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Abstract

In this paper, we consider an economy having access to two different energy sources. The first one is coming from natural polluting resources; and second one is coming from a backstop natural resource. There are two productive sectors in the economy. The first one is dedicated to manufacturing the backstop resources; the second one is devoted to production of the consumption good. Both sectors are dirty in the sense that both use the polluting resource at any time. The social planner, however, has always the possibility of paying an irreversible fixed cost to switch the consumption sector towards the use of a cleaner technology. Additionally, we assume that the accumulation of the backstop, and the increase in pollution stock are stochastic. Our results imply that the incentives to switch to the cleaner technology depend on the relative importance of fossil fuels in the production of consumption goods after the switch. We also find that technological improvement in the solar panels sector is of some importance in order to switch to cleaner technologies.

Keywords: Two sectors economy, Cleaner technologies, Switching, Uncertainty, Irreversibility

1 Introduction

The Intergovernmental Panel on Climate Change [IPCC, 2007] and the International Energy Agency [IEA, 2008] estimate that in order to limit the rise of average global temperature to 2 degrees Celsius, the concentration of greenhouse gases (GHG) should not exceed 450 parts per million (ppm) CO_2. This translates to a peak of global emissions in 2015 and at least a 50 per cent cut in global emissions by 2050, compared with 2005 [UNEP, 2011]. This objective is consistent with that of many developed countries. For instance, France has been committed in 2003 to divide by four the 1990 national level of GHG, while the U.S. and Canada aim at reducing GHG emissions by more than 80 percent by 2050.

To achieve this goal, one of the policies commonly undertaken by many countries is to substitute dirty energy sources, such as coal, oil and gas, with a cleaner and renewable energy source, such as solar and wind energy. For instance, the Directive 2009/28/EC on renewable energy, implemented by Member States by December 2010, sets ambitious targets for all Member States, such that the European Union will reach a 20 percent share of energy from renewable sources by 2020. In spite of this, fossil fuels will continue to be an important part of the energy mix around the world even by 2050. In particular, as long as renewable energies are not very advanced and widespread, (i) industry will still need a percentage of energy that derives from dirty resources, and (ii) the provision of clean energy itself will require dirty resources at least as materials to build the plants (think of solar panels for instance). This is the main idea that we address here. We seek to account for the need of dirty resources even if a clean energy can be used.

In this paper, we consider an economy having access to two different energy sources. The first one comes from a natural polluting resource, such as fossil fuels. The second one comes from a backstop natural resource, such as solar radiation. In particular, we consider the case of solar radiation being converted into energy by means of solar panels. There are two productive sectors in the economy. The first one is dedicated to manufacturing the backstop resources. At any time, this sector requires both fossil fuels and the energy provided by the backstop already available. We therefore account for the need of fossil fuels to provide clean energy. The second sector is devoted to production of the consumption good. Initially it uses energy coming exclusively from fossil fuels. However, it has always the possibility of switching towards a new technology in which energy comes from both types of resources. As the backstop is being accumulated such a switch becomes more attractive. In particular, it gets worth paying a fixed cost to use the existing stock of new solar panels and avoid — at least partially— the use of the polluting input. With this specification, the economy becomes cleaner after the switch although not completely clean, in the sense that the clean energy cannot fully replace fossil fuels to produce the consumption good. Therefore we account for the fact that even if the new technology is used, fossil fuels are still required in the industry. While taking into account these two levels of dependence with respect to the fossil fuels (namely (i) to run the economy, and (ii) to produce clean energy) after the switch, we pay particular attention to the optimal timing of the switching decision, and on the factors influencing the decision to switch.

In modelling this switching decision we include three important characteristics that must be taken into account to evaluate the adoption of any environmental policy [Pindyck, 2000, 2002]. First, we account for the uncertainty over the future costs and benefits. In particular, we assume that the accumulation of the backstop, and the increase in pollution stock —which in our case is equal to the resource extraction— are stochastic. Then, the future availability of the backstop, and the future levels of pollution —affecting the utility function— are not completely known. Second, we introduce the irreversibilities associated with environmental policy. Specifically, adoption of the cleaner technology imposes sunk cost on the consumption sector. Finally, we take into account the fact that technology adoption is rarely a now or never proposition, such that, in most cases, it is feasible to delay action and wait for new information. As the adoption of the new technology is difficult to reverse, the sunk costs are incurred over a long period of time, even if the original rationale for the switching disappears. These kind of sunk costs create an opportunity cost of adopting the new technology now, rather than waiting for more information. Our results imply that the incentives to switch to the cleaner technology depend on the relative importance of fossil fuels in the production of consumption goods after the switch. Specifically, if fossil fuels are relatively less important than solar panels to produce consumption, the central planner tends to wait more in order to switch to the new technology. This is because the solar panels sector needs to be sufficiently developed to prevent some consumption loss once the new technology is adopted. But, if fossil fuels are relatively more important than solar panels to produce consumption, switching to the new technology is easier —smoother—, and then the incentives for the central planner to wait vanishes. We also find that technological improvement in the solar panels sector is of some importance in the decision to switch to cleaner technologies. If the technological change implies that the backstop can be produced with relatively less of the fossil fuels, the adoption occurs sooner .

The rest of the paper is as follows. In Section 2 we describe the assumptions and equations governing our economy. In Section 3 we develop the general equilibrium framework once the cleaner technology has been adopted by the consumption sector. In section 4 we solve the model before the technological switch by assuming that the discount rate is zero, and derive the socially optimal adoption timing. We additionally perform a comparative statics exercise in Section 5. In Section 6 we relax the assumption of a zero discount rate and then solve the model by using numerical methods. We conclude in section 7.

2 The model

We consider an economy with access to two different energy sources: one dirty, and another one clean. Dirty energy comes from a natural polluting resource, R_t, such as fossil fuels (e.g. oil). Clean energy comes from a backstop natural resource, such as solar radiation. Specifically, we consider the case of solar radiation being converted into energy by means of solar panels, S_t.

There are two productive sectors in this economy. The first one is devoted to production of the consumption good. Initially, it uses energy coming exclusively from dirty inputs to run a given constant stock of capital $\overline{K_1}$. At some point, however, the backstop becomes more developed in the economy, such that the consumption sector is more interested to switch to a new technology using both types of energy, i.e. electricity from solar panels and oil, to run the capital. Such a switch becomes more attractive as the backstop is being accumulated: it gets worth paying a fixed cost to use the existing stock of solar panels. In particular, we assume that:

$$C_t = B_1(\kappa_t R_t)\overline{K_1}, \tag{1}$$

for $t < T$, and:

$$C_t = A_1(\kappa_t R_t)^{\eta}(\lambda_t S_t)^{1-\eta}\overline{K_1}, \tag{2}$$

for $t \geq T$, where T is the time of the switching, i.e. when the backstop becomes more active —sufficiently developed— in the economy. In equations (1) and (2), B_1, $A_1 > 0$ are technological parameters, and η ($0 \leq \eta \leq 1$) is the share of the polluting resource in the consumption function. Notice that after the switch, the smaller the parameter η, the cleaner the consumption sector. However, even in the limit case of $\eta = 0$, the economy is not completely "pollution-free" due to the fact that solar panels still require fossil fuels to be produced by the other sector (see below equations (3) and (4)).

The second sector is dedicated to manufacturing the backstop resource. This sector requires both fossil fuels and the energy provided by the backstop already available. On can think of solar panels whose fabrication requires some given constant stock of capital $\overline{K}_2$, as well as solar panels (for electricity provision) and oil as a source of energy or of materials to be built. This particular assumption is in line with a physician view of environmental economics that stresses the need for oil in order to turn to a new energy (and some of them even doubting that current reserves are sufficient for this energy change). Moreover, efficiency in this sector is stochastic, since there is a lot of uncertainty surrounding the productivity of solar panels in energy provision and the maintenance costs of these panels. Uncertainty is assumed to be multiplicative, meaning that the larger the number of solar panels already built the larger future uncertainty on solar panel accumulation. We assume that the backstop is accumulated according to:

$$dS_t = B_2\left[\beta(1-\kappa_t)R_t + (1-\beta)S_t\right]\overline{K}_2 dt + \sigma_S \overline{K}_2 S_t dz_S, \tag{3}$$

for $t < T$, and:

$$dS_t = A_2\left[\alpha(1-\kappa_t)R_t + (1-\alpha)(1-\lambda_t)S_t\right]\overline{K}_2 dt + \sigma_S \overline{K}_2 S_t dz_S, \tag{4}$$

for $t \geq T$, with $S_t > 0$, and S_0 given. In equations (3), and (4) B_2, $A_2 > 0$ as well as α and β ($0 \leq \alpha, \beta \leq 1$) are technological parameters, and dz_S is the standard increment of a Wiener process. The parameters κ_t and λ_t ($0 \leq \kappa_t, \lambda_t \leq 1$) are endogenously chosen fractions of the polluting and backstop resources, respectively, used in the consumption sector. We assume that a 100 percent of the extracted polluting resources, and a 100 percent of the backstop already available are used in the economy. Hence, by choosing optimally κ_t and λ_t, the central planer is implicitly choosing $(1-\kappa_t)$ and $(1-\lambda_t)$ to be the fractions of the polluting and backstop resources, respectively, used in the backstop production sector. It is worth noting that if $\kappa_t = \lambda_t = 1$, there is no production of solar panels after the switch —though they are still stochastically evolving as a simple Brownian motion, then we go back to a one sector formulation. It is also important to notice that before T the backstop resource is not used in the consumption sector, though it is accumulated according to equation (3). Additionally, uncertainty affects solar panels accumulation in the same way irrespective of whether panels are used in the consumption sector or not.

Our formulation in equations (1) to (4) also implies that from period T the economy becomes cleaner in the sense that the clean resource can be used to provide the consumption good, but there is also a switch in technology since the accumulation process of solar panels changes. Particularly, technology in solar panel accumulation improves after the switch as long as $A_2 > B_2$. Additionally, the green effect of the switching is reinforced by assuming that $\alpha < \beta$, such that the backstop production sector is less polluting-resource dependent after the switch. We analyse this case, and the less general case in which there is no technological improvement ($A_2 = B_2$ and $\alpha = \beta$) in Section 5.

We assume that the increase in the pollution stock is equal to the resource extraction. It is also subject to some multiplicative uncertainty that for instance takes into account that Nature assimilation of CO_2 released after oil combustion is not well-known. This is described by the following equation:

$$dP = R_t dt + \sigma_P P_t dz_P, \tag{5}$$

with $P_t \geq 0$, and P_0 given. dz_P is another standard increment of a Wiener process. For simplicity, we assume that dz_S and dz_P are uncorrelated.

The social preferences derived from consumption and environmental quality can be represented by the lifetime expected utility:

$$\mathbb{E}_0 \left[\int_0^\infty e^{-\rho\tau} U(C_\tau, P_\tau) d\tau \right] = \mathbb{E}_0 \left[\int_0^\infty e^{-\rho\tau} \frac{\left(C_\tau P_\tau^\phi\right)^{1-\varepsilon}}{1-\varepsilon} d\tau \right], \tag{6}$$

where $\phi < -1$, and $\rho \geq 0$ is the rate of time preference. This specification satisfies some conditions that are now common in the literature, and takes into account the fact that the combustion of fossil fuels is responsible for an important part of CO_2 emissions and other pollutants, and provides a (negative) amenity to households. The cross derivative U_{cP} is negative which means that utility exhibits a "distaste effect", in the terminology of Michel and Rotillon [1995]: a decrease in pollution increases the marginal utility of consumption and implies that households have a higher desire to consume.

Since there are two arguments in the utility function, it is not immediately obvious what risk aversion or intertemporal substitution means (see Debreu [1976] and Kihlstrom and Mirman [1974] for the literature on multivariate risk aversion). Equation (6) can be rewritten as:

$$\mathbb{E}_0\left[\int_0^{\infty} e^{-\rho\tau}\frac{\left(C_\tau^{\frac{1}{1+\phi}}P_\tau^{\frac{\phi}{1+\phi}}\right)^{1-\Gamma}}{1-\varepsilon}d\tau\right]$$

Debreu [1976] calls the function in the braces the "least concave utility function". The exponents of this function may be interpreted as governing ordinal preferences between the two goods in the absence of risk. The transforming function $[\cdot]^{1-\Gamma}$ can then be interpreted as governing aversion to risk. A simple calculation then reveals that the appropriate measure of risk relative aversion is Γ. Then, following the terminology in Smith [1999] or Pommeret and Schubert [2009] we will call Γ the effective coefficient of relative risk aversion and $\mathcal{E}$ the inverse of the effective elasticity of intertemporal substitution Since Γ depends on ϕ, pollution changes risk aversion:

$$\begin{aligned}\Gamma &= 1-(1-\varepsilon)(1+\phi)\\ \mathcal{E} &= 1-(1-\varepsilon)=\varepsilon.\end{aligned}$$

From now on, we keep the notation ε for the inverse of the effective elasticity of intertemporal substitution. We let Ω denote the set of admissible plans, that is the set κ_t, λ_t, extraction rates and dates of adoption $(\kappa,\ \lambda, R, T)$, such that:

$$\mathbb{E}_0\left[\int_0^{\infty} e^{-\rho\tau}\left|U(C_\tau,P_\tau)\right|d\tau\right]<\infty.$$

In this case, we can write the value function of the central planner as:

$$V(S_0,P_0)=\sup_{(\kappa_\tau,\lambda_\tau,R_\tau,T)\in\Omega}\mathbb{E}_0\left\{\int_0^{T} e^{-\rho\tau}\frac{\left(C_\tau P_\tau^{\phi}\right)^{1-\varepsilon}}{1-\varepsilon}d\tau+e^{-\rho T}W(S_T-IP_T^{-\phi},P_T)\right\},$$

where $W(\cdot)$ is the value function after the switch and $IP_t^{-\phi}$ is the switching cost. Notice that this cost is increasing with the level of pollution, and can be expressed in terms of solar panels by means of some constant $I>0$. In this sense, the cost of switching to a new production technology in the consumption sector is assimilated to a lost of some solar panels. This program can be solved in two stages. We first solve for the problem for the representative agent assuming that the backstop energy is used actively. We next determine the optimal time for adopting the backstop in the consumption sector.

3 The optimal path after the switch, and the case without an option to switch

3.1 The after the switch case

We assume that the backstop energy is used actively in the economy. The set of admissible plans collapses to the set $(\kappa,\, \lambda, R)$ such that:

$$\mathbb{E}_t\left[\int_t^\infty e^{-\rho(\tau-t)}\left|U(C_\tau,P_\tau)\right|d\tau\right]<\infty.$$

The value function of the central planner is:

$$W\left(S_t,P_t\right)=\sup_{(\kappa_\tau,\lambda_\tau,R_\tau)\in\Omega}\mathbb{E}_t\left\{\int_t^\infty e^{-\rho\tau}\frac{\left(C_\tau P_\tau^\phi\right)^{1-\varepsilon}}{1-\varepsilon}d\tau\right\},\ t\geq T.$$

Then, the Hamilton-Jacobi-Bellman equation can be written as:

$$\rho W\left(S_t,P_t\right)=\max_{\kappa_t,\lambda_t,R_t}\left\{\frac{\left(C_tP_t^\phi\right)^{1-\varepsilon}}{1-\varepsilon}+\mathbb{E}_t\left[W\left(S_{t+dt},P_{t+dt}\right)\right]\right\},\ t\geq T. \tag{7}$$

After maximizing the right hand side of equation (7), we can rewrite the Hamilton-Jacobi-Bellman equation by defining the following *pollution-adjusted* version of the variables:

$$c_t\ :\ =C_tP_t^\phi \tag{8}$$
$$s_t\ :\ =S_tP_t^\phi \tag{9}$$
$$r_t\ :\ =R_tP_t^\phi. \tag{10}$$

Then our problem is simplified to one of solving the following second order differential equation in *one* variable:

$$\rho\omega\left(s_t\right)=\mathbf{a}_1\left[\omega'\left(s_t\right)\right]^{\frac{\varepsilon-1}{\varepsilon}}+\mathbf{a}_2\omega'\left(s_t\right)s_t+\mathbf{a}_3\omega''\left(s_t\right)s_t^2,$$

where:

$$\begin{aligned}
\mathbf{a}_1 &= \frac{\varepsilon}{1-\varepsilon}\left[\frac{(1-\eta)\Theta_A}{A_2(1-\alpha)\overline{K_2}}\right]^{\frac{1-\varepsilon}{\varepsilon}}\\
\mathbf{a}_2 &= A_2(1-\alpha)\overline{K_2}+\frac{1}{2}\phi(\phi-1)\sigma_P^2\\
\mathbf{a}_3 &= \frac{1}{2}\left(\sigma_S^2\overline{K_2}+\phi^2\sigma_P^2\right),\\
\Theta_A &= A_1\left(\frac{1-\alpha}{\alpha}\frac{\eta}{1-\eta}\right)^\eta\overline{K_1},
\end{aligned}$$

and:

$$W(S_t, P_t) \equiv \omega\left(s_t\right).$$

This formulation leads us to the following Proposition.

Proposition 1 *If the clean energy is used actively in the consumption sector, the value of the pollution-adjusted solar panels is:*

$$\omega(s_t) = \mathbf{A}\frac{1}{1-\varepsilon}s_t^{1-\varepsilon}, \tag{11}$$

where:

$$\mathbf{A} = \left[\frac{1}{\mathbf{a}_1}\left(\frac{\rho}{1-\varepsilon} - \mathbf{a}_2 + \varepsilon\mathbf{a}_3\right)\right]^{-\varepsilon}.$$

The optimal consumption, and the optimal amount of the dirty input used in the consumption sector are:

$$c_t^* = \Theta_A \lambda^* s_t \tag{12}$$

$$\left(\kappa_t r_t\right)^* = \frac{1-\alpha}{\alpha}\frac{\eta}{1-\eta}\lambda^* s_t, \tag{13}$$

where:

$$\lambda^* = \lambda_t^* = \left[\frac{(1-\eta)\Theta^{1-\varepsilon}}{A_2(1-\alpha)\mathbf{A}}\right]^{\frac{1}{\varepsilon}}$$

is a constant.

Proof See A. ∎

Therefore, we obtain that the repartition of the stock of solar panel between the consumption sector and the backstop manufacturing sector is constant over time. Notice that it is not necessarily the case for κ_t that governs the repartition of fossil fuel extraction between the two sectors. Moreover, (pollution-adjusted or not) consumption is a constant fraction of (pollution-adjusted or not) solar panels. The latter result follows from the fact that $\left(\kappa_t r_t\right)^*$, i.e. the fossil fuel input in the consumption good process, is a constant fraction of s_t.

In Proposition 1 we require that $\mathbf{A} > 0$, so we impose:

$$\begin{array}{ll} \frac{\rho}{1-\varepsilon} - \mathbf{a}_2 + \varepsilon\mathbf{a}_3 > 0, & \text{if } \varepsilon < 1 \\ \frac{\rho}{1-\varepsilon} - \mathbf{a}_2 + \varepsilon\mathbf{a}_3 < 0, & \text{if } \varepsilon > 1. \end{array} \tag{14}$$

The transversality condition requires the convergence of the value function, i.e.:

$$\lim_{t\to\infty} \mathbb{E}_0\left[\omega(s_t)\right] = 0.$$

This condition is satisfied if $\omega(s_t)$ does not grow too fast in expectation. This requires that:

$$\mathbb{E}\left[d\omega(s_t)\right] = \omega_s(s_t)\mathbb{E}\left(ds_t\right) + \frac{1}{2}\omega_{ss}(s_t)\mathbb{E}\left(ds_t\right)^2 < 0.$$

Hence:

$$(\phi+1)\left\{\frac{1-\alpha}{\alpha}\frac{\eta}{1-\eta}\left(-\frac{\alpha A_2}{\phi}\right)\frac{\lambda^*}{\kappa^*}+\frac{1}{2}\sigma_P^2\left[\phi-\varepsilon\left(\phi+1\right)\right]\right\}<0. \tag{15}$$

As $\phi+1<0$, guaranteeing condition (15) to be satisfied requires the term inside the curly brackets to be strictly positive. We can show that sufficient conditions are:

$$\phi<\frac{\varepsilon}{1-\varepsilon}$$

in the case of $\varepsilon>1$, and:

$$\frac{1-\alpha}{\alpha}\frac{\eta}{1-\eta}\left(-\frac{\alpha A_2}{\phi}\right)\lambda^*>\left|\frac{1}{2}\sigma_P^2\left[\phi-\varepsilon\left(\phi+1\right)\right]\right|$$

otherwise.

3.2 The no option to switch case

Having solved the program once the clean energy has been adopted in the consumption sector, one can easily deduce the solution of the central planner's problem in an economy in which this kind of energy is never available to this sector. We will consider the fictive case in which —even though there is no possibility of switching— there is still a second sector which produces solar panels according to equation (3). The hypothetical results —although not intuitively relevant— will be theoretically useful for what follows. We let $W_0(S_t,P_t)$ be the value function of the central planner of the economy with no clean energy used in the consumption sector, with:

$$W_0\left(S_t,P_t\right)=\sup_{(\kappa_\tau,R_\tau)\in\Omega}\mathbb{E}_0\left\{\int_t^\infty e^{-\rho(\tau-t)}\frac{\left(C_\tau P_\tau^{\phi}\right)^{1-\varepsilon}}{1-\varepsilon}d\tau\right\}$$

Following the above steps and definitions, we can show that the value function in this case can be written as:

$$\rho\omega_0\left(s_t\right)=\mathbf{b}_1\left[\omega_0'\left(s_t\right)\right]^{\frac{\varepsilon-1}{\varepsilon}}+\mathbf{b}_2\omega_0'\left(s_t\right)s_t+\mathbf{b}_3\omega_0''\left(s_t\right)s_t^2,$$

with:

$$\begin{aligned}
\mathbf{b}_1 &= \frac{\varepsilon}{1-\varepsilon}\left[\frac{B_1\overline{K_1}}{\beta B_2\overline{K_2}}\right]^{\frac{1-\varepsilon}{\varepsilon}}\\
\mathbf{b}_2 &\equiv \mathbf{a}_2\\
\mathbf{b}_3 &\equiv \mathbf{a}_3.
\end{aligned}$$

and:

$$W_0(S_t,P_t)\equiv\omega_0\left(s_t\right).$$

The solution is as in the following Proposition.

Proposition 2 *If the economy cannot use the clean energy in the consumption sector, the value of the pollution-adjusted panels is:*

$$\omega_0(s_t) = \mathbf{B}\frac{1}{1-\varepsilon}s_t^{1-\varepsilon}, \tag{16}$$

where:

$$\mathbf{B} = \left[\frac{1}{\mathbf{b}_1}\left(\frac{\rho}{1-\varepsilon} - \mathbf{b}_2 + \varepsilon\mathbf{b}_3\right)\right]^{-\varepsilon}.$$

The optimal consumption, and the optimal amount of the dirty input used in the consumption sector are:

$$\begin{aligned} c_t^* &= B_1\Theta_B s_t, \\ (\kappa r_t)^* &= \Theta_B s_t, \end{aligned}$$

where:

$$\Theta_B = \left[\frac{\left(B_1\overline{K_1}\right)^{1-\varepsilon}}{B_2\beta\mathbf{B}\overline{K_2}}\right]^{\frac{1}{\varepsilon}}.$$

Proof See B. ∎

In Proposition 2 we require that $\mathbf{B} > 0$. We then impose:

$$\begin{array}{l} \frac{\rho}{1-\varepsilon} - \mathbf{b}_2 + \varepsilon\mathbf{b}_3 > 0, \quad \text{if } \varepsilon < 1 \\ \frac{\rho}{1-\varepsilon} - \mathbf{b}_2 + \varepsilon\mathbf{b}_3 < 0, \quad \text{if } \varepsilon > 1. \end{array} \tag{17}$$

For any s_t the value function in equation (16) cannot be greater than the lifetime utility of the agent in an economy with the clean energy available in the consumption sector. Then, we must have:

$$\omega_0(s_t) \leq \omega(s_t). \tag{18}$$

This condition ensures that there exists an optimal switching date; that is, in the absence of costs of switching to the cleaner energy, the central planner would choose to immediately switch for any current level of pollution-adjusted capital accumulation. A necessary and sufficient condition for equation (18) to be satisfied is:

$$\begin{aligned} \mathbf{A} &\geq \mathbf{B}, \text{ if } \varepsilon < 1 \\ \mathbf{A} &\leq \mathbf{B}, \text{ if } \varepsilon > 1, \end{aligned}$$

which we impose.

4 The optimal switching time, the undiscounted case

The choice of an optimal consumption plan (through the choice of the optimal extraction rate, and the variables κ_t and λ_t) and of an optimal adoption time, is given by the maximization of the intertemporal utility function subject to the laws of solar panels and pollution accumulations. Once the new energy has been adopted, the central planner optimally follows the consumption plan described by equation (12). Therefore, the value function at the time of the switch is given by the following value-matching and smooth pasting conditions:

$$V(S_T, P_T) = W(S_T - IP_T^{-\phi}, P_T), \tag{19}$$

$$V_S(S_T, P_T) = W_S(S_T - IP_T^{-\phi}, P_T), \tag{20}$$

where $V(S_T, P_T)$ is the value function before the switch —evaluated at T, and subscripts denote partial derivatives. The central planner's problem becomes then:

$$V(S_0, P_0) = \sup_{(\kappa_\tau, \lambda_\tau, R_\tau)\in\Omega} \mathbb{E}_0 \left\{ \int_0^T e^{-\rho\tau} \frac{\left(C_\tau P_\tau^{\phi}\right)^{1-\varepsilon}}{1-\varepsilon} d\tau + e^{-\rho T} W(S_T - IP_T^{-\phi}, P_T) \right\},$$

subject to equations (1), (4), (5), and conditions (19), and (20). Notice that the value function before the switch depends on the current stock of solar panels even though these panels are not used before T. This is because solar panels have some value due to the existence of an opportunity to switch in the future.

By using the notation in equations (8) to (10), we can show that after maximization, the problem collapses to one of solving the following differential equation:

$$\rho\upsilon\left(s_t\right) = \mathbf{b}_1 \left[\upsilon'\left(s_t\right)\right]^{\frac{\varepsilon-1}{\varepsilon}} + \mathbf{b}_2 \upsilon'\left(s_t\right) s_t + \mathbf{b}_3 \upsilon''\left(s_t\right) s_t^2, \tag{21}$$

with:

$$V(S_t, P_t) \equiv \upsilon\left(s_t\right),$$

and the following boundary conditions:

$$\upsilon(s_T) = \omega(s_T - I), \tag{22}$$

$$\upsilon_s(s_T) = \omega_s(s_T - I), \tag{23}$$

which represent the pollution-adjusted version of the value matching and smooth pasting conditions. In the problem above, s_T is the level of the pollution-adjusted solar panels stock for which it is optimal to switch. This value implicitly determines the optimal switching time T. Additionally, given that the central planner can always choose not to switch to the technology using panels to produce consumption, another condition that must be satisfied is:

$$\omega_0(s_t) \leq \upsilon(s_t)\ \forall t. \tag{24}$$

The problem in equations (21) to (23) has an analytical solution only if the discount rate ρ is equal to zero. Let us assume that it is the case. Then, we can find an expression for the marginal value of the pollution-adjusted capital before the switch:

$$\upsilon_s(s_t) = \left(\frac{D_1}{s_t} + D_2 s_t^{D_3}\right)^{\varepsilon}, \tag{25}$$

where:

$$
\begin{aligned}
D_1 &= \mathbf{B}^{\frac{1}{\varepsilon}}, \\
D_3 &= -\frac{\mathbf{b}_2}{\varepsilon \mathbf{b}_3},
\end{aligned}
$$

and D_2 is a constant that must be determined using the smooth pasting condition, equation (23). We can show that this is:

$$
D_2 = \frac{1}{s_T^{D_3}} \left(\frac{\mathbf{A}^{\frac{1}{\varepsilon}}}{s_T - I} - \frac{D_1}{s_T} \right).
$$

We define:

$$
G(s_t, s_T) := D_2 s_t^{D_3}
$$

as the part of the value function due to the option to switch. Notice that in the absence of such an option, i.e. $G(s_t, s_T) \equiv 0$, the marginal value of the pollution-adjusted capital reduces to $\omega_{0s}(s_t)$. We deduce from the following Propositions that the value of $G(s_t, s_T)$, and hence the value function and the optimal switch time crucially depend on the value of ε.

Proposition 3 *If $\varepsilon < 1$, then $G(s_t, s_T) > 0$, and $\upsilon_s(s_t)$ is always defined. As a consequence the stock of the pollution-adjusted panels s_T can be found by solving:*

$$
\int_0^{s_T} \upsilon_s(s_t) dt = \omega(s_T). \tag{26}
$$

Proof See C. ▮

Equation (26) can be solved numerically. Numerical resolution is driven using the parameters in Table 1. Figure 1 shows the three value functions: before the switch, $\upsilon(s_t)$, after the switch, $\omega(s_t - I)$, and without the option to switch, $\omega_0(s_t)$. The threshold that triggers the switch is $s_T = 0.4825$.

Proposition 4 *If $\varepsilon > 1$ the option $G(s_t, s_T)$ collapses to zero. In this case, the value function before the switch can be found to be:*

$$
\upsilon(s_t) = \mathbf{B} \frac{1}{1-\varepsilon} s_t^{1-\varepsilon} + \mathbf{B}_0, \tag{27}
$$

for:

$$
\mathbf{B}_0 = \frac{1}{1-\varepsilon} \left[\mathbf{A} \left(s_T - I \right)^{1-\varepsilon} - \mathbf{B} s_T^{1-\varepsilon} \right].
$$

The optimal switching is then:

$$
s_T = \frac{I}{1 - \left(\frac{\mathbf{b}_1}{\mathbf{a}_1} \right)^{\frac{1}{\varepsilon}}} := s^*. \tag{28}
$$

The optimal consumption, and the optimal amount of the dirty input used in the consumption sector remain as in Proposition (2).

Table 1: Base Case Parameters, $\varepsilon < 1$

ϕ	-2.0
ε	0.7
σ_S	0.5
σ_P	0.05
A_1	1.0
A_2	0.1
α	0.5
η	0.5
B_1	1.0
B_2	0.1
β	0.7
I	0.1

Proof See C. ∎

As before, (pollution-adjusted or not) consumption is a constant fraction of (pollution-adjusted or not) solar panels. This is because the fossil fuel input in the consumption good process, $(\kappa_t r_t)^*$, is still a constant fraction of s_t.

As an example, we drive a numerical resolution by using the values in Table 2. Figure 2 shows the three value functions: before the switch, $\upsilon(s_t)$, after the switch, $\omega(s_t - I)$, and without the option to switch, $\omega_0(s_t)$. The threshold that triggers the switch is $s_T = 0.2514$.

Table 2: Base Case Parameters, $\varepsilon < 1$

ϕ	-2.0
ε	1.7
σ_S	0.2
σ_P	0.2
A_1	1.0
A_2	0.6
α	0.5
η	0.5
B_1	1.0
B_2	0.6
β	0.7
I	0.1

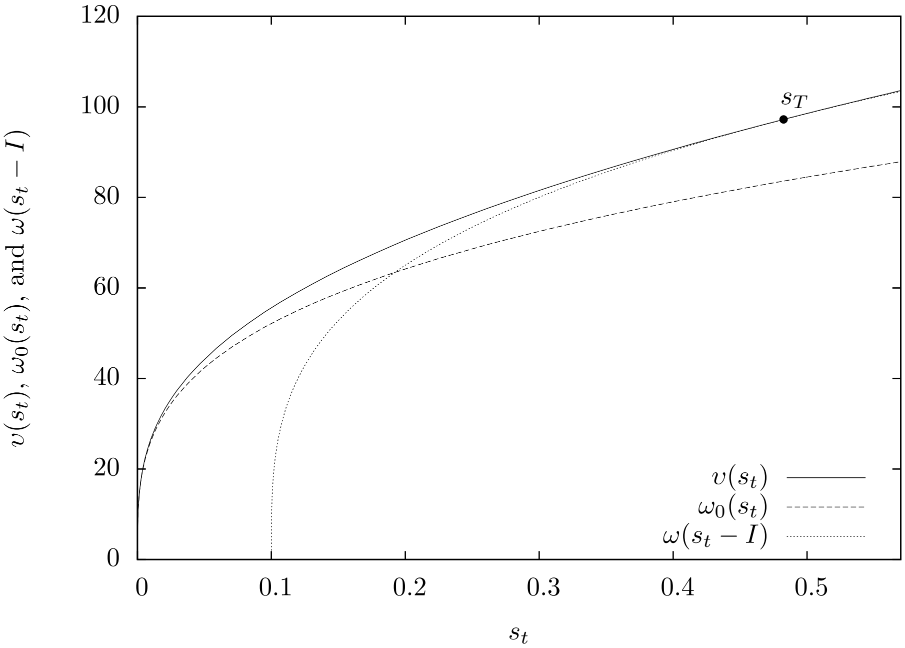

Figure 1: Optimal switching level of s_t, $\varepsilon < 1$.

5 Comparative Statics

In this section we begin our analysis by considering the simplifying assumptions of $A_2 = B_2$, and $\alpha = \beta$. In this case, there is no technological improvement in the backstop production sector, nor a reinforcement of the "green effect" after the switch. By imposing these assumptions, the only advantage of the backstop production sector when switching is that solar panels are to be shared with the consumption sector. Next, we compare our results with those of the more general case of $A_2 > B_2$, and $\alpha < \beta$.

5.1 No technological improvement in the solar panel process

As a base case, we use the same parameters as in Table 1, and Table 2, in which it is already assumed that $A_2 = B_2$, and we set $\alpha = \beta = 0.85$. Then we get that $s_T = 1.4378$ when $\varepsilon < 1$ (Figure 3), and $s_T = 1.4451$ when $\varepsilon > 1$ (Figure 4). We next consider the effect of each of the parameters on the optimal switching value of the pollution-adjusted panels.

We consider first the effect of ϕ on the level of the pollution-adjusted solar panels stock triggering their adoption by the consumption sector. As we can de-

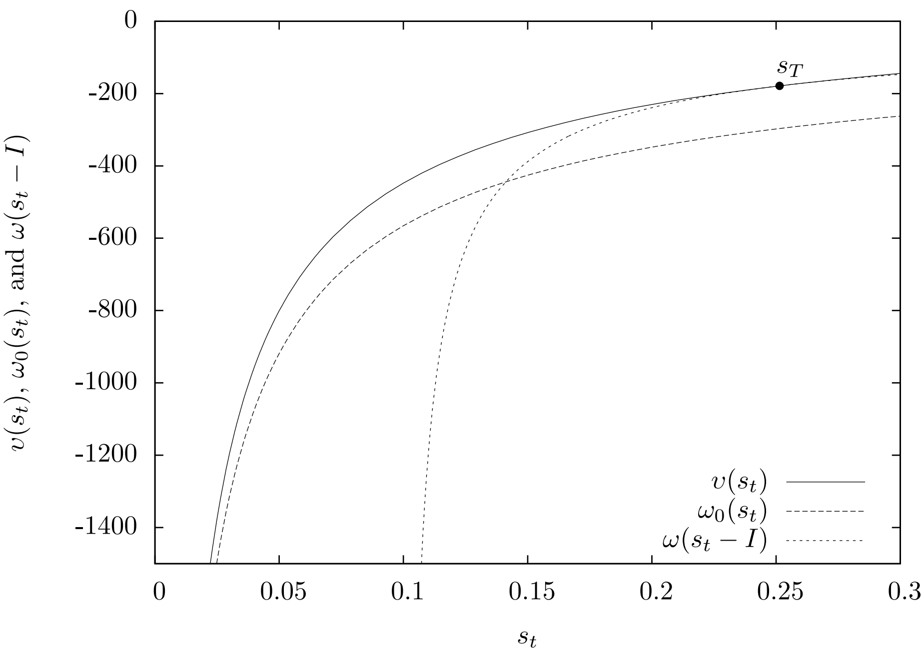

Figure 2: Optimal switching level of s_t, $\varepsilon > 1$.

duce from Table 3, s_T is a decreasing (and convex) function of ϕ: the larger (less negative) ϕ, the technology using solar panels is adopted by the consumption sector for a smaller pollution-adjusted panels stock. This is a priori counter-intuitive: more negative values of ϕ means that the central planner cares more about pollution affecting the utility of households and adoption should occur for a smaller solar panel stock. However, it has to be kept in mind that, from the definition of s_T, more negative values of ϕ, and then smaller values of s_T, may perfectly correspond to higher levels of non-pollution-adjusted panels, S_T, since pollution may be smaller. Moreover, there exists another effect of ϕ through the effective risk aversion $\Gamma = 1-(1-\varepsilon)(1+\phi)$ (see also equation (11) in which the two effects of this parameter clearly appear through the constants $\mathbf{a}_2$ and $\mathbf{a}_3$). The larger ϕ, the smaller the risk aversion. This may explain that a smaller accumulated stock of pollution-adjusted solar panels is required to switch. In the case $\varepsilon > 1$, the switch is triggered by the equality between the marginal values before and after the switch that depend in the same way from ϕ; therefore this parameter does not affect s_T. Again, it does not mean that it does not affect S_T.

We now consider the effect of ε. As we know, this parameter is the inverse of the effective intertemporal elasticity of substitution. On the one hand, larger values of ε reduce the effective intertemporal elasticity of substitution. On the

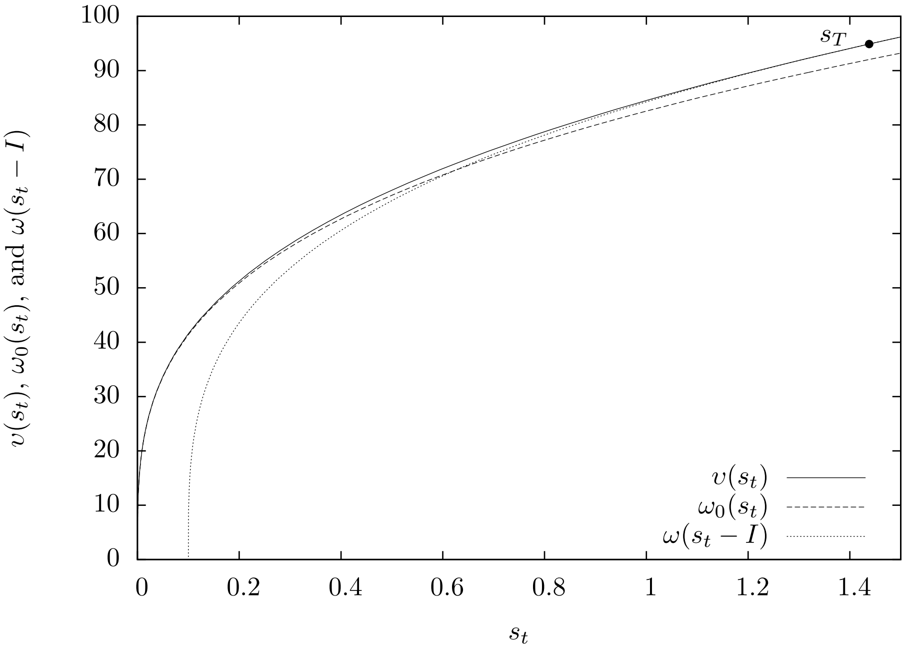

Figure 3: Optimal switching level of s_t without technological change, $\varepsilon < 1$.

other hand, larger values of ε increase the effective coefficient of risk aversion $\Gamma = 1 - (1 - \varepsilon)(1 + \phi)$. We deduce from Table 3 that the optimal level of the pollution-adjusted solar panels is a decreasing function of ε: less taste for intertemporal substitution erodes the option value to wait and therefore induces an adoption for a smaller stock of pollution adjusted panels.

Uncertainty plays an interesting role in the decision to switch, particularly in the case of $\varepsilon < 1$. The level of the pollution-adjusted solar panels at which it is optimal to switch is a decreasing function of the uncertainty on the accumulation of solar panels. This result on "economic" uncertainty fully reverses that of the partial equilibrium literature (e.g. Pindyck [2000]), in which higher levels of uncertainty increase the incentives to wait rather than adopt the policy now. What happens here is that this uncertainty reduces the value before the switch more than the value after it, therefore reducing the level of pollution adjusted panels stock that triggers the switch. On the contrary, uncertainty on pollution accumulation is consistent with the usual partial equilibrium effect of uncertainty. The effect of both σ_S and σ_P disappears in the case of $\varepsilon > 1$ because uncertainties affect in the same way the marginal values before and after the switch and therefore do not affect s_T (it does not mean that it does not affect S_T) as can be seen in equations (11) and (27). This is in standard result in general equilibrium (see, for instance, Pommeret and Schubert [2009]).

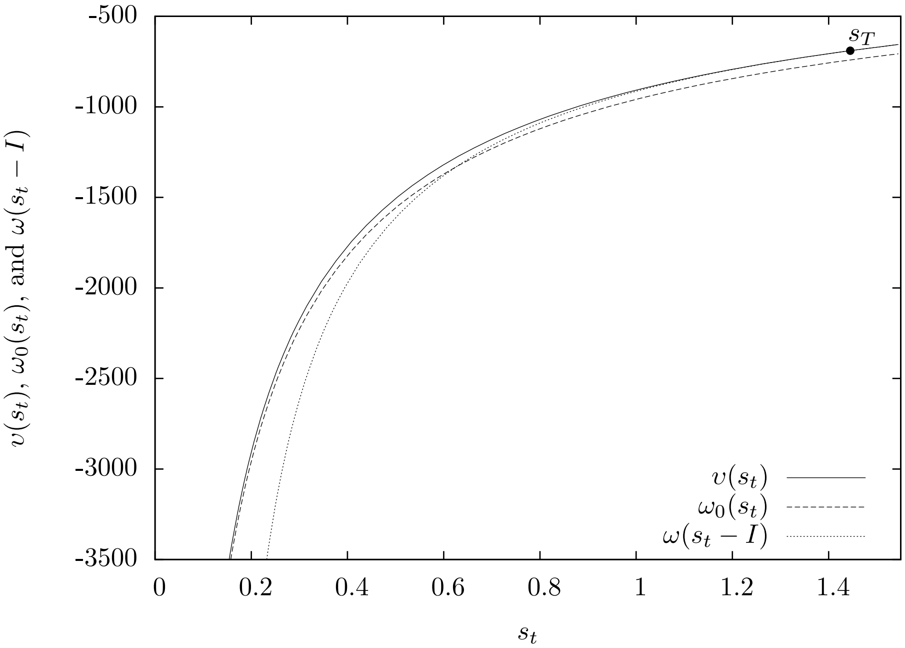

Figure 4: Optimal switching level of s_t without technological change, $\varepsilon < 1$.

We also have that the level of the pollution-adjusted solar panels at which it is optimal to switch is a decreasing function of the technological parameter A_1, and an increasing function of the technological parameter B_1: the larger the technology gain due to the switch, the smaller the pollution adjusted panels stock that triggers adoption. Again this is due to an increase of the value after the switch compared to that before the switch. On the other hand, s_T is an increasing function of A_2: the more the level of technology in the solar panels production sector, the later the adoption. As $A_2 = B_2$, the larger the level of technology in this sector the less the incentives to switch. Such an effect necessarily arises from the effect of solar panels technology on the option value to switch. This last effect, however, disappears in the case of $\varepsilon > 1$ because value functions before and after the switch are affected in the same way.

Our simulations show that s_T is an increasing (and convex) function of η: as the participation of the polluting resource in the production of the consumption good after the switch increases the central planner will choose to adopt for a larger s_T; the larger this parameter, the less the incentive to switch. On the opposite, the larger α, the share of the polluting resource required to accumulate solar panels (before and after the switch), the most important it is to use less of the fossil fuel in the production of the consumption good and therefore the smaller the s_T that triggers the switch. Finally, the central planner will decide to adopt for a higher s_T if the irreversible investment cost is higher.

Table 3: Comparative statics without technological improvement

	$\varepsilon < 1$		$\varepsilon > 1$	
	$f'(\cdot)$	$f''(\cdot)$	$f'(\cdot)$	$f''(\cdot)$
ϕ	≤ 0	≥ 0	$= 0$	$= 0$
ε	≤ 0	≥ 0	≤ 0	≥ 0
σ_S	≤ 0	≥ 0	$= 0$	$= 0$
σ_P	≥ 0	≥ 0	$= 0$	$= 0$
A_1	≤ 0	≥ 0	≤ 0	≥ 0
A_2	≥ 0	≥ 0	$= 0$	$= 0$
α	≤ 0	≥ 0	≤ 0	≤ 0
η	≥ 0	≥ 0	≥ 0	≥ 0
B_1	≥ 0	≥ 0	≥ 0	≤ 0
I	≥ 0	$= 0$	≥ 0	$= 0$

5.2 Technological improvement in the solar panels sector

We now relax the assumptions of $\alpha = \beta$. and $A_2 = B_2$, and see how much the results change in the presence of technological improvement, i.e. $\alpha < \beta$ and $A_2 > B_2$, starting from the parameters of Table 1, and Table 2. Some effects are quite similar to those found in the previous section. For instance, the optimal level of the pollution-adjusted solar panels is still a decreasing (but now convex) function of ε, a decreasing function of A_1, and an increasing function of B_1. We also get that the central planner will decide to adopt for a higher s_T the higher the irreversible investment cost. However, most of the results are inverted. Let us consider each of them.

As before, we first consider the effect of ϕ on the level of the pollution-adjusted solar panels stock triggering their adoption by the consumption sector. As we can see in Table 4, s_T is an *increasing* function of ϕ: the larger (less negative) ϕ, the higher the value of the adjusted-solar panels in order the technology using solar panels to be adopted by the consumption sector. This result seems to be more intuitive than previously. In this particular case, i.e. if the switch allows using less polluting resource for both consumption and solar panels accumulation, it is the direct effect of ϕ on utility the one that matters the most: more negative values of ϕ mean that the central planer cares more about pollution affecting the utility of households and can increase the intertemporal utility thanks to the technology improvement.

We now consider the effect of uncertainty. The role played by uncertainty on the accumulation of solar panels still depends on the value of ε relative to unity, and they are now reversed for $\varepsilon < 1$. To explain these new results we can focus on the effect of the technological improvement after the switch. Whatever the effective intertemporal elasticity of substitution, the pollution-adjusted solar panels stock that triggers the switch is a decreasing function of the uncertainty on pollution accumulation. This comes from the fact that the central planner tries to mitigate the bad effect of an increasing pollution uncertainty by adopting the new technology sooner. Uncertainty on solar panels accumulations has a different effect. The role played by uncertainty on the accumulation of solar panels depends again on the value of ε relative to unity. In particular, for $\varepsilon < 1$, a larger σ_S now leads to a larger s_T and such a result is consistent with the existing literature on technology adoption under uncertainty in partial equilibrium. But the effect is reversed for $\varepsilon > 1$. What happens is that more uncertainty on solar panels accumulation unambiguously reduces the value after the switch, but may increase the value before the switch through the option part of the value. This should trigger adoption for a larger s_T; this is what occurs if the agent likes to substitute in time, but it is no longer the case for $\varepsilon > 1$ for which there is no option part in marginal value before the switch.

Moreover, the level of the pollution-adjusted solar panels at which it is optimal to switch is a decreasing function of A_2 and an increasing function of B_2 whatever ε: the more the gain in technology thanks to the switch the smaller the adoption threshold s_T. This results confirm that technological improvement in either sector is an important incentive (absent in the previous section) to switch. It is also clear that s_T is an increasing function of α, and a decreasing function of β. The more important is the polluting resource to produce solar panels after the switch, the later the adoption. The more important is the polluting resource to produce solar panels before the switch the sooner the adoption. Of course, the fact that β is larger than α and that $A_2 > B_2$ provide the central planner with an additional incentive to switch, as solar panels production process is more efficient after the switch and, in particular, it requires less of the polluting resource in their production process. In other words, this sector becomes "greener"".

Our simulations on η are as in Figure 5. Notice that s_T is an increasing (decreasing) function of η as long as $\eta \leq 0.5$ ($\eta > 0.5$). This is the result of the constant returns to scale in the production of the consumption good after the switch. If fossil fuels are relatively less important than solar panels to produce consumption, the central planner tends to wait for a larger value of s_T in order to switch to the new technology. This is because the solar panels sector needs to be sufficiently developed to not loosing consumption once the new technology is adopted. But, if fossil fuels are relatively more important than solar panels to produce consumption, switching to the new technology is easier (smoother), and then the incentives to wait for the central planner start vanishing.

The effect of all of the parameters on s_T is summarized in Table 4.

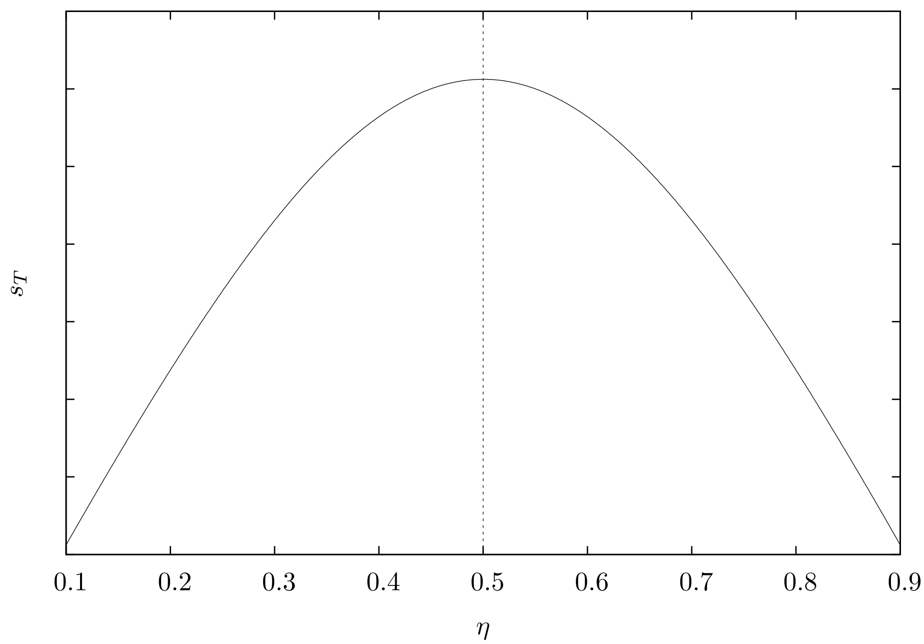

Figure 5: The effect of η on the optimal switching level of s_t.

6 The optimal switching time, the discounted case

When $\rho \neq 0$ there is no analytical solution to the problem described by equations (21) to (24). Hence, numerical methods are necessary to calculate the value function in this case. As suggested by Judd [1992, 1998], we can use the approximation properties of Chebyshev polynomials to compute stable non-diverging solution of the Hamilton–Jacoby–Bellman equation (21). In this section, we follow Mosiño (2012) in transforming the value function and the given conditions into matrix equations with unknown Chebyshev coefficients. By using this representation, our original problem of solving a partial differential equation reduces to a problem of solving a simple system of algebraic equations. Interested readers can also follow Dangl and Wirl [2004], which propose an algorithm using Newton's method.

6.1 A numerical approximation of the value function

In the computations that follow we suppress time subscripts as they are not necessary for clarity. Suppose that $\widehat{v}(s) \approx v(s)$ has a Chebyshev series solution of the form:

Table 4: Comparative statics with technological improvement

	$\varepsilon < 1$		$\varepsilon > 1$	
	$f'(\cdot)$	$f''(\cdot)$	$f'(\cdot)$	$f''(\cdot)$
ϕ	≥ 0	$= 0$	≥ 0	≤ 0
ε	≤ 0	≤ 0	≤ 0	≤ 0
σ_S	≥ 0	≥ 0	≤ 0	≤ 0
σ_P	≤ 0	≤ 0	≤ 0	≤ 0
A_1	≤ 0	≥ 0	≤ 0	≥ 0
A_2	≤ 0	≥ 0	≤ 0	≥ 0
α	≥ 0	≥ 0	≥ 0	≥ 0
η	≤ 0	≤ 0	≤ 0	≤ 0
B_1	≥ 0	≥ 0	≥ 0	≥ 0
B_2	≥ 0	≥ 0	≥ 0	≥ 0
β	≤ 0	≥ 0	≤ 0	≥ 0
I	≥ 0	$= 0$	≥ 0	$= 0$

$$\widehat{v}\,(s) = \frac{1}{2}\gamma_0 T_0(s) + \sum_{i=1}^{N} \gamma_i T_i(s), \tag{29}$$

for $\underline{s} \leq s \leq s_T$. In equation (29), $\underline{s}$ is an artificial lower bound for s, and $T_i(s)$, $i = 0, 1, \ldots, N$, is the general i-th Chebyshev polynomial of the first kind. This can be obtained from the recurrence relation:

$$\begin{aligned} T_0(h(s)) &= 1, \\ T_1(h(s)) &= s, \text{ and} \\ T_{n+1}(h(s)) &= 2hT_n(h(s)) - T_{n-1}(h(s)), \end{aligned}$$

where:

$$h(s) = \frac{2s - (\underline{s} + s_T)}{s_T - \underline{s}}. \tag{30}$$

In equation (29), γ_i, $i = 0, 1, \ldots, N$, are the Chebyshev coefficients to be determined, and $N + 1$ is the degree of approximation. We also assume that:

$$\widehat{v}^{(n)}\,(s) = \frac{1}{2}\gamma_0^{(n)} T_0(s) + \sum_{i=1}^{N} \gamma_i^{(n)} T_i(s), \tag{31}$$

where $\widehat{v}^{(n)}\,(s)$ is the n-th derivative of $\widehat{v}\,(s)$ with respect to s, and $\gamma_i^{(n)}$ are also Chebyshev coefficients. Obviously $\gamma_i^{(0)} = \gamma_i$, and $\widehat{v}^{(0)}\,(s) = \widehat{v}\,(s)$.

Equations (29) and (31) can also be expressed in matrix form:

$$\widehat{v}\,(s) = \mathbf{T}(s)\mathbf{\Gamma}, \tag{32}$$

$$\widehat{v}^{(n)}\,(s) = 2^n \mathbf{T}(s)(\mathbf{M}^g)^n \mathbf{\Gamma}, \tag{33}$$

where:

$$\begin{aligned}\mathbf{T}(s) &= [T_0(s)\ T_1(s)\ \cdots\ T_N(s)]\,,\\ \mathbf{\Gamma} &= \left[\frac{1}{2}\gamma_0\ \gamma_1\ \cdots\ \gamma_N\right]',\end{aligned}$$

and $\mathbf{M}^g$ is as defined in Mosiño (2012).

To obtain a Chebyshev solution of equation (21) in the form of (32), we first linearise the non-linear equation (21):

$$\rho\widehat{v}_{k+1}(s_i) = \left[\mathbf{b}_1\left(\widehat{v}_k^{(1)}(s_i)\right)^{-\frac{1}{\varepsilon}} + \mathbf{b}_2 s_i\right]\widehat{v}_{k+1}^{(1)}(s_i) + \mathbf{b}_3\widehat{v}_{k+1}^{(2)}(s_i)\,s_i^2, \qquad (34)$$

where $k = 0, 1, 2, \ldots$ refers to the k-th iteration on equation (34). Also:

$$s_i = \frac{\widehat{s_T} - \underline{s}}{2}(h_i + 1) + \underline{s},$$

for $\widehat{s_T}$ being an initial guess of s_T, and h_i being the i-th collocation point defined as:

$$h_i = \cos\left(\frac{i\pi}{N}\right),$$

where $i = 0, 1, \ldots, N$, and π is the standard mathematical constant. We also write the "iterative" version of equation (22):

$$\widehat{v}_{k+1}(\widehat{s_T}) = \mathbf{T}(\widehat{s_T})\mathbf{\Gamma} = \omega(\widehat{s_T} - I). \qquad (35)$$

Notice that we are not taking the smooth pasting condition, equation (23), into account. This condition will be useful only at the end of the process.[1]

To start iterating, we take the following initial guess:

$$\widehat{v}_0(s) = \omega(s - I), \qquad (36)$$

which satisfies equation (35) as long as $s = \widehat{s_T}$. Inserting equation (36) into equation (34) we get:

$$\begin{aligned}\rho\widehat{v}_1(s_i) &= \left[\mathbf{b}_1\left(\widehat{v}_0^{(1)}(s_i)\right)^{-\frac{1}{\varepsilon}} + \mathbf{b}_2 s_i\right]\widehat{v}_1^{(1)}(s_i) + \mathbf{b}_3\widehat{v}_1^{(2)}(s_i)\,s_i^2, \qquad (37)\\ \widehat{v}_1(\widehat{s_T}) &= \omega(\widehat{s_T} - I). \qquad (38)\end{aligned}$$

The linear differential problem of equations (37) and (38) can be easily solved by using the Chebyshev matrix method in Mosiño (2012). The resulting approximation $\widehat{v}_1$ is then used to solve:

$$\begin{aligned}\rho\widehat{v}_2(s_i) &= \left[\mathbf{b}_1\left(\widehat{v}_2^{(1)}(s_i)\right)^{-\frac{1}{\varepsilon}} + \mathbf{b}_2 s_i\right]\widehat{v}_2^{(1)}(s_i) + \mathbf{b}_3\widehat{v}_2^{(2)}(s_i)\,s_i^2,\\ \widehat{v}_2(\widehat{s_T}) &= \omega(\widehat{s_T} - I),\end{aligned}$$

[1] Also notice that the transversality condition does not play an important role in the computations. We can say that this condition is satisfied as long as the system is stable.

and so on. In general, the result of the k-th iteration is used to activate the $(k+1)$-th iteration. If the process is convergent, a fixed point will be reached after several iterations. The process is ended when the maximum absolute value of the difference between two consecutive estimates is less than a tolerance error ϵ, i.e.:

$$\widetilde{E}_{k+1} = \max_{\underline{s} \leq s \leq s_T} |\widehat{v}_{k+1}(\widehat{s_T}) - \widehat{v}_k(\widehat{s_T})| \leq \epsilon.$$

Finally, assume that $\widehat{v}_{k+1}$ has reached a fixed point. Hence:

$$\widehat{v}_k(\widehat{s_T}) = \widehat{v}(\widehat{s_T}).$$

The last step is to evaluate our resulting expression by using the smooth pasting condition:

$$\widehat{v}^{(1)}(\widehat{s_T}) = \omega_s(\widehat{s_T} - I). \tag{39}$$

If equation (39) is satisfied, we conclude that $\widehat{s_T} = s_T$ is the optimal threshold value. Otherwise, we have to guess another value for $\widehat{s_T}$ and start the whole process again.[2]

6.2 Results

Table 5: Optimal switching time - Discounted Case

ρ	0.005	0.01	0.015
$\varepsilon < 1$	0.6570	0.9336	1.3263
$\varepsilon > 1$	0.2530	0.2593	0.2656

In our computations we are using the base case parameters of Tables 1 and 2, and $N = 15$. Figures 6 and 7 illustrate the particular example of $\rho = 0.01$ in the more general case of technology improvement.

By running some simulations we can show that the qualitative results of Section 5 remain the same. Then, we focus on the comparative statics with respect to ρ, whose results are shown in Table 5. As we can see, the level of the pollution-adjusted solar panels at which it is optimal to switch is an increasing function of ρ: the higher the discount rate, the later the adoption. This result fully reverses the results of previous literature (e.g. Hugonnier et al. [2008], and Charlier et al. [2011]). Our intuition suggests that, as the social planner is becoming more concerned about the present, she prefers waiting the solar panels sector to be more developed before switching. This is because before any action, the consumption sector can take advantage of the higher productivity of the polluting resource.

[2] If $\widehat{s_T} = s_T$ is not satisfied, we can find the optimal threshold value by using a simple search algorithm.

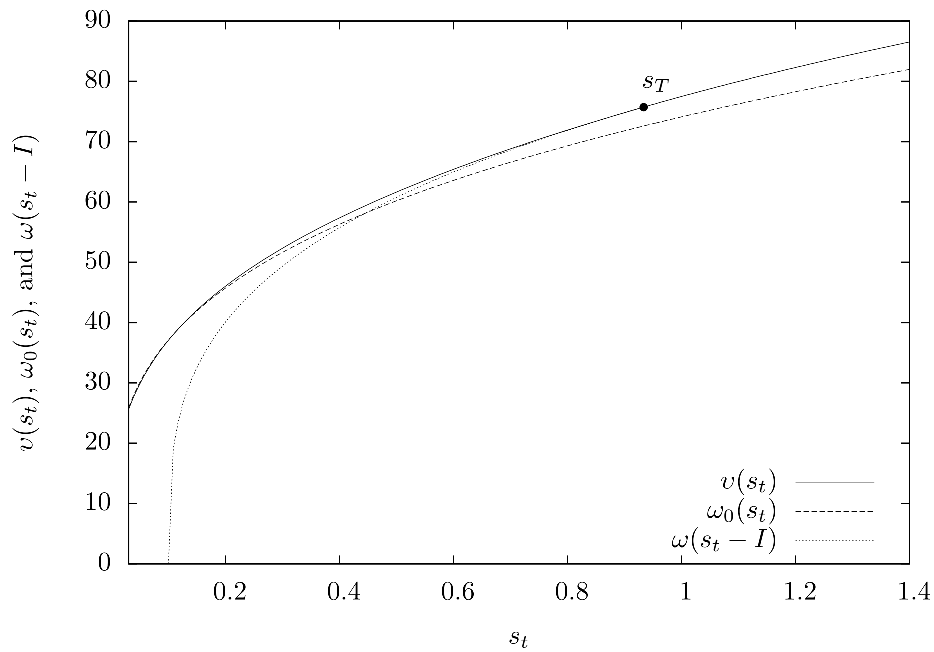

Figure 6: Optimal switching level of s_t, $\rho \neq 0$ $\varepsilon < 1$.

7 Conclusions

In this paper we consider a model in which two sectors interact to produce consumption. The first sector is dedicated to manufacturing a backstop resource —solar panels for instance. At any time, this sector requires both fossil fuels and the energy provided by the backstop already available. The second sector is the one that produces the consumption good. Initially it uses energy coming exclusively from fossil fuels. However, it has always the possibility of switching to a new technology in which energy comes from both types of resources. Using fossil fuels pollutes the economy. We assume that the accumulation of pollution, as well as the accumulation of the backstop, are stochastic. We also assume that, as this backstop resource is being accumulated, it gets worth paying a fixed and irreversible cost to use the existing stock of new solar panels and avoid —at least partially— the use of the polluting input. With this specification, the economy becomes cleaner after the switch —although not completely clean. Particularly, we account for the fact that even if the new technology is used, fossil fuels are still required in the industry.

We find that the threshold triggering adoption crucially depends on technological parameters. In particular, the incentives to switch to the cleaner technology depend on the relative importance of fossil fuels in the production

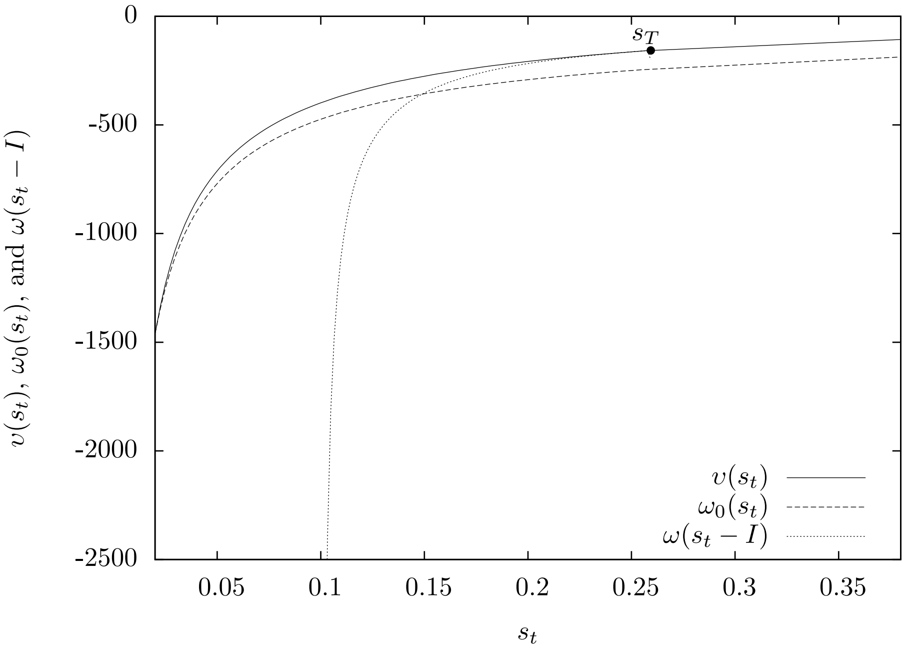

Figure 7: Optimal switching level of s_t, $\rho \neq 0$ $\varepsilon < 1$.

of consumption goods after the switch. Technological improvement in the solar panels sector is also important in order to switch to cleaner technologies. If the technological change implies that the backstop can be produced with relatively less of the fossil fuels, the adoption occurs sooner. These conclusions are important in terms of economic policy. They imply that policy is to be focused on (i) reducing the dependence of countries on fossil fuels — which is particularly important for oil-dependent developing countries, and on (ii) innovation.

We also find that the effect of uncertainty depends on the existence of technological improvement in the backstop production sector, and on the value of inverse of the effective intertemporal elasticity of substitution relative to unity. If the inverse of the effective intertemporal elasticity of substitution is less than unity, and in the absence of any technological improvement, the cleaner technology is adopted sooner as the uncertainty on the accumulation of solar panels increases. The cleaner technology is adopted later for higher levels of uncertainty on pollution. The former result fully reverses that of the partial equilibrium literature, while the latter is fully consistent with it. If the inverse of the effective intertemporal elasticity of substitution is larger than unity, however, both uncertainties affect the the marginal values before and after the switch in the same way, and then their effects disappear. This is a standard result in general equilibrium settings.

When there is technological improvement the effect of uncertainty is even more important. On the one hand, whatever the effective intertemporal elasticity of substitution, the pollution-adjusted solar panels stock that triggers the switch is a decreasing function of the uncertainty on pollution accumulation. This comes from the fact that the central planner tries to mitigate the bad effect of an increasing pollution uncertainty by adopting the new technology sooner. On the other hand, the uncertainty on solar panels accumulation unambiguously reduces the value after the switch, but may increase the value before the switch through the option part of the value.

An extension to this model seems to be particularly relevant. In this paper we assume that the increase in the pollution stock is equal to the resource extraction. However, exhaustibility of the resource is not taken into account explicitly. To deal with this we can either (i) include another process for the resource stock, or (ii) bound the pollution process to take into account the fact that the resource stock cannot be negative. This issue is left for future work.

A Proof of Proposition 1

The Hamilton-Jacobi-Bellman equation after the switch is as in equation (7):

$$\rho W\left(S_t, P_t\right) = \max_{\kappa_t, \lambda_t, R_t} \left\{ \frac{\left(C_t P_t^{\phi}\right)^{1-\varepsilon}}{1-\varepsilon} + \mathbb{E}_t\left[W\left(S_{t+dt}, P_{t+dt}\right)\right] \right\} \tag{A.1}$$

Following the usual techniques (see Dixit and Pindyck [1994] for instance), and using equations (2), (4), and (5) we can rewrite equation (A.1) as:

$$\begin{aligned} \rho W &= \max_{\kappa,\lambda,R} \left\{ \frac{\left(C P^{\phi}\right)^{1-\varepsilon}}{1-\varepsilon} + W_S A_2 \left[\alpha(1-\kappa)R + (1-\alpha)(1-\lambda)S\right] \overline{K_2} dt + W_P R \right\} \\ &+ \frac{1}{2}\left[W_{SS}\sigma_S^2 \overline{K_2}^2 S^2 + W_{PP}\sigma_P^2 P^2\right], \end{aligned} \tag{A.2}$$

where time subscripts and arguments have been suppressed for ease of exposition. Subscripts in equation (A.2) represent partial derivatives. The first order conditions are:

$$\left(C P^{\phi}\right)^{1-\varepsilon} \frac{\eta}{R} + W_S A_2 \left[\alpha(1-\kappa)\right] \overline{K_2} + W_P = 0, \tag{A.3}$$

$$\left(C P^{\phi}\right)^{1-\varepsilon} \frac{(1-\eta)}{\lambda} + W_S A_2 \left[(1-\alpha)S\right] \overline{K_2} = 0, \tag{A.4}$$

$$\left(C P^{\phi}\right)^{1-\varepsilon} \frac{\eta}{\kappa} - W_S A_2 \left[\alpha R\right] \overline{K_2} = 0. \tag{A.5}$$

From equations (A.4) and (A.5) we get that:

$$\kappa R = \frac{1-\alpha}{\alpha} \frac{\eta}{1-\eta} \lambda S. \tag{A.6}$$

Combining this result with equation (2) we obtain:

$$C = \Theta_A \lambda S, \tag{A.7}$$

where Θ_A is as defined in the main text. From this result and equation (A.4) we also get:

$$\lambda S = \left[\frac{(1-\eta)\left(\Theta_A P^{\phi}\right)^{1-\varepsilon}}{W_S A_2 (1-\alpha)\overline{K_2}} \right]^{\frac{1}{\varepsilon}}. \tag{A.8}$$

Using equations (A.7) and (A.8) — and after a few simplifications , we are able to rewrite equation (A.2) as:

$$\begin{aligned} \rho W &= \mathbf{a}_1 \left(W_s P^{-\phi}\right)^{\frac{\varepsilon-1}{\varepsilon}} + A_2(1-\alpha)\overline{K_2} W_S S \\ &+ \frac{1}{2}\left[W_{SS}\sigma_S^2 \overline{K_2}^2 S^2 + W_{PP}\sigma_P^2 P^2\right]. \end{aligned} \tag{A.9}$$

We now consider the following transformation:

$$W(S,P) \equiv \omega(s);\ s := SP^{\phi}.$$

Then:

$$\begin{aligned} W_s &= \omega_s P^{\phi}, \\ W_P &= \frac{\phi}{P}\omega_s s, \end{aligned}$$

and:

$$\begin{aligned} W_{ss} &= \omega_{ss} P^{2\phi}, \\ W_{PP} &= \phi^2 \omega_{ss}\frac{S^2}{P^2} + \phi(\phi-1)\omega_s \frac{S}{P^2}. \end{aligned}$$

This last expressions allow us to rewrite equation (A.9) as in the main text:

$$\rho\omega = \mathbf{a}_1 \omega_s^{\frac{\varepsilon-1}{\varepsilon}} + \mathbf{a}_2 \omega_s s_t + \mathbf{a}_3 \omega_{ss} s_t^2. \tag{A.10}$$

Solution to equation (A.10) can be easily find to be:

$$\omega = \mathbf{A}\frac{1}{1-\varepsilon} s^{1-\varepsilon}, \tag{A.11}$$

which is equation (11) in the main text.

Finally, notice that equation (A.8) can be rewritten as:

$$\lambda S P^{\phi} = \lambda s = \left[\frac{(1-\eta)\Theta_A^{1-\varepsilon}}{\omega_S A_2(1-\alpha)\overline{K_2}}\right]^{\frac{1}{\varepsilon}}.$$

By combining this equation with equation (A.11) we find that:

$$\lambda = \lambda^* = \left[\frac{(1-\eta)\Theta^{1-\varepsilon}}{A_2(1-\alpha)\mathbf{A}}\right]^{\frac{1}{\varepsilon}}$$

is a constant. Direct application of this result on equations (A.6) and (A.7), gives us equations (12) and (13) in the main text.

B Proof of Proposition 2

We now use equations (1), (3), and (5) to write the Bellman equation before the switch as:

$$\begin{aligned} \rho V &= \max_{\kappa,R}\left\{ \frac{\left(CP^{\phi}\right)^{1-\varepsilon}}{1-\varepsilon} + V_S B_2\left[\beta(1-\kappa)R + (1-\beta)S\right]\overline{K_2}dt + V_P R \right\} \\ &+\frac{1}{2}\left[V_{SS}\sigma_S^2\overline{K_2}^2 S^2 + V_{PP}\sigma_P^2 P^2\right], \end{aligned} \tag{B.1}$$

where again time subscripts and arguments have been suppressed for ease of exposition. Subscripts in equation (B.1) represent partial derivatives. The first order conditions are:

$$\left(CP^{\phi}\right)^{1-\varepsilon}\frac{1}{R} + V_S B_2\left[\beta(1-\kappa)\right]\overline{K_2} + V_P = 0, \tag{B.2}$$

$$\left(CP^{\phi}\right)^{1-\varepsilon}\frac{1}{\kappa} - V_S B_2\left[\beta R\right]\overline{K_2} = 0. \tag{B.3}$$

From equations (B.3) and (1) we get that:

$$\kappa R = \left[\frac{\left(B_1\overline{K_1}P^{\phi}\right)^{1-\varepsilon}}{V_S\beta B_2\overline{K_2}}\right]^{\frac{1}{\varepsilon}}, \tag{B.4}$$

and of course:

$$C = B_1\left[\frac{\left(B_1\overline{K_1}P^{\phi}\right)^{1-\varepsilon}}{V_S\beta B_2\overline{K_2}}\right]^{\frac{1}{\varepsilon}}\overline{K_1}. \tag{B.5}$$

These results allow us to rewrite equation (B.1) as:

$$\begin{aligned} \rho V &= \mathbf{b}_1\left(V_s P^{-\phi}\right)^{\frac{\varepsilon-1}{\varepsilon}} + B_2(1-\beta)\overline{K_2}V_S S \\ &+ \frac{1}{2}\left[V_{SS}\sigma_S^2\overline{K_2}^2 S^2 + V_{PP}\sigma_P^2 P^2\right]. \end{aligned} \tag{B.6}$$

We now consider the following transformation:

$$V(S,P) \equiv \upsilon(s);\ s := SP^{\phi}.$$

Then:

$$\begin{aligned} V_s &= \upsilon_s P^{\phi}, \\ V_P &= \frac{\phi}{P}\upsilon_s s, \end{aligned}$$

and:

$$\begin{aligned} V_{ss} &= \upsilon_{ss}P^{2\phi}, \\ V_{PP} &= \phi^2\upsilon_{ss}\frac{S^2}{P^2} + \phi(\phi-1)\upsilon_s\frac{S}{P^2}. \end{aligned}$$

This last expressions allow us to rewrite equation (B.6) as in the main text:

$$\rho \upsilon = \mathbf{b}_1 \upsilon_s^{\frac{\varepsilon-1}{\varepsilon}} + \mathbf{b}_2 \upsilon_s s_t + \mathbf{b}_3 \upsilon_{ss} s_t^2. \tag{B.7}$$

If an option to switch is not available, we redefine:

$$\upsilon(s) = \omega_0(s),$$

and equation (B.7) can be solved directly. Solution can be easily found to be:

$$\upsilon = \mathbf{B} \frac{1}{1-\varepsilon} s^{1-\varepsilon}, \tag{B.8}$$

which is equation (16) in the main text.

Finally, using equation (B.8) on equations (B.4) and (B.5) we get that:

$$\begin{aligned} c_t^* &= B_1 \Theta_B s_t, \\ (\kappa r_t)^* &= \Theta_B s_t, \end{aligned}$$

as in the main text.

C Proof of Propositions 3 and 4

The marginal value of the adjusted solar panels —equation (25) in the main text— is:

$$\upsilon_s(s_t) = \left(\frac{D_1}{s_t} + G(s_t, s_T) \right)^{\varepsilon}. \tag{C.1}$$

Equations (11) and (16), allow us to rewrite equation (C.1) as:

$$\upsilon_s(s_t) = \left([\omega_0'(s_t)]^{\frac{1}{\varepsilon}} + \left([\omega'(s_T - I)]^{\frac{1}{\varepsilon}} - [\omega_0'(s_T)]^{\frac{1}{\varepsilon}} \right) \left(\frac{s_t}{s_T} \right)^{D_3} \right)^{\varepsilon}, \tag{C.2}$$

where:

$$G(s_t, s_T) = \left([\omega'(s_T - I)]^{\frac{1}{\varepsilon}} - [\omega_0'(s_T)]^{\frac{1}{\varepsilon}} \right) \left(\frac{s_t}{s_T} \right)^{D_3},$$

is the part of the value function due to the option to switch.

C.1 Proof of Proposition 3

If $\varepsilon < 1$, it can be easily deduced that:

$$\omega_0'(s_t) \leq \omega'(s_t) < \omega'(s_t - I).$$

This implies that $G(s_t, s_T) > 0$, and hence $\upsilon_s(s_t)$ (> 0) is always defined. Then the stock of the pollution-adjusted solar panels can be found by integrating equation (C.1) and using the value matching condition, equation (22) in the main text.

C.2 Proof of Proposition 4

If $\varepsilon > 1$, the sign of $G(s_t, s_T)$ is ambiguous:

- Assume first that $G(s_t, s_T) < 0$. In this case, there is a value s_{inf}:

$$s_{\text{inf}} = \left(\frac{D_1}{D_2}\right)^{\frac{1}{D_3-1}},$$

  such that $\upsilon_s(s_t) < 0$, and hence the program is not longer defined — because a contradiction with the smooth pasting condition, equation (23) in the main text. Then, this case cannot be considered.

- Now, assume that $G(s_t, s_T) > 0$. In this case the marginal value function is positive as required. However, by integrating this we get that $\upsilon(s_t) > 0$, which is a contradiction with the value matching condition —equation (22) in the main text. Then this case cannot be considered either.

- $G(s_t, s_T) = 0$ ensures both that $\upsilon(s_t) > 0$ and that the value matching condition can be satisfied. Then, this constitutes the sole case we can consider. Equation (27) and s^* in the main text can be easily found by integrating $\upsilon_s(s_t)$ and using the value matching condition, equation (22).

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