Signaling in the shadow of conflict

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Abstract

Informational asymmetries have long been recognized as one of the causes of wasteful conflicts. Signaling has been found to be an effective tool for interested parties to truthfully communicate private information. Can signaling help reduce the risk of conflict? I study this question in a model in which a Sender sends a signal about his privately known cost of conflict, a Receiver makes an offer, and the Sender decides whether or not to start a conflict. I find that an equilibrium with no conflict exists only if the outcome of conflict depends on the Receiver's offer and the Sender's possible costs are sufficiently apart. Importantly, these conditions are never satisfied in the context of wars where belligerents obtain all or none of a dispute territory. In such context, no information is ever transmitted in equilibrium. Overall, this paper establishes that the shadow of conflict renders signaling quite ineffective to resolve informational asymmetry.

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From wars to trials via strikes, conflicts occur various forms despite their cost and the risk they involve all interested parties. This puzzle has led a large literature to investigate the sources of a conflict. One commonly recognized cause is informational asymmetry. But are these informational asymmetries inescapable? Are there means to eliminate them?

In this paper, I consider one potential decentralized mechanism to reduce asymmetry of information: costly signaling. I find that signaling facilitates information transmission only under restrictive conditions on the form of conflict and distribution of private information. Importantly, wars, as they are usually modeled, do not meet these conditions. Signaling never allows for any information transmission prior to military action: the unique equilibrium is a pooling equilibrium. These results contrast with dozens of studies which find that signaling helps information transmission even when players have strong incentives to misrepresent themselves. They, however, can be explained by a single factor: the shadow of conflict.

Formally, I build upon bargaining models of war. A Sender has private information about his cost of conflict, whereas the Receiver's cost is commonly known. In a first stage, the Sender can send a signal at a cost satisfying increasing differences (i.a., increasing his signal becomes more costly as the Sender's cost of conflict increases). After observing the Sender's signal, a Receiver makes an to the Sender. The latter then decides whether to start a conflict. A conflict takes the form of a lottery over various potential outcomes.

In the paper, I distinguish between two different forms of lottery: *fixed-outcome* and *offer-dependent* lotteries. In fixed-outcome lotteries, the possible outcomes of the conflict do not depend on the Receiver's offer. This corresponds to several well-known situations. Armed conflicts are usually modeled as winner-takes-all lotteries in which the payoff from victory does not depend on a country's prior concessions (Ramsay, 2017). Signaling here assumes the form of staging military parade, missile tests, demonstrations, or protestations at the United Nation organization. This representation of conflicts also applies to trials as plea bargain offers do not constraint future legal proceedings (Silveira, 2017). Here, delaying tactics can be thought as signals. Another possible example is strikes whose successes or failures have little to do with the wage offer prior to collective action and for which trade unions can use tracts and posters to signal their ability to mobilize their members. In turn, in offer-dependent lotteries, one or more possible outcomes strictly increases with the Receiver's offer. This type of conflict is less common, but not unknown. Wolton (2017) models outside lobbying activities as an offer-dependent lottery (the Receiver's offer is a bill which is either enacted or abandoned) with informative lobbying and contributions serving as signals.

The form of conflict plays a critical role for information transmission. If the conflict is a fixed-outcome lottery, then the unique equilibrium is a pooling equilibrium. To understand this result, suppose that the Sender's cost can take only two values: high or low. If the Sender were to reveal his cost, the Receiver would make a compromise offer which leaves the Sender indifferent between starting a conflict and accepting the offer (any other offer, I show, is dominated). Now suppose that a low-cost Sender pretends to be a high-type Sender. The Receiver's offer is then less attractive than the compromise offer for a low-cost, and the Sender starts a conflict. Because the conflict is a fixed-outcome lottery, a low-cost Sender's expected payoff from imitating a high-cost is exactly the same as his payoff from revealing his type: the expected payoff from a conflict. A low-cost Sender has thus no gain from differentiating himself from a high type and is not willing to pay any signaling cost. Let us now turn to a high-cost Sender. If he pretends to be a low-cost, he obtains a compromise offer which makes the low-cost Sender indifferent between conflict and peace. This is always more attractive that the compromise offer he obtains from truthfully revealing his type. In other words, a high-cost Sender has a positive benefit from imitating a low-cost Sender and the signaling cost must be positive to avoid him mimicking a low type. Of course, the two conditions cannot be satisfied simultaneously, and information transmission is impossible. The reasoning above applies to all informative equilibria for all possible fixed-outcome lotteries, all (additively separable) continuous utility functions, and all distribution of the Sender's costs. In all cases, there is no information transmission in equilibrium.

When the conflict takes the form of an offer-dependent lottery, a low-cost Sender gets some benefit from differentiation since the outcome of the conflict depends on the offer on the table to begin with. Because of the possibility of conflict, however, a low-cost Sender's benefit from differentiation is strictly lower than a high-cost Sender's benefit from imitation. A low-cost Sender then is only willing to send a relatively low costly signal to reveal his type. For separation to occur, it must be that even this relatively low signal is too expensive for a high-cost Sender. This is possible only if the two types are sufficiently apart (taking advantage of the increasing difference in signaling costs). The reasoning above thus implies that a separating equilibrium never exists when the set of possible Sender's cost contains an interval (as types are too close) and exists only under restrictive conditions when the Sender's costs take discrete values. While information transmission is possible with offer-dependent lottery, it does not occur generically.

A large literature studies the relationship between uncertainty and conflict (for two excellent recent reviews, see Baliga and Sjöström, 2013 and Ramsay, 2017). Few papers, however, consider whether signaling can alleviate informational asymmetries and reduce the risk of conflict. Baliga and Sjöström (2004) and Kydd (2005) show how cheap talk can be effective in term of reducing the risk of arm races. However, their results rely on arm races being a form of prisoner's dilemma and thus players having some commonality of interest. This is not the case in bargaining model of war. In such context, my paper extends the signaling analysis in Arena (2013). While Arena considers a single utility function (linear), single lottery (winner-takes-all), and binary signals, I look at the effectiveness of signaling for all continuous utility functions, all lotteries, and all possible signals. Finally, my finding can be related to recent works on mediation (Hörner, Morelli, and Squintani, 2015; Meirowitz et al., 2017). These papers show that in the optimal mediation equilibrium, the probability of war is strictly positive to induce participants to truthfully reveal their private information. In turn, I establish that there is no decentralized mechanism which guarantees information revelation because of the shadow of war.

1 Set-up

I study a one-period game with a Sender (S) and a Receiver (R). The receiver makes an offer $a_R \in [\underline{a}, \overline{a}]$, with (without loss of generality) $0 \leq \underline{a} < \overline{a}$. Depending on contexts, the offer is a division of a disputed territory, a plea bargain offer, a wage rise, or the content of a bill. After observing a_R, the Sender can decide whether to start a conflict: $f_S \in \{0, 1\}$, with 1 denoting conflict. The conflict corresponds to a war, going to trial, going on strike, or outside lobbying activities. A conflict has two consequences. First, a conflict affects the outcome of the game, denoted z: no conflict leads to outcome $z = a_R$ with probability 1, whereas conflict generates a lottery $\mathcal{L}(a_R)$ over possible outcomes. Second, a conflict imposes a cost on both players. I describe these two aspects in turn.

Denote $\mathbf{z}(a_R) = \{z_1(a_R), \ldots, z_N(a_R)\} \in [\underline{a}, \overline{a}]^N$, $N \geq 2$, the set of possible outcomes in a conflict ($f_S = 1$) with $z_j(\cdot) < z_{j+1}(\cdot)$ for all $j \in \{1, \ldots, N-1\}$. Denote as well $\mathbf{p} = \{p_1, \ldots, p_N\} \in \Delta(\mathbf{z}(a_R))$ the associated probability distribution over outcomes (i.e., $Pr(z = z_j(\cdot)) = p_j$ for all j). The lottery

$\mathcal{L}$ can be represented as $\mathcal{L}(a_R) =< \mathbf{z}(a_R), \mathbf{p} >$. Throughout, I suppose that $\mathcal{L}(\cdot)$ is non-degenerate (i.e., there exists $j \in \{1, \ldots, N\}$ such that $p_j \in (0,1)$). I distinguish between a fixed-outcome lottery in which $z_j(a_R)$ is constant in a_R for all $j \in \{1, \ldots, N\}$ and an offer-dependent lottery in which there exists $j \in \{1, \ldots, N\}$ such as $z_j(a_R)$ is strictly increasing in a_R. A commonly used fixed-outcome lottery in the literature is the winner-takes-all lottery: $\mathbf{z}(a_R) = \{\underline{a}, \overline{a}\}$ and $\mathbf{p} = \{p, 1-p\}$ for all $a_R \in [\underline{a}, \overline{a}]$. An example of offer-dependent lottery is $\mathbf{z} = \{\underline{a}, a_R\}$ and $\mathbf{p} = \{p, 1-p\}$ so that the Receiver either obtains $\underline{a}$ or his offer a_R.

The cost of conflict for the Receiver is common knowledge and equals $k_R > 0$. The cost for the Sender is his private information (type) and denoted k_S. Types are drawn from a commonly known type space $\mathcal{K}_S \subseteq [\underline{k}_S, \overline{k}_S]$ with $0 < \underline{k}_S < \overline{k}_S$ according to the common knowledge cumulative distribution function $F(\cdot)$. Throughout, I assume that $F(\cdot)$ is non-degenerate and without loss of generality $\underline{k}_S, \overline{k}_S \in \mathcal{K}_S^2$. Note that $\mathcal{K}_S$ can be an interval (i.e., $F(\cdot)$ is continuous) or a discrete set (i.e., $F(\cdot)$ exhibits discontinuities). At the beginning of the game, the Sender can send a signal $s_S \in \mathbb{R}_+$ to reveal his type. The cost of signal s_S is $C(s_S, k_S)$. I assume that $C(\cdot, k_S)$ is strictly increasing in s_S with $\lim_{s_S \to \infty} C(s_S, k_S) = \infty$ for all $k_S \in \mathcal{K}_S$.

Turning to payoffs, the Receiver, without loss of generality, prefers higher outcome z. Her utility can be represented as:

$$U_R(a_R, f_S) = r(z) - f_S \times k_R, \tag{1}$$

In turn, the Sender prefers lower outcome and his utility assumes the following form:

$$U_S(a_R, f_S, s_S) = -v(z) - f_S \times k_S - C(s_S, k_S), \tag{2}$$

I impose that both $r(\cdot)$ and $v(\cdot)$ are bounded, and strictly increasing over $[\underline{a}, \overline{a}]$. Further, I assume that $v(\cdot)$ is $\mathcal{C}^1$ over $[\underline{a}, \overline{a}]$ (the reasoning can be extended to a semi-continuous $v(\cdot)$ or $v'(\cdot)$ at the cost of complicating the analysis). Observe that the model allows for players to be risk-seeking on some subset of $[\underline{a}, \overline{a}]$ (i.e., $r(\cdot)$ convex and/or $v(\cdot)$ concave).

To summarize, the game proceeds as follows:

0. Nature draws k_S from $\mathcal{K}_S$ according to the distribution $F(\cdot)$;
1. Sender privately observes k_S and sends signal $s_S \geq 0$;
2. Receiver observes s_S and chooses $a_R \in [\underline{a}, \overline{a}]$;
3. Sender chooses $f_S \in \{0, 1\}$;

4. Nature determines outcome, the game ends, and payoffs are realized.

The equilibrium concept is Perfect Bayesian Equilbrium (henceforth 'equilibrium'). Observe that I do not impose any equilibrium refinement (e.g., Intuitive Criterion). This means that out-of-equilibrium beliefs are unrestricted to facilitate information transmission. I also assume that the cost of signaling $C(\cdot,\cdot)$ is $\mathcal{C}^1$ and exhibits strict increasing differences. That is, for all $k_S^l, k_S^h \in \mathcal{K}_S^2,\ k_S^l < k_S^h$ and $s_S^l, s_S^h \in \mathbb{R}_+^2,\ s_S^l < s_S^h$:

$$C(s_S^h, k_S^h) - C(s_S^l, k_S^h) > C(s_S^h, k_S^l) - C(s_S^l, k_S^l)$$

This assumption is meant to increase the chances that the Sender has incentives to truthfully signal his type at the signaling stage (stage 1.) and thus goes against the paper's main findings (Malaith, 1987). An example of signaling cost function satisfying strict increasing differences is $C(s_S, k_S) = s_S \times k_S$.

For the Sender, a signaling strategy takes the form of a mapping from his type to some real positive value $s_S : [\underline{k}_S, \overline{k}_S] \to \mathbb{R}_+$. Following the usual definition, a strategy is *separating* if for all $k_S^l, k_S^h \in \mathcal{K}_S^2,\ k_S^l < k_S^h,\ s_S(k_S^l) \neq s_S(k_S^h)$. A conflict strategy is a mapping from R's offer and S's type to a conflict decision $f_S : [\underline{a}, \overline{a}] \times \mathcal{K}_S \to \{0, 1\}$. For the Receiver R, his strategy is a mapping from the sender's signal to an offer $a_R : \mathbb{R}_+ \to [\underline{a}, \overline{a}]$. Throughout, I use the subscript $*$ to denote equilibrium actions.

2 Analysis

I study whether signaling can resolve informational asymmetries at the source of wasteful conflicts. To do so, I focus on the most interesting cases for which conflicts would be avoided, should information asymmetries be eliminated.

Formally, denote $a_R^c(k_S)$ the Receiver's offer which leaves the Sender indifferent between starting a conflict ($f_S = 1$) or peace ($f_S = 0$). I label $a_R^c(k_S)$ the *compromise offer* and assume it is unique in the text (the Appendix deals with the general case). I assume that the Sender's potential cost of conflicts are such that compromise is always necessary and possible: $\underline{a} \leq a_R^c(k_S) \leq \overline{a}$ for all $k_S \in \mathcal{K}_S$.[1] Further, I assume that if the Receiver learns the Sender's type, he is willing to

[1]Note that if $\underline{a} > a_R^c(k_S)$ for some $k_S \in \mathcal{K}_S$, then conflict is unavoidable for some types. In turn, if $a_R^c(k_S) > \overline{a}$, some types never start a conflict. The two restrictions are with little loss of generality since we can always focus on the subset of types for which they hold.

compromise: $r(a^c_R(k_S)) \geq E_{\mathcal{L}}\big(r(z)|a_R = \overline{a}\big) - k_R$ for all $k_S \in \mathcal{K}_S$ (where the left-hand side is the payoff from compromise and the right-hand side the highest possible expected payoff from a conflict with expectation over lottery outcomes). Finally, absent any information at the signaling stage, I suppose that there is a risk of conflict: $a^p_R = \arg\max_{a_R \in [\underline{a},\overline{a}]} E\big(U_S(a_R; f_S)|k_S \in \mathcal{K}_S\big)$ (with expectation over Sender's type and lottery outcomes) satisfies $a^p_R > a^c_R(k_S)$ for some $k_S \in \mathcal{K}_S$.

I first consider fixed-outcome lottery. The first proposition states that signaling *never* reduces *by any amount* the informational asymmetry.

Proposition 1. *For all fixed-outcome lotteries and all type-space $\mathcal{K}_S$, in any equilibrium, the Sender plays a pooling strategy: $s^*_S(k_S) = s^p_S$ for all $k_S \in \mathcal{K}_S$.*

Proof. All proofs are collected in the Appendix □

To understand this result, recall that, under the assumptions, after learning that the Sender's cost is k_S, the Receiver chooses the compromise offer $a^c_R(k_S)$, which leaves the Sender indifferent between conflict and peace. Since conflict is a fixed-outcome lottery, $a^c_R(k_S)$ satisfies: $-v(a^c_R(k_S)) = -E_{\mathcal{L}}\big(v(z)\big) - k_S$ for all $k_S \in \mathcal{K}_S$. Consider now the strategy of two types $k^l_S, k^h_S \in \mathcal{K}^2_S,\ k^l_S < k^h_S$. For a separating equilibrium to exist, it must be that (i) a Sender with cost k^h_S does not want to imitate a Sender with cost k^l_S *and* (ii) a type-k^l_S is willing to differentiate himself from a type-k^h_S. When a high-cost Sender (k^h_S) imitates a low-cost Sender (k^l_S), he obtains a better compromise offer and a strictly positive benefit from imitation $v(a^c_R(k^l_S)) - v(a^c_R(k^h_S))$. Thus, to discourage imitation by a high-cost, it must be that mimicking a low cost is costly: $s_S(k^l_S) > s_S(k^h_S)$. In turn, if a low-cost pretends to be a high-cost, the Receiver offers $a^c_R(k^h_S) > a^c_R(k^l_S)$ and the Sender starts a conflict. His expected payoff is then $E_{\mathcal{L}}\big(v(z)\big) - k^l_S$, the exact same payoff as from revealing his type. A low-cost Sender's benefit from differentiation is thus null and he is never willing to pay a signaling cost to reveal his type. Consequently, we can never satisfy conditions (i) and (ii) simultaneously. The result extends to all possible equilibria with information transmission at the signaling stage since I can always find two types such that the reasoning above applies.

As noted in the introduction, many different forms of conflict can be understood as fixed-outcome lotteries: trials, strikes, and especially wars. In all these cases, Proposition 1 indicates that there is no decentralized mechanism which permits information transmission. This result may provide a rationale for why armed conflict actually break out. One of the criticism of bargaining models of war is that as we get closer to the onset of war, uncertainty should be resolved. Proposition 1 shows that this need not be the case. Any action by a belligerent prior to his opponent's final offer has no informative content. Only the beginning or not of military actions is informative about an interested party's type (one—important—limitation to the whole analogy is that the unique pooling equilibrium features no type engaging in costly signaling: $s_S^*(k_S) = 0$ for all $k_S \in \mathcal{K}_S$).

I now turn to offer-dependent lottery. First, I consider whether conflict can always be avoided, that is, whether a separating equilibrium exists. Unlike fixed-outcome lotteries, the answer is positive, though under specific conditions. In particular, it is necessary, but not sufficient, that the type-space is discrete.

Proposition 2. *Denote K the cardinality of $\mathcal{K}_S$. For all offer-dependent lotteries, a separating equilibrium exists if and only if*

1. *The type-space $\mathcal{K}_S$ is discrete;*
2. *There exist a K-dimension vector $\mathbf{s}_\mathbf{S}^* = (s_S^*(k_S^1), \ldots, s_S^*(k_S^K))$ satisfying for all $j \in \{1, \ldots, K-1\}$:*

$$C(s_S^*(k_S^j), k_S^j) - C(s_S^*(k_S^{j+1}), k_S^j) + (k_S^{j+1} - k_S^j) \leq v(a_R^c(k_S^{j+1})) - v(a_R^c(k_S^j))$$
$$\leq C(s_S^*(k_S^j), k_S^{j+1}) - C(s_S^*(k_S^{j+1}), k_S^{j+1}) \quad (3)$$

To understand this result, consider again he strategy of two types $k_S^l, k_S^h \in \mathcal{K}_S^2,\ k_S^l < k_S^h$. As above, a separating equilibrium exists only if a type-k_S^h does not want to imitate a type-k_S^l and a type-k_S^l is willing to distinguish himself from its higher cost counterpart. For a high-cost Sender, the benefit of imitation is still the gain from a more favorable compromise offer: $v(a_R^c(k_S^l)) - v(a_R^c(k_S^h))$ with $a_R^c(k_S)$ such that $-v(a_R^c(k_S)) = -E_\mathcal{L}\big(v(z)|a_R = a_R^c(k_S)\big) - k_S$ (note the dependence of lottery outcomes on $a_R^c(k_S)$). In turn, if the low-cost Sender pretends to be a type k_S^h, it starts a conflict after the Receiver offers the compromise $a_R^c(k_S^h)$. This means that a low-cost Sender's benefit from differentiation is: $v(a_R^c(k_S^l)) - \big(E_\mathcal{L}\big(v(z)|a_R = a_R^c(k_S^h)\big) + k_S^l\big) < v(a_R^c(k_S^l)) - v(a_R^c(k_S^h))$. Observe that since conflict is an offer-dependent lottery, a low-cost Sender's benefit from differentiation is no longer null. Nonetheless, due to the possibility of starting a conflict, it is strictly lower than a high-cost Sender's benefit from imitation. The signaling cost to guarantee separation must thus be sufficiently large to encourage a type-k_S^h Sender not to imitate a lower cost, but also sufficiently small to guarantee that a type-$k - S^l$ Sender reveals his type. When the Senders' possible costs of conflict are very close (e.g., $\mathcal{K}_S$ contains an interval), there never exists a signaling strategy $\mathbf{s}_\mathbf{S}$ which satisfies both conditions. Hence, a separating equilibrium can only exist if the Sender's possible costs take discrete values. In addition, Condition 3 also needs to be satisfied. This condition imposes additional restrictions on the distance between the Sender's possible costs. To better interpret Condition 3, the next example provides a useful illustration.

Example 1. *Suppose* $\mathbf{z}(a_R) = \{\underline{a}, a_R\}$, *with* $Pr(z = a_R) = 1 - p \in (0, 1)$, *and* $C(s_S, k_S) = s_S \times k_S$. *The compromise offer* $a^c_R(k^j_S)$ *satisfies* $v(a^c_R(k^j_S)) = v(\underline{a}) + \frac{k^j_S}{p}$. *A separating equilibrium exists if and only if for all* $j \in \{1, \ldots, K-1\}$ $k^j_S \leq (1-p)k^{j+1}_S$.

Under the assumptions, a separating equilibrium guarantees no conflict. Can we obtain a similar outcome when we allow for coarser information transmission at the signaling stage (i.e., the Sender plays a semi-separating strategy)? As the next proposition shows, in addition to an amended Condition 3, conflict is avoided in a semi-separating equilibrium only if the Receiver is, in some sense, conflict-adverse and compromises with all types playing the same signaling strategy. Before stating formally the result, it is useful to introduce the following notation. For all $K^m_S \subseteq \mathcal{K}_S$, denote $k^m_S := \min K^m_S$.

Proposition 3. *For all offer-dependent lottery, there exists a semi-separating equilibrium with no conflict if and only if there exists a partition of the type space* $\mathcal{K}_S$ *into* $M \geq 2$ *subsets* $K^1_S, \ldots, K^M_S$ *and* $\mathbf{s^{ss}_S} = \{s^1_S, \ldots, s^M_S\}$ *such that for all* $m \in \{1, \ldots, M-1\}$*:*

1. $\max K^m_S < \min K^{m+1}_S$
2. $v(a^c_R(k^{m+1}_S)) - v(a^c_R(k^m_S)) \leq C(s^m_S, k^{m+1}_S) - C(s^{m+1}_S, k^{m+1}_S)$;
3. *For all* $k_S \in K^m_S$, $C(s^m_S, k_S) - C(s^{m+1}_S, k_S) + k^{m+1}_S - k_S \leq v(a^c_R(k^{m+1}_S)) - v(a^c_R(k^m_S))$;
4. *For all* s^m_S, $a^c_R(k^m_S) \in \arg\max E\big(U_R(a_R, f_S) | k_S \in K^m_S\big)$.

Condition 1 in Proposition 3 documents that in any semi-separating equilibrium, partition sets are ordered. Condition 2 establishes a simple condition so that a high-cost Sender does not imitate a Sender with a lower cost. More interestingly, Condition 3 highlights the difficulty to sustain a semi-separating equilibrium (an appropriately modified Condition 3 applies to all semi-separating equilibria). A Sender's incentive compatibility constraint must be satisfied not just for the extrema in the partition sets of the type space, but also for almost all interior costs (i.e., for all $k_S \in K^m_S$, $m \in \{1, \ldots, M-1\}$) . This result boils down again to the shadow of conflict. As in traditional signaling games, the cost of differentiation—i.e., signaling cost—is increasing with a Sender's cost of conflict k_S: $C(s^m_S, k_S) - C(s^{m+1}_S, k_S)$ is increasing in k_S by the increasing differences assumption. In addition, unlike traditional signaling games, in this set-up, the benefit from differentiation is also strictly increasing in the Sender's cost of conflict k_S since a relatively low-cost Sender starts a conflict when it mimics a relatively high type (i.e., send signal s_S^{m+1} instead of s_S^m): $k_S^{m+1} - k_S$ decreases with k_S. The combination of these two effects imply that it is a priori unclear which type in the set K_S^m has the greatest incentive to imitate a relatively high-cost Sender. Finally, Condition 4 stresses that a no-conflict semi-separating equilibrium requires strong assumption on the *Receiver's utility function*. There must exist a partition which satisfies Conditions 1-3 and induces full compromise on the Receiver's part. Overall, Proposition 3 suggests that only separating equilibria (when they exist) can be expected to bring peace.

3 Conclusion

This paper establishes that signaling is unlikely to resolve one of the main sources of conflict: informational asymmetries. A separating equilibrium exists only under restrictive conditions including: (i) the outcome of the conflict must depend on the Receiver's previous offer and (ii) the Sender's privately known cost of conflict must take discrete values. Conditions for existence of a semi-separating equilibrium are no less stringent and such equilibria are unlikely to guarantee no conflict on path. I further establish a strong negative result. If the outcomes of conflict are independent of previous actions, no information is ever revealed in equilibrium for all possible form of uncertainty and payoffs. This suggest that in various settings, such as war or plea bargaining, no decentralized mechanism permits any form of information transmission between interested parties.

In Appendix B and C, I explore the robustness of this striking result. I show that the impossibility to transmit any information with signaling remains when the Sender has better information about his winning probabilities rather than his cost of conflict (Proposition B.1). A similar negative conclusion holds when both the Sender and Receiver face a positive recognition probability to make a take-it-or-leave-it offer (Proposition C.1). Some information can be transmitted, however, if we allow for a bargained solution between the Sender and Receiver, though much depends on the Sender's bargaining power (Propositions C.2 and C.3). This last finding provides an interesting counterpoint to Banks (1990). In his seminal contribution, Banks establishes general properties of bargained outcomes in the shadow of wars for all possible bargaining protocols. My results highlight that no such general property exists when it comes to information transmission.

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A Proofs

Define $A^c(k_S) := \{a_R \in [\underline{a}, \overline{a}] : -v(a_R) = -E_{\mathcal{L}}\big(v(z)|a_R\big) - k_S\}$ the set of actions which leave a type-k_S Sender indifferent between conflict ($f_S = 1$) and no conflict ($f_S = 0$). Further, define $A^p := \Big\{a_R^p \in [\underline{a}, \overline{a}] : a_R^p \in \arg\max_{a_R \in [\underline{a}, \overline{a}]} E\big(U_R(a_R, f_S)|k_S \in \mathcal{K}_S\big)\Big\}$ the set of offers which maximize the Sender's expected utility when no information is transmitted at the signaling stage. I formalise the assumptions in the main text as:

Assumption 1. $-E_{\mathcal{L}}\big(v(z)|a_R = \underline{a}\big) - \underline{k}_S \leq -v(\underline{a})$

Assumption 2. $-E_{\mathcal{L}}\big(v(z)|a_R = \overline{a}\big) - \overline{k}_S \geq -v(\overline{a})$

Assumption 3. *For all $k_S \in \mathcal{K}_S$, $\max A^c(k_S)$ satisfies $r(\max A^c(k_S)) \geq E_{\mathcal{L}}\big(r(z)|a_R = \overline{a}\big) - k_R$*

Assumption 4. *There exist $k_S \in \mathcal{K}_S$ such that $\min A^p > \max A^c(k_S)$.*

Assumption 1 guarantees that there always exists a Receiver's action a_R that the Sender prefers to a conflict. Assumption 2 implies that the Receiver can (almost) never offer the highest action and avoids a conflict. Assumption 3 guarantees that the Receiver is willing to compromise after learning the Sender's cost of conflict. These three assumptions are necessary for a separating equilibrium to exist at least when the type-space is an interval (see Lemma A.1). Finally, Assumption 4 implies that informational asymmetries are a source of conflict. These assumptions guarantee that I focus on cases when signaling plays an important role to avoid a conflict. As explained in the main text, these assumptions are with little loss of generality since I can always redefine the type space so they hold.

Lemma A.1. *Suppose $\mathcal{K}_S = [\underline{k}_S, \overline{k}_S]$ and Assumption 1, 2, or 3 does not hold, then there is no equilibrium in which the Sender plays a separating strategy.*

Before proving the result, I prove two preliminary claims.

Claim 1. *In a separating equilibrium, after signal $s_S^*(k_S)$, the Receiver's best response $a_R^*(s_S^*(k_S))$ is either $a_R^*(s_S^*(k_S)) \in A^c(k_S)$ or $a_R^*(s_S^*(k_S)) = \overline{a}$.*

Proof. To see that, observe first that if $-E_{\mathcal{L}}\big(v(z)|a_R = \overline{a}\big) - k_S \leq -v(\overline{a})$, the Sender does not start a conflict when the Receiver chooses her preferred action. The Receiver's best response is then $a_R^*(s_S^*(k_S^1)) = \overline{a}$.

Assume now that $-E_{\mathcal{L}}\big(v(z)|a_R = \overline{a}\big) - k_S < -v(\overline{a})$ and $A^c(k_S)$ is non-empty. We now show that any action $a_R \notin A^c(k_S) \cup \{\overline{a}\}$ is (weakly) dominated by some other offer. First take $a_R \notin A^c(k_S) \cup \{\overline{a}\}$ so that $f_S(a_R; k_S) = 0$. Under the assumption ad by definition of $A^c(k_S)$, there exists $a_R^c(k_S) \in A^c(k_S)$ such that (i) $a_R^c(k_S) > a_R$ and (ii) $f(a_R^c(k_S); k_S) = 0$ and so provides a higher payoff than a_R. Now take $a_R \notin A^c(k_S) \cup \{\overline{a}\}$ so that $f_S(a_R; k_S) = 1$. Recall that $E_{\mathcal{L}}\big(r(z)|a_R\big)$ is increasing in a_R (strictly if the lottery is offer-dependent) so that offer $\overline{a}$ dominates a_R (strictly if the lottery is offer-dependent). In a fixed-outcome lottery, the Receiver may be indifferent between $a_R \notin A^c(k_S) \cup \{\overline{a}\}$ so that $f_S(a_R; k_S) = 1$ and $\overline{a}$. However, it is without loss of generality to impose $a_R^*(s_S^*(k_S)) = \overline{a}$. Suppose now that $A^c(k_S)$ is empty. This means that the Receiver cannot avoid a challenge with the Sender. By the reasoning above, her best response is then $a_R^*(s_S^*(k_S)) = \overline{a}$. □

Claim 2. *In a separating equilibrium, the Receiver's best response $a_R^*(s_S^*(k_S))$ must be such that for all $k_S^l, k_S^h \in \mathcal{K}_S^2$, $k_S^l < k_S^h$, $a_R^*(s_S^*(k_S^l)) < a_R^*(s_S^*(k_S^h))$.*

Proof. The proof proceeds by contradiction.

First, suppose that $a_R^*(s_S^*(k_S^l)) \in A^c(k_S^l)$ and $a_R^*(s_S^*(k_S^h)) \in A^c(k_S^h)$, with $a_R^*(s_S^*(k_S^l)) > a_R^*(s_S^*(k_S^h))$ (this is possible since the set $A^p(k_S)$ may not be a singleton).[2] A type-k_S^1 and a type-k_S^h Sender's incentive compatibility (IC) constraints must then satisfy respectively:

$$-v(a_R^*(s_S^*(k_S^l))) - C(s_S^*(k_S^l), k_S^l) \geq -E_{\mathcal{L}}\big(v(z)|a_R = a_R^*(s_S^*(k_S^h))\big) - k_S^l - C(s_S^*(k_S^h), k_S^l)$$

$$-v(a_R^*(s_S^*(k_S^h))) - C(s_S^*(k_S^h), k_S^h) \geq -v(a_R^*(s_S^*(k_S^l))) - C(s_S^*(k_S^l), k_S^h)$$

By definitions of $A^c(k_S)$, a type-k_S^l Sender engages in a challenge when he mimics a type-k_S^h; that is, $-E_{\mathcal{L}}\big(v(z)|a_R = a_R^*(s_S^*(k_S^h))\big) - k_S^l > -v(a_R^*(s_S^*(k_S^h)))$. A necessary conditions for the two (IC)

[2] If $a_R^*(s_S^*(k_S^l)) = a_R^*(s_S^*(k_S^h))$, then it can be checked that both types should send the same signal, contradicting the equilibrium is separating.

constraints to simulatenously hold is then:

$$C(s_S^*(k_S^h), k_S^h) - C(s_S^*(k_S^l), k_S^h) \leq v(a_R^*(s_S^*(k_S^l))) - v(a_R^*(s_S^*(k_S^h))) < C(s_S^*(k_S^h), k_S^l) - C(s_S^*(k_S^l), k_S^l)$$

Given $v(a_R^*(s_S^*(k_S^l))) - v(a_R^*(s_S^*(k_S^h))) > 0$ under the assumption, we must have $s_S^*(k_S^h) > s_S^*(k_S^l)$ and $C(s_S^*(k_S^h), k_S^h) - C(s_S^*(k_S^l), k_S^h) < C(s_S^*(k_S^h), k_S^l) - C(s_S^*(k_S^l), k_S^l)$, which is impossible by the increasing differences assumption.

Second, suppose that $a_R^*(s_S^*(k_S^l)) = \overline{a}$ and $a_R^*(s_S^*(k_S^h)) \in A^c(k_S^h)$. The (IC) constraints of respectively a type-k_S^l and a type-k_S^h senders are, respectively:

$$-E_{\mathcal{L}}\big(v(z)|a_R = \overline{a}\big) - k_S^l - C(s_S(k_S^l), k_S^l) \geq -E_{\mathcal{L}}\big(v(z)|a_R = a_R^*(s_S(k_S^h))\big) - k_S^l - C(s_S(k_S^h), k_S^l)$$
$$-E_{\mathcal{L}}\big(v(z)|a_R = \overline{a}\big) - k_S^h - C(s_S(k_S^l), k_S^h) \leq -v(a_R^*(s_S(k_S^h))) - C(s_S(k_S^h), k_S^h)$$

Using the definition of $A^c(k_S)$, the second inequality is equivalent to:

$$-E_{\mathcal{L}}\big(v(z)|a_R = \overline{a}\big) - k_S^h - C(s_S(k_S^l), k_S^h) \leq -E_{\mathcal{L}}\big(v(z)|a_R = a_R^*(s_S(k_S^h))\big) - k_S^h - C(s_S(k_S^h), k_S^h)$$

Both (IC) constraints are satisfied only if

$$\begin{aligned} C(s_S(k_S^h), k_S^h) - C(s_S(k_S^l), k_S^h) \leq E_{\mathcal{L}}\big(v(z)|a_R = \overline{a}\big) - E_{\mathcal{L}}\big(v(z)|a_R = a_R^*(s_S(k_S^h))\big) \\ \leq C(s_S(k_S^h), k_S^l) - C(s_S(k_S^l), k_S^l) \end{aligned}$$

Given $E_{\mathcal{L}}\big(v(z)|a_R = \overline{a}\big) - E_{\mathcal{L}}\big(v(z)|a_R = a_R^*(s_S(k_S^h))\big) \geq 0$, we must have $s_S^*(k_S^h) > s_S^*(k_S^l)$ (if the signals are equal, the equilibrium cannot be separating) and $C(s_S^*(k_S^h), k_S^h) - C(s_S^*(k_S^l), k_S^h) < C(s_S^*(k_S^h), k_S^l) - C(s_S^*(k_S^l), k_S^l)$, which is impossible by the increasing differences assumption.

Finally, suppose $a_R^*(s_S^*(k_S')) = \overline{a} = a_R^*(s_S^*(k_S''))$. The equilibrium then cannot be separating since both types send the same signal. □

Proof of Lemma A.1

If Assumption 1 does not hold, then there exists $k_S^f > \underline{k}_S$ such that in any equilibrium, for all $k_S \in [\underline{k}_S, k_S^f)$ and $a_R \in [\underline{a}, \overline{a}]$, $f_S^*(a_R; k_S) = 1$. As a result, the Receiver's best response to $s_S^*(k_S)$ for all $k_S \in [\underline{k}_S, k_S^f)$ is $a_R^*(s_S^*(k_S)) = \overline{a}$ leading to a contradiction by Claim 2.[3]

If Assumption 2 is violated, there exists $k_S^{nf} < \overline{k}_S$ such that for all $k_S \in [k_S^{nf}, \overline{k}_S]$, the Receiver's best response to $s_S^*(k_S)$ for all $k_S \in [\underline{k}_S, k_S^f)$ is $a_R^*(s_S^*(k_S)) = \overline{a}$ leading to a contradiction by Claim 2.

Finally, suppose Assumption 3 is violated. First note that the strategy profile $a_R(s_S^*(\overline{k}_S)) = \overline{a}$ and $a_R(s_S^*(k_S)) < \overline{a}$ for all $k_S < \overline{k}_S$ and $k_S \in \mathcal{K}_S$ cannot be an equilibrium strategy profile if Assumption 2 holds. Indeed, if the Receiver prefers to generate a conflict after learning the Sender's type is $\overline{k}_S$, she also prefers to generate a conflict for $k_S < \overline{k}_S$ since she then must offer a lower offer a_R to avoid a challenge. So if there is a positive ex-ante probability of conflict on the equilibrium path, it must be that for some $k_S \in \mathbf{int}(\mathcal{K}_S)$, $a_R^*(s_S^*(k_S)) = \overline{a}$. But this violates Claim 2. □

Corollary 1. *If Assumptions 1-3 hold, in a separating equilibrium, the Receiver's best response is* $a_R^*(s_S^*(k_S)) = \max A^c(k_S)$ *for all* $k_S \in \mathcal{K}_S$.

Proof. Given Assumption 3, if $A^c(k_S)$ is not empty, then $a_R^*(s_S^*(k_S)) \in A^c(k_S)$. By Assumption 1 and 2, $A^c(k_S)$ is not empty. The Receiver then obviously chooses the highest possible offer in $A^c(k_S)$. □

In what follows, I use the shorthand $a_R^c(k_S) := \max A^c(k_S)$ to denote the Receiver's best-response after signal $s_S^*(k_S)$—$a_R^*(s_S^*(k_S))$—in a separating equilibrium. Recall that I label $a_R^c(k_S)$ the compromise offer.

Proof of Proposition 1

The proof proceeds in three steps. First, I prove that the Sender never plays a separating strategy in equilibrium. Second, I establish that there never exists a semi-separating equilibrium. Finally,

[3] Note that if the conflict is a fixed-outcome lottery, all types $k_S \in [\underline{k}_S, k_S^f)$ gets the same expected payoff for all $a_R \in [\underline{a}, \overline{a}]$ and no separation is possible.

I show that there does not exist any mixed strategy equilibrium.

Step 1. The proof proceeds by contradiction. Suppose first that there exists a separating equilibrium and denote a type-k_S Sender's signal $s^*_S(k_S)$ for all $k_S \in \mathcal{K}_S$ and all possible type-space $\mathcal{K}_S$. Consider two types $k^l_S, k^h_S \in \mathcal{K}^2_S$ with $k^l_S < k^h_S$. The (IC) constraint of a type-k^h_S Sender relative to a type k^l_S is:

$$-v(a^c_R(k^h_S)) - C(s^*_S(k^h_S), k^h_S) \geq -v(a^c_R(k^l_S)) - C(s^*_S(k^l_S), k^h_S) \tag{4}$$

Since $a^c_R(k^h_S) > a^c_R(k^l_S)$, it is necessary that $s^*_S(k''_S) > s^*_S(k'_S)$ to satisfy the high type's (IC) constraint.

Consider now the (IC) constraint of a type-k^l_S relative to a type-k^h_S.

$$-v(a^c_R(k^l_S)) - C(s^*_S(k^l_S), k^l_S) \geq -E_{\mathcal{L}}\big(v(z)\big) - k^l_S - C(s^*_S(k^h_S), k^l_S) \tag{5}$$

Observe that since $a^c_R(k^l_S) < a^c_R(k^h_S)$ (Claim 2 and Corollary 1), when a type-k^l_S mimics a type-k^h_S Sender by sending signal $s^*_S(k^h_S)$, he starts a conflict after the Receiver offers $a^c_R(k^h_S)$ and obtains $-E_{\mathcal{L}}\big(v(z)\big) - k^l_S$ since the conflict is a fixed-outcome lottery. By definition of $a^c_R(k^l_S)$, $-v(a^c_R(k^l_S)) = -E_{\mathcal{L}}\big(v(z)\big) - k^l_S$. Hence, we can rewrite Equation 5 as:

$$-E_{\mathcal{L}}\big(v(z)\big) - k^l_S - C(s^*_S(k^l_S), k^l_S) \geq -E_{\mathcal{L}}\big(v(z)\big) - k^l_S - C(s^*_S(k^h_S), k^l_S)$$
$$\Leftrightarrow C(s^*_S(k^l_S), k^l_S) \leq C(s^*_S(k^h_S), k^l_S)$$

It is thus necessary that $s^*_S(k^h_S) \leq s^*_S(k^l_S)$ to satisfy the low-type's (IC) constraint. Both (IC) constraints cannot be satisfied simultaneously and a separating equilibrium does not exist.

Step 2. Consider the following semi-separating assessment in which there exists a partition of the type space $\mathcal{K}_S$ into $M \geq 2$ subsets—$K^1_S, \ldots, K^M_S$—such that for all $k_S \in K^m_S$, $m \in \{1, \ldots, M\}$ $s_S(k_S) = s^m_S$. Denote $K^c_S := \{K^m_S :$ there exists $k_S \in \mathbf{int}K^m_S$ s.t. $a^*_R(s^m_S) = a^c_R(k_S)\}$. That is, K^c_S is the set of subsets K^m_S such that the Receiver compromises with some of the types in K^m_S. Using the same reasoning as in step 1, it can be checked that if a semi-separating assessment is an equilibrium, the cardinality of K_S^c is 1 (using the types $\{k_S\}$ such that $a_R^*(s_S^m) = a_R^c(k_S)$). Hence, in any semi-separating equilibrium, $M = 2$ and $a_R^*(s_S^m) = \overline{a}$ for $m \in \{1,2\}$. However, by Assumption 3, for any $m \in \{1,2\}$, $r(a_R^c(\min K_S^m)) \geq E_{\mathcal{L}}\big(r(z)|a_R = \overline{a}\big) - k_R = E_{\mathcal{L}}\big(r(z)|a_R = a_R^c(\max K_S^m)\big) - k_R$ (using the fixed-outcome property of the lottery $\mathcal{L}$). Hence, $a_R^*(s_S^m) < a_R^c(\max K_S^m)$ for all $m \in \{1,2\}$ which implies $\mathbf{card} K_S^c > 1$, a contradiction.

*Step 3.*For a mixed strategy equilibrium to exist, the Sender must be indifferent between two signals. Hence, there must exist a partition of the type space with cardinality greater than 1. But we have seen that this is impossible by step 2.

Combining the three steps, the unique equilibrium is a pooling equilibrium.[4] □

In what follows, I consider offer-dependent lottery. Recall that $\mathcal{K}_S$ is not discrete if there exist $k_S^a < k_S^b$ such that $[k_S^a, k_S^b] \subset \mathcal{K}_S$. I first state a preliminary lemma which states some properties of the Sender's signaling strategy.

Lemma A.2. *Suppose $\mathcal{K}_S$ is not discrete. For all offer-dependent lotteries, in a separating equilibrium, the Sender's signaling strategy $s_S^*(k_S)$ is continuous and strictly increasing for all $k_S \in [k_S^a, k_S^b]$.*

Proof. Consider two types $k_S^l, k_S^h \in [k_S^a, k_S^b]^2$ with $k_S^l < k_S^h$. The (IC) constraint of a type-k_S^h relative to a type-k_S^l Sender is Equation 4. The (IC) constraint of a type-k_S^l relative to a type k_S^h.

$$-v(a_R^c(k_S^l)) - C(s_S^*(k_S^l), k_S^l) \geq -E_{\mathcal{L}}\big(v(z)|a_R = a_R^c(k_S^h)\big) - k_S^l - C(s_S^*(k_S^h), k_S^l) \tag{6}$$

by a similar reasoning as in the proof of Proposition 1 (note, however, that the outcome of the lottery now depends on the Receiver's offer a_R).

Observe that $-v(a_R^c(k_S^l)) > -v(a_R^c(k_S^h))$ (Corollary 1) so to satisfy Equation 4, it is necessary that $s_S^*(k_S^l) > s_S^*(k_S^h)$. Further, by definition of $a_R^c(k_S^l)$, $\lim\limits_{k_S^l \uparrow k_S^h} -E\big(v(z)|a_R = a_R^c(k_S^h)\big) - k_S^l = -v(a_R^c(k_S^h)$. Using Equation 4 and Equation 6, it can then be checked that if $s_S(k_S)$ is not continuous, there exists a profitable deviation for types close enough to the discontinuity. □

[4]Notice that given the Sender's signaling strategy, the Receiver may mix between different actions. However, this does not affect the proposition.

Proof of Proposition 2

The proof proceeds in three steps. I first prove that Condition 1 is necessary. Second, I show that Condition 2 is necessary. I finally prove sufficiency.

Step 1. The proof proceeds by contradiction. Suppose $\mathcal{K}_S$ is not discrete and a separating equilibrium exists with a type-k_S Sender's signal denoted $s_S^*(k_S)$. From Lemma A.2, $s_S^*(k_S)$ is continuous and strictly increasing, so it is differentiable almost everywhere for all $k_S \in [k_S^a, k_S^b]$.

Take $k_S^{mid} \in (k_S^a, k_S^b)$ such that $s_S^*(\cdot)$ is differentiable at k_S^{mid}. Further define: $k_S^h = k_S^{mid} + \delta$ and $k_S^l = k_S^{mid} - \delta$ with $\delta > 0$. A necessary condition for existence of a separating equilibrium is that a type-k_S^{mid} Sender does not want to mimic a type-k_S^h and a type-k_S^l. The (IC) constraints of a type-k_S^{mid} Sender must satisfy using Equation 4 and Equation 6:

$$-v(a_R^c(k_S^{mid})) - C(s_S^*(k_S^{mid}), k_S^{mid}) \geq -v(a_R^c(k_S^l)) - C(s_S^*(k_S^l), k_S^{mid})$$
$$-v(a_R^c(k_S^{mid})) - C(s_S^*(k_S^{mid}), k_S^{mid}) \geq -E_{\mathcal{L}}\big(v(z)|a_R = a_R^c(k_S^h)\big) - k_S^{mid} - C(s_S^*(k_S^h), k_S^{mid})$$

Using $-v(a_R^c(k_S)) = -E_{\mathcal{L}}\big(v(z)|a_R = a_R^c(k_S)\big) - k_S$, we can rewrite the (IC) constraints as:

$$-E_{\mathcal{L}}\big(v(z)|a_R = a_R^c(k_S^{mid})\big) - k_S^{mid} - C(s_S^*(k_S^{mid}), k_S^{mid}) \geq -E_{\mathcal{L}}\big(v(z)|a_R = a_R^c(k_S^l)\big) - k_S^l - C(s_S^*(k_S^l), k_S^{mid})$$
$$-E_{\mathcal{L}}\big(v(z)|a_R = a_R^c(k_S^{mid})\big) - k_S^{mid} - C(s_S^*(k_S^{mid}), k_S^{mid}) \geq -E_{\mathcal{L}}\big(v(z)|a_R = a_R^c(k_S^h)\big) - k_S^{mid} - C(s_S^*(k_S^h), k_S^{mid})$$

Rearranging and using the definitions of k_S^l and k_S^h, I obtain:

$$C(s_S^*(k_S^{mid} - \delta), k_S^{mid}) - C(s_S^*(k_S^{mid}), k_S^{mid}) \geq E_{\mathcal{L}}\big(v(z)|a_R = a_R^c(k_S^{mid})\big) - E_{\mathcal{L}}\big(v(z)|a_R = a_R^c(k_S^{mid} - \delta)\big) + \delta \tag{7}$$

$$C(s_S^*(k_S^m), k_S^{mid}) - C(s_S^*(k_S^{mid} + \delta), k_S^{mid}) \leq E_{\mathcal{L}}\big(v(z)|a_R = a_R^c(k_S^{mid} + \delta)\big) - E_{\mathcal{L}}\big(v(z)|a_R = a_R^c(k_S^{mid})\big) \tag{8}$$

Recall that $E_{\mathcal{L}}\big(v(z)|a_R\big) = \sum_{j=1}^{N} p_j v(z_j(a_R))$ and is differentiable in a_R since $v(\cdot)$ is $\mathcal{C}^1$. This also implies that $a_R^c(k_S)$ is differentiable in k_S for $k_S \in [k_S^a, kS^b]$. Further, by definition, $s_S^*(\cdot)$ is differentiable at k_S^{mid} so $\lim_{k_S \uparrow k_S^{mid}} \frac{\partial s_S^*(k_S)}{\partial k_S} = \lim_{k_S \downarrow k_S^{mid}} \frac{\partial s_S^*(k_S)}{\partial k_S} := \frac{\partial s_S^*(k_S^{mid})}{\partial k_S}$. Dividing both Equation 7 and Equation 8 by δ, taking the limits as δ goes to 0, and using the definition of derivatives, I obtain:

$$-\frac{\partial s_S^*(k_S^{mid})}{\partial k_S}\frac{\partial C(s_S^*(k_S^{mid}),k_S)}{\partial s_S} \geq \frac{\partial E_{\mathcal{L}}\big(v(z)|a_R = a_R^c(k_S^{mid})\big)}{\partial a_R}\frac{\partial a_R^c(k_S^{mid})}{\partial k_S} + 1$$
$$-\frac{\partial s_S^*(k_S^{mid})}{\partial k_S}\frac{\partial C(s_S^*(k_S^{mid}),k_S)}{\partial s_S} \leq \frac{\partial E_{\mathcal{L}}\big(v(z)|a_R = a_R^c(k_S^{mid})\big)}{\partial a_R}\frac{\partial a_R^c(k_S^{mid})}{\partial k_S}$$

Both inequalities clearly cannot be satisfied simultaneously, and I have thus reached a contradiction.

Step 2. Suppose that $\mathcal{K}_S := \{k_S^1, \ldots, k_S^K\}$. Using Equation 6 with $k_S^l = k_S^j$ and $k_S^h = k_S^{j+1}$ yields the following (IC) constraint of a type-k_S^j relative to a type-k_S^{j+1} for all $j \in \{1, \ldots, K-1\}$:

$$\begin{aligned} &E_{\mathcal{L}}\big(v(z)|a_R = a_R^c(k_S^{j+1})\big) - v(a_R^c(k_S^j)) \geq C(s_S^*(k_S^j), k_S^j) - C(s_S^*(k_S^{j+1}), k_S^j) - k_S^j \\ \Leftrightarrow\ & v(a_R^c(k_S^{j+1})) - v(a_R^c(k_S^j)) \geq C(s_S^*(k_S^j), k_S^j) - C(s_S^*(k_S^{j+1}), k_S^j) + k_S^{j+1} - k_S^j \end{aligned} \tag{9}$$

The second line uses $E_{\mathcal{L}}\big(v(z)|a_R = a_R^c(k_S^{j+1})\big) = v(a_R^c(k_S^{j+1})) - k_S^{j+1}$.

In turn, Equation 4 yields the following (IC) constraint of a type-k_S^{j+1} relative to a type-k_S^j:

$$C(s_S^*(k_S^j), k_S^{j+1}) - C(s_S^*(k_S^{j+1}), k_S^{j+1}) \geq v(a_R^c(k_S^{j+1})) - v(a_R^c(k_S^j)) \tag{10}$$

The only choice variable in Equation 9 and Equation 10 is the signaling strategy: $\mathbf{s}_\mathbf{S}^* = (s_S^*(k_S^1), \ldots, s_S^*(k_S^K))$. Hence, it is necessary that there exist signals $s_S^*(k_S^j)$ for all $j \in \{1, \ldots, K\}$ satisfying Equation 3 as claimed.

Step 3. I now show sufficiency. First, I show that if the local (IC) constraint (i.e., type-k_S^j and type-k_S^{j+1}'s incentives to deviate) holds for all $j \in \{1, \ldots, K-1\}$, then all other (IC) constraints are satisfied.

First, I show that if Equation 9 holds for all $j \in \{1, \ldots, K-1\}$ then

$$v(a_R^c(k_S^{j+m})) - v(a_R^c(k_S^j)) \geq C(s_S^*(k_S^j), k_S^j) - C(s_S^*(k_S^{j+m}), k_S^j) + k_S^{j+m} - k_S^j,$$

for all $m \in \{1, \ldots, K-j\}$. To see this, suppose $j < K-1$ and $m = 2$:

$$\begin{aligned}
v(a_R^c(k_S^{j+2})) - k_S^{j+2} - \big(v(a_R^c(k_S^j)) - k_S^j\big) =& \Big(v(a_R^c(k_S^{j+2})) - k_S^{j+2} - \big(v(a_R^c(k_S^{j+1})) - k_S^{j+1}\big)\Big) \\
&+ \Big(v(a_R^c(k_S^{j+1})) - k_S^{j+1} - \big(v(a_R^c(k_S^j)) - k_S^j\big)\Big) \\
\geq& \Big(C(s_S^*(k_S^{j+1}), k_S^{j+1}) - C(s_S^*(k_S^{j+2}), k_S^{j+1})\Big) \\
&+ \Big(C(s_S^*(k_S^j), k_S^j) - C(s_S^*(k_S^{j+1}), k_S^j)\Big) \\
=& C(s_S^*(k_S^j), k_S^j) - C(s_S^*(k_S^{j+2}), k_S^j) \\
&+ \Big(C(s_S^*(k_S^{j+1}), k_S^{j+1}) - C(s_S^*(k_S^{j+2}), k_S^{j+1})\Big) \\
&+ \Big(C(s_S^*(k_S^{j+2}), k_S^j) - C(s_S^*(k_S^{j+1}), k_S^j)\Big)
\end{aligned}$$

The inequality comes from the assumption that Equation 9 holds for all j. By a similar reasoning as in the proof of Lemma A.2, in a separating equilibrium $s_S^*(k_S^j) > s_S^*(k_S^{j+1})$ for all j. Using the assumption that $C(\cdot, \cdot)$ exhibits strict increasing differences, we thus have:

$$\Big(C(s_S^*(k_S^{j+1}), k_S^{j+1}) - C(s_S^*(k_S^{j+2}), k_S^{j+1})\Big) - \Big(C(s_S^*(k_S^{j+1}), k_S^j) - C(s_S^*(k_S^{j+2}), k_S^j)\Big) > 0$$

This implies that

$$v(a_R^c(k_S^{j+2})) - k_S^{j+2} - \big(v(a_R^c(k_S^j)) - k_S^j\big) > C(s_S^*(k_S^j), k_S^j) - C(s_S^*(k_S^{j+2}), k_S^j).$$

I can then use the same reasoning for $m > 2$ to prove the claim.

Finally, I show that if Equation 10 holds for all $j \in \{1, \ldots, K-1\}$ then

$$C(s_S^*(k_S^j), k_S^{j+m}) - C(s_S^*(k_S^{j+m}), k_S^{j+m}) \geq v(a_R^c(k_S^{j+m})) - v(a_R^c(k_S^j)),$$

for all $m \in \{1, \ldots, K-j\}$. To see this, suppose again $j < K-1$ and $m = 2$:

$$\begin{aligned} v(a_R^c(k_S^{j+2})) - v(a_R^c(k_S^j)) =& \Big(v(a_R^c(k_S^{j+2})) - v(a_R^c(k_S^{j+1})) \Big) \\ &+ \Big(v(a_R^c(k_S^{j+1})) - v(a_R^c(k_S^j)) \Big) \\ \leq& \Big(C(s_S^*(k_S^{j+1}), k_S^{j+2}) - C(s_S^*(k_S^{j+2}), k_S^{j+2}) \Big) \\ &+ \Big(C(s_S^*(k_S^j), k_S^{j+1}) - C(s_S^*(k_S^{j+1}), k_S^{j+1}) \Big) \\ =& C(s_S^*(k_S^j), k_S^{j+2}) - C(s_S^*(k_S^{j+2}), k_S^{j+2}) \\ &+ \Big(C(s_S^*(k_S^{j+1}), k_S^{j+2}) - C(s_S^*(k_S^j), k_S^{j+2}) \Big) \\ &+ \Big(C(s_S^*(k_S^j), k_S^{j+1}) - C(s_S^*(k_S^{j+1}), k_S^{j+1}) \Big) \end{aligned}$$

Using again the strict increasing difference properties of $C(\cdot, \cdot)$, I obtain:

$$\Big(C(s_S^*(k_S^j), k_S^{j+1}) - C(s_S^*(k_S^{j+1}), k_S^{j+1}) \Big) - \Big(C(s_S^*(k_S^j), k_S^{j+2}) - C(s_S^*(k_S^{j+1}), k_S^{j+2}) \Big) < 0.$$

Therefore,

$$v(a_R^c(k_S^{j+2})) - v(a_R^c(k_S^j)) < C(s_S^*(k_S^j), k_S^{j+2}) - C(s_S^*(k_S^{j+2}), k_S^{j+2})$$

I can then use the same reasoning for $m > 2$ to prove the claim.

Second, I show that a separating equilibrium exists if there exists a vector of signals such that Equation 3 holds. Consider the following assessment:

- For all $j \in \{1, \ldots, K\}$, a type-k_S^j Sender sends signal $s_S(k_S^j)$ with $s_S(k_S^K) = s_S(\overline{k}_S) = 0$;
- $\mathbf{s_S} = \{s_S(k_S^1), \ldots, s_S(k_S^K)\}$ is such that Equation 3 holds;
- Upon observing $s_S(k_S^j)$ for all j, the Receiver's belief is that the Sender's type is k_S^j with probability 1 and her strategy is $a_R(s_S(k_S^j)) = a_R^c(k_S^j)$;

  Upon observing $\widehat{s_S} \notin \mathbf{s_S}$, the Receiver's out-of-equilibrium is that the Sender's type is $\overline{k}_S$ with probability 1 and her strategy is then $a_R(\widehat{s_S}) = a_R^c(\overline{k}_S)$;
- For all j, a type-k_S^j Sender chooses $f_S = 0$ if and only if $a_R \leq a_R^c(k_S^j)$ and $f_S = 1$ otherwise.

It can be checked that (i) the Receiver's beliefs satisfy Bayes' Rule on the equilibrium path, (ii) the Receiver's strategy is a best response given her (in or out-of-equilibrium) belief, (iii) the Sender's (IC) constraints hold for all types (in particular, the Receiver's strategy implies that for all $j \in \{1, \dots, K\}$, any signal $\widehat{s_S} \notin \mathbf{s_S}$ is not a profitable deviation if Equation 3 holds), and (iv) the Sender's conflict strategy is a best response to any Receiver's offer a_R.[5] Hence, the assessment described above is a PBE. □

Details for Example 1

Under the parametrization, the compromise offer for a type-k_S^j satisfies: $-v(a_R^c(k_S^j)) = -pv(\underline{a}) - (1-p)v(a_R^c(k_S^j)) - k_S^j$, or equivalently $v(a_R^c(\kappa_S^j)) = v(\underline{a}) + \frac{k_S^j}{p}$. Plugging the value of $v(a_R^c(k_S^j))$ into Condition 3, I obtain that a necessary and sufficient condition for the existence of a separating equilibrium is for all $j \in \{1, \dots, K-1\}$:

$$k_S^j(s_S^*(k_S^j) - s_S^*(k_S^{j+1})) + k_S^{j+1} - k_S^j \leq \frac{k_S^{j+1} - k_S^j}{p} \leq k_S^{j+1}(s_S^*(k_S^j) - s_S^*(k_S^{j+1}))$$

This is equivalent to:

$$\frac{k_S^{j+1} - k_S^j}{p} \frac{1}{k_S^{j+1}} \leq s_S^*(k_S^j) - s_S^*(k_S^{j+1}) \leq \frac{k_S^{j+1} - k_S^j}{p} \frac{1-p}{k_S^j}$$

Hence, we can always find signaling values $\mathbf{s_S^*}$ such that a separating equilibrium exists if and only if for all $j \in \{1, \dots, K-1\}$:

$$\frac{1}{k_S^{j+1}} \leq \frac{1-p}{k_S^j} \Leftrightarrow k_S^j \leq (1-p)k_S^{j+1}$$

Proof of Proposition 3

I only prove necessity. Sufficiency follows from a similar argument as in the proof of Proposition 2.

Condition 1. I prove the result for all semi-separating equilibria. Under the assumption on

[5] Observe that the signaling strategy satisfies $s_S^*(\overline{k}_S) = 0$ in any separating equilibrium.

the Receiver's utility function, after s_S^m, the Receiver's best response must satisfy $a_R^{ss}(s_S^m) \in [a_R^c(k_S^m), a_R^c(\max K_S^m))$. Denote $k_S^{ss}(m)$ such that $a_R^c(k_S^{ss}(m)) = a_R^{ss}(s_S(m))$ (such $k_S^{ss}(m)$ exists, otherwise the Receiver can increase her offer without increasing the probability of conflict by a similar reasoning as in Claim 1, a profitable deviation). Suppose that there exists a semi-separating equilibrium in which there exists $m \in \{1, \dots, M-1\}$ such that $\max K_S^m > \min K_S^{m+1} = k_S^{m+1}$. For simplicity, assume that K_S^{m+1} is a convex set and denote $\overline{k_S^m} = \max K_S^m$. We need to consider two cases: (a) $k_S^{ss}(m) < \min K_S^{m+1}$ and (b) $k_S^{ss}(m) > \min K_S^{m+1}$.

Case (a): For a semi-separating equilibrium to exist, it must be that $k_S^{ss}(m+1)$ prefers signal s_S^{m+1} to signal s_S^m, or equivalently: $v(a_R^{ss}(m+1))) - v(a_R^{ss}(m)) \leq C(s_S^m, k_S^{ss}(m+1)) - C(s_S^{m+1} k_S^{ss}(m+1))$. In turn, a type-$\overline{k_S^m}$ must prefer s_S^m to s_S^{m+1}. Notice that a Sender with cost $\overline{k_S^m}$ does not start a conflict when he sends signal s_S^m or signal s_S^{m+1}, hence his incentive compatibility constraint is: $v(a_R^{ss}(m+1)) - v(a_R^{ss}(m)) \geq C(s_S^m, \overline{k_S^m}) - C(s_S^{m+1}, \overline{k_S^m})$. The two (IC) constraints imply $C(s_S^m, k_S^{ss}(m+1)) - C(s_S^{m+1}, k_S^{ss}(m+1)) \geq C(s_S^m, \overline{k_S^m}) - C(s_S^{m+1}, \overline{k_S^m})$, but this contradicts the strict increasing differences assumption.

Case (b): Consider now types k_S^m and k_S^{m+1}. Their (IC) constraints are respectively:

$$-E_{\mathcal{L}}\big(v(z)|a_R^{ss}(m)\big) - k_S^m - C(s_S^m, k_S^m) \geq -E_{\mathcal{L}}\big(v(z)|a_R^{ss}(m+1)\big) - k_S^m - C(s_S^{m+1}, k_S^m)$$
$$-E_{\mathcal{L}}\big(v(z)|a_R^{ss}(m+1)\big) - k_S^{m+1} - C(s_S^{m+1}, k_S^{m+1}) \geq -E_{\mathcal{L}}\big(v(z)|a_R^{ss}(m)\big) - k_S^{m+1} - C(s_S^m, k_S^{m+1})$$

as both start a conflict (or k_S^{m+1} is indifferent) after signals s_S^m and signals s_S^{m+1}. The two (IC) imply that (i) $s_S^m < s_S^{m+1}$ (since $E_{\mathcal{L}}\big(v(z)|a_R^{ss}(m)\big) > E_{\mathcal{L}}\big(v(z)|a_R^{ss}(m+1)\big)$) and (ii) $C(s_S^{m+1}, k_S^m) - C(s_S^m, k_S^m) \geq C(s_S^{m+1}, k_S^{m+1}) - C(s_S^m, k_S^{m+1})$. But this last condition violates the strict increasing differences assumption, a contradiction.

Condition 2. Observe that by the assumption of strict increasing difference, if $v(a_R^c(k_S^{m+1})) - v(a_R^c(k_S^m)) \leq C(s_S^m, k_S^{m+1}) - C(s_S^{m+1}, k_S^{m+1})$, then for all $k_S \in K_S^{m+1}$, $v(a_R^c(k_S^{m+1})) - v(a_R^c(k_S^m)) \leq C(s_S^m, k_S) - C(s_S^{m+1}), k_S)$.

Condition 3. By the now usual reasoning, the upward (IC) constraint of a type $k_S \in K_S^m$ is for all $m \in \{1, \ldots, M-1\}$ is (see the proof of Proposition 2):

$$-v(a_R^c(k_S^m)) - C(s_S^m, k_S) \geq -E_{\mathcal{L}}\big(v(z)|a_R^c(k_S^{m+1}))\big) - k_S - C(s_S^{m+1}, k_S)$$
$$\Leftrightarrow \; C(s_S^m, k_S) - C(s_S^{m+1}, k_S) + k_S^{m+1} - k_S \leq v(a_R^c(k_S^{m+1})) - v(a_R^c(k_S^m))$$

The second line uses $-E_{\mathcal{L}}\big(v(z)|a_R^c(k_S^{m+1}))\big) = -v(a_R^c(k_S^{m+1})) + k_S^{m+1}$. To see that it is not enough that the (IC) constraint is satisfied by the extrema of set K_S^m, suppose that K_S is continuous $\mathcal{L} =< \{0, a_R\}, \{p, 1-p\} >$ and $C(s, k_S) = s \times g(k_S)$ with $g'(\cdot) > 0$. It can be checked then that $s_S^m - s_S^{m+1} = \frac{k_S^{m+1} - k_S^m}{p} \frac{1}{g(k_S^{m+1})}$. The (IC) constraint of a type-$k_S \in K_S^m$ is then: $\frac{k_S^{m+1} - k_S^m}{p} \frac{g(k_S)}{g(k_S^{m+1})} - k_S \leq \frac{(1-p)k_S^{m+1} - k_S^m}{p}$. Denote $\widehat{K_S^m} = [k_S^m, k_S^{m+1}]$ (the smallest closed interval containing K_S^m using Condition 1) and $k_S^{max}(m) = \arg\max_{k_S \in \widehat{K_S^m}} \frac{k_S^{m+1} - k_S^m}{p} \frac{g(k_S)}{g(k_S^{m+1})} - k_S$. Depending on $g(\cdot)$, $k_S^{max}(m)$ can take any interior or extreme values in $\widehat{K_S^m}$.

Condition 4. If the condition does not hold, then it must be that there exists m such that if $k_S = k_S^m$, the Sender's best response is to start a conflict after signal s_S^m and offer $a_R^{ss}(s_S^m)$. Hence, conflict occurs on the equilibrium path. □

B Uncertainty about winning probabilities

In this variation of the baseline model, I suppose that the Sender's cost of conflict is common knowledge and denoted k_S. In turn, I suppose that the Sender has better information about the probability that he wins the conflict. Formally, there exist a set $\Lambda \subseteq [\underline{\lambda}, \overline{\lambda}]$, $\underline{\lambda}, \overline{\lambda} \in \Lambda^2$ such that the probability of outcome z_j occurs is $p_j(\lambda)$, $j \in \{1, \ldots, N\}$ for all $\lambda \in \Lambda$. The lottery now takes the form of $\mathcal{L}(a_R; \lambda) =< \mathbf{z}(a_R), \mathbf{p}(\lambda) >$. Assume that λ's are ordered such that for all $\lambda, \lambda' \in \Lambda$ and $\lambda < \lambda'$, $E_{\mathcal{L}}(v(z)|a_R, \lambda) < E_{\mathcal{L}}(v(z)|a_R, \lambda')$ for all a_R. Further, the signaling cost now satisfies $C(s_S, \lambda)$ with the strict increasing differences still holding on s_S and λ. The amended timing satisfies

0. Nature draws λ according to the distribution $G(\cdot)$;

1. Sender privately observes λ and sends signal $s_S \geq 0$;
2. Receiver observes s_S and chooses $a_R \in [\underline{a}, \overline{a}]$;
3. Sender chooses $f_S \in \{0, 1\}$;
4. Nature determines outcome, game ends, and payoffs are realized.

Denote $a_R^c(\lambda)$ the compromise offer as a function of λ. That is, in the amended setting, $a_R^c(\lambda)$ satisfies $-v(a_R^c(\lambda)) = -E_{\mathcal{L}}\big(v(z)|a_R^c(\lambda), \lambda\big) - k_S$. Throughout this appendix I assume without loss of generality that $a_R^c(\lambda)$ is unique. Further, as in the main text, I impose $a_R^c(\underline{\lambda}) \geq \underline{a}$ and $a_R^c(\overline{\lambda}) \leq \overline{a}$ so compromise is always possible and needed. Finally, as in the main text (Assumption 3), I also assume that the Receiver is always willing to compromise: $\min_{\lambda \in \Lambda} r(a_R^c(\lambda)) - \Big(E_{\mathcal{L}}\big(r(z)|a_R^c(\lambda), \lambda\big) - k_R\Big) \geq 0$. The next result extends Proposition 1 to this setting.

Proposition B.1. *For all fixed-outcome lotteries and all type-space Λ, in any equilibrium, the Sender plays a pooling strategy: $s_S^*(\lambda) = s_S^p$ for all $\lambda \in \Lambda$.*

Proof. Suppose there exists an equilibrium in which there exists $\lambda^l, \lambda^h \in \Lambda^2$ with $\lambda^l < \lambda^h$ such that $s_S^*(\lambda^l) \neq s_S^*(\lambda^h)$. Assume without loss of generality that $a_R^*(s_S^*(\lambda^l)) = a_R^c(\lambda^l)$ and $a_R^*(s_S^*(\lambda^h)) = a_R^c(\lambda^h)$ (such types must exist since the Receiver, otherwise, can increase her offer without increasing the probability of conflict, a profitable deviation). A type-λ^l is willing to play signaling strategy $s_S^*(\lambda^l)$ only if:

$$
\begin{aligned}
&-v(a_R^c(\lambda^l)) - C(s_S^*(\lambda^l), \lambda^l) \geq -E_{\mathcal{L}}\big(v(z)|\lambda^l\big) - k_S - C(s_S^*(\lambda^h), \lambda^l) \\
\Leftrightarrow\ &-E_{\mathcal{L}}\big(v(z)|\lambda^l\big) - k_S - C(s_S^*(\lambda^l), \lambda^l) \geq -E_{\mathcal{L}}\big(v(z)|\lambda^l\big) - k_S - C(s_S^*(\lambda^h), \lambda^l) \\
\Leftrightarrow\ &C(s_S^*(\lambda^h), \lambda^l) - C(s_S^*(\lambda^l), \lambda^l) \geq 0
\end{aligned}
$$

The first line comes from the fact that a type-λ^l starts a conflict when the Receiver proposes $a_R^c(\lambda^h)$ after signal $s_S^*(\lambda^h)$. The second line comes from the definition of $a_R^c(\lambda^l)$.
In turn, a type-λ^h is willing to play signaling strategy $s_S^*(\lambda^h)$ only if:

$$
\begin{aligned}
&-v(a_R^c(\lambda^h)) - C(s_S^*(\lambda^h), \lambda^h) \geq -v(a_R^c(\lambda^l)) - C(s_S^*(\lambda^l), \lambda^h) \\
\Leftrightarrow\ &v(a_R^c(\lambda^l)) - v(a_R^c(\lambda^h)) \geq C(s_S^*(\lambda^h), \lambda^l) - C(s_S^*(\lambda^l), \lambda^h)
\end{aligned}
$$

Since $a^c_R(\lambda^l) < a^c_R(\lambda^h)$, the two inequalities cannot be satisfied simultaneously, a contradiction. $\square$

C Different bargaining protocols

In this section, I assume as in the baseline model that the Receiver is uncertain about the cost of conflict $k_S \in \mathcal{K}_S$. Assumptions 1-4 of the baseline model hold in this Appendix. I further impose that if the Sender were to make an offer, he would need to compromise as well: $r(\underline{a}) < E_{\mathcal{L}}(r(z)|a_R = \underline{a}) - k_R$. I move away from tradition signaling games and consider two different bargaining protocols in turn: 1) the Sender has a positive probability to make a take-it-or-leave-it offer and 2) the offer is the result of some form of bargaining between the Receiver and Sender.

C.1 Different recognition probabilities

In this subsection, I assume that at the bargaining stage (stage 3 in the timing), there is a probability $\beta \in (0, 1)$ (resp. $1-\beta$) that the Receiver (resp. Sender) makes a take-it-or-leave-it offer (i.e., the bargaining protocol is a form of Rubinstein model with interior probability of recognition and immediate breakdown into conflict upon offer rejection). If the offer is rejected, conflict ensues. The baseline model corresponds to the case $\beta = 1$.

To account for the model amendment, I denote $a_B(J) \in [\underline{a}, \overline{a}]$ the offer as a function of the player recognized to make an offer $J \in \{R, S\}$. After the offer of $J \in \{S, R\}$, the other player $-J$ decides whether to start a conflict: $f_{-J} \in \{0, 1\}$. I denote $a^c_B(R; k_S)$ the compromise offer when R is recognized and a Sender's cost is k_S (i.e., $a^c_B(R; k_S) = a^c_R(k_S)$). The compromise offer when S is recognized is: $a^c_B(S; k_R)$ satisfying $r(a^c_B(S; k_R)) = E_{\mathcal{L}}(r(z)|a_R = a^c_B(S; k_R)) - k_R$ (assuming existence and uniqueness). Throughout, I impose the equivalent of Assumption 3 for the Sender: $-v(a^c_B(S; k_R)) > -E_{\mathcal{L}}(v(z)|\overline{a}) - \underline{k}_S$. The rest of the model remains the same and the amended timing is:

0. Nature draws k_S according to the distribution $F(\cdot)$;
1. Sender privately observes k_S and sends signal $s_S \geq 0$;
2. Receiver observes s_S. Nature recognizes player $J \in \{R, S\}$ to make offer $a_B(J) \in [\underline{a}, \overline{a}]$;

3. Player $-J$ chooses $f_{-J} \in \{0, 1\}$;
4. Nature determines outcome, game ends, and payoffs are realized.

The next result shows that Proposition 1 extends to this setting.

Proposition C.1. *For all fixed-outcome lotteries and all type-space $\mathcal{K}_S$, in any equilibrium, the Sender plays a pooling strategy: $s_S^*(k_S) = s_S^p$ for all $k_S \in \mathcal{K}_S$.*

Proof. Denote $a_B(R; s_S)$ the offer strategy of the Receiver if recognized as a function of the signal s_S. In turn, denote $a_B(S; k_S)$ the strategy of the Sender if recognized as a function of his type. Under the assumptions, $a_B(S; k_S) = a_B^c(S; k_R)$.

Suppose there exists an equilibrium in which there exist $k_S^l, k_S^h \in \mathcal{K}_S^2$ with $k_S^l < k_S^h$ such that $s_S^*(k_S^l) \neq s_S^*(k_S^h)$. Assume without loss of generality that $a_B^*(R; s_S^*(k_S^l)) = a_B^c(R; k_S^l)$ and $a_B^*(R; s_S^*(k_S^h)) = a_B^c(R; k_S^h)$ (such types must exist since the Receiver, otherwise, can increase her offer without increasing the probability of conflict, a profitable deviation). A type-k_S^l is willing to play signaling strategy $s_S^*(k_S^l)$ only if:

$$
\begin{aligned}
-\Big(\beta v(a_B^c(R; k_S^l)) &+ (1-\beta) v(a_B^c(S; k_R))\Big) - C(s_S^*(k_S^l), k_S^l) \\
&\geq -\Big(\beta\big(E_{\mathcal{L}}\big(v(z)\big) + k_S^l\big) + (1-\beta) v(a_B^c(S; k_R))\Big) - C(s_S^*(k_S^h), k_S^l) \\
\Leftrightarrow -\beta\Big(E_{\mathcal{L}}\big(v(z)\big) + k_S^l\Big) - C(s_S^*(k_S^l), k_S^l) &\geq \beta\Big(E_{\mathcal{L}}\big(v(z)\big) + k_S^l\Big) - C(s_S^*(k_S^h), k_S^l) \\
\Leftrightarrow C(s_S^*(k_S^h), k_S^l) - C(s_S^*(k_S^l), k_S^l) &\geq 0
\end{aligned}
$$

The first line comes from the fact that a type-k_S^l starts a conflict when the Receiver proposes $a_B^c(R; k_S^h)$ after signal $s_S^*(k_S^h)$. The second line comes from $a_B^c(R; k_S^l) = a_R^c(k_S^l)$.

In turn, a type-k_S^h is willing to play signaling strategy $s_S^*(k_S^h)$ only if:

$$
\begin{aligned}
-\Big(\beta v(a_B^c(R; k_S^h)) &+ (1-\beta) v(a_B^c(S; k_R))\Big) - C(s_S^*(k_S^h), k_S^h) \\
&\geq -\Big(\beta v(a_B^c(R; k_S^l)) + (1-\beta) v(a_B^c(S; k_R))\Big) - C(s_S^*(k_S^l), k_S^h) \\
\Leftrightarrow \beta\Big(v(a_B^c(R; k_S^l)) - v(a_B^c(R; k_S^h))\Big) &\geq C(s_S^*(k_S^h), k_S^h) - C(s_S^*(k_S^l), k_S^h)
\end{aligned}
$$

By the usual reasoning, the two inequalities cannot be satisfied simultaneously, a contradiction. □

C.2 Bargained compromise offer

In this subsection, I exogenously assume that at the bargaining stage, the offer is some action strictly between R's preferred offer (conditional on his information) and S's preferred offer. I leave the protocol which leads to this offer unmodeled.

Using the notation $a_B(J;\cdot)$, denote $a_B^{max}(R;s_S) = \arg\max_{a\in[\underline{a},\overline{a}]} E(U_R(a_R, f_S)|s_S)$ the Receiver's optimal offer following signal s_S (with expectations over types and conflict outcomes in case of conflict). For simplicity and without loss of generality, I assume that $a_B^{max}(R;s_S)$ is unique. As in the previous section, we assume that the Sender's optimal offer for all costs $k_S \in \mathcal{K}_S$ is $a_B^{max}(S;k_S) = a_B^c(S;k_R)$ the compromise offer. More formally, at stage 2, the offer on the table is: $a_B(s_S(k_S)) = \beta a_B^{max}(R;s_S(k_S)) + (1-\beta)a_B^c(S;k_R)$ for $\beta \in (0,1)$. The baseline model corresponds to $\beta = 1$. Using the assumptions, if $s_S(k_S)$ fully reveals the Sender's type, I denote the bargained offer $a_B^s(k_S) := \beta a_B^c(R;k_S) + (1-\beta)a_B^c(S;k_R)$. The amended timing is:

0. Nature draws k_S according to the distribution $F(\cdot)$;
1. Sender privately observes k_S and sends signal $s_S \geq 0$;
2. Receiver observes s_S. Nature offers $a_B(s_S)$;
3. Sender and Receiver jointly choose $f_S, f_R \in \{0,1\}^2$;
4. Nature determines outcome, game ends, and payoffs are realized.

I now show that this form of bargaining protocol renders separation possible. The first result of this subsection is that as long as the types are not too much apart and/or the bargained offer does not vary much with the Sender's cost of conflict (formally, $a_B^s(\overline{k}_S) \leq a_B^c(R;\underline{k}_S)$), a separating equilibrium always exists. The sufficient condition for existence, however, is relatively unsurprising and quite stringent. It is relatively unsurprising because when the Sender has all bargaining power ($\beta \to 0$), there is never any conflict in equilibrium, and the condition stated in the Proposition has the same spirit. It is quite stringent because it does not reduce the odds of conflict. Indeed, as Corollary C.1 establishes, when the sufficient condition holds, there is no conflict on path in a pooling equilibrium.

Proposition C.2. *For all fixed-outcome lotteries and all type-spaces $\mathcal{K}_S$, a separating equilibrium always exists if $a_B^s(\overline{k}_S) \leq a_B^c(R;\underline{k}_S)$.*

Proof. First, observe that the Receiver always chooses no conflict so we can focus on the Sender. Suppose now the condition holds. This implies that for any two costs of conflict k_S^l and $k_S^h > k_S^l$ in $\mathcal{K}_S$, the bargained offer satisfies: $a_B^s(k_S^h) \leq a_B^c(R; k_S^l)$. Hence a type-$k_S^l$ Sender's best response in stage 3 when he imitates a type-k_S^h is $f_S(a_B^s(k_S^h); k_S^l) = 0$. In this case, a type-$k_S^l$'s (IC) constraint is:

$$-v(a_B^s(k_S^l)) - C(s_S(k_S^l), k_S^l) \geq -v(a_B^s(k_S^h)) - C(s_S(k_S^h), k_S^l)$$

In turn, by the usual reasoning, a type-k_S^h's (IC) constraint is:

$$-v(a_B^s(k_S^h)) - C(s_S(k_S^h), k_S^h) \geq -v(a_B^s(k_S^l)) - C(s_S(k_S^l), k_S^h)$$

Using the increasing differences assumption, it can be checked that we can always find a signaling function $s_S(k_S)$ which satisfies both (IC) constraints. Using a similar reasoning as Proposition 2, I can then construct a separating equilibrium. □

Corollary C.1. *If $a_B^s(\overline{k}_S) \leq a_B^c(R; \underline{k}_S)$, for all fixed-outcome lotteries and all type-spaces $\mathcal{K}_S$, in a pooling equilibrium, there is no conflict on path.*

Proof. Consider a pooling equilibrium in which $s_S^*(k_S) = s_S^p$. Observe that the Receiver's optimal offer $a_B^{max}(R; s_S^p) = \arg\max_{a \in [\underline{a}, \overline{a}]} E(U_R(a_R, f_S)|s_S^p)$ satisfies $a_B^{max}(R; s_S^p) \in [a_B^c(\underline{k}_S), a_B^c(\overline{k}_S))$ under Assumption 3. This implies that in a pooling equilibrium, the bargained offer satisfies for all $k_S \in \mathcal{K}_S$, $a_B(s_S^p) < a_B^s(\overline{k}_S)$. Under the condition, this implies that there is no conflict on path. It remains to show that a pooling equilibrium exists. To do so, I can construct a pooling equilibrium by choosing appropriate out-of-equilibrium belief as in the proof of Proposition 2. □

The next proposition considers whether a separating equilibrium can exist if $a_B^s(\overline{k}_S) > a_B^c(R; \underline{k}_S)$. Before stating the result, it is helpful to introduce some additional pieces of notation. Denote $k_S^b(\overline{k}_S)$ the unique solution to $a_B^s(\overline{k}_S) = a_B^c(R; k_S)$ and $K_S^b = [\underline{k}_S, k_S^b(\overline{k}_S)]$. Notice that for all $k_S < k_S^b(\overline{k}_S)$, a type-$k_S$ Sender starts a conflict if he imitates the type-$\overline{k}_S$. In turn, for all $k_S^l \in K_S^b$ denote $k_S^t(k_S^l)$ the unique solution to $a_B^s(k_S) = a_B^c(R; k_S^l)$ and $K_S^t(k_S^l) = [k_S^t(k_S^l), \overline{k}_S]$. Finally, naturally extending $k_S^b(\cdot)$ denote for all $k_S^h \in K_S^t(\underline{k}_S)$ $k_S^b(k_S^h)$ the unique solution to $a_B^s(k_S^h) = a_B^c(R; k_S)$. I then obtain:

Proposition C.3. *Suppose $a^s_B(\overline{k}_S) > a^c_B(R; \underline{k}_S)$. For all fixed-outcome lotteries and all type-spaces $\mathcal{K}_S$, a separating equilibrium exists if and only if there exists a strictly decreasing signaling function $s^*_S(k_S)$ satisfying for all $k^l_S \in K^b_S \cap \mathcal{K}_S$ and all $k^h_S \in K^t_S(k_S) \cap \mathcal{K}_S$*

$$\begin{aligned} C(s^*_S(k^l_S), k^l_S) - C(s^*_S(k^h_S), k^l_S) + k^b_S(k^h_S) - k^l_S &\leq v(a^s_B(k^h_S)) - v(a^s_B(k^l_S)) \\ &\leq C(s^*_S(k^l_S), k^h_S) - C(s^*_S(k^h_S), k^h_S) \end{aligned} \quad (11)$$

Before proving the proposition, notice that Equation 11 takes a similar form as Equation 3 (or Condition 3 of Proposition 3). The presence of a bargained offer generates a benefit from differentiation, which is absent when the Receiver makes a take-it-or-leave-it offer. However, when a relatively low-cost Sender ($k^l_S \in K^b_S \cap \mathcal{K}_S$) is willing to start a conflict after imitating a relatively high-cost Sender ($k^h_S \in K^t_S(k_S) \cap \mathcal{K}_S$), this benefit is limited and is equal to $v(a^s_B(k^h_S)) - k^b_S(k^h_S) - \big(v(a^s_B(k^l_S)) - k^l_S\big)$. In turn, the benefit from imitation for a relatively high-cost Sender is always large and equal to $v(a^s_B(k^h_S)) - v(a^s_B(k^l_S)) > v(a^s_B(k^h_S)) - k^b_S(k^h_S) - \big(v(a^s_B(k^l_S)) - k^l_S\big)$. Hence, the existence of a separating equilibrium is not always guaranteed when the shadow of conflict looms large (i.e., $a^s_B(\overline{k}_S) > a^c_B(R; \underline{k}_S)$).

Proof. I start with necessity. Take two types $k^l_S \in K^b_S \cap \mathcal{K}_S$ and $k^h_S \in K^t_S(k_S) \cap \mathcal{K}_S$ (the two sets are not empty since $\underline{k}_S, \overline{k}_S \in \mathcal{K}^2_S$). A type-$k^l_S$'s (IC) constraint is then:

$$\begin{aligned} &-v(a^s_B(k^l_S)) - C(s_S(k^l_S), k^l_S) \geq -E_{\mathcal{L}}(v(z)) - k^l_S - C(s_S(k^h_S), k^l_S) \\ \Leftrightarrow &-v(a^s_B(k^l_S)) - C(s_S(k^l_S), k^l_S) \geq -v(a^s_B(k^h_S)) + k^b_S(k^h_S) - k^l_S - C(s_S(k^h_S), k^l_S) \\ \Leftrightarrow &\; v(a^s_B(k^h_S)) - v(a^s_B(k^l_S)) \geq C(s^*_S(k^l_S), k^l_S) - C(s^*_S(k^h_S), k^l_S) + k^b_S(k^h_S) - k^l_S \end{aligned}$$

The first line follows from a type-k^l_S Sender starting a conflict after pretending to be a type k^h_S. The second line comes from the definition of $k^b_S(k^h_S)$: $v(a^s_B(k^h_S)) = v(a^c_B(R; k^b_S(k^h_S)) = E_{\mathcal{L}}(v(z)) + k^b_S(k^h_S)$. By the usual reasoning, a type-k^h_S (IC) constraint is:

$$v(a^s_B(k^h_S)) - v(a^s_B(k^l_S)) \leq C(s^*_S(k^l_S), k^h_S) - C(s^*_S(k^h_S), k^h_S)$$

For sufficiency, observe that if $k_S^l \notin K_S^b \cap \mathcal{K}_S$ or $k_S^h \notin K_S^t(k_S) \cap \mathcal{K}_S$, the (IC) constraints can be satisfied by increasing differences following a similar reasoning as in the proof of Proposition C.2. I can then use a similar argument as in the proof of Proposition 2 to finish the proof. $\square$