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Abstract

This paper is an up-to-date survey of the state-of-the art in consumer demand modelling. We review and evaluate advances in a number of related areas, including different approaches to empirical demand analysis, such as the differential approach, the locally flexible functional forms approach, the semi-nonparametric approach, and a nonparametric approach. We also address estimation issues, including sampling theoretic and Bayesian estimation methods, and discuss the limitations of the currently common approaches. We also highlight the challenge inherent in achieving economic regularity, for consistency with the assumptions of the underlying neoclassical economic theory, as well as econometric regularity, when variables are nonstationary.

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1 Introduction

With the exceptions of Blundell (1988) and Lewbel (1997), there has not been a survey of the consumer demand systems literature, since appearance of the authoritative survey by Brown and Deaton (1972). That major survey was followed by the highly influential book by Deaton and Muellbauer (1980b). But that literature has advanced dramatically since then, with significant theoretical and empirical advances. This paper is an up-to-date survey of the state-of-the art in consumer demand analysis. We review and evaluate advances in a number of related areas, including the duality between direct and indirect utility functions, the theory of multistage optimization, and different approaches to empirical demand analysis, including the differential approach, the locally flexible functional forms approach, the semi-nonparametric globally flexible approach, and a nonparametric revealed preference approach first associated with Varian's (1982, 1983). Applications to elasticity calculations are also discussed. Important issues that are not covered here include the effects of demographic or other variables that affect demand, welfare comparisons across households (e.g., equivalence scales), changing tastes, and some of the many issues concerning aggregation over consumers. See, e.g., Lewbel (1991), Kirman (1992), and Stoker (1993).

We also address estimation issues, including sampling theoretic and Bayesian estimation methods. We emphasize the need for economic theory to inform econometric research and argue that the usefulness of the currently popular parametric approach to empirical demand analysis depends on whether the theoretical regularity conditions of neoclassical microeconomic theory (positivity, monotonicity, and curvature) are satisfied. We also discuss econometric regularity and address integration and cointegration issues in consumer demand system estimation. We argue that many demand system studies fail to address the non-stationarity of prices and income, and we highlight the challenge inherent in achieving both economic regularity as well as econometric regularity.

We only deal with consumer choice in a static framework, ignoring dynamic models of consumer choice. However, the static neoclassical theory of consumer choice can be extended to accommodate choice problems in intertemporal environments. See, for example, Blundell (1988) for a survey of the theory of dynamic consumer behavior. Under intertemporal separability, static modelling can be applied to the efficient allocation of current period consumption expenditure over current period goods consumption. As a result, the right hand side variable in the budget constraints in this paper are not income or wealth, but realized total consumption expenditure allocated to the current period in a prior intertemporal allocation decision, which may include risk regarding future prices. We assume that current period prices are known with certainty, so that the current period realized total consumption expenditure can be modelled as allocated under certainty over current period goods.[1]

[1] If there is perfect foresight regarding future prices under intertemporal separability, the allocation of wealth over periods is a first-stage decision under perfect certainty. But so long as current period prices are known with certainty, future price uncertainty does not invalidate current period allocation under perfect

The static neoclassical model of consumer choice can also be extended to accommodate technological change, the introduction of new goods, and changes in the characteristics of the available goods. One of these widely used extensions is the theory of household production, which integrates consumer choice theory with the theory of the firm. See Becker (1965), Lancaster (1966), and Barnett (1977). Finally, merging household production theory with the theory of intertemporal consumer choice gives rise to dynamic household production theory. These and other important extensions of the static neoclassical theory of consumer choice are beyond the objectives of this paper.[2]

2 Neoclassical Demand Theory

Consider n consumption goods that can be selected by a consuming household. The household's problem is

$$\max_{\boldsymbol{x}} u(\boldsymbol{x}) \quad \text{subject to} \quad \boldsymbol{p}'\boldsymbol{x} = y, \tag{1}$$

where $\boldsymbol{x}$ is the $n \times 1$ vector of goods; $\boldsymbol{p}$ is the corresponding vector of prices; and y is the household's total expenditure on goods (often just called "nominal income" in this literature).

2.1 Marshallian Demands

The solution of the first-order conditions for utility maximization are the Marshallian ordinary demand functions,

$$\boldsymbol{x} = \boldsymbol{x}(\boldsymbol{p}, y). \tag{2}$$

Demand systems are often expressed in budget share form $\boldsymbol{s}$, where $s_j = p_j x_j(\boldsymbol{p}, y)/y$ is the expenditure share of good j, and $\boldsymbol{s} = (s_1, \cdots, s_n)'$.

Marshallian demands satisfy the following properties: (i) positivity; (ii) adding up (or summability), $\boldsymbol{p}'\boldsymbol{x}(\boldsymbol{p}, y) = y$; (iii) homogeneity of degree zero in $(\boldsymbol{p}, y)$, implying the absence of money illusion; and iv) the matrix of substitution effects, $\boldsymbol{S} = [\partial \boldsymbol{x}(\boldsymbol{p}, y)/\partial \boldsymbol{p}' + (\partial \boldsymbol{x}(\boldsymbol{p}, y)/\partial y)\, \boldsymbol{x}(\boldsymbol{p}, y)']$, is symmetric and negative semidefinite.

These properties of the demand system are the 'integrability conditions,' since they permit the reconstruction of the preference preordering from the demand system. See, for example, Hurwicz and Uzawa (1971). If the properties are tested empirically and cannot be rejected, then we can infer that there exists a utility function that generates the demand system.

certainty, with total current-period consumption expenditure being the measured realized value. The fact that future uncertainty does not invalidate this contemporaneous allocation under perfect certainty was proved in Barnett (1995, section 4).

[2] See LaFrance (2001) for a summary of the current status of household production theory, dynamic household production theory, and of the microeconomic theory of consumer choice in an intertemporal framework with an emphasis on the role of expectations.

2.2 Indirect Utility

The maximum level of utility at given prices and income, $h(\boldsymbol{p}, y) = u[\boldsymbol{x}(\boldsymbol{p}, y)]$, is the indirect utility function. The direct utility function and the indirect utility function are equivalent representations of the underlying preference preordering. Using h, we can derive the demand system by straightforward differentiation, without having to solve a system of simultaneous equations, as would be the case with the direct utility function first order conditions. In particular, Roy's identity,

$$\boldsymbol{x}(\boldsymbol{p}, y) = -\frac{\partial h(\boldsymbol{p}, y)/\partial \boldsymbol{p}}{\partial h(\boldsymbol{p}, y)/\partial y}, \tag{3}$$

allows us to derive the demand system, provided there is an interior solution and that $\boldsymbol{p} > \mathbf{0}$ and $y > 0$. Alternatively, the logarithmic form of Roy's identity,

$$\boldsymbol{s}(\boldsymbol{p}, y) = -\left(\partial \log h(\boldsymbol{p}, y)/\partial \log \boldsymbol{p}\right) / \left(\partial \log h(\boldsymbol{p}, y)/\partial \log y\right),$$

or Diewert's (1974, p. 126) modified version of Roy's identity,

$$s_j(\boldsymbol{v}) = \frac{v_j \nabla h(\boldsymbol{v})}{\boldsymbol{v}' \nabla h(\boldsymbol{v})}, \tag{4}$$

can be used to derive the budget share equations, where $\boldsymbol{v} = [v_1, \cdots, v_n]'$ is a vector of expenditure normalized prices, with the jth element being $v_j = p_j/y$, and $\nabla h(\boldsymbol{v}) = \partial h(\boldsymbol{v})/\partial \boldsymbol{v}$. The indirect utility function is continuous in $(\boldsymbol{p}, y)$ and has the following properties: (i) positivity; (ii) homogeneity of degree zero in $(\boldsymbol{p}, y)$; (iii) decreasing in $\boldsymbol{p}$ and increasing in y; (iv) strictly quasi-convex in $\boldsymbol{p}$; and (v) satisfies Roy's identity, (3).

Together, properties (i)-(iv) are called the 'regularity conditions.' In the terminology of Caves and Christensen (1980), an indirect utility function is 'regular' at a given $(\boldsymbol{p}, y)$, if it satisfies the above properties at that $(\boldsymbol{p}, y)$. Similarly, the 'regular region' is the set of prices and income at which an indirect utility function satisfies the regularity conditions.

2.3 Hicksian Demands

Dual to the utility maximization problem is the problem of minimizing the cost or expenditure necessary to obtain a fixed level of utility, u, given market prices, $\boldsymbol{p}$,

$$C(\boldsymbol{p}, u) = \min_{\boldsymbol{x}} \ \boldsymbol{p}'\boldsymbol{x} \quad \text{subject to} \quad u(\boldsymbol{x}) \geq u.$$

If the cost function is differentiable with respect to $\boldsymbol{p}$, then Shephard's (1953) lemma,

$$\widetilde{\boldsymbol{x}}(\boldsymbol{p}, u) = \frac{\partial C(\boldsymbol{p}, u)}{\partial \boldsymbol{p}}, \tag{5}$$

can be applied to get the expenditure minimizing demands, $\widetilde{\boldsymbol{x}}(\boldsymbol{p}, u)$, which are the Hicksian compensated demand functions. Hicksian demands are positive valued and have the following properties: (i) homogeneous of degree zero in $\boldsymbol{p}$; and (ii) the Slutsky matrix, $[\partial \widetilde{\boldsymbol{x}}(\boldsymbol{p}, u)/\partial \boldsymbol{p}']$, is symmetric and negative semidefinite.

Finally, the cost or expenditure function, $C(\boldsymbol{p}, u) = \boldsymbol{p}'\widetilde{\boldsymbol{x}}(\boldsymbol{p}, u)$, has the following properties: (i) continuous in $(\boldsymbol{p}, u)$; (ii) homogeneous of degree one in $\boldsymbol{p}$; (iii) increasing in $\boldsymbol{p}$ and u; (iv) concave in $\boldsymbol{p}$; and (v) satisfies Shephard's lemma, (5).

2.4 Elasticity Relations

The elasticity measures can be calculated from the Marshallian demand functions, $\boldsymbol{x} = \boldsymbol{x}(\boldsymbol{p}, y)$. In particular, the income elasticity of demand, $\eta_{iy}(\boldsymbol{p}, y)$, for $i = 1, \ldots, n$, are

$$\eta_{iy}(\boldsymbol{p}, y) = \frac{\partial x_i(\boldsymbol{p}, y)}{\partial y} \frac{y}{x_i(\boldsymbol{p}, y)}.$$

If $\eta_{iy}(\boldsymbol{p}, y) > 0$, the ith good is classified as normal at $(\boldsymbol{p}, y)$, and as inferior if $\eta_{iy}(\boldsymbol{p}, y) < 0$. Also, if $\eta_{iy}(\boldsymbol{p}, y) > 1$, the ith good is classified as a luxury, and as a necessity, if $\eta_{iy}(\boldsymbol{p}, y) < 1$.

The uncompensated (Cournot) price elasticities, $\eta_{ij}(\boldsymbol{p}, y)$, for i, $j = 1, \ldots, n$, are

$$\eta_{ij}(\boldsymbol{p}, y) = \frac{\partial x_i(\boldsymbol{p}, y)}{\partial p_j} \frac{p_j}{x_i(\boldsymbol{p}, y)}.$$

If $\eta_{ij}(\boldsymbol{p}, y) > 0$, the goods are Cournot gross substitutes. If $\eta_{ij}(\boldsymbol{p}, y) < 0$, they are gross complements; and if $\eta_{ij}(\boldsymbol{p}, y) = 0$, they are independent.

The Slutsky equation is

$$\frac{\partial x_i(\boldsymbol{p}, y)}{\partial p_j} = \frac{\partial \widetilde{x}_i(\boldsymbol{p}, u)}{\partial p_j} - x_j(\boldsymbol{p}, y)\frac{\partial x_i(\boldsymbol{p}, y)}{\partial y},$$

for all $(\boldsymbol{p}, y)$, $u = h(\boldsymbol{p}, y)$, and i, $j = 1, \cdots, n$, where $\partial x_i(\boldsymbol{p}, y)/\partial p_j$ is the total effect of a price change on demand, $\partial \widetilde{x}_i(\boldsymbol{p}, u)/\partial p_j$ is the substitution effect of a compensated price change on demand, and $-x_j(\boldsymbol{p}, y)\partial x_i(\boldsymbol{p}, y)/\partial y$ is the income effect. According to Hicks (1936), $\partial \widetilde{x}_i(\boldsymbol{p}, u)/\partial p_j > 0$ indicates substitutability, $\partial \widetilde{x}_i(\boldsymbol{p}, u)/\partial p_j < 0$ indicates complementarity, and $\partial \widetilde{x}_i(\boldsymbol{p}, u)/\partial p_j = 0$ indicates independence.

Slutsky symmetry can be written in elasticity terms, as follows

$$\eta_{iy}(\boldsymbol{p}, y) + \frac{\eta_{ij}(\boldsymbol{p}, y)}{s_j(\boldsymbol{p}, y)} = \eta_{jy}(\boldsymbol{p}, y) + \frac{\eta_{ji}(\boldsymbol{p}, y)}{s_i(\boldsymbol{p}, y)},$$

or using Allen elasticities of substitution, the equation can be written as

$$\sigma^a_{ij}(\boldsymbol{p}, y) = \eta_{iy}(\boldsymbol{p}, y) + \frac{\eta_{ij}(\boldsymbol{p}, y)}{s_j(\boldsymbol{p}, y)} = \eta_{jy}(\boldsymbol{p}, y) + \frac{\eta_{ji}(\boldsymbol{p}, y)}{s_i(\boldsymbol{p}, y)} = \sigma^a_{ji}(\boldsymbol{p}, y),$$

where $\sigma^a_{ij}(\boldsymbol{p}, y)$ denotes the Allen (1938) elasticity of substitution between goods i and j. If $\sigma^a_{ij}(\boldsymbol{p}, y) > 0$, goods i and j are Allen substitutes and if $\sigma^a_{ij}(\boldsymbol{p}, y) < 0$, then the goods are Allen complements.

The Allen elasticity of substitution is the traditional measure. There are, however, other measures. See Davis and Gauger (1996) and Blackorby and Russell (1989) for more details. For example, the Morishima (1967) elasticity of substitution,

$$\sigma^m_{ij}(\boldsymbol{p}, y) = s_i(\boldsymbol{p}, y)\left(\sigma^a_{ji}(\boldsymbol{p}, y) - \sigma^a_{ii}(\boldsymbol{p}, y)\right),$$

measures the net change in the compensated demand for good j when the price of good i changes. Goods will be Morishima complements (substitutes) if an increase in the price of i causes x_i/x_j to decrease (increase).

Either $\sigma^a_{ij}(\boldsymbol{p}, y)$ or $\sigma^m_{ij}(\boldsymbol{p}, y)$ can be used to classify assets as substitutes or complements. But in general, the two elasticity of substitution formulas will yield different stratification sets. If two goods are Allen substitutes, $\sigma^a_{ji}(\boldsymbol{p}, y) > 0$, they must also be Morishima substitutes, $\sigma^m_{ij}(\boldsymbol{p}, y) > 0$. However, two goods may be Allen complements, $\sigma^a_{ji}(\boldsymbol{p}, y) < 0$, but Morishima substitutes. It has thereby been argued that the Allen elasticity of substitution overstates the complementarity relationship. But while the Allen elasticity of substitution matrix is symmetric, the Morishima elasticity of substitution matrix is not. Blackorby and Russell (1989) show that the Morishima elasticity of substitution matrix is symmetric only when the aggregator function is a member of the constant elasticity of substitution family.

3 The Differential Approach to Demand Analysis

One model that has been frequently used to test and use the theory is the Rotterdam model, introduced by Theil (1965). To review this modeling approach, take the total differential of the logarithmic form of the Marshallian demand function for good i, $x_i = x_i(\boldsymbol{p}, y)$, to obtain

$$d\log x_i = \eta_{iy} d\log y + \sum_{j=1}^{n} \eta_{ij} d\log p_j,$$

where η_{iy} is the income elasticity and η_{ij} is the price elasticity of good i with respect to the price of good j. Using the Slutsky decomposition in elasticity terms, $\eta_{ij} = \eta^*_{ij} - \eta_{iy} s_j$, where η^*_{ij} is the compensated cross-price elasticity, the above equation can be written as

$$d\log x_i = \eta_{iy}\left(d\log y - \sum_{j=1}^{n} s_j d\log p_j\right) + \sum_{j=1}^{n} \eta^*_{ij} d\log p_j. \tag{6}$$

Multiplying by s_i yields

$$s_i d\log x_i = b_i d\log \bar{y} + \sum_{j=1}^{n} c_{ij} d\log p_j, \tag{7}$$

where $b_i = s_i \eta_{iy} = p_i \partial x_i / \partial y$ is the marginal budget share of the ith use of money income with $c_{ij} = s_i \eta^*_{ij}$, and $d \log \bar{y} = d \log y - \sum_{j=1}^{n} s_j d \log p_j$ is change in real total expenditure.[3] Replacing the differentials in (7) by finite approximations and treating the b_i's and c_{ij}'s as constant parameters, equation (7) can be estimated, and the theory can be tested. See Barnett and Serletis (2009b) for a detailed discussion of these issues.

The Rotterdam model was a turning point in empirical demand analysis, since that model offers many features not available in prior modeling efforts, such as the logarithmic demand functions and Stone's (1954) linear expenditure system. The Rotterdam model is directly derived from consumer demand theory and has the ability to model the full substitution matrix, the model's parameters can directly be related to underlying theoretical restrictions, and the model is easily estimated, since it is is linear in parameters. Of particular importance is the fact that the model can be derived under weak assumptions on aggregation over consumers. In addition, the model has a particularly well behaved error structure.

Since the publication of Diewert's (1971) paper on duality, much of the demand systems literature has modelled the utility function of a representative consumer. While all microeconomic theory is then easily available at the aggregate level, the representative consumer does not exist under reasonable assumptions. In contrast the Rotterdam model does not require existence of a representation agent, but is therefore not integrable to the aggregate utility function of such a representative consumer. Regarding the Rotterdam model's derivation under weak aggregation assumptions, see Barnett (1979a,b). The next two sections deal with the application of the representative agent approach to demand modeling.

4 The Parametric Approach to Demand Analysis

For many years, the literature concentrated on the use of globally regular functional forms, such as the Cobb-Douglas and the constant elasticity of substitution (CES) functional forms. These forms globally satisfy the theoretical regularity conditions for rational neoclassical economic behavior . In particular, there was a focus on means to generalize the CES to permit different pairwise elasticities of substitution. However, that approach ran into a dead end, when Uzawa (1962) proved that it is not possible to produce a model that simultaneously can have pairwise elasticities of substitution that are independent of quantities consumed (i.e. CES) but also can attain arbitrary constant elasticities of substitution for different pairs of goods. Among the alternatives are the flexible functional forms, to which we now turn.

[3] This model is called the absolute price version of the Rotterdam model. There also exists a more complicated nonlinear version, called the relative price version, which is less widely used. The primary advantage of the relative price version is its easy ability to test for blockwise strong separability.

4.1 Locally Flexible Functional Forms

A locally flexible functional form is a second-order local approximation to an arbitrary function. In the demand systems literature, there are two different definitions of second-order approximations, one by Diewert (1971) and another by Lau (1974). Barnett (1983a) has identified the relationship of each of those definitions to existing definitions in mathematics of local approximation orders.

In what follows, we briefly discuss three popular functional forms: the generalized Leontief, translog, and the almost ideal demand system. The first two are locally flexible functional forms. The third, in its most common parameterization, is perhaps best viewed as a cross between a locally flexible functional form and the Rotterdam model.

4.1.1 The Generalized Leontief

The generalized Leontief (GL) functional form was introduced by Diewert (1973) in the context of cost and profit functions. Diewert (1974) also introduced the GL reciprocal indirect utility function

$$h(\boldsymbol{v}) = \alpha_0 + \sum_{i=1}^{n} \alpha_i v_i^{1/2} + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} v_i^{1/2} v_j^{1/2}, \tag{8}$$

where $\boldsymbol{B} = [\beta_{ij}]$ is an $n \times n$ symmetric matrix of parameters and a_0 and a_i are other parameters, for a total of $(n^2 + 3n + 2)/2$ parameters.

Applying to (8) Diewert's (1974) modified version of Roy's identity, (4), the following share equations result (for $i = 1, \cdots, n$):

$$s_i = \left(\alpha_i v_i^{1/2} + \sum_{j=1}^{n} \beta_{ij} v_i^{1/2} v_j^{1/2} \right) \Bigg/ \left(\sum_{j=1}^{n} \alpha_j v_j^{1/2} + \sum_{k=1}^{n} \sum_{m=1}^{n} \beta_{km} v_k^{1/2} v_m^{1/2} \right). \tag{9}$$

Since the share equations are homogeneous of degree zero in the parameters, the model requires a parameter normalization. Barnett and Lee (1985) use the normalization $2 \sum_{i=1}^{n} \alpha_i + \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} = 1$.

Caves and Christensen (1980) have shown that the generalized Leontief has satisfactory local properties when preferences are nearly homothetic and substitution is low. However, when preferences are not homothetic or substitution is high, the generalized Leontief has a small regularity region.

4.1.2 The Translogs

The basic translog (BTL) flexible functional form was introduced by Christensen *et al.* (1975). The BTL reciprocal indirect utility function replaces the $v_j^{1/2}$ terms in (8) by

$\log v_j$ and is written as

$$\log h(\boldsymbol{v}) = \alpha_0 + \sum_{k=1}^{n} \alpha_k \log v_k + \frac{1}{2} \sum_{k=1}^{n} \sum_{j=1}^{n} \beta_{jk} \log v_k \log v_j, \tag{10}$$

where α_0 is a scalar, $\boldsymbol{\alpha}' = [\alpha_1, \cdots, \alpha_n]$ is a vector of parameters, and $\boldsymbol{B} = [\beta_{ij}]$ is an $n \times n$ symmetric matrix of parameters, for a total of $(n^2 + 3n + 2)/2$ parameters.

The share equations, derived using the logarithmic form of Roy's identity (for $i = 1, \cdots, n$), are

$$s_i = \left(\alpha_i + \sum_{j=1}^{n} \beta_{ij} \log v_j \right) \Bigg/ \left(\sum_{j=1}^{n} \alpha_j + \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} \log v_j \right). \tag{11}$$

Since the share equations are homogeneous of degree zero in the parameters, a normalization is needed. The usual parameter normalization with the translog is $\sum_{i=1}^{n} \alpha_i = -1$. By imposing the restrictions, $\sum_{i=1}^{n} \beta_{ij} = 0$, for all $j = 1, \cdots, n$, on the BTL, the homothetic translog model is obtained. The homothetic translog is a generalization of the Cobb-Douglas and reduces to it when all of the β_{ij} are zero. The basic translog is a special case of the generalized translog, proposed by Pollak and Wales (1980).

4.1.3 The Almost Ideal Demand System

Deaton and Muellbauer (1980a) start with the 'price-independent generalized logarithmic' (PIGLOG) class of preferences, which satisfies the necessary and sufficient conditions for consistent aggregation across consumers. The log of the cost or expenditure function is

$$\log C(\boldsymbol{p}, u) = (1-u) \log a(\boldsymbol{p}) + u \log b(\boldsymbol{p}), \tag{12}$$

where

$$\log a(\boldsymbol{p}) = \alpha_0 + \sum_{k=1}^{n} \alpha_k \log p_k + \frac{1}{2} \sum_{k=1}^{n} \sum_{j=1}^{n} \gamma_{kj}^{*} \log p_k \log p_j,$$

$$\log b(\boldsymbol{p}) = \log a(\boldsymbol{p}) + \beta_0 \prod_{k=1}^{n} p_k^{\beta_k},$$

and where α, β, and γ^* are parameters. Substituting $\log a(\boldsymbol{p})$ and $\log b(\boldsymbol{p})$ into (12), the AIDS demand system in budget shares follows from Shephard's lemma, and is given by (for $i = 1, \cdots, n$):

$$s_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \log p_j + \beta_i \log \left(\frac{y}{P} \right), \tag{13}$$

where the price deflator of the logarithm of income is $\log P = \alpha_0 + \sum_{k=1}^{n} \alpha_k \log p_k + \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} \gamma_{kj} \log p_k \log p_j$, and the parameters γ_{ij} are defined by $\gamma_{ij} = .5 \left(\gamma_{ij}^{*} + \gamma_{ji}^{*} \right)$.

Symmetry requires $\gamma_{ij} = \gamma_{ji}$ for all i, j. Since $c(\boldsymbol{p}, u)$ must be linearly homogeneous and strictly increasing in $\boldsymbol{p}$, the resulting theoretical restrictions on (13) are $\sum_{i=1}^{n} \alpha_i = 1$ and $\sum_{i=1}^{n} \gamma_{ij} = \sum_{j=1}^{n} \gamma_{ij} = \sum_{i=1}^{n} \beta_i = 0$.

The nonlinearity of the AIDS model is commonly circumvent by using a linear approximation to the income deflator, $\log P$. See Barnett and Seck (2008) regarding the various linear approximations of the AIDS model. Deaton and Muellbauer (1980a) suggest using Stone's price index, $\log P = \sum_{k=1}^{n} s_k \log p_k$, where s_k is the expenditure share of good k, to generate a linear approximation to the AIDS model (known as LA-AIDS). With n goods, the linear approximation to the AIDS model's share equations contains $(n^2 + 3n - 2)/2$ free parameters. However, the linear approximation of the AIDS model is not a flexible functional form by Diewert's definition, as recently noted by LaFrance (2004). In fact, LA-AIDS, in first difference form, closely resembles the Rotterdam model with a modified left hand side endogenous variable. In particular, the first difference form uses the change in expenditure share as the left hand variable, while the Rotterdam model uses the quantity component of the share growth rate as its left hand variable. The right hand sides of the two models, in growth rate form, are identical.

The fully nonlinear form of the AIDS model is a flexible functional form and fundamentally different from the Rotterdam model. But in its usual linearized version, the AIDS model was recently found to perform more poorly than the Rotterdam model in Monte Carlo comparisons by Barnett and Seck (2008).

4.2 Effectively Globally Regular Flexible Functional Forms

Locally flexible demand models escape from the dead end of Uzawa's (1962) impossibility theorem, by providing the ability to attain arbitrary elasticities of substitution, although at only one point. These models provide access to all neoclassical microeconomic theory at a point. However, as argued by Caves and Christensen (1980), Guilkey and Lovell (1980), Barnett and Lee (1985), and Barnett *et al.* (1985, 1987), most popular locally flexible functional forms have very small regions of theoretical regularity. These models thereby violate the conditions for the duality theory from which the models were derived, except at points within the regular region.

Problems exist in choosing among the available inflexible globally-regular models, such as the Cobb-Douglas and the CES, and the locally flexible functional forms that are not globally regular. A result was the development of locally flexible functional forms that have large (but not global) regular regions. Cooper and McLaren (1996) classify those models as 'effectively globally regular' flexible functional forms. These functions typically have regular regions that include almost all data points in the sample. In addition, the regularity regions increase as real expenditure levels grow, as is often the case with time series data.

Examples of these functions include Barnett's minflex Laurent (ML) models, based on the Laurent series expansion, the quadratic AIDS (QUAIDS) model of Banks *et al.* (1996), and the general exponential form (GEF) of Cooper and McLaren (1996). In this section, we discuss two of these effectively globally regular flexible functional forms: the minflex Laurent and the quadratic AIDS.

4.2.1 The Minflex Laurent

The minflex Laurent model, introduced by Barnett (1983a, 1985), Barnett and Lee (1985), and Barnett *et al.* (1985, 1987), is a special case of the full Laurent model also introduced by Barnett (1983b). Following Barnett (1983b), the full Laurent reciprocal indirect utility function, based on a Laurent series expansion of the second order about $\boldsymbol{v}^{1/2} = \mathbf{0}$), is

$$h(\boldsymbol{v}) = a_0 + 2\boldsymbol{a}'\boldsymbol{v}^{1/2} + \boldsymbol{v}'^{1/2}\boldsymbol{A}\boldsymbol{v}^{1/2} - 2\boldsymbol{b}'\boldsymbol{v}^{-1/2} - \boldsymbol{v}'^{-1/2}\boldsymbol{B}\boldsymbol{v}^{-1/2}, \tag{14}$$

where a_0 is a scalar constant, $\boldsymbol{a}' = [a_1, \cdots, a_n]$ and $\boldsymbol{b}' = [b_1, \cdots, b_n]$ are vectors of parameters, and $\boldsymbol{A} = [a_{ij}]$ and $\boldsymbol{B} = [b_{ij}]$ are $n \times n$ symmetric matrices of parameters.

Because the full Laurent reciprocal indirect utility function has far more parameters than are needed to acquire a specification that is locally flexible, a particularly useful special case of the full Laurent model is acquired by letting $\boldsymbol{b} = \mathbf{0}$, $a_{ij}b_{ij} = 0$ for all $(i,j) \in S$, where $S = \{(i,j) : i \neq j; i,j = 1, \cdots, n\}$, letting the diagonal elements of $\boldsymbol{B}$ be zero, and constraining the off diagonal elements of both $\boldsymbol{A}$ and $\boldsymbol{B}$ to be nonnegative. Imposing these restrictions, equation (14) reduces to the minflex Laurent reciprocal indirect utility function,

$$h(\boldsymbol{v}) = a_0 + 2\boldsymbol{a}'\boldsymbol{v}^{1/2} + \sum_{i=1}^{n} a_{ii}v_i + \sum\sum_{(i,j)\in S} a_{ij}^2 v_i^{1/2} v_j^{1/2} - \sum\sum_{(i,j)\in S} b_{ij}^2 v_i^{-1/2} v_j^{-1/2}. \tag{15}$$

Note that the off diagonal elements of $\boldsymbol{A}$ and $\boldsymbol{B}$ have been replaced by their squares, which satisfy the nonnegativity constrain.

The minflex Laurent is "parsimonious," defined to mean that the model has no more parametric freedom than is needed to satisfy the definition of local flexibility. Hence the model is no more flexible than the more-widely-used second-order Taylor series models, such as the translog and generalized Leontief. But the minflex Laurent has better regularity properties than the more widely-known models, as shown by Barnett, Lee, and Wolfe (1985,1987) and Barnett and Lee (1985),

By applying Roy's identity to (15), the share equations of the minflex Laurent demand system (for $i = 1, \cdots, n$) are

$$s_i = \frac{a_i v_i^{1/2} + a_{ii}v_i + \sum_{j:j\neq i} a_{ij}^2 v_i^{1/2} v_j^{1/2} + \sum_{j:j\neq i} b_{ij}^2 v_i^{-1/2} v_j^{-1/2}}{\boldsymbol{a}'\boldsymbol{v}^{1/2} + \sum_{k=1}^{n} a_{kk}v_k + \sum\sum_{(j,k)\in S} a_{jk}^2 v_j^{1/2} v_k^{1/2} + \sum\sum_{(j,k)\in S} b_{jk}^2 v_j^{-1/2} v_k^{-1/2}}. \tag{16}$$

Since the share equations are homogenous of degree zero in the parameters, Barnett and Lee (1985) recommend the following identifying normalization,

$$\sum_{i=1}^{n} a_{ii} + 2\sum_{i=1}^{n} a_i + \sum_{(i,j)\in S}\sum a_{jk}^2 - \sum_{(i,j)\in S}\sum b_{ij}^2 = 1. \tag{17}$$

There are $1+n+n(n+1)/2+n(n-1)/2$ parameters in (15). But the $n\,(n-1)\,/2$ equality restrictions, $a_{ij}b_{ij} = 0$ for all $(i,j) \in S$, and the normalization (17) reduce the number of parameters in equation (16) to $(n^2+3n)\,/2$.

If instead of using $v_i^{1/2}$, we use $\log \bar{v}_i$, where $\bar{v}_i = v_i + \theta_i$ with θ_i being a constant, the minflex Laurent translog model is obtained. See, for example, Barnett (1985) and Barnett *et al.* (1987). Also, as shown by Barnett (1983b, Theorem A.3), equation (15) is globally concave for every $\boldsymbol{v} \geq \mathbf{0}$, if all parameters are nonnegative, since in that case (15) would be a sum of concave functions. If the initially estimated parameters of the vector $\boldsymbol{a}$ and matrix $\boldsymbol{A}$ are not nonnegative, curvature can be imposed globally by replacing each unsquared parameter by a squared parameter, as in Barnett (1983b). But this approach, while sufficient for regularity, is not necessary; and better methods of imposing regularity on the model now are available.

4.2.2 The Quadratic AIDS

Since Engel curves for consumption data appear to be more nonlinear than the AIDS and translog models permit, Banks *et al.* (1997) develop an extension of the AIDS model, the quadratic AIDS (QUAIDS) model. The indirect utility function for the quadratic AIDS model is

$$\log h(\boldsymbol{p}, y) = \left\{ \left[\frac{\log y - \log a(\boldsymbol{p})}{b(\boldsymbol{p})} \right]^{-1} + \lambda(\boldsymbol{p}) \right\}^{-1}, \tag{18}$$

where $\lambda(\boldsymbol{p})$ is a differentiable, homogeneous function of degree zero in prices $\boldsymbol{p}$. The functions $a(\boldsymbol{p})$ and $b(\boldsymbol{p})$ are defined as in (12), and $\lambda(\boldsymbol{p}) = \sum_{i=1}^{n} \lambda_i \log p_i$, where $\sum_{i=1}^{n} \lambda_i = 0$. By Roy's identity the budget shares of the QUAIDS model (for $i = 1, \cdots, n$) are given by

$$s_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \log p_j + \beta_i \log \left[\frac{y}{a(\boldsymbol{p})} \right] + \frac{\lambda_i}{b(\boldsymbol{p})} \left\{ \log \left[\frac{y}{a(\boldsymbol{p})} \right] \right\}^2. \tag{19}$$

4.3 Normalized Quadratic Flexible Functional Forms

The effectively-globally-regular flexible functional forms violate the theoretically appropriate regularity conditions less often than the usual locally flexible functional forms, which are based upon second order Taylor approximations. Nevertheless, effectively-globally-regular flexible functional forms do exhibit regions within which the regularity conditions are violated. This problem led Diewert and Wales (1988) to propose two locally flexible functional forms, for which the theoretical curvature conditions can be imposed globally. The first system is derived from a normalized quadratic (NQ) reciprocal indirect utility function and the second is derived from a NQ expenditure function. But these models, even with curvature globally imposed, are not necessarily globally regular, since monotonicity cannot be imposed on thesse models simultaneously with curvature, without losing flexibility. Moreover, as noted by Barnett (2002), the imposition of global curvature on such models may induce spurious violations of monotonicity. See also Barnett and Usui (2007) for a Monte Carlo study of the global regularity properties of the NQ model.

4.3.1 The Normalized Quadratic Reciprocal Indirect Utility Function

Following Diewert and Wales (1988), the NQ reciprocal indirect utility function is defined as

$$h(\boldsymbol{v}) = b_0 + \boldsymbol{b}'\boldsymbol{v} + \frac{1}{2}\frac{\boldsymbol{v}'\boldsymbol{B}\boldsymbol{v}}{\boldsymbol{\alpha}'\boldsymbol{v}} + \boldsymbol{\theta}'\log\boldsymbol{v}, \tag{20}$$

where b_0, $\boldsymbol{b}' = [b_1, \cdots, b_n]$, $\boldsymbol{\theta}' = [\theta_1, \cdots, \theta_n]$, and the elements of the $n \times n$ symmetric $\boldsymbol{B}$ $= [\beta_{ij}]$ matrix are the unknown parameters to be estimated. It is important to note that the quadratic term in (20) is normalized by dividing through by a linear function, $\boldsymbol{\alpha}'\boldsymbol{v}$, and that the nonnegative vector of parameters $\boldsymbol{\alpha}' = (\alpha_1, \cdots, \alpha_n)$ is assumed to be predetermined.[4]

Diewert and Wales (1988) pick a reference (or base-period) vector of expenditure normalized prices, $\boldsymbol{v}^* = \mathbf{1}$, and assume that $\boldsymbol{\alpha}$ satisfies

$$\boldsymbol{\alpha}'\boldsymbol{v}^* = 1, \tag{21}$$

where each of the elements of $\boldsymbol{\alpha}$ is nonnegative. Moreover, they assume that $\boldsymbol{B}$ satisfies the following n restrictions:

$$\sum_{j=1}^{n} \beta_{ij} v_j^* = 0, \qquad i = 1, \cdots, n. \tag{22}$$

Using the modified version of Roy's identity (4), the NQ demand system (for $i = 1, \cdots, n$)

[4] According to Diewert and Wales (1988), the selection of the "weights," $\boldsymbol{\alpha}$, in that normalization is arbitrary. Various selections have been proposed for $\boldsymbol{\alpha}$ in applications. While those weights often are selected to be equal, as in Diewert and Wales (1988), $\boldsymbol{\alpha}$ sometimes is based upon the weights in an index number from index number theory. In principal, $\boldsymbol{\alpha}$ is estimable, but normally viewed as best selected in advance.

can be written

$$s_i = v_i \frac{b_i + (\boldsymbol{\alpha}'\boldsymbol{v})^{-1}\left(\sum_{j=1}^{n}\beta_{ij}v_j\right) - \frac{1}{2}\alpha_i (\boldsymbol{\alpha}'\boldsymbol{v})^{-2}\, \boldsymbol{v}'\boldsymbol{B}\boldsymbol{v} + \theta_i}{\boldsymbol{b}'\boldsymbol{v} + \frac{1}{2}(\boldsymbol{\alpha}'\boldsymbol{v})^{-1}\, \boldsymbol{v}'\boldsymbol{B}\boldsymbol{v} + \sum_{j=1}^{n}\theta_j}. \tag{23}$$

Finally, as the share equations are homogeneous of degree zero in the parameters, Diewert and Wales (1988) suggest that we impose the normalization

$$\sum_{j=1}^{n} b_j = 1. \tag{24}$$

Hence, there are $n(n+5)/2$ parameters in (23), but the imposition of the $(n-1)$ restrictions in (22) and (24) reduces the number of parameters to be estimated to $(n^2+3n-2)/2$.

The NQ reciprocal indirect utility function will be globally concave over the positive orthant, if $\boldsymbol{B}$ is a negative semidefinite matrix and $\boldsymbol{\theta} \geq \boldsymbol{0}$. See Diewert and Wales (1988, Theorem 3). Diewert and Wales (1988) also show that the NQ reciprocal indirect utility function defined by (20), (21), and (22) is locally flexible, if there are no restrictions on its free parameters, but loses flexibility if the curvature conditions need to be imposed. Although curvature conditions can be imposed globally, the imposition of global curvature destroys the flexibility of the NQ reciprocal indirect utility function.

4.3.2 The Normalized Quadratic Expenditure Function

Diewert and Wales (1988) also proposed the NQ expenditure function $C(\boldsymbol{p}, u)$,

$$C(\boldsymbol{p}, u) = \boldsymbol{a}'\boldsymbol{p} + \left(\boldsymbol{b}'\boldsymbol{p} + \frac{1}{2}\frac{\boldsymbol{p}'\boldsymbol{B}\boldsymbol{p}}{\boldsymbol{\alpha}'\boldsymbol{p}}\right)u, \tag{25}$$

where the estimated parameters of the model consist of $\boldsymbol{a}' = [a_1, \cdots, a_n]$, $\boldsymbol{b}' = [b_1, \cdots, b_n]$, and the $n \times n$ symmetric $\boldsymbol{B} = [\beta_{ij}]$ matrix. As with the NQ reciprocal indirect utility function, the additional nonnegative vector of parameters $\boldsymbol{\alpha}' = (\alpha_1, \cdots, \alpha_n)$ is predetermined and assumed to satisfy

$$\boldsymbol{\alpha}'\boldsymbol{p}^* = 1, \qquad \alpha_j \geq 0 \text{ for } j = 1, \cdots, n, \tag{26}$$

where p_j^* is the jth element of the reference vector.[5] The following restrictions are also imposed

$$\sum_{j=1}^{n} a_j p_j^* = 0, \tag{27}$$

$$\sum_{j=1}^{n} \beta_{ij} p_j^* = 0, \qquad i = 1, \cdots, n. \tag{28}$$

There are $n(n+5)/2$ parameters in (25), but the imposition of the above restrictions reduces the number of parameters to $(n^2 + 3n - 2)/2$.[6]

Applying Shephard's lemma (5) to (25) yields the share equations of the NQ expenditure system (for $i = 1, \cdots, n$),

$$s_i = a_i v_i + \frac{(1 - \boldsymbol{\alpha}'\boldsymbol{v}) \left(b_i + (\boldsymbol{\alpha}'\boldsymbol{v})^{-1} \boldsymbol{B}\boldsymbol{v} - \frac{1}{2} (\boldsymbol{\alpha}'\boldsymbol{v})^{-2} \boldsymbol{v}'\boldsymbol{B}\boldsymbol{v}\boldsymbol{\alpha} \right) v_i}{\boldsymbol{b}'\boldsymbol{v} + \frac{1}{2} (\boldsymbol{\alpha}'\boldsymbol{v})^{-1} \boldsymbol{v}'\boldsymbol{B}\boldsymbol{v}}. \tag{29}$$

Since the share equations in (29) are homogeneous of degree zero in the parameters, Diewert and Wales (1988) impose the normalization $\sum_{j=1}^{n} b_j = 1$.

The NQ expenditure function is locally flexible in the class of expenditure functions satisfying local money-metric scaling, and retains this flexibility when concavity is imposed, but not when monotonicity also is imposed. See Diewert and Wales (1988) for more details.

5 The Semi-nonparametric Approach

The flexible functional forms considered so far are capable of approximating an arbitrary function only locally (at a single point). A path-breaking innovation in this area was provided by Gallant (1981) in his introduction of the semi-nonparametric inference approach, which uses series expansions in infinite dimensional parameter spaces. The idea behind the semi-nonparametric approach is to expand the order of the series expansion, as the sample size increases, until the semi-nonparametric function converges asymptotically to the true function generating the data.

Semi-nonparametric functional forms are globally flexible in the sense that the model asymptotically can reach any continuous function. Since inferences do not maintain a

[5] As with the NQ reciprocal indirect utility function, $\boldsymbol{\alpha}$ is arbitrary and is selected in a manner similar to that for the NQ reciprocal indirect utility function.

[6] The NQ expenditure function defined by (25)-(28) is in the Gorman polar form, so the preferences that are dual to the NQ are quasi-homothetic.

specification containing a finite number of parameters, asymptotic inferences are free from specification error. Two globally flexible functional forms in general use are the Fourier flexible functional form, introduced by Gallant (1981), and the Asymptotically Ideal Model (AIM), introduced by Barnett and Jonas (1983) and employed and explained in Barnett and Yue (1988), Barnett, Geweke, and Wolfe (1991), and Barnett, Geweke, and Yue (1999).

5.1 The Fourier

Gallant (1981) expands the indirect utility function using the Fourier series,

$$h(\boldsymbol{v}) = u_0 + \boldsymbol{b}'\boldsymbol{v} + \frac{1}{2}\boldsymbol{v}'\mathbf{C}\boldsymbol{v} + \sum_{\alpha=1}^{A}\left(u_{0\alpha} + 2\sum_{j=1}^{J}\left[u_{j\alpha}\cos(j\boldsymbol{k}'_\alpha\boldsymbol{v}) - w_{j\alpha}\sin(j\boldsymbol{k}'_\alpha\boldsymbol{v})\right]\right), \tag{30}$$

in which

$$\mathbf{C} = -\sum_{\alpha=1}^{A} u_{0\alpha}\boldsymbol{k}_\alpha\boldsymbol{k}'_\alpha,$$

where $\boldsymbol{v}$ denotes income normalized prices, $\boldsymbol{k}_\alpha$ is a multi-index — an n-vector with integer components — and u_0, $\{b\}$, $\{u\}$, and $\{w\}$ are parameters to be estimated. As Gallant (1981) shows, the length of a multi-index, $|\boldsymbol{k}_\alpha|^* = \sum_{i=1}^{n}|k_{i\alpha}|$, reduces the complexity of the notation required to denote high-order partial differentiation and multivariate Fourier series expansions. The parameters A (the number of terms) and J (the degree of the approximation) determine the degree of the Fourier polynomials. The Fourier flexible functional form has the ability of achieving global approximations in Sobolev norm.

By applying to (30) Roy's modified identity (4), we obtain the Fourier demand system (for $i = 1, \cdots, n$)

$$s_i = \frac{v_i b_i - \sum_{\alpha=1}^{A}\left(u_{0\alpha}\boldsymbol{v}'\boldsymbol{k}_\alpha + 2\sum_{j=1}^{J} j\left[u_{j\alpha}\sin(j\boldsymbol{k}'_\alpha\boldsymbol{v}) + w_{j\alpha}\cos(j\boldsymbol{k}'_\alpha\boldsymbol{v})\right]\right)k_{i\alpha}v_i}{\boldsymbol{b}'\boldsymbol{v} - \sum_{\alpha=1}^{A}\left(u_{0\alpha}\boldsymbol{v}'\boldsymbol{k}_\alpha + 2\sum_{j=1}^{J} j\left[u_{j\alpha}\sin(j\boldsymbol{k}'_\alpha\boldsymbol{v}) + w_{j\alpha}\cos(j\boldsymbol{k}'_\alpha\boldsymbol{v})\right]\right)\boldsymbol{k}'_\alpha\boldsymbol{v}}. \tag{31}$$

Eastwood and Gallant (1991) show that Fourier functions produce consistent and asymptotically normal parameter estimates when the number of parameters to be estimated equals the number of effective observations raised to the power of 2/3. With the normalization $b_n = \sum_{j=1}^{n-1} b_j$ imposed, the Fourier demand system has $(n-1)$ parameters b, A parameters $u_{0\alpha}$, AJ parameters $u_{j\alpha}$, and AJ parameters $w_{j\alpha}$ to be estimated, for a total of $(n-1) + A(1+2J)$ free parameters.

As a Fourier series is a periodic function in its arguments but the indirect utility function is not, the scaling of the data is also important. In empirical applications, to avoid the approximation from diverging from the true indirect utility function the data should be rescaled so that the expenditure normalized prices v_j $(j = 1, \cdots, n)$ lie on $0 \leq v_j \leq 2\pi$.[7]

5.2 The AIM Model

The basis functions with which the Fourier model seeks to span the neoclassical function space are sines and cosines, although such trigonometric functions are periodic and, hence, are far from neoclassical. Motivated by this limitation of the Fourier model, Barnett and Jonas (1983) originated a multivariate version of the Müntz-Szatz series expansion, which is globally flexible in the same sense as Gallant's Fourier model. The demand system derived from the Müntz-Szatz series expansion has been named by Barnett and Yue (1988) the 'asymptotically ideal model' (AIM).

The multivariate version of the Müntz-Szatz series model proposed by Barnett and Jonas (1983) is

$$h_K(\boldsymbol{v}) = a_0 + \sum_{k=1}^{K}\sum_{i=1}^{n} a_{ik} v_i^{\lambda(k)} + \sum_{k=1}^{K}\sum_{m=1}^{K}\left[\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ijkm} v_i^{\lambda(k)} v_j^{\lambda(m)}\right] + \cdots,$$

with $\lambda(k) = 2^{-k}$ for $k = 1, \cdots, \infty$, where $a_0, a_{ik}, a_{ijkm}, \cdots$, are parameters to be estimated for $i, j = 1, \cdots, n$ and $k, m = 1, \cdots, \infty$. When, for example, the number of goods is three $(n = 3)$ and the degree of approximation is two $(K = 2)$, the reciprocal indirect utility function for the asymptotically ideal model becomes

$$h_{K=2}(\boldsymbol{v}) = a_0 + \sum_{k=1}^{2}\sum_{i=1}^{3} a_{ik} v_i^{\lambda(k)} + \sum_{k=1}^{2}\sum_{m=1}^{2}\left[\sum_{i=1}^{3}\sum_{j=1}^{3} a_{ijkm} v_i^{\lambda(k)} v_j^{\lambda(m)}\right]$$

$$+ \sum_{k=1}^{2}\sum_{m=1}^{2}\sum_{g=1}^{2}\left[\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{h=1}^{3} a_{ijhkmg} v_i^{\lambda(k)} v_j^{\lambda(m)} v_h^{\lambda(g)}\right], \tag{32}$$

with $\lambda(z) = 2^{-z}$, where a_0, a_{ik}, a_{ijkm}, and a_{ijhkmg} are the parameters to be estimated. The number of parameters is reduced by deleting the diagonal elements of the parameter arrays so that $i \neq j$, $j \neq h$ and $i \neq h$. This deletion does not alter the span of the model's approximation.

To avoid the extensive multiple subscripting in the coefficients a_{ijhkmg}, Barnett and Yue (1988) reparameterize by stacking the coefficients as they appear in (32) into a single vector

[7] The income normalized prices are typically rescaled as $v_j \times [(2\pi - \varepsilon) / \max\{v_j : j = 1, \cdots, n\}]$, with $(2\pi - \varepsilon)$ set equal to 6, as in Gallant (1982). In cases where the income normalized prices are already between 0 and 2π, such rescaling should not be performed.

of parameters, $\boldsymbol{b} = (b_0, \cdots, b_{26})'$, containing the 27 coefficients in (32), as follows

$$\begin{aligned}
h_{K=2}(\boldsymbol{v}) = {} & b_0 + b_1 v_1^{1/2} + b_2 v_2^{1/2} + b_3 v_3^{1/2} + b_4 v_1^{1/4} + b_5 v_2^{1/4} + b_6 v_3^{1/4} + b_7 v_1^{1/2} v_2^{1/2} + b_8 v_1^{1/2} v_2^{1/4} \\
& + b_9 v_1^{1/4} v_2^{1/2} + b_{10} v_1^{1/4} v_2^{1/4} + b_{11} v_1^{1/2} v_3^{1/2} + b_{12} v_1^{1/2} v_3^{1/4} + b_{13} v_1^{1/4} v_3^{1/2} + b_{14} v_1^{1/4} v_3^{1/4} \\
& + b_{15} v_2^{1/2} v_3^{1/2} + b_{16} v_2^{1/2} v_3^{1/4} + b_{17} v_2^{1/4} v_3^{1/2} + b_{18} v_2^{1/4} v_3^{1/4} + b_{19} v_1^{1/2} v_2^{1/2} v_3^{1/2} + b_{20} v_1^{1/4} v_2^{1/2} v_3^{1/2} \\
& + b_{21} v_1^{1/2} v_2^{1/4} v_3^{1/2} + b_{22} v_1^{1/2} v_2^{1/2} v_3^{1/4} + b_{23} v_1^{1/2} v_2^{1/4} v_3^{1/4} + b_{24} v_1^{1/4} v_2^{1/2} v_3^{1/4} \\
& + b_{25} v_1^{1/4} v_2^{1/4} v_3^{1/2} + b_{26} v_1^{1/4} v_2^{1/4} v_3^{1/4}. \qquad (33)
\end{aligned}$$

Applying the modified version of Roy's identity, (4), to (33) yields the AIM(2) demand system,

$$\begin{aligned}
s_1 = \Big(& 2b_1 v_1^{1/2} + b_4 v_1^{1/4} + 2b_7 v_1^{1/2} v_2^{1/2} + 2b_8 v_1^{1/2} v_2^{1/4} + b_9 v_1^{1/4} v_2^{1/2} + b_{10} v_1^{1/4} v_2^{1/4} \\
& + 2b_{11} v_1^{1/2} v_3^{1/2} + 2b_{12} v_1^{1/2} v_3^{1/4} + b_{13} v_1^{1/4} v_3^{1/2} + b_{14} v_1^{1/4} v_3^{1/4} + 2b_{19} v_1^{1/2} v_2^{1/2} v_3^{1/2} \\
& + b_{20} v_1^{1/4} v_2^{1/2} v_3^{1/2} + 2b_{21} v_1^{1/2} v_2^{1/4} v_3^{1/2} + 2b_{22} v_1^{1/2} v_2^{1/2} v_3^{1/4} + 2b_{23} v_1^{1/2} v_2^{1/4} v_3^{1/4} \\
& + b_{24} v_1^{1/4} v_2^{1/2} v_3^{1/4} + b_{25} v_1^{1/4} v_2^{1/4} v_3^{1/2} + b_{26} v_1^{1/4} v_2^{1/4} v_3^{1/4} \Big) /D, \qquad (34)
\end{aligned}$$

$$\begin{aligned}
s_2 = \Big(& 2b_2 v_2^{1/2} + b_5 v_2^{1/4} + 2b_7 v_1^{1/2} v_2^{1/2} + b_8 v_1^{1/2} v_2^{1/4} + 2b_9 v_1^{1/4} v_2^{1/2} + b_{10} v_1^{1/4} v_2^{1/4} \\
& + 2b_{15} v_2^{1/2} v_3^{1/2} + 2b_{16} v_2^{1/2} v_3^{1/4} + b_{17} v_2^{1/4} v_3^{1/2} + b_{18} v_2^{1/4} v_3^{1/4} + 2b_{19} v_1^{1/2} v_2^{1/2} v_3^{1/2} \\
& + 2b_{20} v_1^{1/4} v_2^{1/2} v_3^{1/2} + b_{21} v_1^{1/2} v_2^{1/4} v_3^{1/2} + 2b_{22} v_1^{1/2} v_2^{1/2} v_3^{1/4} + b_{23} v_1^{1/2} v_2^{1/4} v_3^{1/4} \\
& + 2b_{24} v_1^{1/4} v_2^{1/2} v_3^{1/4} + b_{25} v_1^{1/4} v_2^{1/4} v_3^{1/2} + b_{26} v_1^{1/4} v_2^{1/4} v_3^{1/4} \Big) /D, \qquad (35)
\end{aligned}$$

$$\begin{aligned}
s_3 = \Big(& 2b_3 v_3^{1/2} + b_6 v_4^{1/4} + 2b_{11} v_1^{1/2} v_3^{1/2} + b_{12} v_1^{1/2} v_3^{1/4} + 2b_{13} v_1^{1/4} v_3^{1/2} + b_{14} v_1^{1/4} v_2^{1/4} \\
& + 2b_{15} v_1^{1/2} v_3^{1/2} + b_{16} v_1^{1/2} v_3^{1/4} + 2b_{17} v_2^{1/4} v_3^{1/2} + b_{18} v_2^{1/4} v_3^{1/4} + 2b_{19} v_1^{1/2} v_2^{1/2} v_3^{1/2} \\
& + 2b_{20} v_1^{1/4} v_2^{1/2} v_3^{1/2} + 2b_{21} v_1^{1/2} v_2^{1/4} v_3^{1/2} + b_{22} v_1^{1/2} v_2^{1/2} v_3^{1/4} + b_{23} v_1^{1/2} v_2^{1/4} v_3^{1/4} \\
& + b_{24} v_1^{1/4} v_2^{1/2} v_3^{1/4} + 2b_{25} v_1^{1/4} v_2^{1/4} v_3^{1/2} + b_{26} v_1^{1/4} v_2^{1/4} v_3^{1/4} \Big) /D, \qquad (36)
\end{aligned}$$

where now D is the sum of the numerators in equations (34), (35), and (36).

Recently, Serletis and Shahmoradi (2008) estimate the AIM(1), AIM(2), and AIM(3) demand systems for $K = 3$ and argue that the AIM(3) model, estimated subject to global curvature, currently provides the best specification for research in semiparametric modeling of consumer demand systems.

6 The Nonparametric Approach to Demand Analysis

An alternative approach to demand analysis is fully nonparametric, in the sense that it requires no specification of the form of the demand functions. This approach, advocated by Varian (1982, 1983), addresses three issues concerning consumer behavior: (i) consistency of observed behavior with the preference maximization model; (ii) recovery of preferences, given observations on consumer behavior; (iii) forecasting of demand for different price configurations; and (iv) testing for separability.

6.1 The Revealed Preference Concept

Consider the n-vector, $\boldsymbol{x}$, of goods and its corresponding n-vector of prices, $\boldsymbol{p}$. Let $\boldsymbol{x}^i = (x_1^i, \cdots, x_n^i)'$ denote the ith observation of $\boldsymbol{x}$ and $\boldsymbol{p}^i = (p_1^i, \cdots, p_n^i)'$ that of $\boldsymbol{p}$. Suppose that we have T observations on these quantities and prices. A basic question which the nonparametric approach attempts to answer is whether $(\boldsymbol{p}^i, \boldsymbol{x}^i)$, $i = 1, \cdots, T$, is consistent with maximization of a well-behaved utility function. The following definitions from Varian (1982, 1983).

Definition 1 *An observation $\boldsymbol{x}^i$ is directly revealed preferred to a bundle $\boldsymbol{x}$, written $\boldsymbol{x}^i R^0 \boldsymbol{x}$, if $\boldsymbol{p}^{i\prime}\boldsymbol{x}^i \geq \boldsymbol{p}^{i\prime}\boldsymbol{x}$. An observation $\boldsymbol{x}^i$ is revealed preferred to a bundle $\boldsymbol{x}$, written $\boldsymbol{x}^i R \boldsymbol{x}$, if there is a sequence of observations $\left(\boldsymbol{x}^j, \boldsymbol{x}^k, \cdots, \boldsymbol{x}^l\right)$ such that $\boldsymbol{x}^i R^0 \boldsymbol{x}^j$, $\boldsymbol{x}^j R^0 \boldsymbol{x}^k$, $\cdots$, $\boldsymbol{x}^l R^0 \boldsymbol{x}$.*

Definition 2 *The data satisfies the Generalized Axiom of Revealed Preference (GARP), if $\boldsymbol{x}^i R \boldsymbol{x}^j$ implies $\boldsymbol{p}^j \boldsymbol{x}^j \leq \boldsymbol{p}^j \boldsymbol{x}^i$.*

What this definition tells us is that the set of choices $\boldsymbol{x}^i$ is revealed to be preferred to $\boldsymbol{x}^j$, if the expenditures on $\boldsymbol{x}^i$ exceed or are equal to those on $\boldsymbol{x}^j$ evaluated at the original set of prices. Here i and j could refer to dates. Note from the above definition that GARP is a necessary and sufficient condition for observed demand data to be consistent with utility maximization.

6.2 The Maximization Hypothesis

We can use GARP to determine whether there is a utility function that could have generated a given set of data.

Definition 3 *A utility function $u(\boldsymbol{x})$ rationalizes the data $(\boldsymbol{p}^i, \boldsymbol{x}^i)$, $i = 1, ..., T$, if $u(\boldsymbol{x}^i) \geq u(\boldsymbol{x})$ for all $\boldsymbol{x}$ such that $\boldsymbol{p}^i \boldsymbol{x}^i \geq \boldsymbol{p}^i \boldsymbol{x}$, for $i \geq 1, \cdots, T$.*

This definition states that $u(\cdot)$ is consistent with the data, if observed consumption would be optimal under $u(\cdot)$. Varian (1982) developed methods for examining whether any such utility function exists for a given data set, based on the following theorem due to Afriat (1967).

Theorem 1 *(Afriat's theorem). The following conditions are equivalent:*

(i) there exists a concave, monotonically increasing, continuous utility function which rationalizes the data,

(ii) the data satisfies GARP,

(iii) there exist numbers $U^i, \lambda^i > 0$, $i = 1, \cdots, T$ that satisfy the Afriat inequalities,

$$U^i \leq U^j + \lambda^j \boldsymbol{p}^j \left(\boldsymbol{x}^i - \boldsymbol{x}^j\right), \qquad \text{for } i, j = 1, \cdots, T$$

Conditions, (ii) and (iii) have been used by Varian (1982) to provide a basis for systematically testing finite data sets for consistency with the utility maximization hypothesis. Condition (iii) has also been suggested to test for utility maximization by Diewert and Parkan (1985). Varian (1985) developed a nonparametric computational package that is capable of testing, not only for consistency with utility maximization, but also for homotheticity, weak separability, and homothetic separability.

6.3 Nonparametric Tests of Consumer Behavior

As already noted, the nonparametric revealed preference approach to demand analysis imposes no functional form restrictions and requires only actual market data (quantities and prices). For example, in the case of GARP, the nonparametric approach takes advantage of the formulation,

$$\text{if } \boldsymbol{x}^i R \boldsymbol{x}^j \text{ then } \boldsymbol{p}^j \boldsymbol{x}^j \leq \boldsymbol{p}^j \boldsymbol{x}^i,$$

and evaluates all pairs in the data (which are, of course, finite), in order to see if the expenditures on $\boldsymbol{x}^i$, evaluated at $\boldsymbol{p}^j$, are greater than those on $\boldsymbol{x}^j$, evaluated at the same prices, for all $i, j = 1, \cdots, T$. In doing so, the number of violations (reversals of the inequality) is reported. The number of reversals can be considerable, if the data set contains goods held by different sorts of economic agents, as possibly by business firms as well as consumers, or if the data is aggregated over economic agents, of if the data is noisy.

The nonparametric approach has been used and refined in numerous recent contributions, such as, for example, by Swofford and Whitney (1987, 1988, 1994), Fleissig and Whitney (2005), de Peretti (2005), Jones and de Peretti (2005), and Elger *et al.* (2008). See also Jones *et al.* (2007) for a survey of the recent literature. Note, however, that this approach to demand analysis is not without problems. As Fleissig *et al.* (2000, p. 329) explain, the main disadvantage is that the tests are non-stochastic. Violations are all or nothing; either there is a utility function that rationalizes the data or there is not. Thus, establishing consistency with preference maximization by Varian's (1982, 1983) nonparametric techniques uses a very strong standard. There is much ongoing research on approaches to extending the nonparametric approach to the stochastic case permitting noise in the data.

7 Engle Curves and the Rank of Demand Systems

Applied demand analysis uses two types of data, time series data and cross sectional data. Time series data offer substantial variation in relative prices and less variation in income whereas cross sectional data offer limited variation in relative prices and substantial variation in income levels. Household budget data are useful in exploring Engel curves.

Lewbel (1991), extending earlier results by Muellbauer (1975, 1976) and Gorman (1981), defined the rank of any demand system to be the dimension of the space spanned by its Engel curves, holding demographic or other nonincome consumer characteristics fixed. Formally, the rank of any given demand function system, $\boldsymbol{x}(\boldsymbol{p}, y)$, is the smallest value of M such that each s_i can be written as

$$s_i = \sum_{m=1}^{M} \phi_{im}(\boldsymbol{p}) f_m(\boldsymbol{p}, y), \tag{37}$$

for some functions ϕ_{im} and f_m. The rank of the system is the number of linearly independent vectors of price functions. All demand systems have rank $M \leq n$, where n is the number of goods. Hence, any demand system has rank M, if there exist M goods such that the Engel curve of any good equals a weighted average of the Engel curves of those M goods. The rank of an integrable demand system determines the number of price functions upon which the indirect utility function and the cost or expenditure function depend.

Demand systems produced from homothetic utility functions have rank one. Rank-one demand systems, such as the Cobb-Douglas, CES, and homothetic translog, exhibit expenditure proportionality, so that the budget share of every good is independent of total expenditure. This contradicts Engel's law, according to which the budget share of food is smaller for rich than for poor households. Rank-one demand systems can be written as

$$x_i(\boldsymbol{p}, y) = b_i(\boldsymbol{p}) y.$$

A demand system having all linear Engel curves (but not through the origin) is rank two and can be written as

$$x_i(\boldsymbol{p}, y) = c_i(\boldsymbol{p}) + b_i(\boldsymbol{p}) y.$$

Gorman (1961) showed that any demand system that is consistent with utility maximization and linear in expenditure must be of the form

$$x_i(\boldsymbol{p}, y) = f_i(\boldsymbol{p}) - \frac{g_i(\boldsymbol{p})}{g(\boldsymbol{p})} f(\boldsymbol{p}) + \frac{g_i(\boldsymbol{p})}{g(\boldsymbol{p})} y = f_i(\boldsymbol{p}) + \frac{g_i(\boldsymbol{p})}{g(\boldsymbol{p})} \left[y - f(\boldsymbol{p}) \right],$$

where $g(\boldsymbol{p})$ and $f(\boldsymbol{p})$ are functions homogeneous of degree one, and $g_i(\boldsymbol{p})$ and $f_i(\boldsymbol{p})$ denote the partial derivative of $g(\boldsymbol{p})$ and $f(\boldsymbol{p})$ with respect to the ith price. Such demand systems are generated by an indirect utility function of the 'Gorman polar form,'

$$h(\boldsymbol{p}, y) = \frac{y - f(\boldsymbol{p})}{g(\boldsymbol{p})}. \tag{38}$$

Demand functions that have linear Engel curves not intersecting the origin are produced from utility functions that are affine homothetic, also called quasi-homothetic.

Linearity in expenditure implies marginal budget shares that are independent of the level of expenditure, requiring that poor and rich households spend the same fraction of an extra dollar on each good. This hypothesis at the margin is too restrictive for the analysis of household budget data, having a large range of incomes. Another class of demand systems is linear in the logarithm of total expenditure. Such demand systems have been called 'price-independent generalized logarithmic' (PIGLOG) by Muellbauer (1976) and can be written as

$$x_i(\boldsymbol{p}, y) = c_i(\boldsymbol{p}) + b_i(\boldsymbol{p}) \log y.$$

PIGLOG demand systems are generated by indirect utility functions of the form

$$h(\boldsymbol{p}, y) = G(\boldsymbol{p}) \Big[\log y - \log g(\boldsymbol{p}) \Big], \tag{39}$$

where $G(\boldsymbol{p}) = G(\lambda \boldsymbol{p})$ and $g(\lambda \boldsymbol{p}) = \lambda g(\boldsymbol{p})$, and the share equations are given by

$$s_i = \frac{p_i g_i(\boldsymbol{p})}{g(\boldsymbol{p})} - \frac{p_i G_i(\boldsymbol{p})}{G(\boldsymbol{p})} \Big[\log y - \log g(\boldsymbol{p}) \Big].$$

Examples of PIGLOG demand systems are the basic translog (11) and the AIDS (13). However, most of the commonly used PIGLOG specifications are rank two, and thus do not have enough flexibility in modelling the curvature of Engel curves with large variations in income.

One way to relax the assumption that demand systems are linear in expenditure is to specify demand systems that are quadratic in expenditure. The first such proposed functional form was the quadratic AIDS (known as QUAIDS) that we discussed in Section 4.2.2. The QUAIDS is an extension of the simple AIDS, having expenditure shares linear in log income and in another smooth function of income, as follows:

$$s_i = c_i(\boldsymbol{p}) + b_i(\boldsymbol{p}) \log y + a_i(\boldsymbol{p}) g \left(\frac{y}{a(\boldsymbol{p})} \right), \tag{40}$$

where $c_i(\boldsymbol{p})$, $b_i(\boldsymbol{p})$, $a_i(\boldsymbol{p})$ and $g(y/a(\boldsymbol{p}))$ are differentiable functions. The $a_i(\boldsymbol{p})g(y/a(\boldsymbol{p}))$ term in (40) allows for nonlinearities in empirical Engel curves, where $a_i(\boldsymbol{p}) = 0$ in the case of PIGLOG preferences.[8]

[8] The rank of demand system (40) corresponds to the rank of the $n \times 3$ matrix of Engel curve coefficients $[\boldsymbol{c}(\boldsymbol{p})\ \boldsymbol{b}(\boldsymbol{p})\ \boldsymbol{a}(\boldsymbol{p})]$, whose maximum rank is three. In fact, the uniqueness of a rank-three demand system in the form of equation (40) is guaranteed by a theorem in Banks *et al.* (1997), according to which all theoretically plausible exactly aggregable demand systems in the form of equation (40) either have $a_i(\boldsymbol{p}) = d(\boldsymbol{p})b_i(\boldsymbol{p})$ for some function $d(\boldsymbol{p})$ (so the rank is less than 3), or they are rank 3 quadratic logarithmic budget share systems, having indirect utility functions of the form (18).

Following Banks *et al.* (1997), Ryan and Wales (1999) modify the translog, GL, and NQ demand systems and introduce three new (rank-three) demand systems, having expenditure shares quadratic in expenditure. The three new demand systems are referred to as the 'translog-quadratic expenditure system,' 'GL-quadratic expenditure system,' and 'NQ-quadratic expenditure system.' These models are locally flexible and are rank-three demand systems, allowing more flexibility in modelling Engel curves than the AIDS, translog, GL, and NQ models.

Most parametric demand systems can encompass close-to-linear or quadratic Engel curves. But recent empirical work reveals Engel curves with significant curvature and variation across goods. To capture this variety in Engel curve shapes, Lewbel and Pendakur (2007) offer a new methodology for the modelling of demand systems exhibiting a high degree of functional flexibility in Engel curves. They propose the 'exact affine Stone index' (EASI) demand system, which can accommodate a broad range of Engel curve shapes, in contrast to rank-three demand systems, which allow only for quadratic Engel curves. The EASI demand system has budget shares linear in parameters, can accommodate heterogeneity in preferences, can have Engel curves that are polynomials or splines of any order in expenditure, and can have any rank up to $n-1$, where n is the number of goods.

8 Estimation Issues

In order to estimate share equation systems such as (9), (11), (13), (16), (19), (23), (29), (31), and (34)-(36), a stochastic version must be specified. Demand systems are usually estimated in budget share closed form, in order to minimize heteroskedasticity problems, with only exogenous variables appearing on the right-hand side. It often is assumed that the observed share in the ith equation deviates from the true share by an additive disturbance term u_i. Furthermore, it is usually assumed that $\boldsymbol{u} \sim N\left(\mathbf{0}, \boldsymbol{\Omega}\otimes\mathbf{I}_T\right)$, where $\mathbf{0}$ is a null vector, $\boldsymbol{\Omega}$ is the $n \times n$ symmetric positive definite error covariance matrix, and $\mathbf{I}$ is the identity matrix.

With the addition of additive errors, the share equation system can be written in matrix form as

$$\boldsymbol{s}_t = \boldsymbol{g}(\boldsymbol{v}_t, \boldsymbol{\vartheta}) + \boldsymbol{u}_t, \tag{41}$$

where $\boldsymbol{s} = (s_1, \cdots, s_n)'$, $\boldsymbol{g}(\boldsymbol{v}, \boldsymbol{\vartheta}) = \left(\boldsymbol{g}_1\left(\boldsymbol{v}, \boldsymbol{\vartheta}\right), \cdots, \boldsymbol{g}_n\left(\boldsymbol{v}, \boldsymbol{\vartheta}\right)\right)'$, $\boldsymbol{\vartheta}$ is the parameter vector to be estimated, and $\boldsymbol{g}_i\left(\boldsymbol{v}, \boldsymbol{\vartheta}\right)$ is given by the right-hand side of systems such as (9), (11), (13), (16), (19), (23), (29), (31), and (34)-(36).

The assumption made about $\boldsymbol{u}_t$ in (41) permits correlation among the disturbances at time t, but rules out the possibility of autocorrelated disturbances. This assumption and the fact that the shares satisfy an adding up condition imply that the errors across all equations are linearly related and that the error covariance matrix is singular. Barten (1969) has shown that this problem can be handled by arbitrarily deleting any equation from the system. When the errors are homoskedastic and non-autocorrelated, the resulting estimates are invariant to the equation deleted, and the parameter estimates of the deleted equation can be recovered from the restrictions imposed.[9]

8.1 Maximum Likelihood

If the disturbances in (41) are multivariate normally distributed as assumed above, then maximum likelihood estimation of (41) can be used. The log likelihood function for a sample of T observations is

$$\log L(\boldsymbol{s}\,|\boldsymbol{\theta}) = -\frac{MT}{2}\ln(2\pi) - \frac{T}{2}\ln|\boldsymbol{\Omega}| - \frac{1}{2}\sum_{i=1}^{T}\boldsymbol{u}_t'\boldsymbol{\Omega}^{-1}\boldsymbol{u}_t, \tag{42}$$

where $\boldsymbol{\theta} = (\boldsymbol{\vartheta}, \boldsymbol{\Omega})$. In the relevant class of ("seemingly unrelated regression") models, maximization of $\log L(\boldsymbol{s}\,|\boldsymbol{\theta})$ is equivalent to minimization of $|\boldsymbol{\Omega}|$, as shown by Barnett (1976), who provided the relevant asymptotics for the maximum likelihood estimator within the relevant class of nonlinear systems under the customary assumptions.

8.2 Under-researched Complications

The usual assumption that the error terms in (41) are multivariate normally distributed may not hold in practice. We are dealing with shares, such that $0 \leq s_i \leq 1$, so the error terms cannot be exactly normally distributed, and a multivariate logistic distribution might be a better assumption. But Davidson and MacKinnon (1993) argue that if the sample does not contain observations which are near 0 or 1, the normal distribution can be used as an approximation in the inference process. There also are potential problems of simultaneity bias, since systems of demand equations usually are estimated as closed form systems with all right hand variables treated as exogenous, despite the fact that income and prices, while exogenous for individuals, may not be at the aggregate level in general equilibrium. In addition, representative agent models at the aggregate level assume away distribution effects, since only the mean of the income distribution appear in the model and not the higher order moments of the income distribution.

Changing tastes has become the subject of much research, but usually only through the habit formation mechanism. Habit formation specifications are usually motivated by one of two possible reasons: (1) to convert homothetic utility function models that violate Engel's law into affine homothetic (also called quasihomothetic) utility models, and (2) to remove the assumption of intertemporal separability that has been argued to produce such paradoxes

[9] Barten's proof requires a closed form solution for quantities demanded or shares, as is the case with the models described in this paper. An exception is Barnett's (1977) extension of the household production function approach to a general equilibrium system of nonlinear simultaneous equations.

as the equity premium puzzle. More general explorations of time varying tastes have been rare, with a notable exception being Basmann, Molina, and Slottje (1983).

While there has been published research on each of these problems, these often-overlooked areas remain open for potential additional research.

8.3 Bayesian Estimation

Following Judge *et al.* (1988), under the assumption that $\boldsymbol{u}_t$ is multivariate normally distributed, the likelihood function of the whole sample can be written as

$$p(\boldsymbol{s}\,|\boldsymbol{\vartheta}, \boldsymbol{\Omega}) \propto |\boldsymbol{\Omega}|^{-T/2} \exp\left[-\frac{1}{2}(\boldsymbol{s}-\boldsymbol{g})'\left(\boldsymbol{\Omega}^{-1}\otimes \mathbf{I}_T\right)(\boldsymbol{s}-\boldsymbol{g})\right]$$

$$\propto |\boldsymbol{\Omega}|^{-T/2} \exp\left[-\frac{1}{2}\mathrm{tr}\left(\mathbf{A}\boldsymbol{\Omega}^{-1}\right)\right], \tag{43}$$

where $\mathbf{A}$ is a $n\times n$ symmetric matrix with $a_{ij} = (\boldsymbol{s}_i - \boldsymbol{g}_i)'\left(\boldsymbol{s}_j - \boldsymbol{g}_j\right)$. It is to be noted here that there also are new Bayesian approaches, including Zellner's (1997) Bayesian method of moments (BMOM) approach. With many equations containing many parameters and limited data, these new approaches are potentially relevant to the consumer demand systems literature.

The Bayesian model in (43) requires that we choose priors for the parameters $\boldsymbol{\vartheta}$ and $\boldsymbol{\Omega}$. For example, assume that $\boldsymbol{\vartheta}$ and $\boldsymbol{\Omega}$ are independent of each other, assume a flat prior probability density function for $\boldsymbol{\vartheta}$, and a limiting form of the inverted Wishart density for $\boldsymbol{\Omega}$, $p(\boldsymbol{\Omega}) \propto |\boldsymbol{\Omega}|^{-n/2}$. Then the joint prior probability density function for all the unknown parameters can be written as

$$p(\boldsymbol{\vartheta}, \boldsymbol{\Omega}) \propto |\boldsymbol{\Omega}|^{-n/2}. \tag{44}$$

Using Bayes' theorem, the joint posterior probability density function for all the parameters can be written as

$$p(\boldsymbol{\vartheta}, \boldsymbol{\Omega}\,|\,\boldsymbol{s}) \propto |\boldsymbol{\Omega}|^{-(T+n)/2} \exp\left[-\frac{1}{2}\mathrm{tr}\left(\mathbf{A}\boldsymbol{\Omega}^{-1}\right)\right]. \tag{45}$$

Equation (45) can be used to obtain the marginal posterior probability density function for the parameters,

$$p(\boldsymbol{\vartheta}\,|\,\boldsymbol{s}) = \int p(\boldsymbol{\vartheta}, \boldsymbol{\Omega}\,|\,\boldsymbol{s})\, d\boldsymbol{\Omega} \propto |\mathbf{A}|^{-T/2}, \tag{46}$$

which can be usesd to calculate the posterior means and corresponding standard deviations.

However, equation (45) is too complicated for analytical integration. One solution to this problem is the use of simulation techniques, such as Gibbs sampling, introduced by Geman and Geman (1984), and the Metropolis-Hastings algorithm, from early work by

Metropolis *et al.* (1953) and Hastings (1970). Such simulation techniques provide a means of drawing observations from the joint posterior probability density function. When the number of draws is large enough, the chain will converge to the targeted distribution (45). These generated observations are then used to construct histograms and calculate sample means and variances to provide consistent estimates of the marginal posterior probability density functions and thereby consistent estimates of the posterior means and variances of the elements in $\boldsymbol{\vartheta}$. See Chib and Greenberg (1995, 1996) for a detailed discussion.

Gibbs sampling is particularly suitable for linear seemingly-unrelated regression models. But given the nonlinear nature of typical demand systems, the Metropolis-Hastings algorithm can be used to sample the posterior probability density function (46), proceeding iteratively as follows[10]:

- *Step* 1: Select initial values for $\boldsymbol{\vartheta}$, say $\boldsymbol{\vartheta}_0$. Perform the remaining steps with τ set equal to 0.
- *Step* 2: Compute a value for $p(\boldsymbol{\vartheta}_\tau\,|\boldsymbol{s})$, based on equation (46).
- *Step* 3: Generate $\boldsymbol{d}$ from $N(\mathbf{0}, \kappa\boldsymbol{V})$, where $\boldsymbol{V}$ is an adjusted covariance matrix of the maximum likelihood estimates and κ is chosen by experimentation.
- *Step* 4: Compute $\boldsymbol{\vartheta}^* = \boldsymbol{\vartheta}_\tau + \boldsymbol{d}$.
- *Step* 5: Compute a value for $p(\boldsymbol{\vartheta}^*\,|\boldsymbol{s})$ and the ratio of the probability density functions $r = p(\boldsymbol{\vartheta}^*\,|\boldsymbol{s})\,/p(\boldsymbol{\vartheta}_\tau\,|\boldsymbol{s})$.
- *Step* 6: If $r \geq 1$, set $\boldsymbol{\vartheta}_{\tau+1} = \boldsymbol{\vartheta}^*$ and return to Step 2; otherwise proceed to Step 7.
- *Step* 7: Generate a uniform random variable y from the interval $(0, 1)$. If $y \leq r$, set $\boldsymbol{\vartheta}_{\tau+1} = \boldsymbol{\vartheta}^*$; otherwise set $\boldsymbol{\vartheta}_{\tau+1} = \boldsymbol{\vartheta}_\tau$, and return to Step 2.

The Metropolis-Hastings algorithm provides a means for drawing observations consistent with the marginal posterior probability density function for the parameters, $p(\boldsymbol{\vartheta}\,|\boldsymbol{s})$. In particular, the vector $\boldsymbol{d}$ in Step 3 represents a potential change from the last drawing of $\boldsymbol{\vartheta}$ and the potential new value $\boldsymbol{\vartheta}^*$ is given by the random walk process in Step 4. In Step 6 a new observation is accepted, if it is more probable than the previous one. If it is less probable, it is accepted in Step 7 with probability given by the ratio of the two probability density functions.

[10] See Griffiths and Chotikapanich (1997, p. 333) for more details.

9 Theoretical Regularity

The usefulness of flexible functional forms depends on whether they satisfy the theoretical regularity conditions of positivity, monotonicity, and curvature. But in the empirical demand systems literature there has been a tendency to ignore regularity or not to report the results of regularity checks. For example, Serletis and Shahmoradi (2007) found that only three out of fourteen studies in the monetary asset demand literature since 1983 have addressed theoretical regularity issues. Moreover, there is a tendency to remove the word 'regularity' from the empirical literature and replace it with 'curvature.' As Barnett (2002, p. 199) put it in his *Journal of Econometrics* Fellow's opinion article, "monotonicity is rarely even mentioned in that literature. But without satisfaction of both curvature and monotonicity, the second-order conditions for optimizing behavior fail, and duality theory fails. The resulting first-order conditions, demand functions, and supply functions become invalid."

Once a demand system is estimated, the regularity conditions can be checked as follows:

- Positivity is checked by direct computation of the estimated indirect utility function $\widehat{h}(\boldsymbol{v})$. Positivitiy is satisfied, if $\widehat{h}(\boldsymbol{v}) > 0$, for all t.
- Monotonicity is checked by direct computation of the values of the first gradient vector of the estimated indirect utility function. Monotonicity is satisfied, if $\nabla\widehat{h}(\boldsymbol{v}) < 0$.
- Curvature requires that the Slutsky matrix be negative semidefinite and can be checked by performing a Cholesky factorization of that matrix. Curvature is satisfied, if the Cholesky values are nonpositive. See Lau (1978, Theorem 3.2). Curvature can also be checked by examining the Allen elasticities of substitution matrix, if the monotonicity condition holds. Curvature requires that this matrix be negative semidefinite.

If regularity is not attained, the models could be estimated subject to imposed regularity, thereby treating the curvature and monotonicity properties as maintained hypotheses. In the case of the locally flexible functional forms, curvature alone could be imposed using the procedure recently suggested by Ryan and Wales (1998). In the context of the globally flexible functional forms, both curvature and monotonicity can be imposed using the procedures suggested by Gallant and Golub (1984). In what follows, we discuss each of these methods for imposing theoretical regularity.

9.1 Curvature and Locally Flexible Functional Forms

Ryan and Wales (1998) suggest a relatively simple procedure for imposing local curvature conditions. Their procedure applies to those locally flexible demand systems for which, at the point of approximation ($\boldsymbol{v}^* = \mathbf{1}$), the $n \times n$ Slutsky matrix $\boldsymbol{S}$ can be written as

$$\boldsymbol{S} = \boldsymbol{B} + \boldsymbol{C}, \tag{47}$$

where $\boldsymbol{B}$ is an $n \times n$ symmetric matrix, containing the same number of independent elements as the Slutsky matrix, and $\boldsymbol{C}$ is an $n \times n$ matrix whose elements are functions of the other parameters of the system. Curvature requires the Slutsky matrix to be negative semidefinite. Ryan and Wales (1998) draw on related work by Lau (1978) and Diewert and Wales (1987) and impose curvature by replacing $\boldsymbol{S}$ in equation (47) with $-\boldsymbol{K}\boldsymbol{K}'$, where $\boldsymbol{K}$ is an $n \times n$ lower triangular matrix, so that $-\boldsymbol{K}\boldsymbol{K}'$ is by construction a negative semidefinite matrix. Then solving explicitly for $\boldsymbol{B}$ in terms of $\boldsymbol{K}$ and $\boldsymbol{C}$ yields

$$\boldsymbol{B} = -\boldsymbol{K}\boldsymbol{K}' - \boldsymbol{C}.$$

The models can be reparameterized by estimating the parameters in $\boldsymbol{K}$ and $\boldsymbol{C}$ instead of the parameters in $\boldsymbol{B}$ and $\boldsymbol{C}$, thereby ensuring that $\boldsymbol{S}$ is negative semidefinite at the point of approximation.

Ryan and Wales (1998) applied their procedure to three locally flexible functional forms: the almost ideal demand system, the normalized quadratic, and the linear translog. Moschini (1999) suggested a possible reparameterization of the basic translog to overcome some problems noted by Ryan and Wales (1998) and also imposed curvature conditions locally in the basic translog. More recently, Serletis and Shahmoradi (2007) build on Ryan and Wales (1998) and Moschini (1999) to impose curvature conditions locally on the generalized Leontief model. In doing so, they exploit the Hessian matrix of second order derivatives of the reciprocal indirect utility function.[11] But if monotonicity as well as curvature are imposed globally on a parsimonious, locally-flexible functional form, the flexibility is severely damaged. For example, the translog becomes Cobb Douglas.

9.2 Curvature and Globally Flexible Functional Forms

When a globally flexible functional form is used, such as the Fourier or the AIM, the simple method of imposing curvature based on the Cholesky decomposition of the Slutsky matrix is not practical. In this case, the nonlinear constrained optimization approach, initially proposed by Gallant and Golub (1984), and recently used by Serletis and Shahmoradi (2005), can be applied.

The indirect utility function, $h(\boldsymbol{v}, \boldsymbol{\vartheta})$, should be a quasi-convex function in income normalized prices, v_j $(j = 1, \cdots, n)$. Gallant and Golub (1984), following Diewert *et al.* (1977), argue that a necessary and sufficient condition for quasi-convexity of $h(\boldsymbol{v}, \boldsymbol{\vartheta})$, in some region $\widetilde{\boldsymbol{C}}$, is

$$\min_{\boldsymbol{v} \in \widetilde{\boldsymbol{C}}} w(\boldsymbol{v}, \boldsymbol{\vartheta}) \geq 0 \tag{48}$$

where

$$w(\boldsymbol{v}, \boldsymbol{\vartheta}) = \min_{\boldsymbol{z}} \left\{ \boldsymbol{z}' \nabla^2 h(\boldsymbol{v}, \boldsymbol{\vartheta}) \boldsymbol{z} : \boldsymbol{z}' \nabla h(\boldsymbol{v}, \boldsymbol{\vartheta}) = 0,\ \boldsymbol{z}'\boldsymbol{z} = 1 \right\}.$$

[11] See Serletis and Shahmoradi (2007) for a detailed discussion regarding the imposition of curvature on locally flexible functional forms.

As before, $\nabla h(\boldsymbol{v}, \boldsymbol{\vartheta}) = \partial h(\boldsymbol{v}, \boldsymbol{\vartheta})/\partial \boldsymbol{v}$ and $\nabla^2 h(\boldsymbol{v}, \boldsymbol{\vartheta}) = \partial^2 h(\boldsymbol{v}, \boldsymbol{\vartheta})/\partial \boldsymbol{v} \partial \boldsymbol{v}'$. Gallant and Golub (1984) further show that the critical issue in imposing (48) is the computation of $w(\boldsymbol{v}, \boldsymbol{\vartheta})$ and its analytical derivatives. Using a Householder transformation, Gallant and Golub (1984) show that $w(\boldsymbol{v}, \boldsymbol{\vartheta})$ is the smallest eigenvalue of an $(N-1)\times(N-1)$ matrix $\boldsymbol{K}_{22}$. The matrix $\boldsymbol{K}_{22}$ is obtained by deleting the first row and column of $\boldsymbol{K}$, an $N \times N$ matrix related to $\nabla^2 g(\boldsymbol{v}, \boldsymbol{\vartheta})$ and $\nabla g(\boldsymbol{v}, \boldsymbol{\vartheta})$. Gallant and Golub (1984) also derive the analytical derivatives of the smallest eigenvalue. For more details concerning the construction of the matrix $\boldsymbol{K}_{22}$, see Gallant and Golub (1984) or Serletis and Shahmoradi (2005). Thus, when a log likelihood function $\log L(\boldsymbol{s} \,|\boldsymbol{\theta})$ is used as an objective function, the constrained optimization problem becomes

$$\max_{\{\boldsymbol{\theta}\}} \ \log L(\boldsymbol{s} \,|\boldsymbol{\theta}),$$

subject to

$$\text{the smallest eigenvalue of } \boldsymbol{K}_{22} \geq 0.$$

With the constrained optimization method, one can impose curvature restrictions at any arbitrary set of points — at a single data point, over a region of data points, or fully (at every data point in the sample). If $h(\boldsymbol{v}, \boldsymbol{\vartheta})$ is a reciprocal indirect utility function, then it must be quasi-concave in $\boldsymbol{v}$, instead of quasi-convex, and the Gallant and Golub (1984) method can be modified to impose quasi-concavity. In that case, we need to replace $h(\boldsymbol{v}, \boldsymbol{\vartheta})$ by $-h(\boldsymbol{v}, \boldsymbol{\vartheta})$, which is quasi-convex in $\boldsymbol{v}$.

9.3 Monotonicity Matters

In addition to the imposition of curvature, the imposition of monotonicity (when violated) is important. There is, however, a tendency to remove 'regularity' from the empirical literature and replace it solely with 'curvature.' But curvature alone, without monotonicity, is not regularity and does not satisfy the assumptions of the relevant duality theorems, from which demand systems commonly are derived.

Moreover, the imposition of curvature may induce violations of monotonicity, which might not occur otherwise. See Barnett (2002). As Barnett and Pasupathy (2003) argue, research on models permitting imposition of both curvature and monotonicity remains at an early stage. While a difficult literature, we believe that models permitting flexible imposition of true regularity, that is, of both monotonicity and curvature, should expand.

In this regard, the constrained optimization approach can be used to impose full regularity. In fact, the monotonicity constraints can be treated in the same way as the curvature constraints. That is, the above constrained optimization problem can be modified by adding the monotonicity constraints as follows:

$$\max_{\{\boldsymbol{\theta}\}} \ \log L(\boldsymbol{s} \,|\boldsymbol{\theta}),$$

subject to

$$\text{the smallest eigenvalue of } \boldsymbol{K}_{22} \geq 0,$$

$$\nabla \widehat{h}\,(\boldsymbol{v}, \boldsymbol{\vartheta}) < 0.$$

10 Econometric Regularity

Most empirical demand system studies make the implicit assumption that the pattern of demand adjusts to a change in exogenous variables instantaneously, although many studies report results with serially correlated residuals, suggesting that the underlying models are dynamically misspecified. Autocorrelation in the disturbances has commonly been modelled by assuming a first-order autoregressive process in the error terms of equation (41), as follows

$$\boldsymbol{u}_t = \boldsymbol{R}\boldsymbol{u}_{t-1} + \boldsymbol{e}_t,$$

where $\boldsymbol{R} = [R_{ij}]$ is a matrix of unknown parameters and $\boldsymbol{e}_t$ is a non-autocorrelated vector disturbance term with constant covariance matrix. In this case, estimates of the parameters can be obtained by using a result developed by Berndt and Savin (1975). They showed that if there is no autocorrelation across equations (i.e., $\boldsymbol{R}$ is diagonal), the autocorrelation coefficients for each equation must be identical. Consequently, by writing equation (41) for period $t-1$, multiplying by $\boldsymbol{R}$, and subtracting from (41), one can estimate stochastic budget share equations given by

$$\boldsymbol{s}_t = \boldsymbol{g}(\boldsymbol{v}_t, \boldsymbol{\vartheta}) + \boldsymbol{R}\boldsymbol{s}_{t-1} - \boldsymbol{R}\boldsymbol{g}(\boldsymbol{v}_{t-1}, \boldsymbol{\vartheta}) + \boldsymbol{e}_t. \tag{49}$$

In this regard, it should be noted that time series of prices and income are usually nonstationary. See Ng (1995) and Lewbel and Ng (2005) regarding nonstationarity in time series of demand system variables and Stock (1994) and Watson (1994) for a review of the econometric issues relating to nonstationary variables. In fact, standard unit root and stationarity testing procedures reveal that quantity, expenditure, and price variables, but not shares, are typically integrated of order one, i.e, I(1). It follows that quantities, prices, and income must be cointegrated in levels, if demand functions are linear. Hence, quantity demand equation errors must be stationary, although unit root test results on the residuals of most demand systems indicate that they are nonstationary. Nevertheless, most estimated demand models do not have quantities as the left had variables, but rather shares or components of the share growth rates (e.g., the Rotterdam model); and most modern demand models are nonlinear.

Lewbel and Ng (2005) argue that the vast majority of the existing empirical demand system studies, with either household or aggregate-level data, have failed to cope with the issue of nonstationary variables, mainly because standard methods for dealing with nonstationarity in linear models cannot be used with nonstationary data and nonlinear estimation in demand systems. In fact the usual proofs of consistency, asymptotic normality, and asymptotic efficiency require stationarity assumptions as "regularity conditions" on certain functions of the model's variables. In particular, see those proof's assumptions in Barnett (1977). For this reasons, the problem of nonstationarity has usually either been ignored, treating the data as if stationary, or dealt with using cointegration methods that apply to linear models, as in Ogaki (1992) and Attfield (1997). More recently, Lewbel and Ng (2005) propose a reformulation of the translog model that can be written in a linear form to handle nonstationary relative prices. With nonlinear demand systems, a modification of the linear model cointegration methods is needed, as is extension of the commonly utilized asymptotic inference proofs to the case of nonstationarity.

Another possible source of nonstationarity of demand system errors is the omission of nonstationary variables. Anderson and Blundell (1982) deal with dynamically misspecified models by developing unrestricted dynamic formulations to accommodate short-run disequilibrium situations, through the inclusion of lagged endogenous and exogenous variables. That approach to dynamic specification follows in the spirit of the error correction models and stands in contrast to the theoretical approach which maintains specific theories of dynamic adjustment. The theoretical approach to dynamically misspecified models is to develop microtheoretic dynamic generalizations of the traditional static models. At present, no theoretical approach can deal with all sources of demand system dynamics, including habit persistence, adjustment costs, the formation of expectations, misinterpretation of real price changes, and complications from aggregation across consumers.[12]

11 Conclusion

We have observed that flexible functional forms have given researchers the ability to model consumer preferences with no local restrictions on the nature of the substitutability/complementarity relationship between pairs of goods. Unfortunately, however, theoretical regularity restrictions met by simpler forms, such as the Cobb-Douglas and the constant elasticity of substitution, or by globally regular flexible functional forms, might not be satisfied with most locally-flexible functional forms. We have argued that unless economic regularity is attained by chance, locally-flexible functional forms should be estimated subject to regularity within the region of the data. But imposition of both curvature and monotonicity can seriously

[12]It also is possible that the econometric irregularity is caused by the manner in which real income is treated, as noted by Diewert and Wales (1993). They argue that locally flexible functional forms work reasonably well with short time series and small variations in relative prices and income, but are inadequate with cross sectional data or with time series data involving significant real income variations. To address this issue, they combine spline techniques with locally flexible functional form techniques.

damage the flexibility of parsimonious, locally-flexible functional forms. We describe other modelling approaches that seek to address these problems, such as the seminonparametric approach.

We have also emphasized the close relationship between theoretical and empirical developments and addressed estimation issues in nonlinear demand systems, including the problems presented by nonstationary variables.[13] A potentially productive area for future research is the modification of linear cointegration methods to accommodate nonlinear estimation of demand systems.

We have omitted many aspects of consumer behavior and emphasized the systemwide parametric and semi-nonparametric approaches to demand analysis. See Blundell (1988) and Barnett and Serletis (2009a) for discussion of issues not covered in this survey. For example, we have not dealt with choice problems in intertemporally nonseparable environments and point the reader to Blundell (1988) for a survey of that literature. We have also omitted household production theory and refer the reader to Barnett (1977) and the recent survey by LaFrance (2001). We do not cover the application of demand systems to welfare comparisons across households (e.g., equivalence scales). See Lewbel (1997) for a summary of that area of research. We discuss only briefly the deep problems of, and approaches to, aggregation over consumers, when second and higher order moments of the income distribution are relevant.

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[13] An exception is the Rotterdam demand system, which uses log differenced data and therefore possibly stationary series.

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