Opening Hours, Store Quality, and Social Welfare

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Abstract

This paper examines opening hours, store quality, and price decisions by retailers. We consider a scenario in which the investment in store quality is more costly for longer opening hours. This scenario is suitable for a case where a retailer invests in the brand proliferation in order to attract consumers. This is because longer opening hours cause additional wages and administrative costs in order to handle the brand proliferation. We show that a retailer with shorter opening hours chooses higher brand proliferation and charges lower prices. We also examine the impact of deregulated opening hours on social welfare. We find that the liberalization on opening hours is desirable in view of social welfare.

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Keywords: Duopoly; Opening hours; Multi-dimensional product differentiation; Social welfare

[*] All remaining errors are my own. Comments welcome.

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1 Introduction

This paper focuses on the deregulation of opening hours in the retail industry. In Japan, the complete liberalization of opening hours has advanced since 2000, by the current law (*Daiten-Ricchi-Ho*).[1] Many Japanese retailers open for longer (See Table 1).[2] The previous law (*Daiten-Ho*) regulated opening hours of large-scale retailers such as department stores and major supermarkets, and provided a mechanism for adjusting the interests between large-scale retailers, small-scale retailers, and medium sized retailers.[3] However, the current law does not focus on adjusting the interests between such retailers. How does the liberalization of opening hours in the retail industry affects competition between retailers? Can such liberalization improve social welfare? Although the deregulation of opening hours is a trend, not only in Japan, but in many countries, these are debatable issues.[4]

Category	# stores	less than 12 hours	12 to 14 hours	14 to 24 hours	24 hours
General merchandise store	1413	339	422	597	55
Grocery store	14768	5994	5086	2373	1315
Convenience store	35096	-	-	4852	30244

Table 1: Liberalized opening hours in Japanese retail sector (in 2014)

To address these issues, we provide a symmetric duopoly model based on that of Inderst and Irmen (2005). In contrast to their model, we assume that retailers invest in store quality, and incur the cost of investment in that. We consider two scenarios on the cost of investment in store quality: (1) the cost of investment in store quality is independent of opening hours; and (2) the cost of investment in store quality is more costly for longer opening hours. The first scenario captures a case where retailers invest in quality of goods, such as private-label products, in order to attract consumers.[5] This is because the cost of investment in quality of goods is independent of opening hours. The second scenario captures a case where retailers invest in the brand proliferation in order to attract consumers. This is because longer opening hours cause additional wages and administrative costs in order to handle the brand proliferation. To examine whether the deregulation of opening hours can improve social welfare, we also compare the outcome under the deregulated opening hours with that under the regulated opening hours.

The basic structure of the model is as follows. There are two dimensions of product differentiation: the first dimension represents location and the second represents time. To represent the first dimension, we assume a Hotelling line, and consider two symmetric retailers at both ends of the unit interval. Consumers are distributed uniformly on the Hotelling line. To represent the second dimension, we assume a time line, and consider that the two retailers choose

[1]The purpose of this law is to foster the development of the retailing sector as a whole, and thus, the development of the national economy and regional society and the elevation of the national standard of living.

[2]Table 1 is based on the report released by *Ministry of Economy, Trade, and Industry* on March 9, 2016.

[3]Large-scale retail stores are classified by retail floor space into the following two types: Class 1 large-scale retail stores; retail floor space $3,000m^2$ or more (23 special wards in Tokyo and 12 ordinance-designated cities are $6,000m^2$ or more); Class 2 large-scale retail stores: retail floor space greater than $500m^2$ and less than $3,000m^2$ (23 Special Wards in Tokyo and 12 Ordinance-designated cities are more than $500m^2$ and less than $6,000m^2$).

[4]In Norway and France, strict restrictions on opening hours are imposed on retailers. On the other hand, in the UK, Sweden, and Germany, opening hours have become liberalized in recent years.

[5]Japanese major convenience stores, such as 7-Eleven, promote the investment in quality of private-label products.

their closing time on the time line. Consumers with time constraints are uniformly located on the time line, and they cannot buy beyond closing time of the two retailers. On the other hand, consumers without time constraints are located at the initial point of the time line, and they can buy at any time and compare the two retailers. Each consumer can buy one or zero unit of the product from a retailer that maximizes his/her utility. Also, they charge a single price, regardless of their opening hours. In these situations, the game runs as follows. In the first stage, retailers decide their opening hours and store quality. In the second stage, retailers compete in prices.

Within the above framework, we find that retailers' behavior depends on the cost structure of investment in store quality. In the first scenario, the retailer with longer opening hours chooses higher-quality goods and charges higher prices. In the second scenario, the retailer with shorter opening hours chooses higher brand proliferation level and charges lower prices. In both the first and second scenarios, the liberalization of opening hours is desirable for social welfare.

As shown by Inderst and Irmen (2005), the deregulation of opening hours may decrease social welfare if longer opening hours cause additional costs. However, taking the cost of investment in the brand proliferation into account, the result is opposite: the deregulation of opening hours is desirable for social welfare, even if longer opening hours cause additional costs.

Several theoretical literatures highlight the effects of the deregulated opening hours on retail prices (Morrison and Newman, 1983; Clemenz, 1990; Tanguay et al., 1995).[6] However, these papers do not consider opening hours as a strategic variable among retail stores. Since 2000, several papers have endogenized the opening hours of retailers in oligopoly models (Inderst and Irmen, 2005; Shy and Stenbacka, 2008).[7] Extending Inderst and Irmen (2005), we consider the investment in store quality to attract consumers, which have not been considered in previous related papers.

The remainder of the paper is organized as follows. Section 2 presents the basic model. Section 3 presents the results in equilibrium. Section 4 shows the results on social welfare. Section 5 discusses the extended model. Section 6 discusses the findings, and Section 7 concludes the paper.

2 The basic model

The model presented here is based on that of Inderst and Irmen (2005).[8] We consider two dimensions of product differentiation: the first dimension represents location and the second represents time.

To represent the first dimension, we consider a continuum of consumers to be distributed uniformly on a Hotelling line segment $[0, 1]$ with mass 1. The location of consumer $x \in [0, 1]$ is associated with his/her preferences. There are two symmetric competing retailers in this market. Let x_i $(i \in 1, 2)$ be the location of firm i. The retailers are located

[6] Empilical literatures also examine the effect of deregulated opening hours on retail prices. Kay and Morris (1987) perform simulation by using UK data, and show that retailers charge lower prices under the deregulation of opening hours. On the other hand, Tanguay et al., (1995) show that in Quebec, Canada, the deregulated opening hours lead to higher retail prices in large stores.

[7] The related models are extended by taking into account heterogeneous retailers (Wenzel, 2011) and customer loyalty for retailers (Wenzel, 2016). Wenzel (2011) shows that the deregulation of opening hours can improve social welfare.

[8] We simplify the setting of disutility of consumers in the time dimension, but allow for a continuous choice of opening hours. The main insights in Inderst and Irmen (2005) remain valid in this continuous choice setting.

at either end of the unit interval. Retailer 1 is located at 0 and retailer 2 is located at 1. A consumer living at $x \in [0, 1]$ incurs a transportation cost of $t(x - x_i)^2$ when purchasing products from retailer i, where t is a positive constant.

To represent the second dimension, we assume that the length of the time line is 1. We denote by y the point of consumers on the time line located at a distance from 0. Let y_i $(i \in 1, 2)$ be the location of retailer i. In this model, y_i represents the closing time of retailer i. For instance, if all retailers stay open until $y_i = 5/6$, then consumers on the line from $5/6$ to 1 cannot buy any goods. Each retailer chooses its closing time, y_i, given the impact on another retailer.

We assume the existence of two types of consumers. Type 1 consumers can buy at any time; that is, they can adjust their shopping hours. In contrast, type 2 consumers face time constraints and cannot buy beyond closing time; that is, they cannot adjust their shopping hours. To model time constraints, we use a linear city that is similar to that in Cancian et al. (1995), which uses a duopoly setting Hotelling model with a directional constraint.[9] We suppose that type 2 consumers are uniformly located on the line and that type 1 consumers are located at $y = 0$. Let $K \in [1/2, 1)$ be the mass of type 1 consumers and $1 - K$ be that of type 2 consumers. The bold line in Fig. 1 shows the mass of type 1 consumers, K (see Fig. 1).

Consumers have a conditional indirect utility function $V_i(x, y), i = 1, 2$. If a consumer buys from retailer i, his/her utility is equal to

$$V_i(x, y) = \begin{cases} S + q_i - p_i - t(x - x_i)^2 & \text{if } y \leq y_i, \\ 0 & \text{if } y > y_i, \end{cases}$$

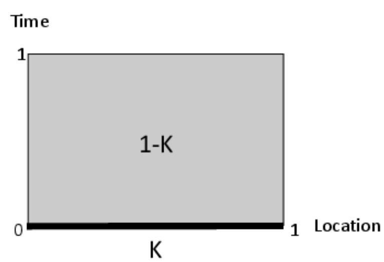

Figure 1: The horizontal axis represents location and the vertical axis represents time.

where S denotes the gross surplus consumers enjoy from a retailer, q_i denotes store quality of retailer i, and p_i denotes price charged by retailer i. We assume $S > 2t$, which means a retailer with longer opening hours does not exclude type 1 consumers by setting a higher price. Consumers have unit demands, i.e., each consumes one or zero unit of the product. Each consumer buys a product from a retailer that maximizes his/her indirect utility.

Each retailer is constrained to charge a single price, regardless of its shopping time.[10] Each retailer incurs the cost of investment in store quality q_i. We consider two scenarios on the cost of investment in store quality: it is independent of opening hours (scenario 1); and it is positively correlated with opening hours (scenario 2). In the first scenario, the

[9]To consider a TV schedule problem, they consider a duopoly setting Hotelling model with a directional constraint. In their model, a consumer can move in only one direction on the Hotelling line. The directional constraint is represented by time. For instance, if a consumer locates at 8 p.m., he/she can watch a TV program after 8 p.m., but cannot watch it before 8 p.m.

[10]As mentioned in Inderst and Irmen (2005), high menu costs may justify this assumption.

cost of investment in store quality of retailer i is given by

$$I(q_i) = r{q_i}^2 \quad i = 1, 2, \tag{1}$$

where r $(> \frac{1}{6t})$ represents the parameter of the cost of investment in store quality. Eq.(1) indicates that the cost of investment in store quality is independent of opening hours. The cost structure such as Eq.(1) is suitable for a case where retailers invest in quality of goods such as private-label products in order to attract consumers. This is because the cost of investment in quality of goods is independent of opening hours. In the second scenario, the cost of investment in store quality of retailer i is given by

$$I(q_i) = (K + (1 - K)y_i)r{q_i}^2 \quad i = 1, 2, \tag{2}$$

where r $(> \frac{1}{6t})$ represents the parameter of the cost of investment in store quality. Eq.(2) indicates that the investment in store quality is more costly for longer opening hours. Retailers incur the cost to prepare for potential demands of retailer i, $K+(1-K)y_i$, and thus, this cost increases according to opening hours, y_i. The cost structure such as Eq.(2) is suitable for a case where retailers invest in the brand proliferation in order to attract consumers. This is because longer opening hours cause additional wages and administrative costs to handle the brand proliferation.

The game runs as follows. In the first stage, retailers determine their location in time y_i and choose their store quality, q_i, simultaneously. In the second stage, after observing retailers' location in time and their choices of store quality, each retailer competes in price p_i.

3 Equilibrium

To derive equilibrium outcomes in scenarios 1 and 2, without loss of generality, we suppose that the opening hour of retailer 1 is larger than or equal to that of retailer 2; that is, $0 \leq y_2 \leq y_1 \leq 1$. Three categories of consumers exist: (i) consumers who are able to buy from retailers 1 and 2 (consumer $y \in [0, y_2]$); (ii) consumers who are able to buy only from retailer 1 (consumer $y \in (y_2, y_1]$); and (iii) consumers who cannot buy any good (consumer $y \in (y_1, 1]$).

The mass of category (i) is $K + (1 - K)y_2$, that of category (ii) is $(1 - K)(y_1 - y_2)$, and that of category (iii) is $(1 - K)(1 - y_1)$. Given p_i and q_i $(i = 1, 2)$, the indifferent consumer in category 1 is

$$\bar{x} = \frac{p_2 - p_1}{2t} + \frac{q_1 - q_2}{2t} + \frac{1}{2}. \tag{3}$$

The demands of retailers 1 and 2 are, respectively,

$$D_1 = (K + (1 - K)y_2)\bar{x} + (1 - K)(y_1 - y_2), \quad D_2 = (K + (1 - K)y_2)(1 - \bar{x}).$$

The profit functions of retailers 1 and 2 are, respectively,

$$\pi_1 = p_1[(K + (1 - K)y_2)\bar{x} + (1 - K)(y_1 - y_2)] - I(q_1), \tag{4}$$

$$\pi_2 = p_2[(K + (1 - K)y_2)(1 - \bar{x})] - I(q_2), \tag{5}$$

where $\bar{x}$ is given by Eq.(3).

3.1 *Scenario 1:* The cost of investment in store quality is independent of opening hours

First, we consider scenario 1 where the cost of investment in store quality is given by Eq. (1). We assume $rt > \frac{1}{6}$ to secure the second-order condition with respect to q_i, $\frac{\partial^2 \pi_i}{\partial q_i^2} < 0$, and the condition for $0 < \bar{x} < 1$.

Equilibrium in the restriction on opening hours

In the restriction on opening hours, no retailers can extend opening hours to attract type 2 consumers with time constraints, who locate uniformly on the time line. They target type 1 consumers who locate at the initial point of the time line. That is, they compete for demand K in the restriction on opening hours. We assume that regulating opening hours restricts retailers' location choice in time to $y_i = 0$, for $i = 1, 2$. The first-order conditions with respect to p_i in the second stage lead to

$$p_1 = \frac{q_1 - q_2}{3} + t, \tag{6}$$

$$p_2 = \frac{q_2 - q_1}{3} + t. \tag{7}$$

Substituting p_1 and p_2 in Eqs.(6)-(7) into $\bar{x}$ in Eq.(3), we have

$$\bar{x} = \frac{q_1 - q_2}{6t} + \frac{1}{2}. \tag{8}$$

Substituting p_1 and p_2 in Eqs.(6)-(7) and $\bar{x}$ in Eq.(8) into Eqs.(4)-(5), we have

$$\pi_1 = \frac{K(q_1 - q_2 + 3t)^2}{18t} - rq_1^2, \tag{9}$$

$$\pi_2 = \frac{K(q_2 - q_1 + 3t)^2}{18t} - rq_2^2. \tag{10}$$

Solving the profit maximization with respect to q_i, we obtain the reaction functions as follows:

$$q_1 = R(q_2) = \frac{K}{18rt - K}(3t - q_2), \tag{11}$$

$$q_2 = R(q_1) = \frac{K}{18rt - K}(3t - q_1). \tag{12}$$

Because $R'(q_j) < 0$, quality of goods are strategic substitutes. If retailer i increases its quality of goods, q_i, given q_j, the demand of retailer j decrease. Solving Eq.(11)-(12), quality of goods is given by

$$q_1^* = q_2^* = \frac{K}{6r}. \tag{13}$$

Substituting q_1^* and q_2^* in Eq.(13) into Eq.(6)-(10), we have:

$$p_1^* = p_2^* = t, \;\; \bar{x} = \frac{1}{2},$$

$$D_1^* = D_2^* = \frac{K}{2},$$

$$\pi_1^* = \pi_2^* = \frac{K(18rt - K)}{36r}.$$

This yields the following proposition.

Proposition 1 *In the restriction on opening hours, when the cost of investment in store quality of retailer i is given by Eq.(1), quality of goods, prices, demands, and profits are equal among retailers.*

In the restriction on opening hours, when retailers invest in quality of goods, they do not differentiate itself in equilibrium.

Equilibrium in the liberalization of opening hours

In the liberalization of opening hours, the first-order conditions with respect to p_i in the second stage lead to

$$p_1 = \frac{q_1 - q_2}{3} + \frac{t(3K + (1-K)(4y_1 - y_2))}{3(K + (1-K)y_2)}, \tag{14}$$

$$p_2 = \frac{q_2 - q_1}{3} + \frac{t(3K + (1-K)(2y_1 + y_2))}{3(K + (1-K)y_2)}. \tag{15}$$

Substituting p_1 and p_2 in Eqs.(14)-(15) into $\bar{x}$ in Eq. (3), we have

$$\bar{x} = \frac{q_1 - q_2}{6t} - \frac{K + (1-K)y_1}{3(K + (1-K)y_2)} + \frac{5}{6}. \tag{16}$$

Substituting p_1 and p_2 in Eqs.(14)-(15) and $\bar{x}$ in Eq.(16) into Eqs.(4)-(5), we have

$$\pi_1 = \frac{[K(q_1 - q_2 + 3t) + (1-K)(t(4y_1 - y_2) + y_2(q_1 - q_2))]^2}{18t(K + (1-K)y_2)} - rq_1^2, \tag{17}$$

$$\pi_2 = \frac{[K(q_2 - q_1 + 3t) + (1-K)(t(2y_1 + y_2) + y_2(q_2 - q_1))]^2}{18t(K + (1-K)y_2)} - rq_2^2. \tag{18}$$

Solving the profit maximization with respect to q_i, we obtain the reaction functions as follows:

$$q_1 = R(q_2) = \frac{-(K + (1-K)y_2)q_2 + 3Kt + (1-K)t(4y_1 - y_2)}{18rt - (K + (1-K)y_2)}, \tag{19}$$

$$q_2 = R(q_1) = \frac{-(K + (1-K)y_2)q_1 + 3Kt + (1-K)t(2y_1 + y_2)}{18rt - (K + (1-K)y_2)}. \tag{20}$$

Because $R'(q_j) < 0$, quality of goods are strategic substitutes. If retailer i increases its quality of goods, q_i, given q_j, the demand of retailer j decrease. The first derivative with respect to y_i, $\frac{\partial \pi_i}{\partial y_i}$, is given by

$$\frac{\partial \pi_1}{\partial y_1} = \frac{4(1-K)[(K + (1-K)y_2)(q_1 - q_2) + (1-K)t(4y_1 - y_2) + 3Kt]}{9(K + (1-K)y_2)},$$

$$\frac{\partial \pi_2}{\partial y_2} = \frac{(1-K)}{18t}[(q_2 - q_1 + t)^2 - \frac{4t^2(K + (1-K)y_1)^2}{(K + (1-K)y_2)^2}].$$

The second derivative with respect to y_i, $\frac{\partial^2 \pi_i}{\partial y_i^2}$, is given by

$$\frac{\partial^2 \pi_1}{\partial y_1^2} = \frac{16(1-K)^2 t}{9(K + (1-K)y_2)} > 0,$$

$$\frac{\partial^2 \pi_2}{\partial y_2^2} = \frac{4(1-K)^2 t(K + (1-K)y_1)^2}{9(K + (1-K)y_2)^3} > 0.$$

Because $\frac{\partial^2 \pi_i}{\partial y_i^2} > 0$, the optimal opening hours are corner solutions. Thus, we have the following three possible corner solutions: $y_1 = 1$ and $y_2 = 0$; $y_1 = 0$ and $y_2 = 0$; and $y_1 = 1$ and $y_2 = 1$. Only the pair of $y_1 = 1$ and $y_2 = 0$ appears in equilibrium (the proof is presented in the Appendix). This yields the following lemma.

lemma 1 *In the liberalization of opening hours, when the cost of investment in store quality is given by Eq.* (1)*, only the pair of* $y_1 = 1$ *and* $y_2 = 0$ *appears in equilibrium.*

When the pair of $y_1 = 1$ and $y_2 = 0$ appears in equilibrium, a retailer locating at $y = 1$ (retailer 1) targets consumers who face time constraints, and another retailer locating at $y = 0$ (retailer 2) targets consumers who can buy at any time. This presents the robustness of Inderst and Irmen (2005).

Solving Eq.(19)-(20), quality of goods of retailer 1 locating at $y = 1$, q_1^{**}, and that of retailer 2 locating at $y = 0$, q_2^{**} are, respectively,

$$q_1^{**} = \frac{3rt(4-K)-K}{6r(9rt-K)}, \tag{21}$$

$$q_2^{**} = \frac{3rt(2+K)-K}{6r(9rt-K)}. \tag{22}$$

Considering $rt > \frac{1}{6}$ and $K \in [1/2, 1)$, $q_1^{**} > q_2^{**}$. Substituting $y_1 = 1$, $y_2 = 0$, q_1^{**}, and q_2^{**} into Eq.(14)-(18), we have:

$$p_1^{**} = \frac{t(3rt(4-K)-K)}{K(9rt-K)}, \;\; p_2^{**} = \frac{t(3rt(2+K)-K)}{K(9rt-K)}, \;\; \bar{x} = \frac{(3rt(5K-2)+K(1-2K))}{2K(9rt-K)}$$

$$D_1^{**} = \frac{3rt(4-K)-K}{2(9rt-K)}, \;\; D_2^{**} = \frac{3rt(2+K)-K}{2(9rt-K)},$$

$$\pi_1^{**} = \frac{(18rt-K)(3rt(4-K)-K)^2}{36Kr(9rt-K)^2}, \;\; \pi_2^{**} = \frac{(18rt-K)(3rt(2+K)-K)^2}{36Kr(9rt-K)^2}.$$

Considering $rt > \frac{1}{6}$ and $K \in [1/2, 1)$, $p_1^{**} > p_2^{**}$, $D_1^{**} > D_2^{**}$ and $\pi_1^{**} > \pi_2^{**}$. We can have the following proposition.

Proposition 2 *In the liberalization of opening hours, when the cost of investment in store quality is given by Eq.*(1)*, quality of goods, prices, demand, and profits of a retailer who locates at* $y = 1$ *are higher than those of a retailer who locates at* $y = 0$*.*

The intuition is the following. A retailer who commits to open for longer can monopolize consumers who face time constraints. Thus, although the retailer sets a higher price, it can meet the relatively high demand. This promotes the investment in its quality of goods. Another retailer, who commits to open for shorter, sets lower quality of goods because it meets lower demand. However, the retailer sets lower prices to attract non-time constrained type 1 consumers who can compare the two retailers .

3.2 *Scenario 2:* The cost of investment in store quality is positively correlated with opening hours

Second, we consider scenario 2 where the cost of investment in store quality is given by Eq. (2). As above, we assume $rt > \frac{1}{6}$ in order to secure the second-order condition with respect to q_i, $\frac{\partial^2 \pi_i}{\partial q_i^2} < 0$, and the condition for $0 < \bar{x} < 1$. We use the same calculation procedures as in scenario 1, but replace the cost of investment in store quality with Eq.(2).

Equilibrium in the restriction on opening hours

Substituting p_1 and p_2 in Eqs.(6)-(7) and $\bar{x}$ in Eq.(8) into Eqs.(4)-(5), we have

$$\pi_1 = \frac{K(q_1 - q_2 + 3t)^2}{18t} - Krq_1^2, \tag{23}$$

$$\pi_2 = \frac{K(q_2 - q_1 + 3t)^2}{18t} - Krq_2^2. \tag{24}$$

Solving the profit-maximization problem with respect to q_i, we obtain the reaction functions as follows:

$$q_1 = R(q_2) = \frac{1}{18rt - 1}(3t - q_2), \tag{25}$$

$$q_2 = R(q_1) = \frac{1}{18rt - 1}(3t - q_1). \tag{26}$$

Because $R'(q_j) < 0$, the brand proliferation are strategic substitutes. If retailer i increases its brand proliferation level, q_i, given q_j, the demand of retailer j decrease. Solving the reaction functions (25) and (26), the brand proliferation level is given by

$$q_1^* = q_2^* = \frac{1}{6r}. \tag{27}$$

Substituting q_1^* and q_2^* in Eq.(27) into Eq.(6)-(8), (23) and (24), we have:

$$p_1^* = p_2^* = t, \;\; \bar{x} = \frac{1}{2},$$

$$D_1^* = D_2^* = \frac{K}{2},$$

$$\pi_1^* = \pi_2^* = \frac{K(18rt - 1)}{36r}.$$

We have the following proposition.

Proposition 3 *In the restriction on opening hours, when the cost of investment in store quality is given by Eq.*(2)*, brand proliferation levels, prices, demands, and profits are equal among retailers.*

In the restriction on opening hours, when retailers invest in the brand proliferation, they do not differentiate itself in equilibrium.

Equilibrium in the liberalization of opening hours

Substituting p_1 and p_2 in Eqs.(14) and (15) and $\bar{x}$ in Eq.(16) into Eqs.(4)-(5), we have

$$\pi_1 = \frac{[K(q1 - q2 + 3t) + (1-K)(t(4y_1 - y_2) + y_2(q_1 - q_2))]^2}{18t(K + (1-K)y_2)} - (K + (1-K)y_1)rq_1^2, \tag{28}$$

$$\pi_2 = \frac{[K(q_2 - q_1 + 3t) + (1-K)(t(2y_1 + y_2) + y_2(q_2 - q_1))]^2}{18t(K + (1-K)y_2)} - (K + (1-K)y_2)rq_2^2. \tag{29}$$

Solving the profit-maximization problem with respect to q_i, we obtain the reaction functions as follows:

$$q_1 = R(q_2) = \frac{-(K + (1-K)y_2)q_2 + 3Kt + (1-K)t(4y_1 - y_2)}{K(18rt - 1) + (1-K)(18rty_1 - y_2)}. \tag{30}$$

$$q_2 = R(q_1) = \frac{-(K+(1-K)y_2)q_1 + 3Kt + (1-K)t(2y_1+y_2)}{K(18rt-1)+(1-K)(18rty_2-y_2)}. \quad (31)$$

Because $R'(q_j) < 0$, the brand proliferation are strategic substitutes. If retailer i increases the brand proliferation level, q_i, given q_j, the demand of retailer j decrease. The first derivative with respect to y_i, $\frac{\partial \pi_i}{\partial y_i}$, is given by

$$\frac{\partial \pi_1}{\partial y_1} = \frac{(1-K)}{9}[4(q_1-q_2) - 9rq_1^2 - 4t + \frac{16t(K+(1-K)y_1)}{K+(1-K)y_2}],$$

$$\frac{\partial \pi_2}{\partial y_2} = \frac{(1-K)}{18t}[2t(q_1-q_2) + (q_1-q_2)^2 - 18rtq_2^2 - t^2 - \frac{4t^2(K+(1-K)y_1)^2}{(K+(1-K)y_2)^2}].$$

The second derivative with respect to y_i, $\frac{\partial^2 \pi_i}{\partial y_i^2}$, is given by

$$\frac{\partial^2 \pi_1}{\partial y_1^2} = \frac{16(1-K)^2 t}{9(K+(1-K)y_2)} > 0,$$

$$\frac{\partial^2 \pi_2}{\partial y_2^2} = \frac{4(1-K)^2 t(K+(1-K)y_1)^2}{9(K+(1-K)y_2)^3} > 0.$$

Because $\frac{\partial^2 \pi_i}{\partial y_i^2} > 0$, the optimal opening hours are corner solutions. We have the following three possible corner solutions: $y_1 = 1$ and $y_2 = 0$; $y_1 = 0$ and $y_2 = 0$; and $y_1 = 1$ and $y_2 = 1$. Only the pair of $y_1 = 1$ and $y_2 = 0$ appears in equilibrium (the proof is presented in the Appendix). Thus, we have the following lemma.

lemma 2 *In the liberalization of opening hours, when the cost of investment in store quality is given by Eq.*(2)*, only the pair of* $y_1 = 1$ *and* $y_2 = 0$ *appears in equilibrium.*

When the pair of $y_1 = 1$ and $y_2 = 0$ appears in equilibrium, retailer 1, locating at $y = 1$, targets consumers who face time constraints, and retailer 2, locating at $y = 0$, targets consumers who can buy at any time. This presents the robustness of Inderst and Irmen (2005) as lemma 1.

Solving Eq.(30)-(31), the brand proliferation level of retailer 1 locating at $y = 1$, q_1^{**}, and that of retailer 2 locating at $y = 0$, q_2^{**} are, respectively,

$$q_1^{**} = \frac{3Krt(4-K)-K}{3Kr(18rt-1-K)}, \quad (32)$$

$$q_2^{**} = \frac{3rt(2+K)-K}{3Kr(18rt-1-K)}. \quad (33)$$

Considering $rt > \frac{1}{6}$ and $K \in [1/2, 1)$, $q_1^{**} < q_2^{**}$. Substituting $y_1 = 1$, $y_2 = 0$, q_1^{**}, and q_2^{**} in Eq. (32)-(33) into (6)-(8), (28) and (29), we have:

$$p_1^{**} = \frac{2t(3rt(4-K)-1)}{K(18rt-1-K)}, \;\; p_2^{**} = \frac{2t(3rt(2+K)-K)}{K(18rt-1-K)}, \;\; \bar{x} = \frac{3rt(5K-2)-K^2}{K(18rt-1-K)},$$

$$D_1^{**} = \frac{3rt(4-K)-1}{18rt-1-K}, \;\; D_2^{**} = \frac{3rt(2+K)-K}{18rt-1-K},$$

$$\pi_1^{**} = \frac{(18rt-K)(3rt(4-K)-1)^2}{9Kr(18rt-1-K)^2}, \;\; \pi_2^{**} = \frac{(18rt-1)(3rt(2+K)-K)^2}{9Kr(18rt-1-K)^2}.$$

Considering $rt > \frac{1}{6}$ and $K \in [1/2, 1)$, $p_1^{**} > p_2^{**}$, $D_1^{**} > D_2^{**}$ and $\pi_1^{**} > \pi_2^{**}$. We have the following proposition.

Proposition 4 *In the liberalization of opening hours, when the cost of investment in store quality is given by Eq.*(2)*, the brand proliferation level of a retailer who locates at* $y=0$ *is higher than that of a retailer who locates at* $y=1$*. Prices, demands, and profits of a retailer who locates at* $y=1$ *are higher than those of a retailer who locates at* $y=0$*.*

The intuition is as follows. Although a retailer who commits to open for longer may have an incentive to promote investment in the brand proliferation because of the relatively high demand, however, because longer opening hours cause additional wages and administrative costs in order to handle the brand proliferation, it sets a lower brand proliferation level. Nevertheless, the retailer can obtain a greater profit by setting higher prices because it can monopolize consumers who face time constraints. On the other hand, a retailer who commits to open for shorter promotes investment in the brand proliferation, even though it meets a lower demand. Because shorter opening hours cause lower wages and administrative costs in order to handle the brand proliferation, it promotes investment in the brand proliferation. Also, it sets lower prices to attract non-time constrained consumers who can compare the two retailers.

4 Welfare Analysis

In this section, we examine whether the deregulation of opening hours in the retail industry can improve social welfare. We define social welfare as the sum of consumer utility and profits of retailers. Prices are irrelevant in social welfare analysis because they are transferred between consumers and retailers. Therefore, social welfare consists of the following three factors: the store quality level, the transportation costs of consumers, and the costs of investments in store quality.

In scenario 1, social welfare under the regulation of opening hours, W_{R1}, is given by

$$W_{R1}=K\int_0^{\bar{x}}(S+q_1-tm^2)dm+K\int_{\bar{x}}^1(S+q_2-t(1-m)^2)dm-rq_1^2-rq_2^2,$$

and social welfare under the deregulation of opening hours, W_{D1}, is given by

$$W_{D1}=\int_0^{\bar{x}}(S+q_1-tm^2)dm+K\int_{\bar{x}}^1(S+q_2-t(1-m)^2)dm-rq_1^2-rq_2^2.$$

Substituting the equilibrium values in the above equations and comparing W_{R1}^* with W_{D1}^*, $\triangle W_1\equiv W_{R1}^*-W_{D1}^*$, we obtain the following value:

$$\begin{aligned}\triangle W_1 = {}& \frac{-6K^5+14K^6-2K^8-6K^9+(3K^3+141K^4-342K^5-24K^6+57K^7+165K^8)rt}{72K^3r(9rt-K)^3}\\
&+\frac{(-54K^2-999K^3+2583K^4+558K^5-549K^6-1539K^7)r^2t^2}{72K^3r(9rt-K)^3}\\
&+\frac{(324K+1296K^2-4455K^3-4293K^4+2025K^5+5103K^6)r^3t^3}{72K^3r(9rt-K)^3}\\
&+\frac{(-648+5508K-11178K^2+10611K^3-2106K^4-2187K^5)r^4t^4}{72K^3r(9rt-K)^3}\\
&+\frac{(-36K^5+108K^6-36K^7-36K^8)rS+(864K^4-2700K^5+864K^6+972K^7)r^2tS}{72K^3r(9rt-K)^3}\\
&+\frac{(-6804K^3+22356K^4-6804K^5-8748K^6)r^3t^2S+(17496K^2-61236K^3+17496K^4+26244K^5)r^4t^3S}{72K^3r(9rt-K)^3}.\end{aligned}$$

Considering $rt > 1/6$ and $K \in [1/2, 1)$, the sign of $\triangle W_1$ is negative, $W^*_{R1} < W^*_{D1}$. That is, social welfare is higher under the deregulated opening hours.

In scenario 2, social welfare under the regulation of opening hours, W_{R2}, is given by

$$W_{R2} = K\int_0^{\bar{x}} (S + q_1 - tm^2)dm + K\int_{\bar{x}}^1 (S + q_2 - t(1-m)^2)dm - Krq_1^2 - Krq_2^2,$$

and social welfare under the deregulation of opening hours, W_{D2}, is given by

$$W_{D2} = \int_0^{\bar{x}} (S + q_1 - tm^2)dm + K\int_{\bar{x}}^1 (S + q_2 - t(1-m)^2)dm - (K + (1-K)y_1)rq_1^2 - (K + (1-K)y_2)rq_2^2.$$

Substituting the equilibrium values in the above equations, and comparing W^*_{R2} with W^*_{D2}, $\triangle W_2 \equiv W^*_{R2} - W^*_{D2}$, we obtain the following value:

$$\begin{aligned}
\triangle W_2 = {} & \frac{4K^3 - 12K^4 - 8K^5 + 12K^6 + 4K^7 + (-72K^2 + 36K^3 + 549K^4 - 261K^5 - 249K^6 - 3K^7)rt}{36K^3r(1+K-18rt)^3} \\
& + \frac{(1872K^2 - 4968K^3 - 1638K^4 + 4536K^5 + 198K^6)r^2t^2}{36K^3r(1+K-18rt)^3} \\
& + \frac{(-7776K^2 + 32400K^3 - 21060K^4 - 3564K^5)r^3t^3}{36K^3r(1+K-18rt)^3} \\
& + \frac{(2592 - 22032K + 44712K^2 - 42444K^3 + 17172K^4)r^4t^4}{36K^3r(1+K-18rt)^3} \\
& + \frac{(-36K^4 - 36K^5 + 36K^6 + 36K^7)rS + (-216K^2 + 324K^3 + 2052K^4 - 324K^5 - 1836K^6)r^2tS}{36K^3r(1+K-18rt)^3} \\
& + \frac{(7776K^2 - 19440K^3 - 19440K^4 + 31104K^5)r^3t^2S + (-69984K^2 + 244944K^3 - 174960K^4)r^4t^3S}{36K^3r(1+K-18rt)^3}.
\end{aligned}$$

Considering $rt > 1/6$ and $K \in [1/2, 1)$, the sign of $\triangle W_2$ is negative, $W^*_{R2} < W^*_{D2}$. That is, social welfare is higher under the deregulated opening hours. Therefore, we can have the following proposition.

Proposition 5 *In scenarios* 1 *and* 2*, social welfare in the liberalization of opening hours is larger than that in the restriction on opening hours.*

The desirability of the deregulation of opening hours is higher regardless of the cost structure of investment in store quality. Therefore, promoting the deregulation of opening hours may be desirable for social welfare. Although Inderst and Irmen (2005) discuss that the deregulation of opening hours may decrease social welfare if longer opening hours cause additional costs, taking the cost structure of investment in store quality as shown by scenario 2 into account, the opposite result is obtained.

5 The Extended Model

In this section, we consider a scenario in which retailers incur not only the cost of investment in store quality as scenario 1, but also the cost of investment in store quality as scenario 2. That is, we consider an extended case where retailers invest in both quality of goods and the brand proliferation. In the first stage, retailers simultaneously choose their location in time y_i and choose quality of goods q_{iA} and the brand proliferation level q_{iB}. In the second stage, each retailer competes in price p_i.

Consumer utility is equal to

$$V_i(x,y) = \begin{cases} S + (q_{iA} + q_{iB}) - p_i - t(x - x_i)^2 & \text{if } y \leq y_i, \\ 0 & \text{if } y > y_i. \end{cases}$$

We assume $S > 2t$, which means that a retailer with longer opening hours does not exclude type 1 consumers by setting a higher price.

In this extended case, the cost of investment in store quality is given by

$$I(q_{iA}, q_{iB}, y_i) = rq_{iA}^2 + \{K + (1-K)y_i\}rq_{iB}^2 \quad i = 1, 2, i \neq j,$$

where $r(> \frac{1}{6t})$ represents the parameter of the cost of investment in store quality. This parameter secures the second-order conditions with respect to q_{iA} and q_{iB}, namely $\frac{\partial^2 \pi_i}{\partial q_{iA}^2} < 0$ and $\frac{\partial^2 \pi_i}{\partial q_{iB}^2} < 0$, and the condition for $0 < \bar{x} < 1$.

Because we use the same mathematical procedures as in the basic model, we mention only some findings and results. We find that quality of goods of retailer i, q_{iA}, is a strategic complement for the brand proliferation level of retailer i, q_{iB}. Increasing in quality of goods of retailer i leads to higher demand, so that increasing in its brand proliferation level leads to higher consumer's marginal utility. In the restriction on opening hours, both retailers set $q_{iA}^* < q_{iB}^*, i = 1, 2, i \neq j$, that is, their investment in the brand proliferation are larger than those in quality of goods. Defining $Q_{iR} \equiv q_{iA}^* + q_{iB}^*$ and $Q_{jR} \equiv q_{jA}^* + q_{jB}^*$, we can obtain $Q_{iR} = Q_{jR}$, that is, the store quality is equal among retailers. Furthermore, prices, profits, and demands are equal among retailers. In the case of the deregulation opening hours, the second derivative with respect to y_i, $\frac{\partial^2 \pi_i}{\partial y_i^2}$, is positive. Thus, the optimal opening hours are corner solutions. As in the basic model, we generate an asymmetric configuration in opening hours in equilibrium: retailer i locates at $y_i = 1$, and retailer j locates at $y_j = 0$. Retailer i with longer opening hours, sets $q_{iA}^{**} = q_{iB}^{**}$. That is, its investment in quality of goods is equal to that in the brand proliferation. On the other hand, the retailer j with shorter opening hours sets $q_{jA}^{**} < q_{jB}^{**}$. That is, its investment in the brand proliferation is larger than that in quality of goods. Although the investment in the brand proliferation is more costly for longer opening hours, retailer j with shorter opening hours can increase its brand proliferation level. We now define $Q_{iD} \equiv q_{iA}^{**} + q_{iB}^{**}$ and $Q_{jD} \equiv q_{jA}^{**} + q_{jB}^{**}$. If $\frac{1}{2} < K < \frac{2}{3}$, we have $Q_{iD} < Q_{jD}$. This means that retailer i with longer opening hours decreases quality of its goods and its brand proliferation level than those of retailer j with shorter opening hours when there are fewer consumers who can buy at any time. This intuition is simply. Because there are many consumers who face time constraints, retailer i with longer opening hours can obtain greater profits without increasing quality of its goods and its brand proliferation level. If $\frac{2}{3} < K < 1$, we have $Q_{iD} > Q_{jD}$. This means that retailer i with longer opening hours promotes quality of its goods and its brand proliferation level when there are many consumers who can buy at any time. Because consumers who can buy at any time can compare the two retailers, retailer i with longer opening hours increases quality of its goods and its brand proliferation level in order to attract consumers. Prices, profits, and demands of retailer i with longer opening hours are higher, regardless of the value of K.

Proposition 6 *When retailers invest in both quality of goods and the brand proliferation, in the case of regulated* *opening hours, store quality, prices, profits, and demands are equal among retailers. In the case of deregulated opening hours, store quality of a retailer who locates at $y = 0$ is higher than that of a retailer who locates at $y = 1$ when $\frac{1}{2} < K < \frac{2}{3}$. When $\frac{2}{3} < K < 1$, store quality of a retailer who locates at $y = 1$ is higher than that of a retailer who locates at $y = 0$. Prices, profits, and demand of a retailer i who locates at $y = 1$ are higher, regardless of the value of K.*

We find that retailers' strategies related to store quality depend on the mass of consumers who can buy at any time when retailers invest in both quality of goods and the brand proliferation.

In this extended model, we examine whether the deregulation of opening hours in the retail industry can improve social welfare. Social welfare under the regulated opening hours W_{RE} is given by

$$W_{RE} = K\int_0^{\bar{x}}(S + Q_1 - tm^2 dm) + K\int_{\bar{x}}^1 (S + Q_2 - t(1-m)^2 dm) - rq_{1A}^2 - Krq_{1B}^2 - rq_{2A}^2 - Krq_{2B}^2,$$

and social welfare under the deregulated opening hours W_{DE} is given by

$$W_{DE} = \int_0^{\bar{x}}(S + Q_1 - tm^2 dm) + K\int_{\bar{x}}^1 (S + Q_2 - t(1-m)^2 dm) - rq_{1A}^2 - (K + (1-K)y_1)rq_{1B}^2 - rq_{2A}^2 - (K + (1-K)y_2)rq_{2B}^2.$$

Substituting in the equilibrium values in the above equations and comparing W_{RE}^* with W_{DE}^*, we have $W_{RE}^* < W_{DE}^*$ by considering $rt > 1/6$ and $K \in [1/2, 1)$. Therefore, we can have the following proposition.

Proposition 7 *If retailers invest in both quality of goods and the brand proliferation, social welfare in the liberalization of opening hours is larger than that in the restriction on opening hours.*

6 Discussion

This paper does not consider differences in store size. Morrison and Newman (1983), Tanguay et al. (1995), Inderst and Irmen (2005), and Wenzel (2011) analyze competition in opening hours between large and small retailers. Considering the difference in store sizes in our model, we explain the behavior of Japanese retailers. We associate a large store with higher store quality and lower prices. Based on this association, the large store is a retailer with shorter opening hours as shown by our proposition 4 in which the retailer with shorter opening hours sets higher brand proliferation level and lower prices. This may be consistent with the behavior of Japanese grocery stores. For instance, Ito-Yokado affiliate grocery stores, which is Japanese major retailer and targets consumers without time constraints, promotes investment in store quality such as the brand proliferation and sets reasonable prices for consumers.

Although we assume the two types of consumers in the main body (those without time constraints and those with time constraints consumers), we now consider the existence of loyal customers who do not switch to competitors. If many loyal consumers are workers who have time constraints, all retailers may open for longer and promote store quality to attract more consumers. On the other hand, if many loyal consumer are people who have no time-constraints, all retailers may open for shorter and promote store quality to attract more consumers.

Although we assume the duopoly model in the main body, we now consider the triopoly case. In the scenario where the cost of investment in store quality is independent of opening hours, all retailers may open for longer and not differentiate itself in the time dimension. In addition, they may set a higher store quality. The intuition is simple. We suppose that a retailer locates at $y = 1$ and the other two retailers locate at $y = 0$. The former sets a higher store quality, and then, charges higher prices and obtains monopoly rent. Although the other two retailers set a lower store quality and compete in prices, one of the two retailers deviates to $y = 1$ in order to obtain the demand of consumers who face time constraints, and sets a higher quality level to attract consumers who can compare the three retailers. Then, the retailer charges lower prices because the deviation to $y = 1$ leads to competition among the retailers who locate at $y = 1$. The retailer locating at $y = 0$ has difficulty obtaining the demand of consumers who can compare the three retailers. In order to obtain the demand of consumers who face time constraints, it also deviates to $y = 1$ and sets a higher quality, yielding to higher competition among the three retailers locating at $y = 1$. As a result, all retailers may open for longer and set higher store quality and lower prices in equilibrium. However, in the scenario where the cost is positively correlated with opening hours, the result is ambiguous. We leave an analysis of this triopoly market for future research.

In this paper, we assume that consumers can move in only one direction on the Hotelling line, as in Cancian et al. (1995). That is, consumers face symmetric directional constraint. However, in reality, some consumers can postpone their shopping hours, while others can advance their shopping hours. Nilssen (1997) considers an asymmetric directional constraint, which we will do as well in future research.

Although we assume that each retailer is constrained to charge a single price regardless of opening hours, some retailers set "afternoon (lunch) special", in which some goods are cheaper during certain hours. For example, some retailers change prices of fresh foods and daily dishes at lunch time or in the afternoon. Assuming that menu costs are high for retailers, however, with computerized management of price labels, retailers can change prices of goods easily at a low cost. In future research, considering low menu costs, we re-examine retailers' strategies related to opening hours, store quality, and prices.

Although our paper focus on competition among retailers, the empilical analysis by Jacobsen and Kooreman (2005) focuses on consumers' reactions to liberalized opening hours. Using Dutch data from 1995, 1997, 1999, and 2000, they show that the deregulated opening hours not only had a positive effect on consumers' shopping time, but also on consumers' labor time. We will consider as well in future research by taking store quality into account.

7 Conclusion

Using a duopoly model with symmetric retailers, this paper examines the retail strategies related to opening hours, store quality, and prices. The basic setting is based on that in Inderst and Irmen (2005). We assume that retailers invest in store quality. They incur the cost of investment in store quality. We consider two scenarios on the cost of investment in store quality: the cost of investment in store quality is independent of opening hours; the cost of investment in store quality is positively correlated with opening hours. In the former scenario, a retailer with longer opening hours chooses a higher store quality and then charges higher prices. In the latter scenario, a retailer with shorter opening hours chooses a higher store quality and then charges lower prices. We also find that the deregulation of opening hours can improve social welfare, regardless of the cost structures of investment in store quality.

Appendix

Proof of Lemma 1

In scenario 1 where the cost of investment in store quality is given by Eq.(1), when opening hours are deregulated, we examine whether retailers have incentives to deviate in three possibilities of corner solutions: (a) $y_1 = 1$ and $y_2 = 0$; (b) $y_1 = 0$ and $y_2 = 0$; (c) $y_1 = 1$ and $y_2 = 1$.

In the pattern (a), we obtain:

$$q_1^{**} = \frac{3rt(4-K)-K}{6r(9rt-K)}, \;\; q_2^{**} = \frac{3rt(2+K)-K}{6r(9rt-K)},$$

$$\pi_1^{**} = \frac{(18rt-K)(3rt(4-K)-K)^2}{36Kr(9rt-K)^2}, \;\; \pi_2^{**} = \frac{(18rt-K)(3rt(2+K)-K)^2}{36Kr(9rt-K)^2}$$

We assume retailer 1 deviates to $y_1 = 0$ given $y_2 = 0$ and $q_2^{**} = \frac{3rt(2+K)-K}{6r(9rt-K)}$. Substituting $y_1 = 0$ into the first-order condition with respect to q_1, we have:

$$q_{1d} = \frac{K(6rt(27rt-1)-K(21rt-1))}{6r(18rt-K)(9rt-K)}, \;\; \pi_{1d} = \frac{K(6rt(27rt-1)-K(21rt-1))^2}{36r(18rt-K)(9rt-K)^2}.$$

Comparing π_1^{**} with π_{1d}, we have:

$$\pi_1^{**} - \pi_{1d} = \frac{4(1-K)t(K^2-9Krt(2+K)+54(2+K)r^2t^2)}{3K(18rt-K)(9rt-K)}.$$

Considering $rt > \frac{1}{6}$ and $K \in [\frac{1}{2}, 1)$, $\pi_1^{**} > \pi_{1d}$. This indicates that retailer 1 has no incentives to deviate to $y_1 = 0$. Next, we assume retailer 2 deviates to $y_2 = 1$ given $y_1 = 1$ and $q_1^{**} = \frac{3rt(4-K)-K}{6r(9rt-K)}$. Substituting $y_2 = 1$ into the first-order condition with respect to q_2, we have:

$$q_{2d} = \frac{6rt(27rt-2)-K(15rt-1)}{6r(18rt-1)(9rt-K)}, \;\; \pi_{2d} = \frac{(6rt(27rt-2)-K(15rt-1))^2}{36r(18rt-1)(9rt-K)^2}.$$

Comparing π_2^{**} with π_{2d}, we have:

$$\pi_2^{**} - \pi_{2d} = \frac{t(-(1-K)K^2+18K(1-K^2)rt-27(4+K(8-(13-K)K))r^2t^2+486(4-K)(1-K)r^3t^3)}{6K(18rt-1)(9rt-K)^2}.$$

Considering $rt > \frac{1}{6}$ and $K \in [\frac{1}{2}, 1)$, $\pi_2^{**} > \pi_{2d}$. This indicates that retailer 2 has no incentives to deviate to $y_2 = 1$. Thus, the pair of $y_1 = 1$ and $y_2 = 0$ is sustainable in equilibrium.

In the pattern (b), we have:

$$q_1^{**} = q_2^{**} = \frac{K}{6r},$$

$$\pi_1^{**} = \pi_2^{**} = \frac{K(18rt-K)}{36r}.$$

We assume retailer 1 deviates to $y_1 = 1$ given $y_2 = 0$ and $q_2^{**} = \frac{K}{6r}$. Substituting $y_1 = 1$ into the first-order condition with respect to q_1, we have:

$$q_{1d} = \frac{6rt(4-K)-K^2}{6r(18rt-K)}, \;\; \pi_{1d} = \frac{(6rt(4-K)-K^2)^2}{36Kr(18rt-K)}.$$

Comparing π_1^{**} with π_{1d}, we have:

$$\pi_1^{**} - \pi_{1d} = \frac{4(1-K)t(K^2-6rt(2+K))}{3K(18rt-K)}.$$

Considering $rt > \frac{1}{6}$ and $K \in [\frac{1}{2}, 1)$, $\pi_1^{**} < \pi_{1d}$. This indicates that retailer 1 has incentives to deviate to $y_1 = 1$. Thus, the pair of $y_1 = 0$ and $y_2 = 0$ does not appear in equilibrium.

In the pattern (c), we have:

$$q_1^{**} = q_2^{**} = \frac{1}{6r},$$
$$\pi_1^{**} = \pi_2^{**} = \frac{18rt - 1}{36r}.$$

We assume retailer 2 deviates to $y_2 = 0$ given $y_1 = 1$ and $q_1^{**} = \frac{1}{6r}$. Substituting $y_2 = 0$ into the first-order condition with respect to q_2, we have:

$$q_{2d} = \frac{6rt(2+K) - K}{6r(18rt - K)}, \;\; \pi_{2d} = \frac{(6rt(2+K) - K)^2}{36Kr(18rt - K)}.$$

Comparing π_2^{**} with π_{2d}, we have:

$$\pi_2^{**} - \pi_{2d} = \frac{(1-K)t(K - 6(4-K)rt)}{6K(18rt - K)}.$$

Considering $rt > \frac{1}{6}$ and $K \in [\frac{1}{2}, 1)$, $\pi_2^{**} < \pi_{2d}$. This indicates that retailer 2 has incentives to deviate to $y_2 = 0$. Thus, the pair of $y_1 = 1$ and $y_2 = 1$ does not appear in equilibrium.

Proof of Lemma 2

In scenario 2 where the cost of investment in store quality is given by Eq.(2), when opening hours are deregulated, we examine whether retailers have incentive to deviate in the following three possibilities of corner solutions: (a) $y_1 = 1$ and $y_2 = 0$; (b) $y_1 = 0$ and $y_2 = 0$; (c) $y_1 = 1$ and $y_2 = 1$.

In the pattern (a), we have:

$$q_1^{**} = \frac{3Krt(4-K) - K}{3Kr(18rt - 1 - K)}, \;\; q_2^{**} = \frac{3rt(2+K) - K}{3Kr(18rt - 1 - K)},$$
$$\pi_1^{**} = \frac{(18rt - K)(3rt(4-K) - 1)^2}{9Kr(18rt - 1 - K)^2}, \;\; \pi_2^{**} = \frac{(18rt - 1)(3rt(2+K) - K)^2}{9Kr(18rt - 1 - K)^2}$$

We assume retailer 1 deviates to $y_1 = 0$ given $y_2 = 0$ and $q_2^{**} = \frac{3rt(2+K)-K}{3Kr(18rt-1-K)}$. Substituting $y_1 = 0$ into the first-order condition with respect to q_1, we have:

$$q_{1d} = \frac{K(3rt(54rt - 3K - 4) + 1) - 6rt}{3Kr(18rt - 1)(18rt - 1 - K)}, \;\; \pi_{1d} = \frac{(K(3rt(54rt - 3K - 4) + 1) - 6rt)^2}{9Kr(18rt - 1)(18rt - 1 - K)^2}.$$

Comparing π_1^{**} with π_{1d}, we have:

$$\pi_1^{**} - \pi_{1d} = \frac{(1-K)(K - 6(3 + K(3K+8))rt + 9(80 + K(116 + K(9K + 32)))r^2t^2 - 162(64 + K(17K + 48))r^3t^3 + 23328(2+K)r^4t^4)}{9Kr(18rt - 1)(18rt - 1 - K)^2}.$$

Considering $rt > \frac{1}{6}$ and $K \in [\frac{1}{2}, 1)$, $\pi_1^{**} > \pi_{1d}$. This indicates that retailer 1 has no incentives to deviate to $y_1 = 0$. Next, we assume retailer 2 deviates to $y_2 = 1$ given $y_1 = 1$ and $q_1^{**} = \frac{3Krt(4-K)-K}{3Kr(18rt-1-K)}$. Substituting $y_2 = 1$ into the first-order condition with respect to q_2, we have:

$$q_{2d} = \frac{3rt(54rt - 2K - 7) + 1}{3r(18rt - 1)(18rt - 1 - K)}, \;\; \pi_{2d} = \frac{(3rt(54rt - 2K - 7) + 1)^2}{9r(18rt - 1)(18rt - 1 - K)^2}.$$

Comparing π_2^{**} with π_{2d}, we have:

$$\pi_2^{**} - \pi_{2d} = \frac{(1-K)(-K + 30Krt + 9(K(4K-29)+4)r^2t^2 - 324(4-K)r^3t^3 + 2916(4-K)r^4t^4)}{9K(18rt-1)(18rt-1-K)^2}.$$

Considering $rt > \frac{1}{6}$ and $K \in [\frac{1}{2}, 1)$, $\pi_2^{**} > \pi_{2d}$. This indicates that retailer 2 has no incentives to deviate to $y_2 = 1$. Thus, the pair of $y_1 = 1$ and $y_2 = 0$ is sustainable in equilibrium.

In the pattern (b), we have:

$$q_1^{**} = q_2^{**} = \frac{1}{6r},$$
$$\pi_1^{**} = \pi_2^{**} = \frac{K(18rt-1)}{36r}.$$

We assume retailer 1 deviates to $y_1 = 1$ given $y_2 = 0$ and $q_2^{**} = \frac{1}{6r}$. Substituting $y_1 = 1$ into the first-order condition with respect to q_1, we have:

$$q_{1d} = \frac{6rt(4-K) - K}{6r(18rt-K)}, \quad \pi_{1d} = \frac{(6rt(4-K)-K)^2}{36Kr(18rt-K)}.$$

Comparing π_1^{**} with π_{1d}, we have:

$$\pi_1^{**} - \pi_{1d} = \frac{(1-K)(-K^2 + 6K(3K+8)rt - 288(2+K)r^2t^2)}{36Kr(18rt-K)}.$$

Considering $rt > \frac{1}{6}$ and $K \in [\frac{1}{2}, 1)$, $\pi_1^{**} < \pi_{1d}$. This indicates that retailer 1 has incentives to deviate to $y_1 = 1$. Thus, the pair of $y_1 = 0$ and $y_2 = 0$ does not appear in equilibrium.

In the pattern (c), we have:

$$q_1^{**} = q_2^{**} = \frac{1}{6r},$$
$$\pi_1^{**} = \pi_2^{**} = \frac{18rt-1}{36r}.$$

We assume retailer 2 deviates to $y_2 = 0$ given $y_1 = 1$ and $q_1^{**} = \frac{1}{6r}$. Substituting $y_2 = 0$ into the first-order condition with respect to q_2, we have:

$$q_{2d} = \frac{6rt(2+K) - K}{6Kr(18rt-1)}, \quad \pi_{2d} = \frac{(6rt(2+K)-K)^2}{36Kr(18rt-1)}.$$

Comparing π_2^{**} with π_{2d}, we have:

$$\pi_2^{**} - \pi_{2d} = \frac{(1-K)(K - 12Krt - 36(4-K)r^2t^2)}{36Kr(18rt-1)}.$$

Considering $rt > \frac{1}{6}$ and $K \in [\frac{1}{2}, 1)$, $\pi_2^{**} < \pi_{2d}$. This indicates that retailer 2 has incentives to deviate to $y_2 = 0$. Thus, the pair of $y_1 = 1$ and $y_2 = 1$ does not appear in equilibrium.

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