Okun's Law, Dead or Alive: A Fundamental Approach

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DLF Macro Economic Research

2014

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Marcel R. de la Fonteijne

Abstract

Okun's Law, which relates changes in output to changes in unemployment has been debated since Okun revealed it in 1962. By introducing a simple but plausible process of hiring and firing workers we will show a fundamental flaw in Okun's Law. It is interesting to see that this process can generate lookalike plots to empirical quarterly data. Translation of the model to an estimator is comparable with a VAR method with lag 2 and will yield higher accuracy. Okun's Law is not a suitable tool to estimate unemployment from GDP growth or unemployment change from GDP growth.

JEL Classification B31 · C02 · C13 · C53 · D84

Keywords: sustainability, Okun, unemployment estimator, GDP growth, real business cycle

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The author declares no potential conflicts of interest with respect to the research, authorship, and/or publication of this article. The author received no financial support for the research, authorship, and/or publication of this article.
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1. Introduction

Okun's Law is an empirically-observed relation between GDP and unemployment. Arthur Okun (1962) first noticed and described this relation and Okun's Law is since then a common tool for policy makers. Okun's Law always has been debated, but after the great recession a serious reconsideration was triggered because of great discrepancies between predicted unemployment change when using Okun's Law and measured data. The question rose whether or not this empirical rule of thumb was still valid under the new circumstances introduced by the financial crisis of 2007. A lot of papers are written with, to my knowledge, all an empirical focus on the subject, but the opinions diverse and it is hard to judge the pro's and con's for all the given arguments.

This paper will try to shed some light on this topic from a fundamental and theoretical point of view. First we will examine the variables which are effecting the relation between GDP and unemployment, next we introduce a theoretical mechanism that describes this relation and show that this mechanism can generate some remarkable similarities with historical data. Moreover we prove the fact that scattered data around Okun's Law has a mainly fundamental background and that it is not due to e.g. statistical fluctuation. Finally we will introduce an new estimator as an alternative and make a suggestion for an adaptive estimator.

Let's start with Okun's Law itself.

From the quarterly or yearly collected data points the growth rate of GDP g and the changes in unemployment Δu are calculated. Regressing the growth rate and changes in unemployment will result in the difference or growth rate form of Okun's Law, which relates changes in output to changes in unemployment:

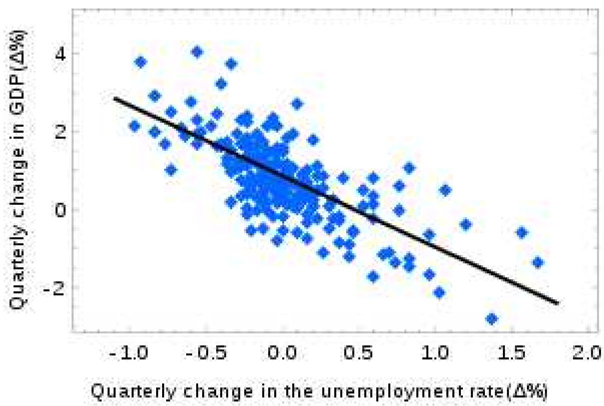
$$g = \frac{\Delta Y}{Y} = k - c\Delta u \qquad (1)$$

where

g is the growth rate of GDP Y

u is the unemployment rate.

k is the mean growth rate



Graph of US quarterly data (not annualized) from 1947 through 2002 estimates a form of the difference version of Okun's Law: g = .856 - 1.827Δu.
R^2 = .504
Differences from other results are partly due to the use of quarterly data.
Wikipedia (2013)

Fig. 1 Okun's Law describes the best fit through the data points. In this case a 1 % change in unemployment will result in a -1.8 % change in GDP.

From a theoretical point of view you would expect c to be approximately equal to the workforce share (1-α) in GDP Y. This is easy to prove for, for example, a Cobb-Douglas production function with α as the power to capital K. Such a relation can of course be found for every type of production function. From this relation you can calculate ceteris paribus g from Δu or Δu from g and it is a first order approximation.

In case of empirical data points and using Okun's Law to estimate Δu from g will not provide a satisfactory solution, because the fit is not that good. We will show that this is not simply caused by errors in measurements or other random fluctuations, but is a fundamental effect caused by the behavior of the economic process itself.

To understand where this might come from we go into more detail.

There are a few reasons why the growth rate g will not follow theory:

- the hoarding of workers in the downturn of the economy
- the search for new workforce in the upswing of the economy
- the creation of product stock
- the change in workforce participation rate
- the population growth rate
- the growth rate of hours worked
- the growth rate of Total Factor Productivity (TFP)
- capital change, delays in capital change

Hoarding of workforce, in literature also referred to as 'sticky workforce', is the effect of not firing people straight away on first notice of a lower demand. Firms will probably stay at the same production level for a while and create a larger product stock. After a while they start to fire a fraction of the workforce not needed still in the hope the time will change for the better and after each period they re-evaluate their position and take that decision. Meanwhile, the fraction retained but not needed is supposed not to work and could be considered as an temporal overall lowering of productivity. The fraction of fired/retained workforce is also subject to other influences such as union regulations, contracts, etc.

Searching for people in the upswing of the economy has the effect that when demand is increasing you may not find new workforce immediately and it will take some time to find the right workers, considering expertise and experience needed. Once you find those new workers you need additional capital, training on the job, etc., so it will take a while before the new recruits work at full productivity. On the other hand, one could increase working hours to compensate on the short run. We will use the term 'flexy workforce' when we consider the mechanism of hiring new workforce.

Several studies investigated the asymmetric behavior of this phenomena (Harris, 2000). We implemented this sticky and flexy workforce in a Real Business Cycle (RBC) and growth model we developed, and we will use it throughout this paper.

The fraction f_{sticky} indicates the fraction of the workforce no longer needed which remains in the workforce, and (1-f_{sticky}) will become unemployed. The fraction f_{flexy} indicates the fraction of the unemployed in-demand but which remains unemployed and (1-f_{flexy}) will become employed.

From here we start our analyses by showing the influence of fluctuation in demand.

2. Analyses of Okun's Law.

As an example we do a simulation with f_{sticky}=.9/quarter and f_{flexy}=.9/quarter and we let production follow demand which is a sine wave superimposed on a steady growing demand. The amplitude of the sine wave is .03 of the steady growing demand and the frequency is .25/year. Figure 2 shows the time function of the growth of GDP and the change in unemployment rate. Plotting growth of GDP vs changes in unemployment results in the elliptical graph of fig. 3, which would have been a better function to describe this phenomena than Okun's Law. However we don't deal with a single frequency with a fixed amplitude.

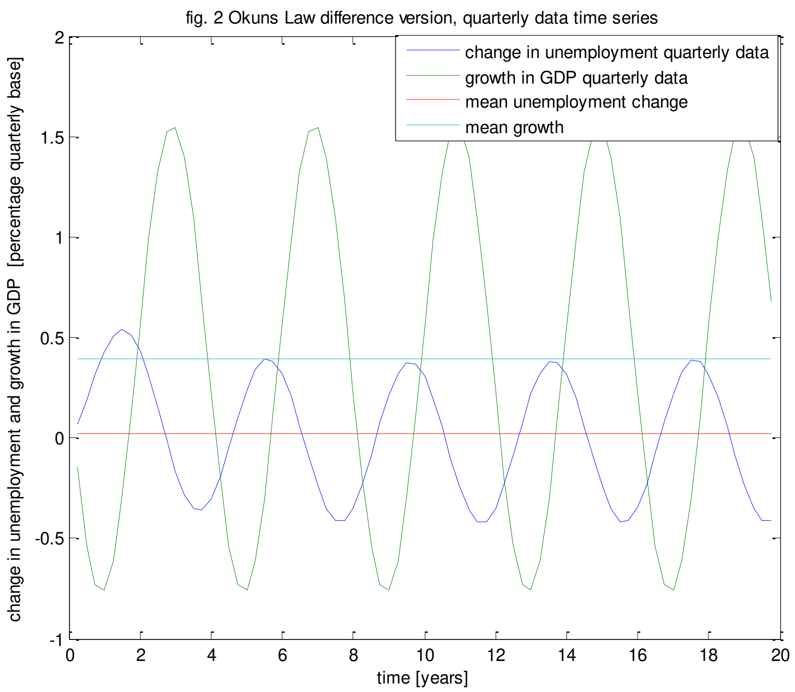


Fig. 2 Plot of the quarterly data g and Δu as a function of time (f_{sticky}=.9/quarter and f_{flexy}=.9/quarter).

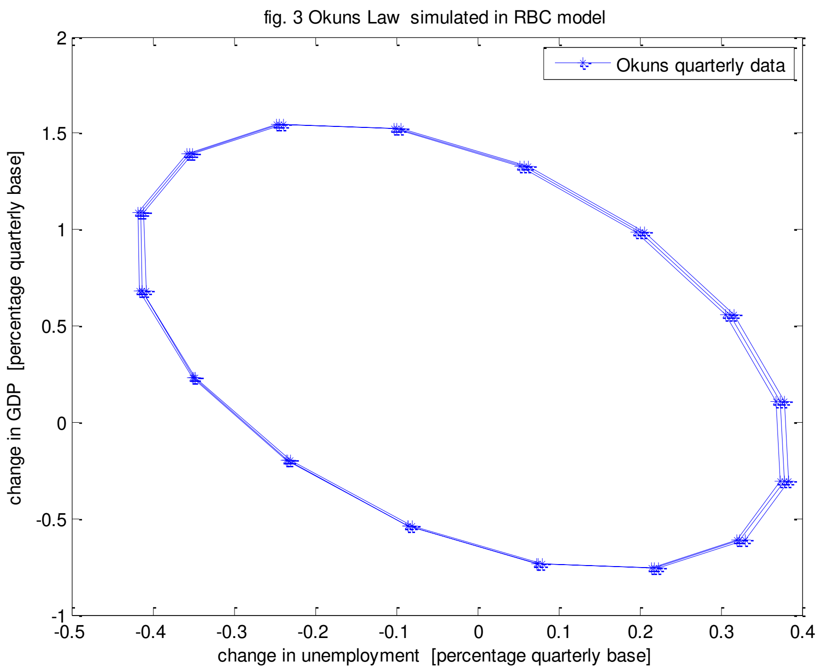


Fig 3 Plot of the quarterly data g vs Δu, stabilized after the first few years (f_{sticky}=.9/quarter and f_{flexy}=.9/quarter).

From these generated data points we can calculate Okun's factor c as the ratio of the range of growth g and the range of change in unemployment, c = 2.8 and the mean growth is calculated to be k = 1.5 % /year. The data points in fig. 3 are connected by a line to show how the points are related in time.

If $g_p = \frac{\Delta p}{p}$ is the relative growth of Total Factor Productivity p (Donselaar, 2011) the average growth is approximately $\bar{g} = \frac{g_p}{1-\alpha}$ under the condition that capital K is increasing under optimal conditions, see for more information De la Fonteijne (2012), then k in equation (1) is equal to the mean productivity growth $\overline{g}$ and corresponds closely to the data used.

In case changes in time are so fast that capital cannot built up then mean growth will be equal to TFP $\bar{g} = g_p$ and there are, of course, cases in between.

We know already that in case f_{sticky}= 0 and f_{flexy}= 0 the elliptical graph in fig. 3 would reduce to a straight line with c = $1 - \alpha$ = .7 where $\alpha = .3$

The mechanism of workers put to work or getting unemployment seems reasonable and is asymmetrical in time. So for fundamental reasons in general Okun's Law cannot be used as an unbiased estimator for predicting changes in unemployment from growth of GDP.

You could argue the graph presented in fig. 3 does not resemble the empirical data, but this graph is only presented to show the fundamental effect of one frequency.

To make it more realistic we will leave out the superimposed sine wave and instead introduce some random walk noise in addition to demand and again we take demand equal to GDP. This means that apart from the fixed growth, future growth of GDP is independent from the past and is white noise distributed. In fig. 4 you can find GDP as a function of time, the stars are indicating recessions (minimum 2 quarters of diminishing GDP).

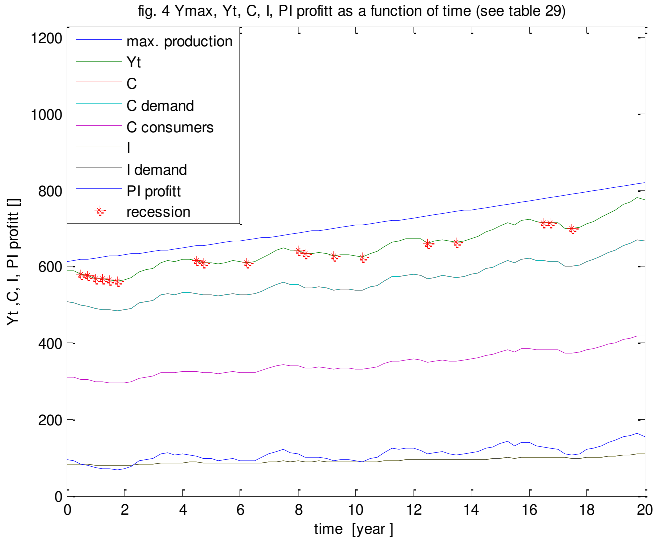


Fig. 4 GDP as a function of time. Random walk noise in GDP superimposed on fixed growth rate (f_{sticky}=.9/quarter and f_{flexy}=.9/quarter).

Similar we can use the gap to show the same information (fig.5)

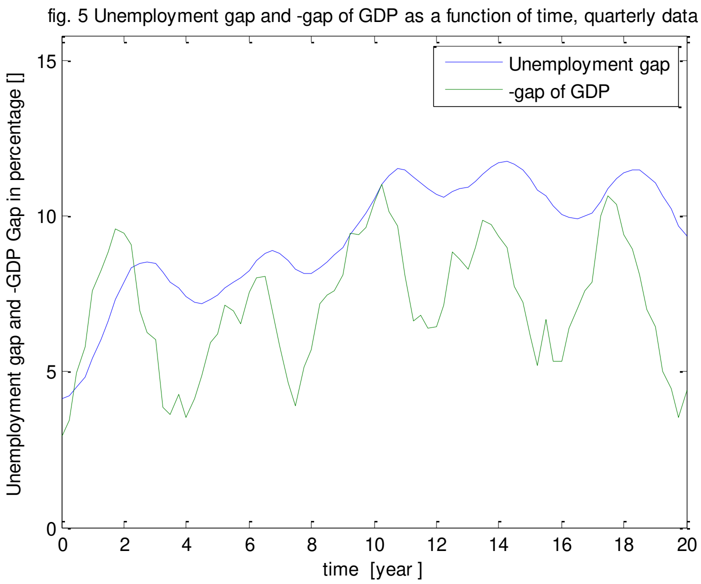


Fig. 5 GDP gap and unemployment. Random walk noise in GDP superimposed on fixed growth rate (f_{sticky}=.9/quarter and f_{flexy}=.9/quarter).

Now comes the interesting part. This exercise is generating lookalike pictures (fig. 6) for growth vs change in unemployment as seen in empirical data, suggesting that this behavior depends heavily on the character of GDP volatility even when f_{sticky} and f_{flexy} are not changing over time. The straight line shown in fig. 6 is Okun's Law, and is the best fit for the data points and results in c = 1.83 and k = 1.68 % /year. It is interesting to see the same counterclockwise looping behavior in the data in fig. 6 as in the recent empirical data analyses of Daly et al. (2014).

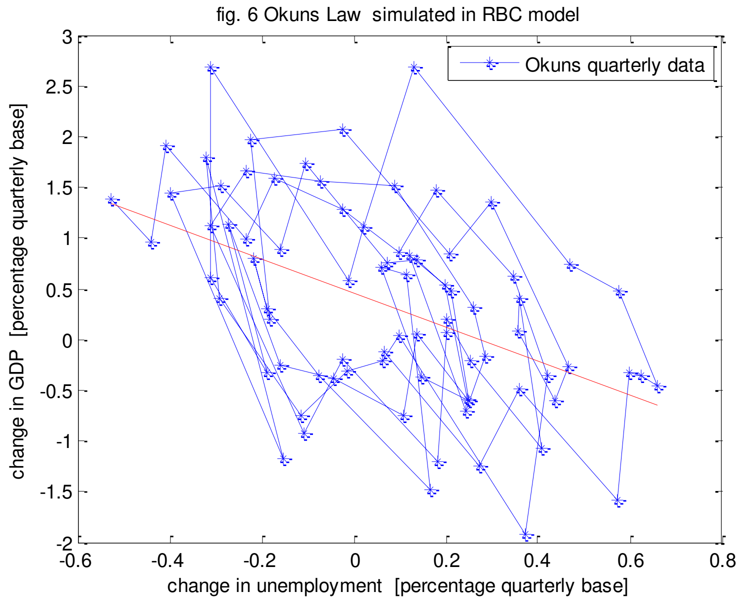


Fig. 6 Growth of GDP vs. changes in unemployment. Random walk noise in GDP superimposed on fixed growth rate (f_{sticky}=.9/quarter and f_{flexy}=.9/quarter).The straight line is Okun's Law with c = 1.83 and k = 1.68 % /year.

If we do ten times this simulation we obtain the results shown in table 1.

table 1 Parameters in Okun's Law

	k	c
Simulation 1	1,79	1,88
Simulation 2	1,56	1,90
Simulation 3	1,89	2,05
Simulation 4	1,81	2,15
Simulation 5	1,53	1,57
Simulation 6	1,77	2,50
Simulation 7	1,87	1,77
Simulation 8	1,36	2,81
Simulation 9	1,29	2,67
Simulation 10	1,59	2,12
Mean μ	1,646	2,142
St. deviation σ	0,212	0,402

In the simulation we used $g_p = 1.1$ % so average productivity growth $\bar{g} = \frac{g_p}{1-\alpha}$ = 1.57 %.

As you will notice there are remarkable variations in the value of *c*.

We will now take another approach to figure out how this process is working. For that we will calculate the transfer function, which calculates the response in actual Δu on a frequency input in the theoretical change of the unemployment without sticky and flexy workforce Δu_{basic}.

3. Transfer function

The production process itself is very non-linear and so is the influence of sticky and flexy workforce. However if we consider only small changes in GDP and we take $f_{sticky} = f_{flexy}$ (symmetrical in up and down swing of the economy) we can describe the process of sticky and flexy workforce in a linear way by estimating the response to an impulse in Δu_{basic} to be approximately:

$$z(t) = a\, e^{-at} \text{ for } t \geq 0 \tag{2}$$

$$\text{with } a = -\ln(f_{sticky})\,/t_{sticky} \tag{3}$$

where t_{sticky} is the time period corresponding to f_{sticky}. For example, if f_{sticky} = .9 and t_{sticky} is a quarter than a = -ln(.9)/.25 = .42/year.

The transfer function Z is the Fourier transform of z.

$$Z(x) = \int_{-\infty}^{\infty} z(t) e^{-i2\pi x t} dt \tag{4}$$

$$Z(x) = \frac{a^2}{a^2+(2\pi x)^2} + \frac{2a\pi x i}{a^2+(2\pi x)^2} \tag{5}$$

The amplitude of the transfer function is

$$|Z(x)| = \frac{1}{\sqrt{1+\left(\frac{2\pi x}{a}\right)^2}} \tag{6}$$

and is shown in fig 7.

This function describes how much the amplitude of a sine wave at a certain frequency is reduced compared to a zero frequency. The reciprocal value at certain frequency can be used as a multiplication factor for the theoretical *c* resulting in the practical value of *c*.

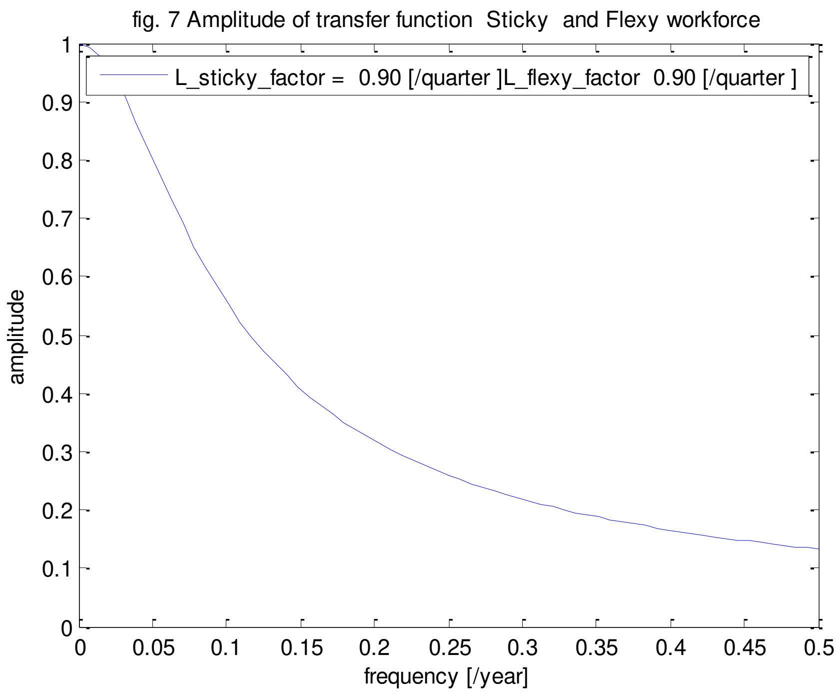


Fig. 7 The amplitude of the transfer function as a function of frequency for sticky and flexy workforce.

The amplitude of the resulting sine wave of the change in unemployment is reduced by the factor $|Z(x)|$, so *c* has to be multiplied by that factor. We can calculate *c* from

$$c = \frac{(1-\alpha)}{|Z(x)|} \tag{7}$$

In our case $|Z(x)| = .26$ at frequency *x* = .25/year resulting in *c* = 2.69 with $\alpha = .3$ which is close to the sine wave calculation of *c* = 2.8 we made earlier. This outcome implies that we really understand how the process is working. Furthermore if we realize that *Z(x)* is a complex function introducing a phase shift in the sine wave, we understand the elliptical graph of fig. 3.

4. Okun's Law as an estimator of unemployment from GDP growth *g*.

From the above we can see that *c* not only depends on f_{sticky} and f_{flexy} but *c* depends also on frequency as is to be expected. In other words it depends on the way GDP is evolving. Okun's Law is more an indirect way of describing the volatility of GDP, the effect of f_{sticky} and f_{flexy} and the variation of the other items mentioned in the list by means of one simple formula. This argument makes Okun's Law as a tool to estimate unemployment from growth *g* on a quarterly or year base a bad predictor, see Theeuwes (2010) and Gordon (2010). Also Owyang (2012) is reporting big variation in *c* when using different time windows. Another critique is from Meyer (2012) reporting that sometimes the law holds and sometimes it doesn't. Of course the low correlation rate of the data does not help either.

Although Okun's Law is not a very useful law for predicting, the measured scattered data are what they are and Okun's Law is what it is, but the economic meaning of *c* becomes quite vague.

The factors f_{sticky} and f_{flexy} depend on governmental and union regulations, on contracts and the expectation of future demand. As changes in stock are included in GDP those changes itself will not influence the relation between Δu and g directly, but only indirect via f_{sticky} and f_{flexy}.

Even estimating the minimum GDP growth rate needed to decrease unemployment (Theeuwes (2010), Bernanke (2012)) on the short run is not possible because Okun's Law is unstable in time and based on an economic unclear dependency of GDP growth. This results in a very poor estimate, because by using Okun's Law you don't take into account actual GDP dependency nor actual prior information.

The formula Gordon (2010) used was implemented for the Netherlands by Theeuwes (2010) and is, in fact, a recursive formulation of such a Sticky-Flexy model. This gives a better result and higher correlation, because you are also using prior information i.e. Δu_{t-1} and g_{t-1}, with the advantage that it is recent and valid information on the behavior of u and g, although the other weak points still remain unchanged as well as the fact that using only data over a small range make estimation sensitive to disturbances. Be aware that they left out the more troublesome points of 2008 and 2009. Unclear is the causal connection from g_{t-1} to Δu_t but one might e.g. think of Keynesian stimulating programs when unemployment is high.

In order to develop an estimator using sticky and flexy workforce we could e.g. calculate the convolution of GDP growth *g(t)* with the impulse response *z(t)* which results in the unemployment change as a function of time (now including the production process in *z(t)*.

$$\Delta u(t) = (g * z)(t) = \int_0^t g(\tau)z(t-\tau)d\tau \quad \text{for g, z} : [0,\infty) \rightarrow \mathcal{R} \qquad (8)$$

Or equivalent as a multiplication in the Fourier domain.

However, because we defined the process as a simple recurrence relation we better stick to this idea. In the next paragraph we will show how this can be done.

5. Derivation of the estimator.

Now you understand why Okun's Law is not a very useful tool, but before introducing an alternative, it will become a little bit worse. Okun's Law is not, even when considering the most simple production process an unbiased estimator if we consider it more precise.

In calculating $\frac{\Delta L}{L}$ and relating it to Δu we use the workforce L_w is constant. Be aware that you use the same units for U and L_w.

$$\frac{\Delta L}{L} = \frac{\Delta(L_w - U)}{(L_w - U)} = -\frac{L_w}{(L_w - U)}\frac{\Delta(U)}{L_w} = -\frac{1}{(1-u)}\Delta u \qquad (9)$$

This means that if there are no complicating effects like sticky and flexy workforce the estimator has to be modified to

$$g = k - c\frac{\Delta u}{(1-u)} \qquad (10)$$

and is suitable for all kind of production functions as a first order approximation.

Let us now return to our process of building the estimator. To make it more general we allow f_{sticky} to vary in time.

Let $\Delta u_{t-1} = u_{t-1} - u_{t-2}$ be the change in unemployment at time t-1 and suppose it is positive. Let $\Delta u_{basic\ ext}$ be the change in unemployment (without sticky or flexy process) caused by factors external not related to companies. The difference is the part which was fired

$$u_{fired\ t-1} = \Delta u_{t-1} - \Delta u_{basic\ ext\ t-1} \qquad (11)$$

and the fraction under consideration to be fired was

$$u_{pot\ t-1} = \frac{u_{fired\ t-1}}{(1 - f_{sticky\ t-1})} \qquad (12)$$

and the fraction still in the workforce but effectively not needed is

$$u_{latent\ t-1} = u_{pot\ t-1}\, f_{sticky\ t-1} \qquad (13)$$

Let $\Delta u_{basic\ int}$ be the change in unemployment (without sticky or flexy process) due to a change in demand (GDP) and other company-related issues than the new potential level from where the company has to take its decision is

$$u_{pot\ t} = u_{latent\ t-1} + \Delta u_{basic\ int\ t} \qquad (14)$$

And the new Δu_t can be calculated from

$$\Delta u_t = \left(1 - f_{sticky\ t}\right) u_{pot\ t} + \Delta u_{basic\ ext\ t} \qquad (15)$$

And this concludes the recursive part. We only have to calculate $\Delta u_{basic\ int\ t}$ and $\Delta u_{basic\ ext\ t}$ which are the change in unemployment from growth without the sticky and flexy process only due to company-related issues and non-company-related issues, respectively. This will be clarified in detail in the next part. We can use as a first order approximation the inverse of equation (10), or, in general, the first order approximation of the production function.

Let *Y = F(p, L, K)* be a production function and *L* is the actual number of workers (including factor for hours of work) in the production process and L_0 is the number of people in the population at time *t=0*

$$L = f_h f_u f_{part} f_{pop} L_0 \qquad (16)$$

where

f_h is the factor due to working hours

f_u is the factor due to unemployment

f_{part} is participation rate

f_{pop} is the factor of people in the population

The potential workforce is $L_{pot} = \frac{L}{f_h}$

The first order approximation is

$$\frac{\Delta Y}{Y} = \frac{\frac{\partial Y}{\partial L}}{\frac{Y}{L}} \frac{\Delta L}{L} + \frac{\frac{\partial Y}{\partial K}}{\frac{Y}{K}} \frac{\Delta K}{K} + \frac{\frac{\partial Y}{\partial p}}{\frac{Y}{p}} \frac{\Delta p}{p} \quad (17)$$

$$\frac{\Delta L}{L} = \frac{\Delta f_h}{f_h} + \frac{\Delta f_u}{f_u} + \frac{\Delta f_{part}}{f_{part}} + \frac{\Delta f_{pop}}{f_{pop}} \quad (18)$$

Combining equation (17) and (18) and using $\frac{\Delta f_u}{f_u} = -\frac{\Delta u_{basic}}{(1-u)}$ this results in

$$\Delta u_{basic} = (1-u)(g_h + g_{part} + g_{pop} - (g - a_K g_K - a_p g_p)/a_L) \quad (19)$$

where

g_h is growth rate of number of hours worked

g_{part} is growth rate of the participation

g_{pop} is growth rate of the population

$$a_K = \frac{\frac{\partial Y}{\partial K}}{\frac{Y}{K}} \qquad a_p = \frac{\frac{\partial Y}{\partial p}}{\frac{Y}{p}} \qquad a_L = \frac{\frac{\partial Y}{\partial L}}{\frac{Y}{L}}$$

We split Δu_{basic} into 2 parts, $\Delta u_{basic\ ext}$ for participation and population growth which are not direct related to companies and the rest $\Delta u_{basic\ int}$ will influence the part u_{pot} over which the company can decide.

$$\Delta u_{basic} = \Delta u_{basic\ int} + \Delta u_{basic\ ext} \quad (20)$$

$$\Delta u_{basic\ int} = (1-u)\left(g_h + \frac{-g + a_K g_K + a_p g_p}{a_L}\right) \quad (21)$$

$$\Delta u_{basic\ ext} = (1-u)(g_{part} + g_{pop}) \quad (22)$$

Combining equation (11), (12), (13), (14), (15), (21) and (22) results finally in our estimator for Δu_t

$$\Delta u_t = (1 - f_{sticky\ t})\{\left(\Delta u_{t-1} - (1 - u_{t-1})(g_{part\ t-1} + g_{pop\ t-1})\right)\frac{f_{sticky\ t-1}}{1 - f_{sticky\ t-1}} +$$

$$+(1 - u_t)(g_{h\ t+}\frac{-g_t + a_K g_{K\ t} + a_p g_{p\ t}}{a_L})\} + (1 - u_t)(g_{part\ t} + g_{pop\ t}) \qquad (23)$$

As an example take a Cobb-Douglas production function where $a_K = \alpha, a_L = 1 - \alpha, a_p = 1$, $f_{sticky} = 0$ and let $g^e_{h\ t} = g^e_{part\ t} = g^e_{pop\ t}$=0, $g^e_t = g$, $g^e_{K\ t} = \frac{g_p}{1-\alpha}$ (optimal) and $g^e_{p\ t} = g_p$ then equation (23) reduces to

$$\Delta u = (1 - u)(-\frac{g}{1-\alpha} + \frac{g_p}{(1-\alpha)^2}) \qquad (24)$$

Remember that $k = \bar{g} = \frac{g_p}{(1-\alpha)}$ so equation (24) will become

$$\Delta u = \frac{(1-u)}{(1-\alpha)}(-g + k) \qquad (25)$$

Inverting equation (25) will give us the equation (10) which was the first order approximation in the Cobb-Douglas case without sticky and flexy workforce.

If you want to use this technique be aware that it is also possible that f_{sticky} is bigger than 1 (or even smaller than 0), e.g. if the estimated $g^{estimate}_{t+1}$ is much bigger than g_t and entrepreneurs decide not only not to fire workers they don't need at time t but in contrary employ even more workers, because of future expectations. Also notice that if we would write u_t in terms u, we need u_{t-1} and u_{t-2}. Furthermore is would be more precise to use $(1 - u_t)$ instead of the approximation $(1 - u_{t-1})$ throughout the derivation and to solve for u_t, but the difference will probably negligible from an economic point of view.

It is also possible to estimate $f_{sticky\ t}$ as a function of time from equation (23) and we will come back to that later.

It is time for an example to show you the robustness of the method on generated data.

6. Example with computer generated data.

We let demand be a sine wave superimposed onto a steady increasing demand and on top of that a random walk demand. We introduce some random noise to simulate the error in predicting future growth in demand before using the estimator ceteris paribus. The standard deviation of the noise is $\sigma_{growth} = .005$ /quarter relative to demand. We estimate 1, 2 and 3 quarters ahead. The estimated change in unemployment is shown in fig. 8.

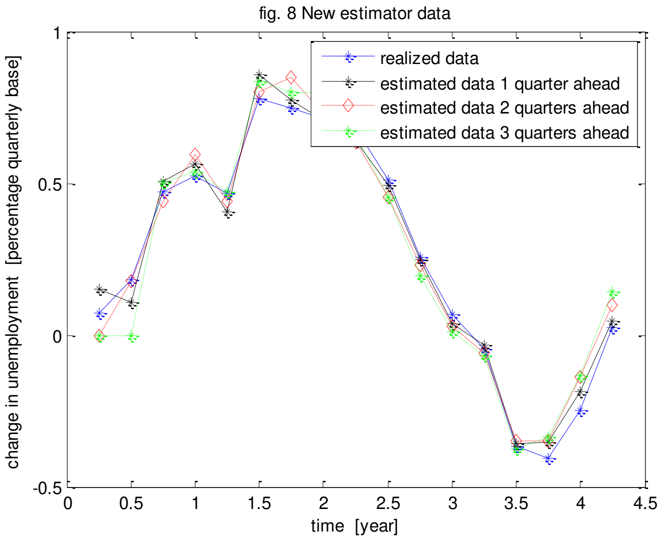


Fig. 8 Estimated change in unemployment 1, 2 and 3 quarters ahead with noise on growth g to simulate uncertainty in predicting g.

The errors of the method are shown in fig. 9.

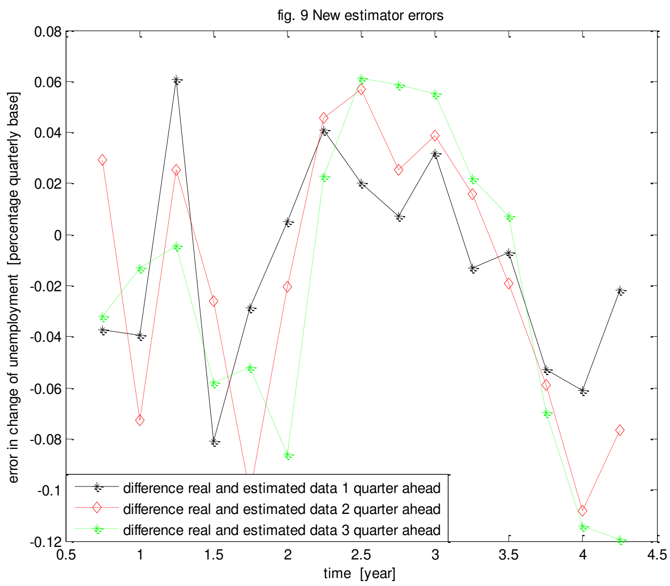


Fig. 9 Errors in estimated change in unemployment 1, 2 and 3 quarters ahead with noise on growth g to simulate uncertainty in predicting g.

The standard deviation of the error 1 quarter ahead is, of course, approximately equal to

$$\sigma_{error\ 1\ quarter} = \frac{(1-f_{sticky})}{1-\alpha}(1-u)\sigma_{growth}.$$

$\sigma_{error\ 1\ quarter}$= .135$\sigma_{growth}$= .07 % /quarter.

The method is more sensitive for changes in f_{sticky}.

7. Example with empirical data.

The next example is on empirical data of The Netherlands over the period 2003-2013. The interesting point in the graph is the year 2009 of the recession and its effect on unemployment. We calculate $f_{sticky\ t}$ (fig. 11) from equation (23) assuming Δu, the calculated change in unemployment is equal to the realized value, see fig. 10.

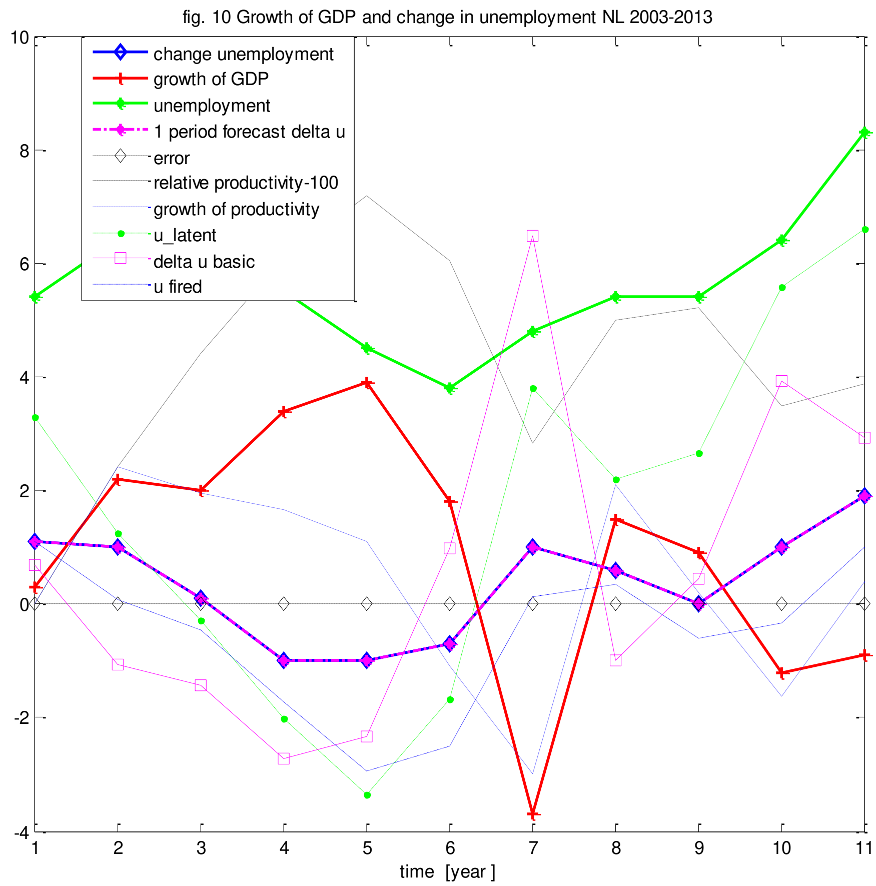


Fig. 10 Growth of GDP, unemployment, Δu_t, $\Delta u_{basic\ t}$, $u_{fired\ t}$ and $u_{latent\ t}$ as a function of time with estimated f_{sticky}. The Netherlands case from 2003-2013. Data is gathered from Statistics Netherlands (CBS), Netherlands Bureau for Economic Policy Analysis (CPB), OECD and IMF. Total Factor Productivity is fixed at 0.75 %. We cannot guarantee the consistency between all the data used.

If $f_{sticky\ t}$ would have been 0 then the estimated value for the unemployment change would have been equal to $\Delta u_{estimate} = \Delta u_{basic}$ and $u_{latent} = 0$, because the reaction of companies are very fast and there is no delay in firing or hiring workers . In 2009 (point 7) you can see a theoretical increase in unemployment Δu of over 6.4 % but due to f_{sticky} this is reduced to 1.0 % (see table 2 and 3 or the graph in fig. 10). At the same time the part of the workers not really needed u_{latent} is increasing and productivity (/hour) is dropping. Here productivity is calculated as the growth minus growth in hours, population, participation and unemployment, so only due to growth in TFP, capital growth and the influence of latent workers. The value of f_{sticky} is even slightly higher than 1 indicating that employers are not only not firing workers but even do a little hiring, probably expecting higher demand in the year to come. However, demand is not returning at the pace needed to lower unemployment, while employers stick to their strategy and u_{latent} keeps on rising.

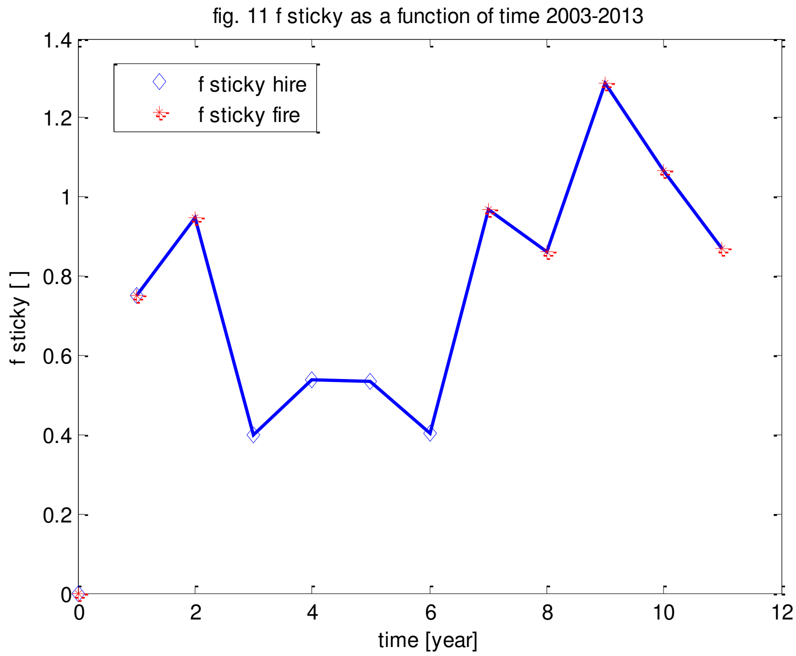


Fig. 11 Estimated f_{sticky} as a function of time with $\Delta u_{estimate\ t} = \Delta u_t$. The Netherlands case from 2003-2013.

Note the difference in procedure between hiring and firing, i.e. the value of f_{sticky}.

The given example is only meant to give you an different approach to look at unemployment data. We are convinced it can be improved a lot by using data on a quarterly base, cross-correlating these data with e.g. measured productivity, alternative ways to estimate f_{sticky} and other procedures.

A simple use of $f_{sticky} = .9$ and $f_{flexy} = .4$ resulted in a reduction of the estimated unemployment changes as compared to the estimated value by Okun's Law using the regression line. The standard deviation in the error reduced by 30 %. In this case especially the extreme differences were reduced regarding the year 2009. There are however to less data point in this exercise to conclude how we can improve our strategy. An adaptive approach with respect to for f_{sticky} and f_{flexy} might open up such an improvement. I leave it to the interested researcher to do so with more historical data points.

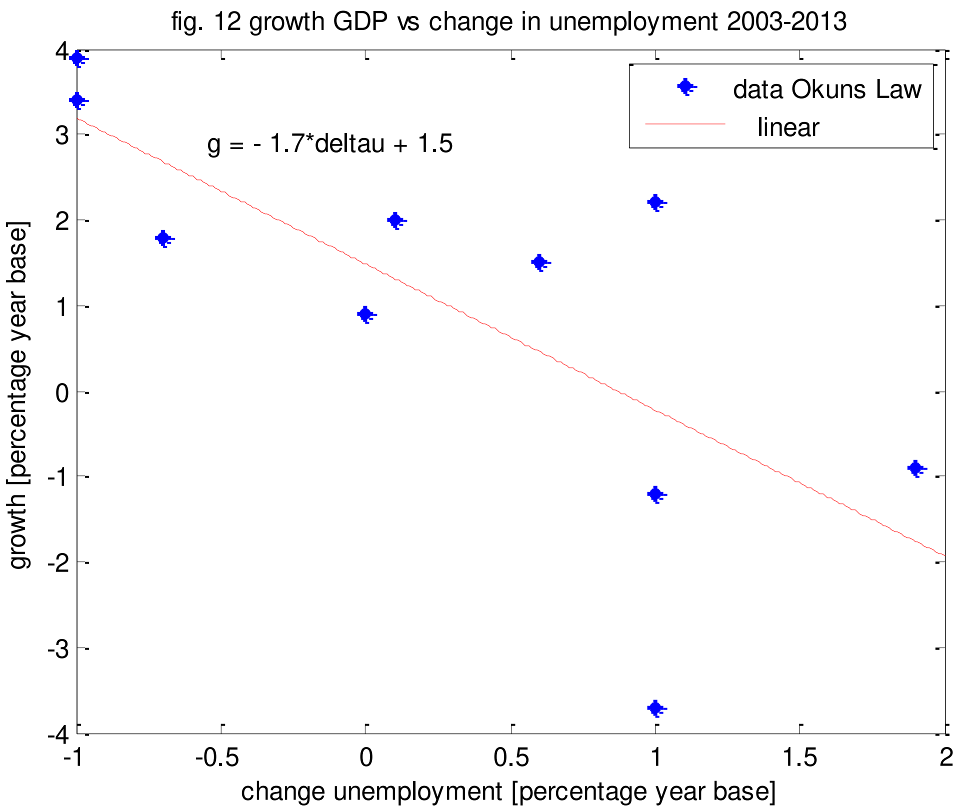


Fig. 12 Okun's Law for The Netherlands data 2003-2013, year base, c = 1.7 /year and k = 1.5 % /year

8. Gap version of Okun's Law.

As a last point we like to point out that Bernanke (2005) introduced the gap version of Okun's Law stating that c is equal to the ratio of the output gap and the unemployment gap.

$$gap = \frac{(Y_{pot} - Y)}{Y_{pot}} \tag{26}$$

$$gap_u = u - \bar{u}_{pot} \tag{27}$$

$$c = \frac{gap}{gap_u} \tag{28}$$

where $\bar{u}_{pot}$ is the natural rate of unemployment and $u = \bar{u}_{pot}$ corresponds to $Y = Y_{pot}$.

Bernanke proved that the gap version is equivalent to the difference version of Okun's Law as a first order approximation. The problem is that the correctness of the law is already supposed to be true. That means he assumed that c is a constant in time. This is only the case when using a fixed number of data points. Apparently this is not the case in general, e.g. in case of using a moving window c is changing in time and then the equivalence in first order approximation is only true per fixed number of data points. It is difficult to judge the consequence of this issue, because c in Okun's Law does not have a useful economic meaning, in general, in the first place. We can make better use of the introduced estimator and not use Okun's Law at all.

9. Conclusion

- From a theoretical point of view Okun's Law is not a suitable tool to estimate unemployment from growth g, because the slope of the curve depends strongly on the evolution of the frequency distribution of GDP.
- The effect of f_{sticky} and f_{flexy} shows that Okun's Law is, in general, not an accurate unbiased estimator and that the economic meaning of the slope factor c is vague.
- The estimator, we developed, which include sticky and flexy workforce will be more accurate, although to incorporate the influence of the expectation of future demand in f_{sticky} and f_{flexy} is more difficult to deal with. The coefficients in the estimator are all derived from theory. The simple sticky and flexy process itself might be replaced by a better one to improve the estimator, e.g. with an adaptive approach .
- It is interesting to see that with random walk demand it was possible to generate lookalike plots to empirical quarterly data, making it very likely that the spread in empirical data is mainly caused by the sticky and flexy process itself, apart from other statistical fluctuations.
- A better and more accurate method is suggested, though we like to stick to our computer model as a simulating tool, where it is also possible to take into account non-linearities, consumer- and capital-goods companies, etc.
- The derivation of the difference version from the gap version of Okun's Law by Bernanke is a first order approximation and holds under the condition that c is time independent.

10. Acknowledgement

This paper, including the economic model we developed to calculate and plot a few of the pictures we presented in this paper, is part of a study to reduce unemployment in a sustainable way whilst keeping governmental debt within sustainable limits and improve prosperity. In our opinion this is one of the most important things to achieve in society from a macro economic and from a participation society point of view. We are convinced we can provide a blueprint to the solution of this problem as part of a sustainable society.

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Table 2 New estimator for change in unemployment and estimator for f_sticky for The Netherlands 2003-2013

Year	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
unemployment	5.4000	6.4000	6.5000	5.5000	4.5000	3.8000	4.8000	5.4000	5.4000	6.4000	8.3000
growth GDP	0.3000	2.2000	2.0000	3.4000	3.9000	1.8000	-3.7000	1.5000	0.9000	-1.2000	-0.9000
growth capital	1.0541	1.0431	1.0324	1.3281	1.5192	1.6866	1.0609	0.8043	0.8076	0.5807	1.0000
growth TFP	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
growth hours	0.0000	-0.1428	-0.4283	-0.0714	-0.2855	0.2855	-0.5710	-0.2141	0.0714	0.0714	-0.2141
delta u basic int	0.6977	-1.9883	-1.9903	-3.4446	-4.2894	-0.8166	5.6012	-1.2279	-0.1456	2.5729	2.0306
growth part	0.0000	0.5926	0.2946	0.5874	1.8978	1.5759	0.4231	-0.2809	0.1408	0.9845	0.6964
growth pop	0.0000	0.4043	0.2921	0.1759	0.1456	0.2898	0.4900	0.5411	0.4875	0.4476	0.2942
delta u basic ext	0.0000	0.9331	0.5485	0.7213	1.9515	1.7948	0.8693	0.2461	0.5945	1.3405	0.9084
delta u	1.1000	1.0000	0.1000	-1.0000	-1.0000	-0.7000	1.0000	0.6000	0.0000	1.0000	1.9000
estim delta u	1.1000	0.9999	0.1006	-0.9998	-1.0004	-0.6997	0.9999	0.6005	0.0001	0.9999	1.8996
u pot	4.4000	1.3127	-0.7456	-3.7420	-6.3092	-4.1748	3.9246	2.5624	2.0634	5.2298	7.6043
u pot n	0.0000	1.3117	-0.7446	-3.7416	-6.3101	-4.1743	3.9213	2.5660	2.0629	5.2307	7.6009
u fired	1.1000	0.0669	-0.4485	-1.7213	-2.9515	-2.4948	0.1307	0.3539	-0.5945	-0.3405	0.9916
u latent	3.3000	1.2457	-0.2970	-2.0207	-3.3578	-1.6799	3.7939	2.2085	2.6578	5.5702	6.6127
error delta u	0.0000	-0.0001	0.0006	0.0002	-0.0004	0.0003	-0.0001	0.0005	0.0001	-0.0001	-0.0004
g productivity	0.0000	2.4143	1.9485	1.6499	1.0950	-1.0789	-2.9917	2.0882	0.2002	-1.6351	0.3955
delta u basic	0.6977	-1.0553	-1.4418	-2.7233	-2.3380	0.9783	6.4706	-0.9818	0.4488	3.9134	2.9390
f_sticky	0.7500	0.9490	0.3984	0.5400	0.5322	0.4024	0.9667	0.8619	1.2881	1.0651	0.8696
estim f_sticky	0.0000	0.9490	0.3984	0.5400	0.5322	0.4024	0.9667	0.8619	1.2881	1.0651	0.8696

Remarks:

- g_h is a measure for the growth of productivity per hour worked
- growth of TFP is estimated to be .75 %

Table 3 New estimator for change in unemployment and estimator for f_sticky for The Netherlands 2003-2013

Year	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
L population	100.0000	100.4043	100.6976	100.8747	101.0216	101.3143	101.8108	102.3617	102.8607	103.3211	103.6251
L part	67.5000	68.1745	68.5750	69.0992	70.5131	71.8319	72.4893	72.6768	73.1340	74.1846	74.9210
L h	67.5000	68.0772	68.1835	68.6553	69.8588	71.3704	71.6097	71.6393	72.1422	73.2314	73.7980
L werkend	63.8550	63.8113	64.1177	65.2987	67.3400	69.1023	69.0098	68.7522	69.1847	69.4368	68.7025
L werkend h	63.8550	63.7202	63.7515	64.8792	66.7151	68.6584	68.1724	67.7708	68.2465	68.5446	67.6727
L werkloos part	3.6450	4.3632	4.4574	3.8005	3.1731	2.7296	3.4795	3.9245	3.9492	4.7478	6.2184
L werkloos h	3.6450	4.3569	4.4319	3.7760	3.1436	2.7121	3.4373	3.8685	3.8957	4.6868	6.1252
L delta u basext	0.0000	0.6361	0.3762	0.4984	1.3760	1.2893	0.6302	0.1789	0.4347	0.9944	0.6806
L u fired	0.7425	0.0456	-0.3076	-1.1894	-2.0812	-1.7921	0.0947	0.2572	-0.4347	-0.2526	0.7429
L u latent	2.2275	0.8493	-0.2037	-1.3963	-2.3677	-1.2067	2.7502	1.6051	1.9438	4.1323	4.9543
L u pot	2.9700	0.8949	-0.5113	-2.5857	-4.4488	-2.9988	2.8449	1.8623	1.5090	3.8797	5.6972
L u latent h	2.2275	0.8481	-0.2025	-1.3873	-2.3457	-1.1990	2.7168	1.5822	1.9174	4.0792	4.8801
L u effectief h	61.6275	62.8722	63.9541	66.2665	69.0608	69.8573	65.4556	66.1886	66.3291	64.4655	62.7927
Productivity y_h	0.5639	0.5775	0.5888	0.5982	0.6044	0.5979	0.5799	0.5921	0.5932	0.5835	0.5857
Growth y_h	-	2.4162	1.9499	1.6027	1.0409	-1.0812	-3.0136	2.1016	0.1967	-1.6298	0.3768
p	1.0050	1.0100	1.0151	1.0202	1.0253	1.0304	1.0355	1.0407	1.0459	1.0511	1.0564
K	10.1054	10.2108	10.3162	10.4532	10.6121	10.7910	10.9055	10.9932	11.0820	11.1464	11.2578
GDP	36.0060	36.7981	37.5341	38.8102	40.3238	41.0496	39.5308	40.1238	40.4849	39.9991	39.6391

Remarks:

- K0 = 10, p0 = 1 and L0 = 100 arbitrary chosen.
- y_h is the productivity of GDP per contract hour worked
- $\alpha = .3$