) updates the Sherwig data and uses linear interpolation to generate a quarterly series.

In previous research, Nachane and Hatekar (1995) use the data in levels to test for unit roots and

[19] The details of the estimation procedure are in the appendix.

[20] The original source of this data were weekly data from the *Resumption Report* (1819) and the *Bank Charter Report* (1932).

cointegration. Officer (2000), like the previous authors, finds evidence of unit roots, but no evidence of cointegration. Thus, Officer uses first differenced data to estimate his models. This paper considers an alternative hypothesis. While the Bank Restriction Period lasted from 1797 to 1821, the Bank of England resumed redemptions on its own volition in 1817. As a result, this paper proposes that there may have been a structural break that occurred in 1817, in which the trend of the price level, the supply of bank notes, and the exchange rate reversed. To test this hypothesis, the following regression is estimated for the variables used in this paper:

$$x_t = \alpha + \beta_1 t + \beta_2(\text{1817 Dummy}) + \beta_3(t * \text{1817 Dummy}) + u_t$$

where x_t denotes the variable of interest, t denotes time and the 1817 Dummy takes on a value of 1 when the Bank of England resumed redemption. The residual, u_t, is then the de-trended value of the variable. One can then estimate the following regression:

$$u_t = \rho u_{t-1} + \sum_{i=1}^{k} \Delta\beta_i u_{t-k} + e_t$$

Estimates of ρ can then be used to examine the unit root hypothesis. The results of the first regression are shown in Table 1. Standard errors are shown in parentheses. In each case, the coefficients on the trend, the dummy, and the interaction term are each the sign that one would expect and statistically significant.

To determine whether or not these variables have a unit root or whether they are stationary around a changing trend, the second regression is estimated. The results are shown in Table 2. The results show the estimated AR(1) coefficient, ρ, and the corresponding t-statistic associated with testing the null hypothesis that $\rho = 1$. Given where the assumed structural break occurs in the sample, the critical value that the t-stat must exceed to reject the null hypothesis is -4.04.[21] As shown, the null hypothesis of a unit root can be rejected for the price level and the supply of bank notes, but not the exchange rate. Thus, in the empirical analysis that follows, these two variables are expressed in deviations from a changing trend whereas the exchange rate and real government spending are expressed in first differences.[22]

[21] See Perron (1989).

[22] A similar hypothesis was considered for real government spending for the initial end of military conflict and for Napoleon's defeat at Waterloo. There is no evidence of a structural break in the trend at these break points.

5.2 Functional Forms and Prior Distribution

In order to estimate the model two functional forms must be specified. First, it is assumed that the functional form of $u(q)$ for debtors is given as $u(q) = q^{1-\delta}/(1-\delta)$. Second, the disutility cost function for creditors is given as $f(q) = (1/2)q^2$. Some of the model parameters are fixed prior to estimation. The discount factor, β, is set at 0.99. The parameters α and θ are set to 0.1 and 0.9, respectively. These parameters are fixed because they appear as a product in the model and therefore would be difficult to estimate. In addition, there are restrictions on these parameters that are required to produce a steady state in which all quantities are positive. Also, the total number of agents $NI = 228$. This is chosen such that the steady state real interest rate is approximately equal to 3%.[23]

The prior distributions of the estimated parameters are shown in Table 3. The parameter δ from the debtor utility function is assumed to follow a gamma distribution with a mean of 2.0 and a standard deviation of 0.5. The parameter γ, which measures the response of the bank to changes in spending follows a uniform distribution on the interval $[-2, 2]$. The choice of a uniform distribution that is symmetric around zero is used to ensure the bank's credit policy is identified through estimation and is not determined by a tight prior. The priors for the AR(1) coefficients on the stochastic processes are assumed to follow a beta distribution with a mean of 0.5. The standard errors of the four shocks in the model are assumed to follow an inverse gamma distribution with a mean of 0.01. Finally, since the empirical measure of government spending is an imperfect measure of spending as it is presented in the model, it is assumed that there is a measurement error. The prior distribution of the measurement error is shown in the final row of Table 3.

5.3 Estimation Results

Table 4 shows the mean, the 10th, and the 90th percentiles of the parameters obtained with the Metropolis-Hastings algorithm. The mean estimate of δ is 1.29. The standard deviation of the posterior distribution of δ is considerably smaller than the prior distribution, which provides evidence that this parameter is well-identified. The estimated mean of the parameter that measures the bank's response to changes in spending, γ is 1.98. This estimate suggests that the Bank of England monetized

[23] This figure is broadly consistent with interest rate data in the aftermath of the Bank Restriction Period. Data on interest rates during this period are not available. However, it is broadly reported that nominal interest rates were at the usury limit of 5% for much of the Bank Restriction period. Thus, even calculating ex post real interest rates for this period seems dubious.

some amount of government spending during this period. Again, the posterior distribution of this parameter is tight around the mean whereas the prior density was flat thereby providing evidence that the parameter is well-identified. This finding is of particular interest because it is consistent with previous estimates by Barro (1987). In particular, Barro (1987) found that a 1% increase in temporary military expenditures increased money growth by 1.12% during bank suspension periods. The present model implies that a 1% increase in the government expenditures causes a 0.1% increase in the level of the money supply. The magnitude of the estimates are not directly comparable since one is measuring the effect on the growth rate and the other is measuring the change in the level. In addition, the measure of government expenditures in this model is more broad than that used by Barro. Also, Barro's data includes the period from 1914 - 1918 in his sample as well. Nonetheless, the sign and significance are consistent. Finally, the relationship between the log of the money supply and temporary government spending during the Restriction Period shown in Barro's (1987) Figure 6 is qualitatively consistent with the estimates in this paper.

The estimates of the AR(1) coefficients on the stochastic processes indicate persistence in the supply of bank notes and real government spending. The estimated shocks to the supply of bank notes are nearly twice as large as shocks to real government spending. The standard deviation of the measurement error for real government spending is of considerable magnitude relative to the shocks. This likely reflects the large, temporary variations in the growth rate of real government spending during this period.[24]

5.4 Impulse Response Functions

Figures 3 and 4 plot the estimated impulse response functions for shocks to the supply of bank notes and to real government spending, respectively. The thick solid line is the impulse response function associated with the mean parameter estimates of the posterior distribution. The dashed lines correspond to the 10 percent and 90 percent posterior intervals. As shown in Figure 3, a shock to the supply of bank notes causes the price level to rise and the exchange rate to depreciate. Also, it is important to note that the impulse response function at the 10% posterior interval is above zero for the price level and less than zero at the 90% posterior interval for the exchange rate. Each of these results provides support for the Bullionist position.

The consolidated budget constraint motivates intuition for the response of the variables to a shock

[24]This is evident in the data used in this paper as well as in Figure 3 of Barro (1987: 225).

to real bank (government) spending shown in Figure 4. As shown, a shock to real bank (government) spending causes a persistent increase in the supply of bank notes. This increase in the money supply causes the price level to rise. Thus, real government spending increases by more than the real supply of money. As a result, bank profits must rise to finance some of the increased spending and therefore the real interest rate rises. The increase in the real interest rate creates both an income and a substitution effect for creditors with random preferences. The higher real interest rate means that for a given amount of production by creditors, the less consumption they will be able to do at night. This is the substitution effect. The higher real interest implies that to maintain the same level of consumption, creditors with random preferences will have to produce more. This is the income effect. Evidently, the substitution effect initially exceeds the income effect since the quantity produced by creditors with random preferences immediately declines in response to the shock. Production subsequently rises. This initial decline in production reduces the demand for real money balances at the same time that the money supply is increasing. As a result, the home price level rises and the exchange rate depreciates.

The response of the price level to a shock to real government spending appears to be consistent with the Anti-Bullionists since the price level rises, the exchange rate depreciates, and the quantity of private debt declines following a shock to real government spending. This is not necessarily the case. It is important to consider the magnitude of the effects and the mechanisms through which the shock influenced the price level and exchange rate. First, it is important to note that the shock to government spending causes the real interest rate to increase, which has a crowding out effect on private debt. However, the changes in both the real interest rate and the amount of private debt are small in magnitude. Also, given that the changes in the real interest rate and the quantity of private debt are in opposite directions, this suggests from the consolidated budget constraint that a large portion of these shocks to government spending were being monetized by the bank. This is evident in the large and persistent effect that shocks to real government spending have on the supply of bank notes. Evidently, the Bank of England monetized some significant fraction of government spending. Thus, at least some of the increase in the price level and the depreciation of the exchange rate should be attributed to the change in the supply of bank notes. In fact, to illustrate this, Figure 5 plots the impulse response function of the price level and the exchange rate from a simulation assuming the parameters are equal to the mean estimates of the posterior distribution. In addition, the figure also plots the impulse response functions from a simulation that assumes that γ is an order of magnitude smaller. As shown, the monetization

of real government spending appears to be the driving force behind the effects of the real government spending on the price level and the exchange rate.

The impulse response functions provide support for the Bullionist position without providing corresponding support for the Anti-Bullionist position. This is in contrast to previous empirical results.

5.5 Granger Causality

Despite the evidence from the estimated model shown above, previous authors have argued that Granger causality tests support the Anti-Bullionst position with either no evidence or mixed evidence in favor of the Bullionist position. Thus, an open question is why this paper identifies such strong evidence in favor of the Bullionist position when previous authors could find relatively little support for the Bullionist position. It is argued below, using simulated data from the model, that Granger causality tests are uninformative.

Granger causality tests examine the null hypothesis that the lags of a particular variable do not have an effect on the variable of interest. It is possible that the results of Granger causality tests are sensitive to the timing of the correlation between the supply of bank notes and the price level. As a method of comparison, the theoretical model presented above is used to simulate data. The simulation consists of 100,000 time series each for the supply of bank notes, the price level, and real government spending using the mean of the posterior distribution of parameters shown in Table 4. A trivariate VAR consisting of each of the three simulated series is estimated for each of the 100,000 simulations.

The first panel of Table 5 presents the results of Granger causality tests from a trivariate VAR model using the same data that was used to estimate the model. The results are printed such that $X \rightarrow Y$ corresponds with the null hypothesis that the lags of variable X have no effect on variable Y. The second column of the table presents the p-value associated with that hypothesis test. As shown, the hypothesis that lags of the supply of bank notes do not affect the price level cannot be rejected.

Consider the same test using data simulated from the model. The second panel of Table 5 lists the average p-value for the relevant Granger causality test from 100,000 estimates of the VAR. As shown, one cannot reject the null hypothesis that bank notes do not affect the price level.

The consistency of the results are important for the following reason. For the simulated data, the true data generating process is known. In the true data generating process, a shock to the supply of bank notes causes the price level to rise and the price level remains higher for several quarters thereafter. Relying

solely on Granger causality tests, one would be led to incorrectly conclude that the supply of bank notes are unimportant for explaining the price level. If the true data generating process is consistent with that of the model, the simulation results imply that Granger causality tests would not be useful to determine whether changes in the supply of bank notes had an effect on the price level. Thus, at most, the evidence from the simulations suggests that Granger causality tests are not informative. At the very least, the simulation results explain how the results in this paper can be reconciled with previous research.

6 Conclusion

The Bullionist Controversy represents one of the first debates about the determination of the price level and the exchange rate under a paper money standard. The Bullionists argued, on what would now be considered quantity theoretic grounds, that the increase in the price level and the depreciation of the exchange rate were caused by an excess supply of bank notes. In contrast, the Anti-Bullionists argued that changes in the exchange rate and the price level were due to supply shocks, shocks to the balance of trade, and shocks to military expenditures.

Despite the significance of the debate in the history of economic thought, there is little empirical evidence that exists using modern time series analysis. The evidence that does exist favors the Anti-Bullionist position. The purpose of this paper is to use an explicit theoretical model to analyze the positions of the Bullionists and Anti-Bullionists. The model is estimated using Bayesian procedures.

The estimated results suggest that shocks to the supply of bank notes caused a significant increase in the price level and a significant depreciation of the exchange rate and that the Bank of England monetized some portion of government spending. There is little support for the Anti-Bullionist position. The evidence supports the Bullionist position.

Appendix

Proof of Proposition 1

A creditor with deterministic preferences can use bills of exchange or bank notes as a medium of exchange at night. Thus, this proof requires that it be shown that these creditors will never be better off choosing bank notes.

First, consider the case in which $g_t^* = 0$. In this case, $q_t = \varrho_t$.

When creditors and debtors are matched in the morning the expected surplus from trade that goes to creditors is given as

$$(1-\theta)\Big[u(q_t) - f(q_t)\Big]$$

It follows that the expected surplus is maximized if

$$\frac{u'(q_t)}{f'(q_t)} = 1$$

Given that $u(q) = q^{1-\delta}/(1-\delta)$ and $f(q) = (1/2)q^2$, it follows that $q^* = 1$, where q^* denotes the optimal quantity of trade.

For creditors that use bills of exchange at night, it is shown in section 3.2 that $f'(q_t) = 1$. Thus, for creditors using bills of exchange, $q_t = q_t^* = 1$.

For creditors that use bank notes at night, it is shown in section 3.3 that q_t satisfies:

$$E_{t-1}\frac{\phi_{t-1}}{\beta\phi_t} = \alpha\theta E_{t-1}\left[u'(q_t) - \frac{1}{R_t}\right] + 1$$

The allocation is optimal if $q^* = 1$. Thus, for individuals to hold bank notes, it must be true that $E_{t-1}u'(q_t) = 1$:

$$\frac{1}{\alpha\theta}E_{t-1}\frac{\phi_{t-1}}{\beta\phi_t} + E_{t-1}\frac{1}{R_t} = 1 + \frac{1}{\alpha\theta}$$

This is true if

$$E_{t-1}\frac{\phi_{t-1}}{\beta\phi_t} = E_{t-1}\frac{1}{R_t} = 1 \tag{28}$$

From (16), the bank consumes g_t according to

$$g_t = \phi_t(M_t - M_{t-1}) + (R_t - 1)b_t^R$$

Similarly from equation (21), it follows that

$$\frac{\phi_t}{\phi_{t-1}} = \frac{[(1-\theta)u(q_t) + \theta f(q_t)]}{[(1-\theta)u(q_{t-1}) + \theta f(q_{t-1})]} \frac{M_{t-1}}{M_t}$$

If the allocation is optimal, it will be true that

$$\frac{\phi_{t-1}}{\phi_t} = \frac{M_t}{M_{t-1}} \equiv \mu_t$$

Thus, for (28) to hold, it must be true that $E_{t-1}\mu_t = \beta$ and $E_{t-1}R_t = 1$. Supposing that this is true, (16) can be re-written as

$$g_t = \left(1 - \frac{1}{\beta}\right) M_t$$

However, this implies that $g_t < 0$, which violates the assumption that $g_t \geq 0$. Thus, creditors with deterministic preferences will always be better off using bills of exchange as a medium of exchange at night.

Now consider the case in which $g_t^* > 0$. Now the size of the surplus is given as

$$(1-\theta)\Big[u(\varrho_t) - f(\varrho_t)\Big]$$

A creditor using bills of exchange will produce a quantity $q_t = 1$. In addition, $q_t = \varrho_t + (1/NI)g_t^*$. It follows that $\varrho_t = 1 - (1/NI)g_t^*$. A creditor using bank notes will produce a quantity $q_t = 1/R_t$. It follows that $\varrho_t = 1/R_t - (1/NI)g_t^*$. Thus, a creditor will prefer to use bills of exchange if

$$(1-\theta)[u(1-(1/NI)g_t^*) - f(1-(1/NI)g_t^*)] \geq (1-\theta)[u((1/R_t) - (1/NI)g_t^*) - f((1/R_t) - (1/NI)g_t^*)]$$

or

$$u(1-(1/NI)g_t^*) - u((1/R_t) - (1/NI)g_t^*) \geq 1 - \frac{1}{R_t}$$

For $R = 1$, this clearly holds with equality. Thus, it needs to be determined whether this holds as R

increases. Differentiating the condition above yields

$$\frac{1}{R_t^2}u'((1/R_t) - g_t^*)dR_t|_{g_t^*=g^*} \geq \frac{1}{R^2}dR_t|_{g_t^*=g^*}$$

Creditors will therefore choose to carry bills of exchange if

$$u'[(1/R_t) - (1/NI)g^*] - 1 \geq 0$$

Given the functional form, $u(q) = q^{1-\delta}/(1-\delta)$, and the fact that $g^* = 1$ in the steady state, this implies that

$$\frac{1}{(1/R_t - 1/NI)^\delta} - 1 \geq 0$$

This will be true for any $R_t > 1$. Thus, creditors with deterministic preferences will always choose to hold bills of exchange rather than bank notes.

Proof of Proposition 2

Constraint (6) will be binding if creditors spend all bank notes on consumption at night. Creditors will spend all of their bank notes at night if the real value of those bank notes in period t is greater than or equal to the real, discounted expected value of the banks notes in period $t+1$. This is true if $\phi_t \geq E_t\beta\phi_{t+1}$. The proof of Proposition 3 requires proving that this condition always holds in an equilibrium in which money is held.

Recall the debtor's optimization problem:

$$V_t(m_t) = \max_{m_{t+1}} \left(\phi_t m_t - \phi_t m_{t+1} + \phi_t m_t^c + b_t^l(j) \right) + \beta E_t W_{t+1}(m_{t+1})$$

The Kuhn-Tucker condition is given as

$$-\phi_t + \beta E_t W'_{t+1}(m_{t+1}) \leq 0$$

where this condition holds with equality if $m_t > 0$. Consider the case in which $m_t > 0$. Using the

definition of W, the equilibrium condition is given as

$$E_t \frac{\phi_t}{\beta \phi_{t+1}} = \alpha \theta E_t \left[u'(\varrho_{t+1}^{CR}) - f'(\varrho_{t+1}^{CR}) \right] + 1$$

If $\phi_t < \beta E_t \phi_{t+1}$, this implies that the surplus from trade would have be negative, which is inconsistent with utility maximization and therefore no agent would hold the bank notes. Thus, in any equilibrium in which bank notes are held it must be true that $\phi_t \geq \beta \phi_{t+1}$.

Details of the Estimation Procedure

The model presented in Section 4 has a rational expectations solution that can be written as

$$Y_t = \Gamma S_t$$

$$S_t = \Psi S_{t-1} + \Theta \varepsilon_t$$

where Y_t is a vector of control variables, S_t is a vector of state variables, and ε_t is a vector of structural shocks. Define $Z_t = [Y_t' S_t']$ It follows that one can re-write the system of equations above as

$$Z_t = A Z_{t-1} + B \varepsilon_t \tag{29}$$

where A and B are functions of Γ, Ψ, and Θ. Now define $\mathcal{Y}$ as a vector of observable variables. It follows that the observables can be written as

$$\mathcal{Y}_t = C + D Z_t + \xi_t \tag{30}$$

where C is a vector to match the means of the observables, D is a matrix of zeroes and ones to match the observables to the definitions in the model, and ξ is a vector of measurement errors. Let Φ denote the vector of parameters in the model. The likelihood function of the model can be written as

$$\mathcal{L}(\mathcal{Y}_T | \Phi) = \prod_{t=1}^{T} \mathcal{L}(\mathcal{Y}_t | \mathcal{Y}_1, \ldots, \mathcal{Y}_{t-1}, \Phi)$$

The Kalman filter can be used to compute the likelihood of the model using the state space system given by equations (29) and (30).

The posterior distribution of the model parameters is then characterized by the Metropolis-Hastings algorithm. The algorithm can be described as follows. The algorithm begins with an initial parameter vector $\Phi_{1,0}$. The Kalman filter is then used to obtain and estimate of $\mathcal{L}(\mathcal{Y}_t|\Phi_{1,0})$. The Metropolis-Hastings algorithm updates the initial parameter vector according to

$$\Phi_{1,1} = \Phi_{1,0} + \omega\psi\epsilon_1$$

where $\Phi_{1,1}$ is the updated parameter vector, ω is a "jump" scalar, ψ is the Choleski decomposition of the variance-covariance matrix of Φ_1, and ϵ_1 is normally distributed.

The updated parameter vector is fed through the Kalman filter to calculate the likelihood function of the model. The Metropolis-Hastings decision rule is then used to determine whether to accept the initial parameter vector or the proposed update. This process is carried out N times and is used to generate a posterior density, $p(\Phi|\mathcal{Y}_T)$.

Of particular importance for the characterization of the posterior distribution is the choice of the "jump" scalar ω. This parameter determines the size of the "jump" in the updated parameter vector for a given ϵ. A larger value of ω implies a lower acceptance rate for the algorithm.

For the estimation of the model, a sample of $250{,}000$ draws was created. The first $50{,}000$ draws were discarded. The jump scalar was set to 0.225, which produced an acceptance rate of 32%.

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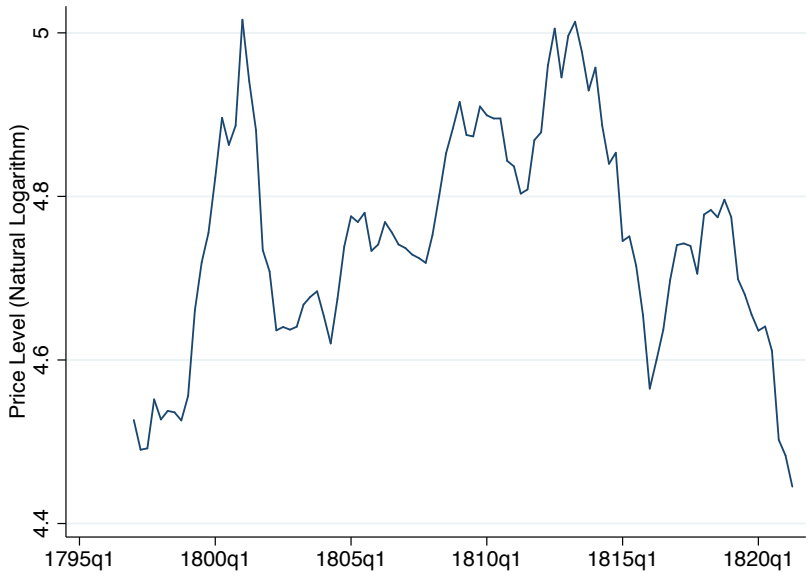

Figure 1: **Price Level, 1797 - 1821**. Source: Gayer, Rostow, and Schwartz (1953)

Table 1: Trend Regression Results

	Price Level	Bank Notes	Exchange Rate
Constant	6.47	9.35	-13.80
	(0.34)	(0.27)	(8.04)
Time Trend	0.003	0.01	-0.08
	(0.001)	(0.0004)	(0.01)
1817 Dummy	-11.28	-9.14	165.26
	(2.93)	(0.50)	(69.86)
(Time Trend * 1817 Dummy)	-0.02	-0.02	0.28
	(0.005)	(0.001)	(0.12)

Table 2: Unit Root Regression Results

	Price Level	Bank Notes	Exchange Rate
ρ	0.83	0.83	0.88
	(-4.25)	(-4.25)	(-2.67)

t-stats are in parentheses

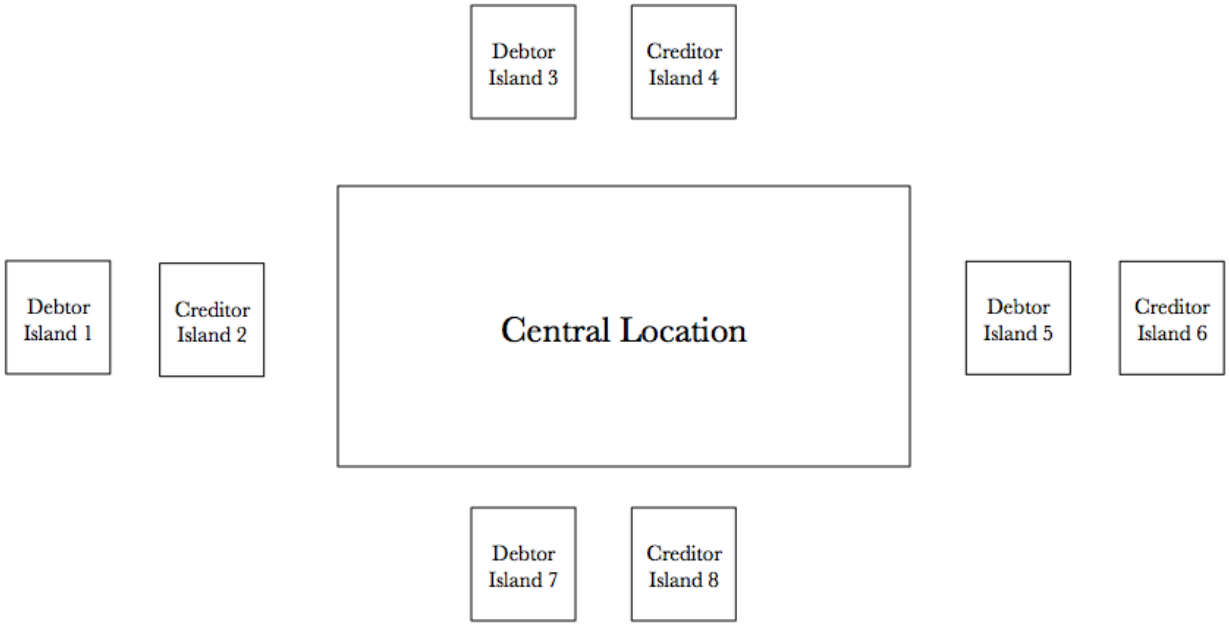

Figure 2: Model Setup

Table 3: Prior Distribution of the Parameters

Parameter	Distribution	Mean	Standard Deviation
δ	Gamma	2.00	0.50
γ	Uniform	0.00	1.15
ρ_m	Beta	0.50	0.10
ρ_g	Beta	0.50	0.10
ρ_{em}	Beta	0.50	0.10
ρ_{eg}	Beta	0.50	0.10
ρ_m^*	Beta	0.50	0.20
ρ_g^*	Beta	0.50	0.20
$\sigma_{\varepsilon m}$	Inverse Gamma	0.01	Inf
$\sigma_{\varepsilon g}$	Inverse Gamma	0.01	Inf
$\sigma_{\varepsilon m*}$	Inverse Gamma	0.01	Inf
$\sigma_{\varepsilon g*}$	Inverse Gamma	0.01	Inf
σ_g	Uniform	2.50	1.44

Table 4: Posterior Distribution of the Parameters

Parameter	Posterior Mean	10th percentile	90th percentile
δ	1.292	1.288	1.297
γ	1.98	1.96	2.00
ρ_m	0.95	0.946	0.953
ρ_g	0.952	0.951	0.953
ρ_{em}	0.67	0.65	0.69
ρ_{eg}	0.952	0.952	0.953
ρ_m^*	0.993	0.989	0.998
ρ_g^*	0.64	0.60	0.66
$\sigma_{\varepsilon m}$	0.055	0.050	0.060
$\sigma_{\varepsilon g}$	0.029	0.025	0.033
$\sigma_{\varepsilon m*}$	0.255	0.192	0.312
$\sigma_{\varepsilon g*}$	0.010	0.002	0.018
σ_g	0.59	0.51	0.66

Table 5: Granger Causality Tests

Granger causality	p-value
British Data	
Bank Notes $\rightarrow$ Price Level	0.44
Simulated Data, $\gamma = 1.98$	
Bank Notes $\rightarrow$ Price Level	0.32

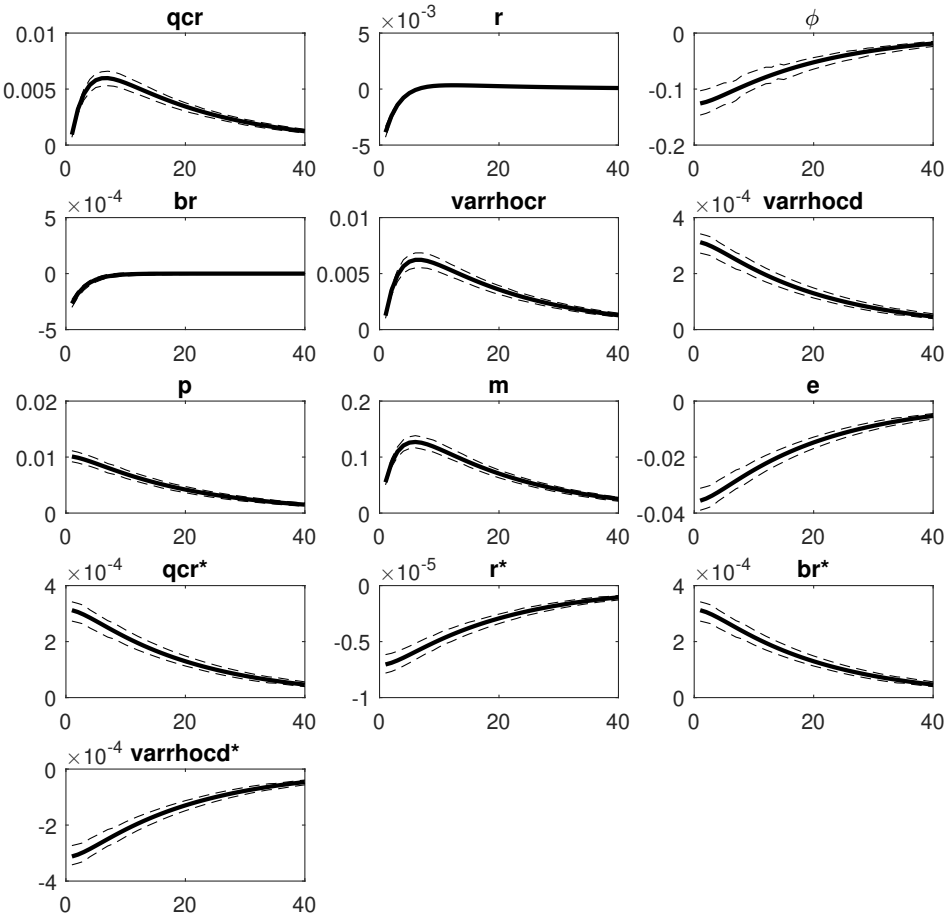

Figure 3: Estimated Impulse Response Functions: Shock to the Supply of Bank Notes

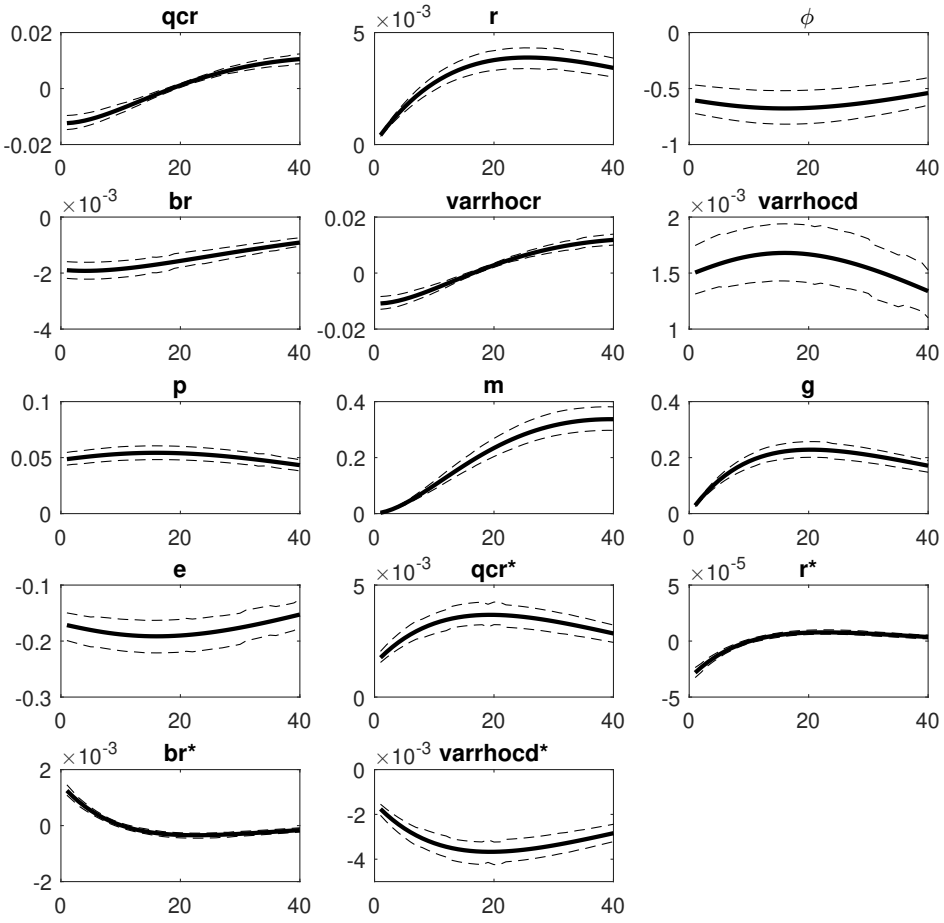

Figure 4: Estimated Impulse Response Functions: Shock to Real Bank (Government) Spending

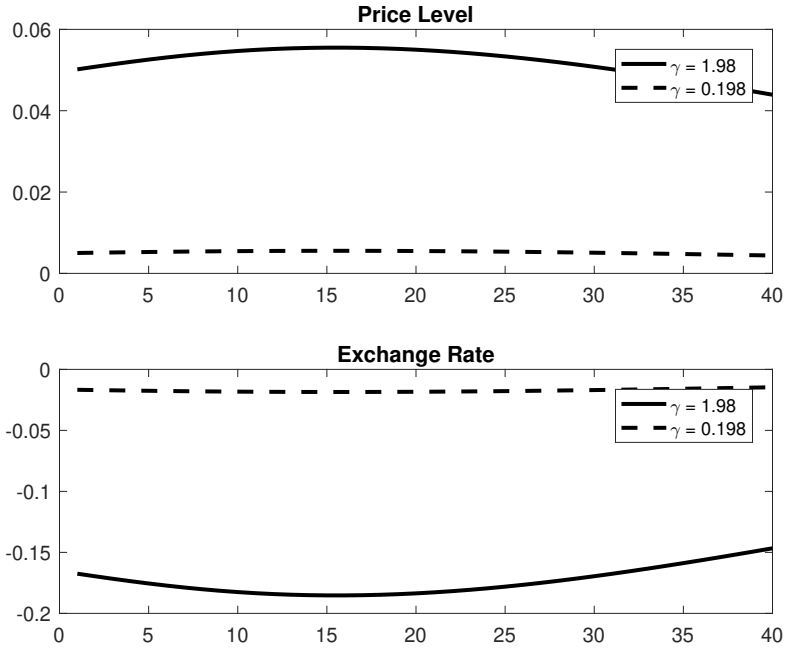

Figure 5: Simulated Impulse Response Functions: Shock to Real Bank (Government) Spending Under Different Assumptions About Monetization