Dynamic Analysis of a Disequilibrium Macroeconomic Model with Dual Labor Markets

Ogawa, Shogo

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Shogo Ogawa*

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Abstract

We extend the general disequilibrium model of Malinvaud (1980) by using dual labor market theory. By considering two tiers of workers, we find that while the duality of the labor market expands an equilibrium regime in the short term, it does not always keep an equilibrium in the medium term. In the medium term, the business cycle converges toward a disequilibrium regime unless the goods market is potentially in equilibrium. Employment and wages at the steady state are affected by the size of the government, and the stability of wage bargaining is only a sufficient condition of the local stability of our dynamic system. Therefore, involuntary unemployment can be remedied only when goods demand is sufficiently large.

Keywords: Disequilibrium macroeconomics, Non-Walrasian analysis, Segmented labor markets, Business cycles

JEL Classification: E12, E24, E32, J42

*Graduate School of Economics, Kyoto University, Yoshida Honmachi, Sakyo-ku, Kyoto (Email: ogawa.shougo.54e@st.kyoto-u.ac.jp).

1 Introduction

Involuntary unemployment has become a contentious subject of macroeconomics, and why the labor market does not clear is a worrisome problem for theoretical analyses. Today, so-called new Keynesian economists present an answer to this problem.[1] According to the new Keynesian school, involuntary unemployment is a consequence of wage rigidity. When an economy suffers an exogenous shock, the market detaches itself from the equilibrium. This disequilibrium is not solved immediately since prices are sticky. The analyses of new Keynesian economists derive this stickiness from strict micro foundations, and this approach has provided many implications for labor markets and business cycles. However, new Keynesian theory overlooks an important macroeconomics concept, namely "effective demand" in the words of Keynes (1936). In this school, the cause of involuntary unemployment is not the shortage of effective labor demand, but rather disturbance factors (e.g., price rigidity, information asymmetry) in the labor market.[2] Such a fact implies that this school does not include unemployment from effective demand in its theory, necessitating another tool with which to analyze involuntary unemployment and the labor market equilibrium.

General disequilibrium economics (or non-Walrasian economics) is one solution to this problem.[3] To illustrate the principle of effective demand, this approach uses the concept of "quantity constraints," which works on the assumption that the quantity adjustment is completed more quickly than the price adjustment.[4] In disequilibrium economics, because transactions of goods, services, and labor are conducted at the prevailing prices, the supplier (or the demander) may not deal with goods as much as it would like. Since this constraint makes it revise the actual demand or supply in another market, a constraint in one market might cause another disequilibrium. Thus, this process can serve to illustrate involuntary unemployment as the shortage of effective labor demand caused by the goods market. Furthermore, this approach comprehends an equilibrium by the price adjustment(Benassy, 1986). Therefore, we might derive an important implication for the persistence of involuntary unemployment.

Malinvaud (1980) builds a disequilibrium macroeconomic model that consists of the goods market, labor market, and capital market to analyze the persistence of involuntary unemployment. He carries out a dynamic analysis of Keynesian unemployment,[5] concluding that involuntary unemployment sustains and that full employment is not achieved automatically. Since Malinvaud's analysis is limited to the "Keynesian" regime, which is characterized by excess supply in the goods market, Osumi (1992) extends his model by taking regime switching into account. Osumi argues that the persistence of involuntary unemployment depends on the size of potential excess demand in the goods market. He also derives a business cycle among disequilibrium regimes and finds that convergence into an equilibrium regime happens only when the goods market potentially clears. Although the above-mentioned analyses draw suggestive conclusions about unemployment and the

[1] Mankiw (1990) summarizes some such analyses.

[2] Tobin (1993) insists that involuntary unemployment is caused by demand constraints rather than nominal price rigidity.

[3] After Clower (1965) analyzed the rationed supply of goods, Barro and Grossman (1971, 1976), Benassy (1975), Malinvaud (1977), and Muellbauer and Portes (1978) presented the general disequilibrium model. Ito (1985), Benassy (1986), and Heijdra (2009, Chapter5) introduce studies of this approach. For dynamic analyses, see Böhm (1978), Ito (1980), and Honkapohja and Ito (1983).

[4] Leijonhufvud (1967) upholds this assumption.

[5] Specifically, he classifies involuntary unemployment into *Keynesian unemployment*, which is caused by a constraint on goods supply, and *classical unemployment*, which is compatible with the excess demand of goods. The latter is caused by insufficient productive capacity.

equilibrium, these models are still too limited: labor supply is fixed and the labor transaction is independent of wages. In their models, single labor supply is constant and labor demand is determined by actual output, meaning that unemployment is only a matter of the output. Indeed, some workers are employed independently of wages in the short term, even though some liquid workers (e.g., part-time workers, temporary employees) do exist. To treat these contrary employment types simultaneously, we need a new tool for the labor market, namely *dual labor market* theory.

Despite the plethora of jobs in the market, economists have typically built models that have a single type of labor, which is sufficient to research macroeconomic topics. However, the importance of several labor markets is rising. Against this background, Doeringer and Piore (1985) introduce *dual labor market* theory, which contains two tiers of labor; primary workers and secondary workers.[6] Primary workers are highly skilled and they join a union to enjoy high wages, high benefits, and employment security, while secondary ones do not. Moreover, primary workers are protected by the union and thus their employment is *internal*. By contrast, the market for secondary workers is competitive and thus *external*.

How does this duality work in macroeconomic models? The first focal point is the effect on the income distribution among classes. Analyses of distributions usually focus on workers and capitalists; however, some studies extend this class conflict and distinguish workers into two types by introducing the middle class or manager class.[7]

The second focal point is employment forms or wage settings. According to dual labor market theory, different employment forms exist simultaneously and labor markets affect each other. For example, Saint-Paul (1996) argues that the duality of labor markets could stabilize the economy. In his analysis, primary workers are a quasi-fixed factor of production; however, the liquidity of secondary workers provides flexibility to production and prices. Hence, the economy will be stabilized by such duality. Osumi (1999) also researches several *internal* employment forms and examines how duality works in the face of an exogenous shock. Although the duality of the labor market seems to reduce the business cycle in these studies, the situations are limited. In other words, they see stabilization by duality only when the economy suffers an exogenous shock.

Recent studies have analyzed how labor market duality works in an endogenous business cycle model. For example, Flaschel and Greiner (2011) and Flaschel et al. (2012) extend Goodwin (1967) and conclude that minimum and maximum wage regulations mitigate business cycles. Sasaki et al. (2013) presents a Kaleckian model with regular and non-regular workers and studies how the wage gap and wage regulations affect wages and employment at the steady state. These works explicitly illustrate the interaction between the goods market and heterogeneous employment.

In this study, we build a general disequilibrium model with dual labor markets to examine how stabilization works in a macroeconomic system and the persistence of unemployment. We use Malinvaud's (1980) model, reformulated by Osumi (1992), in which the profitability of the firm determines the dynamics (or investment function). In our model, wage bargaining also defines the dynamics, allowing us to show that the conflict between the firm and workers affects the stability of macroeconomic dynamics.[8]

The rest of this paper is organized as follows. In the next section, we present a

[6]This duality is empirically analyzed by using a switching regression model in Dickens and Lang (1985), Arthur and Chen (1991) and Ishiwaka (1999). Kalantzis et al. (2012) shows the existence of this duality in the Lost Decade in Japan.

[7]See Lavoie (2009), Palley (2014), Tavani and Vasudevan (2014), Dutt et al. (2015), and Palley (2016). Sasaki et al. (2013) and Sasaki (2015) use another distinction of workers: regular and non-regular.

[8]Charpe et al. (2015) also build a dynamic Keynesian model with dual labor markets; however, their work focuses on investment and policy.

disequilibrium model with two types of workers. In section 3, transactions in a static economy are investigated. In our model, wages and the productive capacity of capital characterize the economic regimes. We also see that duality expands an equilibrium regime in Section 3. In Sections 4 and 5, we formulate the dynamics of the variables and implement a dynamic analysis. Since our model has a "medium-term" framework, the importance of which is emphasized by Solow (1988), we analyze adjustments to productive capacity and nominal prices in this section. We also induce a business cycle in the medium term and analyze the steady states. Section 6 summarizes the analysis.

2 The model

In this section, we build a macroeconomic disequilibrium model based on Malinvaud (1980), Osumi (1992), and Yoshikawa (1995, Chapter 5).

We suppose three types of economic agents: a representative firm, households, and the government. These all deal with goods and labor in a closed economy.

We first present equations that express the transactions of goods. Let Y denote the realized net output, which is aggregated. We assume that this is equal to the minimum of effective demand and supply:

$$Y = \min(Y^d, Y^s), \tag{2.1}$$

where Y^d and Y^s are the aggregated effective demand and supply of goods, respectively. The former includes consumption demand C, investment demand I, and government demand G, meaning that

$$Y^d = C + I + G. \tag{2.2}$$

Effective supply is determined by existing capital and labor.

In the following, we formulate the activities of the three units and then specify the system to analyze the short-term equilibria.

2.1 Households

We assume that there are homogeneous households that consume goods and supply labor.

First, their consumption is assumed to be the simplest Keynesian consumption function:

$$C = cY, \ 0 < c < 1, \tag{2.3}$$

where c is the consumption propensity and constant. Although this formulation seems crude, it is useful for analyzing the multiplier effect.[9]

Second, households supply two types of labor, namely primary workers L_1 and secondary workers L_2:

$$L_1^s = \bar{L}_1 = \text{const}, \tag{2.4}$$

$$L_2^s = L_2^s\left(\frac{w_2}{w_1}\right), \ L_2^{s\prime} > 0, \ \text{where } w_i = \frac{W_i}{P}. \tag{2.5}$$

[9]The assumption of the Keynesian consumption function is compatible with disequilibrium models. For instance, Clower (1965) argues that consumption depends on the realized income because of the dual nature of the decision and Benassy (1986) derives this form from a household's optimization.

The subscript s is supply, W_i is the nominal wage of L_i, and P is the price.

Since L_1^s is the supply of primary workers, we suppose that this is constant. Further, their wages are determined by wage bargaining between the firm and union, and therefore W_1 is also constant in the short term. We suppose that the real primary wage w_1 is bounded or $w_1 \in [\underline{w}_1, \bar{w}_1]$. On the contrary, secondary labor is additive for households, and thus the supply of such workers depends on relative wages.

2.2 The firm

The representative firm produces single goods by using capital and labor and maximizes its profit. We suppose that its production function is a fixed coefficient function:

$$Y = \min(\bar{Y}, L), \tag{2.6}$$

where $\bar{Y}$ and L are the productive capacity of capital and labor, respectively.

We use a fixed proportion production function for two reasons. First, because our analysis focuses on the short- and medium-term economy, we should abstract the substitution between capital and labor. Second, this production function enables us to specify the states of the two markets easily since we need only one equation to compare productive capacity, labor supply, and aggregate demand.

The firm controls the employment of both types of workers to maximize its profit since the capital stock is given in the short term. Thus, we should first analyze the relationship between labor and profit.

2.2.1 Employment and profit maximization

The firm can employ two types of workers and adjust them to maximize real profit. Real profit in the short term π is

$$\pi = Y - w_1 L_1 - w_2 L_2, \tag{2.7}$$

where w_i is the real wage of L_i. To derive the effective labor output, we analyze the labor allocation here.

First, we suppose that the labor output function can be expressed by ϕ, which is a sufficiently smooth function of the two types of workers and

$$L = \phi(L_1, L_2), \phi_i > 0,\ \phi_{ii} < 0,\ \phi_{12} = \phi_{21} > 0, \tag{2.8}$$

where $\phi_i = \frac{\partial \phi}{\partial L_i}$, $\phi_{ii} = \frac{\partial^2 \phi}{\partial L_i^2}$, $i = 1, 2$.

In the case with no output constraint, output is determined by L and profit maximization shows that there is an optimum:

$$Y_M(w_1, w_2) \equiv \underset{Y=L}{\arg\max}\{Y - w_1 L_1 - w_2 L_2\} \text{ subject to } L_1 \le \bar{L}_1. \tag{2.9}$$

If the capital capacity is sufficient, then the firm desires this quantity. Hence, this is the notional optimal output.

When the output quantity is confined, on the contrary, the firm solves the following cost minimization problem:

$$\min\{w_1 L_1 + w_2 L_2\} \text{ subject to } \phi(L_1, L_2) = Y. \tag{2.10}$$

The first-order condition is

$$\frac{w_1}{w_2} = \frac{\phi_1(L_1, L_2)}{\phi_2(L_1, L_2)}. \tag{2.11}$$

Figure 1 shows that the employment of both types of workers depends on output and relative wages.

To begin the labor allocation analysis, we examine the secondary labor market. Eq. (2.11) shows that effective demand for L_2 is decreasing in w_2 and increasing in Y and w_1. Hence, the secondary labor market can be expressed as shown in Figure 2. The market clearing condition is

$$L_2^s(w_2^*; w_1) = L_2^d(w_2^*; w_1, Y). \tag{2.12}$$

The wage w_2^* is increasing in Y and w_1, while employment L_2^* is independent of w_1 as long as Eq. (2.11) holds. Therefore, the realized relative wage $\frac{w_1}{w_2^*}$ is fixed when $L_1 \le \bar{L}_1$. This implies that effective primary labor demand depends only on output Y as long as we consider secondary labor market clearing.

By contrast, the primary labor transaction is conducted at the given w_1:

$$L_1 = \min\left(\bar{L}_1, L_1^d(Y)\right), \text{ where } \phi\left(L_1^d, L_2^*(Y)\right) = Y. \tag{2.13}$$

Primary labor demand is increasing in output, and thus an output quantity ensures full employment for L_1. We denote this as Y_f:

$$\bar{L}_1 = L_1^d(Y_f). \tag{2.14}$$

We now return to the notional optimum $Y_M(w_1, w_2^*(w_1))$. This output level depends only on w_1 and is decreasing in w_1, and we suppose

$$Y_M(\bar{w}_1) < Y_f < Y_M(\underline{w}_1), \tag{A1}$$

This inequality is intuitive: when they earn a low wage, employing all primary workers is optimal; however, this is not the case for a too high wage. Eq. (A1) indicates

$$Y_f = Y_M(w_1^*), \ \exists w_1^* \in [\underline{w}_1, \bar{w}_1], \tag{2.15}$$

according to the intermediate value theorem. At w_1^*, full utilization for primary workers is notionally optimal; hence, this wage rate can be seen as suitable.

Hence, the optimized labor output is

$$L = \min(\bar{Y}, Y_M(w_1)), \tag{2.16}$$

and thus we have found the "effective" supply of goods:

$$Y^s = \min(\bar{Y}, L) = \min(\bar{Y}, Y_M(w_1)). \tag{2.17}$$

Our effective supply differs slightly from that of other disequilibrium models. While it is generally rationed by labor supply, this is not the case in this study. Indeed, in our model, profit maximization alternatively confines supply to $Y_M(w_1)$.

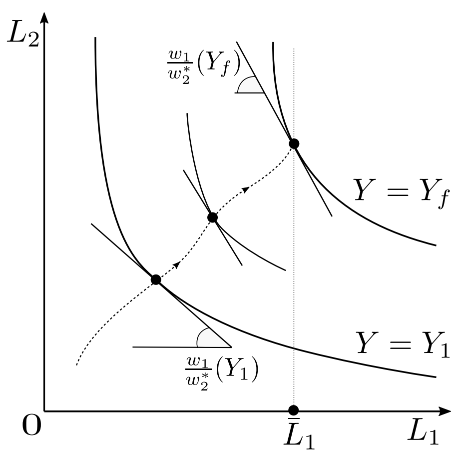

Figure 1: The isoquant curve for labor output

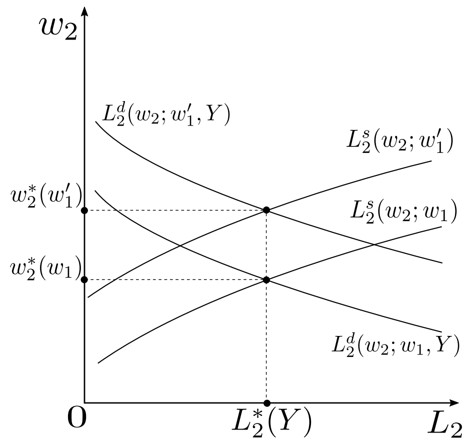

Figure 2: Secondary labor market at fixed Y and $w_1 < w_1'$

2.2.2 Investment under uncertainty

The capital stock is fixed in the short term but adjustable by a net investment in the medium term. Thus, the firm adjusts its capital stock as a net investment to maximize real future profit; such an adjustment is independent of the optimization of real profit π since it is a dynamic problem. In this study, we formulate the investment following Malinvaud (1980), considering the uncertainty of the goods market:[10]

$$I = \alpha[\min(Y^d, Y_M(w_1)) - \bar{Y}] + \beta(w_1^* - w_1), \alpha, \beta > 0. \tag{2.18}$$

This investment function implies that the firm tends to expand its equipment both (1) when its productive capacity is less than that desired and (2) when the rent from a primary worker measured by $w_1^* - w_1$ is large.

2.3 Government

The government only implements its fiscal policy: in our model, it only demands goods $G > 0$ and is an exogenous variable. Note that G might be interpreted variously. As this is the remainder of effective demand, it can include some autonomous goods demand such as autonomous consumption and investment. By way of a simplification, these are

[10] We introduce his resolution. First, define a new variable:

$$\hat{Y} = \min(Y^d, Y_M(w_1)).$$

The variable $\hat{Y}$ can be considered to be the desired productive capacity and is a random variable.

Real profit π ignores the capital cost. Considering the idle equipment cost, we define the future net return R as follows:

$$R = \pi - q\max(0, \bar{Y} - \hat{Y}),\ q > 0.$$

The parameter q is the unit cost of idle equipment. The firm, which is risk-neutral, sets its productive capacity to maximize the net expected return in the future $E[R]$, meaning that the optimization problem in dynamics (P) is

$$\max_{\bar{Y}} E[R]. \tag{P}$$

In the linear economy, the solution for (P) shows the first term of Eq. (2.18).

gathered as G herein, which represents the scale of the intervention of the government. The comparative statics of G are used to assess whether the government is large or limited.

2.4 Disequilibrium system in the short term

According to the above formulations, we have gained the static macroeconomic system:

$$
\begin{aligned}
Y &= \min(Y^d, \bar{Y}, Y_M(w_1)), && (2.19a)\\
Y^d &= C + I + G, && (2.19b)\\
C &= cY, && (2.19c)\\
I &= \alpha[\min(Y^d, Y_M(w_1)) - \bar{Y}] + \beta(w_1^* - w_1), && (2.19d)\\
L_1 &= \min\left(\bar{L}_1, L_1^d(Y, w_1)\right), && (2.19e)\\
L_2 &= L_2^s(w_2^*; w_1) = L_2^d(w_2^*; w_1, Y), && (2.19f)\\
w_2 &= w_2^*(Y, w_1), && (2.19g)\\
\bar{Y} &= \text{const}, && (2.19h)\\
w_1 &= \text{const}. && (2.19i)
\end{aligned}
$$

There are nine independent equations and nine endogenous variables (Y, Y^d, $\bar{Y}$, C, I, L_1, L_2, w_1, w_2), meaning that the system is closed. In the last two equations, we suppose that two variables, $\bar{Y}$ and w_1, are constant in the short term. These variables characterize the economy in each time period.

3 Transactions in the short term

We attained the static disequilibrium system in the preceding section. In this section, we distinguish economic regimes by using the pair of variables w_1 and $\bar{Y}$. Our economic model has three disequilibrium regimes and two equilibrium regimes, which are displayed as domains on the $\bar{Y}$–w_1 plane.

We first distinguish which variable determines the realized output.

1. $Y = Y^d$
   The cases $Y^d \leq \bar{Y} \leq Y_M$ and $Y^d \leq Y_M \leq \bar{Y}$ are necessary and sufficient for the condition $Y = Y^d$. Since output is determined by effective demand, effective demand Y^{d*} has a multiplier effect here:

   $$Y^{d*} = \frac{1}{k}[\beta(w_1^* - w_1) + G - \alpha\bar{Y}], \text{ where } k = 1 - c - \alpha \tag{3.1}$$

   and the condition of $Y = Y^d$ is that $Y^{d*} \leq Y_M$ and $Y^{d*} \leq \bar{Y}$. We assume $0 < k < 1$, which is the stability condition of the multiplier process.

2. $Y = \bar{Y}$
   In this case, aggregate demand Y^d changes depending on the other variables:

   $$
   \begin{aligned}
   Y^d &= \tfrac{1}{1-\alpha}[\beta(w_1^* - w_1) + G + (c-\alpha)\bar{Y}] && \text{if } \bar{Y} \leq Y^d \leq Y_M, && (3.2)\\
   Y^d &= \beta(w_1^* - w_1) + G + (c-\alpha)\bar{Y} + \alpha Y_M && \text{if } \bar{Y} \leq Y_M \leq Y^d, && (3.3)
   \end{aligned}
   $$

   and the region $Y = \bar{Y}$ is the union of the regions $\bar{Y} \leq Y^d \leq Y_M$ and $\bar{Y} \leq Y_M \leq Y^d$. We assume $c - \alpha > 0$, which implies that effective demand increases as productive capacity rises when capacity is low.

3. $Y = Y_M$
   In this case, Y^d has a unique form:

$$Y^d = \beta(w_1^* - w_1) + G - \alpha\bar{Y} + (c+\alpha)Y_M, \tag{3.4}$$

   and we should illustrate the region of the intersection $Y_M \leq Y^d$ and $Y_M \leq \bar{Y}$ as the region $Y = Y_M$.

The three cases distinguished to determine the realized output are shown as regions in Figure 3.[11] Two variables are "effective" to determine the realized output on the boundary lines between two regions. The point E^M means $Y^d = \bar{Y} = Y_M(w_1)(= Y)$. On the right side of the figure, there exists excess supply for goods. Note that the exogenous variable G shifts the boundary lines up and down. Here, we assume the following equation:

$$\beta + kY_M'(w_1) > 0, \ \forall w_1 \in [\underline{w}_1, \bar{w}_1]. \tag{A2}$$

This inequality means that the rise in primary wages restrains investment. Under this assumption, the expansion of G, which means that effective demand expands, enlarges the region of $Y^s < Y^d$, as shown by Osumi (1992).

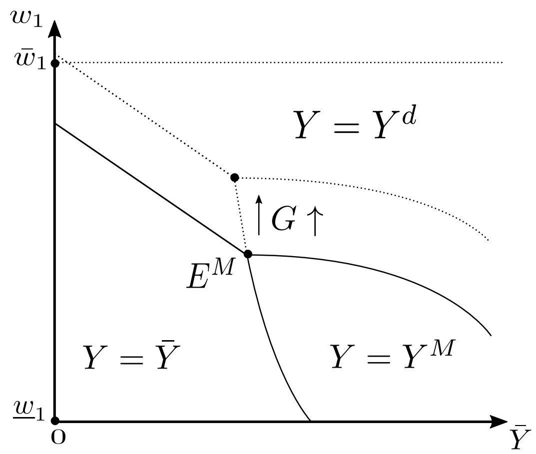

Figure 3: Determinants of output on the $\bar{Y}$–w_1 plane

Furthermore, aggregate demand Y^d changes depending on the magnitude of the correlations among Y^d, $\bar{Y}$, and $Y_M(w_1)$ because the multiplier process working in the model differs according to effective supply and demand.

Next, we analyze the equilibrium or disequilibrium regimes in the static system. We formulate the conditions of each regime and depict each as a region on the $\bar{Y}$–w_1 plane.

3.1 Disequilibrium regimes

3.1.1 Keynesian unemployment

In this regime, the realized output is rationed by aggregate demand and primary workers are not completely employed. In other words, involuntary unemployment occurs because of the lack of effective demand in the goods market. This disequilibrium regime is expressed by the following equation:

$$Y = Y^d < Y_f. \tag{3.5}$$

[11] This figure is the same as the regime dividing in Osumi (1992), which treats a single labor.

This condition is the correlation between w_1 and $\bar{Y}$, and it implies that the region is a subset of the region $Y = Y^d$.

3.1.2 Classical unemployment

In the classical unemployment regime, involuntary unemployment occurs even though excess goods demand claims more production. In other words, excess demand exists in the goods market and excess supply exists in the primary labor market, meaning that

$$Y = Y^s < Y_f. \tag{3.6}$$

3.1.3 Repressed inflation

In this regime, the economy has excess labor demand and excess goods demand. Hence, the firm would no longer employ more workers despite it being physically possible. The repressed inflation region is the interior of the region $Y = Y_M$:

$$Y_f \le Y = Y_M(w_1) < Y^d, \bar{Y}. \tag{3.7}$$

3.2 Equilibrium regimes

We call the states in which primary workers are completely employed and the goods market is rationed by either productive capacity or demand, the "equilibrium." Therefore, the condition of the equilibrium regime is

$$Y_f \le \min(Y^d, \bar{Y}) \le Y_M(w_1). \tag{3.8}$$

This condition implies that the equilibrium region is the one of the union set of the closures of the subsets of $Y = Y^d$ and $Y = \bar{Y}$.

In the equilibrium regime, the firm employs all primary workers and operates to maximize profit under its physical constraints (i.e., capital stock and goods demand).

Considering the relevance to Walrasian economics, we divide this regime into two: the full equilibrium, which is the same as the general equilibrium in standard Walrasian economics, and the quasi-equilibrium, in which the goods market does not clear.

3.2.1 Full equilibrium

We name the state in which both markets clear as the "full equilibrium." The condition is

$$Y_f \le Y^d = Y^s. \tag{3.9}$$

In this regime, productive capacity is equal to goods demand and labor markets also clear; thus, the regime is equivalent to the general equilibrium in Walrasian economics.[12]

Indeed, this full equilibrium is incompatible with full utilization when G is too large: if potential aggregate demand is larger than $Y_M(w_1^*)$, then excess capacity happens in the full equilibrium. To see that, define w_1^M as the wage at point E^M. Then, the following equation holds:

$$\frac{1}{k'}G \gtreqless Y_M(w_1^*) \iff w_1^M \gtreqless w_1^*, \tag{3.10}$$

[12]We do not call this equilibrium regime the Walrasian equilibrium since we omit the dynamic process here. In this study, the Walrasian equilibrium is defined as the steady state, which is the full equilibrium.

where $k' = 1 - c$. $\frac{1}{k'}G$ in the first comparison could be called potential aggregate demand or aggregate demand at the steady state, since it is attained when $I = 0$ and $Y = Y^{d*}$. As w_1^M is the minimum wage that satisfies $Y^d = \bar{Y}$, the full equilibrium does not include full utilization when $w_1^M > w_1^*$.

3.2.2 Quasi-equilibrium

The quasi-equilibrium regime is the remainder of the equilibrium regime with the full equilibrium removed. The condition is

$$Y_f \leq Y, \; Y^d \neq Y^s. \tag{3.11}$$

3.3 Dividing the economy into regimes

From the above arguments, we can display these regimes on the $\bar{Y}$–w_1 plane, as shown in Figure 4. The left side is the case with small G and the right one is the case with large G.

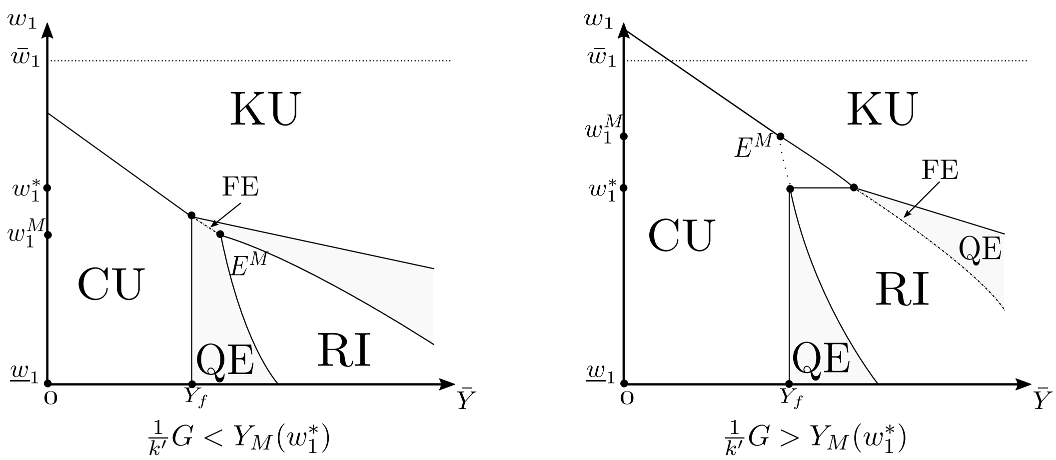

Figure 4: Dividing the regimes on the $\bar{Y}$–w_1 plane

This plane is confined to the upper limit of w_1 or $\bar{w}_1$, but this is unimportant. Thus, we suppose that $\bar{w}_1$ is sufficiently large to omit its limitation.

From Figure 4, we can characterize each regime. First, the Keynesian unemployment regime has high wages and abundant productive capacity. In this regime, the low profit rate and plentiful capital stock restrain investment, and the low effective demand for goods causes the involuntary unemployment of primary workers.

Second, the classical unemployment regime has low productive capacity. In this regime, the low capital stock reduces labor demand and some workers would not be hired even if wages are low.

Third, the repressed inflation regime has low wages and large productive capacity. The substitution of workers promotes the employment of primary workers because of their low wages. Moreover, the low wage rate (high profit rate) accelerates investment. Therefore, effective demand rises and the capital stock is accumulated excessively. In this regime, the firm maximizes its notional profit.

Finally, the equilibrium region lies around repressed inflation. This fact implies that this regime is characterized by a moderate wage and an adequate capital stock, which seems reasonable. In particular, when $\frac{1}{k'}G \neq Y_M(w_1^*)$, the full equilibrium regime is expressed as the curve.[13]

We should note that a too high wage ($w_1 > w_1^*$) causes unemployment, whereas a fall in wages does not always solve unemployment: if capital is insufficient, wage lowering brings about the classical unemployment regime because unemployment is not a problem for the labor market, but rather a problem for the whole economy.

For the competitive statics, we should compare the two figures. As G becomes large or the government expands, the boundary lines shift upward, indicating that Keynesian unemployment shrinks, while classical unemployment expands. Further, the equilibrium and repressed inflation, which show full employment, also expand. This finding implies that a large government solves unemployment with low goods demand.

4 Preparation for the dynamic analysis

In this section, we formulate the dynamics of two variables, w_1 and $\bar{Y}$, which were supposed to be constant in the short term.

4.1 Investment and productive capacity

Net investment I has a dual effect: it is a component of effective demand in the short term, while it adjusts the capital stock in the medium term. Eq. (2.18) indicates that the dynamics of capital productivity $\bar{Y}$ are expressed as the following equation:

$$\dot{\bar{Y}} = I = \alpha[\min(Y^d, Y_M(w_1)) - \bar{Y}] + \beta(w_1^* - w_1), \tag{4.1}$$

where t is time and $\dot{X} \equiv \frac{dX}{dt}$ is the dynamics of a time variable $X(t)$.

4.2 Wage bargaining and nominal wage dynamics

To formulate the dynamics of real wages w_1, we consider the wage bargaining of nominal wage W_1 because the firm and primary workers are supposed to bargain with the price as given. In addition, such bargaining determines only the wage since we adopt the "right to manage" model for the primary labor market: employment is given when bargaining is conducted. Considering our assumptions, we adopt static arguments such as McDonald and Solow (1985) to assess the wage dynamics.

The factors that affect this wage bargaining include the payoffs of the two economic units (the firm and primary workers) and relative bargaining power of the union, meaning that

$$\frac{\dot{W}_1}{W_1} = \frac{\dot{W}_1}{W_1}(\sigma, w_2, w_1^*),\ \frac{\partial}{\partial \sigma}\left(\frac{\dot{W}_1}{W_1}\right) > 0,\ \frac{\partial}{\partial w_2}\left(\frac{\dot{W}_1}{W_1}\right) > 0,\ \frac{\partial}{\partial w_1^*}\left(\frac{\dot{W}_1}{W_1}\right) > 0, \tag{4.2}$$

and σ is the relative bargaining power of the union.[14] Then, we analyze each variable and build a simple equation.

[13] The line of the full equilibrium is peculiar to our model: conventional disequilibrium models depict the full equilibrium as a point on a plane. On the contrary, the present study supposes the equilibrium of the secondary labor market and therefore output can represent a variable number under the complete employment of L_1. The segment implies that the general equilibrium might not have uniqueness in a static system. This indeterminacy would be solved in a dynamic analysis.

[14] The background of this equation is presented in Appendix A.

Suppose that σ depends on the number of union members; then, the variable is affected by the unemployment rate u:

$$\sigma = \sigma(u),\ \sigma' < 0 \text{ where } u = \frac{\bar{L}_1 - L_1}{\bar{L}_1}. \tag{4.3}$$

Employment for L_1 expands as output increases as long as $Y < Y_f$; therefore, we rearrange the equation above:

$$\sigma = \sigma(Y),\ \sigma' \begin{cases} > 0 & \text{if } Y < Y_f \\ = 0 & \text{if } Y \geq Y_f. \end{cases} \tag{4.4}$$

The wages of secondary workers w_2 are determined by output Y in each time period, which is an increasing function of L_2:

$$w_2 = w_2^*(Y),\ w_2^{*\prime} > 0 \tag{4.5}$$

These two variables are the factors that affect the attitude of workers. The equations above indicate that the wage w_1 aims to rise when the realized output Y is large.

Finally, we assume that the wage rate desired by the firm w_1^* is the objective rate in the medium term, which implies that the firm would prefer not to diverge from it.

The discussions above revise the formulation of Eq. (4.2):

$$\begin{aligned} \frac{\dot{W}_1}{W_1} &= \frac{\dot{W}_1}{W_1}\left(\sigma(Y), w_2(Y), w_1^*\right) \\ &= \frac{\dot{W}_1}{W_1}(Y, w_1^*),\ \frac{\partial}{\partial Y}\left(\frac{\dot{W}_1}{W_1}\right) > 0,\ \frac{\partial}{\partial w_1^*}\left(\frac{\dot{W}_1}{W_1}\right) > 0. \end{aligned} \tag{4.6}$$

We use a linear dynamic function for simplification:

$$\frac{\dot{W}_1}{W_1} = \omega_1 \left[Y - Y_f\right] + \omega_2(w_1^* - w_1),\ \omega_1,\ \omega_2 > 0, \tag{4.7}$$

where ω_i is constant. The first term on the right side indicates that employment raises workers' bargaining power and that the wage rises after they are completely employed.[15] The second term shows that the firm wants the desired wage rate w_1^* and bargains to achieve it.

4.3 Price dynamics in the goods market

The price of goods P changes according to a disequilibrium in the goods market. That is, we suppose that a Walrasian adjustment process exists in the goods market:

$$\frac{\dot{P}}{P} = \rho(Y^d - Y^s),\ \rho > 0, \tag{4.8}$$

where ρ is the adjustment speed. Therefore, we can now formulate the dynamics of the real wage w_1:

$$\frac{\dot{w}_1}{w_1} = \frac{\dot{W}_1}{W_1} - \frac{\dot{P}}{P} = \omega_1(Y - Y_f) + \omega_2(w_1^* - w_1) - \rho(Y^d - Y^s). \tag{4.9}$$

[15] Goodwin (1967) uses a similar equation: his wage dynamics also depend on the present level of employment. Considering the proportional production function, moreover, we can say that our formulation concurs with that of Dutt (1992) since the employment rate and output have a positive correlation.

Although this is a linear function, the real wage dynamics are somewhat complicated: they depend on the realized output and effective supply and demand, and thus they differ in each regime.

We now have a macroeconomic system in the medium term, which consists of Eqs. (2.19a)–(2.19g) from Section 2 as well as Eqs. (4.1) and (4.9) from this section. In the next section, we analyze the stability and cycles in this dynamic system.

5 Dynamic analysis

We formulated the dynamics of the two variables $\bar{Y}$ and w_1 in Section 4. In this section, we use them to analyze the dynamics of our disequilibrium model.

We first examine the dynamics of these two variables with Eqs. (4.1) and (4.9) and then analyze the dynamics of our model by using phase diagrams.

5.1 Dynamics of the two variables

Productive capacity is adjusted along with Eq. (4.1) in the medium term. This equation includes effective demand Y^d and differs in each regime.

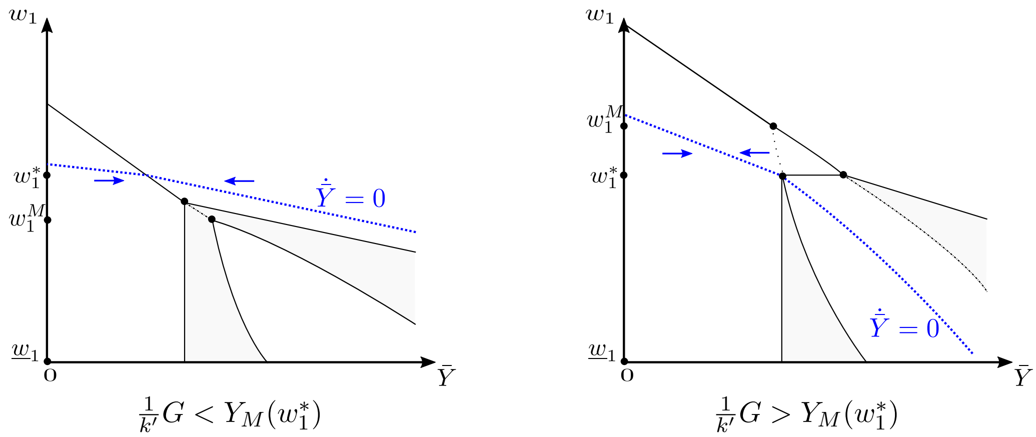

Figure 5: The dynamics of $\bar{Y}$

To create a phase diagram, we illustrate the set of points that fulfills $\dot{\bar{Y}} = 0$ on the $\bar{Y}$–w_1 plane (see Figure 5). That is downward-sloping, and productive capacity is adjusted to this nullcline.[16]

The scale of G transforms this nullcline and we present two cases: $\frac{1}{k'}G > Y_M(w_1^*)$ and $\frac{1}{k'}G < Y_M(w_1^*)$. In the former, potential aggregate demand is sufficient for notional profit maximization. This is a profitable case for the firm. Capital equipment reduces when $Y = Y^d$ and is adjusted to an excess-demand point when $w_1 = w_1^*$. In the latter, the line $\dot{\bar{Y}} = 0$ crosses the region $Y = Y^d$ and $\bar{Y}$ is adjusted to the equilibrium of the goods market when $w_1 = w_1^*$.

The diagrams show that if effective goods demand is high, the firm does not need to invest as much to create demand.

The dynamics of w_1, or the wages of primary workers, are shown in Eq. (4.9). This equation also includes the variables that are effective, which differ by regime. Figure 6 shows the $\dot{w}_1$–nullclines, where w_1 rises on the right side of the curve and falls on the

[16]We can confirm that $\frac{\partial}{\partial \bar{Y}}\dot{\bar{Y}} < 0$ for any coordinates.

opposite side. This diagram indicates that abundant capital equipment is profitable for workers during wage bargaining; thus, the nominal wage aims to rise and large supply in the goods market aims to lower prices.

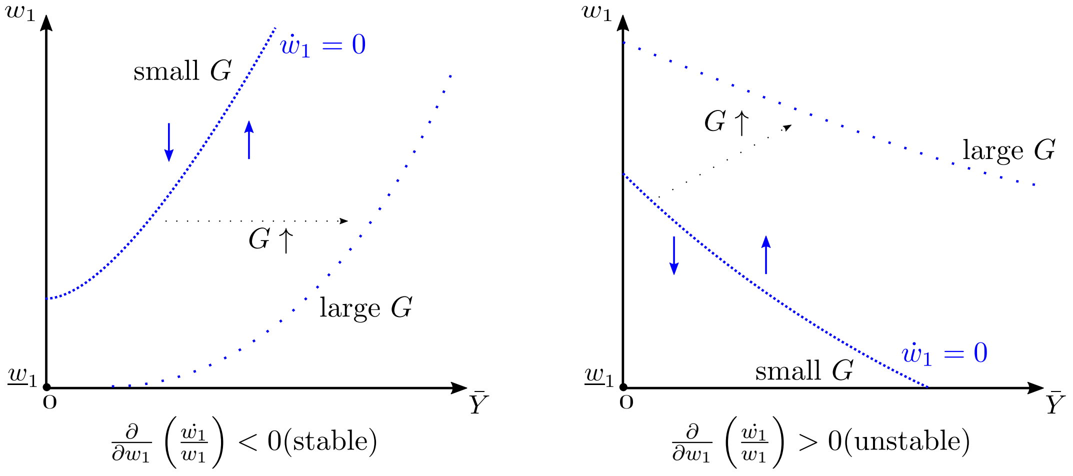

Figure 6: The dynamics of w_1

The scale of G changes this diagram: as G rises, the curve expands to the right side because high effective demand in the goods market restrains prices and a price drop raises the real wage. Note that when $\frac{1}{k'}G = Y_M(w_1^*)$, or the goods market is potentially in equilibrium, the isocline $\dot{w}_1 = 0$ crosses E^M.

We divide the dynamics of w_1 into two cases relating to wage adjustment stability.[17] We should see the sign of

$$\frac{\partial}{\partial w_1}\left(\frac{\dot{w}_1}{w_1}\right). \tag{5.1}$$

If this is negative, the wage dynamics are stable. In other words, wages have a centripetal force if you ignore the goods market.

5.2 Macroeconomic dynamics: cycles and convergences

We can now depict the phase diagrams. Our dynamics differ in the scale of G and the stability of wage bargaining; concerning the latter, we focus on the stable case.

To see the convergence, we define the steady state as that in which both our subjective variables are constant: on the $\bar{Y}$–w_1 plane, the steady state is expressed as the intersection between $\dot{\bar{Y}} = 0$ and $\dot{w}_1 = 0$.

Figure 7 illustrates the phase diagrams with two cases: small G (left) and large G (right). These diagrams show that our model has a counterclockwise cycle : excess capacity reduces investment, which then also reduces effective demand in the goods market. Then, the realized output decreases and the price rises, which is disadvantageous for workers as real wages fall. The low wage expands profit and thus the firm invests to produce more, rising wages again because of the large output.

The business cycle is at the steady state and sometimes falls into dysfunction; as a consequence, output reduces to zero. This failure of wages occurs even if wage bargaining is stable.

[17] The isoclines in Figure 6 are not general: we can illustrate another form when the magnitude relation among the parameters changes. We adopt the simple and typical case in this study (see Appendix B).

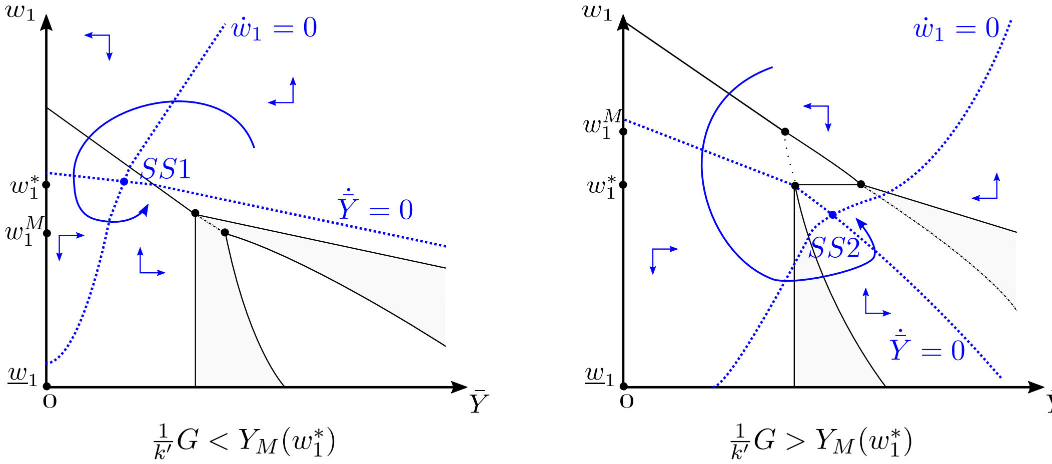

Figure 7: Phase diagrams with stable wage bargaining

Our dynamic system has a unique steady state in each case. In the small G case, we call it $SS1$. $SS1$ is in the domains $Y^s < Y^d$ and $w_1 > w_1^*$, which suggests that the cycle converges toward the classical unemployment regime. Low goods demand weakens the firm, so the wage rises and low profitability restrains investment.

On the contrary, the economy with large G reaches the steady state in the domains $Y = Y^M$ and $w_1 < w_1^*$, or in the repressed inflation regime. We call this $SS2$. In this scenario, the firm is aggressive in its wage bargaining since there is abundant goods demand and notional profit maximization is achieved.

Note that the equilibrium regime is unstable in these cases: bargaining and capital adjustment have a centrifugal force from the equilibrium unless notional goods supply and potential goods demand coincide.

5.2.1 Walrasian equilibrium

As long as $\frac{1}{k'}G \neq Y_M(w_1^*)$, the steady state is in a disequilibrium regime, as shown above. However, we attain the steady state in an equilibrium regime when G is adequate:

$$\frac{1}{k'}G = Y_M(w_1^*). \tag{5.2}$$

Under this condition, the cycle converges toward the full equilibrium regime. We call this steady state the Walrasian equilibrium, which achieves the market equilibriums, notional profit maximization, and full utilization for capital capacity.

5.2.2 Wage stability and macroeconomic stability

From the above analysis, we can conclude that the scale of G or demand of the government affects the steady state. On the contrary, the stability of wage bargaining affects the stability or cycle of the economy. The important result is this:

Proposition. *If wage bargaining is stable, the steady state is locally stable.*

Proof. At the steady state, the Jacobian matrix of our dynamic system is as follows:

$$\mathbf{J} = \begin{pmatrix} -\alpha & j_{12} \\ j_{21} & \frac{\partial}{\partial w_1}\left(\frac{\dot{w}_1}{w_1}\right) \end{pmatrix}, \text{ where } j_{12} < 0,\ j_{21} \geq 0. \tag{5.3}$$

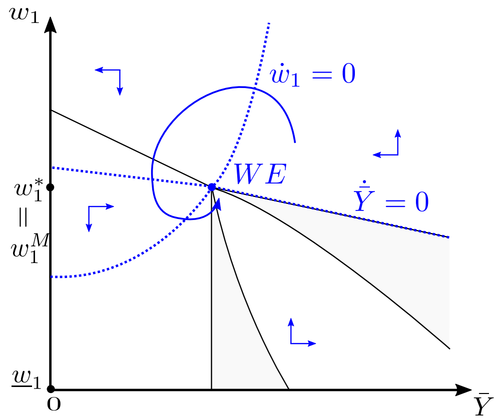

Figure 8: Walrasian equilibrium

If wage bargaining is stable, the diagonal elements are both negative and thus

$$\det\mathbf{J} > 0 \text{ and } \mathrm{tr}\mathbf{J} < 0.$$

This condition shows that the steady state is locally asymptotically stable in each regime. Since the dynamic equations are continuous on every boundary, the local stability in each regime indicates that our dynamic system is locally stable, as proved by Eckalbar (1980). □

The proposition claims that wage bargaining affects the stability of the economy, whereas global stability is not ensured: even if wage bargaining is stable, the cycle might diverge from the steady state because of the disequilibrium in the goods market. This partial effect of bargaining stability shows that the interconnection between markets and adjustment process are important factors in economic stability.

Figure 9 shows the unstable case. The cycle is similar to that in the stable case, but it often diverges.

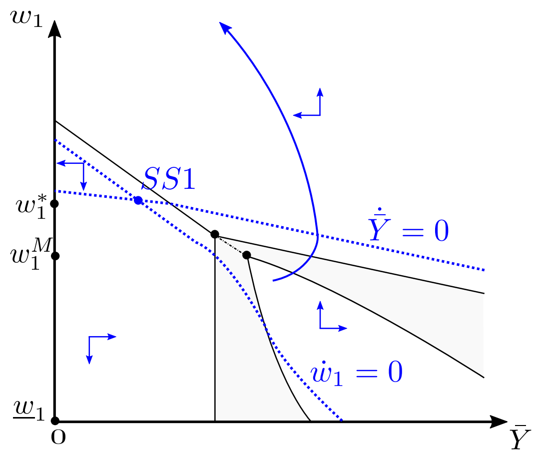

Figure 9: The phase diagram with unstable wage bargaining and $\frac{1}{k'}G < Y_M(w_1^*)$

5.2.3 Implications for government policy

As we have seen, the size of government (or the size of fiscal policy) G affects the steady states. In this subsection, we evaluate this and derive further policy implications.

First, the size of G determines the regime and wage at the steady state. When bargaining is stable, the isocline $\dot{w}_1 = 0$ is upward-sloping and large G lowers this curve as well as $\dot{\bar{Y}} = 0$. Therefore, the wage at the steady state is decreasing in G. This shift also changes the regime of the steady state. As we have seen, small G induces classical unemployment. Hence, we can conclude that a large government ensures full employment but that this employment is accompanied with low wages.

When bargaining is unstable, on the contrary, the evaluation of G is complicated: as G changes, the wage at the steady state changes non-monotonically. In this case, there is an optimal value for G to maximize the wage.

In our model, the government can implement another policy: wage regulation. The primary wage w_1 has an effective range or $[\underline{w}_1, \bar{w}_1]$. Because this is exogenous, the government can impose some regulation on this range. Obviously, suitable regulation mitigates the business cycle, since this cycle is about the primary wage.[18] As the wage at the steady states changes with the size of G, if the government implements a large fiscal policy, the minimum wage regulation should not be strict (or, rather, a maximum wage regulation is needed).

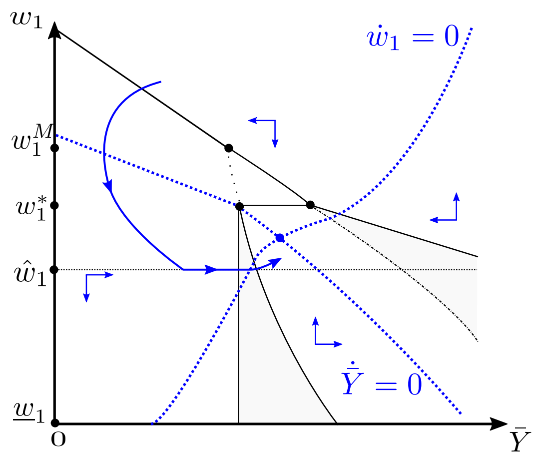

Figure 10: A mitigated cycle with a suitable minimum wage regulation $\hat{w}_1$

6 Conclusion

In this study, we analyzed the effect of dual labor markets on the disequilibrium macroeconomic model. We conclude our analysis as follows:

1. A substitutable dual labor force generates equilibrium regions (full equilibrium and quasi-equilibrium) among the disequilibrium regions, and thus the economy can stay in an equilibrium region comparatively easily in the short term.

2. In the medium term, a business cycle about productive capacity and wages occurs and sometimes the economy converges to the steady state.

[18] This result is the same as Flaschel and Greiner (2011).

3. The steady states differ in economic characteristics; when the government is sufficiently large, involuntary unemployment is solved in the medium term.

4. The equilibrium regime is unstable when the goods market is not potentially in equilibrium.

These findings imply that a dual labor market stabilizes the economy in the short term but that such stability is subtle in the medium term. Keynesian unemployment turns into another disequilibrium regime, and low goods demand induces persistent unemployment. Our analysis evaluated the effect of duality on the economic model throughout and is limited: such duality does not allow us to regard the Walrasian equilibrium as given, since the equilibrium is hardly stable. Theoretically, this limited function of the duality comes from the staticity of equilibrium theory. We suppose an excessively high adjustment speed for the secondary labor market, and this characteristic does not contribute to the dynamics, implying that the disequilibrium plays a substantial role in economic dynamics or business cycles.

In addition, as we omit the distribution here, the stability effect of the labor market is further doubtful. Liquid employment sometimes implies unstable employment, meaning that each wage would differ in its distribution effect. This fact implies that labor substitution from primary to secondary might lower goods consumption and this underconsumption might cause further unemployment. The investigation of this point remains a future research issue.

Finally, our model calls for more strict settings for dual labor markets. As Saint-Paul (1996) insists, protection for primary workers such as the firing cost is an important aspect of this labor market model. Since the firm in our model can hire and fire both types of workers freely, this characteristic might derive another conclusion. In this respect, our analysis should aid further studies of dual labor market theory and involuntary unemployment.

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A Nash bargaining and the dynamics of W_1

Here, we solve a Nash bargaining problem and derive Eq. (4.2). The bargaining on L_1 is a right-to-manage model in the present study. Hence, we can control only the variable W_1 to solve the following problem:

$$\max_{W_1} \left[u(W_1) - u(W_2)\right]^{\sigma} \left[\Gamma(W_1^* - W_1)\right]^{1-\sigma}, \tag{A.1}$$

where u is a payoff function of the union and $u' > 0, u'' < 0$, while Γ is the payoff function of the firm and we assume $\Gamma' > 0, \Gamma'' \leq 0$. $\sigma \in (0, 1)$ is the relative bargaining power of the union. The first-order condition is

$$\sigma \frac{du}{dW_1}\Gamma - (1-\sigma)(u_1 - u_2)\Gamma' = 0, \tag{A.2}$$

where $u_1 - u_2 = u(W_1) - u(W_2)$. By using the total difference, we can see the relationship among the variables:

$$\begin{aligned} &\left\{\sigma\left[u''\Gamma - u'\Gamma'\right] - (1-\sigma)\left[-\Gamma''(u_1 - u_2) + \Gamma'u'\right]\right\} dW_1 - \left[(1-\sigma)\Gamma'u'\right] dW_2 \\ &\qquad + \left\{u'\Gamma + \Gamma'(u_1 - u_2)\right\} d\sigma + \left\{\sigma u'\Gamma' - (1-\sigma)(u_1 - u_2)\right\} dW_1^* = 0. \end{aligned} \tag{A.3}$$

Therefore, it can be said that

$$\frac{dW_1}{dW_2} > 0, \ \frac{dW_1}{d\sigma} > 0, \ \frac{dW_1}{dW_1^*} > 0. \tag{A.4}$$

Recall that the price is constant in each time period; therefore, we can rewrite nominal wages as real ones. Furthermore, the wages in each time period are given and thus the direction of the dynamics of w_1 is the same as the equation above:

$$\frac{d\dot{W}_1}{dw_2} > 0, \ \frac{d\dot{W}_1}{d\sigma} > 0, \ \frac{d\dot{W}_1}{dw_1^*} > 0. \tag{A.5}$$

B Depicting the isocline $\dot{w}_1 = 0$

To complete the phase diagrams, we should know the form of the isocline $\dot{w}_1 = 0$ or $\dot{w}_1$–nullcline. For the former, note that the size of G affects this equation and we should consider the reference point $\frac{1}{k'}G = Y_M(w_1^*) = Y_f$. If this condition holds, the nullcline crosses the point E^M on the Y–w_1 plane:

$$\frac{\dot{w}_1}{w_1} = \omega_1 (Y - Y_f) + \omega_2(w_1^* - w_1^M) - \rho(Y^d - Y^s) = 0, \tag{B.1}$$

since all three terms in the middle one become zero at E^M under this condition. Next, we analyze five cases for the dynamics since the dynamic equation includes Y, Y^d, and Y^s.

By dividing the cases, we expand it as follows:

$$\frac{\dot{w_1}}{w_1}_A = \frac{\beta\omega_1 + k\omega_2 - \beta\rho}{k}(w_1^* - w_1) + \frac{\omega_1 - \rho}{k}G + \frac{-\alpha\omega_1 + k'\rho}{k}\bar{Y} - \omega_1 Y_f \tag{B.2a}$$

$$\frac{\dot{w_1}}{w_1}_B = \frac{\beta\omega_1 + k\omega_2 - \beta\rho}{k}(w_1^* - w_1) + \frac{\omega_1 - \rho}{k}G + \frac{-\alpha\omega_1 + \alpha\rho}{k}\bar{Y} + \rho Y_M(w_1) - \omega_1 Y_f \tag{B.2b}$$

$$\frac{\dot{w_1}}{w_1}_C = \frac{(1-\alpha)\omega_2 - \beta\rho}{1-\alpha}(w_1^* - w_1) - \frac{\rho}{1-\alpha}G + \frac{(1-\alpha)\omega_1 + k'\rho}{1-\alpha}\bar{Y} - \omega_1 Y_f \tag{B.2c}$$

$$\frac{\dot{w_1}}{w_1}_D = (\omega_2 - \beta\rho)(w_1^* - w_1) - \rho G + (\omega_1 + (k' + \alpha)\rho)\bar{Y} - \alpha\rho Y_M(w_1) - \omega_1 Y_f \tag{B.2d}$$

$$\frac{\dot{w_1}}{w_1}_E = (\omega_2 - \beta\rho)(w_1^* - w_1) - \rho G + \alpha\rho\bar{Y} - (\omega_1 + k\rho)Y_M(w_1) - \omega_1 Y_f, \tag{B.2e}$$

where the subscripts correspond to the regions in Figure 11. The graphs of $\dot{w}_1 = 0$ are a compound of the line $w_1 = 0$ and the curves derived by letting the right sides of the equations above be zero. Since w_1 is positive, we ignore the former.

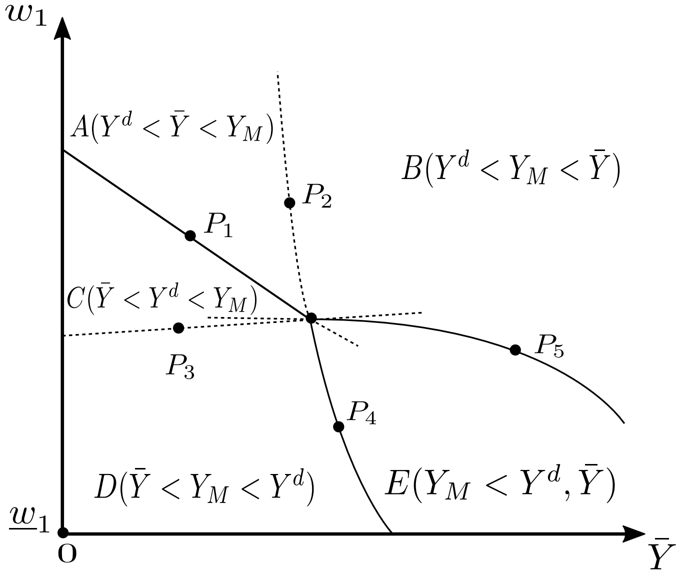

Figure 11: Region dividing and crossing points

By using the total differentials, we can check the direction of w_1, which is not on the curve $\dot{w}_1 = 0$, the slope of the curve, and the shifts when G changes. About the last one, we assume the following condition for simplification:

$$\omega_1 < \rho, \tag{A4}$$

which implies that the friction of the speed of bargaining is smaller than the whole adjustment speed of the goods market. Under this assumption, the sign of $\frac{dw_1}{dG}$ and slopes of the nullcline are common in all regions. Furthermore, we assume that the slope of the curve is always positive or negative.

To depict the curve, we should know whether the set $\dot{w}_1 = 0$ is valid in each region. For instance, can the right side of Eq. (B.2a) be zero in region A? Therefore, we check the crossing points of the region boundaries and curves $\dot{w}_1 = 0$.[19] Each region has two boundaries and if both the crossing points on them are valid, the curve $\dot{w}_1 = 0$ crosses the region. In other words, if P_1 and P_2 satisfy the validity condition, then the isocline $\dot{w}_1 = 0$ emerges in region A. As the curve crosses E^M when $\frac{1}{k'}G = Y_M(w_1^*)$ and the boundaries shift as G changes, we should know the relationship between the wage at the crossing point P_i or w_1^i and the reference wage w_1^M. In other words, we should know the signs of

$$\frac{dw_1^i}{dG} - \frac{dw_1^M}{dG}, \ i = 1, 2, 3, 4, 5. \tag{B.3}$$

If this is positive, the crossing point rises along the boundary relative to E^M as G grows. Therefore, we can check which crossing point is valid in the cases $\frac{1}{k'}G < Y_M(w_1^*)$ and $\frac{1}{k'}G > Y_M(w_1^*)$.

[19]The curves $\dot{w}_1 = 0$ are continuous on all the crossing points since the minimum function of continuous functions is explicitly continuous.