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Arturo Garcia and Mariel Leal and Sang-Ho Lee

Technologico de Moterrey, Technologico de Moterrey, Chonnam National University

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Endogenous timing with a socially responsible firm

Arturo García\textsuperscript{a}, Mariel Leal\textsuperscript{a}, Sang-Ho Lee\textsuperscript{b,}\textsuperscript{*}

\textsuperscript{a}School of Engineering and Sciences, Tecnológico de Monterrey, Campus Monterrey, Mexico
\textsuperscript{b}Department of Economics, Chonnam National University, Gwangju, South Korea

Abstract

This study considers a mixed duopoly in which a socially responsible firm competes with a private firm by incorporating environmental externality and clean technology. We analyze the endogenous market structure in which both firms strategically decides quantities sequentially or simultaneously, which also affects abatement activities. We show that depending on the relative concerns on environment and consumers surplus, the socially responsible firm can be less or more aggressive in the production and abatement. Thus, not only the significance of externality but also the instrumental conflict of social concerns are crucial factors in determining the equilibrium of endogenous timing game, in which the socially responsible firm might earn higher profits.

Keywords: endogenous timing; socially responsible firm; mixed duopoly; clean technology; environmental externality

JEL classification: L13; L31; Q5

1. Introduction

Conventional economic theory regards firms as entities whose sole objective is to maximize their profits. In the real world, however, many private firms have voluntarily and increasingly paid attention to corporate social responsibility (CSR)\textsuperscript{1} Due to the current expansion of CSR, in most countries, many industries are characterized by the simultaneous presence of for-profit firms and not-for-profit firms. It represents that the heterogeneity of objectives among the firms emerges as an important research topic in the literature\textsuperscript{2}.

\textsuperscript{*}Corresponding author

\textit{Email addresses:} aru.gmtz@hotmail.com (Arturo García), mariellealc@gmail.com (Mariel Leal), sangho@chonnam.ac.kr (Sang-Ho Lee)

\textsuperscript{1}A large number of companies participated in greenhouse gas reduction programs and issued various statements on CSR and outlined activities in their annual reports. For example, see CSR trend report by PricewaterhouseCoopers (2010) and KPMG (2013, 2015).

\textsuperscript{2}Chirco et al. (2013) show that behavioral heterogeneity may be the equilibrium outcome of the strategic interaction of ex-ante identical agents, while Matsumura and Ogawa (2014) investigate that the heterogeneity may produce the different market structure.
The recent topic on CSR has received increasing attention from broad research in both empirical and theoretical economics. Numerous studies have also formulated theoretical approaches on the CSR in the field of applied microeconomic theory. They analyzed different competition models of oligopolies where profit-maximizing firms compete with their rival firms that adopt CSR activities. In particular, they utilized a model in which the firm adopts consumer surplus as a proxy of its own CSR concerns. That is, a CSR initiative includes both profitability and consumer surplus, and thus the objective of CSR firm is a combination of consumers surplus and its own profits. They found that the CSR firm may achieve a higher profit and welfare in a quantity-setting competition. However, these results put aside the concern on environmental problem, which is becoming an essential part of CSR behavior and thus it is a more realistic representation of how CSR firms operate.

In this paper, we define a socially responsible (SR) firm that takes into account its profits and social concerns, which include not only consumers surplus but environmental externality. In fact, these two social concerns have opposite effects on production and abatement. While the concern on environment restrains the production of an SR firm and increase abatement activities, the concern on consumer surplus expands the production and decreases costly abatement. Thus, the commitment to social concerns may allow the SR firm to include different production strategy, which induces the SR firm to be more or less aggressive compared to its rival firm, and thus the competition in a mixed duopoly leads the different market structure.

We consider a mixed duopoly model in which an SR firm with social concerns competes against a for-profit (FP) private firm by incorporating environmental externality and clean technology. We then analyze the endogenous market structure and examine whether the SR firm strategically decides quantities sequentially or simultaneously, which also affects abatement activities. In specific, as formulated by Hamilton and Slutsky (1990), we investigate an observable delay game in which the role of a leader or a follower in a mixed duopoly is determined endogenously. In the presence of clean technology, we show that

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3For the intensive discussions on the empirical works on CSR, see Schreck (2011) and Crifo and Forget (2015). Lyon and Maxwell (2004) and Kitzmueller and Shimshack (2012) provided fruitful discussions on the practical and academic issues on CSR.


5Note that committing to being a CSR firm can be one way of strategically committing to higher output. This formulation of CSR firms is related to the managerial delegation contract with sales targets, in which firms have incentives to commit to putting a higher weight on output to induce rivals to reduce their outputs in quantity-setting oligopolies. See, Vickers (1985) and Pershman and Judd (1987).

6Recent analysis has emphasized that environmental concern is critical in the recent CSR codes of conduct. See the discussions in Lamberti and Tampieri (2015), Liu et al. (2015), Hirose et al. (2017), Lee and Park (2017) and so on.

7The observable delay game was extended to a mixed market by Pal (1998), Lu (2006), Barcena-Ruiz (2007), Lu and Poddar (2009) and Heywood and Ye (2009). For more extensive
depending on the relative concerns on environment and consumers surplus, the SR firm can be less or more aggressive in the production and abatement. Therefore, not only the instrumental conflict between the two social interests but also the significance of externality are crucial factors in determining the equilibrium of endogenous timing game.

On the one hand, when the SR firm does not concern for consumer surplus but cares for the environment, regardless of how significant the externality is a simultaneous-move game is a unique equilibrium. This result is consistent with [Hamilton and Slutsky (1990)], who first formulated an observable delay game in a private duopoly between two homogeneous FP firms. Thus, we extend their analysis into the case with externality and show that in a mixed duopoly with an SR firm having a high concern on the environment, the analysis of a simultaneous-move game is still useful. However, this result sharply contrasts with the case of price competition with externality. For example, [Lee and Xu (2017)] considered environmental tax and showed that a sequential-move (simultaneous-move) game emerges in equilibrium when the externality is insignificant (significant) in a private duopoly while the results are reversed in a mixed duopoly. We further show that a simultaneous-move game emerges in equilibrium if the \textit{ex-ante} initial level of pollution emission is not significant and the SR firm concerns for consumers surplus is not high, but entirely accounts for the environmental externality it solely causes. This result also includes the analysis of [Matsumura and Ogawa (2017)], who considered environmental externality in a mixed duopoly and showed that a simultaneous-move (sequential-move) game emerges in equilibrium when the externality is significant (insignificant). In this case, the SR firm reduces output and thus always obtains lower profits than FP firm in the equilibrium.

On the other hand, when the SR firm concerns for consumer surplus significantly, two sequential-move games might be equilibria. In particular, if the SR firm concerns for environment and accounts entirely for consumers surplus, then regardless of externality two sequential-movements are the equilibrium of the endogenous timing game. This result is consistent with [Pal (1998)], who analyzed an observable delay game in a mixed duopoly between an FP firm and a welfare-maximizing public firm. Thus, we extend his analysis into the case with externality and show that the equilibrium does hold no matter how significant the externality is. Further, we also show that the SR firm increases output if it does not concern for externality significantly and thus it can obtain higher

\footnote{In the literature on endogenous timing game in duopolies, the results are mostly reversed depending on both whether firms compete in price or quantity and whether firms compete in a private or mixed duopoly. For example, in a private duopoly with symmetric payoffs firms decide simultaneously under quantity competition and sequentially under price competition, whereas in a mixed duopoly with asymmetric payoffs firms decide sequentially under quantity competition and simultaneously under price competition. See [Hamilton and Slutsky (1990), Pal (1998) and Barcena-Ruiz (2007)] among others.}
profits than its profit-seeking competitor in both leadership equilibria.\footnote{Recent instrumental approach on the CSR also supports that firms can increase its profits under the strategic CSR. For example, see Kopel and Brand (2012), Brand and Grothe (2013, 2015), Liu et al. (2015), Lambertini and Tampieri (2015) and Lee and Park (2017).} This result supports the analysis of Lambertini and Tampieri (2015), who considered a mixed duopoly where an SR firm competes with an FP firm and showed that the SR firm could obtain higher profits than the FP firm if the market size is large enough and the SR firm accounts for consumers surplus completely. However, their analysis is confined to a simultaneous Cournot game, which is not an equilibrium in the endogenous timing game in our setting where the SR firm invests in pollution abatement. Thus, our finding implies that in a mixed duopoly with a SR firm having a high concern on consumer surplus, the analysis of a simultaneous-move game is problematic. However, if the SR firm never concerns for the environment but cares for consumers surplus, then three different equilibria emerge in the endogenous timing game. Further, regardless of the externality, if the SR firm does not account for consumers surplus significantly, a simultaneous-movement also emerges in equilibrium. In these three cases, we can also show that the SR firm increases output and thus it always obtains higher profits than FP firm in the equilibrium. Thus, the analysis in Lambertini and Tampieri (2015) holds in an endogenous choice game only when the SR firm does not account for consumers surplus significantly regardless of the externality in our setting.

The remainder of this paper is organized as follows. In section 2, we formulate a Cournot duopoly model with an SR firm and an FP firm. Sections 3 analyzes a fixed timing game and Section 4 analyzes an endogenous timing game, respectively and examines illustrative cases. Finally, Section 5 concludes the paper.

2. Model

We consider there are two firms in a quantity-setting game. One of the firms is an SR firm, (hereafter referred to as firm 0). This firm maximizes not only its profits but makes an effort to decrease the pollution generated by its production and cares for consumers surplus as well. The other is an FP firm (hereafter referred to as firm 1) that maximizes only its profits.

Both polluting firms produce homogeneous goods and compete in quantities. Firms sell their output \( q_0 \geq 0 \) and \( q_1 \geq 0 \), respectively, at the market clearing price \( p(Q) = 1 - Q \) where \( Q = q_0 + q_1 \). We assume that both firms have identical technologies and the production cost function takes a quadratic form, \( c(q_i) = \frac{1}{2} q_i^2 \), \( i = 0, 1 \).

Suppose that each unit of production generates \( e \) pollution emissions. However, the SR firm can make an abatement effort \( a \) per unit production to reduce the pollution emissions. This \( e \) tells us how significant the pollution externality is in a particular industry. For example, the emissions of the financial sector are...
not as substantial as the mining, textiles or clothing industries. We also assume that both products in the same industry emit the same type of ex-ante pollutants. Unlike other studies which consider end-of-pipe abatement technologies, however, we consider a cleaner production technology in our analysis. In specific, the pollution generated by firm 0 after abatement effort is \( E_0 = (e - a) \cdot q_0 \).

The expenditure function of this abatement effort is \( a^2 \). As in practice, most pollutants cannot be decreased completely; therefore we assume \( e > a \geq 0 \). Because pollution abatement is costly, profit-maximizing firm 1 makes no effort in the absence of environmental regulation and thus its pollution is \( E_1 = e \cdot q_1 \).

The profit of SR firm is given by \( \pi_0 = p \cdot q_0 - \frac{1}{2} q_0^2 - \frac{a^2}{2} \). We assume that the SR firm maximizes profits plus a fraction of consumer surplus (CS), and as it cares for the environment, it places a weight on the pollution that its production generates. Thus, the payoff that SR firm maximizes is as follows:

\[
V_0 = \pi_0 - \gamma E_0 + \theta CS
\]  

where \( CS = \frac{Q^2}{2} \). The parameters \( \theta \in [0, 1] \) and \( \gamma \in [0, 1] \) measure the degree of concern for consumers and the degree of environmental concern, respectively, that the SR firm has. Both concerns are exogenously given.

The FP firm seeks only for profit maximization:

\[
\pi_1 = p \cdot q_1 - \frac{1}{2} q_1^2
\]  

The analysis of the observable two-stage delay game by Hamilton and Slutsky (1990) is considered. In the first stage, each firm simultaneously chooses whether to move early or late. In the second stage, the game played is simultaneous if both firms choose the same period, and sequential otherwise.

3. Fixed timing

In this section, we first consider a fixed-timing game in which both firms, SR and the FP, compete in quantities in a simultaneous-move game and a sequential-move game, respectively. In the following analysis, we assume that the three fixed timing games have interior solutions.

**Assumption:** For any \( \gamma \in [0, 1] \) and \( \theta \in [0, 1] \), \( e \in (e, \tau) \) where \( e \equiv max\{\frac{2(3+\theta)}{21-4\theta}, \frac{\gamma(2+\theta)}{2(1-\theta)}\} \) and \( \tau \equiv min\{\frac{5+2\theta-\theta^2-\gamma^2(2+\theta)}{\gamma(8-3\gamma^2-2\theta)}, \frac{2(3+\theta)}{9\gamma}\} \).

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10End-of-pipe technology refers to an equipment setup by a firm that can reduce gross pollution but leave the firm’s output unchanged. For example, see Wang and Wang (2009), Pal and Sahai (2015), Xu et al. (2016) and Lee and Xu (2017). On the other hand, clean technology involves a change in a firm’s production process whereby it generates less pollution per output, and therefore its output and abatement decisions are intertwined. See Chiou and Hu (2001), Ulph and Ulph (2007), Jinji (2013) and Tsai et al. (2016).
3.1. Simultaneous-move game

In this game, the SR firm independently chooses its abatement effort level \((a)\) and output \((q_0)\); and the FP firm chooses its production \((q_1)\). Solving the first-order conditions for maximizing the payoffs of both firms in (1) and (2), respectively, we obtain the following equilibrium quantities and abatement level:

\[
q_0^* = \frac{2 - 3e\gamma + \theta}{8 - 3\gamma^2 - 2\theta}, \quad a^* = \gamma \cdot q_0^*, \quad q_1^* = \frac{2 + e\gamma - \gamma^2 - \theta}{8 - 3\gamma^2 - 2\theta} \tag{3}
\]

where superscript ‘c’ denotes the Cournot game. The equilibrium profits and payoffs are, respectively:

\[
\pi_0^c = \sigma_1 + (2 + \theta) \left( 6 - 5\theta - \gamma^2 (6 + \theta) \right) \over 2 (8 - 3\gamma^2 - 2\theta)^2,
\]

\[
V_0^c = \sigma_2 + 12\theta + \gamma^4 \theta - 5\theta^2 - \gamma^2 (4 + 8\theta - \theta^2) + 12 \over 2 (8 - 3\gamma^2 - 2\theta)^2,
\]

\[
\pi_1^c = \frac{3 (2 + e\gamma - \gamma^2 - \theta)^2}{2 (8 - 3\gamma^2 - 2\theta)^2} \tag{4}
\]

where \(\sigma_1\) and \(\sigma_2\) are as presented in Appendix A.\(^{11}\)

3.2. SR firm as a Stackelberg leader

In this case, first firm 0 chooses its output and abatement levels and then firm 1 chooses its output level sequentially. Then, we have the following equilibrium quantities and abatement level:

\[
q_0^{sl} = \frac{6 - 9e\gamma + 2\theta}{21 - 9\gamma^2 - 4\theta}, \quad a^{sl} = \gamma \cdot q_0^{sl}, \quad q_1^{sl} = \frac{5 + 3e\gamma - 3\gamma^2 - 2\theta}{21 - 9\gamma^2 - 4\theta} \tag{5}
\]

where superscript ‘sl’ denotes the equilibrium outcome in the Stackelberg game with SR firm leadership. The resulting profits and payoffs are, respectively:

\[
\pi_0^{sl} = \frac{\sigma_3 + 4 (21 - 8\theta - 5\theta^2 - \gamma^2 (27 + 12\theta + \theta^2))}{2 (21 - 9\gamma^2 - 4\theta)^2},
\]

\[
V_0^{sl} = \frac{9e^2\gamma^2 - 4e\gamma (3 + \theta) + (5 - \gamma^2) \theta + 4}{42 - 18\gamma^2 - 8\theta},
\]

\[
\pi_1^{sl} = \frac{3 (5 + 3e\gamma - 3\gamma^2 - 2\theta)^2}{2 (21 - 9\gamma^2 - 4\theta)^2} \tag{6}
\]

\(^{11}\)For the sake of expositional convenience, we provide \(\sigma_i\) \((i = 1, ..., 5)\) in Appendix A.
3.3. FP firm as a Stackelberg leader

In this case, first firm 1 chooses its output level and then firm 0 chooses its output and abatement levels. Then, we have the following equilibrium quantities and abatement level:

\[ q_{f0} = \frac{5 + 2\theta - \gamma^2(2 + \theta) - e\gamma(8 - 3\gamma^2 - 2\theta)}{(2 + \theta)^2}, \quad a_{fl} = \gamma \cdot q_{f0}, \quad q_{f1} = \frac{2 + e\gamma - \gamma^2 - \theta}{\gamma^2 - 2\theta} \]

(7)

where superscript ‘fl’ denotes the equilibrium outcome in the Stackelberg game with FP firm leadership. The resulting profits and payoffs are, respectively:

\[ \pi_{f0} = -\sigma_4 + \eta_1 \]
\[ V_{f0} = \frac{\sigma_5 + \eta_2}{2(3 - \gamma^2 - \theta)^2(7 - 3\gamma^2 - \theta)} \]
\[ \pi_{f1} = \frac{(2 + e\gamma - \gamma^2 - \theta)^2}{2(3 - \gamma^2 - \theta)(7 - 3\gamma^2 - \theta)} \]

(8)

where \( \eta_1 \) and \( \eta_2 \) are as presented in Appendix B.

4. Equilibrium in the Endogenous Timing Game

We now discuss the first-stage choice in an endogenous timing game. Each firm \( i \) \((i = 0, 1)\) simultaneously chooses whether to move early \((t_i = 1)\) or late \((t_i = 2)\). If both firms choose the same period, the equilibrium is a simultaneous-move game. Otherwise, the equilibrium is a sequential move game. Table 1 provides the payoff matrix of the observable delay game.

Table 1: Payoff Matrix of the Observable Delay Game

<table>
<thead>
<tr>
<th>Firm 0/1</th>
<th>( t_1 = 1 )</th>
<th>( t_1 = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_0 = 1 )</td>
<td>((V_{0f}, \pi_{0f}^{fl}))</td>
<td>((V_{0c}^{sl}, \pi_{0c}^{fl}))</td>
</tr>
<tr>
<td>( t_0 = 2 )</td>
<td>((V_{0f}, \pi_{0f}^{fl}))</td>
<td>((V_{0c}, \pi_{0c}^{fl}))</td>
</tr>
</tbody>
</table>

Using the revealed fact that the payoff of a firm when it is the leader is never smaller than its payoff in the simultaneous-move game, \( V_{0f}^{sl} \geq V_{0c} \) and \( \pi_{0f}^{fl} \geq \pi_{0c} \), we have that \((t_0, t_1) = (2, 2)\) never constitutes an equilibrium unless \( V_{0f}^{sl} = V_{0c} \) and \( \pi_{0f}^{fl} = \pi_{0c} \). We can show that \( V_{0f}^{sl} = V_{0c} \) and \( \pi_{0f}^{fl} = \pi_{0c} \) never hold simultaneously. Under these conditions, the equilibrium outcomes are as follows.

\(^{12}\text{For the sake of expositional convenience, we provide } \eta_i \ (i = 1, ..., 5) \text{ in Appendix B}\)
(a) \((t_0, t_1) = (2, 1)\) emerges as an equilibrium if \(V_0^c \leq V_0^{fl}\);
(b) \((t_0, t_1) = (1, 2)\) emerges as an equilibrium if \(\pi_1^c \leq \pi_1^{sl}\); and
(c) \((t_0, t_1) = (1, 1)\) emerges as an equilibrium if \(V_0^c \geq V_0^{fl}\) and \(\pi_1^c \geq \pi_1^{fl}\).

Let \(\bar{\theta}_0(\gamma) \equiv \frac{\gamma}{2(\gamma - \gamma^2)}\), defined on the interval \([0, 1]\), and \(e_0(\theta, \gamma) \equiv \frac{\gamma - \theta}{\gamma(3\gamma - 2\theta)}\), where \(e_0 : [0, 1] \times (0, 1] \to \mathbb{R}\) monotonically decreases on \(\theta\) such as \(e_0(\theta, \gamma) > 0\) if \(0 \leq \theta < \bar{\theta}_0\), \(e_0(\bar{\theta}_0, \gamma) = 0\) and \(e_0(\theta, \gamma) < 0\) if \(\bar{\theta}_0 < \theta \leq 1\). Comparing \(V_0^c\) and \(V_0^{fl}\), we obtain the following lemma.

**Lemma 1.** \(V_0^c \geq V_0^{fl}\) for any \(\gamma \in (0, 1]\), if \(0 \leq \theta < \bar{\theta}_0(\gamma)\) and \(0 < e \leq e_0(\theta, \gamma)\). When \(\gamma = 0\), it holds for any \(e \in (0, \infty)\) if \(0 \leq \theta < \bar{\theta}_0(0)\).

**Proof.** See Appendix C.

Let \(\bar{\theta}_1(\gamma) \equiv \frac{\gamma}{3 - \gamma}\), defined on the interval \([0, 1]\), and \(e_1(\theta, \gamma) \equiv \frac{2 - (3 - \gamma^2)\theta}{\gamma(3 - 2\theta)^2}\), where \(e_1 : [0, 1] \times (0, 1] \to \mathbb{R}\) monotonically decreases on \(\theta\) such as \(e_1(\theta, \gamma) > 0\) if \(0 \leq \theta < \bar{\theta}_1(\gamma)\), \(e_1(\bar{\theta}_1, \gamma) = 0\) and \(e_1(\theta, \gamma) < 0\) if \(\bar{\theta}_1(\gamma) < \theta \leq 1\). Comparing \(\pi_1^c\) and \(\pi_1^{sl}\), we obtain the following lemma.

**Lemma 2.** \(\pi_1^c \geq \pi_1^{sl}\) for any \(\gamma \in (0, 1]\), if \(0 \leq \theta < \bar{\theta}_1(\gamma)\) and \(0 < e \leq e_1(\theta, \gamma)\). When \(\gamma = 0\), it holds for any \(e \in (0, \infty)\) if \(0 \leq \theta < \bar{\theta}_1(0)\).

**Proof.** See Appendix C.

Then, we also obtain the following lemma.

**Lemma 3.** a) \(e_1(\theta, \gamma) > e_0(\theta, \gamma)\) for any \(\theta \in [0, 1]\) and \(\gamma \in (0, 1]\)
b) \(e_1(1, \gamma) = e_0(1, \gamma) = \gamma - \frac{1}{\gamma} \leq 0\) and \(\gamma \in (0, 1]\)
c) \(\bar{\theta}_1(\gamma) > \bar{\theta}_0(\gamma)\), for any \(\gamma \in [0, 1]\)

**Proof.** See Appendix C.

From Lemmas 1-3, we obtain the following main result.

**Proposition 1.** In a mixed duopoly with a socially responsible firm, we have the followings:

(i) For any \(\gamma \in (0, 1]\) and \(e \in (e^*, \overline{\pi})\):

(a) If \(0 \leq \theta < \bar{\theta}_0(\gamma)\) and \(e < e_0(\theta, \gamma)\), the only equilibrium of the game is the simultaneous movement, that is, \((t_0, t_1) = (1, 1)\);
(b) If \(0 \leq \theta < \bar{\theta}_0(\gamma)\) and \(e = e_0(\theta, \gamma)\), either the simultaneous movement, \((t_0, t_1) = (1, 1)\) or the sequential-move outcome in which the FP firm is the leader, \((t_0, t_1) = (2, 1)\), are equilibrium outcomes.
(c) If \(\bar{\theta}_1(\gamma) \leq \theta \leq 1\) or \(e \geq e_1(\theta, \gamma)\), either the SR firm or the FP firm could be the Stackelberg leader of the game, that is, \((t_0, t_1) = (1, 2)\) and \((t_0, t_1) = (2, 1)\), are the equilibrium outcomes.
(d) Otherwise, one sequential-move outcome in which the FP firm is the leader, \((t_0, t_1) = (2, 1)\), is the unique equilibrium outcome.
(ii) When $\gamma = 0$, the equilibrium outcomes for any $e \in (0, \infty)$ are:
(a) If $0 \leq \theta < \bar{\theta}_0(0)$, then $(t_0, t_1) = (1, 1)$;
(b) If $\theta = \bar{\theta}_0(0)$, then $(t_0, t_1) = (1, 1)$ and $(t_0, t_1) = (2, 1)$.
(c) If $\bar{\theta}_1(0) \leq \theta \leq 1$, then $(t_0, t_1) = (1, 2)$ and $(t_0, t_1) = (2, 1)$.
(d) Otherwise, $(t_0, t_1) = (2, 1)$.

Proof. See Appendix D.

For economic explanations on Proposition 1, we examine four interesting cases for illustration. First, consider a case with $\theta = 0$ where the SR firm does not care for consumers surplus but care for the environment. Then, Proposition 1(i) states that regardless of how significant the externality is, the only equilibrium of the game is the simultaneous movement. (See Appendix E) This result is consistent with the observable delay game in a private duopoly without environmental externality, as formulated by Hamilton and Slutsky (1990). Thus, this case extends their analysis into the case with externality and shows that does not matter how significant the externality is the simultaneous-move outcome emerges in the equilibrium. This implies that in a mixed duopoly with an SR firm having a high concern for the environment, the analysis of a simultaneous-move game is very useful. However, this result sharply contrasts with the case of price competition with externality, which is examined by Lee and Xu (2017). They considered environmental tax under price competition and showed that a sequential-move (simultaneous-move) game emerges in equilibrium when the externality is insignificant (significant) in a private duopoly while the results are reversed in a mixed duopoly. In our setting, furthermore, the SR firm reduces output to reduce its pollution and thus we can show that $\pi_1 > \pi_0$ for $\gamma \in (0, 1]$. Therefore, the SR firm always obtains lower profits than its profit-seeking competitor at the equilibrium.

Second, consider a case with $\gamma = 1$ where the SR firm concerns for consumers surplus but entirely accounts for the environmental externality it solely causes (Note that we assume the marginal environmental damage is 1). Then, Proposition 1(i) states that depending on how much the SR firm cares for consumers surplus three different equilibria emerge in the endogenous timing game. (See Appendix E) This result includes the analysis of Matsumura and Ogawa (2017), who considered environmental externality in a mixed duopoly where a welfare-maximizing public firm competed with an FP firm and showed that a simultaneous-move (sequential-move) game emerges in equilibrium when the externality is significant (insignificant). In particular, we can show that the conditions for having the simultaneous-move outcome are $0 \leq \theta < \bar{\theta}_0(1)$ and $e < e \leq e_0(\theta, 1)$. That is if the concern on consumers surplus is not high and the ex-ante initial level of pollution emission is not significant but the SR firm concerns for environment significantly, both firms choose to move early, that is a simultaneous-movement emerges in equilibrium. Otherwise, sequential-movements are equilibrium outcomes. In these three cases, we can show that the SR firm reduces output and thus $\pi_1 > \pi_0$ for $\theta \in (0, 1]$ and $e \in (e, \bar{\pi})$. That is, the SR firm always obtains lower profits than FP firm in the equilibrium.
Third, consider a case with $\theta = 1$ where the SR firm concerns for the environment but accounts for consumer surplus entirely. Then, Proposition 1(i) states that regardless of how significant the externality is two sequential-movements are the equilibrium outcomes of the game. (See Appendix E) This result coincides with Pal (1998), who consider a mixed duopoly without environmental externality. Thus, this case extends his analysis into the case with externality and shows that does not matter how significant the externality is two sequential-move outcomes emerge in the equilibrium. Further, we can also show that the SR firm increases output if it does not concern for externality significantly and thus $\pi_0 > \pi_1$ only if $\gamma < 0.577$ for both leadership equilibria. Thus, a necessary condition for the SR firm to obtain higher profits than its profit-seeking competitor in both leadership equilibria is that it does not concern for externality significantly. This result supports the analysis of Lambertini and Tampieri (2015), who considered a mixed Cournot oligopoly where an SR firm competes with FP firms and showed that the SR firm can produce higher output and thus obtain higher profits than the FP firms if the market size is large enough and the SR firm accounts for consumer surplus completely. However, their analysis is confined to a simultaneous Cournot game, which is not an equilibrium in the endogenous timing game in our setting where the SR firm invests in pollution abatement.

Finally, consider a case with $\gamma = 0$ where the SR firm never concerns for the environment but cares for consumers surplus. Then, Proposition 1(ii) states that depending on how much the SR firm concerns for consumers surplus three different equilibria emerge in the endogenous timing game. (See Appendix E) Further, we can show that the condition for having the simultaneous-move outcome is $\theta < \bar{\theta}_0(0)$. That is, regardless of the externality, if the SR firm does not account for consumers surplus significantly, both firms choose to move early, that is a simultaneous-movement emerges in equilibrium. Otherwise, sequential-movements are equilibrium outcomes. In specific, the condition for having both sequential-move outcomes is $\bar{\theta}_1(0) < \theta \leq 1$. Thus, regardless of the externality, if the SR firm accounts for consumers surplus significantly, one firm chooses to move early and the other firm later in the equilibrium. In the three different equilibria, we can also show that the SR firm increases output and thus $\pi_0 > \pi_1$ for $\theta \in (0, 1]$. Thus, the SR firm always obtains higher profits than FP firm in the equilibrium. This result includes the findings of Lambertini and Tampieri (2015), but we can show that their results hold in an endogenous choice game only when the SR firm does not account for consumers surplus significantly regardless of the externality.

5. Concluding remarks

This paper considered the heterogeneity of objectives among the firms in which an SR firm concerns for not only consumers surplus but environmental externality in the presence of clean technology. We examined how behavioral heterogeneity and the significance of externality induce the equilibrium outcome of the endogenous choice on the different market structure. We found that two
social concerns on consumers surplus and environment have opposite effects on production and abatement, and thus the commitment to social concerns may allow the SR firm to include different market structure in the equilibrium of endogenous timing game. In particular, when the SR firm concerns for externality more, depending on the significance of ex-ante initial level of pollution emission, a simultaneous-movement emerges in equilibrium and the SR firm reduces output and always obtains lower profits than the FP firm. However, when the SR firm concerns for consumers surplus significantly, two sequential-move games might also be equilibria and the SR firm increases output and obtains higher profits than FP firm.

As a future research, it is also important to analyze the effects of the equilibrium of the endogenous timing game on the environment and welfare consequences. Further, governmental intervention on the CSR behaviors and its implications on the endogenous choice of a market structure including pricing and production strategies are also challenging policy issues.

References


Appendix A. The values of $\sigma_i$

\[
\begin{align*}
\sigma_1 & = 3e^2\gamma^2 \left(7 + 3\gamma^2\right) + 2e\gamma \left(2 - 11\theta - 3\gamma^2(4 + \theta)\right) \\
\sigma_2 & = e\gamma^2 \left(27 - 9\gamma^2 - 8\theta\right) - 2e\gamma \left(18 + \theta - 2\theta^2 - 2\gamma^2(3 + \theta)\right) \\
\sigma_3 & = -27e^2\gamma^2 \left(7 + 3\gamma^2\right) + e\gamma \left(132\theta + 36\gamma^2(6 + \theta)\right) \\
\sigma_4 & = e^2\gamma^2 \left(144 + 9\gamma^6 - 68\theta + 8\theta^2 - 3\gamma^4(9 - 4\theta) - \gamma^2 \left(46 + 6\theta - 4\theta^2\right)\right) \\
& \quad + 2e\gamma \left(15 - 87\theta + 41\theta^2 - 5\theta^3 - 3\gamma^6(4 + \theta) + \gamma^4 \left(64 - 8\theta - 5\theta^2\right) - \gamma^2 \left(91 - 74\theta + 5\theta^2 + 2\theta^3\right)\right) \\
\sigma_5 & = e^2\gamma^2 \left(8 - 3\gamma^2\right)^2 - 29\theta + 11\gamma^2\theta + 3\theta^2\right) - 2e\gamma \left(40 - 7\theta^2 + \theta^3 + 2\gamma^4(3 + \theta) - \gamma^2 \left(31 + 5\theta - 3\theta^2\right)\right)
\end{align*}
\]
Appendix B. The values of $\eta_i$

$$\eta_1 \equiv 75 - 50\theta - 32\theta^2 + 22\theta^3 - 3\theta^4 - \gamma^6 (12 + 8\theta + \theta^2) + \gamma^4 (72 + 28\theta - 11\theta^2 - 2\theta^3)$$
$$- \gamma^2 (135 - 46\theta^2 + 6\theta^3 + \theta^4)$$

$$\eta_2 \equiv 25 + 32\theta - \gamma^6 \theta - 22\theta^2 + 3\theta^3 + \gamma^4 (4 + 11\theta - 2\theta^2) - \gamma^2 (20 + 34\theta - 14\theta^2 + 3\theta^3)$$

Let $\alpha = 19755 - 67932\gamma^2 + 100440\gamma^4 - 83146\gamma^6 + 42020\gamma^8 - 13236\gamma^{10} + 2531\gamma^{12} - 268\gamma^{14} + 12\gamma^{16}$. Then

$$\eta_3 \equiv 44 - 32\gamma^2 + 6\gamma^4 - \frac{2\sqrt{2}}{3} \left( 99 - 175\gamma^2 + 113\gamma^4 - 31\gamma^6 + 3\gamma^8 \right)$$
$$\sqrt{96 - 399\gamma^2 + 531\gamma^4 - 316\gamma^6 + 87\gamma^8 - 9\gamma^{10} + \sqrt{- (14 - 13\gamma^2 + 3\gamma^4)^2} \alpha}$$

$$- 2^{2/3} \sqrt{96 - 399\gamma^2 + 531\gamma^4 - 316\gamma^6 + 87\gamma^8 - 9\gamma^{10} + \sqrt{- (14 - 13\gamma^2 + 3\gamma^4)^2} \alpha}$$

$$\eta_4 \equiv 82 - 63\gamma^2 + 12\gamma^4 - 3\theta (55 - 52\gamma^2 + 17\gamma^4 - 2\gamma^6) + (66 - 48\gamma^2 + 9\gamma^4) \theta^2$$
$$- (7 - 3\gamma^2) \theta^3$$

$$\eta_5 \equiv 6 (4 (1 - \gamma^2) + \gamma^4) + (1 - \theta) (12\gamma^4 + 4\theta^2 + (1 - \theta) (41 - 15\gamma^2) + 62 - 57\gamma^2) > 0$$

Appendix C. Proofs of Lemmas

Lemma 1

$$V_0^c - V_0^{fl} = \frac{(1-\theta)(e\gamma+1-\gamma^2+1-\theta)}{2(3-\gamma^2-\theta)(7-3\gamma^2-\theta)^2(8-3\gamma^2-2\theta)^2} (\eta_4 - e\gamma\eta_5).$$

a) If $\gamma \in (0, 1]$ the sign of the difference $V_0^c - V_0^{fl}$ is the sign of $\eta_4 - e\gamma\eta_5$ which is positive if and only if $0 \leq \theta < \theta_0(\gamma) \equiv \frac{\eta_4}{e\gamma\eta_5}$ and $0 < e < e_0 \equiv \frac{\eta_4}{\gamma\eta_5}$.

b) If $\gamma = 0$, the sign of the difference $V_0^c - V_0^{fl}$ is the sign of $\eta_4(\gamma = 0) = 82 - 165\theta + 66\theta^2 - 7\theta^3$, which does not depend on $e$ and is positive if and only if $0 \leq \theta < \theta_0(0) \approx 0.658$.

Lemma 2

$$\pi_1^i - \pi_1^{fl} = \frac{3(2-(1-\gamma^2)\theta-e\gamma(3-2\theta))(e\gamma(45-18\gamma^2-10\theta)+(1-\theta)(47-25\gamma^2-8\theta)+(35-18\gamma^2)(1-\gamma^2))}{2(8-3\gamma^2-2\theta)^2(21-9\gamma^2-4\theta)^2}.$$

a) If $\gamma \in (0, 1]$, the sign of the difference $\pi_1^i - \pi_1^{fl}$ is the sign of $2 - (3 - \gamma^2) \theta - e\gamma(3 - 2\theta)$ which is positive if and only if $0 \leq \theta < \theta_1(\gamma) \equiv \frac{2}{3-\gamma^2}$ and $0 < e < e_1 \equiv \frac{2-3\theta+\gamma^2\theta}{\gamma(3-2\theta)}$.

b) If $\gamma = 0$, the sign of the difference $\pi_1^i - \pi_1^{fl}$ is the sign of $2 - (3 - \gamma^2) \theta$, which does not depend on $e$ and is positive if and only if $0 \leq \theta < \theta_1(0) = \frac{2}{3}$.
Lemma 3

a) For any \( \gamma \in (0, 1] \),
\[
e_1 - e_0 = \frac{(1-\theta)^2(8-3\gamma^2-2\theta)(1+(1-\gamma^2)\theta)}{\gamma(3-2\theta)} > 0 \text{ for any } 0 \leq \theta < 1.
\]
b) By substitution of \( \theta = 1 \), \( e_1(1, \gamma) = e_0(1, \gamma) = \gamma - \frac{1}{2} \).
c) From Lemma 3a) and b), \( e_1(\bar{\theta}_0, \gamma) > e_0(\bar{\theta}_0, \gamma) = 0 = e_1(\bar{\theta}_1, \gamma) \); since \( e_1 \) monotonically decreases on \( \theta \), we have \( \bar{\theta}_1(\gamma) > \bar{\theta}_0(\gamma) \).

Appendix D. Proof of proposition

Using Lemmas 3a, 3b we obtain the following table:

<table>
<thead>
<tr>
<th>( \theta/e )</th>
<th>( e \leq e_0 )</th>
<th>( e_0 &lt; e \leq e_1 )</th>
<th>( e &gt; e_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; ( \theta &lt; \bar{\theta}_0 )</td>
<td>( V^c_0 \geq V'^f_l ) &amp; ( \pi^c_1 \geq \pi'^l_1 )</td>
<td>( V^c_0 &lt; V'^f_l ) &amp; ( \pi^c_1 \geq \pi'^l_1 )</td>
<td>( V^c_0 &lt; V'^f_l ) &amp; ( \pi^c_1 &lt; \pi'^l_1 )</td>
</tr>
<tr>
<td>( \bar{\theta}_0 &lt; \theta &lt; \bar{\theta}_1 )</td>
<td>( V^c_0 &lt; V'^f_l ) &amp; ( \pi^c_1 \geq \pi'^l_1 )</td>
<td>( V^c_0 &lt; V'^f_l ) &amp; ( \pi^c_1 \geq \pi'^l_1 )</td>
<td>( V^c_0 &lt; V'^f_l ) &amp; ( \pi^c_1 &lt; \pi'^l_1 )</td>
</tr>
<tr>
<td>( \theta &gt; \bar{\theta}_1 )</td>
<td>( V^c_0 &lt; V'^f_l ) &amp; ( \pi^c_1 &lt; \pi'^l_1 )</td>
<td>( V^c_0 &lt; V'^f_l ) &amp; ( \pi^c_1 &lt; \pi'^l_1 )</td>
<td>( V^c_0 &lt; V'^f_l ) &amp; ( \pi^c_1 &lt; \pi'^l_1 )</td>
</tr>
</tbody>
</table>

Appendix E. Cases

E1. Case with \( \theta = 0 \)

A graphical representation is shown in Figure E.1

![Figure E.1: Endogenous Timing Game Equilibrium with \( \theta = 0 \)](image-url)
Remark 1. When $\theta = 0$, the only equilibrium of the game is the simultaneous movement, that is, $(t_0, t_1) = (1, 1)$.

Remark 2. When $\theta = 0$, $\pi_1^c > \pi_0^c$ for $\gamma \in (0, 1]$.

E2. Case with $\gamma = 1$

Remark 3. When $\gamma = 1$, the equilibrium outcomes of the endogenous timing game are as follows:

(a) If $0 \leq \theta < \bar{\theta}_0(1)$ and $e \leq e_0(\theta, 1)$, then $(t_0, t_1) = (1, 1)$;
(b) if $\bar{\theta}_1(1) < \theta \leq 1$ or $e > e_1(\theta, 1)$, then $(t_0, t_1) = (1, 2)$ and $(t_0, t_1) = (2, 1)$.
(c) otherwise, $(t_0, t_1) = (2, 1)$.

A graphical representation is shown in Figure E.2.

Figure E.2: Endogenous Timing Game Equilibrium with $\gamma = 1$

Remark 4. When $\gamma = 1$, we have

(a) $\pi_1^c > \pi_0^c$.
(b) $\pi_1^l > \pi_0^l$.
(c) $\pi_1^s > \pi_0^s$.

E3. Case with $\theta = 1$

A graphical representation is shown in Figure E.3.
Remark 5. When \( \theta = 1 \), the equilibrium outcomes of the game are either the SR firm or the FP firm could be the Stackelberg leader of the game, that is, \((t_0, t_1) = (1, 2)\) and \((t_0, t_1) = (2, 1)\).

Remark 6. When \( \theta = 1 \), we have

(a) \( \pi_{s1}^{0} > \pi_{s1}^{1} \) for \( \gamma \in (0, 0.577) \) and \( e \in (\max\{\varepsilon, \xi_1\}, \xi_2) \), and \( \pi_{s1}^{0} > \pi_{s1}^{0} \) otherwise.

(b) \( \pi_{f1}^{0} > \pi_{f1}^{1} \) for \( \gamma \in (0, 0.577) \) and \( e \in (\xi_3, \xi_4) \), and \( \pi_{f1}^{1} > \pi_{f1}^{0} \) otherwise.

Proof. When \( \theta = 1 \).

(a) Let \( \xi_1 = \frac{13+51\gamma^2-\sqrt{289-1173\gamma^2+999\gamma^4-243\gamma^6}}{9(8+3\gamma^2)} \) and \( \xi_2 = \frac{13+51\gamma^2+\sqrt{289-1173\gamma^2+999\gamma^4-243\gamma^6}}{9(8+3\gamma^2)} \).

Then \( \pi_{s1}^{0} - \pi_{s1}^{1} = \frac{5-106\gamma^2-27\gamma^4-27e^2\gamma^2(8+3\gamma^2)+6e(13+51\gamma^2)}{2(17-9\gamma^2)^2} > 0 \) for \( \gamma \in (0, 0.577) \) and \( \max\{\varepsilon, \xi_1\} < e < \xi_2 \).

(b) Let \( \xi_3 = \frac{2(1+3\gamma^2)-\sqrt{4-16\gamma^2+13\gamma^4-3\gamma^6}}{\gamma(8+3\gamma^2)} \) and \( \xi_4 = \frac{2(1+3\gamma^2)+\sqrt{4-16\gamma^2+13\gamma^4-3\gamma^6}}{\gamma(8+3\gamma^2)} \).

Then \( \pi_{f1}^{0} - \pi_{f1}^{1} = \frac{\gamma(-\gamma(5+\gamma^2)+4e(1+3\gamma^2)-e^2\gamma(8+3\gamma^2))}{6(2-\gamma^2)^2} > 0 \) for \( \gamma \in (0, 0.577) \) and \( \xi_3 < e < \xi_4 \).

E4. Case with \( \gamma = 0 \)

A graphical representation is shown in Figure E.4.
Remark 7. Let $\theta \in (0, 1]$. When $\gamma = 0$, we have that $\pi_0 > \pi_1$ at every equilibrium of the endogenous timing game.

Proof. When the SR firm sets $\gamma = 0$. Then

(a) $\pi_0^c - \pi_1^c = \frac{(3-\theta)\theta}{(4-\theta)^2} > 0$ for any $0 < \theta < \bar{\theta}_0(0)$

(b) $\pi_0^s - \pi_1^s = \frac{9+(149-32\theta)\theta}{2(21-4\theta)^2} > 0$ for any $\bar{\theta}_1(0) < \theta \leq 1$

(c) $\pi_0^f - \pi_1^f = \frac{-1+(21-4\theta)\theta}{4(7-\theta)^2} > 0$ for any $\bar{\theta}_0(0) \leq \theta \leq 1$

$\square$