# Bargaining over Collusion Profits under Cost Asymmetry and Demand Uncertainty

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# Bargaining over Collusion Profits under Cost Asymmetry and Demand Uncertainty

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**Abstract.** In this paper we borrow from Ciarreta and Gutiérrez-Hita (2012) a duopolistic industry structure with cost asymmetry and demand uncertainty, and using this structure we build a bargaining model to study the division of collusion profits –obtained from the joint selection of supply functions– under the possibility of side payments. In our model, we consider potential disagreement points obtained from the non-cooperative equilibrium of either the quantity competition or the supply function competition, and potential bargaining solutions splitting the gains from agreement either equally or proportionally according to the relative disagreement payoffs of the duopolists. Given any of these disagreement points and any of these bargaining solutions, we find that each duopolist has always incentive to join a collusive agreement. On the other hand, irrespective of whether the bargaining solution splits the gains from agreement equally or proportionally respecting the relative disagreement payoffs, the more efficient firm (the less efficient firm) in the cartel always obtains a higher agreement payoff when the disagreement point is obtained from the equilibrium of supply function competition (quantity competition). Given the studied disagreement points and bargaining solutions, we also find that bargaining over collusion profits always makes the more efficient firm worse off and the less efficient firm better off in comparison to a collusive agreement equalizing the marginal costs of these two firms.

**Keywords:** Duopoly; collusion; bargaining; Cournot competition; supply function competition; uncertainty

**JEL Codes:** D43; L13

# 1 Introduction

A voluminous literature has extensively studied the basic problem of oligopolistic cartels as to how to divide collusion profits under cost asymmetries. While a strand of this literature followed Patinkin's (1947) efficiency concern that the cartel should allocate the collusive outcome to equalize the marginal costs of oligopolists, another strand accepted Bain's (1948) criticism that the division of collusion profits must respect the relative bargaining power of the oligopolists obtained from the threat of playing their non-cooperative equilibrium strategies (see, for example, Osborne and Pitchik, 1983, and Schmalansee, 1987). On the other hand, both strands of literature have mainly focused on types of collusion where the oligopolistic firms compete in prices or in quantities. An exception is the recent work of Ciarreta and Gutiérrez-Hita (2012), which allows collusive firms in a duopolistic industry to compete in supply functions, as well. However, since that type of competition may yield uncountably many equilibria in deterministic environments (as shown by Grossman, 1981), Ciarreta and Gutiérrez-Hita (2012) had to ensure the uniqueness of equilibrium by allowing in their model for the possibility of demand uncertainty as well, a remedy which was earlier proposed by Klemperer and Meyer (1989). One of the results Ciarreta and Gutiérrez-Hita (2012) obtained from their sophisticated model suggests that the less efficient firm has no incentive to reach a collusive agreement in either supply functions or in quantities when the cost asymmetry in the duopoly is sufficiently high, because in that case the expected profits of this firm are higher at the non-cooperative equilibria of these two types of competition. This result, though quite interesting, may not be general, as it is based on the assumed impossibility of side payments between duopolists, forcing them to share the collusive outcome according to cost efficiency considerations. In this paper we replace this assumption with an opposite assumption that allows for side payments between the duopolists, enabling them to freely bargain over collusion profits within the cooperative framework of Nash (1950b).

In our bargaining model, the bargaining set consists of all possible (not necessarily efficient) divisions of the collusion profits obtained from the joint selection of supply functions of the duopolists. Given a bargaining set, we consider potential disagreement points obtained from the non-cooperative equilibrium of either the quantity competition or the supply function competition, and potential bargaining solutions splitting the gains from agreement either equally or proportionally according to the relative disagreement payoffs of the duopolists. Given any of these disagreement points and any of these bargaining solutions, we find that each duopolist has always incentive to join a collusive agreement. On the other hand, irrespective of whether the bargaining solution splits the gains from agreement equally or proportionally respecting the relative disagreement payoffs, the more efficient firm (the less efficient firm) in the cartel always obtains a higher agreement payoff when the disagreement point is obtained from the equilibrium of supply function competition (quantity competition). Given the studied disagreement points and bargaining solutions, we also find that bargaining over collusion profits always makes the more efficient firm worse off and the less efficient firm better off in comparison to a collusive agreement equalizing the marginal costs of these two firms.

The rest of the paper is organized as follows: In Section 2, we present some preliminaries, involving the duopolistic model of Ciarreta and Gutiérrez-Hita (2012) and some of their results relevant for our purpose. Using these preliminaries, we construct in Section 3 a bargaining model for collusive duopolists. Section 4 contains our results and Section 5 concludes. Finally, the Appendix contains the proofs of all results in Section 4.

## 2 Preliminaries

We will present here the duopolistic industry structure considered by Ciarreta and Gutiérrez-Hita (2012) along with some of their results that will be relevant for our bargaining model and results in Sections 3 and 4 respectively. This structure involves a duopolistic industry model with a single homogeneous good produced under the possibility of cost asymmetry and demand uncertainty. The duopolistic firms have quadratic cost functions such that firm $i = 1, 2$ producing a quantity of output $q_i \geq 0$ incurs the cost

$$C_i(q_i) = (1 + \theta_i(c))\, q_i^2/2, \tag{1}$$

where $0 \leq c \leq 1$ and

$$\theta_i(c) = \begin{cases} -c & \text{if } i = 1, \\ +c & \text{if } i = 2, \end{cases} \tag{2}$$

implying that firm 1 is (generally) more efficient than firm 2. The industry demand curve is given by

$$D(p, \alpha(\mu)) = \alpha(\mu) - p, \tag{3}$$

where $p \geq 0$ is the market price of the good and $\alpha(\mu)$ is a scalar random variable that takes the values $1 - \mu$ and $1 + \mu$ with equal probability.[1] It is assumed that $0 \leq \mu \leq 1$. The distribution of $\alpha(\mu)$, the curves $C(q)$ and $D(p, \alpha(\mu))$, and the ranges of the parameters $c$ and $\mu$ are all common knowledge.

## 2.1 Two Types of Competition

For the duopolistic industry described above, two types of competition will be considered. In one of the types, the duopolistic firms compete in supply functions. That is, a strategy for firm $i = 1, 2$ is to non-cooperatively choose - before it learns the realization of the demand uncertainty $\alpha(\mu)$- a linear function mapping price into a quantity of output, i.e., $S_i = v_i p$ where $v_i \geq 0$.[2] Given

[1] In Ciarreta and Gutiérrez-Hita (2012), $\alpha(\mu)$ can take (with equal probability) the values $\alpha - \mu$ and $\alpha + \mu$, where $\alpha \in \mathbb{R}_+$. We set $\alpha = 1$ because the parameter $\alpha$ does not affect our results qualitatively.

[2] In Ciarreta and Gutiérrez-Hita (2012), the supply functions are affinely linear; however, in both collusive and non-cooperative equilibria the constant part of the supply function equals zero for each firm. Given this fact, in order to simplify the notation and analysis, we will proceed with linear supply functions.

the supply functions of the firms, the market clears at a realization $\alpha(\mu)$ of the demand uncertainty if

$$D(p(\alpha(\mu)), \alpha(\mu)) = S_1(p(\alpha(\mu)) + S_2(p(\alpha(\mu)) = (v_1 + v_2)p(\alpha(\mu)). \tag{4}$$

So, inserting (3) into (4) yields the market clearing price

$$p(\alpha(\mu), v_1, v_2) = \frac{\alpha(\mu)}{1 + v_1 + v_2}. \tag{5}$$

We say that a pair of linear supply functions $(S_1^*(p), S_2^*(p)) = (v_1^* p, v_2^* p)$ constitutes a Nash equilibrium (Nash, 1950a) if for each $i = 1, 2$ the function $S_i^*(p)$ maximizes the expected profits of firm $i$ when the remaining firm $j \neq i$ produces according to its supply function $S_j^*(p)$. Formally, this implies that for each $i, j \in \{1, 2\}$ with $j \neq i$, the parameter $v_i^*$ solves

$$\begin{aligned} \max_{v_i \geq 0} \frac{1}{2} \sum_{\alpha(\mu)} \Big[ & p(\alpha(\mu), v_i, v_j^*) S_i^*(p(\alpha(\mu), v_i, v_j^*)) - \\ & (1 + \theta_i(c)) \left( S_i^*(p(\alpha(\mu), v_i, v_j^*)) \right)^2 / 2 \Big]. \end{aligned} \tag{6}$$

**Proposition 1 (Ciarreta and Gutiérrez-Hita, 2012).** *Competition in linear supply functions has a unique Nash equilibrium characterized by $S_i^*(p) = v_i^* p$ for each $i = 1, 2$, where*

$$v_i^* = \frac{1}{2} \left( \frac{v(c) - 2\theta_i(c)}{3 - c^2} - 1 \right), \tag{7}$$

*with*

$$v(c) = \sqrt{45 - c^2(14 - c^2)}. \tag{8}$$

*At this equilibrium, the expected profits of firm $i = 1, 2$ are given by*

$$\pi_i^S(c, \mu) = \frac{1 + \mu^2}{4} \left( \frac{(9 - c^2) + \theta_i(c)(5 - c^2)}{v(c)} - (1 + \theta_i(c)) \right). \tag{9}$$

Proposition 1 implies that the equilibrium supply function of the more efficient firm (firm 1) in the duopoly is always steeper. Therefore, the more efficient firm always has higher output and expected profits in equilibrium. On the other hand, demand uncertainty affects the two firms in the same direction: the equilibrium expected profits of both firms become higher when the demand uncertainty becomes higher.

The second type of competition we will consider is the well-known quantity competition of Cournot (1838). Under this competition, a strategy for firm $i = 1, 2$ is to choose –before it learns the realization of the demand uncertainty $\alpha(\mu)$– a fixed quantity of output, $q_i \geq 0$. Given the quantities chosen by the firms, the market clears at a realization $\alpha(\mu)$ of the demand uncertainty if

$$D(p(\alpha(\mu)), \alpha(\mu)) = q_1 + q_2. \tag{10}$$

Inserting the demand equation (3) into (10) yields the market clearing price

$$p(\alpha(\mu), q_1, q_2) = \alpha(\mu) - q_1 - q_2. \tag{11}$$

We say that a pair of quantities $(q_1^*, q_2^*)$ constitutes a (Cournot) Nash equilibrium if for each $i = 1, 2$ the quantity $q_i^*$ maximizes the expected profits of firm $i$ when the remaining firm $j \neq i$ produces according to the quantity $q_j^*$. Formally, for each $i, j \in \{1, 2\}$ with $j \neq i$, the quantity $q_i^*$ solves

$$\max_{q_i \geq 0} \frac{1}{2} \sum_{\alpha(\mu)} \left[ p(\alpha(\mu), q_i, q_j^*) q_i - (1 + \theta_i(c))\, {q_i}^2/2 \right]. \tag{12}$$

**Proposition 2 (Ciarreta and Gutiérrez-Hita, 2012).** *Competition in quantities has a unique (Cournot) Nash equilibrium characterized by*

$$q_i^*(c) = \left( \frac{2 - \theta_i(c)}{8 - c^2} \right), \quad i = 1, 2. \tag{13}$$

*At this equilibrium, the expected profits of firm $i = 1, 2$ are given by*

$$\pi_i^C(c) = \frac{1}{2} \left[ \frac{(3 + \theta_i(c))(2 - \theta_i(c))^2}{(8 - c^2)^2} \right]. \tag{14}$$

The above proposition shows that under the quantity competition, the more efficient firm always produces more in comparison to the less efficient firm, like in the supply function competition; hence, it obtains higher expected profits in equilibrium. However, unlike under the supply function competition, the expected profits of the duopolistic firms are independent of the demand uncertainty under the quantity competition. One may immediately ask whether any of these two competitions can always yield more expected profits to the more efficient firm or to the less efficient firm in the duopoly. We will give a non-trivial answer to this question in Section 4.

## 2.2 Two Types of Collusion

We will now consider two types of collusion. According to the first type, the duopolistic cartel chooses the linear supply functions of the two firms jointly to maximize the industry (cartel) profits. Let $\pi^I(\alpha(\mu), v_1, v_2)$ denote the industry profits at the demand realization $\alpha(\mu)$ when the supply functions chosen by the cartel for firm 1 and firm 2 are $S_1(p) = v_1 p$ and $S_2(p) = v_2 p$ respectively. These profits can be calculated as

$$\begin{aligned}\pi^I(\alpha(\mu), v_1, v_2) \quad = \quad & \sum_{i=1,2} \Big[ p(\alpha(\mu), v_1, v_2) S_i(p(\alpha(\mu), v_i)) - \\ & (1 + \theta_i(c)) \left(S_i(p(\alpha(\mu), v_i))\right)^2/2 \Big], \end{aligned} \tag{15}$$

where $p(\alpha(\mu), v_1, v_2)$ is given by (5). We say that a pair of supply functions, $(\hat{S}_1(p), \hat{S}_2(p)) = (\hat{v}_1 p, \hat{v}_2 p)$, leads to collusion if for all $i, j \in \{1, 2\}$ with $j \neq i$ the parameter $\hat{v}_i$ solves

$$\max_{v_i \geq 0} \frac{1}{2} \sum_{\alpha(\mu)} \pi^I(\alpha(\mu), v_i, \hat{v}_j). \tag{16}$$

**Proposition 3 (Ciarreta and Gutiérrez-Hita, 2012).** *Collusion in linear supply functions arises only if $\hat{S}_i(p) = \hat{v}_i p$ for each $i = 1, 2$, where*

$$\hat{v}_i = \frac{1 - \theta_i(c)}{3 - c^2}. \tag{17}$$

*Under this collusion, the expected industry profits are equal to*

$$\pi^{I,S-Col}(c, \mu) = \frac{1 + \mu^2}{5 - c^2}. \tag{18}$$

*Moreover, if the output is allocated between the firms to equalize their marginal costs, then the expected profits of firm $i = 1, 2$ become*

$$\pi_i^{S-Col}(c, \mu) = \frac{1+\mu^2}{2}\left[\frac{1-\theta_i(c)}{5-c^2}\right]. \tag{19}$$

Note in the above proposition that the supply curve chosen by the duopolistic cartel for the more efficient firm is always steeper than the supply curve chosen for the less efficient firm. Consequently, the output allocated to, and the expected profits obtained by, the more efficient firm in the cartel are higher. On the other hand, the expected profits of both firms are increasing in the size of demand uncertainty.

Now, we will consider the type of collusion where the duopolistic cartel chooses the fixed quantities of the two firms jointly to maximize the industry profits. Let $\pi^I(\alpha(\mu), q_1, q_2)$ denote the industry profits at the demand realization $\alpha(\mu)$ when the quantities chosen for firm 1 and firm 2 are $q_1$ and $q_2$ respectively. Then, we must have

$$\pi^I(\alpha(\mu), q_1, q_2) = \sum_{i=1}^{2}\left[(\alpha(\mu) - q_1 - q_2)q_i - (1+\theta_i)q_i^2/2\right]. \tag{20}$$

We say that a pair of quantities, $(\hat{q}_1, \hat{q}_2)$, leads to collusion if for all $i, j \in \{1, 2\}$ with $j \neq i$ the quantity $\hat{q}_i$ solves

$$\max_{q_i \geq 0} \frac{1}{2} \sum_{\alpha(\mu)} \pi^I(\alpha(\mu), q_i, \hat{q}_j). \tag{21}$$

**Proposition 4 (Ciarreta and Gutiérrez-Hita, 2012).** *Collusion in quantities arises only if for each $i = 1, 2$ the quantity of output produced by firm $i$ is equal to*

$$q_i^{C-Col}(c) = \frac{1-\theta_i(c)}{5-c^2}. \tag{22}$$

*Under this collusion, the expected industry profits are always given by*

$$\pi^{I,C-Col}(c) = \frac{1}{5-c^2}. \tag{23}$$

*Moreover, if the output is allocated between the two firms to equalize their marginal costs, then the expected profits of firm $i = 1, 2$ become*

$$\pi_i^{C-Col}(c) = \frac{1}{2}\left[\frac{1-\theta_i(c)}{5-c^2}\right]. \tag{24}$$

The above proposition shows that -like in the case of collusion in supply functions- the output allocated to, and the expected profits obtained by, the more efficient firm in the cartel are always higher. Comparing Propositions 3 and 4, we also observe that the expected profits of each firm, hence the industry profits, are always higher when the cartel commits to supply functions, instead of quantities, provided that there is any size of demand uncertainty.

# 3 The Model

Using the duopolistic industry presented above, Ciarreta and Gutiérrez-Hita (2012) studied the formation and stability of a collusive agreement (or a cartel). However, they only considered agreements without side payments, and following the suggestion of Patinkin (1947) they restricted themselves to collusive allocations that equalize the duopolists' marginal costs of production. In this paper, we will get rid of this restriction and study the problem of dividing collusion profits under the possibility of bargaining with side payments. We will formulate this division problem using the two-person cooperative bargaining model of Nash (1950), where the two persons, namely the duopolists in our problem, are allowed to choose a payoff allocation in a bargaining set of payoffs according to any rule they agree upon.

Formally, in the bargaining model of Nash (1950), a bargaining set is any nonempty subset of $\mathbb{R}^2_+$, representing von Neumann-Morgenstern utilities attainable through the cooperative actions of two agents. Given any bargaining set $S \in \mathbb{R}^2_+$, a point $d = (d_1, d_2)$ in $S$ is called the disagreement point. If the two persons fail to agree on any point in $S$, then they receive the gains $d_1$ and $d_2$ respectively. The bargaining set $S$ and a disagreement point $d$ in this set together define a bargaining problem. It is assumed that $S$ is compact and convex and there exists $x \in S$ such that $x \geq d$.[3] In addition, $S$ is d-comprehensive (allowing free disposal of utility); that is, for all $x, y \in \mathbb{R}^2_+$, if $x \in S$ and $x \geq y \geq d$, then $y \in S$. Let $WP(S)$ denote the set of weakly Pareto optimal points in $S$; i.e. $WP(S) = \{y \in S : \text{if } x > y, \text{then } x \notin S\}$, and let $P(S)$ denote the set of (strongly) Pareto optimal points in $S$; i.e. $P(S) = \{y \in S : \text{if } x \geq y, \text{then } x \notin S\}$.

Given the above assumptions and definitions, let $\Sigma^2_+$ denote the set of all two-person bargaining problems with a non-negative disagreement point. We can now formulate the bargaining problem between the firms in a duopolistic cartel. First, we will construct the bargaining set for this cartel. Here, we assume that the two firms jointly choose -among the two alternatives in Section 2.2- the type of collusion that yields the highest industry profits. Comparison of Propositions 3 and 4 to this end immediately reveals that the two firms, when they desire to form a cartel, should always choose to collude in supply functions since the expected profits of the cartel would be never lower under that type of collusion and would be always higher as long as there is any uncertainty in demand. Thus, for the rest of this study, we assume that the expected collusion profits to be shared by the duopolistic firms are equal to $\pi^{I,S-Col}(c, \mu)$, satisfying equation (18), for any $c \in [0, 1]$ and $\mu \in [0, 1]$. We will denote this profit level by $\pi^I(c, \mu)$.

Since any division of collusion profits is allowed under the possibility of side payments, for any $c$ and $\mu$ in $[0, 1]$ the corresponding bargaining set $S(c, \mu)$ must be equal to $\{s_1, s_2 \in \mathbb{R}^2_+ : 0 \leq s_1 + s_2 \leq \pi^I(c, \mu)\}$. Note that $S(c, \mu)$ always has a linear frontier, i.e. $P(S(c, \mu)) = WP(S(c, \mu))$. Also, $S(c, \mu)$ is always compact, convex, and also d-comprehensive given any $d \in S(c, \mu)$. Let us denote by $\Sigma^{2,\mathcal{D}}_+$ the set of all problems $(S(c, \mu), d)$ in $\Sigma^2_+$ faced by the duopolistic firms for some $c$ and $\mu$ in $[0, 1]$. Clearly, $\Sigma^{2,\mathcal{D}}_+ \subset \Sigma^2_+$.

Given the definition for bargaining problems, a solution $F$ on $\Sigma^2_+$ is a mapping from $\Sigma^2_+$ to $\mathbb{R}^2_+$ such that for any $(S, d) \in \Sigma^2_+$, $F(S, d) \in S$. (Given the solution point $F(S, d)$, we will denote by $F_i(S, d)$ the payoff of firm $i = 1, 2$.)

[3] Given two vectors $x$ and $y$ in $\mathbb{R}^2_+$, $x > y$ means $x_i > y_i$ for $i = 1, 2$ and $x \geq y$ means $x_i \geq y_i$ for $i = 1, 2$.

Obviously, there exist uncountably many solutions on $\Sigma_+^2$. However, we will only consider solutions that will divide up the cartel's gains from agreement either always equally or always proportionally according to the ratio of the disagreement payoffs. To formally describe these solutions, we say that

(i) a bargaining solution $F$ on $\Sigma_+^2$ is called the Equal Split solution on $\Sigma_+^{2,\mathcal{D}}$ if for any $c \in [0,1]$, $\mu \in [0,1]$, and $d \in S(c,\mu)$ it satisfies

$$F_i(S(c,\mu),d) - d_i = \frac{1}{2}[\pi^I(c,\mu) - d_1 - d_2], \quad i = 1,2, \tag{25}$$

(ii) a bargaining solution $F$ on $\Sigma_+^2$ is called the $d$-Proportional Split solution on $\Sigma_+^{2,\mathcal{D}}$ if for any $c \in [0,1]$, $\mu \in [0,1]$, and $d \in S(c,\mu)$ it satisfies

$$F_i(S(c,\mu),d) - d_i = \phi_i(d)\,[\pi^I(c,\mu) - d_1 - d_2], \quad i = 1,2, \tag{26}$$

where

$$\phi_i(d) = \begin{cases} \dfrac{1}{2} & \text{if } \; d_1 + d_2 = 0, \\ \dfrac{d_i}{d_1 + d_2} & \text{if } \; 0 < d_1 + d_2 \leq \pi^I(c,\mu). \end{cases} \tag{27}$$

In the oligopoly literature, the solution that splits the gains from agreement equally was referred by Schmalansee (1987) as the Equal Gains solution of Roth (1979) while he was dealing with problems where the bargaining sets are linear and the disagreement point is set to the Nash equilibrium payoffs obtained under the quantity (Cournot) competition, whereas for similar problems the solution that splits the gains from agreement d-proportionally was referred by Fischer and Normann (2016) as the Equal Relative Gains solution of Roth (1979). One can check that many well known solutions, including the Nash (1950) solution, the Kalai and Smorodinsky (1975) solution, and the Egalitarian solution (Kalai, 1977) among others, reduce to the Equal Split solution on $\Sigma_+^{2,\mathcal{D}}$. One can also check that both the Equal Split and the Proportional Split solution can be obtained from some members of the class of reference function solutions, defined and characterized by Anbarci (1995).

In the next section, we will study the implications of using the Equal Split and d-Proportional Split solutions in dividing up the collusion profits. For notational simplicity, we will sometimes use $S$, $\pi^S$, $\pi^C$, and $\pi^I$ in place of $S(c,\mu)$, $\pi^S(c,\mu)$, $\pi^C(c)$, and $\pi^I(c,\mu)$, respectively.

# 4 Results

The bargaining model constructed in the previous section requires the duopolistic firms to determine a disagreement point in their bargaining set. The characterization results of Ciarreta and Gutiérrez-Hita (2012), presented in Section 2, suggest two natural candidates: The expected profits obtained by the duopolistic firms from the supply function competition, $\pi^S$, and from the Cournot competition, $\pi^C$. The firms may agree upon using any of these equilibrium points as a disagreement (status quo) point $d$, provided that they *mutually* believe that if an agreement fails to occur, each firm will be playing its non-cooperative strategy at the equilibrium strategy profile inducing the payoffs at $d$. Below, we will show that, for all values of the parameters $c$ and $\mu$, the payoff allocations $\pi^S(c,\mu)$ and $\pi^C(c)$ are always inside the bargaining set $S(c,\mu)$.

**Lemma 1.** *For any $c \in [0,1]$ and $\mu \in [0,1]$, if $d \in \{\pi^S(c,\mu), \pi^C(c)\}$ then $d \in S(c,\mu) \backslash WP(S(c,\mu))$.*

Lemma 1 shows that each of the points $\pi^S(c,\mu)$ and $\pi^C(c)$ is admissible as a disagreement point. However, it also implies that neither of these points can be selected by any bargaining solution that satisfies efficiency, i.e., that leaves no part of the collusion profits undistributed between the duopolists. This lemma has a direct implication on the formation of a collusive agreement.

**Theorem 1.** *Let $F$ be any solution on $\Sigma_+^2$ that reduces to either the Equal Split solution or the d-Proportional Split solution on $\Sigma_+^{2,\mathcal{D}}$. Then, for any $c \in [0,1]$,*

*$\mu \in [0,1]$, and $d \in \{\pi^S(c,\mu), \pi^C(c))$ each duopolist has incentive to collude under the solution $F$, i.e., $F_i(S(c,\mu), d) > d_i$ for each $i = 1,2$.*

We should note that given a disagreement point $d$, both the Equal Split solution and the d-Proportional Split solution offer to each duopolist a bargaining payoff which is equal to its disagreement payoff plus a positive fraction of the gains from agreement $\pi^I(c,\mu) - d_1 - d_2$. So, Theorem 1 can be valid if and only if for all values of the parameters $c$ and $\mu$ the gains from agreement are positive both when $d = \pi^S(c,\mu)$ and when $d = \pi^C(c)$. Lemma 1 ensures that this is indeed the case, since neither of these two potential disagreement points is on the weak Pareto frontier of $S(c,\mu)$.

We should recall that in case the bargaining between the duopolistic firms fails, each firm gains its disagreement payoff irrespective of the bargaining solution. Given this, our next concern is to compare the disagreement payoffs of each duopolist induced by the potential disagreement points $\pi^S(c,\mu)$ and $\pi^C(c)$. To this aim, we will first compare $\pi_1^S(c,\mu) - \pi_1^C(c)$ and $\pi_2^S(c,\mu) - \pi_2^C(c)$ in the following lemma.

**Lemma 2.** *For any $\mu$ in $[0,1]$, $\pi_1^S(c,\mu) - \pi_1^C(c) = \pi_2^S(c,\mu) - \pi_2^C(c)$ if $c = 0$ and $\pi_1^S(c,\mu) - \pi_1^C(c) > \pi_2^S(c,\mu) - \pi_2^C(c)$ if $c \in (0,1]$.*

Lemma 2 shows that if the disagreement point is changed from $\pi^C(c)$ to $\pi^S(c,\mu)$, the absolute gain –obtained when the bargaining fails– becomes higher for the firm which is more efficient. Noticing that the inequality in this lemma can be rewritten as $\pi_1^S(c,\mu) - \pi_2^S(c,\mu) > \pi_1^C(c) - \pi_2^C(c)$, one can also observe that as long as there exists cost asymmetry, the welfare inequality between the duopolists is always higher under the supply function competition. Lemma 2 will be instrumental to prove the next lemma where we will deal with the existence, uniqueness, and several other properties of some critical $\mu$ values.

**Lemma 3.** *For any $i \in \{1, 2\}$ and $c \in [0, 1]$, there exists a unique $\mu_i(c) \in (0, 1)$ such that*

$$\pi_i^S(c, \mu_i(c)) = \pi_i^C(c). \tag{28}$$

*It is also true that $\mu_1(c)$ is always decreasing in $c$ whereas $\mu_2(c)$ is always increasing. Moreover, $\mu_1(0) = \mu_2(0)$ and $\mu_1(c) < \mu_2(c)$ for any $c \in (0, 1]$.*

Using Lemma 3, we can make the following comparisons.

**Theorem 2.** *Given any $c \in [0, 1]$ and $\mu \in [0, 1]$, the disagreement payoffs of each duopolist at the disagreement points $\pi^S(c, \mu)$ and $\pi^C(c)$ can be compared as follows:*

$$\begin{aligned}
&\pi_1^S(c,\mu) < \pi_1^C(c) \quad \textit{and} \quad \pi_2^S(c,\mu) < \pi_2^C(c) \quad \textit{if} \quad 0 \le \mu < \mu_1(c),\\
&\pi_1^S(c,\mu) = \pi_1^C(c) \quad \textit{and} \quad \pi_2^S(c,\mu) < \pi_2^C(c) \quad \textit{if} \quad \mu = \mu_1(c),\\
&\pi_1^S(c,\mu) > \pi_1^C(c) \quad \textit{and} \quad \pi_2^S(c,\mu) < \pi_2^C(c) \quad \textit{if} \quad \mu_1(c) < \mu < \mu_2(c),\\
&\pi_1^S(c,\mu) > \pi_1^C(c) \quad \textit{and} \quad \pi_2^S(c,\mu) = \pi_2^C(c) \quad \textit{if} \quad \mu = \mu_2(c),\\
&\pi_1^S(c,\mu) > \pi_1^C(c) \quad \textit{and} \quad \pi_2^S(c,\mu) > \pi_2^C(c) \quad \textit{if} \quad \mu_2(c) < \mu \le 1.
\end{aligned} \tag{29}$$

Theorem 2 implies that in any bargaining situation where bargaining fails and the disagreement payoffs are realized, both firms in the cartel would regret if they had agreed, before the bargaining takes place, on the disagreement point $\pi^C(c)$ instead of $\pi^S(c, \mu)$ (on the disagreement point $\pi^S(c, \mu)$ instead of $\pi^C(c)$) provided that the demand uncertainty is sufficiently high (sufficiently low) with respect to the cost asymmetry.

Figure 1 illustrates our findings in Theorem 2. Note that $\mu_2(c)$ is the curve separating the purple and blue colored areas respectively at the top and the middle of the figure whereas $\mu_1(c)$ is the curve separating the blue and yellow colored areas respectively at the middle and the bottom. One can see that an increase in the cost parameter $c$ increases $\mu_2(c)$ and decreases $\mu_1(c)$, raising the difference $\mu_2(c) - \mu_1(c)$ or widening the blue colored region towards the right. In situations where the uncertainty parameter $\mu$ is random, the rise in $\mu_2(c)-\mu_1(c)$ would imply an increase in the likelihood that $\pi^S(c,\mu)$ and $\pi^C(c)$ are Pareto non-comparable. On the other hand, as we can see in the purple (yellow) colored region the disagreement welfares of both firms in the duopolistic cartel are higher (lower) at $\pi^S(c,\mu)$ than at $\pi^C(c)$ if the demand uncertainty is sufficiently high (low).

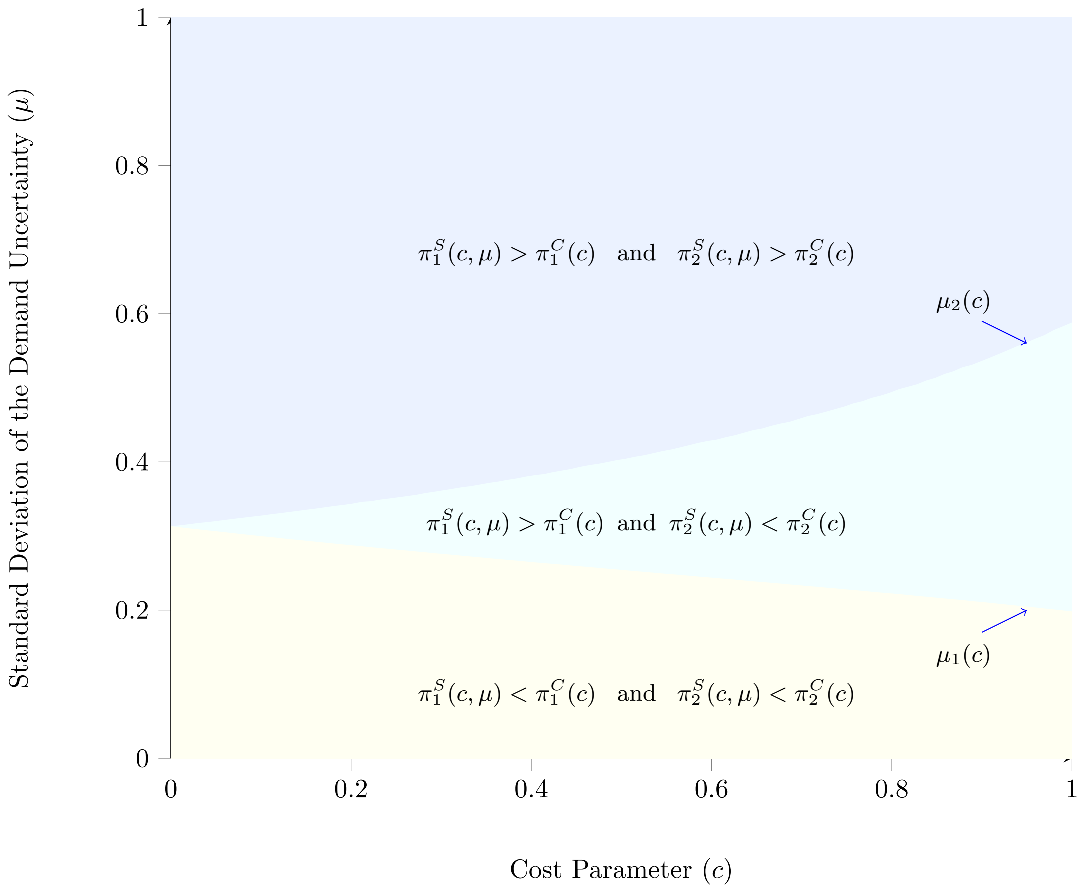



**Figure 1.** The effects of $\mu$ and $c$ on the Pareto comparison of the disagreement points $\pi^S(c,\mu)$ and $\pi^C(c)$.

Hereafter, we will study the *agreement* payoffs obtained by the duopolistic firms when the disagreement point is either $\pi^S(c,\mu)$ or $\pi^C(c)$. We will first consider the simple case of symmetric costs, i.e., $c = 0$. It should be clear from (8), (9), and (14) that $\pi_1^S(0,\mu) = \pi_2^S(0,\mu)$ for any $\mu \in [0,1]$ and $\pi_1^C(0) = \pi_2^C(0)$. These equalities trivially imply that if there is no cost asymmetry, then irrespective of the size of the demand uncertainty the duopolistic firms will always share the collusion profits equally, both when the solution splits the gains from agreement equally and when it splits them $d$-proportionally regardless whether the disagreement point is obtained from the equilibrium of the supply function or the quantity competition.

**Theorem 3.** *Let $F$ be any solution on $\Sigma_+^2$ that reduces to either the Equal Split solution or the d-Proportional Split solution on $\Sigma_+^{2,\mathcal{D}}$. If there is no cost asymmetry in the industry ($c = 0$), then for any $\mu \in [0,1]$ and $d \in \{\pi^S(0,\mu), \pi^C(0)\}$, the collusion profits are split equally under the solution $F$, i.e., $F_i(S(0,\mu),d) = \pi^I(0,\mu)/2$ for any $i \in \{1,2\}$.*

In the rest of our paper, we will be interested in the more involving case of asymmetric costs, i.e., $c \in (0,1]$. The below lemma will shorten our analysis substantially. In the proof of many results, after ensuring some payoff comparison for firm 1, we will use Lemma 4 to simply infer the relevant comparison for firm 2.

**Lemma 4.** *Let $F$ be any solution on $\Sigma_+^2$ that reduces to either the Equal Split solution or the d-Proportional Split solution on $\Sigma_+^{2,\mathcal{D}}$. Then, for any $c \in (0,1]$, $\mu \in [0,1]$, and $d, d' \in S(c,\mu)$, we have $F_i(S(c,\mu),d) > F_i(S(c,\mu),d')$ if and only if $F_j(S(c,\mu),d) < F_j(S(c,\mu),d')$ for any $i,j \in \{1,2\}$ with $j \neq i$.*

Now, we can start studying solutions that split the gains from agreement equally.

**Lemma 5.** *Let $F$ be any solution on $\Sigma_+^2$ that reduces to the Equal Split solution on $\Sigma_+^{2,\mathcal{D}}$. For any $c \in (0,1]$, $\mu \in [0,1]$, and $d, d' \in S(c,\mu)$, we have $F_i(S(c,\mu), d) > F_i(S(c,\mu), d')$ if and only if $d_i - d_j > d'_i - d'_j$ for any $i, j \in \{1,2\}$ with $j \neq i$.*

Using Lemma 5, we will next show that if the bargaining solution splits the gains from agreement equally, then the bargaining payoff of the more efficient firm in the cartel is always higher when the disagreement point is $\pi^S$ than when it is $\pi^C$, while the opposite is true for the payoff of the less efficient firm.

**Theorem 4.** *Let $F$ be any solution on $\Sigma_+^2$ that reduces to the Equal Split solution on $\Sigma_+^{2,\mathcal{D}}$. Then for any $c \in (0,1]$ and $\mu \in [0,1]$, we have $F_1(S(c,\mu), \pi^S(c,\mu)) > F_1(S(c,\mu), \pi^C(c))$ and $F_2(S(c,\mu), \pi^S(c,\mu)) < F_2(S(c,\mu), \pi^C(c))$.*

To see why Theorem 4 is true, we should note by Lemma 2 that the payoff difference $\pi_1^S(c,\mu) - \pi_2^S(c,\mu)$ is always higher than $\pi_1^C(c) - \pi_2^C(c)$ when there is cost asymmetry. Then, using Lemma 5, one can simply show that for any solution $F$ that splits the gains from agreement equally, $F_1(S(c,\mu), \pi^S(c,\mu))$ must be higher than $F_1(S(c,\mu), \pi^C(c))$. A negative implication of Theorem 4 is that any size of cost asymmetry would always lead to a conflict between the duopolistic firms regarding whether –in a pre-bargaining stage– they should set the disagreement point to $\pi^S(c,\mu)$ or $\pi^C(c)$. The firms may resolve this conflict if they manage to agree upon an alternative disagreement point which is compromising for both of them. A natural candidate for such a point may be a weighted average of $\pi^S(c,\mu)$ and $\pi^C(c)$, i.e., the point $\pi^\omega(c,\mu) = \omega\pi^S(c,\mu) + (1-\omega)\pi^C(c)$ where $\omega \in (0,1)$. Note that the allocation $\pi^\omega(c,\mu)$ lies in $S(c,\mu)$ because $S(c,\mu)$ is convex.

**Corollary 1.** *Let $F$ be any solution on $\Sigma_+^2$ that reduces to the Equal Split solution on $\Sigma_+^{2,\mathcal{D}}$. Then for any $c \in (0,1]$, $\mu \in [0,1]$, and $\omega \in (0,1)$, we have*

*$F_1(S(c,\mu),\pi^S(c,\mu)) > F_1(S(c,\mu),\pi^\omega(c,\mu)) > F_1(S(c,\mu),\pi^C(c))$ and $F_2(S(c,\mu),$ $\pi^S(c,\mu)) < F_2(S(c,\mu),\pi^\omega(c,\mu)) < F_2(S(c,\mu),\pi^C(c))$.*

Corollary 1 shows that when the bargaining solution splits the gains from agreement equally, the disagreement point $\pi^\omega$ becomes, for any $\omega \in (0,1)$, always superior to each firm's worst alternative in $\{\pi^C, \pi^S\}$ and always inferior to each firm's best alternative in the same set, always implying some degree of compromise for both firms. We should also note that the higher the weight parameter $\omega$, the higher the agreement payoff of firm 1 and the lower the agreement payoff of firm 2, pointing to a new conflict between the two firms as to the determination of $\omega$. Definitely, there is no way to predict which value of $\omega$ the firms would select. However, if they can agree upon choosing their bargaining solution under some form of symmetry condition leading to a solution splitting the gains from agreement equally, it might not be unreasonable to assume that they could impose a similar condition of symmetry also when they have to bargain over the parameter $\omega \in (0,1)$, leading to the disagreement point $\pi^{1/2}$, which is the equally weighted average of $\pi^S$ and $\pi^C$ at all parameter values.

Now, we will consider bargaining solutions that split the gains from agreement in any bargaining problem d-proportionally.

**Theorem 5.** *Let $F$ be any solution on $\Sigma_+^2$ that reduces to the d-Proportional Split solution on $\Sigma_+^{2;\mathcal{D}}$. Then for any $c \in (0,1]$ and $\mu \in [0,1]$, $F_1(S(c,\mu),\pi^S(c,\mu))$ $> F_1(S(c,\mu),\pi^C(c))$ and $F_2(S(c,\mu),\pi^S(c,\mu)) < F_2(S(c,\mu),\pi^C(c))$.*

Theorem 5 shows that when the bargaining solution splits the gains from agreement proportionally respecting the relative payoffs at the disagreement point, the more efficient firm always prefers –with respect to the induced agreement payoffs– the disagreement point $\pi^S$ to the disagreement point $\pi^C$, while the opposite is true for the less efficient firm. Recall that the same conclusion was also reached in Theorem 4, where the bargaining solution splits the gains from agreement equally. We should also observe that both in Theorem 4 and in Theorem 5, we have picked and fixed a bargaining solution and then compared the agreement payoffs of the duopolistic firms induced by the disagreement points $\pi^S$ and $\pi^C$. These two theorems together imply that the preferences of the duopolistic firms over $\pi^S$ and $\pi^C$ are not affected by whether the gains from agreement are split equally or d-proportionally. But, there remains a question we have not answered yet. Which of the two solutions, considered in Theorems 4 and 5, would be preferred by the more efficient firm, or the less efficient firm, in the cartel if the disagreement point were fixed at either $\pi^S$ or $\pi^C$? The answer to this question will be implied by the following theorem that builds a bridge between the results in Theorems 4 and 5.

**Theorem 6.** *Let $F^{d-PS}$ and $F^{ES}$ be bargaining solutions on $\Sigma_+^2$ that respectively reduce to the d-Proportional Split solution and the Equal Split solution on $\Sigma_+^{2,\mathcal{D}}$. Then, for any $c \in (0,1]$ and $\mu \in [0,1]$, $F_1^{d-PS}(S(c,\mu),\pi^C(c)) > F_1^{ES}(S(c,\mu),\pi^S(c,\mu))$ and $F_2^{d-PS}(S(c,\mu),\pi^C(c)) < F_2^{ES}(S(c,\mu),\pi^S(c,\mu))$.*

We know from Theorem 2 that in cases where the size of demand uncertainty is sufficiently large, the disagreement payoff of the more efficient firm is always lower at $\pi^C$ than at $\pi^S$. However, even in such cases it is true –by Theorem 6– that the more efficient firm prefers a bargaining environment where the disagreement point is $\pi^C$ and the gains from agreement are split d-proportionally to an environment where the disagreement point is $\pi^S$ and the gains from agreement is split equally. Theorems 4, 5, and 6 together allow us to observe the following.

**Corollary 2.** *Let $F^{d-PS}$ and $F^{ES}$ be bargaining solutions on $\Sigma_+^2$ that respectively reduce to the d-Proportional Split solution and the Equal Split solution on $\Sigma_+^{2,\mathcal{D}}$. Then, for any $c \in (0,1]$ and $\mu \in [0,1]$,*

$$F_1^{d-PS}(S,\pi^S) > F_1^{d-PS}(S,\pi^C) > F_1^{ES}(S,\pi^S) > F_1^{ES}(S,\pi^C), \quad \textit{and}$$

$$F_2^{d-PS}(S,\pi^S) < F_2^{d-PS}(S,\pi^C) < F_2^{ES}(S,\pi^S) < F_2^{ES}(S,\pi^C)$$

*where* $S = S(c, \mu)$*,* $\pi^S = \pi^S(c, \mu)$*, and* $\pi^C = \pi^C(c)$.

To summarize the findings we have obtained so far, consider at each $c \in [0, 1]$ and $\mu \in [0, 1]$, the set of bargaining environments

$$\mathcal{E}(c, \mu) = \{(F^{d-PS}, \pi^S(c, \mu)), (F^{d-PS}, \pi^C(c)), (F^{ES}, \pi^S(c, \mu)), (F^{ES}, \pi^C(c))\},$$

where each environment involves a bargaining solution and a disagreement point we have studied. To compare the environments in $\mathcal{E}(c, \mu)$ for the duopolistic firms, we can define their preference relations. Given any $(F, d)$ and $(F', d')$ where $F$ and $F'$ are bargaining solutions on $\Sigma^2_+$ and $d, d' \in S(c, \mu)$, we say that in terms of the induced agreement payoffs firm $i$ prefers $(F, d)$ to $(F', d')$, denoted by $(F, d) \succ_i (F', d')$, if and only if $F_i(S(c, \mu), d) > F'_i(S(c, \mu), d')$ and firm $i$ is indifferent between $(F, d)$ and $(F', d')$, denoted by $(F, d) \sim_i (F', d')$, if and only if $F_i(S(c, \mu), d) = F'_i(S(c, \mu), d')$.

Given the above definitions, Theorem 3 implies that in case $c = 0$, firm $i = 1, 2$ has the preference ordering

$$(F^{d-PS}, \pi^S) \sim_i (F^{d-PS}, \pi^C) \sim_i (F^{ES}, \pi^S) \sim_i (F^{ES}, \pi^C) \tag{30}$$

at any $\mu \in [0, 1]$. On the other hand, Corollary 2 implies the following orderings

$$(F^{d-PS}, \pi^S) \succ_1 (F^{d-PS}, \pi^C) \succ_1 (F^{ES}, \pi^S) \succ_1 (F^{ES}, \pi^C) \tag{31}$$

$$(F^{ES}, \pi^C) \succ_2 (F^{ES}, \pi^S) \succ_2 (F^{d-PS}, \pi^C) \succ_2 (F^{d-PS}, \pi^S) \tag{32}$$

for any $c \in (0, 1]$ and $\mu \in [0, 1]$.

The preference orderings above show that in situations where the firms in the duopolistic cartel restrict themselves at any $c \in (0, 1]$ and $\mu \in [0, 1]$ to the set of bargaining environments in $\mathcal{E}(c, \mu)$, the more efficient firm would desire to be in the environment $(F^{d-PS}, \pi^S(c, \mu))$, which is the worst environment for the less efficient firm. Oppositely, the best environment in $\mathcal{E}(c, \mu)$ from the viewpoint of the less efficient firm, namely $(F^{ES}, \pi^C(c))$, is the worst environment according to the more efficient firm, pointing to a conflict of choice between the two firms.

The firms may resolve this conflict by using a disagreement point, $\pi^{1/2}(c, \mu)$, which averages $\pi^C(c)$ and $\pi^S(c, \mu)$, along with a moderating solution $F^M$ on $\Sigma_+^2$ that will equally split for the two firms the sum of payoffs generated by $(F^{d-PS}, \pi^S(c, \mu))$ and $(F^{ES}, \pi^C(c))$, i.e.,

$$F_i^M(S(c,\mu), \pi^{1/2}(c,\mu)) = \frac{1}{2}\Big(F_i^{d-PS}(S(c,\mu), \pi^S(c,\mu)) + F_i^{ES}(S(c,\mu), \pi^C(c))\Big) \quad (33)$$

for any $i \in \{1, 2\}$. We should observe that for any $c \in (0, 1]$ and $\mu \in [0, 1]$, the environment $(F^M, \pi^{1/2}(c, \mu))$ lies in the preference ordering of each firm between the most desirable and the least desirable environments in $\mathcal{E}(c, \mu)$, since we have

$$(F^{d-PS}, \pi^S(c,\mu)) \succ_1 (F^M, \pi^{1/2}(c,\mu)) \succ_1 (F^{ES}, \pi^C(c)) \quad (34)$$

and

$$(F^{ES}, \pi^C(c)) \succ_2 (F^M, \pi^{1/2}(c,\mu)) \succ_2 (F^{d-PS}, \pi^S(c,\mu)). \quad (35)$$

Having noted that $(F^M, \pi^{1/2}(c, \mu))$ may be a plausible bargaining environment, we wonder whether its outcome can be more desirable for any of the duopolistic firms in comparison to the division of collusion profits in the absence of bargaining, as considered by Ciarreta and Gutiérrez-Hita's (2012). Recall from (19) that when there is no bargaining, firm $i = 1, 2$ obtains a cost-based share of collusion profits, amounting $\pi_i^{S-Col}(c, \mu) = (1 - \theta_i(c))\pi^I(c, \mu)/2$. Since this payoff must also be attainable when bargaining is possible, we will –for the sake of notational harmony– define for every $c$ and $\mu$ in $[0, 1]$, a bargaining environment $(F^{S-Col}, \pi^{S-Col}(c, \mu))$, where $F^{S-Col}$ is a solution on $\Sigma_+^2$ such that

$$F^{S-Col}(S(c,\mu), \pi^{S-Col}(c,\mu)) = \pi^{S-Col}(c,\mu). \quad (36)$$

Since $\pi^{S-Col}(c, \mu) \in WP(S(c, \mu))$, $F^{S-Col}$ can be any solution on $\Sigma_+^2$ as long as it satisfies on $\Sigma_+^{2,L}$ an axiom of individual rationality requiring that the solution must not be below the disagreement point in any bargaining set.

**Theorem 7.** *Let $F^{d-PS}$ and $F^{ES}$ be bargaining solutions on $\Sigma_+^2$ that respectively reduce to the d-Proportional Split solution and the Equal Split solution on*

$\Sigma_+^{2,\mathcal{D}}$. *Then for any* $c \in (0,1]$ *and* $\mu \in [0,1]$, $F_1^{S-Col}(S(c,\mu), \pi^{S-Col}(c,\mu)) > F_1^{d-PS}(S(c,\mu), \pi^S(c,\mu))$ *and* $F_2^{S-Col}(S(c,\mu), \pi^{S-Col}(c,\mu)) < F_2^{d-PS}(S(c,\mu), \pi^S(c,\mu))$.

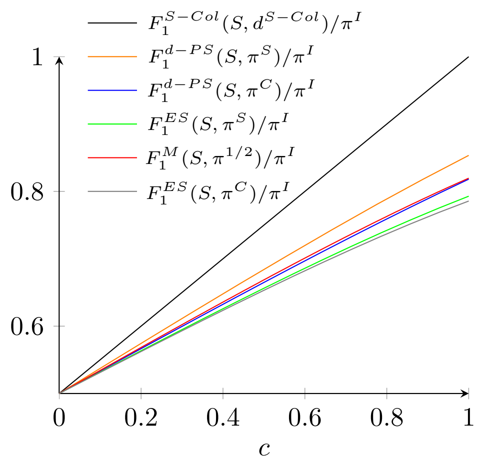
Theorem 7 and Corollary 2 together imply that for all values of $c \in (0,1]$ and $\mu \in [0,1]$, the collusion payoff obtained by the more efficient firm in the cartel when bargaining (with side payments) is not possible is above the highest payoff this firm can expect to gain in any bargaining environment of $\mathcal{E}(c,\mu)$. On the other hand, even the lowest payoff the less efficient firm can expect to obtain in $\mathcal{E}(c,\mu)$ is higher than its collusion payoff in the absence of bargaining. These results allow us to compare for each firm the payoff obtained in the absence of bargaining with the payoff obtained in the bargaining environment $(F^M, \pi^{1/2})$. We find that in this moderating environment (only) the more efficient firm in the cartel would wish that the cartel did not have the bargaining possibility.

**Corollary 3.** *Let* $F^{d-PS}$ *and* $F^{ES}$ *be bargaining solutions on* $\Sigma_+^2$ *that respectively reduce to the d-Proportional Split solution and the Equal Split solution on* $\Sigma_+^{2,\mathcal{D}}$. *Then for any* $c \in (0,1]$ *and* $\mu \in [0,1]$, $F_1^{S-Col}(S(c,\mu), \pi^{S-Col}(c,\mu)) > F_1^M(S(c,\mu), \pi^{1/2}(c,\mu))$ *and* $F_2^{S-Col}(S(c,\mu), \pi^{S-Col}(c,\mu)) < F_2^M(S(c,\mu), \pi^{1/2}(c,\mu))$.

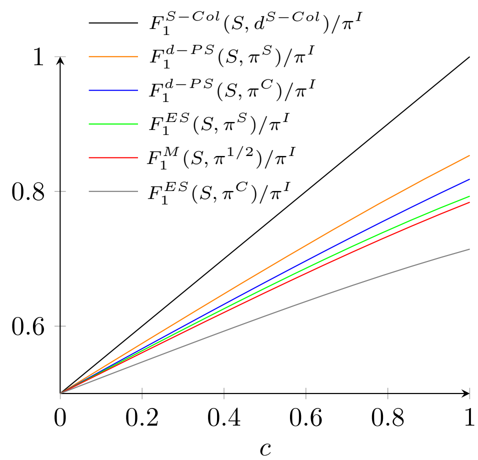
So far, we have considered the rankings of the agreement payoffs in various bargaining environments. One may also wonder how the sizes of these agreement payoffs are affected from the changes in the cost asymmetry and demand uncertainty. To explore this, in Figure 2 we plot the bargaining payoff of the more efficient firm in the cartel as a share of collusion profits under all bargaining environments studied in this paper. The value of $\mu^2$, the variance of the demand uncertainty, is set to 0, 1/3, 2/3, and 1 in panels (i), (ii), (iii), and (iv), respectively. Comparing all panels, we observe that the welfare of the more efficient firm in the cartel (and consequently the welfare of the less efficient firm) is more sensitive to the variations in the bargaining solution and the disagreement point when the demand uncertainty is higher.

Note that in Figure 2 the orange curve represents the bargaining payoff of the more efficient firm –as a share of the collusion profits– in the bargaining environment $(F^{d-PS}, \pi^S(c,\mu))$, the most desirable environment for this firm in $\mathcal{E}(c,\mu)$. The only curve above the orange curve is the black curve which represents the share of collusion profits the more efficient firm in the cartel can secure when bargaining with side payments is not possible. Also note that the profit shares of the more efficient and the less efficient firm always sum up to 1. Therefore, the significantly positive distance between the black and orange curves at medium to large sizes of cost symmetry implies that the possibility of bargaining may increase the welfare of the less efficient firm in the cartel substantially even when it has to bargain in its least desirable environment in $\mathcal{E}(c,\mu)$, namely $(F^{d-PS}, \pi^S(c,\mu))$.

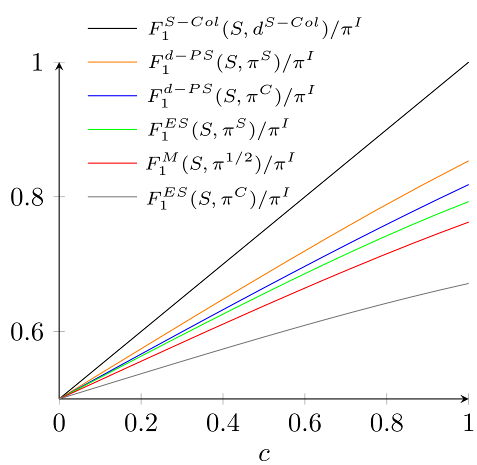
Comparing the four graphs in Figure 2, we also observe that an increase in the level of demand uncertainty increases at all cost levels the distance between the orange and gray lines, widening the size of the payoff conflict between the duopolistic firms whenever they restrict themselves to choose a bargaining environment from the set of alternatives $\mathcal{E}(c,\mu)$. Regarding this conflict, our remedy, suggesting the use of the bargaining environment $(F^M, \pi^{1/2}(c,\mu)$, generates agreement payoffs represented by the red curve. Note that by definition, the payoffs represented by the red curve are obtained by taking the average of the payoffs obtained under $(F^{d-PS}, \pi^S(c,\mu))$ and $(F^{ES}, \pi^C(c))$, that is why the red curve always lies below the orange curve but above the gray curve. In fact, if $\mu^2$ is equal to $1/3$ or higher as in panels (ii)-(iv), the red curve always stays under all other curves except for the gray curve, implying that the bargaining environment we have suggested dominates for the less efficient firm all bargaining environments in the set $\mathcal{E}(c,\mu)$, except for its favorite environment, namely $(F^{ES}, \pi^C(c))$.



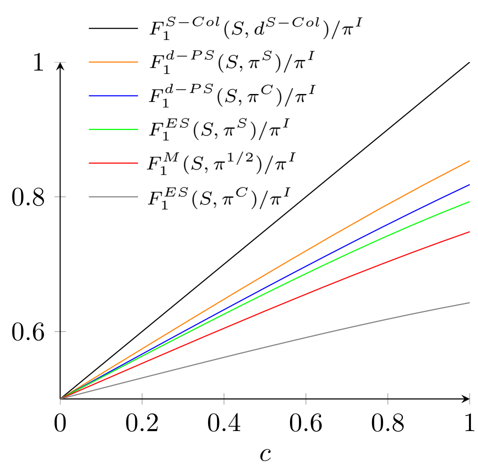

(i) $\mu^2 = 0$




(ii) $\mu^2 = 1/3$




(iii) $\mu^2 = 2/3$




(iv) $\mu^2 = 1$

**Figure 2.** The payoff of firm 1 as a share of collusion profits in various bargaining environments.

## 5 Conclusion

In this paper, we have dealt with the basic problem of a duopolistic cartel regarding how to divide up collusion profits. We have borrowed the structures of our duopolistic industry from Ciarreta and Gutiérrez-Hita's (2012), who studied the formation (and also stability) of duopolistic collusion under cost asymmetry and demand uncertainty. However, differing from Ciarreta and Gutiérrez-Hita's (2012) approach to the problem, we have allowed duopolistic firms to cooperatively bargain with side payments over collusion profits.

Using the bargaining model of Nash (1950), we have identified for our duopolistic cartel, a bargaining set of payoffs, a disagreement point in this set –to be realized only if the bargaining fails– and a solution that selects a point inside the bargaining set possibly taking the disagreement point into consideration. More specifically, we have defined – for each demand and cost realization– all possible divisions of the collusion profits as the bargaining set of the cartel. We have assumed that these collusion profits are always obtained from the joint profit maximization program of the cartel, employing supply functions as production strategies instead of fixed quantities, since the former option yields higher expected profits as long as there is any size of demand uncertainty, as was earlier shown by Ciarreta and Gutiérrez-Hita's (2012).

Given the bargaining set of the cartel, we have set the disagreement point to either the payoff allocation at the (Cournot) Nash equilibrium in quantities, $\pi^C$, or the payoff allocation at the Nash equilibrium in supply functions, $\pi^S$. As for bargaining solutions, we have restricted ourselves mainly to those that split the cartel's total gains from agreement either equally or proportionally with respect to the ratio of the disagreement payoffs. We have called these two classes of solutions, within the domain of duopolistic bargaining problems, the Equal Split solution and the d-Proportional Split solution, respectively.

In our first result, Theorem 1, we have showed that under the possibility of bargaining each firm in the duopolistic industry has incentive to join a collusive agreement if the bargaining solution splits the gains from agreement either equally or d-proportionally and the disagreement point is obtained from the equilibrium of either the supply function competition or the quantity competition. Our result is partially different from the earlier result of Ciarreta and Gutiérrez-Hita (2012) –obtained in the absence of bargaining possibility– showing that when the collusive outcome is allocated between the firms to equalize their marginal costs, the more efficient firm is always willing to collude both under quantity competition and supply function competition, whereas the less efficient firm is willing to collude only if the cost asymmetry is sufficiently small.

Our second result, Theorem 2, compares the disagreement payoffs of each firm at the two disagreement points considered throughout the paper. Basically, this result shows that both firms in the cartel have higher (lower) disagreement payoffs at $\pi^C$ than at $\pi^S$ only if the demand uncertainty is sufficiently low (high). If the demand uncertainty is intermediate, then in terms of the disagreement payoff the more efficient firm may prefer the disagreement point $\pi^S$ to $\pi^C$, while the opposite would be true for the less efficient firm.

Our other findings are related to agreement payoffs. In Theorem 3 we have simply showed that when there is no cost asymmetry, the duopolists always share the collusion profits equally, both when the solution splits the gains from agreement equally and when it splits them $d$-proportionally regardless whether the disagreement point is $\pi^S$ or $\pi^C$. In the subsequent results we have dealt with the case of asymmetric costs. Theorem 4 shows that under any solution that splits the gains from agreement equally the more efficient firm in the cartel always has a higher agreement payoff when the disagreement point is $\pi^S$ than when the disagreement point is $\pi^C$, whereas the opposite is true for the less efficient firm, pointing to a possible conflict of choice between the two firms. We have suggested that this conflict can be moderated if the duopolistic firms set the disagreement point to a weighted average of $\pi^C$ and $\pi^S$. Corollary 1 shows that by using this moderation, instead of setting the disagreement point to either $\pi^C$ or $\pi^S$, it is always possible to reduce the payoff difference of the duopolistic firms.

In Theorem 5, we have extended our result in Theorem 4 to show that the more efficient firm in the cartel prefers the disagreement point $\pi^S$ to $\pi^C$ when the bargaining solution splits the gains from agreement $d$-proportionally, as well. This result holds because the disagreement payoff of the more efficient firm relative to the disagreement payoff of the less efficient firm is always higher at $\pi^S$ than at $\pi^C$, irrespective of the sizes of the demand uncertainty and the cost asymmetry.

In Theorem 6, we have constructed a bridge between the results of Theorems 4 and 5 to show that the more efficient firm prefers a bargaining environment where the disagreement point is $\pi^C$ and the gains from agreement are split d-proportionally to an environment where the disagreement point is $\pi^S$ and the gains from agreement is split equally. We should note that none of the results in Theorems 4, 5, and 6 can be trivially predicted. On the other hand, these three theorems together lead to a predictable conclusion in Corollary 2, implying that the more efficient firm in the cartel always prefers, with respect to the induced agreement payoffs, a solution with d-proportional splitting to a solution with equal splitting regardless whether the disagreement point is obtained from the supply function or the quantity competition.

Finally, in Theorem 7 we have showed that the highest agreement payoff that can be obtained by the more efficient firm in the cartel under any bargaining solution and disagreement point studied in our paper is always below than the share of collusion profits this firm can receive when the collusive agreement equalizes the marginal costs of the duopolistic firms as proposed by Patinkin (1947). Oppositely, the less efficient firm in the cartel always becomes better off when the firms can use their relative bargaining powers to divide up collusion profits as suggested by Bain (1948).

We should note that the bargaining solutions we have considered may be among plausible alternatives for the duopolists. As a matter of fact, the bargaining solutions that split the duopolists' gains from agreement equally may be very rich, also including many well-known solutions for two-person bargaining problems. Besides, the disagreement points we have considered may be among the most natural candidates to represent status quo payoffs. Thus, we believe that the bargaining environments studied in this paper are theoretically sound. On the other hand, we also acknowledge that there may exist other meaningful alternatives, as well. For example, one may include into our set of bargaining environments, environments where the disagreement point is obtained from the equilibrium of Bertrand (1883) competition in prices, the third type of competition studied by Ciarreta and Gutiérrez-Hita (2012) under cost asymmetry and demand uncertainty. However, we are also aware that irrespective from the theoretical completeness or soundness of our model, it is an empirical question whether the duopolists could manage to collude and how they would divide the industry profits in case they collude. For an asymmetric Cournot duopoly these questions were recently studied by Fischer and Normann (2017) in an experimental work, with a focus on the role of explicit communication on collusion. We believe that future research may extend their work to our duopolistic bargaining model where the duopolists are allowed, in the possibility of any disagreement, to compete in either quantities or supply functions.

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# Appendix

**Proof of Lemma 1.** Pick any $c \in [0,1]$ and $\mu \in [0,1]$. First, consider the point $\pi^S(c,\mu)$. We know from equations (8) and (9) that $\pi^S(c,\mu) > 0$. Also, equations (2), (9), and (18) imply that $\pi^I(c,\mu) > \pi_1^S(c,\mu) + \pi_2^S(c,\mu)$ if and only if

$$\frac{1+\mu^2}{5-c^2} > \frac{1+\mu^2}{2}\left(\frac{9-c^2}{v(c)} - 1\right), \tag{37}$$

implying

$$(7-c^2)^2\, v^2(c) > [45 - c^2(14-c^2)]^2. \tag{38}$$

Using equation (8), the above inequality can be reduced to

$$(7-c^2)^2 > 45 - c^2(14-c^2), \tag{39}$$

which can be easily checked to be true. Hence, $\pi^I(c,\mu) > \pi_1^S(c,\mu) + \pi_2^S(c,\mu)$.

Now, consider the point $\pi^C(c)$. Equation (14) implies that $\pi^C(c) > 0$. Also, equations (2), (14), and (18) imply that $\pi^I(c,\mu) > \pi_1^C(c) + \pi_2^C(c)$ if and only if

$$\frac{1+\mu^2}{5-c^2} > \frac{12-c^2}{(8-c^2)^2}, \tag{40}$$

implying

$$(1+\mu^2)(64-16c^2+c^4) > (60-17c^2+c^4), \tag{41}$$

which can be easily checked to be true. Therefore, $\pi^I(c,\mu) > \pi_1^C(c) + \pi_2^C(c)$. Thus, we have proved that for any $d \in \{\pi^S(c,\mu), \pi^C(c)\}$, $0 < d_1 + d_2 < \pi^I(c,\mu)$, implying $d \in S(c,\mu)$ and $d \notin WP(S(c,\mu))$. $\square$

**Proof of Theorem 1.** Let $F$ be any solution on $\Sigma_+^2$ that reduces to either the Equal Split solution or the d-Proportional Split solution on $\Sigma_+^{2,\mathcal{D}}$. Pick any $i \in \{1,2\}$, $c \in [0,1]$, $\mu \in [0,1]$, and $d \in \{\pi^S(c,\mu), \pi^C(\mu)\}$. Then, (25), (26), and (27) imply that

$$F_i(S(c,\mu), d) - d_i = k_i(d)[\pi^I(c,\mu) - d_1 - d_2] \tag{42}$$

for some $k_i(d) \in (0,1)$ such that $k_i(d) = 1/2$ if $F$ reduces to the Equal Split solution on $\Sigma_+^{2,\mathcal{D}}$ and $k_i(d) = \phi_i(d)$ if $F$ reduces to the d-Proportional Split solution on $\Sigma_+^{2,\mathcal{D}}$. By Lemma 1, $d \notin WP(S(c,\mu))$, implying $\pi^I(c,\mu) - d_1 - d_2 > 0$. Then, equation (42) implies $F_i(S(c,\mu),d) - d_i > 0$. $\square$

**Proof of Lemma 2.** Pick any $c \in [0,1]$ and $\mu$ in $[0,1]$. Using (2) and (9) we obtain

$$\pi_1^S(c,\mu) - \pi_2^S(c,\mu) = c\left(\frac{1+\mu^2}{2}\right)\left(1 - \frac{5-c^2}{v(c)}\right). \tag{43}$$

On the other hand, using (2) and (14) we obtain

$$\pi_1^C(c) - \pi_2^C(c) = \frac{c}{8-c^2}. \tag{44}$$

First note that $\pi_1^S(c,\mu) - \pi_2^S(c,\mu) = \pi_1^C(c) - \pi_2^C(c) = 0$ if $c = 0$. This implies $\pi_1^S(c,\mu) - \pi_1^C(c) = \pi_2^S(c,\mu) - \pi_2^C(c)$ if $c = 0$. Now, consider the case where $c \in (0,1]$. Considering (43) and (44) when $\mu = 0$, we observe that $\pi_1^S(c,0) - \pi_2^S(c,0) > \pi_1^C(c) - \pi_2^C(c)$ if and only if

$$\frac{1}{2}\left[1 - \frac{5-c^2}{v(c)}\right] > \frac{1}{8-c^2}, \tag{45}$$

implying

$$v(c) > \frac{(5-c^2)(8-c^2)}{(6-c^2)}. \tag{46}$$

Inserting (8) into the above inequality and taking the square of both sides yield

$$45 - c^2(1-c^2) > \frac{(5-c^2)^2(8-c^2)^2}{(6-c^2)^2}, \tag{47}$$

which can be easily checked to be true. Moreover, since $\pi_1^S(c,\mu) - \pi_2^S(c,\mu)$ is increasing in $\mu$, we have $\pi_1^S(c,\mu) - \pi_2^S(c,\mu) > \pi_1^S(c,0) - \pi_2^S(c,0)$ for any $\mu \in (0,1]$. This implies that for any $\mu \in [0,1]$, we have $\pi_1^S(c,\mu) - \pi_2^S(c,\mu) > \pi_1^C(c) - \pi_2^C(c)$, further implying $\pi_1^S(c,\mu) - \pi_1^C(c) > \pi_2^S(c,\mu) - \pi_2^C(c)$, completing the proof. $\square$

**Proof of Lemma 3.** We can check that the difference $\pi_1^S(c,0) - \pi_1^C(c)$ reaches its minimal value $(-0.00833)$ at $c = 0$, its maximal value $(-0.00690)$ at $c = 1$, and it is increasing everywhere on $[0,1]$. On the other, the difference $\pi_2^S(c,0) - \pi_2^C(c)$ reaches its maximal value ($-0.00833$) at $c = 0$, its minimal value (-0.01049) at $c = 1$, and it is decreasing everywhere on $[0,1]$. So, for any $i \in \{1,2\}$ and $c \in [0,1]$, we have $\pi_i^S(c,0) - \pi_i^C(c) < 0$, implying $\mu_i(c) \neq 0$.

Now, we check that the difference $\pi_1^S(c,1) - \pi_1^C(c)$ reaches its minimal value (0.07707) at $c = 0$, its maximal value (0.16988) at $c = 1$, and it is increasing everywhere on $[0,1]$. On the other, the difference $\pi_2^S(c,1) - \pi_2^C(c)$ reaches its maximal value (0.07707) at $c = 0$, its minimal value (0.01984) at $c = 1$, and it is decreasing everywhere on $[0,1]$. So, for any $i \in \{1,2\}$ and $c \in [0,1]$, we have $\pi_i^S(c,1) - \pi_i^C(c) > 0$, implying $\mu_i(c) \neq 1$.

So far, we have showed that $\mu_i(c) \neq 0$, $\mu_i(c) \neq 1$, $\pi_i^S(c,0) - \pi_i^C(c) < 0$, and $\pi_i^S(c,1) - \pi_i^C(c) > 0$. Since for any $i \in \{1,2\}$ and $c \in [0,1]$, $\pi_i^S(c,\mu) - \pi_i^C(c)$ is continuous in $\mu$, there must exist some $\mu_i(c) \in (0,1)$ such that $\pi_i^S(c,\mu_i(c)) - \pi_i^C(c) = 0$. Above, we have also found that $\pi_1^S(c,0) - \pi_1^C(c)$ is always increasing in $c$ and $\pi_2^S(c,1) - \pi_2^C(c)$ is always decreasing in $c$. Since for any $i \in \{1,2\}$, $\pi_i^S(c,\mu) - \pi_i^C(c)$ is always increasing in $\mu$, it must be true for any $\mu \in [0,1]$ that $\pi_1^S(c,\mu) - \pi_1^C(c)$ is always increasing in $c$ and $\pi_2^S(c,\mu) - \pi_2^C(c)$ is always decreasing in $c$. These results imply that $\mu_i(c)$ is unique for each $i \in \{1,2\}$ and also that $\mu_1(c)$ is decreasing whereas $\mu_2(c)$ is increasing.

Finally, to prove the assertions regarding the comparison of $\mu_1(c)$ and $\mu_2(c)$, consider first $c = 0$. Equation (28) implies $\pi_1^S(0,\mu_1(0)) - \pi_1^C(0) = 0$ and $\pi_2^S(0,\mu_2(0)) - \pi_2^C(0) = 0$, further implying $\pi_1^S(0,\mu_1(0)) - \pi_2^S(0,\mu_2(0)) = \pi_1^C(0) - \pi_2^C(0)$. On the other hand, equation (44) implies $\pi_1^C(0) - \pi_2^C(0) = 0$. So, we must have $\pi_1^S(0,\mu_1(0)) - \pi_2^S(0,\mu_2(0)) = 0$, as well. We can rewrite this last equality as $\pi_1^S(0,\mu_1(0)) - \pi_2^S(0,\mu_1(0)) + \pi_2^S(0,\mu_1(0)) - \pi_2^S(0,\mu_2(0)) = 0$. We know from equation (43) that $\pi_1^S(0,\mu_1(0)) - \pi_2^S(0,\mu_1(0)) = 0$. Inserting this into the previous equality yields $\pi_2^S(0,\mu_1(0)) - \pi_2^S(0,\mu_2(0)) = 0$. Since equations (8) and (9) imply that $\pi_2^S(0,\mu)$ is increasing in $\mu$, the last equality above implies $\mu_1(0) = \mu_2(0)$. Now, let $c \in (0,1]$. Lemma 2 implies $\pi_1^S(c,\mu) - \pi_1^C(c) > \pi_2^S(c,\mu) - \pi_2^C(c)$ for any $\mu \in [0,1]$. So, we must have $\pi_1^S(c,\mu_2(c)) - \pi_1^C(c) > \pi_2^S(c,\mu_2(c)) - \pi_2^C(c)$

since $\mu_2(c) \in (0,1)$. Above, we can replace the difference $\pi_2^S(c, \mu_2(c)) - \pi_2^C(c)$ with $\pi_1^S(c, \mu_1(c)) - \pi_1^C(c)$ since both differences are zero by equation (28). This would yield $\pi_1^S(c, \mu_2(c)) - \pi_1^C(c) > \pi_1^S(c, \mu_1(c)) - \pi_1^C(c)$, implying $\mu_2(c) > \mu_1(c)$ since $\pi_1^S(c, \mu)$ is always increasing in $\mu$. This completes the proof. $\square$

**Proof of Theorem 2.** Pick any $i \in \{1,2\}$ and $c \in [0,1]$. By Lemma 3, we know that the value $\mu_i(c)$ is unique. Moreover, we know from (8), (9), and (14) that $\pi_i^S(c, \mu) - \pi_i^C(c)$ is increasing in $\mu$. Given equation (28), this implies that

$$\pi_i^S(c, \mu) - \pi_i^C(c) > 0 \text{ if and only if } \mu > \mu_i(c). \tag{48}$$

By Lemma 3, we also know that $\mu_1(0) = \mu_2(0)$ and $\mu_1(c) < \mu_2(c)$ for any $c \in (0,1]$, implying $\mu_1(c) \leq \mu_2(c)$ for any $c \in [0,1]$. All of these facts imply that (29) is true. $\square$

**Proof of Theorem 3.** Let $F$ be any solution on $\Sigma_+^2$ that reduces to either the Equal Split solution or the d-Proportional Split solution on $\Sigma_+^{2,\mathcal{D}}$. Pick any $\mu \in [0,1]$ and any $d \in \{\pi^S(0,\mu), \pi^C(0)\}$. Equations (8), (9), and (14) imply $d_1 + d_2 > 0$ and $d_1 = d_2$. On the other hand, Lemma 1 implies $d_1 + d_2 < \pi^I(0,\mu)$. If $F$ reduces to the Equal Split solution on $\Sigma_+^{2,\mathcal{D}}$, then for each $i \in \{1,2\}$, equation (25) implies that

$$F_i(S(0,\mu), d) = \frac{\pi^I(0,\mu)}{2} + \frac{d_1 - d_2}{2} = \frac{\pi^I(0,\mu)}{2}. \tag{49}$$

On the other hand, if $F$ reduces to the d-Proportional Split solution on $\Sigma_+^{2,\mathcal{D}}$, then for each $i \in \{1,2\}$, (26) and (27) imply that

$$F_i(S(0,\mu), d) = \left(\frac{d_1}{d_1 + d_2}\right) \pi^I(0,\mu) = \frac{\pi^I(0,\mu)}{2}. \tag{50}$$

This completes the proof. $\square$

**Proof of Lemma 4.** Let $F$ be any solution on $\Sigma_+^2$ that reduces to either the Equal Split solution or the d-Proportional Split solution on $\Sigma_+^{2,\mathcal{D}}$. Then, equations (25), (26), and (27) imply that for any $c \in (0,1]$, $\mu \in [0,1]$, and $d, d' \in$

$S(c,\mu)$, we have $F_1(S(c,\mu),d) + F_2(S(c,\mu),d) = \pi^I(c,\mu)$ and $F_1(S(c,\mu),d') + F_2(S(c,\mu),d') = \pi^I(c,\mu)$. Thus,

$$F_1(S(c,\mu),d) - F_1(S(c,\mu),d') = -\left[F_2(S(c,\mu),d) - F_2(S(c,\mu),d')\right], \quad (51)$$

implying that $F_i(S(c,\mu),d) > F_i(S(c,\mu),d')$ if and only if $F_j(S(c,\mu),d) < F_j(S(c,\mu),d')$ for any $i,j \in \{1,2\}$ with $j \neq i$. □

**Proof of Lemma 5.** Let $F$ be any solution on $\Sigma_+^2$ that reduces to the Equal Split solution on $\Sigma_+^{2,\mathcal{D}}$. Pick any $c \in (0,1]$, $\mu \in [0,1]$, $d,d' \in S(c,\mu)$, and $i,j \in \{1,2\}$ with $j \neq i$. Equation (25) implies that

$$F_i(S(c,\mu),d) = \frac{\pi^I(c,\mu)}{2} + \frac{d_i - d_j}{2} \quad (52)$$

and

$$F_i(S(c,\mu),d') = \frac{\pi^I(c,\mu)}{2} + \frac{d_i' - d_j'}{2}. \quad (53)$$

Then, (52) and (53) imply

$$F_i(S(c,\mu),d) - F_i(S(c,\mu),d') = \frac{(d_i - d_j)}{2} - \frac{(d_i' - d_j')}{2}. \quad (54)$$

So, $F_i(S(c,\mu),d) > F_i(S(c,\mu),d')$ if and only if $d_i - d_j > d_i' - d_j'$. □

**Proof of Theorem 4.** Let $F$ be any solution on $\Sigma_+^2$ that reduces to the Equal Split solution on $\Sigma_+^{2,\mathcal{D}}$. Pick any $c \in (0,1]$ and $\mu \in [0,1]$. Lemma 2 implies that $\pi_1^S(c,\mu) - \pi_2^S(c,\mu) > \pi_1^C(c) - \pi_2^C(c)$. Then, Lemma 5 implies $F_1(S(c,\mu),\pi^S(c,\mu)) > F_1(S(c,\mu),\pi^C(c))$. By Lemma 4, we also have $F_2(S(c,\mu),\pi^S(c,\mu)) < F_2(S(c,\mu),\pi^C(c))$. □

**Proof of Corollary 1.** Let $F$ be any solution on $\Sigma_+^2$ that reduces to the Equal Split solution on $\Sigma_+^{2,\mathcal{D}}$. Pick any $c \in (0,1]$, $\mu \in [0,1]$, and $\omega \in (0,1)$. Equation (25) implies that

$$F_1(S(c,\mu),\pi^\omega(c,\mu)) = \frac{\pi^I(c,\mu)}{2} + \frac{\pi_1^\omega(c,\mu) - \pi_2^\omega(c,\mu)}{2}$$

$$
\begin{aligned}
&= \frac{\pi^I(c,\mu)}{2} + \left(\frac{\omega\pi_1^S(c,\mu) + (1-\omega)\pi_1^C(c)}{2}\right) - \\
&\quad \left(\frac{\omega\pi_2^S(c,\mu) + (1-\omega)\pi_2^C(c)}{2}\right) \\
&= \omega\left(\frac{\pi^I(c,\mu)}{2} + \frac{\pi_1^S(c,\mu) - \pi_1^S(c,\mu)}{2}\right) + \\
&\quad (1-\omega)\left(\frac{\pi^I(c,\mu)}{2} + \frac{\pi_1^C(c) - \pi_2^C(c)}{2}\right) \\
&= \omega F_1(S(c,\mu), \pi^S(c,\mu)) + (1-\omega) F_1(S(c,\mu), \pi^C(c)). \qquad (55)
\end{aligned}
$$

Since we have $F_1(S(c,\mu), \pi^S(c,\mu)) > F_1(S(c,\mu), \pi^C(c))$ by Theorem 4, equation (55) and the assumption $\omega \in (0,1)$ imply that $F_1(S(c,\mu), \pi^S(c,\mu)) > F_1(S(c,\mu), \pi^\omega(c,\mu)) > F_1(S(c,\mu), \pi^C(c))$. Then, Lemma 4 implies $F_2(S(c,\mu), \pi^S(c,\mu)) < F_2(S(c,\mu), \pi^\omega(c,\mu)) < F_2(S(c,\mu), \pi^C(c))$. □

**Proof of Theorem 5.** Let $F$ be any solution on $\Sigma_+^2$ that reduces to the d-Proportional Split solution on $\Sigma_+^{2,\mathcal{D}}$. Pick any $c \in (0,1]$ and $\mu \in [0,1]$. Equations (8) and (9) imply $\pi_1^S(c,\mu) + \pi_2^S(c,\mu) > 0$ whereas equation (14) implies $\pi_1^C(c) + \pi_2^C(c) > 0$. On the other hand, Lemma 1 implies that $\pi_1^S(c,\mu) + \pi_2^S(c,\mu) < \pi^I(c,\mu)$ and $\pi_1^C(c) + \pi_2^C(c) < \pi^I(c,\mu)$. Then, it follows from equations (26) and (27) that

$$F_1(S(c,\mu), \pi^S(c,\mu)) = \left(\frac{\pi_1^S(c,\mu)}{\pi_1^S(c,\mu) + \pi_2^S(c,\mu)}\right) \pi^I(c,\mu) \qquad (56)$$

and

$$F_1(S(c,\mu), \pi^C(c)) = \left(\frac{\pi_1^C(c)}{\pi_1^C(c) + \pi_2^C(c)}\right) \pi^I(c,\mu). \qquad (57)$$

So, $F_1(S(c,\mu), \pi^S(c,\mu)) > F_1(S(c,\mu), \pi^C(c))$ if and only if $\pi_1^S(c,\mu)/\pi_2^S(c,\mu) > \pi_1^C(c)/\pi_2^C(c)$, implying

$$\frac{(9-c^2) - c(5-c^2) - (1-c)v(c)}{(9-c^2) + c(5-c^2) - (1+c)v(c)} > \frac{(3-c)(2+c)^2}{(3+c)(2-c)^2}, \qquad (58)$$

using (9) and (14). After some algebra, one can show that the last inequality holds if and only if

$$v(c)(10 - c^2)^2 - (66 - 17c^2 + c^4) > 0. \tag{59}$$

Using (8), the left hand side of the above inequality can be reduced to $144 - 56c^2 + 4c^4$, which is always positive since $c \in (0, 1]$. This proves that $F_1(S(c, \mu), \pi^S(c, \mu)) > F_1(S(c, \mu), \pi^C(c))$. Then, Lemma 4 implies that $F_2(S(c, \mu), \pi^S(c, \mu)) < F_2(S(c, \mu), \pi^C(c))$. $\square$

**Proof of Theorem 6.** Let $F^{d-PS}$ and $F^{ES}$ be bargaining solutions on $\Sigma_+^2$ that respectively reduce to the d-Proportional Split solution and the Equal Split solution on $\Sigma_+^{2,\mathcal{D}}$. Pick any $c \in (0, 1]$ and $\mu \in [0, 1]$. Equation (14) implies $\pi_1^C(c) + \pi_2^C(c) > 0$, while Lemma 1 implies $\pi_1^C(c) + \pi_2^C(c) < \pi^I(c, \mu)$. Then, it follows from (26) and (27) that

$$F_1^{d-PS}(S(c, \mu), \pi^C(c)) = \left(\frac{\pi_1^C(c)}{\pi_1^C(c) + \pi_2^C(c)}\right)\pi^I(c, \mu). \tag{60}$$

On the other hand, (25) implies

$$F_1^{ES}(S(c, \mu), \pi^S(c, \mu)) = \frac{\pi^I(c, \mu)}{2} + \frac{\pi_1^S(c, \mu) - \pi_2^S(c, \mu)}{2}. \tag{61}$$

So, $F_1^{d-PS}(S(c, \mu), \pi^C(c)) > F_1^{ES}(S(c, \mu), \pi^S(c, \mu))$ if and only if

$$2\left(\frac{\pi_1^C(c)}{\pi_1^C(c) + \pi_2^C(c)} - \frac{1}{2}\right) > \frac{\pi_1^S(c, \mu) - \pi_2^S(c, \mu)}{\pi^I(c, \mu)}. \tag{62}$$

We calculate

$$\frac{\pi_1^S(c, \mu) - \pi_2^S(c, \mu)}{\pi^I(c, \mu)} = c\left[\frac{5 - c^2}{2}\right]\left[1 - \frac{(5 - c^2)}{v(c)}\right] \tag{63}$$

using (18) and (43). On the other hand, we obtain

$$\frac{\pi_1^C(c)}{\pi_1^C(c) + \pi_2^C(c)} = \frac{(3 - c)(2 + c)^2}{24 - 2c^2}, \tag{64}$$

using (2) and (14). Hence, we have

$$2\left(\frac{\pi_1^C(c)}{\pi_1^C(c) + \pi_2^C(c)} - \frac{1}{2}\right) = \frac{(3 - c)(2 + c)^2}{12 - c^2} - 1 = \frac{c(8 - c^2)}{12 - c^2}. \tag{65}$$

Then, it is easy to check that

$$\frac{c(8-c^2)}{12-c^2} > c\left[\frac{5-c^2}{2}\right]\left[1-\frac{(5-c^2)}{v(c)}\right] \tag{66}$$

for any $c \in (0,1]$. This implies $F_1^{d-PS}(S(c,\mu), \pi^C(c)) > F_1^{ES}(S(c,\mu), \pi^S(c,\mu))$. Then, Lemma 4 implies that $F_2^{d-PS}(S(c,\mu), \pi^C(c)) < F_2^{ES}(S(c,\mu), \pi^S(c,\mu))$. □

**Proof of Corollary 2.** Let $F^{d-PS}$ and $F^{ES}$ be bargaining solutions on $\Sigma_+^2$ that respectively reduce to the d-Proportional Split solution and the Equal Split solution on $\Sigma_+^{2,\mathcal{D}}$. Pick any $c \in (0,1]$ and $\mu \in [0,1]$. Then, we have

$$F_1^{d-PS}(S(c,\mu), \pi^S(c,\mu)) > F_1^{d-PS}(S(c,\mu), \pi^C(c)) \quad \text{by Theorem 5,}$$

$$F_1^{d-PS}(S(c,\mu), \pi^C(c)) > F_1^{ES}(S(c,\mu), \pi^S(c,\mu)) \quad \text{by Theorem 6,}$$

$$F_1^{ES}(S(c,\mu), \pi^S(c,\mu)) > F_1^{ES}(S(c,\mu), \pi^C(c)) \quad \text{by Theorem 4.}$$

The three inequalities together imply for firm 1 the payoff comparisons in the corollary. Then, Lemma 4 implies the payoff comparisons for firm 2. □

**Proof of Theorem 7.** Let $F^{d-PS}$ and $F^{ES}$ be bargaining solutions on $\Sigma_+^2$ that respectively reduce to the d-Proportional Split solution and the Equal Split solution on $\Sigma_+^{2,\mathcal{D}}$. Pick any $c \in (0,1]$ and $\mu \in [0,1]$. Note that (8) and (9) imply $\pi_1^S(c,\mu) + \pi_2^S(c,\mu) > 0$ whereas Lemma 1 implies $\pi_1^S(c,\mu) + \pi_2^S(c,\mu) < \pi^I(c,\mu)$. Then, using (2), (19), and (36) together with (26) and (27) we observe that $F_1^{S-Col}(S(c,\mu), \pi^{S-Col}(c,\mu)) > F_1^{d-PS}(S(c,\mu), \pi^S(c,\mu))$ if and only if

$$\left(\frac{1+c}{2}\right)\pi^I(c,\mu) > \left(\frac{\pi_1^S(c,\mu)}{\pi_1^S(c,\mu)+\pi_2^S(c,\mu)}\right)\pi^I(c,\mu), \tag{67}$$

implying

$$\frac{1+c}{1-c} > \frac{\pi_1^S(c,\mu)}{\pi_2^S(c,\mu)}. \tag{68}$$

Given equation (9), the inequality in (68) becomes

$$\frac{1+c}{1-c} > \frac{(9-c^2)-c(5-c^2)-(1-c)v(c)}{(9-c^2)+c(5-c^2)-(1+c)v(c)}. \tag{69}$$

After some algebra, we can show that the above inequality holds if and only if

$$(14 - 2c^2)^2 - 4[v(c)]^2 > 0. \tag{70}$$

Using (8), the left hand side of the above inequality can be reduced to

$$(196 - 56c^2 + 4c^4) - (180 - 56c^2 + 4c^4) = 16, \tag{71}$$

which is greater than zero. Thus, we have $F_1^{S-Col}(S(c,\mu), \pi^{S-Col}(c,\mu)) > F_1^{d-PS}(S(c,\mu), \pi^S(c,\mu))$. Then, Lemma 4 implies $F_2^{S-Col}(S(c,\mu), \pi^{S-Col}(c,\mu)) < F_2^{d-PS}(S(c,\mu), \pi^S(c,\mu))$. $\square$

**Proof of Corollary 3.** Let $F^{d-PS}$ and $F^{ES}$ be bargaining solutions on $\Sigma_+^2$ that respectively reduce to the d-Proportional Split solution and the Equal Split solution on $\Sigma_+^{2,\mathcal{D}}$. Pick any $c \in (0,1]$ and $\mu \in [0,1]$. Theorem 7 implies that $F_1^{S-Col}(S(c,\mu), \pi^{S-Col}(c,\mu)) > F_1^{d-PS}(S(c,\mu), \pi^S(c,\mu))$. On the other hand, Corollary 2 and equation (33) imply that $F_1^{d-PS}(S(c,\mu), \pi^S(c,\mu)) > F_1^M(S(c,\mu), \pi^{1/2}(c,\mu))$. Therefore, $F_1^{S-Col}(S(c,\mu), \pi^{S-Col}(c,\mu)) > F_1^M(S(c,\mu), \pi^{1/2}(c,\mu))$. Then, Lemma 4 implies $F_2^{S-Col}(S(c,\mu), \pi^{S-Col}(c,\mu)) < F_2^M(S(c,\mu), \pi^{1/2}(c,\mu))$. $\square$