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14 January 2018

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Abstract

In a dynamic storable good market where demand changes over time, we investigate the producer's strategic incentives to hold inventories in response to the possibility of buyer stockpiling. The literature on storable goods has demonstrated that buyer stockpiling in anticipation of higher future prices harms the producer's profitability, particularly when the producer cannot commit to future prices. We show that the producer's inventories act as a strategic device to mitigate the loss from the lack of commitment. Our results provide a rationale for the producer's inventory behavior that sheds new light on the well-documented empirical evidence about inventories.

Keywords: buyer stockpiling, commitment, storable goods, strategic inventories.

JEL Classification: D21, D42, L12.

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1 Introduction

Inventory management is essential for a firm's viability in many industries. Although the share of inventory investment in the gross domestic product (GDP) is relatively small, changes in inventories are a significant component of economic fluctuations. During the recent financial crisis, the reduction in inventories accounted for 29% of the decline in GDP (Wang et al. 2014).

Traditional reasons for inventories are driven by technological features, such as production smoothing over time in the presence of convex production costs and stockout avoidance when production takes time and cannot be immediately adjusted to demand shocks (e.g., Aguirregabiria 1999; Anupindi et al. 2012; Arrow et al. 1951; Arvan and Moses 1982; Holdt et al. 1960; Kahn 1987; Krishnan and Winter 2007, 2010; Nahmias 2008; Zipkin 2000). The empirical evidence indicates that inventories are procyclical and production is more variable than sales (e.g., Blanchard 1983; Blinder 1986; Ramey and West 1999; Wen 2005). Information about the production and inventory activities is generally available in industry reports, financial statements as well as balance sheets. Such information is also collected in accurate databases. For instance, Standard & Poor's Compustat has provided since 1962 financial, statistical and market data about companies throughout the world.[1]

The presence of intermediaries and arbitrageurs in the commodity markets reveals that buyers also exhibit incentives to store. A recent strand of the empirical literature has systematically documented buyer stockpiling in anticipation of higher future prices in markets for various intermediate and final goods (e.g., Erdem et al. 2003; Hall and Rust 2000; Hendel and Nevo 2004, 2006a, 2006b; Pesendorfer 2002).

The storage activities of firms and their customers have been examined separately in the literature so far. In this paper, we provide a unified framework in order to investigate the inventory behavior of a producer vis-à-vis forward-looking buyers that are willing to store in anticipation of higher future prices. Abstracting from the aforementioned classical reasons for inventories, we show that a producer unable to commit to future prices has a strategic incentive to hold inventories when facing the prospect of buyer stockpiling. Our results provide theoretical support for the main stylized facts about the firms' inventory activities.

We focus our attention on storable goods, which are perishable in consumption but can be stored for future consumption. Typical examples are various intermediate goods (e.g., oil, coffee and wheat) and groceries that can be purchased in advance and stored. In order to characterize the strategic role of the producer's inventories, we build on the seminal paper of Dudine et al. (2006), which considers a dynamic storable good market with deterministic time-varying demand where a monopolistic producer cannot commit to future prices and faces a continuum of competitive buyers available to stockpile in anticipation of higher future prices. Dudine et al. (2006) show that the excessively high future prices driven by the producer's lack of commitment trigger buyer stockpiling, which is ex ante profit detrimental in that it reduces future sales occurring at a higher price. As a result of wasteful buyer stockpiling, profits and welfare are lower than under full commitment. In this setting, we introduce the possibility for the firm to accumulate production in the form of inventories available for future

[1] Information about the 100 largest companies traded on the US stock exchanges can be regularly found at https://www.stock-analysis-on.net.

sales. To fix ideas, consider the oil market in the US, where a large oil producer (or refiner) generally supplies competitive distributors (or directly the petrol stations) and accumulates some quantity in its depositories to cover future demand. The distributors (and petrol stations) are also endowed with storage capacities. In many other markets for storable goods – such as bananas, bauxite, coffee, copper, diamonds, iron ore, mercury, phosphates, and tin – supply is relatively concentrated, with some producers possessing large market power. Competitive speculators trade these goods and engage in stockpiling activities.[2]

In our framework, production costs are linear and do not vary over time, while demand evolves deterministically. Therefore, there is no scope for the aforementioned well-documented motives for inventories, and the producer shall engage in inventory activities only for strategic purposes. Under full commitment, the producer does not benefit from inventories, since it can credibly announce a price sequence that removes buyer stockpiling and inventories would only result in a mere loss due to their holding costs. However, this conclusion is no longer valid when the producer is unable to commit to future prices, and inventories can emerge in equilibrium.[3] To understand the rationale for this result, it is important to realize that inventories are produced in the first period and their cost is sunk once the second period has commenced. A producer that cannot commit to future prices holds inventories as a strategic device to reduce future costs, which translates into lower future prices and mitigates the buyer stockpiling incentives.

We find that under certain circumstances, despite the lack of commitment, the producer accumulates the amount of inventories that maximizes the ex ante profits, and the constraint of sequential optimality is slack in equilibrium. As under full commitment, buyer stockpiling is removed, with the only additional cost of holding inventories. Notably, this solution is implementable only if the unit cost of production is large enough. *Prima facie*, this might seem counterintuitive, since one could expect that a more efficient producer finds inventories more attractive. The rationale for this result arises from the strategic nature of inventories. Each level of inventories is associated with a second period price at which they are fully exhausted. When the cost of production is large enough, the ex ante optimal level of inventories is relatively small and in the second period the producer does not succumb to the temptation to discard some inventories and to set a price above the level at which inventories are fully sold. Moreover, as long as the cost of holding inventories is relatively small, the firm does not have any incentive to produce some additional quantity in the second period, either. Additional production would be clearly suboptimal, since producing the entire quantity in the second period allows the firm to avoid the cost of holding inventories. Given that the producer shall charge the second period price at which inventories are fully exhausted, the ex ante optimal inventory level is also sequentially optimal. Anticipating a lower future price associated with the sunkness of inventory costs, buyers abstain from any stockpiling activity. Therefore, inventories constitute a strategic device to mitigate the producer's loss from the lack of commitment. In particular, as long as holding inventories is costless, the full commitment outcome is restored.

If the unit cost of production is small enough, the ex ante optimal level of inventories is

[2] Governments can also implement stockpiling policies, especially in order to protect against future supply disruptions (e.g., Nichols and Zeckhauser 1977).

[3] As discussed in Section 7.3, the introduction of forward or futures contracts cannot restore the full commitment solution.

relatively large and in the second period the producer cannot refrain from discarding a portion of the inventories accumulated in the first period and setting a second period price above the level at which inventories are fully sold. In other words, the ex ante optimal level of inventories is not sequentially optimal and cannot be sustained in equilibrium. We show that, even in this case, inventories can play a strategic role in mitigating the buyer stockpiling incentives. It follows from our previous discussion that inventories must be distorted away from the ex ante optimal level, and the constraint of sequential optimality is binding in equilibrium. As a result, both the producer and the buyers may now engage in storing activities.

When the cost of production is relatively large and therefore the ex ante optimal inventory level is also sequentially optimal, inventories generally lead to lower prices although they involve holding costs. The rationale for this apparently surprising result stems from the strategic nature of inventories, which mitigate the producer's temptation to increase future prices and curb the buyer stockpiling incentives. Hence, despite being used in the producer's private interest, inventories increase consumer surplus and social welfare. This conclusion deserves some qualifications when the cost of production is relatively small and therefore the equilibrium inventory level departs from the ex ante optimal level. As described in Section 5, the comparisons between equilibrium prices are driven by the demand curvature and the inventory cost in a non-trivial manner. Equilibrium prices exhibit peculiar features that merit some attention. For instance, we show in Sections 4 and 5 that a lower inventory cost may lead to higher prices.

Our paper provides a novel strategic rationale for the firms' inventory activities, which does not lie in the specific production technologies assumed by the traditional inventory theories. The predictions of our model lend themselves to an empirically testable validation and can stimulate the empirical or experimental investigation on the firms' inventory management. A relevant market where our results can be applied is the oil market, which exhibits relatively large costs of production and small inventory costs. In 2015, with a crude oil price of around 48 dollars per barrel, in the US the oil production costs were 36 dollars per barrel, while the inventory costs only amounted to about 0.5 dollars per barrel.[4]

Our results also shed new light on the empirical evidence about inventories discussed at the beginning. In particular, since strategic inventories are associated with periods of demand expansion, we establish microfoundations for the well-documented observation of inventory procyclicality. Furthermore, as discussed in Section 7.1, inventories are more likely to emerge in the presence of higher degrees of market concentration or product differentiation. Interestingly, this provides theoretical corroboration for the empirical evidence documented by Amihud and Mendelson (1989) that firms with greater market power hold a larger level of inventories. An additional, more theoretical, implication of our model is that the normalization of production costs to zero usually adopted in the literature is not innocuous in storable good markets, since it undermines the firms' strategic inventory incentives.

Our investigation is conducted in a fairly general setting without imposing any unduly restrictive assumptions on the functional forms. The analysis in Section 7 reveals that the model is robust and the driving force of our results persists in alternative scenarios, such as competition among producers or a higher number of periods. Although our model is deterministic in

[4]Further details are available at https://www.rystadenergy.com.

the spirit of Dudine et al. (2006), our results can be easily generalized to a setting where demand evolves stochastically over time.[5] As argued in Section 8, our results provide potentially significant managerial, empirical and policy implications.

Related literature There exists a recent fast-growing literature on strategic inventories in markets for storable goods. Anand et al. (2008) show that in a dynamic buyer-seller relationship the buyer uses strategic inventories to induce the seller to decrease its future price. Extending the model of Anand et al. (2008), Arya and Mittendorf (2013) investigate the role of consumer rebates in the presence of strategic inventories, while Arya et al. (2015) find that strategic inventories influence the choice between centralization and decentralization. Hartwig et al. (2015) provide experimental support for strategic inventories. Differently from these contributions, we investigate the producer's inventory strategic incentives that arise from the possibility of buyer stockpiling.

It has been recognized long ago in the economic literature that a firm can benefit from the investment in capacities or inventories as an irreversible commitment against the rivals (e.g., Arvan 1985; Dixit 1980; Driver 2000; Mollgaard et al. 2000; Saloner 1986; Ware 1985). Rotemberg and Saloner (1989) identify inventories as a means to sustain collusion. Deneckere et al. (1996) show that a manufacturer facing uncertain demand and selling through a competitive retail market may wish to support an adequate level of retail inventories. More recently, Mitraille and Moreaux (2013) consider a dynamic setting where Cournot competitors, after storing in the first period, may produce and sell in the second period.

As previously discussed, one of the most relevant papers in the literature on buyer stockpiling is Dudine et al. (2006). Hendel et al. (2014) extend the analysis to nonlinear pricing of storable goods. Other important contributions deserve consideration. Anton and Das Varma (2005) show in a Cournot setting that, when buyers are sufficiently patient, competition among firms to attract buyer stockpiling generates an increasing price path.[6] Guo and Villas-Boas (2007) find that in a differentiated good market the preference heterogeneity translates into differential buyer stockpiling propensity, which exacerbates future price competition and may remove buyer stockpiling in equilibrium. Su (2010) incorporates buyer stockpiling into Su (2007)'s analysis of the optimal dynamic strategy of a seller that faces strategic buyers. Differently from our approach, the seller's inventories are carried for standard reasons such as economies of scale and do not have any strategic role. Hendel and Nevo (2013) investigate intertemporal price discrimination when buyers differ in their storage abilities.

Our paper also pertains to the vast literature on durable goods, which share some similarities with storable goods, as Dudine et al. (2006) point out. Among others, two relevant recent contributions are Board (2008), which solves the profit maximization problem of a durable good monopolist with time-varying demand, and Garrett (2016), which addresses the same problem in a setting where buyers arrive over time and have values for the good that evolve stochastically. While these studies are interested in the classical problem of demand postponement, we focus

[5]For instance, a shock may affect the expectation about future demand. The stochastic process may follow a mean reversion pattern, similarly to Antoniou et al. (2017). In Section 8 we discuss the implications of allowing for uncertain demand.

[6]Section 7.1 describes the relation between our paper and Anton and Das Varma (2005) and Mitraille and Moreaux (2013).

on demand anticipation.

The rest of the paper is structured as follows. Section 2 sets out the formal model. Section 3 identifies three relevant benchmarks: the producer's static problem, the producer's dynamic problem under full commitment, and the producer's dynamic problem under limited commitment in the absence of inventories. Section 4 shows the main results of the paper about the producer's strategic incentives to hold inventories under limited commitment. Section 5 investigates price comparisons. Section 6 provides a full characterization of the results with explicit functions. Section 7 discusses the robustness of the model and explores different extensions. Section 8 concludes and illustrates some managerial, empirical and policy implications. The main formal proofs are collected in the Appendix. Additional formal results and associated proofs are relegated to the Supplementary Appendix.

2 The model

2.1 Setting

Buyers We consider a two-period monopoly market for a storable good where in each period $\tau \in \{1, 2\}$ the producer faces a (continuously differentiable) demand $D_\tau(p_\tau)$, which decreases with the price p_τ, i.e., $D_{\tau|\tau} < 0$.[7] In line with Dudine et al. (2006), the demand changes deterministically over time and, as it will be clear in the sequel, we are mainly interested in the case where the demand rises in the second period. For the sake of simplicity, there is no discounting on the second period. In Section 7.4 we discuss the role of the discount factor.

The producer serves a continuum of competitive buyers, which operate as arbitrageurs in the market and purchase the good from the producer in order to resell it to the final consumers at zero profits. Therefore, the buyer demand corresponds to the final consumer demand. Buyers can purchase in advance and stockpile the good in the first period at a unit cost $s_b > 0$.[8]

Denoting by p_2^e the expected price in the second period, the buyer stockpiling demand writes as

$$D_s(p_1) = \begin{cases} D_2(p_1 + s_b) & \text{if } p_1 + s_b < p_2^e \\ [0, D_2(p_1 + s_b)] & \text{if } p_1 + s_b = p_2^e \\ 0 & \text{if } p_1 + s_b > p_2^e \end{cases} \tag{1}$$

If $p_1 + s_b < p_2^e$, the first period price inflated by the buyer stockpiling cost is smaller than the second period expected price, and therefore buyers prefer to purchase in advance and stockpile the good. Conversely, if $p_1 + s_b > p_2^e$, buyer stockpiling is strictly dominated. If $p_1 + s_b = p_2^e$, buyers are indifferent between stockpiling or not. Under buyer rational expectations and perfect foresight (no uncertainty), the second period expected price coincides with the second period equilibrium price.[9]

[7] The subscript τ on the right of the slash denotes the derivative with respect to the price p_τ and the subscript $\tau\tau$ identifies the second-order derivative with respect to p_τ. We follow this notation throughout the paper.

[8] Our results are unaffected when the producer directly faces the final consumers. Note that the arbitrageurs, whose presence is also suggested by Dudine et al. (2006), are more likely to have lower stockpiling costs and superior information about the producer. We refer to Section 2.2 for further details.

[9] As mentioned in the introduction, our qualitative results carry over under demand uncertainty. In Section 8 we provide additional discussion on this point.

Producer In each period the producer decides on the amount of production and the level of sales, or equivalently the price for the good. Let $c \geq 0$ be the (constant) unit cost of production.[10] The quantity produced net of the current sales represents the producer's inventories, which are available for sale in the following period. Since the game consists of two periods, it is straightforward to see that both the producer and the buyers do not store in the second period. Therefore, we restrict our attention to the producer's inventories and buyer stockpiling in the first period. Let $I \geq 0$ be the producer's first period inventory level available for sale in the second period, which involves a unit inventory cost $s_p \geq 0$. We assume that $s_p \leq s_b$. This reflects the natural idea that the producer is more efficient at storing than the buyers and captures the most relevant case for our purposes.[11]

The producer's aggregate profits are $\Pi = \Pi_1 + \Pi_2$, where

$$\Pi_1 = (p_1 - c)\left[D_1(p_1) + D_s(p_1)\right] - (c + s_p)\, I \tag{2}$$

and

$$\Pi_2 = p_2\left[D_2(p_2) - D_s(p_1)\right] - c\left[D_2(p_2) - D_s(p_1) - I\right] \cdot \mathbf{1}_{Q_2} \tag{3}$$

denote the profits in the first and second period, respectively. The indicator function $\mathbf{1}_{Q_2}$ in (3) assumes value 1 if production takes place in the second period, i.e., $D_2(p_2) - D_s(p_1) - I > 0$, and value zero otherwise. The buyer stockpiling demand $D_s(p_1)$ increases the demand in the first period but depresses it in the second period. The aggregate cost $(c + s_p)$ per unit of inventory is incurred by the producer in the first period and therefore it is a sunk cost in the second period.

The producer's profits Π_τ, $\tau \in \{1, 2\}$, satisfy the following standard assumption.

Assumption 1 $\Pi_{\tau|\tau\tau} < 0$, $\tau \in \{1, 2\}$.

This ensures that the producer's profits Π_τ are concave in the price p_τ and the second-order conditions for profit maximization are fulfilled.

2.2 Timing and equilibrium concept

Each period of the game includes the following two stages.
(I) The producer chooses the amount of production and the price for the good.
(II) Buyers purchase a quantity of the good and decide on the amount to be stockpiled.

The solution concept we adopt is the subgame perfect Nash equilibrium. The difference between the quantity produced and the sales in each period determines the producer's inventories available for sale in the following period. In line with the relevant literature (e.g., Anand et al. 2008; Arvan 1985; Arya et al. 2015; Arya and Mittendorf 2013; Mitraille and Moreaux 2013; Mollgaard et al. 2000; Krishnan and Winter 2010; Ware 1985), production and inventory decisions are observable and can therefore affect the buyer stockpiling behavior. As discussed in the

[10] This cost formulation isolates the strategic inventory incentives under investigation and neutralizes further possible inventory reasons. If we allow for decreasing or increasing economies of scale, the inventory incentives are magnified by standard technological motives. In the first case, the firm may engage in production smoothing over time, while in the second case higher production in a given period leads to lower unit costs. A change in production costs over time also affects the inventory behavior in a predictable manner.

[11] An alternative interpretation is that the producer attaches a higher weight on the future than the buyers.

introduction, reliable data about the production and inventory activities are publicly available, at least for large firms.[12] Krishnan and Winter (2010) assume that customers are perfectly informed about the firms' inventories available for sale and characterize in their expanded working paper (Krishnan and Winter 2009) the main channels through which this information can be acquired. For instance, in several markets intermediaries operate to collect information about a firm's inventory performance. Moreover, firms often advertise their availability and allow customers to view inventories online.[13]

Notably, our results hold even when buyers cannot perfectly observe the firm's production and inventory decisions. Following Mitraille and Moreaux (2013), it is sufficient that the producer is endowed with a communication technology that signals its actions in a relatively precise manner (Schelling 1960). The cost of this technology can be captured in our model by the inventory cost. Signals about the firm's production and inventory activities can be inferred from different sources, such as industry reports and midyear financial statements. As Dudine et al. (2006) suggest, buyers can be thought of as competitive arbitrageurs that should possess accurate information about the producer's financial situation.

3 Relevant benchmarks

3.1 Static problem

When buyer stockpiling is not feasible, the producer does not have any interest in holding inventories. In each period the firm produces the amount of the good that meets the current buyer demand. Formally, the producer's problem in period $\tau \in \{1, 2\}$ is given by

$$\max_{p_\tau} \left(p_\tau - c\right) D_\tau \left(p_\tau\right). \tag{4}$$

Setting $\tau = 2$ in the first-order condition $D_\tau \left(p_\tau\right) + \left(p_\tau - c\right) D_{\tau|\tau} \left(p_\tau\right) = 0$ yields the following auxiliary function

$$\phi_2(p_2) \equiv D_2 \left(p_2\right) + \left(p_2 - c\right) D_{2|2} \left(p_2\right), \tag{5}$$

which is helpful for our analysis. The derivative of (5) with respect to p_2 is negative, i.e., $\phi_{2|1} \left(p_2\right) < 0$ (by Assumption 1). The equilibrium static monopoly price in period $\tau \in \{1, 2\}$ is $p_\tau^m = c - \frac{D_\tau^m}{D_{\tau|\tau}^m}$. Note from (1) that for $p_1^m + s_b \geq p_2^m$ buyers do not exhibit any (strict) incentive for stockpiling. The producer's problem is trivial and corresponds to a replica of the static monopoly problem. In line with Dudine et al. (2006), hereafter we impose the following assumption, which ensures that the opportunity of buyer stockpiling affects the producer's intertemporal pricing problem.

Assumption 2 $p_1^m + s_b < p_2^m \Leftrightarrow s_b < \frac{D_1^m}{D_{1|1}^m} - \frac{D_2^m}{D_{2|2}^m}$.

[12] For instance Shell, one of the largest oil companies in the world, systematically publishes information about the production and inventory levels and their variations. Details are available at https://reports.shell.com/annual-report/2016/servicepages/download-centre.php.

[13] Yin et al. (2009) provide some evidence about the firms' inventory disclosure to their customers.

The buyer stockpiling cost s_b must be sufficiently small that buyers prefer to stockpile when the producer charges the static profit maximizing monopoly price in each period. As $s_b > 0$, Assumption 2 requires that the demand (for a given slope) grows over time.

3.2 Full commitment

To better appreciate the forces at play under limited commitment, we first consider the case where the producer is able to commit to a two-period pricing policy (p_1, p_2). The following lemma formalizes the equilibrium under full commitment.

Lemma 1 *The full commitment equilibrium exhibits the following features:*
(i) no producer's inventories, i.e., $I^c = 0$*;*
(ii) no buyer stockpiling, i.e., $D_s^c = 0$*;*
(iii) prices $p_1^c = c - \frac{D_1^c + \phi_2^c}{D_{1|1}^c}$ *and* $p_2^c = p_1^c + s_b$*, where* $p_1^c > p_1^m$ *and* $p_2^c < p_2^m$*.*

Lemma 1 indicates that the producer prefers to commit to a price sequence that fully removes buyer stockpiling. To this end, the producer increases the first period price and reduces the second period price relative to the static monopoly level, i.e., $p_1^c > p_1^m$ and $p_2^c < p_2^m$. The no-arbitrage constraint is binding, i.e., $p_2^c = p_1^c + s_b$, and buyers pay in the second period the cost they would incur for stockpiling, which allows the producer to extract the increased surplus. Indeed, buyers are indifferent between stockpiling or not, but no stockpiling takes place in equilibrium. This is because the producer could slightly reduce the second period price, which fully removes buyer stockpiling and yields a discontinuous increase in profits. The possibility of buyer stockpiling harms the producer, since it cannot equalize marginal revenues and marginal costs in the two periods. For our purposes, it is important to note that, under full commitment, inventories are not profitable for the producer, because they involve a mere loss associated with the inventory costs.

3.3 Limited commitment without inventories

Turning to the case where the producer cannot commit to future prices, the following lemma establishes the results in the absence of producer's inventories.

Lemma 2 *Under limited commitment, in the absence of producer's inventories one of the following outcomes arises:*
(a) buyer stockpiling, i.e., $D_s^{ns} = \phi_2^{ns}$*, and prices* $p_1^{ns} = c - \frac{D_1^{ns} + \phi_2^{ns} - s_b \phi_{2|1}^{ns}}{D_{1|1}^{ns}}$, $p_2^{ns} = p_1^{ns} + s_b$*;*
(b) no buyer stockpiling, i.e., $D_s^{nn} = 0$*, and prices* $p_1^{nn} = p_2^{nn} - s_b$, $p_2^{nn} = p_2^m$*.*

Lemma 2 replicates the results of Dudine et al. (2006), where the producer does not engage in inventory activities.[14] Notably, the full commitment outcome is no longer an equilibrium under limited commitment. To see this, recall from Lemma 1 that the producer finds it optimal to commit to a second period price below the static monopoly level in order to remove buyer stockpiling. When the producer cannot commit to future prices, it succumbs to the temptation

[14]Since we show the results of Dudine et al. (2006) in a manner that better fits our purposes, the proof of Lemma 2 is included in the Appendix.

to charge the second period monopoly price if buyers did not store in the first period. Anticipating this, buyers are eager to store, and the full commitment outcome is not achievable anymore. As Lemma 2(a) reveals, buyer stockpiling emerges in equilibrium. This outcome occurs when the demand in the second period is relatively high or buyer stockpiling is not too costly. To mitigate buyer stockpiling, the producer sets the price in the first period above the full commitment level. Since the no-arbitrage constraint is binding, the price exceeds the full commitment level in the second period as well. The buyer eagerness not to be exploited by the producer leads to higher prices. Buyers are indifferent to the quantity stored in equilibrium, and buyer stockpiling is endogenously determined by taking into account the producer's response.

As Lemma 2(b) indicates, when the demand growth is less pronounced or the buyer stockpiling cost is large enough, buyer stockpiling does not take place in equilibrium, although the no-arbitrage constraint is still binding. The price in the second period is set at the static monopoly level but the price in the first period is distorted upward (by Assumption 2). In line with Lemma 2(a), prices are higher than under full commitment in both periods.

4 Strategic inventories

We are now in a position to address the main issue of this paper and investigate the producer's incentives to hold inventories under limited commitment. Since the producer's inventories correspond to the difference between the quantity produced and the current sales, the producer's problem reduces to the choice of the price per period and the inventory level. Formally, using (2) and (3), we have

$$\begin{aligned} &\max_{p_1, I} \left(p_1 - c\right)\left[D_1\left(p_1\right) + D_s\left(p_1\right)\right] - \left(c + s_p\right) I \\ &+ p_2\left[D_2\left(p_2\right) - D_s\left(p_1\right)\right] - c\left[D_2\left(p_2\right) - D_s\left(p_1\right) - I\right] \cdot \mathbf{1}_{Q_2} \end{aligned} \tag{6}$$

subject to the following constraint of sequential optimality

$$p_2\left(D_s\left(p_1\right), I\right) \equiv \arg\max_{\widetilde{p}_2} \widetilde{p}_2\left[D_2\left(\widetilde{p}_2\right) - D_s\left(p_1\right)\right] - c\left[D_2\left(\widetilde{p}_2\right) - D_s\left(p_1\right) - I\right] \cdot \mathbf{1}_{Q_2}. \tag{7}$$

The relevant feature of the producer's problem is the interdependence of the choice variables across periods. Similarly to the setting of Dudine et al. (2006) described in Lemma 2, the first period price p_1 can influence the second period profits through the buyer stockpiling demand D_s in (1). The innovative aspect of our framework is the possibility that the producer engages in inventory activities. The producer accumulates inventories in the first period, which are available for sale in the second period. Inventories affect the producer's aggregate profits through two channels. A first effect of inventories arises from the fact that the aggregate inventory costs are incurred in the first period and therefore they are sunk in the second period. A second, more subtle, effect of inventories – that we investigate in the sequel – is their impact on the buyer behavior and specifically on the buyer stockpiling demand in equilibrium.

In our parsimonious model with constant marginal costs, the producer's choice of the inventory level reduces to a binary decision. Either the producer abstains from holding inventories at all, i.e., $I = 0$, or it accumulates the amount of inventories that covers the second period demand net of buyer stockpiling, i.e., $I = D_2 - D_s$, which drives the producer's second period costs to zero. Any production in the second period in addition to inventories would entail a marginal cost equal to c, which translates into a price at the static second period monopoly level. Anticipating this, the firm would prefer to produce the whole quantity in the second period and to avoid the inventory costs. Clearly, any inventory level above the second period (net) demand is also suboptimal.

We know from Lemma 1 that inventories are not profitable for a producer with full commitment powers. However, when the producer cannot commit to future prices, it succumbs to the temptation to adjust the price in the second period in response to the buyer stockpiling behavior. In the following proposition, we characterize the producer's incentives to hold inventories as a strategic device to mitigate the loss from the lack of commitment.

Proposition 1 *Suppose $c \geq \widetilde{c}$, where $\widetilde{c}$ is defined by (11) in the Appendix. Then, under limited commitment, there exists a threshold $\widetilde{s}_p > 0$ for the inventory cost such that for $s_p \leq \widetilde{s}_p$ the limited commitment equilibrium exhibits the following features:*

(i) producer's inventories, i.e., $I^* = D_2^*$;

(ii) no buyer stockpiling, i.e., $D_s^* = 0$;

(iii) prices $p_1^* = c - \frac{D_1^* + \phi_2^* - s_p D_{2|1}^*}{D_{1|1}^*}$ and $p_2^* = p_1^* + s_b$.

For $s_p = 0$, the full commitment outcome is restored.

Proposition 1 indicates that under certain circumstances the producer benefits from using inventories for strategic purposes. The amount of inventories covers the second period demand and maximizes the producer's ex ante profits in (6), ignoring the constraint of sequential optimality in (7). In the light of the producer's lack of commitment, this inventory strategy can be sustained in equilibrium only if selling exactly the ex ante optimal amount of inventories maximizes the second period profits, namely, it is sequentially optimal. Put differently, the producer must not have any incentive to revise its decision and to sell in the second period a quantity that differs from the inventory level.

The solution in Proposition 1 is implementable when the cost of production c is above the threshold $\widetilde{c}$ defined by (11) in the Appendix, where $\widetilde{c} > 0$ for values of the inventory cost s_p small enough (by Assumption 2). *Prima facie*, this could seem counterintuitive, since one might expect that a high cost of production undermines the desirability of inventories. The rationale for this result stems from the strategic nature of inventories under limited commitment. In the extreme case where $c = 0$, the producer cannot refrain from charging a second period price at which marginal revenues fall to zero, irrespective of the amount of inventories. For $c > 0$, inventories are potentially beneficial, since they reduce the second period costs, which translates into a lower second period price and mitigates the buyer stockpiling incentives.

As Figure 1 illustrates, any ex ante optimal inventory level I^* such that $D_2(p_2^m) \leq I^* = D_2^* \leq D_2(p_2^m|_{c=0})$ maximizes the second period profits as well. In other terms, the constraint of sequential optimality is slack in equilibrium. As formally shown in the proof of Proposition 1 in the Appendix, when the cost of production is relatively large, i.e., $c \geq \widetilde{c}$, the second period price at which inventories are fully exhausted is higher than the monopoly price at zero costs, i.e., $p_2^* \geq p_2^m|_{c=0}$. A price increase from p_2^* results in a mere loss, since the marginal revenue at

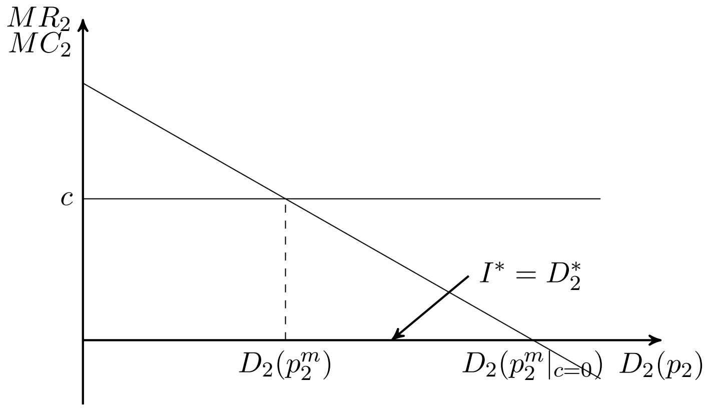


Figure 1: Strategic inventories

$I^* = D_2^* \leq D_2\left(p_2^m\,|_{c=0}\right)$ is positive and therefore the lower sales due to a higher price reduce the producer's revenues, while costs remain at zero. The idea is that a sufficiently large cost of production curbs the amount of inventories so that the producer does not have any incentive to raise its price in the second period and to discard some inventories. Moreover, for $p_2^* \leq p_2^m$ the inventory level $I^* = D_2^* \geq D_2\left(p_2^m\right)$ is such that the marginal cost c of the additional production associated with a price reduction from p_2^* outweighs the corresponding marginal revenue. Hence, in the second period the producer does not have any incentive to reduce its price from the level p_2^* at which inventories are fully exhausted. We know from Lemma 1 that the second period price under full commitment is below the static monopoly level, i.e., $p_2^c < p_2^m$. Since $p_2^* = p_2^c$ for $s_p = 0$, we have that $p_2^* \leq p_2^m$ when s_p is small enough. Consequently, for $c \geq \widetilde{c}$ the full commitment outcome in Lemma 1 can be replicated under limited commitment when holding inventories is costless, i.e., $s_p = 0$. In other terms, the producer is able to allocate the ex ante optimal quantity in the second period at no additional cost. By continuity, as long as s_p is sufficiently small, the producer shall have incentives to hold inventories, although this generates a price distortion above the full commitment level.

Since buyers anticipate that the producer will not revise its price relative to the level at which inventories are fully exhausted and their cost is sunk in the second period, inventories act as a strategic device to credibly reduce future prices. This weakens the buyer stockpiling incentives and mitigates the producer's loss from the lack of commitment.[15] In order to deal with buyer stockpiling, the producer resorts to inventories as a complementary instrument to prices, which is perfect as long as holding inventories is costless. As mentioned in the introduction, the oil market is a particularly suitable example for our purposes, since it exhibits relatively large costs of production and negligible inventory costs.

Given that the no-arbitrage constraint is binding, buyers are indeed indifferent to storing. As under full commitment, no buyer stockpiling occurs in equilibrium, since a slight rise in the first period price fully removes buyer stockpiling while preserving sequential optimality, which yields a discontinuous increase in profits (the constraint of sequential optimality is slack in equilibrium). A similar result would obtain with a slight rise in the producer's inventories, which translates into a lower second period price.

[15] It follows from our previous discussion that introducing a cost at which the producer can discard or destroy its inventories would reinforce this result.

As shown in Figure 1, any ex ante optimal inventory level I^* such that either $I^* < D_2(p_2^m)$ or $I^* > D_2(p_2^m|_{c=0})$ is not sequentially optimal and cannot be sustained in equilibrium. When the cost of production is relatively small, i.e., $c < \widetilde{c}$, the second period price at which inventories are fully exhausted is lower than the monopoly price at zero costs, i.e., $p_2^* < p_2^m|_{c=0}$. The inventory level $I^* > D_2(p_2^m|_{c=0})$ is excessive from the second period perspective, since it entails negative marginal revenues. The producer succumbs to the temptation to increase the second period price at $p_2^m|_{c=0}$ and to discard some inventories. When the inventory cost s_p is sufficiently large, the second period price at which inventories are fully exhausted exceeds the static monopoly level, i.e., $p_2^* > p_2^m$. In this case, the inventory level $I^* < D_2(p_2^m)$ does not suffice to maximize the second period profits, since the marginal revenue at I^* is higher than the marginal cost c. The firm is inclined to reduce the price at p_2^m and to produce some additional quantity.

Since the result in Proposition 1 holds for a sufficiently large cost of production, a natural issue is whether the strategic role of inventories persists when the cost of production is relatively small and therefore the ex ante optimal inventory level violates the constraint of sequential optimality. To this end, we begin with the characterization of the possible inventory outcomes in the following lemma.

Lemma 3 *Suppose $c < \widetilde{c}$. Then, under limited commitment, if the producer holds inventories, one of the following outcomes arises:*

(a) producer's inventories and buyer stockpiling, i.e., $I^{is} = -p_2^{is} D_{2|1}^{is}$, $D_s^{is} = \phi_2^{is} + c D_{2|1}^{is}$, and prices $p_1^{is} = c - \frac{D_1^{is} + \phi_2^{is} - s_p D_{2|1}^{is} - (s_b - s_p)\left(\phi_{2|1}^{is} + c D_{2|11}^{is}\right)}{D_{1|1}^{is}}$, $p_2^{is} = p_1^{is} + s_b$;

(b) producer's inventories but no buyer stockpiling, i.e., $I^{in} = D_2^{in}$, $D_s^{in} = 0$, and prices $p_1^{in} = p_2^{in} - s_b$, $p_2^{in} = p_2^m|_{c=0}$.

We know from the discussion following Proposition 1 that, if the cost of production is relatively small, i.e., $c < \widetilde{c}$, the amount of inventories that maximizes the producer's ex ante profits is excessive from the second period perspective and cannot be sustained in equilibrium. The inventory level must be distorted below the ex ante optimal level, and the constraint of sequential optimality is binding in equilibrium. Inventories still mitigate the buyer stockpiling incentives since they involve a reduction in future prices, but buyer stockpiling is not fully removed. Similarly to Lemma 2(a), buyer stockpiling emerges in Lemma 3(a) when the second period demand is relatively high or the buyer stockpiling cost is small enough. Since the no-arbitrage constraint is binding, buyers are indeed indifferent to storing in equilibrium, and buyer stockpiling is endogenously derived by taking into account the producer's response.

Interestingly, we find that a lower inventory cost leads to higher prices as long as the second period demand is not too convex. To understand the rationale for this surprising result, it is important to note from Lemma 3(a) that a higher price reduces the buyer stockpiling $D_s^{is}(p_1)$, which translates into a higher second period net demand $D_2(p_1 + s_b) - D_s^{is}(p_1)$ when the second period demand D_2 is not too convex. Since the producer's inventories coincide with the second period net demand, cheaper inventories make it more attractive for the producer to expand the second period net demand, which requires higher prices.[16]

[16]Using (5), we find that $D_{s|1}^{is}(p_1) = 2D_{2|1}(p_1 + s_b) + (p_1 + s_b) D_{2|11}(p_1 + s_b) < 0$, where the inequality follows

Lemma 3(b) reveals that, in line with Lemma 2(b), when the second period demand is relatively low or the buyer stockpiling cost is high enough, buyer stockpiling does not take place in equilibrium. Since inventories cover the second period demand, the producer sets the second period price at the monopoly level with zero costs.

We can now present the following result.

Proposition 2 *Suppose $0 < c < \widetilde{c}$. Then, under limited commitment, as long as in the absence of producer's inventories buyer stockpiling arises, holding inventories is profitable for the producer if the inventory cost s_p and the cost of production c are small enough.*

Inventories can be profitable for the producer despite the fact that the solution in Proposition 1 is not implementable. One of the outcomes in Lemma 3 emerges in equilibrium. The benefit of inventories still consists in the reduction in the second period price, which mitigates the buyer stockpiling incentives. Differently from Proposition 1, it follows from Lemma 3 that the cost of holding inventories is not only the mere physical inventory cost but also the loss due to the distortion from the ex ante optimal inventory level to ensure sequential optimality. In other terms, when the cost of production is small enough, inventories are a less effective instrument to mitigate the buyer stockpiling incentives. Consequently, buyer stockpiling can persist in equilibrium. A comparison between the results in Propositions 1 and 2 reveals that a larger cost of production improves the strategic value of inventories.

As formally shown in the Appendix, Proposition 2 provides sufficient, albeit not necessary, conditions for the profit superiority of inventories, which ensure that the producer can replicate the equilibrium pricing policy without inventories described in Lemma 2(a) and be better off due to a reduction in buyer stockpiling. In particular, since from Lemmas 2(a) and 3(a) the buyer stockpiling difference evaluated at p_1^{ns} is $D_s^{ns}\left(p_1^{ns}\right) - D_s^{is}\left(p_1^{ns}\right) = -cD_{2|1}^{ns} > 0$ (for $c > 0$), a sufficiently small (but positive) cost of production c implies that, for any buyer stockpiling level $D_s^{ns}\left(p_1^{ns}\right) > 0$ without inventories, there exists a lower feasible buyer stockpiling level $D_s^{is}\left(p_1^{ns}\right) \geq 0$ with inventories. *A fortiori*, the producer's benefits are larger when equilibrium prices with inventories are considered.

5 Price comparisons

The following proposition formalizes the price comparisons across different scenarios. Some threshold values are derived in the Appendix.

Proposition 3 *For $\tau \in \{1, 2\}$, the following price orderings hold:*

(i) $p_\tau^c \leq p_\tau^* < p_\tau^{ns} < p_\tau^{nn}$, where $p_\tau^c = p_\tau^*$ if and only if $s_p = 0$ and $p_\tau^* < p_\tau^{ns}$ if and only if $s_p < \overline{s}_p$;

from Assumption 1 (second-order condition for the maximization of static second period profits with zero costs). It can be immediately shown that the sign of the derivative of $D_2\left(p_1 + s_b\right) - D_s^{is}\left(p_1\right) = -\left(p_1 + s_b\right) D_{2|1}\left(p_1 + s_b\right)$ with respect to p_1 is positive if and only if $D_{2|11}\left(p_1 + s_b\right) < -\frac{D_{2|1}\left(p_1+s_b\right)}{p_1+s_b}$. This is the same condition under which p_1^{is} decreases with s_p. Differentiating the left-hand side of the first-order condition for p_1^{is} in (12) in the Appendix with respect to s_p yields after some manipulation $\left(p_1 + s_b\right) D_{2|11}\left(p_1 + s_b\right) + D_{2|1}\left(p_1 + s_b\right)$. It follows from the implicit function theorem that $\frac{\partial p_1^{is}}{\partial s_p} < 0$ if and only if this expression is negative, i.e., $D_{2|11}\left(p_1 + s_b\right) < -\frac{D_{2|1}\left(p_1+s_b\right)}{p_1+s_b}$.

(ii) $p_\tau^{is} < p_\tau^{in} \leq p_\tau^{nn}$, *where* $p_\tau^{in} = p_\tau^{nn}$ *if and only if* $c = 0$;

(iii) $p_\tau^{is} < p_\tau^{ns}$ *if and only if one of the following conditions holds: (a)* $D_{2|11}^{ns} \leq 0$ *and* $s_p > \widehat{s}_p$; *(b)* $0 < D_{2|11}^{ns} < \widehat{D}_{2|11}$; *(c)* $D_{2|11}^{ns} \geq \widehat{D}_{2|11}$ *and* $s_p < \widehat{s}_p$.

Proposition 3 delivers results of some interest. We begin with the price comparisons collected in point (i). As discussed in Section 4, the solution in Proposition 1, where the ex ante optimal inventory level is also sequentially optimal, leads to prices distorted above the full commitment level as long as holding inventories is costly, i.e., $p_\tau^c < p_\tau^*$ for $s_p > 0$. This is because the producer passes a part of these costs on to the buyers. The full commitment solution can be replicated if and only if holding inventories is costless, i.e., $p_\tau^c = p_\tau^*$ for $s_p = 0$.

Inventories allow a producer with limited commitment powers to generally charge lower prices, i.e., $p_\tau^* < p_\tau^{ns}$ and $p_\tau^* < p_\tau^{nn}$. The result that, despite being costly, inventories lead to lower prices stems from their strategic role in mitigating future costs. In particular, we find that $p_\tau^* < p_\tau^{ns}$ as long as the inventory cost s_p is small enough, i.e., $s_p < \overline{s}_p$. Using D_s^{ns} in Lemma 2(a), it follows from (13) in the Appendix that this condition corresponds to $s_p D_{2|1}^{ns} > s_b D_{s|1}^{ns}$. The term on the left-hand side captures the inventory cost effect, whose sign is negative. This is because a higher price reduces the second period demand, which coincides with the producer's inventories, and therefore mitigates the aggregate inventory costs. This effect is more pronounced when the unit inventory cost s_p is higher. The term on the right-hand side captures the buyer stockpiling effect, whose sign is also negative ($D_{s|1}^{ns} = \phi_{2|1}^{ns} < 0$ by Assumption 1, where $\phi_2(.)$ is defined by (5)). The idea is that a higher price reduces the buyer stockpiling D_s^{ns} and the associated loss for the producer. The comparison between these two effects implies that, if the inventory cost s_p is sufficiently small, i.e., $s_p < \overline{s}_p$, the buyer stockpiling effect dominates (in absolute terms) and prices are lower in the presence of inventories, as expected. However, we cannot dismiss the opposite case, which seems at first sight less intuitive. To see this, note that when the second period demand is sufficiently convex the price impact on the buyer stockpiling D_s^{ns} in Lemma 2(a) is negligible ($D_{s|1}^{ns} = \phi_{2|1}^{ns} \to 0$). Hence, the inventory cost effect dominates (in absolute terms) even for relatively low values of s_p and prices are higher when the producer holds inventories. The reason is that with a sufficiently convex demand higher prices are more beneficial to the reduction in the aggregate inventory costs than to the reduction in buyer stockpiling. It is important to realize that, as long as the inventory cost is not too large, inventories are profitable for the producer since they remove buyer stockpiling, although equilibrium prices may rise. The fact that buyer stockpiling disappears despite higher prices is only apparently a contradiction. In equilibrium, the no-arbitrage constraint is binding irrespective of the producer's inventories, and therefore buyers are indeed indifferent between stockpiling or not.

We can also see from point (i) that $p_\tau^* < p_\tau^{nn}$, where $p_2^{nn} = p_2^m$ by Lemma 2(b). As discussed after Proposition 1, the equilibrium second period price with inventories must be lower than the static monopoly level in order to be implementable.[17] Moreover, we have $p_\tau^{ns} < p_\tau^{nn}$, since buyer stockpiling reduces the demand (and the price) in the second period. An immediate implication of the results in point (i) is that, when the cost of production is sufficiently large that the ex

[17]Note that a lower price in one period implies a lower price in the other period as well, since the no-arbitrage constraint is always binding in equilibrium, i.e., $p_2 = p_1 + s_b$.

ante optimal level of inventories is also sequentially optimal, the producer's inventories lead to lower prices as long as the inventory cost is small enough. Therefore, consumer surplus and profits are higher, and inventory activities unambiguously enhance social welfare.

As point (ii) of Proposition 3 reveals, when the solution in Proposition 1 is not implementable and the producer's inventories must be distorted from the ex ante optimal level to ensure sequential optimality, prices are still lower with respect to the situation where no storing occurs in the economy. In particular, when the producer holds inventories, buyer stockpiling leads to lower prices, i.e., $p_\tau^{is} < p_\tau^{in}$, since it dampens the demand (and the price) in the second period. Moreover, in the absence of buyer stockpiling, the producer's inventories ensure lower prices unless production is costless, i.e., $p_\tau^{in} \leq p_\tau^{nn}$, where the equality holds if and only if $c = 0$.

Point (iii) of Proposition 3 indicates that the welfare conclusions drawn from point (i) deserve some qualifications when inventories are distorted from the ex ante optimal level. Using $D_s^{ns}(p_1)$ and $D_s^{is}(p_1)$ in Lemmas 2(a) and 3(a), it follows from (14) in the Appendix that $p_\tau^{is} < p_\tau^{ns}$ if and only if $s_p \left[D_{2|1}(p_1^{ns}) - D_{s|1}^{is}(p_1^{ns})\right] > s_b \left[D_{s|1}^{ns}(p_1^{ns}) - D_{s|1}^{is}(p_1^{ns})\right]$. Differently from point (i), the inventory cost effect on the left-hand side is now positive as long as the second period demand is not too convex, i.e., $D_{2|11}^{ns} < \widehat{D}_{2|11}$, where $\widehat{D}_{2|11} > 0$. To understand why, recall from the discussion following Lemma 3 that higher prices decrease buyer stockpiling and, provided that the second period demand is not too convex, this translates into a higher second period demand net of buyer stockpiling, which coincides with the producer's inventories. Hence, lower prices mitigate the aggregate inventory costs. A higher s_p strengthens the inventory cost effect. Furthermore, differently from point (i), since buyer stockpiling persists with the producer's inventories, the buyer stockpiling effect on the right-hand side now concerns the buyer stockpiling difference, whose amount evaluated at p_1^{ns} is $D_s^{ns}(p_1^{ns}) - D_s^{is}(p_1^{ns}) = -cD_{2|1}^{ns}$, as it follows from Lemmas 2(a) and 3(a). The positive (negative) sign of this effect means that a higher price is more (less) helpful for the reduction in buyer stockpiling when the producer holds inventories. If the second period demand is concave, i.e., $D_{2|11}^{ns} \leq 0$, the buyer stockpiling effect is also positive. The comparison between the two effects implies that the inventory cost effect dominates and prices are lower when the producer holds inventories if the second period demand is concave and the unit inventory cost is high enough, i.e., $p_\tau^{is} < p_\tau^{ns}$ if $D_{2|11}^{ns} \leq 0$ and $s_p > \widehat{s}_p$, as point (iii-a) of Proposition 3 indicates.

If the second period demand is convex, i.e., $D_{2|11}^{ns} > 0$, the buyer stockpiling effect is negative. This implies that, if the second period demand is only moderately convex, i.e., $0 < D_{2|11}^{ns} < \widehat{D}_{2|11}$, the two effects push in the same direction and prices are lower when the producer holds inventories, i.e., $p_\tau^{is} < p_\tau^{ns}$, as point (iii-b) reveals.

If the second period demand is sufficiently convex, i.e., $D_{2|11}^{ns} \geq \widehat{D}_{2|11}$, the producer's inventories decrease rather than increase with prices, and the inventory cost effect is now negative. Since the buyer stockpiling effect is also negative with convex demand, we find that the latter effect dominates (in absolute terms) and the producer sets lower prices when holding inventories if the second period demand is sufficiently convex and the unit inventory cost is small enough, i.e., $p_\tau^{is} < p_\tau^{ns}$ if $D_{2|11}^{ns} \geq \widehat{D}_{2|11}$ and $s_p < \widehat{s}_p$, as point (iii-c) establishes. Note that, similarly to point (i), inventories may lead to higher prices. The difference is that, since inventories are now distorted from the ex ante optimal level, buyer stockpiling is not fully removed, which makes price comparisons more convoluted.

6 Linear demand

We now consider a linear demand of the form $D_\tau(p_\tau) = \alpha_\tau - p_\tau$, $\tau \in \{1, 2\}$, which allows an explicit characterization of the results. It follows from the producer's problem in (4) that the equilibrium static monopoly price in period τ is $p_\tau^m = \frac{\alpha_\tau + c}{2}$. Assumption 2 becomes the following.

Assumption 2$'$ $p_1^m + s_b < p_2^m \Leftrightarrow s_b < \frac{\alpha_2 - \alpha_1}{2}$.

Since $s_b > 0$, Assumption 2$'$ requires that the demand must be higher in the second period, i.e., $\alpha_2 > \alpha_1$.

The formal results are collected in the Supplementary Appendix. The condition about the costs of production in Proposition 1 is $c \geq \widetilde{c} \equiv \frac{\alpha_2 - \alpha_1 - 2s_b - s_p}{2}$. This ensures that any ex ante optimal inventory level is not excessive from the second period perspective. Notably, a higher buyer stockpiling cost s_b reduces the threshold $\widetilde{c}$. This is because more expensive buyer stockpiling induces a higher second period price, which alleviates the producer's temptation to increase the price above the level at which inventories are fully exhausted and to discard a portion of them in the second period. In the same vein, a higher inventory cost s_p also leads to a reduction in the threshold $\widetilde{c}$. However, recall from the discussion after Proposition 1 that a high inventory cost translates into a second period price above the static monopoly level, which is not sequentially optimal and cannot be sustained in equilibrium.

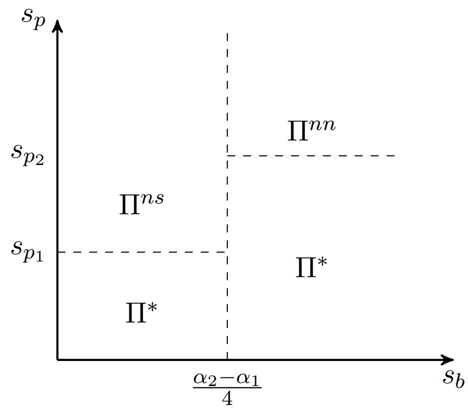
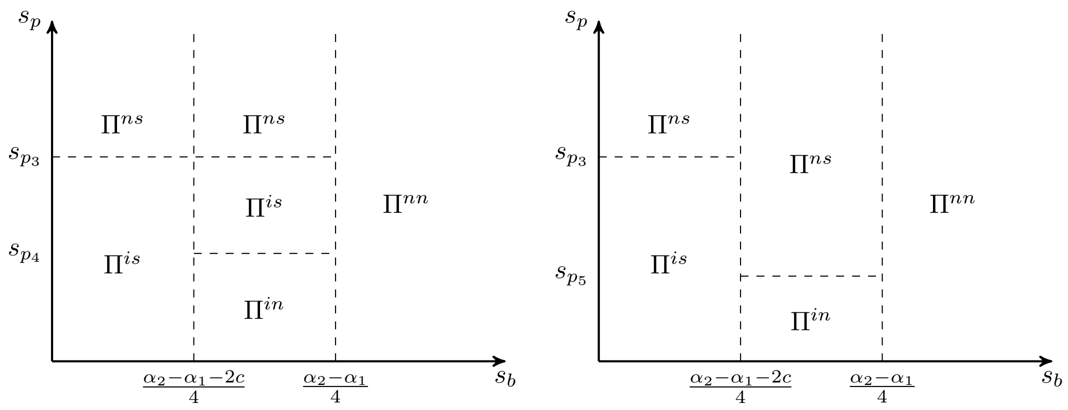


Figure 2: Producer's profits for $c \geq \widetilde{c}$

Figure 2 illustrates the case where the cost of production is relatively large, i.e., $c \geq \widetilde{c}$. Suppose first that the demand growth is sufficiently pronounced or the buyer stockpiling cost is relatively small, i.e., $s_b < \frac{\alpha_2 - \alpha_1}{4}$. If the inventory cost is relatively low, i.e., $s_p \leq s_{p_1}$, the producer holds the ex ante optimal level of inventories and fully removes buyer stockpiling, as Proposition 1 indicates. Otherwise, as in the most interesting scenario in Dudine et al. (2006) formalized in Lemma 2(a), the producer does not engage in inventory activities and buyer stockpiling emerges in equilibrium. When the demand growth is relatively moderate or the buyer stockpiling cost is high enough, i.e., $s_b \geq \frac{\alpha_2 - \alpha_1}{4}$ (still satisfying Assumption 2$'$), buyer stockpiling never occurs in equilibrium. In particular, when the inventory cost is low enough, i.e., $s_p \leq s_{p_2}$, the producer holds inventories in equilibrium. Otherwise, the no-storing outcome characterized in Lemma 2(b) holds.

Note from Figure 2 that a higher buyer stockpiling cost widens the scope for strategic inventories. Since buyers are willing to pay a higher second period price, the surplus from the avoided buyer stockpiling that the producer can extract is larger. As formally shown in the Supplementary Appendix, the threshold values s_{p_1} and s_{p_2} below which holding inventories is profitable for the producer increase with the cost of production c. In line with the discussion following Proposition 1, the idea is that a higher c amplifies the reduction in future prices associated with the producer's inventories, which translates into a wider range of values for s_p that supports inventories in equilibrium.



(a) $\min\{s_{p_4}, s_{p_5}\} = s_{p_4} \Leftrightarrow \alpha_2$ high enough (b) $\min\{s_{p_4}, s_{p_5}\} = s_{p_5} \Leftrightarrow \alpha_2$ low enough

Figure 3: Producer's profits for $c < \widetilde{c}$

Now, we turn to the case where the cost of production is relatively small, i.e., $c < \widetilde{c}$, which implies that the ex ante optimal inventory level is excessive from the second period perspective. As Figure 3 illustrates, there is still scope for strategic inventories, although things become more involved. Suppose first that the demand growth is sufficiently high or the buyer stockpiling cost is relatively small, i.e., $s_b < \frac{\alpha_2-\alpha_1-2c}{4}$. In line with the case $c \geq \widetilde{c}$, the producer finds it optimal to hold inventories when the inventory cost is sufficiently small, i.e., $s_p \leq s_{p_3}$. The key difference is that, since the amount of inventories is distorted from the ex ante optimal level in order to ensure sequential optimality, buyer stockpiling cannot be fully removed. This corresponds to the outcome in Lemma 3(a). When the inventory cost is high enough, i.e., $s_p > s_{p_3}$, only buyer stockpiling emerges in equilibrium.

For intermediate values of the buyer stockpiling cost, i.e., $\frac{\alpha_2-\alpha_1-2c}{4} < s_b < \frac{\alpha_2-\alpha_1}{4}$, the producer can still benefit from using inventories. In particular, if the inventory cost is low enough, i.e., $s_p \leq \min\{s_{p_4}, s_{p_5}\}$, the producer holds inventories and buyer stockpiling vanishes, as Lemma 3(b) predicts. Moreover, when the demand growth is sufficiently pronounced – which means that $\min\{s_{p_4}, s_{p_5}\} = s_{p_4}$, as formally shown in the Supplementary Appendix – the producer prefers to hold inventories and allow buyer stockpiling, provided that the inventory cost is not too high, i.e., $s_{p_4} < s_p \leq s_{p_3}$. Otherwise, only buyer stockpiling materializes.

The comparison between panels (a) and (b) of Figure 3 shows that a higher second period demand enlarges the scope for strategic inventories. This is because a higher demand makes it more profitable for the producer to stimulate the second period sales through a reduction in buyer stockpiling induced by the inventory activities.

When the buyer stockpiling cost is high enough, i.e., $s_b \geq \frac{\alpha_2 - \alpha_1}{4}$, no storing takes place in the economy. It is worth noting that, differently from the case $c \geq \widetilde{c}$, the producer never profits from inventories. Buyer stockpiling does not occur even in the absence of inventories and the full commitment solution cannot be approached since inventories are distorted from the ex ante optimal level to satisfy sequential optimality.

7 Robustness and extensions

7.1 Competition

The presence of competition among producers definitely deserves some attention. As discussed in the introduction, competitive markets for storable goods have been examined in the literature. For our purposes, it is helpful to consider two relevant contributions that focus on storing activities on the two opposite sides of the market. In a two-period Cournot duopoly model that allows for buyer stockpiling, Anton and Das Varma (2005) show that each firm prefers to reduce its price in the first period and to capture the resulting buyer stockpiling rather than share the demand with the rival in the second period. The possibility of buyer stockpiling exacerbates competition and leads to lower prices. The firms' incentives to attract buyer stockpiling are stronger when the demand growth is smaller, since the second period market is less relevant. Conversely, in our setting as well as in Dudine et al. (2006), buyer stockpiling is profit detrimental when the demand growth is high enough. Therefore, we expect that, if the demand does not increase significantly over time, the firms' incentives to compete for buyer stockpiling à la Anton and Das Varma (2005) will prevail. However, when the demand growth is high enough, the firms' incentives to mitigate buyer stockpiling and share the demand in the following period should be dominant, as in our setting. The predictions of our model naturally extend to a competitive environment, and firms will hold inventories to reduce future prices and discourage buyer stockpiling. A natural implication is that, when competition is intense, firms tend to behave aggressively and stimulate buyer stockpiling. In the presence of higher degrees of market concentration or product differentiation, firms can internalize to a larger extent the strategic dynamic benefits of inventories driven by the lower buyer stockpiling incentives. This provides theoretical corroboration for the empirical evidence documented by Amihud and Mendelson (1989) that a larger level of inventories is associated with greater market power.

A second contribution relevant for our purposes is Mitraille and Moreaux (2013), which abstracts from buyer stockpiling and considers a two-period model where identical firms accumulate inventories in the first period and engage in Cournot competition in the second period. Inventories allow some firms to exert endogenously a Stackelberg leadership over the rivals. The main results are derived for a relatively low demand. This condition is likely to fail in our framework, where the demand grows in the second period. Therefore, we provide a complementary explanation for inventories that generally applies to different circumstances from those analyzed in Mitraille and Moreaux (2013). Notably, when the two inventory incentives coexist, they are mutually reinforcing.

7.2 Number of periods

A natural extension of our framework is to allow for a number of periods larger than two. We feel that a two-period model is the most adequate for various reasons aside from its analytical tractability. Demand predictions are likely to be accurate only in the near future. Moreover, storable goods generally depreciate over time and can be stockpiled only for a limited period. Nonetheless, it is of some interest to incorporate a larger time horizon into our analysis.

To fix ideas, consider a three-period model where the demand increases over time so that buyers are willing to stockpile across periods when the producer charges the static monopoly price. Production costs are sufficiently large that the producer does not want to discard some inventories after the second period starts. For the sake of simplicity, suppose for the time being that holding inventories is costless. We know from Proposition 1 that under these conditions the full commitment outcome can be restored in a two-period setting. A novel feature now emerges, since the producer may have an incentive to reallocate the full commitment quantities between the second and the third period given that their associated costs are sunk. When the third period marginal revenues are larger than the second period marginal revenues, i.e., $MR_3\left(D_3^c\right) > MR_2\left(D_2^c\right)$, the producer succumbs to the temptation to carry some quantity from the second to the third period and to increase the second period price. Anticipating this, buyers decide to store in the first period and the full commitment outcome is not feasible anymore. However, things are different in the opposite case where marginal revenues are larger in the second period than in the third period, i.e., $MR_2\left(D_2^c\right) > MR_3\left(D_3^c\right)$. Now, the producer would like to sell more in the second period, but it does not do so since a lower second period price would trigger buyer stockpiling in the same period, which is profit detrimental. Therefore, the full commitment outcome can be restored, as in the baseline model. This is the case when the demand is expected to rise significantly in the third period, since the resulting price pattern $p_2^m < p_2^c < p_3^c < p_3^m$ implies that $MR_2\left(D_2^c\right) > c > MR_3\left(D_3^c\right)$. By continuity, the same ordering of marginal revenues persists for $MR_2\left(D_2^c\right) < c$. Notably, it always holds in the extended version of the linear model described in Section 6.[18] Similar results clearly emerge when the inventory cost is small enough, and therefore the predictions of our model generalize to a larger time horizon.

7.3 Forward contracts

In practice, a potentially valuable instrument to which a firm can resort in some markets to restore its commitment powers is a forward or futures contract that specifies the trade of a good at a given price, with delivery and payment occurring at a future point.[19] We argue that in our framework forward contracting cannot remove the commitment problem. The presence of buyers that act in the market as arbitrageurs or speculators and are not interested in final

[18] Applying the framework formalized in the Supplementary Appendix to a three-period model with demand $D_\tau = \alpha_\tau - p_\tau$, $\tau \in \{1,2,3\}$, we find that the full commitment prices are $p_1^c = \frac{\alpha_1+\alpha_2+\alpha_3+3(c-2s_b)}{6}$, $p_2^c = p_1^c + s_b$ and $p_3^c = p_1^c + 2s_b$. Standard computations yield $MR_2\left(D_2^c\right) - MR_3\left(D_3^c\right) = \alpha_3 - \alpha_2 - 2s_b > 0$, where the inequality follows from the condition $p_2^m + s_b < p_3^m$ (direct extension of Assumption $2'$ in a three-period model).

[19] This may also apply to the setting of Dudine et al. (2006).

consumption generates ex post incentives for contract renegotiation between the producer and the buyers. Indeed, forward contracts are rarely executed in reality.

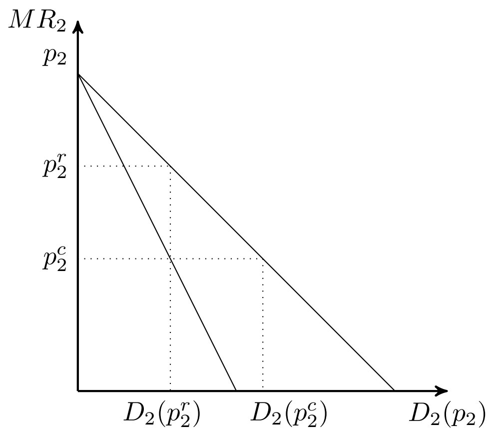


Figure 4: Forward contracts

To understand the rationale for this result, consider a contract that commits the producer to deliver to the buyers in the second period a quantity $D_2(p_2^c)$ at a unit price p_2^c, which potentially replicates the full commitment solution. If the contract is perfectly enforced, buyers clearly make zero profits, since they resell the good to final consumers at the same price p_2^c. With the help of Figure 4 we can see that the contract renegotiation in the second period is mutually beneficial. Suppose that a new contract is proposed according to which the producer retains the quantity $D_2(p_2^r) < D_2(p_2^c)$ in exchange for a two-part tariff specifying a unit price p_2^c and a fixed fee. Since the producer can charge a unit price $p_2^r > p_2^c$ to final consumers, the fixed fee allows the producer to share the resulting gains from trade with the buyers. The residual quantity $D_2(p_2^c) - D_2(p_2^r)$ may be regularly delivered by the producer at p_2^c and sold to final consumers at the same price. The producer is able to engage in price discrimination among final consumers and to increase the second period profits relative to full commitment. Consequently, the full commitment solution collapses.

Notably, the fact that buyers purchase the entire full commitment quantity $D_2(p_2^c)$ through forward contracting is irrelevant. Our rationale carries over even when final consumers also participate in the forward market, although this seems less plausible to occur in practice. The crucial point is that the presence of buyers that are not interested in final consumption prevents forward contracts from restoring the full commitment solution.

7.4 Discount factor

Throughout the analysis, we assume that the producer attaches the same weight to its current and future profits, namely, the discount factor equals 1. In line with Dudine et al. (2006), this captures in a simple manner the economic relevance of the producer's inability to commit to future prices. Our qualitative results clearly extend to a more general discount factor, provided that the producer places enough weight on the future. Otherwise, the producer's prominent attention to its current profits makes buyer stockpiling beneficial and the limited commitment problem becomes inconsequential, which removes the scope for strategic inventories. In our framework, the inventory cost s_p can be interpreted as a measure of the weight on the future in the firm's current production and inventory decisions. A lower s_p reflects a greater interest in accumulating inventories for future sales at a higher price. Therefore, our model predicts that the inventories of a producer with a longer business horizon mitigate to a larger extent the loss from the lack of limited commitment. Note that, while the discount factor generally applies to the entire profits, the inventory cost only directly affects the production and inventory activities. Despite this difference, since a higher discount factor makes the producer's limited commitment problem more significant, we expect that a larger strategic use of inventories will be associated with a higher discount factor (or lower interest rates), analogously to what we find with a lower inventory cost.

The buyer stockpiling cost s_b captures the buyer discount factor in an intuitive manner. The buyer stockpiling behavior affects the producer's intertemporal problem more prominently when buyers are more forward-looking, namely, the buyer stockpiling cost is lower.

8 Conclusions: managerial, empirical and policy implications

A full understanding of the firms' inventory behavior is a challenging task that goes well beyond our study. In this paper, we unveil a strategic channel for inventories that complements the traditional inventory theories. We characterize the producer's strategic incentives to hold inventories in a dynamic storable good monopoly framework à la Dudine et al. (2006) where the producer is unable to commit to future prices and forward-looking buyers can stockpile in expectation of higher future prices. Anticipating that the producer cannot refrain from charging a higher future price than under full commitment, buyers engage in wasteful stockpiling. In this setting, we show that the producer's inventories act as a strategic device to mitigate the loss from the lack of commitment. Since their cost is sunk when being available for sale, inventories reduce future costs, which translates into lower future prices and weakens the buyer stockpiling incentives. When the (unit) cost of production is relatively large, the producer accumulates the level of inventories that maximizes the ex ante profits, and the constraint of sequential optimality is slack in equilibrium. As a result, buyer stockpiling is fully removed, despite the lack of commitment. The only additional cost is the mere inventory cost. When the cost of production is small enough, inventories can be still beneficial for the producer although their amount is distorted from the ex ante optimal level to ensure sequential optimality. This implies that buyer stockpiling can persist in equilibrium. We also provide non-trivial results about the features of equilibrium prices. Although our attention is devoted to storable goods, similar storage incentives apply to durable goods, as Dudine et al. (2006) emphasize. Therefore, the predictions of our model can be extended to durable good markets.

The plainness of our analysis allows us to identify in a transparent and intuitive manner a novel strategic rationale for inventories that leads to potentially significant managerial, empirical and policy implications. Our results recommend as a profitable strategy for managers the inventory accumulation in periods of demand expansion. We also provide theoretical support for the anecdotal evidence about the companies' advertisement and disclosure of their product availability (e.g., Krishnan and Winter 2009, 2010; Yin et al. 2009).

Our analysis sheds new light on some well-established empirical facts, such as inventory procyclicality and the positive relationship between inventories and market power. The empirical implications of our results include the impact of inventories on the firm's performance. As emphasized by Eroglu and Hofer (2011), despite the popularity of the lean inventory practices, there exists limited empirical evidence that lean inventory management improves the firm's performance. Using a panel data set of 54 US manufacturing industries in the period 2003-2008, Eroglu and Hofer (2011) find that the inventory-performance relationship varies significantly across industries and lean inventory practices are not profitable in a number of industries. As they suggest, a relatively large use of inventories can be driven by industry-specific features, such as product and demand conditions. Our study proposes the empirically testable explanation based on buyer product storability and demand expansion. A related financial implication potentially helpful for investors in the stock and commodity markets is the role of inventories as a predictor of higher profitability in storable good markets when the demand is expected to grow.

A stochastic version of our model can capture other relevant pieces of the empirical evidence about inventories. In a setting à la Kahn (1987) where the demand is uncertain and exhibits positive serial correlation, a positive demand shock increases production more than sales to cover the higher expected future demand. A negative demand shock reduces production more than sales in expectation of lower future demand, and the producer will not hold any inventories for the future if expected demand is low enough. This makes the volatility of production larger than the volatility of sales, consistently with the empirical evidence.

We bring to the public debate a number of additional findings that lend themselves to an empirical or experimental validation and can stimulate further research on the crucial business issue of inventory management. Our results suggest that inventories are more likely to be observed in industries with larger costs of production, and they typically lead to higher aggregate production and lower prices over time. This generates policy implications of some relevance, especially in terms of inventory taxation.

Acknowledgments We thank Helmut Bester, Evangelia Chalioti, Evangelos Dioikitopoulos, Ricardo Flores-Fillol, Sotiris Georganas, Jochen Gönsch, Eugen Kovac, Matthias Lang, Carolina Manzano Tovar, Andras Niedermayer, Patrick Rey, Emanuele Tarantino, Bernd Theilen for helpful comments and suggestions. We also thank the participants in the seminars at the University of Ioannina, University of Duisburg-Essen and Free University of Berlin as well as in the Trobada URV-UA 2017 in Alicante, CESC 2017 in Barcelona and CRETE 2017 in Milos.

Appendix

This Appendix collects the proofs.

Proof of Lemma 1. Under full commitment, the producer's maximization problem is

$$\begin{aligned}&\max_{p_1,p_2,I}\,(p_1-c)\left[D_1(p_1)+D_s(p_1)\right]-(c+s_p)\,I\\&+p_2\left[D_2(p_2)-D_s(p_1)\right]-c\left[D_2(p_2)-D_s(p_1)-I\right]\cdot\mathbf{1}_{Q_2}.\end{aligned}$$

Since the objective function decreases with I, we have $I^c = 0$ in equilibrium. Using (1), we identify the following three cases: (a) $p_1 + s_b > p_2$, which implies $D_s(p_1) = 0$; (b) $p_1 + s_b = p_2$, which implies $D_s(p_1) \in [0, D_2(p_1 + s_b)]$; (c) $p_1 + s_b < p_2$, which implies $D_s(p_1) = D_2(p_1 + s_b)$. We examine these cases one by one.

(a) $p_1 + s_b > p_2 \Rightarrow D_s(p_1) = 0$. The producer's maximization problem reduces to the unconstrained problem in (4), which leads to the equilibrium static monopoly prices p_1^m and p_2^m. However, the supposition $p_1^m + s_b > p_2^m$ violates Assumption 2. Since for any price $p_2' < p_2^m$ such that $p_1^m + s_b > p_2'$ there exists a p_2'' such that $p_2'' > p_2'$ and $p_1^m + s_b > p_2''$ which makes the producer better off (the second period profits increase), we conclude that case (a) is irrelevant.

(b) $p_1 + s_b = p_2 \Rightarrow D_s(p_1) \in [0, D_2(p_1 + s_b)]$. Note that in equilibrium it must hold $D_s(p_1) = 0$, otherwise the producer would have an incentive to slightly increase p_1 and induce no buyer stockpiling, which entails a discontinuous increase in profits. The producer's maximization problem becomes

$$\max_{p_1} (p_1 - c) D_1(p_1) + (p_1 + s_b - c) D_2(p_1 + s_b).$$

Taking the first-order condition for p_1 yields

$$D_1(p_1) + (p_1 - c) D_{1|1}(p_1) + \phi_2(p_1 + s_b) = 0, \tag{8}$$

where $\phi_2(.)$ is defined by (5). This implies $p_1^c = c - \frac{D_1^c + \phi_2^c}{D_{1|1}^c}$ and $p_2^c = p_1^c + s_b$.

(c) $p_1 + s_b < p_2 \Rightarrow D_s(p_1) = D_2(p_1 + s_b)$. Buyers store in the first period and abstain from purchasing in the second period. However, the producer can do better by setting $p_2' = p_1 + s_b$, which implies from case (b) that the producer can serve $D_2(p_1 + s_b)$ at $p_1 + s_b$ in the second period rather than at p_1 in the first period. Therefore, case (c) is irrelevant.

It follows from the previous analysis that the full commitment solution reflects the outcome in case (b). We now show that $p_1^c > p_1^m$ and $p_2^c < p_2^m$, where p_τ^m, $\tau \in \{1, 2\}$, is the solution to the producer's maximization problem in (4). To show $p_1^c > p_1^m$, we substitute the first-order condition for p_1^m, i.e., $D_1(p_1) + (p_1 - c) D_{1|1}(p_1) = 0$, into the left-hand side of the first-order condition for p_1^c in (8). This yields $\phi_2(p_1^m + s_b) > 0$, where the inequality follows from Assumption 2. Then, Assumption 1 implies $p_1^c > p_1^m$. To show $p_2^c < p_2^m$, we substitute the first-order condition for p_2^m, i.e., $\phi_2(p_2) = 0$, into the left-hand side of the first-order condition for p_2^c, which corresponds to the first-order condition for p_1^c in (8). This yields $D_1(p_2^m - s_b) + (p_2^m - s_b - c) D_{1|2}(p_2^m - s_b) < 0$, where the inequality follows from Assumption 2. Then, Assumption 1 implies $p_2^c < p_2^m$. ■

Proof of Lemma 2. Consider first $p_1 + s_b = p_2$, which implies $D_s(p_1) \in [0, D_2(p_1 + s_b)]$. It follows from the constraint of sequential optimality $p_2 = c - \frac{D_2(p_2) - D_s(p_1)}{D_{2|2}(p_2)}$ in (7) with $I = 0$ that

$$D_s(p_1) = \max\{\phi_2(p_1 + s_b), 0\},$$

where $\phi_2(.)$ is defined by (5). Suppose first that $D_s(p_1) > 0$. Substituting $D_s(p_1)$ into (6) with $I = 0$, the producer's maximization problem becomes after some manipulation

$$\max_{p_1} (p_1 - c) D_1(p_1) + (p_1 + s_b - c) D_2(p_1 + s_b) - s_b \phi_2(p_1 + s_b).$$

The first-order condition for p_1 is

$$D_1(p_1) + (p_1 - c) D_{1|1}(p_1) + \phi_2(p_1 + s_b) - s_b \phi_{2|1}(p_1 + s_b) = 0, \tag{9}$$

which yields $p_1^{ns} = c - \frac{D_1^{ns} + \phi_2^{ns} - s_b \phi_{2|1}^{ns}}{D_{1|1}^{ns}}$ and $p_2^{ns} = p_1^{ns} + s_b$. Buyer stockpiling is $D_s^{ns} = \phi_2^{ns}$, where $\phi_2^{ns} \equiv \phi_2(p_1^{ns} + s_b)$. This is the result in point (a) of the lemma. If $D_s^{ns} \leq 0$, buyer stockpiling is $D_s^{nn} = 0$, with prices $p_1^{nn} = p_2^{nn} - s_b$ and $p_2^{nn} = c - \frac{D_2^{nn}}{D_{2|2}^{nn}} = p_2^m$. This is the result in point (b) of the lemma.

Following the same rationale as in the proof of Lemma 1 in points (a) and (c), the remaining two cases, i.e., $p_1 + s_b > p_2$ and $p_1 + s_b < p_2$, are irrelevant. ■

Proof of Proposition 1. Suppose that the producer chooses to hold inventories. We characterize the conditions under which this strategy is optimal. Consider first $p_1 + s_b = p_2$, which implies $D_s(p_1) \in [0, D_2(p_1 + s_b)]$. At the end of the proof, we check that the other cases are irrelevant. Since $I = D_2(p_1 + s_b) - D_s(p_1)$, ignoring the constraint of sequential optimality in (7), the producer's maximization program is given by

$$\max_{p_1} (p_1 - c) D_1(p_1) + (p_1 + s_b - c - s_p) D_2(p_1 + s_b) - (s_b - s_p) D_s(p_1).$$

As $s_p \leq s_b$, we have $D_s(p_1) = 0$ in equilibrium. The first-order condition for p_1 is

$$D_1(p_1) + (p_1 - c) D_{1|1}(p_1) + \phi_2(p_1 + s_b) - s_p D_{2|1}(p_1 + s_b) = 0, \tag{10}$$

which yields $p_1^* = c - \frac{D_1^* + \phi_2^* - s_p D_{2|1}^*}{D_{1|1}^*}$ and $p_2^* = p_1^* + s_b$. We now show that selling $I^* = D_2^*$ in the second period satisfies the constraint of sequential optimality if and only if $p_2^m|_{c=0} \leq p_2^* \leq p_2^m$. The second period cost function $C_2(p_2)$ is $c[D_2(p_2) - D_2^*]$ for $p_2 < p_2^*$ and zero for $p_2 \geq p_2^*$, which yields a marginal cost $C_{2|2}(p_2)$ equal to $cD_{2|2}(p_2)$ for $p_2 < p_2^*$ and zero for $p_2 > p_2^*$. The second period revenue function is $R_2(p_2) = p_2 D_2(p_2)$, which entails a marginal revenue $R_{2|2}(p_2) = D_2(p_2) + p_2 D_{2|2}(p_2)$. Assume first that $p_2^* < p_2^m|_{c=0}$, where $R_{2|2}(p_2^m|_{c=0}) = 0$. Since the concavity (by Assumption 1) of the static second period profit function with zero costs implies that $R_{2|2}(p_2)$ decreases with p_2, we have $R_{2|2}(p_2^*) > \lim_{p_2 \to p_2^{*+}} C_{2|2}(p_2) = 0$. Then, charging a price above p_2^* and discarding some inventories increases the producer's second period profits. In other words, $p_2^* < p_2^m|_{c=0}$ is not sequentially optimal. Now, assume that $p_2^* > p_2^m$, where $R_{2|2}(p_2^m) = cD_{2|2}(p_2^m)$. Since the concavity (by Assumption 1) of the static second period profit function with marginal costs c (maximized at p_2^m) implies that $R_{2|2}(p_2^*) < cD_{2|2}(p_2^*)$, we have $R_{2|2}(p_2^*) < \lim_{p_2 \to p_2^{*-}} C_{2|2}(p_2) = cD_{2|2}(p_2^*)$. Then, charging a price below p_2^* and producing some additional quantity increases the producer's second period profits. In other words, a price $p_2^* > p_2^m$ is not sequentially optimal, either. The last step is to show that, if $p_2^m|_{c=0} \leq p_2^* \leq p_2^m$, then p_2^* is sequentially optimal. We have $R_{2|2}(p_2^*) \in [cD_{2|2}(p_2^*), 0]$, where the upper and lower bounds of the interval hold if and only if $p_2^* = p_2^m|_{c=0}$ and $p_2^* = p_2^m$, respectively. A price above p_2^* reduces the producer's second period profits, since $R_{2|2}(p_2^*) \leq \lim_{p_2 \to p_2^{*+}} C_{2|2}(p_2) = 0$. A price below p_2^* also reduces the producer's second period profits, since $0 \geq R_{2|2}(p_2^*) \geq \lim_{p_2 \to p_2^{*-}} C_{2|2}(p_2) = cD_{2|2}(p_2^*)$. Hence, p_2^* is sequentially optimal if and only if $p_2^m|_{c=0} \leq p_2^* \leq p_2^m$.

We now derive the conditions under which $p_2^m|_{c=0} \leq p_2^* \leq p_2^m$. Substituting the first-order condition for $p_2^m|_{c=0}$, i.e., $D_2(p_2) + p_2 D_{2|2}(p_2) = 0$, into the left-hand side of the first-order condition for p_2^*, which corresponds to the first-order condition for p_1^* in (10), and using (5) yields

$$D_1(p_2^m|_{c=0} - s_b) + (p_2^m|_{c=0} - s_b - c) D_{1|2}(p_2^m|_{c=0} - s_b) - (c + s_p) D_{2|2}(p_2^m|_{c=0}) \geq 0$$

$$\Leftrightarrow c \geq \widetilde{c} \equiv \frac{D_1(p_2^m|_{c=0} - s_b) + (p_2^m|_{c=0} - s_b) D_{1|2}(p_2^m|_{c=0} - s_b) - s_p D_{2|2}(p_2^m|_{c=0})}{D_{1|2}(p_2^m|_{c=0} - s_b) + D_{2|2}(p_2^m|_{c=0})}. \quad (11)$$

It follows from Assumption 1 that $p_2^* \geq p_2^m|_{c=0}$ if and only if $c \geq \widetilde{c}$. We find from Assumption 2 that $\widetilde{c} > 0$ for s_p small enough.

Substituting the first-order condition for p_2^m, i.e., $D_2(p_2) + (p_2 - c) D_{2|2}(p_2) = 0$, into the left-hand side of the first-order condition for p_2^*, which corresponds to the first-order condition for p_1^* in (10), and using (5) yields

$$D_1(p_2^m - s_b) + (p_2^m - s_b - c) D_{1|2}(p_2^m - s_b) - s_p D_{2|2}(p_2^m) \leq 0$$

$$\Leftrightarrow s_p \leq \frac{D_1(p_2^m - s_b) + (p_2^m - s_b - c) D_{1|2}(p_2^m - s_b)}{D_{2|2}(p_2^m)},$$

where the expression on the right-hand side is positive by Assumption 2. It follows from Assumption 1 that $p_2^* \leq p_2^m$ if and only if this condition is satisfied. Using (8) and (10), we find that for $c \geq \widetilde{c}$ the full commitment outcome is restored as long as $s_p = 0$. By continuity, there exists a threshold $\widetilde{s}_p > 0$ such that for $s_p \leq \widetilde{s}_p$ the solution characterized in the proposition is optimal.

Finally, we examine the remaining cases and show that they are irrelevant. Consider the case $p_1 + s_b > p_2$, which implies $D_s(p_1) = 0$ and $I = D_2(p_2)$. This yields $p_1 = p_1^m$ and $p_2 < p_1^m + s_b$. However, the producer can do better by holding a lower inventory level $I = D_2(p_2')$, where $p_2' = p_1^m + s_b$ is sequentially optimal and closer to p_2^m (from the left by Assumption 2). It follows from the proof of Lemma 2 that the case $p_1 + s_b < p_2$ is also irrelevant. ■

Proof of Lemma 3. Consider first $p_1 + s_b = p_2$, which implies $D_s(p_1) \in [0, D_2(p_1 + s_b)]$. Since $I = D_2(p_1 + s_b) - D_s(p_1)$, it follows from the constraint of sequential optimality $p_2 = -\frac{D_2(p_2) - D_s(p_1)}{D_{2|2}(p_2)}$ in (7) that

$$D_s(p_1) = \max\left\{\phi_2(p_1 + s_b) + c D_{2|1}(p_1 + s_b), 0\right\}.$$

Suppose first that $D_s(p_1) > 0$. Substituting $D_s(p_1)$ into the producer's maximization problem in (6) yields after some manipulation

$$\max_{p_1} (p_1 - c) D_1(p_1) + (p_1 + s_b - c - s_p) D_2(p_1 + s_b) - (s_b - s_p) D_s(p_1 + s_b).$$

The first-order condition for p_1 is

$$\begin{aligned} &D_1(p_1) + (p_1 - c) D_{1|1}(p_1) + \phi_2(p_1 + s_b) - s_p D_{2|1}(p_1 + s_b) \\ &- (s_b - s_p)\left[\phi_{2|1}(p_1 + s_b) + c D_{2|11}(p_1 + s_b)\right] = 0, \end{aligned} \quad (12)$$

which yields $p_1^{is} = c - \frac{D_1^{is} + \phi_2^{is} - s_p D_{2|1}^{is} - (s_b - s_p)\left(\phi_{2|1}^{is} + c D_{2|11}^{is}\right)}{D_{1|1}^{is}}$ and $p_2^{is} = p_1^{is} + s_b$. The producer's inventories and buyer stockpiling are $I^{is} = D_2^{is} - D_s^{is} = -p_2^{is} D_{2|1}^{is}$ and $D_s^{is} = \phi_2^{is} + cD_{2|1}^{is}$, where $\phi_2^{is} \equiv \phi_2\left(p_1^{is} + s_b\right)$. This is the result in point (a) of the lemma. If $D_s^{is} \leq 0$, the producer's inventories and buyer stockpiling are $I^{in} = D_2^{in}$ and $D_s^{in} = 0$, with prices $p_1^{in} = p_2^{in} - s_b$ and $p_2^{in} = -\frac{D_2^{in}}{D_{2|2}^{in}} = p_2^m\,|_{c=0}$. This is the result in point (b) of the lemma.

Finally, we examine the remaining cases and show that they are irrelevant. Consider the case $p_1 + s_b > p_2$, which implies $D_s\left(p_1\right) = 0$ and $I = D_2\left(p_2\right)$. This yields $p_1 = p_1^m$ and $p_2 = p_2^m\,|_{c=0} < p_1^m + s_b$. It follows from Assumption 2 that $p_2' = p_1^m + s_b$ is such that $p_2^m\,|_{c=0} < p_2' < p_2^m$. Applying the same rationale as in the proof of Proposition 1, we find that p_2' is sequentially optimal and makes the producer better off. It follows from the proof of Lemma 2 that the case $p_1 + s_b < p_2$ is also irrelevant. ■

Proof of Proposition 2. In the light of the proof of Lemma 2, if buyer stockpiling is $D_s^{ns} = \phi_2^{ns} > 0$, the producer's profits are

$$\Pi^{ns}\left(p_1^{ns}\right) = \left(p_1^{ns} - c\right) D_1^{ns} + \left(p_1^{ns} + s_b - c\right) D_2^{ns} - s_b \phi_2^{ns}.$$

It follows from the proof of Lemma 3 that the profits of a producer that holds inventories and replicates the prices without inventories are given by

$$\Pi^{is}\left(p_1^{ns}\right) = \left(p_1^{ns} - c\right) D_1^{ns} + \left(p_1^{ns} + s_b - c - s_p\right) D_2^{ns} - \left(s_b - s_p\right) D_s^{is}\left(p_1^{ns}\right),$$

which is feasible when $D_s^{is}\left(p_1^{ns}\right) = \phi_2^{ns} + cD_{2|1}^{ns} \geq 0$. This yields after some manipulation

$$\begin{aligned} \Pi^{is}\left(p_1^{ns}\right) - \Pi^{ns}\left(p_1^{ns}\right) &= -s_b c D_{2|1}^{ns} - s_p\left(D_2^{ns} - \phi_2^{ns} - cD_{2|1}^{ns}\right) \\ &= \left[s_p\left(p_1^{ns} + s_b\right) - s_b c\right] D_{2|1}^{ns} > 0 \Leftrightarrow s_p < \frac{s_b c}{p_1^{ns} + s_b}. \end{aligned}$$

Since $D_s^{ns}\left(p_1^{ns}\right) - D_s^{is}\left(p_1^{ns}\right) = -cD_{2|1}^{ns} > 0$ (for $c > 0$), we have for any $D_s^{ns}\left(p_1^{ns}\right) > 0$ that $D_s^{is}\left(p_1^{ns}\right) \geq 0$ if c is small enough. ■

Proof of Proposition 3. The results in point (i) of the proposition follow from steps 1-4.

1. Substituting the first-order condition for p_1^c in (8) into the left-hand side of the first-order condition for p_1^* in (10) yields $-s_p D_{2|1}^c \geq 0$, where the equality follows if and only if $s_p = 0$. Assumption 1 implies that $p_1^c \leq p_1^*$. Since $p_1 + s_b = p_2$, we find $p_\tau^c \leq p_\tau^*$, $\tau \in \{1, 2\}$, where the equality follows if and only if $s_p = 0$.
2. Substituting the first-order condition for p_1^{ns} in (9) into the left-hand side of the first-order condition for p_1^* in (10) yields

$$s_b \phi_{2|1}^{ns} - s_p D_{2|1}^{ns}. \tag{13}$$

The sign of (13) is negative if and only if $s_p < \frac{s_b \phi_{2|1}^{ns}}{D_{2|1}^{ns}} \equiv \overline{s}_p$, where $\overline{s}_p > 0$. It follows from Assumption 1 that $p_1^* < p_1^{ns}$ if and only if $s_p < \overline{s}_p$. Since $p_1 + s_b = p_2$, we find $p_\tau^* < p_\tau^{ns}$, $\tau \in \{1, 2\}$, if and only if $s_p < \overline{s}_p$.
3. It follows from the proof in Proposition 1 that $p_2^* \leq p_2^m$. Then, a (necessary) condition for the (strict) optimality of inventories is that $p_2^* < p_2^m$. Since $p_2^m = p_2^{nn}$ from Lemma 2(b) and $p_1 + s_b = p_2$, we find $p_\tau^* < p_\tau^{nn}$, $\tau \in \{1, 2\}$.
4. Substituting the first-order condition for p_2^{ns}, i.e., $\phi_2(p_2) - D_s^{ns}\left(p_1\right) = 0$, into the left-hand

side of the first-order condition for p_2^{nn}, i.e., $\phi_2(p_2) = 0$, yields $D_s^{ns} > 0$, where the inequality follows from Lemma 2(a). Assumption 1 implies $p_2^{ns} < p_2^{nn}$. Since $p_1 + s_b = p_2$, we find $p_\tau^{ns} < p_\tau^{nn}$, $\tau \in \{1, 2\}$.

The results in point (ii) of the proposition follow from steps 5-6.

5. Substituting the first-order condition for p_2^{is}, i.e., $\phi_2(p_2) + cD_{2|2}(p_2) - D_s^{is}(p_1) = 0$, into the left-hand side of the first-order condition for p_2^{in}, i.e., $\phi_2(p_2) + cD_{2|2}(p_2) = 0$, yields $D_s^{is} > 0$, where the inequality follows from Lemma 3(a). Assumption 1 implies $p_2^{is} < p_2^{in}$. Since $p_1 + s_b = p_2$, we find $p_\tau^{is} < p_\tau^{in}$, $\tau \in \{1, 2\}$.

6. Substituting the first-order condition for p_2^{in}, i.e., $\phi_2(p_2) + cD_{2|2}(p_2) = 0$, into the left-hand side of the first-order condition for p_2^{nn}, i.e., $\phi_2(p_2) = 0$, yields $-cD_{2|2}^{in} \geq 0$, where the equality holds if and only if $c = 0$. Assumption 1 implies $p_2^{in} \leq p_2^{nn}$. Since $p_1 + s_b = p_2$, we find $p_\tau^{in} \leq p_\tau^{nn}$, $\tau \in \{1, 2\}$, where the equality holds if and only if $c = 0$.

To show the results in point (iii) of the proposition, we substitute the first-order condition for p_1^{ns} in (9) into the left-hand side of the first-order condition for p_1^{is} in (12), which yields after some manipulation

$$s_p \left[D_{2|1}^{ns} + (p_1^{ns} + s_b) D_{2|11}^{ns} \right] - s_b c D_{2|11}^{ns}. \tag{14}$$

The sign of (14) is negative if and only if one of the following conditions holds:

(a) $D_{2|11}^{ns} \leq 0$ and $s_p > \frac{s_b c D_{2|11}^{ns}}{D_{2|1}^{ns} + (p_1^{ns} + s_b) D_{2|11}^{ns}} \equiv \widehat{s}_p$, where $\widehat{s}_p \geq 0$;

(b) $0 < D_{2|11}^{ns} < -\frac{D_{2|1}^{ns}}{p_1^{ns} + s_b} \equiv \widehat{D}_{2|11}$;

(c) $D_{2|11}^{ns} \geq \widehat{D}_{2|11}$ and $s_p < \widehat{s}_p$.

Assumption 1 implies that under one of these conditions we have $p_1^{is} < p_1^{ns}$, which yields $p_\tau^{is} < p_\tau^{ns}$, $\tau \in \{1, 2\}$, since $p_1 + s_b = p_2$. ■

Supplementary Appendix

This Supplementary Appendix formalizes the results with linear demand in Section 6 and collects the associated proofs. We assume $c > s_b > s_p \geq 0$, which allows us to focus on the most plausible parameter constellations and limits the number of case distinctions. The following remark formalizes the results for $c \geq \widetilde{c}$, where $\widetilde{c} \equiv \frac{\alpha_2 - \alpha_1 - 2s_b - s_p}{2}$. The threshold values for s_p are derived in the proof.

Remark 1 *Suppose $c \geq \widetilde{c}$. Then, the limited commitment equilibrium exhibits the following features.*

A. For $s_b < \frac{\alpha_2 - \alpha_1}{4}$,

(A1) if $s_p \leq s_{p_1}$, then $I^* = \alpha_2 - p_2^*$, $D_s^* = 0$, and $p_1^* = \frac{\alpha_1 + \alpha_2 + 2(c - s_b) + s_p}{4}$, $p_2^* = p_1^* + s_b$;

(A2) if $s_p > s_{p_1}$, then $I^{ns} = 0$, $D_s^{ns} = \alpha_2 + c - 2s_b - 2p_1^{ns}$, and $p_1^{ns} = \frac{\alpha_1 + \alpha_2 + 2c}{4}$, $p_2^{ns} = p_1^{ns} + s_b$.

B. For $s_b \geq \frac{\alpha_2 - \alpha_1}{4}$,

(B1) if $s_p \leq s_{p_2}$, then (A1) applies;

(B2) if $s_p > s_{p_2}$, then $I^{nn} = D_s^{nn} = 0$, and $p_1^{nn} = p_2^{nn} - s_b$, $p_2^{nn} = p_2^m = \frac{\alpha_2 + c}{2}$.

Proof of Remark 1. It follows from the proof of Lemma 2 that the producer's maximization problem in the absence of inventories is given by

$$\max_{p_1} \,(p_1 - c)\left[\alpha_1 - p_1 + D_s(p_1)\right] + (p_1 + s_b - c)\left[\alpha_2 - p_1 - s_b - D_s(p_1)\right].$$

Using the second period first-order condition $p_2 = \frac{\alpha_2 - D_s(p_1)+c}{2}$ and $p_2 = p_1 + s_b$ yields $D_s(p_1) = \max\{\alpha_2 + c - 2s_b - 2p_1, 0\}$. Suppose first $D_s(p_1) > 0$. Then, substituting $D_s(p_1)$ into the producer's maximization problem and taking the first-order condition for p_1 yields $p_1^{ns} = \frac{\alpha_1+\alpha_2+2c}{4}$ and $p_2^{ns} = p_1^{ns} + s_b$. Moreover, $I^{ns} = 0$ and $D_s^{ns} = \alpha_2 + c - 2s_b - 2p_1^{ns} = \frac{\alpha_2-\alpha_1-4s_b}{2}$. Associated profits are

$$\Pi^{ns} = \frac{(\alpha_1 + \alpha_2 - 2c)^2}{8} + s_b^2. \tag{15}$$

If $\alpha_2 - \alpha_1 - 4s_b \leq 0$, then $I^{nn} = D_s^{nn} = 0$, with prices $p_1^{nn} = p_2^{nn} - s_b$ and $p_2^{nn} = p_2^{m} = \frac{\alpha_2+c}{2}$. Associated profits are

$$\Pi^{nn} = \frac{(\alpha_1 - c)(\alpha_2 - c) + 2(\alpha_2 - \alpha_1)s_b - 2s_b^2}{2}. \tag{16}$$

It follows from the proof of Proposition 1 that the producer's maximization problem in the presence of inventories is given by

$$\max_{p_1} \,(p_1 - c)(\alpha_1 - p_1) + (p_1 + s_b - c - s_p)(\alpha_2 - p_1 - s_b),$$

which yields $p_1^* = \frac{\alpha_1+\alpha_2+2(c-s_b)+s_p}{4}$ and $p_2^* = p_1^* + s_b$. Moreover, $I^* = \alpha_2 - p_2^* = \frac{3\alpha_2-\alpha_1-2(c+s_b)-s_p}{4}$ and $D_s^* = 0$. Associated profits are

$$\Pi^* = \frac{(\alpha_1+\alpha_2)^2 - 4\alpha_2(c - s_b) - 4\alpha_1(c + s_b) + 4\left(c^2 - s_b^2\right)}{8} + \frac{2\alpha_1 - 6\alpha_2 + 4(c + s_b) + s_p}{8}s_p. \tag{17}$$

The last step is to compare the producer's profits in each case. We obtain the following results.

A. For $s_b < \frac{\alpha_2-\alpha_1}{4}$, equilibrium profits are $\max\{\Pi^{ns}, \Pi^*\}$. Using (15) and (17) yields $\Pi^{ns} \leq \Pi^* \Leftrightarrow s_p \leq s_{p_1}$, where $s_{p_1} \equiv 3\alpha_2 - \alpha_1 - 2(c + s_b) - \sqrt{(3\alpha_2 - \alpha_1 - 2c)^2 - 8(2\alpha_2 - \alpha_1 - c - 2s_b)s_b}$.

B. For $s_b \geq \frac{\alpha_2-\alpha_1}{4}$, equilibrium profits are $\max\{\Pi^{nn}, \Pi^*\}$. Using (16) and (17) yields $\Pi^{nn} \leq \Pi^* \Leftrightarrow s_p \leq s_{p_2}$, where $s_{p_2} \equiv 3\alpha_2 - \alpha_1 - 2(c + s_b) - 2\sqrt{(\alpha_2 - c)(2\alpha_2 - \alpha_1 - c - 2s_b)}$.

Differentiating s_{p_1} and s_{p_2} with respect to c yields $\frac{\partial s_{p_1}}{\partial c} = \frac{2(3\alpha_2-\alpha_1-2c-2s_b)}{\sqrt{(3\alpha_2-\alpha_1-2c)^2-8(2\alpha_2-\alpha_1-c-2s_b)s_b}} - 2 > 0$ and $\frac{\partial s_{p_2}}{\partial c} = \frac{3\alpha_2-\alpha_1-2c-2s_b}{\sqrt{(\alpha_2-c)(2\alpha_2-\alpha_1-c-2s_b)}} - 2 > 0$, where the inequalities follow from the assumptions on the parameters of the model. ■

The following remark formalizes the results for $c < \widetilde{c}$. The threshold values for s_p are derived in the proof.

Remark 2 *Suppose $c < \widetilde{c}$. Then, the limited commitment equilibrium exhibits the following features.*

A. For $s_b \leq \frac{\alpha_2-\alpha_1-2c}{4}$, *(A1) if* $s_p \leq s_{p_3}$*, then* $I^{is} = \alpha_2 - p_2^{is} - D_s^{is}$*,* $D_s^{is} = \alpha_2 - 2s_b - 2p_1^{is}$*, and* $p_1^{is} = \frac{\alpha_1+\alpha_2+2c-s_p}{4}$*,* $p_2^{is} = p_1^{is} + s_b$*;*

(A2) if $s_p > s_{p_3}$*, then* $I^{ns} = 0$*,* $D_s^{ns} = \alpha_2 + c - 2s_b - 2p_1^{ns}$*, and* $p_1^{ns} = \frac{\alpha_1+\alpha_2+2c}{4}$*,* $p_2^{ns} = p_1^{ns} + s_b$*.*

B. For $\frac{\alpha_2-\alpha_1-2c}{4} < s_b < \frac{\alpha_2-\alpha_1}{4}$*,*

(B1) if $s_p \leq \min\{s_{p_4}, s_{p_5}\}$*, then* $I^{in} = \alpha_2 - p_2^{in}$*,* $D_s^{in} = 0$*, and* $p_1^{in} = p_2^{in} - s_b$*,* $p_2^{in} = p_2^m|_{c=0} = \frac{\alpha_2}{2}$*;*

(B2) if $\min\{s_{p_4}, s_{p_5}\} = s_{p_4}$ *and* $s_{p_4} < s_p \leq s_{p_3}$*, then (A1) applies;*

(B3) otherwise, (A2) applies.

C. For $s_b \geq \frac{\alpha_2-\alpha_1}{4}$*, then* $I^{nn} = D_s^{nn} = 0$*, and* $p_1^{nn} = p_2^{nn} - s_b$*,* $p_2^{nn} = p_2^m = \frac{\alpha_2+c}{2}$*.*

Proof of Remark 2. It follows from the proof of Lemma 3 that the producer's maximization problem in the presence of inventories is given by

$$\max_{p_1} (p_1 - c)[\alpha_1 - p_1 + D_s(p_1)] + (p_1 + s_b - c - s_p)[\alpha_2 - p_1 - s_b - D_s(p_1)].$$

Using the second period first-order condition $p_2 = \frac{\alpha_2 - D_s(p_1)}{2}$ and $p_2 = p_1 + s_b$ yields $D_s(p_1) = \max\{\alpha_2 - 2s_b - 2p_1, 0\}$. Suppose first $D_s(p_1) > 0$. Then, substituting $D_s(p_1)$ into the producer's maximization problem and taking the first-order condition for p_1 yields $p_1^{is} = \frac{\alpha_1+\alpha_2+2c-s_p}{4}$ and $p_2^{is} = p_1^{is} + s_b$. Moreover, $I^{is} = \alpha_2 - p_2^{is} - D_s^{is} = \frac{\alpha_1+\alpha_2+2c+4s_b-s_p}{4}$ and $D_s^{is} = \frac{\alpha_2-\alpha_1-4s_b-2c+s_p}{2}$. Associated profits are

$$\Pi^{is} = \frac{(\alpha_1+\alpha_2)^2 - 4c(\alpha_1+\alpha_2) + 8s_b(c+s_b) + 4c^2}{8} - \frac{2\alpha_1 + 2\alpha_2 + 4c + 8s_b - s_p}{8} s_p. \quad (18)$$

If $\alpha_2 - \alpha_1 - 4s_b - 2c + s_p \leq 0$, then $I^{in} = \alpha_2 - p_2^{in} = \frac{\alpha_2}{2}$ and $D_s^{in} = 0$, with prices $p_1^{in} = p_2^{in} - s_b$ and $p_2^{in} = p_2^m|_{c=0} = \frac{\alpha_2}{2}$. Associated profits are

$$\Pi^{in} = \frac{\alpha_1\alpha_2 - 2\alpha_1(c+s_b) + 2s_b(\alpha_2 - c - s_b) - \alpha_2 s_p}{2}. \quad (19)$$

The last step is to compare the producer's profits. We obtain the following results.

A. For $s_b \leq \frac{\alpha_2-\alpha_1-2c}{4}$, equilibrium profits are $\max\{\Pi^{ns}, \Pi^{is}\}$. Using (15) and (18) yields $\Pi^{ns} \leq \Pi^{is} \Leftrightarrow s_p \leq s_{p_3}$, where $s_{p_3} \equiv \alpha_1 + \alpha_2 + 2c + 4s_b - \sqrt{(\alpha_1+\alpha_2+2c+4s_b)^2 - 8s_b c}$.

B. For $\frac{\alpha_2-\alpha_1-2c}{4} < s_b < \frac{\alpha_2-\alpha_1}{4}$,

(i) if $s_p \leq s_{p_4}$, where $s_{p_4} \equiv \alpha_1 - \alpha_2 + 4s_b + 2c$, equilibrium profits are $\max\{\Pi^{ns}, \Pi^{in}\}$. Using (15) and (19) yields $\Pi^{ns} \leq \Pi^{in} \Leftrightarrow s_p \leq \min\{s_{p_4}, s_{p_5}\}$, where $s_{p_5} \equiv \frac{8s_b(\alpha_2-\alpha_1-c-2s_b)-(\alpha_2-\alpha_1-2c)^2}{4\alpha_2}$;

(ii) if $s_p > s_{p_4}$, equilibrium profits are $\max\{\Pi^{ns}, \Pi^{is}\}$. It follows from (15) and (18) that $\Pi^{ns} \leq \Pi^{is} \Leftrightarrow s_{p_4} < s_p \leq s_{p_3}$ and $\min\{s_{p_4}, s_{p_5}\} = s_{p_4}$. It holds $\min\{s_{p_4}, s_{p_5}\} = s_{p_4} \Leftrightarrow \alpha_2 > \frac{\alpha_1+2c+4s_b+2\sqrt{(\alpha_1+2c)^2+2(4\alpha_1+5c)s_b+16s_b^2}}{3}$;

C. For $s_b \geq \frac{\alpha_2-\alpha_1}{4}$, equilibrium profits are Π^{nn} in (16). ■

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