Optimal Monetary and Fiscal Policy with Migration in a Currency Union

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Abstract

We develop an open economy model of a currency union with frictional goods markets and costly migration to study optimal monetary and fiscal policy for the union. Households finance consumption with a common currency and can migrate across regions given regional differences in goods market characteristics and microstructure. Equilibrium is generically inefficient due to regional spillovers from migration. While monetary policy alone cannot correct this distortion, fiscal policy can help by taxing or subsidizing at the regional level. When households of only one region can migrate, optimal policy entails a deviation from the Friedman rule and a production subsidy (tax) if there is underinvestment (overinvestment) in migration. Optimal policy when households from both region can migrate is the Friedman rule and zero taxes in both regions.

Keywords: currency unions, costly migration, search frictions, optimal monetary and fiscal policy
JEL Classification Codes: C92, D83, E40

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1 Introduction

An old idea dating back to Mundell (1961) emphasizes labor mobility as a precondition for an optimal currency area. Within a common currency area, mobility is still widely regarded as a key adjustment mechanism for absorbing asymmetric shocks (e.g., Bayoumi and Prasad 1997, Silva and Tenreyro 2010, DeGrauwe et al. 2014).[1] However, once a country joins a currency union, migration may be inefficiently high or low if there are regional spillovers or congestion associated with individual migration decisions. As suggested by Lavenex and Ucarer (2002), market inefficiencies may arise from migration patterns if there are spatial differences in income, labor productivity, or migration costs.[2] Given heterogeneity across regions, spatial spillovers may also make a common monetary policy less effective. Indeed by relinquishing monetary autonomy, monetary policy alone may not be sufficient to correct regional distortions arising from individual migration decisions. Regional policy interventions, such as distortionary fiscal policy, may therefore help by taxing or subsidizing at the regional level, thereby correcting regional spillovers associated with migration.

In this paper, we develop an open economy model of a currency union with costly mobility between regions that formalizes the interaction between migration and monetary and fiscal policy. We study three related questions using this framework. First, under what conditions is migration across regions socially optimal? Second, what are the effects of monetary and fiscal policy interventions on migration, trade, and welfare? And finally, what is the optimal mix of monetary and fiscal policy that maximizes social welfare for the union?

The model is based on an open economy version of Rocheteau and Wright (2005) where two regions have a single monetary authority that chooses the money growth rate of a common currency. Each region produces a tradable consumption good that can be financed with the common currency. Regional trade occurs in decentralized goods markets where households and producers meet bilaterally and negotiate the terms of trade. While producers have immobile factors of production, households can invest to temporarily relocate to the other region. However due to search frictions, migration by an individual household creates spillovers for other households in the union. Terms of trade are negotiated through bilateral bargaining, where regional differences in bargaining power and matching efficiency affect ex ante migration decisions.

A key implication of our model is that migration is an endogenous response to institutional differ-

[1]The original Mundellian proposition has been generalized and extended in multiple ways, e.g., to emphasize the importance of income transfers through fiscal policy, integrated financial or capital markets, and public insurance schemes (McKinnon 1963, Kenen 1969, Ingram 1962). For surveys, see Tavlas (1993) and DeGrauwe et al. (2014).

[2]These regional differences are especially prevalent across countries in Europe where annual cross border migrations rates across countries are between 0.3% to 1% in 2010, compared with compared with interstate migrations rates in the U.S. of around 2.4% (Eurofound 2014).

ences among regions. In particular, migration rates decrease with market power of producers in the destination region but increase with mark ups at home. These findings are consistent with evidence that migration is driven by movement to regions with higher expected surpluses.[3] However, whether or not equilibrium migration is socially efficient is more subtle since an individual household also imposes a congestion externality upon moving to other households in the destination region.

Indeed, we show that equilibrium with migration is generically inefficient along both intensive and extensive margins of trade. First, output per trade is inefficiently low due to an asynchronicity between production and trade. Second, the number of trades can be inefficiently high or low since agents do not internalize the externalities created by their migration decisions. Efficiency jointly requires the Friedman rule at the union level and the Hosios condition at the regional level.[4] Intuitively, the Friedman rule eliminates the intertemporal distortion by making the opportunity cost of holding money as low as possible, while the Hosios condition eliminates the matching inefficiency by providing an appropriate division of the trade surplus.

To highlight these mechanisms, we first consider equilibria where only households of one region choose to migrate. For instance, this arises if the destination region has sufficiently lower markups and higher matching efficiency than the home region.[5] Households underinvest (overinvest) in migration if their bargaining power at home is larger (smaller) than their contribution to the matching process. A permanent negative shock to matching efficiency at home can generate a reallocation of migration where households go from underinvesting to overinvesting in relocating. This arises even at the Friedman rule, except in the knife edge case where the Hosios condition is also satisfied.

Given inefficiencies along both intensive and extensive margins of trade, we next consider the roles of monetary and fiscal policy in alleviating these distortions. In general, achieving efficiency on both margins requires the Friedman rule and the Hosios condition are jointly satisfied. Without migration, monetary policy alone is sufficient to achieve the first best since the only distortion is at the intensive margin (output per trade). With endogenous migration, an additional policy instrument is needed to correct the extensive margin distortions arising from agents' migration decisions (number of trades). We therefore introduce regional fiscal policy through a tax or subsidy scheme to local producers. If there is underinvestment (overinvestment) in migration, a production subsidy (tax) in the destination

[3]Kennan and Walker (2010) develop a dynamic migration model where location decisions are driven by expected incomes. See Greenwood (1997) and Molloy et al. (2011) for a summary of U.S. migration patterns. Bonin et al. (2008) show migration patterns in the European Union are mostly driven by income differences, while the effects on labor market outcomes like unemployment are more mixed.

[4]Similar inefficiencies arise in search models of money with both intensive and extensive margins of trade. See Section 1.1 for a discussion.

[5]Equilibrium with migration is not generically unique due to complementarities between migration decisions and production. Intuitively, more production abroad raises the expected surplus for households to relocate which increases migration and the hence total number of trades abroad. This raises the gain to produce abroad, thereby further raising production.

region increases (decreases) equilibrium migration by raising (reducing) output per trade and hence the net gains from migration. Implicit in this analysis is our assumption that the monetary union is also a fiscal union: the government implements regional fiscal policy financed by a common central bank that sets a union wide money growth rate.[6] The optimal policy mix is then a money growth rate and production subsidy or tax that maximizes welfare for the union.

When household bargaining power is higher than their contribution to the matching process, households underinvest in migration. Since a household's incentive to move decreases with their bargaining power at home, a sufficiently large bargaining power induces households to invest too little in relocating. The resulting optimal policy entails a higher money growth rate than the Friedman rule and a production subsidy in the destination region. Intuitively, a deviation from the Friedman rule is optimal since an increase in money growth increases migration provided the destination region has more favorable trading conditions, i.e. higher bargaining power to households or higher matching efficiency. However, higher money growth also decreases production for both regions. To counteract part of this distortion, the policymaker sets a production subsidy in the destination region that increases production and migration to that region. On the other hand, when household bargaining power is too low, households overinvest in migration. The resulting optimal policy is the Friedman rule and a production tax in the destination region. The policymaker therefore deflates at the rate of time preference to minimize the intensive margin distortion, which also has the additional benefit of decreasing migration and bringing it closer to its first best level.

When households from both regions choose to migrate, the optimal policy prescription is the Friedman rule and zero taxes. Intuitively, this arises since monetary policy is more effective at correcting the intensive margin distortion than fiscal policy is at alleviating the extensive margin distortion. Such a policy implies no migration by households of either region.

The paper is organized as follows. Section 1.1 discusses related literature. Section 2 describes the environment. Section 3 studies the social planner problem, and Section 4 characterizes equilibrium. We consider one sided migration in Section 5 and illustrate the optimal monetary-fiscal policy mix for the union. We extend the analysis to include two sided migration in Section 6. Section 7 concludes. Proofs for the paper are collected in the Appendix.

[6] This resonates with a classic idea from Kenen (1969) on the importance of fiscal integration for a monetary union: "It is a chief function of fiscal policy, using both sides of the budget, to offset or compensate for regional differences, whether in earned income or in unemployment rates. The large-scale transfer payments built into fiscal systems are interregional, not just interpersonal..."

1.1 Related Literature

This paper builds on the New Monetarist framework surveyed by Rocheteau and Nosal (2017) and Lagos et al. (2017). As with many papers following this approach, we have search frictions that affect both the frequency of trade (extensive margin) and output per trade (intensive margin). Specifically, the way we model agents' migration decision is similar to models with endogenous search intensity, e.g. Lagos and Rocheteau (2005) and Berentsen et al. (2007), and models with imperfect mobility of workers across sectors, e.g. Chang (2012) and Branch et al. (2016). Relative to these papers, we model migration across regions in an open economy and show how this generates spillovers and additional distortions across regions. We also analyze optimal monetary and fiscal policy whereas these studies take these policies as given.

Our open economy model is similar to Zhang (2014) and Gomis-Porqueras et al. (2017) and relates with recent models of currency unions in the New Monetarist tradition. In contrast to Zhang (2014) and Gomis-Porqueras et al. (2017), our model just has one currency, endogenizes migration, and instead of studying the conditions under which different currency regimes arise, we take the existence of a currency union as given and focus on the positive and normative implications for migration and trade. Bignon et al. (2015) develop a model of money and credit in a currency union and show there are welfare gains from credit market integration but not currency market integration. Herrenbreuck (2015) studies optimal monetary policy in an open economy model with price posting and shows inflation can have nonmonotonic effects due consumers' search intensity. Relative to these studies, we study both optimal monetary and fiscal policy and explicitly model agents' migration decisions.

More recently, Farhi and Werning (2014) also study migration in a currency union, but in a different type of model with nominal rigidities and internal imbalances. They also show how migration out of depressed regions may produce a positive spillover for stayers. Relative to this study, we model a currency union without nominal rigidities and emphasize search and information frictions that generate both a motive for migration and a role for monetary policy. In addition, we focus on household mobility that affects the location where resources are spent, rather than labor mobility that affects the location where income is generated. Moreover, we consider monetary policy in conjunction with one fiscal instrument (a tax on profits in frictional markets), whereas they consider two fiscal instruments (labor and profit taxes) but no monetary policy and design the optimal policy mix to alleviate regional distortions from migration.

This paper is also related to the literature on optimal monetary and fiscal policy in New Monetarist models where the government can commit to their future policies.[7] Aruoba and Chugh (2010)

[7]There is also an extensive literature on monetary and fiscal policy in monetary models with nominal rigidities.

introduce production and capital taxes in the Lagos and Wright (2005) model and study the resulting Ramsey policy. Due to the under-accumulation of capital, optimal policy entails a subsidy on capital income. In a similar vein, Gomis-Porqueras and Peralta-Alva (2010) consider fiscal policy in a similar environment but when the government does not have to finance its consumption. In that case, a production subsidy like the one we consider in the present paper can restore efficiency.[8] Relative to these studies, we highlight how endogenous migration generates additional inefficiencies in an open economy that monetary policy alone cannot correct. Finally, we abstract from commitment issues by the policymaker, i.e. we restrict attention to Ramsey policies. See Martin (2011) for an analysis of monetary and fiscal policy absent commitment in a New Monetarist model with money, nominal bonds, and distortionary taxes.

2 Environment

Time is discrete and continues forever. There are two regions, $i, j \in \{1, 2\}$, each with a continuum of infinitely lived buyers (or households), denoted by $\mathbb{B}_i$ and sellers (or producers), denoted by $\mathbb{S}_j$. Agents are exogenously assigned to one of the regions. Region 1 has a measure $2n$ of agents and region 2 has a measure 2 of agents, where $n \in (0, 1]$ is the relative size of region 1.[9]

Each period consists of two stages. In the first, agents meet bilaterally in decentralized markets (DM) where buyers want to consume a regional good that only sellers from $j \in \{1, 2\}$ can produce. Let $q_j \in \mathbb{R}_+$ denote the quantity produced in region j. Sellers have immobile factors of production and cannot produce the other region's good. There is lack of record keeping, no public information nor communication of individual trading histories, and no enforcement. These frictions preclude unsecured credit arrangements, thus generating a need for a medium of exchange. In the second stage, there is a frictionless centralized market (CM) where all agents can produce and consume a homogenous and perishable numéraire good, $x \in \mathbb{R}$, by supplying labor with a linear production technology in labor, $f(y) = y$. At the end of each CM, buyers return to their region of origin. The discount factor for all agents is $\beta = (1 + r)^{-1}$, where $r > 0$ is the rate of time preference.

See e.g. Corsetti and Pesenti (2001), Beestma and Jensen (2005), Gali and Monacelli (2008), and Ferrero (2009), and references therein. In these papers, there is a single central bank that sets a common interest rate for the union while fiscal policy is determined at the regional level through the choice of government spending. In response to country specific supply shocks, monetary policy is used to stabilize the union-wide economy while fiscal policy is used to stabilize regional inflation differences and the terms of trade.

[8] There are also studies with policy competition between governments in open economies. See e.g. Mendoza and Tesar (2005) for competition between welfare maximizing governments in setting an optimal tax code. Li and Matsui (2009) and Zhang (2014) consider competition between monetary authorities in setting an optimal inflation rate.

[9] There are different interpretations of buyers and sellers, e.g. households purchasing consumption goods from productive firms, or firms acquiring productive capital from suppliers. Here we take the former interpretation though our theory would also apply to the latter.

Migration. In the DM, buyers are mobile while sellers are immobile. For instance, buyers have region specific skills that allow them to move across regions, while sellers have immobile factors of production due to, e.g., regulation or prohibitively costly legal barriers that differ across regions. At the beginning of each period, a buyer $b \in \mathbb{B}_i$ from i can invest $\rho_i^b \in [0,1]$ units of effort to move to region $i' \in \{1,2\} \neq i$. He moves to the other region with probability ρ_i^b. With complementary probability $1-\rho_i^b$, the buyer remains in the same location. The decision ρ_i^b represents a region i buyer's investment to migrate to region i'. When $\rho_i^b = 0$, the buyer remains at the same location, while $\rho_i^b = 1$ means the buyer moves to the other region. When $\rho_i^b \in (0,1)$, the buyer is indifferent between the two regions and follows a mixed strategy where they migrate with probability ρ_i^b and stay with probability $1-\rho_i^b$. Relocating is costly. In particular, a buyer from i investing ρ_i incurs a utility cost $\Phi_i(\rho_i)$. We assume $\Phi' > 0, \Phi'' > 0, \Phi(0) = \Phi'(0) = 0$ and $\Phi'(1) = \infty$. In a stationary equilibrium, these assumptions imply that buyers are indifferent between relocating across regions.[10] Let $\widehat{\rho}_i \equiv \int_{b\in\mathbb{B}_i} \rho_i^b \, db$ denote the average migration rate for buyers in region i. In the following, we use ρ_i to denote ρ_i^b when no confusion may arise.

Matching. Following migration, agents are matched pairwise in the DM by an aggregate matching function. Since sellers are immobile, matches are formed in the seller's location j. Given $\widehat{\rho}_1$ and $\widehat{\rho}_2$, the total number of matches in region j is given by $\mathcal{M}_j(\mathcal{B}_j, \mathcal{S}_j)$, which depends on the measures of active buyers and sellers in region j. The matching function is constant returns to scale, twice continuously differentiable, strictly increasing, strictly concave with respect to each argument and satisfies $\mathcal{M}_j(0,\mathcal{S}_j) = \mathcal{M}_j(\mathcal{B}_j, 0) = 0$ and $\mathcal{M}_j(\mathcal{B}_j,\mathcal{S}_j) \leq \min(\mathcal{B}_j,\mathcal{S}_j)$. In region 1, the total measure of buyers and sellers are $\mathcal{B}_1 = (1-\widehat{\rho}_1)n + \widehat{\rho}_2$ and $\mathcal{S}_1 = n$, respectively. Similarly in region 2, $\mathcal{B}_2 = \widehat{\rho}_1 n + (1-\widehat{\rho}_2)(1-\eta)$ and $\mathcal{S}_2 = 1$. The ratio of sellers to active buyers in region j, or market tightness, is $\vartheta_j \equiv \frac{\mathcal{S}_j}{\mathcal{B}_j}$.

Conditional on migrating, an individual buyer's meeting probability is $\alpha_j(\vartheta_j) = \mathcal{M}_j(\mathcal{B}_j,\mathcal{S}_j)/\mathcal{B}_j = \mathcal{M}_j(1,\vartheta_j)$. The matching probability of a seller in region j is $\alpha_j(\vartheta_j)/\vartheta_j = \mathcal{M}_j(\mathcal{B}_j,\mathcal{S}_j)/\mathcal{S}_j = \mathcal{M}_j\left(\vartheta_j^{-1}, 1\right)$. The dependence of the matching probabilities on market tightness reflects the usual search and congestion externalities. We further assume $\alpha_j(0) = 0$, $\alpha_j'(0) \geq 0$, $\alpha_j(\infty) = 1$, and $\alpha_j'(\infty) = 0$. Table 1 summarizes buyers' meeting probability, $\alpha_j(\vartheta_j)$, across meeting types. Since matches are random, $\alpha_j(\vartheta_j)/\vartheta_j$ is the matching probability of a seller in j. We denote the elasticity

[10] While ρ_i is endogenous and determined by buyers' migration effort (the intensive margin), the measure of buyers at the start of the DM (the extensive margin) is exogenous. In a closed economy, Lagos and Rocheteau (2005) keep the ratio of buyers to sellers fixed and introduce endogenous search intensity. Alternatively, Rocheteau and Wright (2005) have a fixed number of buyers and free entry by sellers while Rocheteau and Wright (2009) have a fixed total number of agents that can choose whether to be buyers or sellers. In either case, constant returns in matching implies a focus on market tightness rather than the overall size of the market.

of the matching function in j as $\epsilon(\vartheta_j) \equiv 1 - \frac{\vartheta_j\ \alpha_j'(\vartheta_j)}{\alpha_j(\vartheta_j)}$.

Table 1: Buyers' Meeting Probabilities

	Seller from 1	Seller from 2
Buyer from 1	$(1-\rho_1)\alpha_1(\vartheta_1)$	$\rho_1\alpha_2(\vartheta_2)$
Buyer from 2	$\rho_2\alpha_1(\vartheta_1)$	$(1-\rho_2)\alpha_2(\vartheta_2)$

Preferences. The period utility of an active buyer in region j originally from i is given by

$$U^b(\rho_i, q_j, x, y) = -\Phi(\rho_i) + u(q_j) + x - y,$$

where ρ_i is the buyer's migration choice, q_j is consumption in DM of region j, x is consumption of numéraire, and y is production of numéraire. We assume $u'(0) = \infty$, $u' > 0$ and $u'' < 0$ for $q_j > 0$. Similarly, the period utility of a seller in region j is

$$U^s(q_j, x, y) = -c(q_j) + x - y,$$

where $c(0) = c'(0) = 0$, $c' > 0$, and $c'' \geq 0$. We assume $c(q_j) = u(q_j)$ for some $q_j > 0$ and let q^* denote the solution to $c'(q^*) = u'(q^*)$.

Money. A single monetary authority issues a common currency for the union. The currency is intrinsically worthless, divisible, storable and recognizable. The aggregate money supply in the CM of period t is M_t and the relative price of money in terms of numéraire, ϕ_t, adjusts to clear the market. The gross growth rate of the money supply is constant over time and equal to $\gamma \equiv M_{t+1}/M_t \geq \beta$. New money is injected if $\gamma > 1$, or withdrawn if $\gamma < 1$, through lump sum transfers or taxes to buyers at the beginning ofthe CM. The budget constraint for the currency union is therefore

$$\phi_t(M_{t+1} - M_t) = \mathcal{T}_t, \tag{1}$$

where $\mathcal{T}_t$ is the lump sum transfer (if $\gamma < 1$) or tax (if $\gamma > 1$) to buyers.

Timing. At the beginning of the DM, all buyers are in their exogenously assigned regions of origin. A buyer from i chooses how much to invest to relocate to region $j \neq i$. Conditional on this choice, buyers are then matched pairwise with sellers from j with probability α_j. After migrating and matching, the buyer is either in region 1 or 2, where terms of trade are determined through bilateral bargaining. At the start of the CM, buyers receive lump sum transfers of the common currency and adjust their portfolios.

3 Social Optimum

As a benchmark, we first consider the social planner's problem. The planner is constrained by the same frictions as private agents and chooses a stationary allocation, $\{(\rho_1, \rho_2), (q_1, q_2)\}$, to maximize total welfare for the union. Given market tightness $\vartheta_1 = \frac{n}{(1-\widehat{\rho}_1)n+\widehat{\rho}_2}$ and $\vartheta_2 = \frac{1}{(1-\widehat{\rho}_2)+\widehat{\rho}_1 n}$, steady state welfare is defined as the sum of agents' utilities in the two regions:

$$\mathcal{W} \equiv n\frac{\alpha_1(\vartheta_1)}{\vartheta_1}[u(q_1) - c(q_1)] + \frac{\alpha_2(\vartheta_2)}{\vartheta_2}[u(q_2) - c(q_2)] - n\Phi_1(\widehat{\rho}_1) - \Phi_2(\widehat{\rho}_2), \tag{2}$$

where $n\frac{\alpha_1(\vartheta_1)}{\vartheta_1}$ and $\frac{\alpha_2(\vartheta_2)}{\vartheta_2}$ are the measure of matches in region 1 and 2, respectively. Consequently, welfare in the union consists of the total trade surplus in DM of the two regions net of buyers' investment in relocating. The social planner's problem is then

$$(q_1, q_2, \rho_1, \rho_2) \in \arg\max \mathcal{W} \tag{3}$$

subject to $\vartheta_1 = \frac{n}{(1-\widehat{\rho}_1)n+\widehat{\rho}_2}$ and $\vartheta_2 = \frac{1}{(1-\widehat{\rho}_2)+\widehat{\rho}_1 n}$.

Lemma 1. *The social optimum is given by $q_1 = q_2 = q^*$, $\rho_1 = \rho_1^*$ and $\rho_2 = \rho_2^*$ that solve*

$$u'(q^*) = c'(q^*), \tag{4}$$

$$\Phi_1'(\rho_1^*) = [\alpha_2(\vartheta_2^*)\epsilon(\vartheta_2^*) - \alpha_1(\vartheta_1^*)\epsilon(\vartheta_1^*)]\,[u(q^*) - c(q^*)], \tag{5}$$

$$\Phi_2'(\rho_2^*) = [\alpha_1(\vartheta_1^*)\epsilon(\vartheta_1^*) - \alpha_2(\vartheta_2^*)\epsilon(\vartheta_2^*)]\,[u(q^*) - c(q^*)], \tag{6}$$

where $\vartheta_1^* \equiv \frac{n}{(1-\rho_1^*)n+\rho_2^*}$ and $\vartheta_2^* \equiv \frac{1}{(1-\rho_2^*)+\rho_1^* n}$ is market tightness at the first best and $\epsilon(\vartheta_j^*) \equiv 1 - \frac{\vartheta_j^*\,\alpha_j'(\vartheta_j^*)}{\alpha_j(\vartheta_j^*)}$ is the elasticity of the matching function at the first best.

As is standard, (4) gives the efficient quantity of production by equating the marginal gain from consuming to the marginal cost of producing. From (5) and (6), efficient migration requires the marginal cost of relocating, $\Phi_1'(\rho_1^*)$ and $\Phi_2'(\rho_2^*)$, equals the difference in the social marginal contribution of relocating times the first-best surplus generated per trade.

4 Monetary Equilibrium

We now describe agents' decision problems in the CM and DM, respectively. We focus on stationary equilibrium where aggregate real balances are constant over time.

At the beginning of the CM, buyers choose consumption of numéraire, labor, and real balances to bring forward next period. The buyer's state is his original location, indexed by $j = \{1, 2\}$, and their current holdings of real balances, $z_j \equiv \phi m_j \in \mathbb{R}_+$. Let W_j^b denote the buyer's value function in CM and V_j^b denote the buyer's value function in the ensuing DM. In what follows, variables with a prime denote next period's variables. The lifetime expected utility for a buyer from j is

$$W_j^b(z_j) = \max_{x,h,z_j^{'} \geq 0} \{x - h + \beta\, V_j^b(z_j^{'})\} \tag{7}$$

$$\text{s.t.} \quad x + \phi m_j^{'} = h + z_j + T, \tag{8}$$

where $z_j^{'}$ is the buyer's portfolio of real balances taken into the next DM, $T \equiv \frac{\mathcal{T}}{1+n}$ is the per capita transfer of common currency (in units of numéraire) and $V_j^b(z_j^{'})$ is the buyer's continuation value in the next DM. Substituting $m_j^{'} = z_j^{'}/\phi$ from (8) into (7) yields

$$W_j^b(z) = z_j + T + \max_{z_j^{'} \geq 0} \left\{-\gamma z_j^{'} + \beta\, V^b(z_j^{'})\right\},$$

where γ is the gross growth rate of the money supply. Accordingly, the buyer's lifetime utility in the CM consists of his current period's real balances, the lump sum transfer, and his continuation value at the start of the next DM net of his investment in real balances. Hence, in order to hold $z_j^{\prime}$ units of real balances next period, the buyer must acquire $\gamma z_j^{\prime}$ units of real balances in the current period. Since $W_j^b(z_j) = z_j + W_j^b(0)$, the buyer's CM value function is linear in his wealth, z_j. In addition, the buyer's choice of real balances next period is independent of his current period's real balances. So long as $\gamma \geq \beta$, sellers have no strict incentive to accumulate real balances in the DM. Consequently, the CM value function of a seller with z_j is $W^s(z_j) = z_j + \beta V_j^s(0)$, which is also linear in z_j.

Following migration, terms of trade in the DM are determined by Kalai (1977)'s proportional bargaining solution. In region j, a buyer acquires output q_j in exchange for payment d_j to the seller and receives a constant share θ_j of the total surplus, where $\theta_j \in (0, 1]$ is the bargaining power of a buyer in j.[11] By the linearity of W_j^b, the surplus of a buyer who gets q_j in exchange for payment d_j is $u(q_j) + W_j^b(z_j - d_j) - W_j^b(z_j) = u(q_j) - d_j$, where the threat point is no trade. Similarly, the seller's

[11] Differences in bargaining power across regions can reflect different laws or market structures between regions.

surplus is $d_j - c(q_j)$. The terms of trade solve the bargaining problem

$$\max_{q_j, d_j} \{u(q_j) - d_j\} \text{ s.t. } u(q_j) - d_j = \frac{\theta_j}{1-\theta_j}[d_j - c(q_j)] \text{ s.t. } d_j \leq z_j.$$

If $z_j \geq (1-\theta_j)c(q^*) + \theta_j u(q^*)$, the buyer is unconstrained and the solution is $q = q^*$ and $d = (1-\theta_j)c(q^*) + \theta_j u(q^*)$. Otherwise, $q_j < q^*$ and the buyer will just hand over all his real balances, $d_j = (1-\theta_j)u(q_j) + \theta_j c(q_j)$. In that case, real balances are

$$z_1 = (1-\theta_1)u(q_1) + \theta_1 c(q_1), \tag{9}$$

$$z_2 = (1-\theta_2)u(q_2) + \theta_2 c(q_2). \tag{10}$$

Using the linearity of W_j^b, the lifetime value of a region 1 buyer in the DM is

$$V_1^b(z_1) = \max_{\rho_1 \in [0,1]} \{-\Phi_1(\rho_1) + (1-\rho_1)\ \alpha_1(\vartheta_1)\ [u(q_1) - d_1] + \rho_1\ \alpha_2(\vartheta_2)\ [u(q_2) - d_2] + z_1 + W_1^b(0)\}.$$

A buyer in region 1 incurs a investment cost $\Phi_1(\rho_1)$ to move to region 2. With probability $(1-\rho_1)\alpha_1(\vartheta_1)$, the buyer meets a seller from 1, in which case he gets q_1 and transfers d_1 in exchange to the seller. With probability $\rho_1\alpha_2(\vartheta_2)$, the buyer moves to region 2 and meets a seller and receives q_2 in exchange for d_2. The term $z_1 + W_1^b(0)$ results from the linearity of the CM value function and is the value of proceeding to the next CM with one's portfolio intact.

Given the bargaining solution, the DM value function for a region 1 buyer is

$$V_1^b(z_1) = \max_{\rho_1 \in [0,1]} \{-\Phi_1(\rho_1) + (1-\rho_1)\ \alpha_1(\vartheta_1)\ \theta_1[u(q_1) - c(q_1)] + \rho_1\ \alpha_2(\vartheta_2)\ \theta_2[u(q_2) - c(q_2)] + z_1 + W_1^b(0)\}.$$

Similarly, the value function for a region 2 buyer is

$$V_2^b(z_2) = \max_{\rho_2 \in [0,1]} \{-\Phi_2(\rho_2) + \rho_2\ \alpha_1(\vartheta_1)\ \theta_1[u(q_1) - c(q_1)] + (1-\rho_2)\ \alpha_2(\vartheta_2)\ \theta_2[u(q_2) - c(q_2)] + z_2 + W_2^b(0)\}.$$

We now turn to buyers' mobility decisions at the beginning of the DM. When making this decision, individuals take as given market tightness and hence aggregate migration rates. For a buyer in region 1, $\rho_1 \in [0,1]$ solves

$$\Phi_1'(\rho_1) = \alpha_2(\vartheta_2)\ \theta_2[u(q_2) - c(q_2)] - \alpha_1(\vartheta_1)\ \theta_1[u(q_1) - c(q_1)]. \tag{11}$$

Since $\Phi_1(\cdot)$ is strictly convex, the buyer's mobility choice is uniquely defined and is continuous by the

Theorem of the Maximum. The left side of (11) is the buyer's marginal cost of moving, $\Phi_1'(\rho_1)$, which must equal the marginal gain from relocating. A similar expression applies to a region 2 buyer, where $\rho_2 \in [0, 1]$ solves

$$\Phi_2'(\rho_2) = \alpha_1(\vartheta_1)\ \theta_1[u(q_1) - c(q_1)] - \alpha_2(\vartheta_2)\ \theta_2[u(q_2) - c(q_2)]. \tag{12}$$

Conditions (11) and (12) equate the private, rather than social, cost and benefit of relocating. The dependence of buyers' matching probabilities on market tightness and the average relocation decisions of other buyers generates an externality typically not internalized in equilibrium. We will revisit this efficiency issue later in the text.

We now describe the buyer's portfolio problem in the CM. Substituting $V_1^b(z_1)$ into $W_1^b(z)$, and using the linearity of W_1^b, the portfolio problem for a buyer chose to move to region 1 is given by

$$\max_{z_1 \in \mathbb{R}_+} \{-\iota z_1 - \Phi(\rho_1) + (1 - \rho_1)\ \alpha_1(\vartheta_1)\ \theta_1[u(q_1) - c(q_1)] + \rho_1\ \alpha_2(\vartheta_2)\ \theta_2[u(q_2) - c(q_2)]\} \tag{13}$$

where $\iota \equiv (1 + \rho)\gamma - 1 = \frac{\gamma - \beta}{\beta}$, which can be interpreted as the nominal interest rate on an illiquid bond denominated in the common currency. Since (13) is continuous and maximizes over a compact set, a solution exists by the Theorem of the Maximum. The first order condition is

$$-\iota + (1 - \rho_1)\ \alpha_1(\vartheta_1)\ \theta_1\ [u'(q_1) - c'(q_1)]\ \frac{\partial q_1}{\partial z_1} + \rho_1\ \alpha_2(\vartheta_2)\ \theta_2\ [u'(q_2) - c'(q_2)]\ \frac{\partial q_2}{\partial z_1} \leq 0, \tag{14}$$

where (14) holds at equality if $z_j > 0$. Kalai bargaining implies

$$\frac{\partial q_j}{\partial z_1} = \frac{1}{\theta_j c'(q_j) + (1 - \theta_j)u'(q_j)}.$$

As a result, $z_1 > 0$ solves

$$\iota = (1 - \rho_1)\ \alpha_1(\vartheta_1)\ \mathcal{L}_1(q_1) + \rho_1\ \alpha_2(\vartheta_2)\ \mathcal{L}_2(q_2), \tag{15}$$

where the term $\mathcal{L}_j(\cdot) \equiv \frac{\theta_j[u'(\cdot) - c'(\cdot)]}{\theta_j c'(\cdot) + (1 - \theta_j)u'(\cdot)}$ is the marginal benefit a buyer receives from using the common currency to trade in the DM of region j. Similarly, $z_2 > 0$ solves

$$\iota = \rho_2\ \alpha_1(\vartheta_1)\ \mathcal{L}_1(q_1) + (1 - \rho_2)\ \alpha_2(\vartheta_2)\ \mathcal{L}_2(q_2) \tag{16}$$

Definition 1. *A stationary monetary equilibrium with migration is a list* $\{(z_1, z_2), (\rho_1, \rho_2), (q_1, q_2)\}$

that solves (9), (10), (11), (12), (15), (16), and market clearing in the money market, $z_1+nz_2 = \phi M$*.*

Equilibrium has a recursive structure. Once ρ_1 and ρ_2 are determined by (11) and (12), q_1 and q_2 are obtained from (15) and (16). Real balances are then pinned down by (9) and (10). We next compare the constrained efficient allocation given by (4), (5) and (6), with the equilibrium outcome. The following proposition describes the conditions under which an equilibrium is constrained efficient.

Proposition 1. *Equilibrium in the currency union achieves the social optimum if and only if*

$$\gamma = \beta, \tag{17}$$

$$\theta_1 = \epsilon(\vartheta_1), \tag{18}$$

$$\theta_2 = \epsilon(\vartheta_2), \tag{19}$$

where $\epsilon(\vartheta_j) = 1 - \frac{\vartheta_j \, \alpha_j'(\vartheta_j)}{\alpha_j(\vartheta_j)}$ *is the elasticity of the matching function in region* j*.*

According to Proposition 1, equilibrium coincides with the social optimum if and only if the Friedman rule holds at the union level and the Hosios condition holds at the regional level. Condition (17) is the Friedman rule, which ensures the efficient quantity of DM output per trade by contracting the money supply at the rate of time preference which drives the associated cost of holding real balances to zero. While necessary, the Friedman rule is not sufficient for efficiency. Equations (18) and (19) are the corresponding Hosios conditions for each region, which ensures individual mobility decisions are socially optimal. Even when the Friedman rule holds, the Hosios condition is typically not satisfied, unless in the knife edge case when buyers' bargaining powers exactly equal to their contributions to the matching process as implied by (18) and (19). Thus, monetary equilibrium is generically inefficient – even at the Friedman rule – due to regional migration externalities. Before introducing additional policy instruments, we next consider the positive implications of agents' migration decisions.

5 One Sided Mobility

To highlight the main mechanisms of the model, we begin by studying one sided mobility where only region 1 buyers choose to migrate while the migration rate in region 2 is fixed at $\bar{\rho}_2$. We first characterize properties of equilibrium with one sided mobility and then compare with the social optimum.

Proposition 2. *Given* $\rho_2 = \overline{\rho}_2$*, a unique steady state monetary equilibrium with one sided migration* *exists and features* $\rho_1 > 0$ *if* $\iota < \bar{\iota} \equiv \min\{\frac{\theta_1}{1-\theta_1}, \frac{\theta_2}{1-\theta_2}\}$ *is small and*

$$\alpha_2 \left(\frac{1}{1-\bar{\rho}_2} \right) \theta_2 > \alpha_1 \left(\frac{n}{n+\bar{\rho}_2} \right) \theta_1. \tag{20}$$

In that case, comparative statics are given by Table 2.

Table 2: Comparative Statics with One Sided Migration

	ι	θ_1	θ_2
ρ_1	+	−	+
q_1	−	+	+
q_2	−	+	+

With one sided migration, there is a unique equilibrium with positive migration by region 1 households if (20) holds and inflation is not too high. A necessary condition for (20) to hold is that trading conditions in region 2, captured by the matching probability and household bargaining power, are sufficiently large. From Table 2, higher inflation decreases trade in both regions and increases migration to region 2 if region 2 has more favorable terms of trade than region 1, i.e. θ_2 is large relative to θ_1. Intuitively, buyers choose to relocate to regions they expect to have larger surpluses. As expected, migration to region 2 increases with θ_2, while the frequency of trades in region 2 decreases. In contrast, migration to region 2 decreases with θ_1 since this makes staying in region 1 more attractive.

In general, stationary equilibria may not be unique for all ι. Multiple steady states may arise due to a complementarity between output produced in the destination region and households' migration decisions. From (15) and (16), output produced in region 1, q_1, is decreasing in the migration rate, ρ_1, while output produced in region 2, q_2, is increasing in ρ_1. Intuitively, higher DM production in region 2 raises the expected surplus of relocating for households in region 1, which increases the migration rate ρ_1 and hence the total number of trades in region 2, $\mathcal{B}_2\alpha_2(\vartheta_2) = [(1-\bar{\rho}_2)+\rho_1 n]\alpha_2(\vartheta_2)$. This makes trade in region 2 more valuable which raises the value of money in region 2, z_2, and DM production, q_2.[12] In what follows, we assume the conditions in Proposition 2 are satisfied.

5.1 Efficiency

Achieving a constrained efficient allocation requires satisfying the conditions in Proposition 1. These imply production levels $q_1 = q_2 = q^*$, and a migration rate for region 1 households ρ_1 that solves (11)

[12] The intuition for this multiplicity is similar to the complementarity between the value of money and agents' entry or search decisions in e.g. Rocheteau and Wright (2005) and Berentsen et al. (2007).

given $\rho_2 = \bar{\rho}_2$. The next proposition summarizes conditions for underinvestment or overinvestment in migration when the monetary authority implements the Friedman rule.

Proposition 3. *Suppose $\iota = 0$, $\rho_2 = \overline{\rho}_2$, and (20) holds. If $\theta_1 > \epsilon(\vartheta_1)$, households underinvest in migration, $\rho_1 < \rho_1^*$, and if $\theta_1 < \epsilon(\vartheta_1)$, households over-invest in migration, $\rho_1 > \rho_1^*$.*

The Friedman rule generates the efficient quantity per trade, $q_1 = q_2 = q^*$, but typically not the efficient migration rate. From Proposition 1, the individual migration rate ρ_1 is socially efficient if and only if $\theta_1 = \epsilon(\vartheta_1)$, i.e. the buyer's bargaining power equals their contribution to the matching process. In equilibrium, a buyer's incentive to move decreases with their bargaining power at home, i.e. $\partial\rho_1/\partial\theta_1 < 0$. Consider a small deviation from $\theta_1 = \epsilon(\vartheta_1)$. If θ_1 increases, i.e. $\theta_1 > \epsilon(\vartheta_1)$, the marginal gain from relocating given by the right side of (11) falls. As a result, ρ_1 decreases and households underinvest in migration, $\rho_1 < \rho_1^*$. On the other hand, if θ_1 decreases, i.e. $\theta_1 < \epsilon(\vartheta_1)$, ρ_1 increases since now the marginal gain from relocating is higher. In this case, households over-invest in migration, $\rho_1 > \rho_1^*$.

Numerical Examples

To illustrate additional properties of equilibrium, we consider numerical examples that demonstrate some of the model's positive implications when there is underinvestment and overinvestment in migration. The matching function in region j is $\mathcal{M}_j(\mathcal{B}_j, \mathcal{S}_j) = \chi_j \frac{\mathcal{B}_j\mathcal{S}_j}{\mathcal{B}_j+\mathcal{S}_j}$, where $\chi_j > 0$ represents the efficiency of the matching process in region j. This implies that buyers' matching probabilities are $\alpha_1(\vartheta_1) = \frac{\chi_1}{(1-\rho_1)+\bar{\rho}_2/n+1}$ and $\alpha_2(\vartheta_2) = \frac{\chi_2}{(1-\bar{\rho}_2)+\rho_1 n+1}$. DM utility and cost functions for production and migration are, respectively, $u(q_j) = \ln(q_j + b) - \ln(b)$ where $b > 0$, $c(q_j) = q_j$, and $\Phi_1(\rho_1) = \frac{\rho_1^2}{1-\rho_1}$.

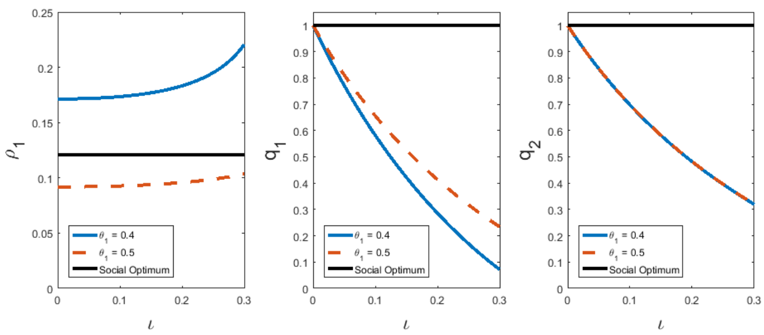
Table 3 summarizes the parameter values used in the examples. We consider two values for household bargaining power in region 1, θ_1, which together with $\epsilon(\vartheta_1)$ determines whether there is under- or overinvestment in migration. We set $\theta_2 = 0.55$ to ensure the condition for uniqueness, (20), is satisfied under both parameterizations. The annual discount rate is set to $\rho = 3\%$, which gives $\beta = 0.97$. As a benchmark, we set $\gamma = \beta$, which is the Friedman rule, but later consider examples with higher values for γ.

Table 3: Parameter Values

β	b	n	χ_1	χ_2	θ_1	θ_2	$\bar{\rho}_2$	γ
0.97	0.001	0.03	1.17	1	0.4, 0.5	0.55	0.02	0.97

When $\theta_1 = 0.5$, the matching elasticity in region 1 is $\epsilon(\vartheta_1) = 0.41$. Since $\theta_1 > \epsilon(\vartheta_1)$, households ,

Figure 1: Effects of Money Growth on Migration and Production with One Sided Migration



undervest in migration, i.e. the equilibrium migration rate $\rho_1 = 0.09$ is below the first best $\rho_1^* = 0.12$. In contrast, when $\theta_1 = 0.4$, $\epsilon(\vartheta_1) = 0.44$, which implies overinvestment in migration, i.e. $\rho_1 = 0.17$.

Figure 1 illustrates the effects of money growth on the migration rate, ρ_1, and regional production, q_1 and q_2, for the two parameterizations of θ_1. As γ increases, DM output in both regions fall while the migration rate increases. In addition, the percent change in DM output is larger when households face a lower bargaining power in region 1. In our example, the percent change in the migration rate in region 1 is smaller compared to DM production in region 1, which suggests the intensive margin (output per trade) responds more to monetary policy than the extensive margin (total number of trades).

While higher money growth increases migration, this effect is only second order while the negative intensive margin effect is first order. Hence the Friedman rule is still optimal. While the Friedman rule delivers efficiency in DM production in both regions, migration is inefficiently high when $\theta_1 = 0.4$ and inefficiently low when $\theta_1 = 0.5$. Under positive nominal interest rates, union wide welfare is always higher when buyer's bargaining power in region 1 is larger. This finding suggests the drop in DM production is larger when buyer's in region 1 face a lower bargaining power, relative to the increase in the migration rate. Moreover, as γ increases, the welfare difference between the two economies increases. We next consider how fiscal instruments may be able to correct this extensive margin distortion.

5.2 Fiscal Policy

From the previous section, efficiency on all margins requires satisfying the Friedman rule at the union level and Hosios conditions at the regional level. Without migration, monetary policy is sufficient, and the Friedman rule achieves the first best. With endogenous migration, buyers do not internalize the externalities their migration decisions have on matching probabilities in the DM. Since monetary policy alone is not enough to correct this distortion, we consider an additional instrument that can target migration rates at the regional level. Here we consider fiscal policy through region specific tax or subsidies in the DM.

In the following, we consider a proportional tax or subsidy on DM production. With one sided mobility, the scheme only applies to producers in region 2 (we consider the two sided case where producers from both regions are subsidized or taxed in Section 6). To implement this policy, we assume the government has access to a costless record keeping technology that keeps track of the identity and production of producers but not identify of households in the DM.[13] For instance, the government can record DM production since producers are in fixed and known locations due to immobile factors of production.[14] Hence while the fiscal authority cannot directly tax household migration decisions, they can indirectly affect migration rates by taxing DM profits of producers, which affects the expected surplus of migrating.

As in Gomis-Porqueras and Peralta-Alva (2010), the government institutes a proportional subsidy or tax on DM goods implemented through lump sum monetary injections/withdrawals in the beginning of the CM. After CM trade but before the next DM, the monetary authority implements changes in the money supply through (a different) lump sum transfer to all households. Implicit in our set up is that the currency union is also a fiscal union: the fiscal authority is taxing (subsidizing) region 2 production but finances it by subsidizing (taxing) both regions through the common inflation tax. As a result, there is redistribution between households and producers in region 2 which will play an important role in internalizing the congestion externality from migration decisions.[15]

Terms of trade in region 2 are still determined through Kalai bargaining, but now includes the subsidy/tax, $\tau_2 \in [-1, 1]$, on region 2 production. Terms of trade now solve

$$\max_{q_2, d_2} \{u(q_2) - c(q_2)\}$$

$$\text{s.t.} \;\; u(q_2) - d_2 = \frac{\theta_2}{1-\theta_2}[d_2(1+\tau_2) - q_2]$$

$$d_2 \leq z_2,$$

where q_2 also depends implicitly on ρ_1. If $z_2 > \frac{(1-\theta_2)u(q^*)+\theta_2 c(q^*)}{1+\tau_2\theta_2}$, the household has enough real

[13] If all agents are anonymous in the DM, the government cannot directly tax productive activities. However, when the identity of *some agents* are known and a record of their production is available to the government, the fiscal authority can tax or subsidize DM activity.

[14] The fact that producers' identities are known does not preclude money being socially useful since all households are still anonymous. In addition, anonymity of some private agents does not preclude the government from raising tax revenues. As in Chari and Kehoe (1993), taxes directly levied on firms are feasible as their output is observable.

[15] This is consistent with a classic proposal from Kenen (1969) on the importance of fiscal integration in a monetary union. See also the literature on fiscal policy in monetary unions, e.g. Sibert (1992), Dixit and Lambertini (2001), Beetsma and Uhlig (1999), Cooper and Kempf (2004) and Chari and Kehoe (2008), and references therein.

balances to purchase the efficient quantity q^*. Otherwise, the household is cash constrained and will just hand over all his real balances to the seller so that $d_2 = z_2$ and q_2 solves

$$d_2 = \frac{(1-\theta_2)u(q_2) + \theta_2 c(q_2)}{1+\tau_2\theta_2}.$$

When $d_2 = z_2$, there are two cases. When $\tau_2 \in (0, 1]$, the fiscal authority enacts a subsidy and

$$\frac{\partial q_2}{\partial \tau_2} = \frac{z_2\theta_2}{(1-\theta_2)u'(q_2) + \theta_2 c'(q_2)} > 0.$$

Hence output produced in region 2 is increasing in the subsidy, $\tau_2 \in (0, 1]$. In addition, notice τ_2 also affects buyers' effective bargaining power, $\tau_2\theta_2$, which can make fiscal policy especially useful.[16] In particular, $\tau_2\theta_2$ affects the magnitude of the effective buyer's surplus in region 2 relative to region 1, which affects the migration decision by buyers in region 1. Since q_2 is increasing in ρ_1, the subsidy increases migration and brings ρ_1 closer to the first best ρ_1^*. Hence, there is a trade off between output produced in DM and the migration rate. This creates room for fiscal and monetary policies to exploit the trade off between intensive and extensive margins.

When the fiscal authority enacts a proportional tax, i.e. $\tau_2 \in [-1, 0)$, DM output decreases with τ_2 ($\frac{\partial q_2}{\partial \tau_2} < 0$) and the migration rate decreases ($\frac{\partial \rho_1}{\partial \tau_2} < 0$). We wil show how this fiscal policy scheme could raise welfare if there is underinvestment in migration.

5.3 Optimal Monetary and Fiscal Policies

We now consider the design of optimal policy following the Ramsey tradition. In this context, the government chooses the monetary-fiscal policy mix to maximize union wide welfare, taking as given the government budget constraint and equilibrium decisions of private agents. The available instruments are the money growth rate for the union, γ, and the production tax/subsidy on region 2's DM production, τ_2. Importantly, the monetary and fiscal authorities sets (γ, τ_2 once and for all and can commit to their policies. The policy problem is given by

$$\max_{\gamma,\tau_2} \left\{ n\frac{\alpha_1(\vartheta_1)}{\vartheta_1} \ [u(q_1) - c(q_1)] - n\Phi(\rho_1) + \frac{\alpha_2(\vartheta_2)}{\vartheta_2} \ [u(q_2) - c(q_2)] - \Phi(\rho_2) \right\} \tag{21}$$

subject to (11), the equilibrium conditions for q_1 and q_2,

$$\iota = (1-\rho_1)\alpha_1(\vartheta_1)\mathcal{L}_1(q_1) + \rho_1\alpha_2(\vartheta_2)(1+\tau_2\theta_2)\mathcal{L}_2(q_2), \tag{22}$$

[16] Instead of a proportional scheme, the government could alternatively propose a lump sum subsidy on region 2 sellers. In that case, there would be less change on production since a lump sum subsidy would only affect the total surplus.

$$\iota = \rho_2\alpha_1(\vartheta_1)\mathcal{L}_1(q_1) + (1-\rho)2)\alpha_2(\vartheta_2)(1+\tau_2\theta_2)\mathcal{L}_2(q_2), \tag{23}$$

the government budget constraint,

$$\phi\mathcal{S} = \tau_2\ \alpha_2(\vartheta_2)\ [\rho_1 + (1-\bar{\rho}_2)]\ z_2, \tag{24}$$

and the market clearing condition,

$$\phi M(\gamma - 1) = \phi\mathcal{S} + \mathcal{T}, \tag{25}$$

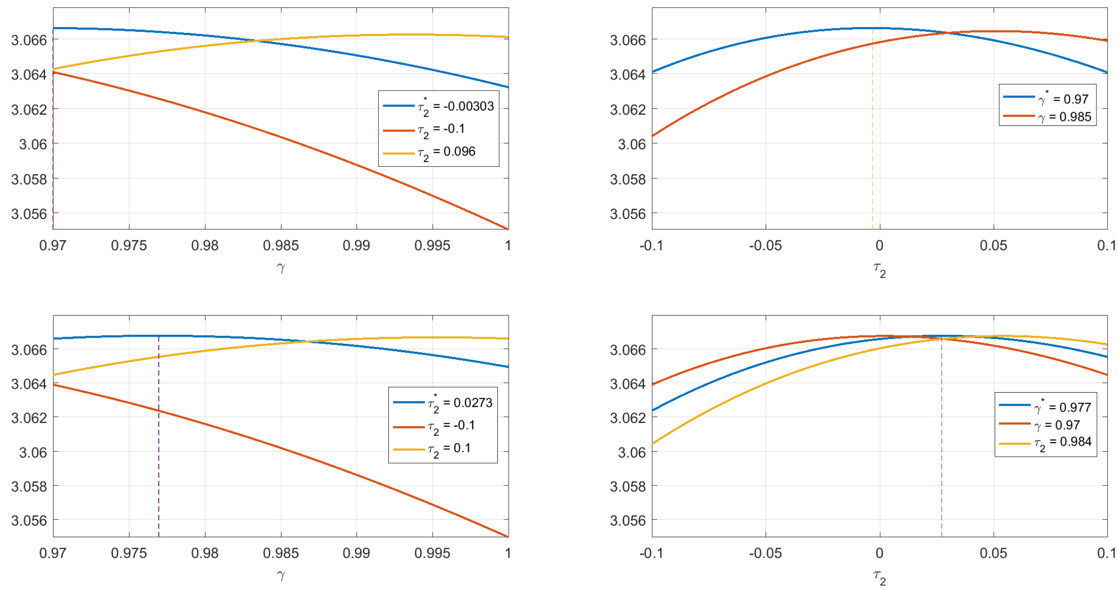
$$\phi M = z_1 + nz_2. \tag{26}$$

The first three constraints correspond to equilibrium decisions of private agents: (11) determines the migration rate, while (22) and (23) determine production in the two regions given taxes/subsidies in region 2. In addition, (24) defines the size of the subsidy paid to sellers in region 2, (25) is the government budget constraint, and (32) is the money market clearing condition. Since the central bank prints money in the CM and distributes it to households, there is a redistribution of resources to producers in the DM when $\tau_2 \neq 0$.

Since the optimal policy mix depends on whether there is under or overinvestment in migration, we consider two cases: $\theta_1 = 0.4$, which implies overinvestment in migration at the Friedman rule, and $\theta_1 = 0.5$, which implies underinvestment. Figure 2 shows the optimal policy mix, (γ, τ_2), for these two scenarios; the top panels assume $\theta_1 = 0.4$ and the bottom panels assume $\theta_1 = 0.5$. The blue lines in left panels plot union welfare against the money growth rate γ, assuming that the fiscal authority follows the optimal policy τ_2^*. Instead, the red (yellow) line assumes a value of τ below (above) the optimal τ_2^*. Similarly, the right panel plot welfare against the tax rate τ_2, in blue we plot the optimal money growth rate γ^* and lower (higher) values of γ below (above) in red (yellow).

When households overinvest in migration ($\theta_1 = 0.4$), optimal monetary policy is the Friedman rule. In contrast, with underinvestment ($\theta_1 = 0.5$), optimal monetary policy is a money growth rate above the Friedman rule, $\gamma = 0.9752$. This is illustrated by the top left panel for $\theta_1 = 0.4$ and the bottom left panel for $\theta_1 = 0.5$. In both cases, there is deflation, $\gamma < 1$, and hence all buyers face lump sum taxes in the CM. These results highlight the importance of having a monetary-fiscal union that implements the union wide inflation tax. Since the deviation from the Friedman rule is not large in this example, the intertemporal distortion may be more relevant than the congestion externality induced by endogenous migration. Indeed, the middle left panel shows the Friedman rule can still be optimal even when other instruments are available to the government. This arises when a production tax in the DM is required, i.e. when $\tau_2 = -0.003$ as in the left panel. In contrast, the Friedman

Figure 2: Welfare and Optimal Policies with One Sided Migration



rule is not optimal when there is a production subsidy, i.e. $\tau_2 = 0.027$ when $\theta_1 = 0.5$ as in the left panel. These results highlight the importance of the redistribution from buyers to sellers of region 2 to correct the congestion externality.

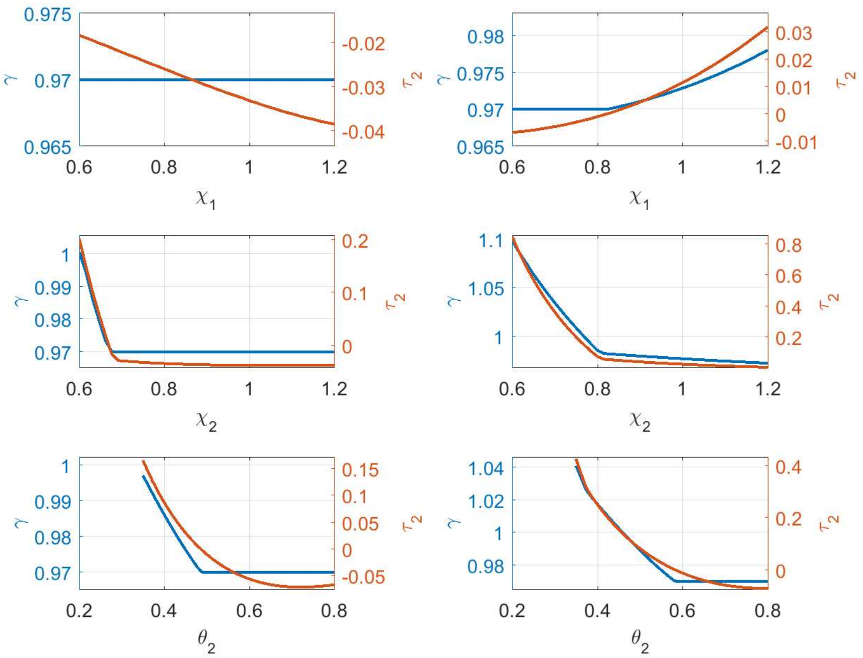
We now study how these optimal policies respond with changes in model parameters, i.e. matching efficiency in both regions and the buyer's bargaining power in region 2. Figure 3 summarizes comparative statics with respect to the parameters governing optimal monetary policy γ (when fiscal policy is fixed at its optimal value) and optimal fiscal policy τ_2 (assuming monetary policy is fixed at its optimal value) when $\theta_1 = 0.4$ (left) and $\theta_1 = 0.5$ (right).

The blue lines denote the optimal money growth rate, γ, while the orange lines denote the optimal tax/subsidy rate in region 2, τ_2. From Figure 4, higher matching efficiency and household bargaining power in region 2 result in larger fiscal responses. These are more accentuated when buyers are under-investing in their migration decisions.

6 Two Sided Migration

We now allow households from both regions to migrate. In this case, ρ_1 and ρ_2 jointly solve the first order conditions associated with the buyers' migration decisions which are given by equations (11) and (12). We first consider equilibrium where monetary policy follows the Friedman rule, $\iota = 0$, and there is no fiscal policy. As a result, changes in bargaining power and matching efficiency do not

Figure 3: Comparative Statics for Optimal Policies with One Sided Migration



produce any distortion along the intensive margin, i.e. on q_1 and q_2. However, the extensive margin, ρ_1 and ρ_2, will still be affected where the effects are summarized by the following table.

Table 4: Comparative Statics Two Sided Migration

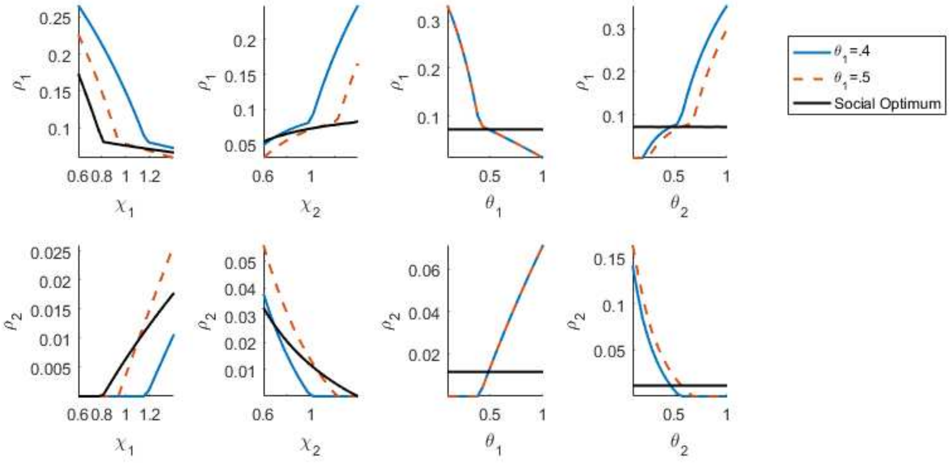
	θ_1	θ_2	χ_1	χ_2
ρ_1	$-$	$+$	$-$	$+$
ρ_2	$+$	$-$	$+$	$-$

These results are consistent with those found in the one sided migration case. More precisely, an increase in household bargaining power or matching efficiency at home decreases migration to the destination region. As in the one sided case, buyers choose to relocate to regions they expect to have larger surpluses, which are associated with regions with increased buyer's bargaining power and matching efficiencies.

Similarly, the response of the migration rates in both regions depend critically on how far equilibrium values are from the first best. To illustrate this effect, Figure 4 shows the effect of changes in bargaining and matching efficiencies on migration rates, ρ_1 and ρ_2, using the same benchmark parametrization as in the one sided case.

From Figure 4, the equilibrium migration response in region 2 are much smaller than the ones observed in region 1. Moreover, the effects of bargaining power and matching efficiency are monotonic

Figure 4: Comparative Statics with Two Sided Migration



and piecewise linear. These depend on whether equilibrium migration rates are above or below the corresponding first best migration rates. The equilibrium migration response in the two regions relative to the first best is much larger when bargaining power changes. This property reflects the importance of the buyer's bargaining power in the determining the expected surplus. Changes in the matching efficiency deliver smaller departures from the first best, indicating that the associated changes in the extensive margin are relatively small. It is also worth highlighting that when buyers of both regions can migrate, the smaller region shows a larger response to changes in matching efficiency. In addition, the differential equilibrium migration response between buyers facing $\theta_1 = 0.4$ and $\theta_1 = 0.5$ is much larger when matching efficiencies change.

6.1 Optimal Monetary and Fiscal Policies

With two sided migration, there is an additional instrument available to the fiscal authority. Now the optimal policy mix consists of a money growth rate for the union and production tax/subsidies in both regions (γ, τ_1, τ_2) that maximize social welfare of the union subject to the equilibrium conditions, government budget constraint, and market clearing conditions. The policy problem with two sided migration is

$$\max_{\gamma,\tau_1,\tau_2} \left\{ n\frac{\alpha_1(\vartheta_1)}{\vartheta_1} \; [u(q_1) - c(q_1)] - n\Phi(\rho_1) + \frac{\alpha_2(\vartheta_2)}{\vartheta_2} \; [u(q_2) - c(q_2)] - \Phi(\rho_2) \right\} \tag{27}$$

subject to (11) and (12), the equilibrium conditions for q_1 and q_2

$$\iota = (1 - \rho_1)\alpha_1(\vartheta_1)(1 + \tau_1\theta_1)\mathcal{L}_1(q_1) + \rho_1\alpha_2(\vartheta_2)(1 + \tau_2\theta_2)\mathcal{L}_2(q_2), \tag{28}$$

$$\iota = \rho_2\alpha_1(\vartheta_1)(1+\tau_1\theta_1)\mathcal{L}_1(q_1) + (1-\rho)2)\alpha_2(\vartheta_2)(1+\tau_2\theta_2)\mathcal{L}_2(q_2), \tag{29}$$

the subsidy size, government budget constraint, and the market clearing condition

$$\phi\mathcal{S} = \tau_1\ \alpha_1(\vartheta_1)\ [(1-\rho_1)n + \rho_2]z_1 + \tau_2\alpha_2(\vartheta_2)[\rho_1 + (1-\rho_2)]z_2, \tag{30}$$

$$\phi M(\gamma - 1) = \phi\mathcal{S} + \mathcal{T}, \tag{31}$$

$$\phi M = z_1 + nz_2. \tag{32}$$

The first four constraints correspond to equilibrium decisions of private agents. In particular, equations (11) and (12) determine the optimal migration rates for buyers in region 1 an d 2 respectively, while (28) and (29) determine production in the two regions given taxes/subsidies in region 1 and 2. In addition, (30) defines the size of the subsidy paid to sellers in region 2, (31) is the government budget constraint, and (32) is the money market clearing condition. Since the central bank prints money in the CM and distributes it to households, there is a redistribution of resources to producers in the DM when $\tau_2 \neq 0$.

As in the one sided case, we numerically compute the optimal policy using the benchmark parametrization in Table 3. The optimal policy is the Friedman rule and zero taxes in both regions. This finding highlights that correcting the potential distortions in the intensive margin is much more important than bringing the extensive margin closer to the efficient level. The qualitative results obtained in the two sided migration case are in line with the results under one sided migration. In both cases, the authorities in the monetary union prioritize the intensive margin rather than the extensive one.

7 Conclusion

In this paper, we constructed an open economy model of a currency union with endogenous migration and studied the roles of monetary and fiscal policies at correcting distortions along the intensive margin – the quantity traded per match – and the extensive margin – the total number of trades. Due to regional spillovers from agents' migration decisions, equilibrium is generically inefficient. While monetary policy can eliminate the intensive margin distortion by running the Friedman rule, migration rates can still be too high or too low, unless in the knife edge case when the Hosios condition is also satisfied. To correct this inefficiency, we introduce fiscal policy that can tax or subsidize production at the regional level.

Key to our analysis is the assumption that the monetary union is also a fiscal union: the fiscal authority is taxing (subsidizing) production but finances it by subsidizing (taxing) both regions through the common inflation tax. Consequently, fiscal policy leads to redistribution between households and producers and hence can be set to minimize the extensive margin distortion. When only households from one region can migrate, the optimal policy mix entails a deviation from the Friedman rule and either a production subsidy if there is underinvestment in migration or a production tax if there is overinvestment. However when households from both regions can migrate, optimal policy is the Friedman rule and zero taxes in both regions. Overall, this finding indicates that the (intensive margin) distortions in quantity traded are more socially beneficial to correct than the (extensive margin) distortions in migration rates. A potential avenue for future work is to allow for further heterogeneity and examine its consequences for trade-off between inflation and migration.

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Appendix

Proof of Lemma 1

To obtain the social optimum, we differentiate the social welfare function (2) with respect to q_1, q_2, ρ_1, and ρ_2. The first order condition with respect to q_j yields $u'(q_j) = c'(q_j)$ and hence $q_1 = q_2 = q^*$. The first order conditions with respect to $\rho_j > 0$ are

$$\left\{n\left(\frac{\alpha_1'(\vartheta_1)}{\vartheta_1} - \frac{\alpha_1(\vartheta_1)}{\vartheta_1^2}\right)\frac{\partial\vartheta_1}{\partial\rho_1} + \left(\frac{\alpha_2'(\vartheta_2)}{\vartheta_2} - \frac{\alpha_2(\vartheta_2)}{\vartheta_2^2}\right)\frac{\partial\vartheta_2}{\partial\rho_1}\right\}[u(q^*) - c(q^*)] = n\Phi_1'(\rho_1)$$

$$\left\{\left(\frac{\alpha_1'(\vartheta_1)}{\vartheta_1} - \frac{\alpha_1(\vartheta_1)}{\vartheta_1^2}\right)\frac{\partial\vartheta_1}{\partial\rho_2} + \left(\frac{\alpha_2'(\vartheta_2)}{\vartheta_2} - \frac{\alpha_2(\vartheta_2)}{\vartheta_2^2}\right)\frac{\partial\vartheta_2}{\partial\rho_2}\right\}[u(q^*) - c(q^*)] = \Phi_2'(\rho_2)$$

where

$$\frac{\partial\vartheta_1}{\partial\rho_1} = \vartheta_1^2 \ ; \ \frac{\partial\vartheta_2}{\partial\rho_2} = \vartheta_2^2$$

$$\frac{\partial\vartheta_1}{\partial\rho_2} = -\frac{\vartheta_1^2}{n} \ ; \ \frac{\partial\vartheta_2}{\partial\rho_1} = -n\vartheta_2^2.$$

Upon substituting and rewriting, we obtain

$$\Phi_1'(\rho_1^*) = \left[\alpha_2(\vartheta_2^*)\left(1 - \frac{\alpha_2'(\vartheta_2^*)\vartheta_2^*}{\alpha_2(\vartheta_2^*)}\right) - \alpha_1(\vartheta_1^*)\left(1 - \frac{\alpha_1'(\vartheta_1^*)\vartheta_1^*}{\alpha_1(\vartheta_1^*)}\right)\right][u(q^*) - c(q^*)],$$

$$\Phi_2'(\rho_2^*) = \left[\alpha_1(\vartheta_1^*)\left(1 - \frac{\alpha_1'(\vartheta_1^*)\vartheta_1^*}{\alpha_1(\vartheta_1^*)}\right) - \alpha_2(\vartheta_2^*)\left(1 - \frac{\alpha_2'(\vartheta_2^*)\vartheta_2^*}{\alpha_2(\vartheta_2^*)}\right)\right][u(q^*) - c(q^*)],$$

which are (5) and (6) in the main text. □

Proof of Proposition 1

To show the Friedman rule, $\iota = 0$, produces the efficient quantity of trade, $q_1 = q_2 = q^*$, consider the equilibrium values for q_1 and q_2 given by (15) and (16). Setting $\iota = 0$ gives $u'(q_j) = c'(q_j)$ and hence, $q_1 = q_2 = q^*$. Equilibrium migration rates are given by (11) and (12) at $\iota = 0$, or

$$\Phi_1'(\rho_1) = [\alpha_2(\vartheta_2)\theta_2 - \alpha_1(\vartheta_1)\theta_1][u(q^*) - c(q^*)], \tag{33}$$

$$\Phi_2'(\rho_2) = [\alpha_1(\vartheta_1)\theta_1 - \alpha_2(\vartheta_2)\theta_2][u(q^*) - c(q^*)]. \tag{34}$$

It is now easy to see the equilibrium migration rates equal the first best allocations if (33) and (34) coincide with (5) and (6), respectively. This is happen if and only if $\theta_j = \epsilon(\vartheta_j)$. □

Proof of Proposition 2

Equilibrium has a recursive structure. Once ρ_1 is determined by 41, we obtain $q_1(\rho_1)$ and $q_2(\rho_1)$ from (39) and (40). Real balances z_1 and z_2 are then pinned down by (9) and (10). Since $\Phi_1(0) = \Phi_1'(0) = 0$ and $\Phi_1'' > 0$, the left side of (41) is increasing in ρ_1. To show the equilibrium exists and is unique, we first establish there is a unique solution, z_1, rewritten as $\max_{z_1 \geq 0} O(z_1; \iota)$ where

$$O(z_1; \iota) \equiv -\iota z_1 - \Phi(\rho_1) + (1 - \rho_1)\ \alpha_1(\vartheta_1)\ \theta_1[u(q_1) - c(q_1)] + \rho_1\ \alpha_2(\vartheta_2)\ \theta_2[u(q_2) - c(q_2)]$$

With no loss in generality, we can restrict the choice for z_1 to the compact interval $[0, z_1^*]$ where $z_1^* \equiv (1 - \theta_1)c(q^*) + \theta_1 u(q^*)$. If $z_1 > z_1^*$, then $O'(z_1; \iota) = -\iota$. Moreover, $O(z_1; \iota)$ is continuous in z_1. Hence a solution exists by the Theorem of the Maximum and $\max_{z_1 \geq 0} O(z_1; \iota)$ is continuous in ι. A similar argument applies to the solution to z_2. Under Kalai bargaining, money is valued if and only if $\iota < \min\{\frac{\theta_1}{1-\theta_1}, \frac{\theta_2}{1-\theta_2}\}$.

Next, we establish there is a unique solution, ρ_1 to (15). Since $\Phi(\cdot)$ is strictly convex, ρ_1 is uniquely defined and continuous by the Theorem of the Maximum. Hence a solution exists. To make sure there exist a unique positive solution for ρ_1 at $\iota = 0$, we need $\Phi_1(0) > 0$ which is satisfied if (20) holds.

Comparative statics for ρ_1 are obtained by applying the implicit function theorem and Cramer's rule:

$$\frac{\partial \rho_1}{\partial \theta_1} = \frac{\alpha_1(\vartheta_1^*)[u(q^*) - c(q^*)]}{\frac{\partial F_1}{\partial \rho_1}|_{\iota=0}} < 0,$$

$$\frac{\partial \rho_1}{\partial \theta_2} = -\frac{\alpha_2(\vartheta_1^*)[u(q^*) - c(q^*)]}{\frac{\partial F_1}{\partial \rho_1}|_{\iota=0}} > 0,$$

where

$$\frac{\partial F_1}{\partial \rho_1}\Big|_{\iota=0} = -\Phi_1''(\rho_1^*) + [\alpha_2'(\vartheta_2^*)(-n\vartheta_2^{*2})\theta_2 - \alpha_1'(\vartheta_1^*)\vartheta_1^{*2}\theta_1][u(q^*) - c(q^*)] < 0. \tag{35}$$

If ι is small and (20) holds, $\partial\rho_1/\partial\theta_1 < 0$ and $\partial\rho_1/\partial\theta_2 > 0$.

At $\iota = 0$, comparative statics with respect to ι are

$$\begin{aligned} \frac{d\rho_1}{d\iota}\Big|_{\iota=0} &= 0 \\ \frac{dq_1}{d\iota}\Big|_{\iota=0} &= \frac{1}{\alpha_1(\vartheta_1^*)\mathcal{L}_1'(q^*)} < 0 \\ \frac{dq_2}{d\iota}\Big|_{\iota=0} &= \frac{1}{\alpha_2(\vartheta_2^*)\mathcal{L}_2'(q^*)} < 0 \end{aligned} \tag{36}$$

where we used that

$$\mathcal{L}_j'(q^*) = \frac{\theta_j(c'u'' - u'c'')}{[\theta_j c' + (1 - \theta_j)u']^2}\Big|_{q=q^*} < 0. \tag{37}$$

□

Proof of Proposition 3

We measure steady state welfare for the union at the start of DM, before households make their migration decisions and portfolio choices.

$$\mathcal{W} \equiv n\frac{\alpha_1(\vartheta_1)}{\vartheta_1}[u(q_1) - c(q_1)] + \frac{\alpha_2(\vartheta_2)}{\vartheta_2}[u(q_2) - c(q_2)] - n\Phi_1(\widehat{\rho}_1) - \Phi_2(\widehat{\rho}_2), \tag{38}$$

where $n\frac{\alpha_1(\vartheta_1)}{\vartheta_1}$ and $\frac{\alpha_2(\vartheta_2)}{\vartheta_2}$ are the measure of matches in DM of regions 1 and 2. Equilibrium values for z_1, z_2, ρ_1, q_1, and q_2 are given by (9), (10), (11), (12), (15), and (16).

At $\iota = 0$, $q_1 = q_2 = q^*$, and social welfare is

$$\mathcal{W}^{FR} \equiv n\frac{\alpha_1(\vartheta_1^{FR})}{\vartheta_1^{FR}}[u(q^*) - c(q^*)] + \frac{\alpha_2(\vartheta_2^{FR})}{\vartheta_2^{FR}}[u(q^*) - c(q^*)] - n\Phi_1(\rho_1^{FR}) - \Phi_2(\rho_2^{FR}),$$

where ϑ_1^{FR} is market tightness at $\iota = 0$ and ρ_1^{FR} solves (33). Defining $\epsilon(\vartheta_1) \equiv 1 - \frac{\vartheta_1\ \alpha_1'(\vartheta_1)}{\alpha_1(\vartheta_1)}$ as the elasticity of the matching function in region 1, the socially optimal migration rates from (5) solves (5). We next establish $\partial\rho_1/\partial\theta_1 < 0$ at $\iota = 0$:

$$\left.\frac{\partial\rho_1}{\partial\theta_1}\right|_{\iota=0} = \frac{-\alpha_1(\vartheta_1)[u(q^*) - c(q^*)]}{\Phi''(\rho_1) + \left[\theta_2\alpha_2'(\vartheta_2)\vartheta_2^2/n - \theta_1\alpha_1'(\vartheta_1)\vartheta_1^2\right][u(q^*) - c(q^*)]} < 0,$$

where we assumed the condition for uniqueness, (20), holds.

If $\theta_1 = \epsilon(\vartheta_1)$, $\rho_1 = \rho_1^*$ from Proposition 1. Now consider a small deviation from $\theta_1 = \epsilon(\vartheta_1)$. If θ_1 increases, i.e. $\theta_1 > \epsilon(\vartheta_1)$, ρ_1 falls since $\partial\rho_1/\partial\theta_1 < 0$. As a result, $\rho_1 < \rho_1^*$. Similarly, if θ_1 decreases, i.e. $\theta_1 < \epsilon(\vartheta_1)$, ρ_1 now increases. Hence, $\rho_1 > \rho_1^*$. □

Equilibrium Conditions Under Functional Forms in Sections 5 and 6

Given the functional forms in Section 5, $q_1(\rho_1)$ and $q_2(\rho_1)$ are given by

$$q_1(\rho_1) = \frac{\theta_1 - (1-\theta_1)F_1(\rho_1)}{\theta_1(1 + F_1(\rho_1))} - b, \tag{39}$$

$$q_2(\rho_1) = \frac{\theta_2 - (1-\theta_2)F_2(\rho_1)}{\theta_2(1 + F_2(\rho_1))} - b, \tag{40}$$

where

$$F_1(\rho_1) \equiv \frac{\iota(1 - \rho_1 - \bar{\rho}_2)}{(1-\bar{\rho}_2)(1-\rho_1) - \rho_1\bar{\rho}_2}\frac{(1-\rho_1) + \bar{\rho}_2/n + 1}{\chi_1},$$

$$F_2(\rho_1) \equiv \frac{\iota(1-\rho_1-\bar{\rho}_2)}{(1-\bar{\rho}_2)(1-\rho_1)-\rho_1\bar{\rho}_2}\frac{(1-\bar{\rho}_2)+\rho_1 n+1}{\chi_2}.$$

The migration rate, ρ_1, solves

$$\frac{\rho_1(2-\rho_1)}{(1-\rho_1)^2} = \frac{\chi_2\theta_2[\ln(q_2(\rho_1)+b)-\ln(b)-q_2(\rho_1)]}{(1-\bar{\rho}_2)+\rho_1 n+1} - \frac{\chi_1\theta_1[\ln(q_1(\rho_1)+b)-\ln(b)-q_1(\rho_1)]}{(1-\rho_1)+\bar{\rho}_2/n+1}. \quad (41)$$

Given the functional forms from Section 6, resulting equilibrium migration rates ρ_1 and ρ_2 solve

$$\frac{\rho_1(2-\rho_1)}{(1-\rho_1)^2} = \frac{\chi_2\theta_2[\ln(q_2(\rho_1,\rho_2)+b)-\ln(b)-q_2(\rho_1,\rho_2)]}{(1-\rho_2)+\rho_1 n+1} - \frac{\chi_1\theta_1[\ln(q_1(\rho_1,\rho_2)+b)-\ln(b)-q_1(\rho_1,\rho_2)]}{(1-\rho_1)+\rho_2/n+1}, \quad (42)$$

$$\frac{\rho_2(2-\rho_1)}{(1-\rho_2)^2} = \frac{\chi_1\theta_1[\ln(q_1(\rho_1,\rho_2)+b)-\ln(b)-q_1(\rho_1,\rho_2)]}{(1-\rho_1)+\rho_2/n+1} - \frac{\chi_2\theta_1[\ln(q_2(\rho_1,\rho_2)+b)-\ln(b)-q_2(\rho_1,\rho_2)]}{(1-\rho_2)+\rho_1 n+1}; \quad (43)$$

where $q_1(\rho_1)$ and $q_2(\rho_1)$ are given by

$$q_1(\rho_1,\rho_2) = \frac{\theta_1-(1-\theta_1)F_1(\rho_1,\rho_2)}{\theta_1(1+F_1(\rho_1,\rho_2))} - b \ , \ q_2(\rho_1,\rho_2) = \frac{\theta_2-(1-\theta_2)F_2(\rho_1,\rho_2)}{\theta_2(1+F_2(\rho_1,\rho_2))} - b,$$

with

$$F_1(\rho_1,\rho_2) \equiv \frac{\iota(1-\rho_1-\rho_2)}{(1-\rho_2)(1-\rho_1)-\rho_1\bar{\rho}_2}\frac{(1-\rho_1)+\rho_2/n+1}{\chi_1};$$

$$F_2(\rho_1,\rho_2) \equiv \frac{\iota(1-\rho_1-\rho_2)}{(1-\rho_2)(1-\rho_1)-\rho_1\rho_2}\frac{(1-\rho_2)+\rho_1 n+1}{\chi_2}.$$