The Reinvestment Rate Assumption Fallacy for IRR and NPV: A Pedagogical Note

Magni, Carlo Alberto and Martin, John D.

University of Modena and Reggio Emilia, Baylor University

December 2017

2017

The Reinvestment Rate Assumption Fallacy for IRR and NPV

A PEDAGOGICAL NOTE

CARLO ALBERTO MAGNI AND JOHN D. MARTIN

Table of Contents

The Reinvestment Rate Assumption Fallacy for IRR and NPV: A Pedagogical Note

1 Introduction

The notion that the internal rate of return (IRR) and net present value (NPV) have reinvestment rate assumptions built into them has long been settled in the academic finance literature.[1] Specifically, there are no reinvestment rate assumptions built into, or implicit to, the computation and use of either the IRR or NPV. Once an investment's cash flows are received they can be distributed to the firm's creditors or shareholders without any necessity to reinvest them. However, there persists the notion that IRR and NPV have *implicit* "reinvestment rate assumptions" embedded in them. For example, the following statement was taken from Investopedia:

. . . the traditional internal rate of return (IRR) assumes the cash flows from a project are reinvested at the IRR.[2]

Here are two more quotes taken from different websites, that focus on student users that make the same point:

The *IRR rule assumes that intermediate cash flows from the project get reinvested at the IRR. Implicit is the assumption that the firm has an infinite stream of projects yielding similar IRRs.*[3]

NPV and PI assume reinvestment at the discount rate. IRR assumes reinvestment at the internal rate of return.[4]

In this brief note, we first review the theoretical underpinnings of the rate of return assumption fallacy. Next, we offer two possible origins from the academic finance literature that may be responsible for the fallacy. Specifically, Ezra Solomon's discussion of the ranking of mutually exclusive investments and Jack Hirshleifer's discussion of the multiple IRR problem. We conclude that the reinvestment assumption is a sufficient condition, not an implicit assumption, for solving the problems of conflicting ranking and multiple IRRs.

2 The fallacy

To illustrate the fallacy of rate of return assumptions behind IRR and NPV we use a simple example. Here's how it works. Consider a security market in which borrowing and lending rates (r) are the same. Assume now that an investor has an opportunity to undertake a project with cash-flow stream $f = (-C_0, f_1, \dots, f_n)$, where $f_t > 0$ for each t and C_0 is the initial cash outlay required to finance the investment. The investor may then borrow an amount equal to the present value of the project's future cashflows, i.e., $V_0 = \sum_{t=1}^{n} f_t(1+r)^{-t}$. By taking these actions, the investor (i) realizes a cash inflow equal to V_0 from the loan proceeds, (ii) pays C_0 to finance the investment, and (iii) obligates the

[1] Dudley (1972, p. 908) put it bluntly, "There is no such assumption implicit in the technique". More recent papers include Rich and Rose (2014) and Walker et. al. (2010).

[2] https://www.investopedia.com/terms/m/mirr.asp.

[3] https://www.coursehero.com/file/p7plecl5/The-IRR-rule-assumes-that-intermediate-cash-flows-on-the-project-get-reinvested/.

[4] https://www.boundless.com/finance/textbooks/boundless-finance-textbook/capital-budgeting-11/introduction-to-capital-budgeting-91/reinvestment-assumptions-394-8295/.

project's future cash flows to repay the loan. The resulting net cash-flow vector then is $(V_0 - C_0, 0,0, \dots ,0) = (NPV, 0,0, \dots ,0)$. Of course, the project is worth undertaking if and only if $NPV = \sum_{t=0}^{n} f_t(1+r)^{-t} > 0$. Note that since all the project's future cash flows are converted to their present value equivalent via the loan, and the project's future cash flows are committed to repaying the loan, there are no future cash flows available to reinvest!

That NPV analysis does not assume reinvestment can be demonstrated in several other ways, abstracting from any consideration of borrowing. Most notably, consider the price V_0 of a portfolio traded in the market replicating the project's cash flow. Shareholders may undertake the project by investing C_0 or buy a replicating portfolio by investing V_0. The sequence of prospective cash flows is the same from both alternatives, so acceptance only depends on the difference between the project's cost and the price of the portfolio (i.e., the NPV), not on reinvestments of cash flows.[5]

As for IRR, assuming $f = (-C_0, f_1, \dots, f_n)$ is a conventional cash flow stream (outflows preceding inflows), the NPV function is monotonically decreasing so that $NPV > 0$ if and only if $IRR > r$. As the condition $NPV > 0$ has been determined with no reinvestment consideration, the condition $IRR > r$ is not tied to reinvestment consideration as well. The IRR is the (assumed constant) rate of return on the invested capital, period by period. The condition that $IRR > r$ only means that if, in any given period, an investor invests an amount of capital equal to the capital that remains invested in the project, then the rate of return earned with the former would be greater than the rate earned with the latter.

3 Potential sources of the reinvestment rate fallacy

How is it that many analysts continue to refer to the IRR as a required "reinvestment rate" for future cash flows when using the IRR as a project evaluation tool, and the cost of capital as the required "reinvestment rate" when using NPV? The answer may lie in some of the early writings regarding the difficulties encountered when

(i) ranking mutually exclusive investment opportunities (where IRR and NPV rankings are often in conflict), and

(ii) multiple IRRs arise in some nonconventional projects.

Early contributions to capital budgeting literature

In the 1950s the finance literature devoted to the analysis of mutually exclusive investment projects and the analysis of multiple IRRs both incorporated consideration for reinvestment rates. The discussion of reinvestment rates in this context, we believe, may well be the source of the confusion about reinvestment rates and project IRRs and NPVs.

For example, it seemed natural to consider reinvesting cash flows as one way to eliminate interim cash flows which were the source of the ranking conflicts between NPV and IRR or to overcome the difficulties encountered in comparing mutually exclusive investment proposals with different initial investments and/or different investment lives and/or different cash-flow patterns, and for projects whose cash flows have multiple IRRs. Even more so considering that, in 1950s, modern finance was not yet fully established, and the concept of *terminal* value seemed still common, as opposed to the idea of

[5] The rate r is the expected rate of return of the replicating portfolio.

a *present* value equal to the price of a portfolio replicating the project's cash flows.[6] Significant contributions to this discussion came from Solomon (1956) and Hirshleifer (1958) and both suggested that incorporating consideration of reinvestment rates for interim project cash flows might prove beneficial.

Ranking Alternative Investments

Solomon (1956) provided an important contribution to the analysis of mutually exclusive investment alternatives in which he raised the question as to implicit reinvestment rates for IRR and NPV.[7] In short, Solomon believed that both NPV and IRR have implicit reinvestment rates, the former at the cost of capital, the latter at the IRR itself. According to the author, the cause of the conflict lies in the different reinvestment assumptions of IRR and NPV. To solve the conflict, he suggested that the analyst make an "explicit" and common assumption regarding the rate at which funds can be reinvested up to the investment's terminal date to arrive at an appropriate ranking: "If a common assumption is adopted, both approaches will always rank projects identically" (p. 126).

The multiple IRR problem

Hirshleifer (1958) provided an early and important discussion of the problem of multiple IRRs that can arise in projects that have non-conventional investment cash flows (see Section III B. of his paper). While he suggested that the IRR assumes reinvestment at the IRR itself, he criticized this implicit assumption, deeming it unrealistic. Consistent with Solomon, Hirshleifer recommended making an explicit reinvestment rate estimate for the interim cash flows, which would turn the project into a course of action with a unique IRR (later called Modified Internal Rate of Return). This unique IRR is compatible with the NPV of the entire course of action, in the sense that NPV is positive if and only if the MIRR is greater than the cost of capital.

The modified IRR

Solomon (1956) and Hirshleifer (1958) may then be considered forerunners of the modified internal rate of return (MIRR) approach. If multiple IRRs occur only in projects with interim cash flows, then Solomon's and Hirshleifer's arguments may be used to solve the multiple-IRR problem: making an NPV-invariant modification of the project such that the modified project has no interim cash flows, and consequently the project's IRR will be unique. Among the infinitely many ways to adjust a cash-flow stream, one may consider the explicit reinvestment of interim cash flows up to the terminal. However, the assumption of reinvestment of interim cash flows at some rate is introduced only for making the multiple IRR problem (and the ranking problem) disappear, not because any reinvestment option is implicit in the IRR procedure, as seen above.

Note, however, that by modifying the cash-flow stream, the meaning of the resulting rate (MIRR) changes. Further, the MIRR does not measure the project's rate of return because it takes account of

[6] Which in turn, is equal to the price that the project would have if it were traded in the security market.

[7] Renshaw (1957) summarized Solomon's position on reinvestment rates as follows: "The contribution of the Solomon article was to point out that the apparent conflict between these two ranking procedures was due to differing implicit assumptions about reinvestment rates (the present-value approach assumes reinvestment of intermediate cash receipts at the discounting rate, while the internal rate of return approach assumes reinvestment at the internal rate) and to suggest that the conflict could be eliminated by making an explicit assumption about the expected return from reinvestment." p. 193 A key observation here is the statement regarding "implicit assumptions regarding reinvestment rates".

the reinvestments of the interim cash flows, which have nothing to do with the original investment opportunity. The MIRR is simply the internal rate of return of a course of action which includes the project being analyzed and other projects that are associated with the reinvestment of interim cash flows. Ross et. al., 2011, p. 250 describes the MIRR as follows: "it's a rate of return on a modified set of cash flows, not the project's actual cash flows". Therefore, the MIRR is the IRR of a portfolio of projects, not the original investment. The key point here is that a project's rate of return should not be affected by the choice of a reinvestment rate for its cash flows. Brealey, Myers and Allen (2011) put it very succinctly "The prospective return on another independent investment should never be allowed to influence the investment decision" (p. 141).

The MIRR also solves the problem of conflicting ranking (if the initial outlays are the same): The comparison of any two MIRRs will result in the same ranking as the comparison of the two associated NPVs. In other words, explicitly incorporating consideration for the reinvestment rate is sufficient to make IRR and NPV consistent and to ensure existence and uniqueness of IRR. As a result, scholars and practitioners have mistaken a *sufficient* condition for an *implicit* assumption.

Scale effects

Keane (1979) supplied an enlightening view that contributes to dispel the misconception of the reinvestment assumption. He attributed the problem of ranking projects as evidence of the "scale effect". That is, the conflict between NPV and IRR ranking is due to difference in scale of the projects under consideration, not to different reinvestment assumptions. By "scale" one should not merely refer to initial outlays, but to the total units of capital outstanding and capital length of the project (see Keane 1979, pp. 53-54). While NPV considers project scale as well as economic efficiency, IRR only measures economic efficiency, so it is not adequate for ranking projects.

It is worth noting that the conflict between the notion of rate of return and NPV disappears if explicit reinvestment assumptions are made on competing projects (if the initial outlays are the same),[8] because this changes the project scale in a way that makes the IRR ranking and the NPV ranking identical (as long as the initial outlays are equal).[9] In other words, reinvestment assumptions can be used to solve the problem of conflicting ranking.[10]

It is worth noting that, while not mentioned by Keane (1979), the "scale effect" is also the essential reason why reinvestment assumption solves the multiple-IRR problem: indeed, it can be shown that, if interim cash flows are reinvested, then the resulting modified project possesses outstanding capital which has the same sign in each period (i.e., a higher scale), thereby ensuring unique a IRR.[11]

[8] The NPV must be computed on the modified cash-flow streams.
[9] It may be proved that a higher IRR is associated with a higher scale. Therefore, the project with higher IRR also has a higher value created.
[10] The role of the scale effect and the relation of scale to rate of return is analyzed in Magni et al. 2017, where a reconciliation between NPV ranking and rate-of-return ranking is supplied with no need of assuming any reinvestment. See also Keefe and Rousch (2001) for a relatively recent paper on the inexistence of reinvestment assumptions.
[11] Multiple IRRs may not arise if capital outstanding has the same sign in each period.

What's the harm of assuming required reinvestment rates?

To illustrate the type of problems that can arise when assuming implicit reinvestment rates consider the following paraphrased conversation that occurred between the CFO and a financial analyst at a major public firm (relayed to the authors via a phone conversation):

"We have an investment opportunity that promises an extremely high IRR of 30%. However, we are worried about the decision to undertake the investment since the firm does not expect to have investment opportunities to reinvest those cash flows to earn 30%."

The implication being that without the opportunity to reinvest the cash thrown off by the 30% project at a similar rate of return, the project's IRR would not be realized. This type of reasoning can be quite misleading and costly to the firm. Specifically, it might lead the firm to avoid very high rate of return projects (paradoxically, the higher the IRR of the project, the more problematic is the project's acceptance).

In fact, the cash flows the investment opportunity promises can simply be distributed *as they are received* to the firm's creditors in the form of interest and/or principal payments or to stockholders via dividends or share repurchases, and the firm will realize the promised 30% return, regardless of the destination of those cash flows, because the IRR is the return generated by one dollar of capital that remains invested in the project. As such, the IRR does not have anything to say about the rate of return of the capital that is divested from the project. Instead IRR speaks to the return earned on the capital that is invested in the project.

Is this really a problem of semantics?

To this point we have been using adjectives like implicit and explicit when referring to reinvestment rates. Is it possible that some of the misunderstanding about reinvestment rates has something to do with language? For example, when an analyst attempts to move project cash flows either forward or backward in time in such a way as to not alter the project's NPV, then the rate of interest that preserves the investment's NPV is the rate used to evaluate the investment (i.e., the cost of capital). Similarly, if the analyst wishes to preserve the project's IRR, the rate of interest that should be used to move cash flows either forward or backward is the IRR of the project itself. Does this observation mean that these rates of interest are implied reinvestment rates? Asked somewhat differently, do interim cash flows have to be reinvested to realize the promised NPV or IRR? The answer is, of course, no. Remember that the investment cash flows do not need to be reinvested to guarantee the NPV or IRR of the project. It is easy to imagine that this relationship between interest rates and preserving NPV or IRR might be misconstrued to mean these rates are "required" or "mandatory" when they are not. Reinvestment is not implicit in the IRR and NPV; rather, it is a sufficient condition for solving ranking conflicts between NPV and IRR and the multiple-IRR problem.

4 Summary remarks

Despite demonstrated theory to the contrary, many continue to believe that NPV and IRR have implicit reinvestment rates. The reinvestment rate for NPV being the cost of capital for the investment and for IRR it is the IRR itself. The notion that project cash flows must be reinvested to assure the investor realizes the IRR as a rate of return (should cash flows materialize as expected) or that these cash flows must earn the cost of capital to earn the estimated NPV is simply not the case. Once received, the

investment cash flows can be distributed to the firm's creditors or shareholders, or simply held in short-term investments awaiting reinvestment at some future date at a yet unknown rate of return.

We offer a possible explanation for the persistence of the reinvestment rate assumption that derive out of some of the foundational work on the evaluation of investments. Specifically, the answer may lie in some of the early writings regarding the ranking of mutually exclusive investment opportunities where IRR and NPV rankings are often in conflict and the problem of multiple IRRs arising in some nonconventional projects. This assumption may solve the problems, which is an explanation of why scholars and practitioners turned a *sufficient condition* into an *implicit assumption*.

The problem with believing in the reinvestment rate assumption is that it can bias corporate investment decisions. Specifically, where using the IRR is presumed to require that the cash thrown off by the investment must be reinvested to earn the IRR, corporate decision makers may shy away from very high IRR projects, believing that a lack of equally high rate of return investments might jeopardize their realization of a high rate on the initial project.

References

R. Brealey, S. Myers, and F. Allen, 2011, Principles of Corporate Finance, 10th ed., (McGraw-Hill Irwin).

C. Dudley, 1972, A note on reinvestment assumptions in choosing between net present value and internal rate of return, The Journal of Finance, Vol. 27, No. 4 (September), pp. 907-15.

J. Hirshleifer, 1958, On the theory of optimal investment decisions, Journal of Political Economy, 66, 329-352.

S.M. Keane 1979, The Internal Rate of Return and the reinvestment fallacy, Abacus, 15(1), 44-55.

S.P. Keef and M.L. Roush, 2001, Discounted cash flow methods and the fallacious reinvestment assumption: a review of recent texts, Accounting Education: An International Journal, 10(1), 105-116.

C.A. Magni, P. Veronese, R. Graziani, 2017, Chisini means and and rational decision-making: Equivalence of investment criteria. Mathematics and Financial Economics. DOI 10.1007/s11579-017-0201-4

S. Rich and J. Rose, 2014, Re-examining an old question: Does the IRR Method Implicitly Assume a Reinvestment Rate? Journal of Financial Education, Spring/Summer, pp. 152-166.

E. Renshaw, 1957, A note on the arithmetic of capital budgeting decisions, The Journal of Business, Vol. XXX, No. 3 (July), pp. 193-201.

S. Ross, R. W Westerfield, and B. D. Jordan, 2011, Essentials of Corporate Finance, 7th ed., (McGraw-Hill Irwin).

E. Solomon, 1956, The Arithmetic of Capital Budgeting Decisions, Journal of Business, 29, pp. 124-29.

J. Walker, H. Check, JR., and K. Randall, 2010, Does the internal rate of return calculation require a reinvestment rate assumption? There is still no consensus, Journal of the Academy of Business Education, 11, Proceedings, November.